May 24, 1838.

FRANCIS BAILY, Esq., V.P. and Treas., in the Chair.

His Imperial and Royal Highness Leopold II., Grand duke of Tuscany, was elected a Fellow.

The reading of the paper by Mr. Ivory, "On the Theory of the Astronomical Refractions," was concluded.

In this communication, the author, after stating that the mean refractions are the object of investigation, and fully defining what he understands by this term, gives an historical review of what has been done up to the present time on this very important subject. Having stated that the foundation of the theory of astronomical refractions was laid by Dominique Cassini, he deduces on Cassini's hypothesis (that of an homogeneous atmosphere) a formula for the refraction, which agrees exactly with that of La Place, employed in computing the first part of the table of mean refractions, published by the French Board of Longitude.

The labours of our immortal countryman Newton, in this vast field of inquiry, are next reviewed. As the density of the atmosphere in ascending decreases gradually, the path described by a ray from a star, in its passage through the atmosphere, is not a straight line, as it would be on Cassini's hypothesis, but is a curve more and more inflected towards the earth's centre. In the Principia there is found whatever is necessary for determining the nature of this curve, and, consequently, for solving the problem of the astronomical refractions, which consists in ascertaining the difference between the direction of light when it enters the atmosphere, and its ultimate direction when it arrives at the earth's surface.

On the principles established in the second section of the Principia, the author deduces equations requisite for the solution of the problem of astronomical refractions, and remarks that these equations are perfectly general, and will apply in any constitution of the atmosphere that may be adopted. In this investigation, in preference to employing functions with peculiar properties to express the molecular action, the manner in which the forces act has been considered. When the light, in passing through the atmosphere, arrives at a surface of increased density, it receives an impulse which may be considered as instantaneous; and this impulse being distributed over the breadth of a stratum of uniform density, ascertains the centripetal force tending to the earth's centre, by the action of which the trajectory is described.

It appears, that Newton himself was the first to apply this new method to the problem of the astronomical refractions. In his first attempt he assumes that the densities decrease in ascending, in the same proportion as the distances from the earth's centre increase. On this supposition the author investigates a formula, which M. Biot has also obtained, and which is equivalent to the construction communicated by Newton to Flamsteed. On this basis a table was computed and communicated to Flamsteed; but Newton subsequent-

ly informed Flamsteed that he did not intend to publish it, in consequence of a serious objection to the supposed scale of densities. Adopting the principles in the twenty-second proposition of the second book of his Principia, Newton, it appears, succeeded at length in computing a second table of refractions, which he likewise communicated to Flamsteed, and which, there is every reason to think, is the same which he gave to Halley, and which was inserted by that astronomer in the Philosophical Transactions for 1721. As the determining whether the two tables are identical is a question of much interest, the author enters very fully into it, and, from the results of elaborate calculations, concludes that Halley's table is no other than the one which Newton calculated on the supposition that the densities in the atmosphere are proportional to the pressures. He remarks that, as far as the mathematics are concerned, the problem of the astronomical refractions was fully mastered by Newton.

After referring to the labours of Brook Taylor, Kramp, and Thomas Simpson, the author again adverts to Newton's views, remarking that, in assigning the rarefaction of the lower region of the atmosphere by heat as the cause why the calculated refractions near the horizon so much exceeded the observed, as was found to be the case, Newton had assigned the true cause; but that he had no clear conception of the manner in which the density in the lower region is altered by the agency of heat; and he considers that nearly the same ignorance in that respect still prevails.

The two atmospheres, with densities decreasing in arithmetical and geometrical progression, which, it now appears, were imagined by Newton, and which have been discussed by Thomas Simpson and other geometers, are found, when the same elements are employed, to bring out horizontal refractions on opposite sides of the observed quantities. La Place conjectured that an intermediate atmosphere which should partake of the nature of both, and should agree with observation in the horizontal refraction, would approach nearly to the true atmosphere. If recourse be had to the algebraical expressions of La Place, it will be found that the atmosphere he proposes is one of which the density is the product of two terms, the one taken from an arithmetical, the other from a geometrical series; the effect of which combination is to introduce a supernumerary constant, by means of which the horizontal refraction is made to agree with the true quantity. The author considers, with Dr. Brinkley, that the French table, founded on La Place's investigation, is only a little less empirical than the other tables, and that the hypothesis of La Place does not appear to possess any superiority over other supposed constitutions of the atmosphere in leading to a better and less exceptionable theory.

After eulogizing Bessel's tables of mean refractions, published in his *Tabulæ Regiomontanæ*, the author refers to his own paper in the Philosophical Transactions for 1823. In this paper the refractions are deduced entirely from the very simple formula,—

$$\frac{1+\beta\tau'}{1+\beta\tau''}=1-f\,(1-\overset{-u}{c})$$

in which β stands for the dilatation of air or gas by heat, τ' and τ'' for the temperature at the earth's surface, and at any height above it, and c^{-u} for the density of the air at that height in parts of its density at the surface. If this formula be verified at the earth's surface in any invariable atmosphere, by giving a proper value to the constant f, it will still hold, at least with a very small deviation from exactness, at a great elevation; and this is immediately shown.

This manner of arriving at the constitution of the atmosphere is contrasted with the procedure of M. Biot of transforming an algebraical formula, for the express purpose of bringing out a given result. As the problem in the *Mécanique Céleste* is solved by means of an interpolated atmosphere between two others; as in Mr. Ivory's paper of 1823, there is no allusion to such an atmosphere; and as the table in that paper is essentially different from all the tables computed by other methods, he contends that all these must be sufficient to stamp an appropriate character on his solution of the problem. But if ingenuity could trace some relation, in respect of the algebraic expression, between the paper of 1823 and La Place's calculations, he considers that it is not difficult to find, between the same paper and the view of the problem taken by the author of the Principia in 1696, an analogy much more simple and striking. Newton having solved the problem, on the supposition that the density of the air is produced solely by pressure, and having found that the refractions thus obtained greatly exceeded the observed quantities near the horizon, inferred, in the true spirit of research, that there must be some cause not taken into account, such as the agency of heat, which should produce, in the lower part of the atmosphere, the proper degree of rarefaction necessary to reconcile the theoretical with the observed refractions. The author's sole intention, in introducing the quantity f in his formula, is to cause the heat at the earth's surface to decrease in ascending, at the same rate that actually obtains in nature, not before noticed by any geometer, but which evidently has the effect of supplying the desideratum of Newton.

The author considers, that the comparison of the table in the paper of 1823, with the best observations that could be procured at the time of publication, was satisfactory; and after the publication of the *Tabulæ Regiomontanæ*, he found that the table agreed with Bessel's observed refractions to the distance of 88° from the zenith, with such small discrepancies as may be supposed to exist in the observations themselves.

The paper in the Philosophical Transactions for 1823, however, takes into account only the rate at which the densities, in a mean atmosphere, vary at the surface of the earth; but, in the present communication, the author proposes to effect the complete solution of the problem, by estimating the effect of all the quantities on which the density at any height depends. For this purpose, he finds it necessary to employ functions of a particular kind; and then gives a formula, one part of which consists of a series of these functions, for the complete expression of the temperature of an atmosphere in equilibrium; the intention of assuming this formula being to ex-

press the temperature in terms of such a form as will produce, in the refraction, independent parts that decrease rapidly. By this means he proceeds in the analytical investigation of the problem in its more comprehensive form, and deduces two equations on which its solution depends.

The first of these contains the law according to which the heat decreases as the height above the earth's surface increases; and the second determines the perpendicular ascent, when the difference of the pressures and of the temperatures at its upper and lower extremity have been found. If the latter, with a slight transformation, be multiplied by the proper factor, representing the variable force of gravity in different latitudes, it becomes identical with the usual barometric formula, all its minutest corrections included; and it has this advantage; that, whereas the usual formula is investigated on the arbitrary assumption, that the temperature is constant at all the points of an elevation, and equal to the mean of the temperatures at the two extremities, this formula is strictly deduced from the general properties of an atmosphere in equilibrium.

Having determined, from experimental results, the values of certain constants in these formulæ,—first, in an atmosphere of dry air, and, secondly, in an atmosphere of air mixed with aqueous vapour, the author remarks, that the analytical theory agrees in every respect with the real properties of the atmosphere, as far as these have been ascertained.

The object of Mr. Ivory's further investigation is to show, that the same theory represents the astronomical refractions with a fidelity that can be deemed imperfect only as far as the values of particular constants, which can only be determined by experiment, are liable to the charge of inaccuracy. He therefore proceeds to determine, from the formulæ previously deduced, the refraction of a star in terms of its apparent zenith distance. For this purpose, the differential equations are transformed by the introduction of new symbols; the limits of certain terms are determined previously to their being neglected; and the equation is finally reduced to a form, in which the remaining operations consist in investigating the integrals of four expressions, and in subsequently assigning their numerical values. Great skill is displayed in conducting these intricate investigations; and after going through the most laborious calculations and computations, the author exhibits a table of theoretical refractions, deduced solely from the phenomena of the atmosphere, for zenith distances, extending from 10° to 89½°. These refractions are compared with those in Bessel's table, in the *Tabulæ Regiomontanæ*, and also with those in the table in the *Connaissance des Temps*. From this comparison, it appears, that the three tables agree within less than 1″, as far as 80° from the zenith: from 80° to 88° of zenith distance, the numbers in the French table exceed those in Bessel's, the excess being 2″ at 84°, and 4″ at 88°; and with a single exception at 88°, (probably, judging from the character of the adjacent number, arising from an error of computation,) the refractions in the new table are nearer to Bessel's than those in the French table;

but when the zenith distance is greater than 80°, the author considers the accuracy of the French table questionable, both on account of the hypothetical law of the densities, and because the quantity assumed for the horizontal refraction is uncertain.

After giving a few examples, illustrative of the use of the new table, the author inquires how far the refractions are likely to be affected by the term which it was found necessary to leave out, because the present state of our knowledge of the phenomena of the atmosphere made it impossible to determine the coefficient by which it is multiplied. For this purpose, the variable part of that term has been computed for every half degree, from 85° to 88°, and the results are exhibited in a table. From this it appears, that this coefficient, although considerably less than that of the preceding term, may still have some influence on the refractions at very low altitudes. The mean refraction in Bessel's table, and in the new table, can hardly be supposed to differ 2″ from the true quantity, which would limit the coefficient in question to be less than one-tenth. It is a matter of some importance to obtain a near value of this coefficient; and it is probable that this can be accomplished in no other way, than by searching out such values of the two coefficients as will best represent many good observed refractions at altitudes less than 5°. If such values were found, our knowledge of the decrease of heat in ascending in the atmosphere would be improved, and the measurement of heights by the barometer would be made more perfect.

At the end of the paper is given a table of mean refractions for the temperature 50° Fahr. and barometric pressure 30 inches, at every degree from 0° to 70° zenith distance, and at every 10′ from 70° to the horizon; and tables of the corrections requisite for variations of the thermometer and barometer are subjoined.

May 31, 1838.

DAVIES GILBERT, Esq., V.P., in the Chair.

The Rev. John Hymers was duly elected a Fellow of the Society.

A paper was read, entitled, "Remarks on the Theory of the Dispersion of Light, as connected with Polarization." By the Rev. Baden Powell, M.A., F.R.S., Savilian Professor of Geometry in the University of Oxford.

The present paper is a sequel to those already presented by the author to the Royal Society, in which he had instituted a comparison of the observations of the refractive indices for the standard rays of light in various media, with the results calculated from theoretical formulæ, deduced from the most improved views of the undulatory hypothesis; the cases discussed including the greatest range of data as yet furnished by experiment. The comparison exhibited an accordance sufficient to warrant the conclusion that the theory af-