that the force which produces reflexion, varies according to a different law in different bodies. The reflective forces of the solid and the fluid may be conceived to decrease in various ways: first, they may respectively extend to different distances from the reflecting surface, and decrease according to the same law. Secondly, they may extend to different distances, and vary according to a different law; or, lastly, they may extend to the same distance, and vary according to different laws. Whether the refracting forces follow the same law in solids and in fluids, it is extremely difficult to determine by direct experiment; but if we assume the mutual dependence of the refracting and reflecting forces, then the experiments recorded in this paper will establish a variation in the law of the refracting forces of different media.

These facts may be explained on the undulatory theory of light, by supposing that the density or elasticity of the ether varies near the surface of different bodies, an hypothesis which has already afforded an explanation of the loss of part of an undulation in several of the phenomena of interference; the part lost being, according to Dr. Young, a variable fraction, depending on the nature of the contiguous media.

The phenomena of periodical colours at the confines of media of the same or of different refractive powers, are evidently dependent on the law of interference, although it may be difficult to point out the precise mode in which they are produced. In combinations where there is much uncompensated refraction, their production is influenced by certain changes, such as the formation of a thin and invisible film on the surface of the solid, the nature and origin of which the author endeavours to investigate, but which he acknowledges he has hitherto been unable to discover. That some unrecognised physical principle is the cause of all these phenomena will, he thinks, appear still more probable from a paper which he intends to present to the Society, on the production of the very same periods of colour, at similar angles of incidence, by the surfaces of metals and transparent solids, when acting singly upon light. He also announces, as the subjects of two other communications, the results of researches in which he has been long engaged; first, on the action of light on the surfaces of bodies as an universal mineralogical character, with the description of a lithoscope for discriminating minerals; and secondly, on the influence of the doubly refracting forces upon the ordinary forces, which reflect and polarize light at the surfaces of bodies.

On the Reduction to a Vacuum of the Vibrations of an Invariable Pendulum. By Captain Edward Sabine, *of the Royal Artillery, Sec. R.S. Communicated by Dr.* Thomas Young, *Secretary of the late Board of Longitude.* Read March 12 *and* 19, 1829. [*Phil. Trans.* 1829, *p.* 207.]

The experiments contained in this paper originated in a notice published by M. Bessel in the *Astronomische Nachrichten,* for

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January 1828, announcing that he had found the theory usually employed for reducing the vibrations of a pendulum in air to the corresponding vibrations *in vacuo*, was incorrect, inasmuch as it omitted the expenditure of a portion of the moving force on the particles of the air which are set in motion by the pendulum in its vibration. In order to ascertain by the most direct mode of experiment the retardation which a pendulum experiences by vibrating in the medium of the atmosphere, the author constructed, at the expense of the late Board of Longitude, an apparatus in which an invariable pendulum could be vibrated alternately in air of the full atmospheric pressure, and in rarefied air approaching nearly to a vacuum. The apparatus was set up and employed in a room assigned for that purpose in the Royal Observatory at Greenwich: its description is given in this paper with reference to plates, and those processes are particularly dwelt upon which were ultimately successful in ensuring the immobility of the suspension of the pendulum, and in rendering the apparatus impermeable when the air was withdrawn from the interior. The arrangements for observing the coincidences of the pendulum with a clock, and for ascertaining the exact pressure of the air, both in its ordinary and rarefied state, are minutely described. When the air was either partially or wholly withdrawn, a correction was found to be required for the indications of the thermometer giving the temperature of the pendulum, to compensate the removal of the pressure of the atmosphere on the exterior of the ball and tube of the thermometer. The value of this correction for different states of exhaustion was ascertained by placing the thermometer in pounded ice under the receiver of an air-pump, and noting the height of the mercury corresponding to different heights in the gauge.

The number of distinct experiments made with the above described apparatus is eleven; of these, the six first were designed exclusively for the purpose of comparing the vibration in air of full pressure, and in rarefied air. Each experiment consisted of three distinct series of vibrations made in succession, and occupying usually the greater part of two days: the first and third series were in air of full pressure, and the second series in rarefied air; the mean of the results of the first and third series gave the vibration in the ordinary atmosphere, which was compared with the result of the intermediate series in rarefied air. The comparison was thus rendered wholly independent of the daily rate of the clock, and in some measure also of its deviations from an uniform rate in intervals of less than 24 hours. The 7th and 8th experiments had a double object: first, to compare the retardations of a pendulum in common air and in hydrogen gas, both under atmospheric pressure; and secondly, to obtain the amount of retardation in both cases, by comparing the vibration in hydrogen gas under thirty inches pressure, with the vibration in the same gas in a highly rarefied state. There are thus eight experiments on the retardation occasioned by the ordinary atmosphere, and two on the retardation in hydrogen gas under atmospheric pressure; the latter furnishing also the comparative influence of hydrogen gas and atmo-

spheric air. The 9th, 10th, and 11th experiments were made to examine whether the vibration of the pendulum in air within the apparatus was the same as in the free air of the apartment; this was accomplished by vibrating the pendulum alternately in free and in confined air, by removing and again replacing such parts of the apparatus as were necessary for that purpose. It was found, by this means, that the confinement of the medium by the glasses produced no sensible effect on the time of vibration.

From the mean of the eight experiments on the retardation in common air, the author obtains 10·36 vibrations *per diem*, as the reduction to a vacuum of the invariable pendulum, vibrating in air of 45° under a pressure of 30 inches of mercury: and by computing the retardation severally for the circumstances of each experiment, and comparing the computed and observed retardations, he shows, that were the amount of the reduction to a vacuum separately derived from each of the eight experiments, it would in no case differ more than 0·14 of a vibration *per diem*, from the conclusion derived from their mean, or one 74th part of the conclusion itself.

The reduction to a vacuum which would have been previously computed for the vibration in air of 45°, and 30 inches pressure, is 6·26 vibrations *per diem*. The retardation in air of that temperature and density is therefore 4·1 vibrations greater than has been hitherto supposed; and the proportion which the experimental reduction bears to that which is now shown to have been erroneous, is, for the invariable pendulum of the ordinary form used in this country, as 1·655 to 1.

From the experiments in hydrogen gas, under a pressure respectively of 30 inches, and of less than one inch, the retardation of a pendulum vibrating in hydrogen gas of 40° under a barometric pressure of 30 inches, is two vibrations *per diem*. The hydrogen gas employed was obtained by the action of zinc upon dilute sulphuric acid, and was passed into the apparatus through a cylinder containing muriate of lime. A portion withdrawn after the experiments were concluded, was examined by Mr. Faraday, and found to contain no appreciable mixture of air.

The two experiments on the comparative retardation in air and in hydrogen gas give the ratio as 10·55 to 2, and as 10·41 to 2; or, generally, as $5\frac{1}{4}$ to 1. But the ratio of the respective densities of atmospheric air and hydrogen gas is about as 13 to 1. Whence the author takes occasion to remark, that if the resistance of the elastic fluids to bodies falling through them were simply as the respective densities of the fluids, the retardation occasioned by air should be 13 times as great as that occasioned by hydrogen gas,—that the difference of this ratio from that shown by experiment is much too great to be ascribed to error of experiment, particularly as repetition produced results almost identical; that it may rather be regarded as indicating an inherent property in the elastic fluids analogous to that of viscidity in liquids, and of resistance to the motions of bodies passing through them independently of their density; and

that as an example, hydrogen gas, compared with atmospheric air, appears to possess this property in a proportion more than double that which would be given by the respective densities of air and hydrogen gas.

The paper concludes with an investigation of the effect which the near reduction to a vacuum will have on the variations of gravity at different parts of the earth's surface, which have been obtained with invariable pendulums; and particularly of the experiments of the author himself, which embrace a greater range and variety of temperature than those of any other experimentalist. It is shown that in consequence of the peculiar mode in which those experiments were reduced to a mean term of comparison, their re-calculation with the more correct elements now known, would have no other effect than that of adding an equal amount to the vibrations of the experimental pendulum at every station, leaving the acceleration at different stations unaltered.

Consideration of the Objections raised against the geometrical Representation of the Square Roots of Negative Quantities. By the Rev. John Warren, *M.A. of Jesus College, Cambridge. Communicated by* Thomas Young, *M.D. For. Sec. R.S.* Read February 19, 1829. [*Phil. Trans.* 1829, *p.* 241.]

It has always appeared a paradox in mathematics, that by employing what are called imaginary or impossible quantities, and subjecting them to the same algebraic operations as those which are performed on quantities that are real and possible, the results obtained should always prove perfectly correct. The author inferring from this fact, that the operations of algebra are of a more comprehensive nature than its definitions and fundamental principles, was led to inquire what extension might be given to these definitions and principles, so as to render them strictly applicable to quantities of every description, whether real or imaginary. This deficiency, he conceives, may be supplied by having recourse to certain geometrical considerations. By taking into account the directions as well as the lengths of lines drawn in a given plane, from a given point, the addition of such lines may admit of being performed in the same manner as the composition of motions in dynamics; and four such lines may be regarded as proportional, both in length and direction, when they are proportionals in length, and, when also the fourth is inclined to the third at the same angle that the second is to the first. From this principle he deduces, that if a line drawn in any given direction be assumed as a positive quantity, and consequently its opposite a negative quantity, a line drawn at right angles to the positive or negative direction will be represented by the square root of a negative quantity; and a line drawn in an oblique direction will be represented by the sum of two quantities, the one either positive or negative, and the other the square root of a negative quantity. On this subject, the author published a treatise in April 1828; since