a and x; and secondly, of x in terms of a and y; the corresponding developements hitherto given being incomplete. He considers the principles employed in this inquiry as presenting a solution of many difficulties, and illustrating peculiarities appertaining to the theory of logarithms of negative quantities; and when applied to geometry, as furnishing the means of tracing the form and developing the properties of curves whose equations involve exponential quantities. He also states that by their means various differential and other formulæ usually exhibited in treatises on logarithms may be rendered complete. An appendix is subjoined containing several examples of these applications of his principles. In the course of his investigations, the author endeavours to explain the remarkable anomaly which frequently presents itself to the analyst, of developements, in which, upon substituting a particular value for the variable in each, there is no approximation to numerical identity between the several resulting series, calculated to any number of terms, and the respective functions which they ought to represent. He combats the paradoxical opinion which has been advanced, that equations, which in particular instances are numerically false, are yet analytically true; and explains the difficulty by reverting to the limitations inherent in the hypothesis upon which the developement is founded. He maintains, in opposition to the opinions of Jean Bernouilli and D'Alembert, that the logarithms of negative and positive numbers are not in general the same; and hence infers that negative numbers have occasionally even real logarithms. The chief novelty of his system consists in showing that any assigned quantity, relatively to a given base, has an infinite number of orders of logarithms, and an infinite number of logarithms in each order.

On the Reflection and Decomposition of Light at the separating Surfaces of Media of the same and of different refractive Powers. By David Brewster, *LL.D. F.R.S. L. & E.* Read February 12, 1829. [*Phil. Trans.* 1829, *p.* 187.]

When white light is incident upon a surface which separates two different media, the portion that is reflected should, according to the Newtonian theory of light, preserve its whiteness, provided the thickness of either of the media exceed the eighty millionth of an inch. But since the dispersive powers of bodies are different, it must follow as a necessary consequence, that reflected light can never under any circumstances retain perfect whiteness, although the modification it experiences is not of sufficient amount to become sensible in ordinary experiments. The author during his investigations of the laws of polarization for light reflected at the separating surface of different media, had occasion to inclose oil of cassia between two prisms of flint glass, and was surprised to find that the light reflected was of a blue colour. The fact was new, but might be readily explained upon the principle that although the refractive density of oil of cassia greatly exceeds that of flint glass for the mean rays, yet the action

of these two bodies is nearly the same on the less refrangible rays: hence it may happen that a larger proportion of the former than of the latter is transmitted, and the pencil formed by reflexion will then appear blue. The partial decomposition thus effected in the incident rays will be the same in kind, though it may vary in degree, at different angles of incidence, and cannot therefore give rise to any variation of colour in the reflected rays, although they may differ in intensity according to the obliquity of the incidence. By using different kinds of glass, and of interposed fluids, the author obtained various analogous results, different rays of the spectrum being separated according to the prevalence, in each particular case, of one or other of the opposite actions exerted upon them by the solid and the fluid medium. The author directed his attention more particularly to those conditions in which the nearest approach could be made to a perfect equilibrium of all the forces which affect the incident rays. The solids which he employed in his experiments were two prisms of plate glass, of which the sections were right-angled isosceles triangles, and differing but very slightly in their refractive indices. The fluids were castor oil and balsam of copivi, the former having a less, and the latter a greater refractive power than the glass prisms; a thin film of either fluid being interposed between them. With castor oil, and within the limit of total reflexion, the reflected light is yellow; on gradually diminishing the angle of incidence, it passes in succession through all the tints of three orders of colours, of which the details are presented in a table exhibiting those which correspond to different angles of incidence. When the incident light is homogeneous, no colours are seen, but the reflected pencils have their maxima and minima of intensity; like the rays of thin plates, or the fringes of inflected light formed by homogeneous rays. When copivi balsam is employed as the fluid medium, the same orders of colours are obtained by reflexion, but at smaller angles of incidence than with castor oil.

Having ascertained that at a temperature of about 94° the mean refractive index of the balsam became equal to that of the glass prisms, the author examined the influence of a gradual elevation of temperature upon the colours of the reflected pencils; and found that no particular change marked the instant when the refractive densities of the two media became equal; although when the temperature was increased considerably, the tints entirely disappeared. Analogous results were obtained by employing prisms of obsidian instead of glass.

The author next engaged in more extensive series of experiments with various fluids interposed between glass prisms; and states their results in the form of a table, showing more especially the periods of colours produced at the separating surfaces by the different kinds of oils. He considers the facts which are there detailed, as establishing the existence of reflecting forces at the confines of media of the same refracting power; and as proving, first, that the reflective and refractive forces in these media do not follow the same law: and, secondly,

that the force which produces reflexion, varies according to a different law in different bodies. The reflective forces of the solid and the fluid may be conceived to decrease in various ways: first, they may respectively extend to different distances from the reflecting surface, and decrease according to the same law. Secondly, they may extend to different distances, and vary according to a different law; or, lastly, they may extend to the same distance, and vary according to different laws. Whether the refracting forces follow the same law in solids and in fluids, it is extremely difficult to determine by direct experiment; but if we assume the mutual dependence of the refracting and reflecting forces, then the experiments recorded in this paper will establish a variation in the law of the refracting forces of different media.

These facts may be explained on the undulatory theory of light, by supposing that the density or elasticity of the ether varies near the surface of different bodies, an hypothesis which has already afforded an explanation of the loss of part of an undulation in several of the phenomena of interference; the part lost being, according to Dr. Young, a variable fraction, depending on the nature of the contiguous media.

The phenomena of periodical colours at the confines of media of the same or of different refractive powers, are evidently dependent on the law of interference, although it may be difficult to point out the precise mode in which they are produced. In combinations where there is much uncompensated refraction, their production is influenced by certain changes, such as the formation of a thin and invisible film on the surface of the solid, the nature and origin of which the author endeavours to investigate, but which he acknowledges he has hitherto been unable to discover. That some unrecognised physical principle is the cause of all these phenomena will, he thinks, appear still more probable from a paper which he intends to present to the Society, on the production of the very same periods of colour, at similar angles of incidence, by the surfaces of metals and transparent solids, when acting singly upon light. He also announces, as the subjects of two other communications, the results of researches in which he has been long engaged; first, on the action of light on the surfaces of bodies as an universal mineralogical character, with the description of a lithoscope for discriminating minerals; and secondly, on the influence of the doubly refracting forces upon the ordinary forces, which reflect and polarize light at the surfaces of bodies.

On the Reduction to a Vacuum of the Vibrations of an Invariable Pendulum. By Captain Edward Sabine, *of the Royal Artillery, Sec. R.S. Communicated by Dr.* Thomas Young, *Secretary of the late Board of Longitude.* Read March 12 *and* 19, 1829. [*Phil. Trans.* 1829, *p.* 207.]

The experiments contained in this paper originated in a notice published by M. Bessel in the *Astronomische Nachrichten*, for