

Newton, he informs us of the circumstances which led him to the present investigation, namely, the occasion he had some years ago to solve a problem which required the rectification of an equilateral hyperbola.

He then enters upon his subject; and in a first section he investigates in nine theorems the several series which apply to this curve, whose different characters, namely, the ratios of their terms, or rather the rates of their convergency and divergency, depend on the relative proportions of their elements. Of these series one only, and that not the best, is all that he has hitherto been able to find in other works. Two are of the form which is called ascending, and six descending. One of them is of a peculiar form, which can only be understood by turning to the paper. Among these series, he observes, may always be found some which will converge, whether the portion of the hyperbolic arch taken from the vertex be long or short, or of a moderate length; but the ascending series always differs from the descending one by a constant quantity.

In a second section the author treats of the methods of computing the values of the constant quantities, by which the ascending series differ from the descending ones. Here he has recourse to two methods, of which he has already given an illustration in his *Mathematical Essays*: the one by computing the value of both an ascending and descending series, taking for the ordinate to the axis some small definite quantity; and the other by comparing the values of those series together, when the ordinate is taken immensely great. The former method he says is more general; but the latter, when it can be applied, usually affords the easiest computation.

In the third section are given five examples, which show the use of these theorems, as well as the manner of choosing such as are best adapted to any particular case. In one of these the author corrects an error in the length of a large arch of an equilateral hyperbola, which was first published in the year 1771, and has been since reprinted by some eminent mathematicians.

Lastly, he concludes with some remarks on former writers, and takes notice of the defects of two series given by the late Dr. Waring for the rectification of an hyperbola.

*Catalogue of 500 new Nebulae, nebulous Stars, planetary Nebulae, and Clusters of Stars; with Remarks on the Construction of the Heavens.*  
By William Herschel, LL.D. F.R.S. Read July 1, 1802. [*Phil. Trans.* 1802, p. 477.]

To this catalogue is prefixed a classification of the multitude of sidereal bodies hitherto discovered, not according to their apparent magnitudes or appearances on our earth, but according to their peculiar nature and arrangement in the heavens. They are divided into the twelve following classes:

1. *Insulated stars*, or such as may be considered out of the reach of mutual attraction; such as our Sun, Arcturus, Capella, Lyra, Si-

rius, &c.—These indeed, as well as all other heavenly bodies, cannot be said to be entirely free from the influence of the stars surrounding them; but the character assigned to them is, that the attraction in one direction is so counteracted by a contrary influence of the same nature, as to be retained for many ages in a state almost equal to undisturbed rest. Dr. Herschel suspects that we are to look for solar systems only among those insulated stars.

2. *Binary sidereal systems*, or double stars.—It is sufficiently obvious that these are not stars seen nearly in the same visual ray, for these rays may be an immense distance from each other; but by these are meant two stars that are connected together by the influence of attraction. It is easy to prove, by the doctrine of gravitation, that two stars thus connected, and sufficiently distant from the influence of other celestial bodies, will perform revolutions round a common centre of motion; that hence they will always move in directions opposite and parallel to each other; and that their system, if not destroyed by some foreign cause, will remain permanent. This kind of rotation is exemplified by the instance of our earth and the moon. Dr. Herschel proposes, on a future occasion, to communicate a series of observations made on double stars, whereby it will be seen that many of them have actually changed their situation with regard to each other, in a progressive course, denoting a periodical revolution round each other, and that the motion of some of them is direct, while that of others is retrograde.

3. *More complicated sidereal systems*, or treble, quadruple, and multiple stars.—From the combination of two stars, it is easy to advance a step further, and allow that three or more stars may be connected in one mutual system of reciprocal attraction; and the computation for determining the common centre of their respective orbits is here exemplified by a variety of hypothetical cases. The author at the same time asserts, that there is not a single night when in passing over the zones of the heavens by sweeping, he does not meet with numerous collections of such multiple stars, apparently insulated from other groups, and probably joined in some small sidereal system of their own.

4. *Clustering stars*.—These are described as great collections of small stars that are profusely scattered over the milky way, by no means uniformly, but unequally dispersed in many separate allotments. An instance of one of these aggregates is given, which in a space of about  $5^{\circ}$  between  $\beta$  and  $\gamma$  Cygni, contains above 331,000 stars. A more particular account of the milky way, we are promised, will be the subject of a future communication.

5. *Groups of stars*.—These differ from the preceding class by being collections of closely, and almost equally compressed stars, of any figure or outline; and from the next following, by showing no particular condensation that seems to point out any ideal centre of attraction.

6. *Clusters of stars*.—These are generally round, and the compression of their stars indicates a gradual accumulation towards their

centre, where they are sufficiently condensed to produce the appearance of a nucleus. These we are told are the most magnificent objects that can be seen in the heavens.

7. *Nebulæ*.—These, it is thought, may be resolved into the three last-mentioned species, only removed to such a distance that they can only be seen by means of the most powerful telescopes.

8. *Stars with burrs, stellar Nebulæ*.—These are thought to be clusters of stars, at great distances, the light of which is gathered so nearly into one point, as to leave but just enough of it visible to produce the appearance of burrs.

9. *Milky nebulosities*.—These phenomena are probably of two different kinds, one of them being deceptions; namely, such as arise from extensive regions of closely connected clustering stars contiguous to each other, like those that compose our milky way: the other, on the contrary, being real, and possibly at no very great distance from us. The milky nebulosity of Orion, discovered by Huygens, is given as an instance of this singular appearance.

10. *Nebulous stars*.—Whether these be the effect of the atmospheres of certain stars remains yet to be determined; and indeed every thing respecting the nature of these appearances is still involved in much doubt and obscurity.

11. *Planetary Nebulæ*; and 12. *Planetary Nebulæ with centres*.—These also, though objects manifestly distinct from the former ones, are as yet so imperfectly known, as to baffle all reasoning concerning their nature and habits; and Dr. Herschel contents himself for the present with merely inserting the few he has observed in his catalogue.

Here follows the copious catalogue of Nebulæ, &c., which being a continuation of two preceding papers of the like nature, and arranged in the same manner, requires no further explanation.

*The Bakerian Lecture. Observations on the Quantity of horizontal Refraction; with a Method of measuring the Dip at Sea. By William Hyde Wollaston, M.D. F.R.S. Read November 11, 1802. [Phil. Trans. 1803, p. 1.]*

In a communication on this subject, published in the volume of the Philosophical Transactions for the year 1800, Dr. Wollaston accounted for various singular phenomena of horizontal refraction by certain gradual changes in the density of the refracting medium. Having since perused what M. Monge has published in the *Mémoires sur l'Égypte*, concerning the appearance known to the French by the name of *Mirage*, where it is ascribed to permanent rarefied strata of air near the surface of the earth; our author, having reconsidered the subject, and finding that the facts related by the French philosopher accord entirely with his own theory, declares here that he still adheres to his former opinion, and assigns his reasons for not departing from it.

The chief of these reasons is, that the definite reflecting surface,