Das Verhältniss der chemischen Verwandtschaft zur galvanischen Elektricität, in Versuchen dargestellt. Von N. W. Fischer, M. et Phil. Doct. 8vo.—*The Author.*

The reading of a Paper, entitled, On the Equilibrium of Fluids; and the Figure of a Homogeneous Planet in a Fluid state. By James Ivory, Esq. M.A. F.R.S.—was commenced.

---

January 20.

HIS ROYAL HIGHNESS THE DUKE OF SUSSEX, President, in the Chair.

William John Blake, Esq. M.A. was elected a Fellow.

The following Presents were received, and thanks ordered for them:—

Transactions of the Society for the Encouragement of Arts, Manufactures, and Commerce, for the year 1829. 8vo.—*Presented by the Society.*

Proceedings of the Committee of Science and Correspondence of the Zoological Society of London. (Nov. 9, to Dec. 4, 1830.) 8vo.—*The Society.*

On the Probable Connection of Rock Basins, in Form and Situation, with an internal Concretionary Structure in the Rocks on which they occur: introduced by Remarks on the alleged Artificial Origin of those Cavities. By E. W. Brayley, Jun., Esq. 8vo.—*The Author.*

Tabulæ Regiomontanæ Reductionum Observationum Astronomicarum ab Anno 1750 usque ad Annum 1850 computatæ. Auctore F. W. Bessel. 8vo.—*The Author.*

The reading of Mr. Ivory's Paper was resumed and concluded.

The author considers the essential property of a fluid, and that on which its definition should be founded, as consisting in the perfect mobility of its particles among one another. If abstraction be made of the force of gravity, or other accelerating force, when a continuous fluid is at rest, and consequently in a state of equilibrium, all its particles are equally pressed in every direction, are equally distant from one another, and are similarly arranged about every interior point. No fluid is absolutely incompressible; but the degree of compressibility may be conceived to be so small as not to affect the results; and it is accordingly disregarded in the investigations which occupy the present paper.

These investigations are built on the assumption that the hydrostatic pressure at every point of the fluid is the same function of the three rectangular co-ordinates of the point drawn to three planes intersecting one another at right angles. The author shows that the algebraical expressions of the accelerating forces producing the pressure are not entirely arbitrary; because they must necessarily be equal to the partial differential co-efficients of a function of three independent variables, and therefore they are likewise the same func-

tions of the co-ordinates of their point of action in every part of the mass. This is one of the conditions required for the equilibrium of a mass of homogeneous fluid; and a second necessary condition is, that these functions of the ordinates are capable of being integrated. When these two conditions are fulfilled, the determination of the figure of equilibrium is reduced to a question purely mathematical. For we can form an equation expressive of an equilibrium between the accelerating forces and the variation of pressure, and by integrating this equation we may obtain the hydrostatic pressure; whence may be deduced the equation of all those points at which there is no pressure, that is, of the outer surface of the fluid. All that is then requisite for securing the permanence of the figure of the fluid, is that the pressures propagated through the mass be either supported, or mutually balance one another. The upper surface, which is at liberty, and where there is no pressure, and all interior surfaces, where the pressure is constant, have the same differential equation; and from this the author infers that such surfaces are perpendicular to the resultant of the accelerating forces acting upon the particles contained in them. These interior surfaces were denominated by Clairaut *level surfaces;* and they are distinguished by the two properties of being equally pressed at all their points, and of cutting the resultant of the forces at right angles.

The author next extends the investigation to heterogeneous fluids, the different parts of which vary in their density, and deduces a similar conclusion to the former with respect to the perpendicularity of the interior level surfaces to the resultant of the accelerating forces, which act upon the particles situated in each surface respectively. He discusses the hypothesis of Clairaut, of narrow canals traversing the mass in various directions, and shows that the same results follow from it as from the general theory.

The conditions laid down by Clairaut, and all other authors, as those which are necessary for the equilibrium of a homogeneous fluid, are these two:—first, the accelerating forces must be expressed by the partial differential co-efficients of a function of three independent co-ordinates; secondly, the resultant of the forces in action at the upper surface at liberty must be perpendicular to that surface. The author shows that the second condition is a consequence of the first; and he states the independent conditions of equilibrium to be these:—first, the expressions of the forces must be the same functions of the co-ordinates in every part of the mass; secondly, the same expressions must be the partial differential co-efficients of a function of three independent co-ordinates.

In a very extensive class of problems, the difference in the two ways of laying down the conditions of equilibrium disappears. But the theory of Clairaut cannot be extended to the cases in which the particles mutually attract or repel one another, or where the accelerating forces depend on the figure of the mass of fluid. Such is the condition of a homogeneous planet in a fluid state, in which there are forces which prevail in the interior parts, but vanish at the surface; and which are, therefore, not taken into account in Clairaut's theory. But since

these forces tend to change the figure of the fluid, that theory is inadequate to give an exact determination of the equilibrium in those cases.

In the second part of the paper, the author applies his theory of the equilibrium of fluids to the determination of the figure of the planets, under the supposition that they are composed wholly of fluid materials. For this purpose he first considers the problem of determining the equilibrium of a homogeneous mass of fluid entirely at liberty, when the accelerating forces are known functions of the co-ordinates at their point of action. In the investigation of this problem, he supposes that the centre of gravity is at rest, and undisturbed by the action of any accelerating force. He then supposes the fluid to be in equilibrium, and that three planes are laid down, intersecting one another at right angles in the centre of gravity of the mass, to which planes the particles of the fluid are referred by rectangular co-ordinates. The algebraical consequences of this supposition are then pursued, the conditions necessary to equilibrium pointed out, and the conclusion deduced, that the resultant of the accelerating forces is perpendicular to the outer surface, and also to the interior level surfaces of the fluid, at every point of which there is the same intensity of pressure. The figure of the fluid being determined, it remains to inquire, whether the equilibrium is secure; and the result of the inquiry furnishes an equation which proves that the particles have no tendency to move, from any inequality of pressure.

A further discussion is entered into in order to prove that the pressures propagated from the surfaces into the interior parts balance and destroy one another, which completely establishes the permanence of the figure of the fluid. It is also shown that the mass of fluid, under these circumstances, has no tendency to turn upon an axis.

To illustrate the foregoing problem, the author applies it to the determination of the figure of equilibrium of a homogeneous mass of fluid entirely at liberty, of which the particles attract one another with a force directly proportional to the distance, at the same time that they are urged by a centrifugal force caused by rotation about an axis.

He then enters upon the investigation of the second problem, in which the law of attraction of the particles is that of the inverse duplicate ratio of the distance; and finally arrives at the conclusion, that the form of the fluid in equilibrium is, exclusively of all other figures, an oblate elliptical spheroid of revolution, and that its axis of rotation is the lesser axis of the spheroid. He also shows that within the spheroid there are no more than two sets of surfaces equally pressed by the action of the exterior fluid; and no more than two different spheroids of equilibrium answering to the same rotatory motion. If the whole spheroid be one of small oblateness, the greatest of the interior surfaces of equable pressure, which is not a level surface, stands upon the equator; and the rest are within this, and are similar to it, and similarly posited. When it is very oblate, the greatest of these surfaces is described about the lesser axis; and the rest are within it, and are similar to it, and similarly posited. The existence

of two sets of interior surfaces, that are equally pressed at all their points by the action of the exterior fluid, is inconsistent with Clairaut's theory, and is a proof of the insufficiency of that theory for determining the figure of a homogeneous planet.

---

January 27.

GEORGE RENNIE, Esq. V.P., in the Chair.

The following Presents were received, and thanks ordered for them:—

Views of the Pelvis, showing the Natural Size, Form and Relations of the Bladder, Rectum, Uterus, &c. in the Infant and in the Adult; taken from Preparations made for the Museum of the Royal College of Surgeons in Ireland. By John Houston, Esq. folio.—*Presented by the Author.*

An Account of two newly discovered Muscles for compressing the Dorsal Vein of the Penis, in Man and other Animals; and also of a similar Provision for compressing the Veins of the Chameleon's Tongue. By the Same. 8vo.—*The Author.*

Connaissance des Tems pour l'An 1833. 8vo.—*The Board of Longitude of France.*

L'Annuaire pour l'An 1831. 12mo.—*The Same.*

Bulletin de la Société Française de Statistique Universelle. Première Livraison. 4to.—*The Society.*

Statuts, et Liste des Membres, de la Société Française de Statistique Universelle. 8vo et folio.—*The Society.*

A paper was read, On the probable electric origin of all the phenomena of Terrestrial Magnetism; with an illustrative experiment. By Peter Barlow, Esq. F.R.S. Corr. Mem. Inst. France, and of the Imp. Acad. St. Petersburgh.

The author begins his paper by a retrospect of the several discoveries on terrestrial magnetism made since the commencement of the present century. Humboldt, by his numerous and accurate observations on this subject, laid the foundation of all the scientific knowledge relating to it, which we hitherto possessed. The task of reducing these observations to definite principles, by subjecting them to calculation, was undertaken by Biot; and the conclusion which he drew from them was, that on the hypothesis of the earth's being a great magnet, the facts would best accord with the supposition that its two poles are coincident, or indefinitely near to each other, at the centre of the globe. The same result was also obtained, though by a different process of reasoning, by M. Kraft of St. Petersburgh. It followed as a necessary consequence, that terrestrial magnetism observes a law different from that of a permanently magnetic body, but identical with that of a body in which transient magnetism is excited by induction. The law which obtains in the case of a sphere of iron rendered magnetic by induction was first investigated, in 1829, by Mr. Barlow; and also, by Mr. Charles Bonnycastle, Professor of