

IV. *On the action of crystallized bodies on homogeneous light, and on the causes of the deviation from NEWTON'S scale in the tints which many of them develope on exposure to a polarised ray.* By J. F. W. HERSCHEL, Esq. F. R. S. Lond. and Edin.

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SINCE the period of the brilliant discovery of MALUS of the polarisation of light by reflection, the investigation of the general laws which regulate the action of crystallized bodies on light, has advanced with a rapidity truly astonishing, and the labours of an ARAGO, a BREWSTER, and a BIOT, have already gone far towards completing the edifice of which that distinguished philosopher laid the foundation. When MALUS wrote, the list of doubly refracting crystals was small, and the most remarkable among them possessing only one axis of double refraction, it seems to have been for some time, tacitly at least, presumed that the law discovered by HUYGENS, and since re-established in the most rigorous manner for that one,* might hold good in all. The discovery, by Dr. BREWSTER, of crystals possessing two axes of double refraction, or two

* The author of the article on Polarisation, in the 63d Number of the Edinburgh Review, just published, is guilty of a most unpardonable mistake, in asserting, (p. 188), as deducible from Dr. BREWSTER'S experiments, that the Huygenian law is *incorrect*, for carbonate of lime. Dr. BREWSTER'S general formulæ for crystals with two axes resolve themselves into the Huygenian law when the axes coincide, of which case it is only an extension. That excellent philosopher, if I understand English, in the paragraph which gave rise to this strange assertion, only means to declare his opinion that it remains undemonstrated.

directions in which a ray may penetrate their substance without separation into distinct pencils, has proved the fallacy of any such generalization, and rendered it necessary to enter on a far more extensive scale of investigation.

There are two methods which may be pursued in observations on double refraction and polarisation, the one direct, the other indirect. The former turns on immediate observations of the angular deviation of the extraordinary pencil, and is, of course, only applicable when the forces which act exclusively on the rays composing it are sufficiently intense to cause a sensible separation of the two pencils. There exist, however, a multitude of crystals in which the force of double refraction is so feeble as to produce scarcely any, or at most a very inconsiderable deviation of the extraordinary ray, and in which, consequently, the laws of double refraction could neither be investigated nor verified, without having recourse to some artificial means of magnifying the quantity to be observed; a thing easy enough in theory, but requiring, in practice, the greatest nicety on the part of the observer, and in many cases altogether impracticable, from the physical constitution of the crystals themselves. The indirect method depends on the discovery of ARAGO, scarcely inferior in intrinsic importance to that of MALUS, of the separation of a polarised ray into complementary portions by the action of a crystallized lamina. It was reserved, however, for the genius of M. BIOT, to trace this striking phenomenon to its ultimate causes, in the action of crystals on the differently coloured rays, and to develope, in a simple and elegant theory, the successive gradations by which the polarisation of a ray in its passage through a doubly refracting crystal is performed;

while, on the other hand, the splendid phenomena of the polarised rings, which we owe to Dr. BREWSTER, have established the connection of the tints so polarised with the force producing the deviation of the extraordinary pencil, and shown the legitimacy of conclusions respecting the intensity of the latter, drawn from observations on the former.

This indirect mode of observation, which consists in noticing the gradations of colour for different positions and thicknesses of the crystal, possesses three capital advantages. The first is its extreme sensibility, which enables us to detect the existence, and measure precisely the intensity of forces, far too feeble to produce any measurable deviation of the extraordinary pencil. It, in fact, affords the rare combination of an almost indefinite enlargement of our scale of measurement, with a possibility of applying it precisely to the object measured, arising from the distinctness of all its parts. Another, no less precious, is the leading us by mere ocular inspection to the *laws* of very complicated phenomena, and enabling us to form, and mould as it were our analytical formulæ, not on a laborious, and sometimes deceptive discussion of tabulated measures, but on the actual form of the curves themselves, which are loci of the functions under consideration. It is true, that a reference to tabulated measures is indispensable to give precision to such first approximations; but the power this mode of observation affords of copying our outline fresh from nature, and from the general impression of the phenomena, brought at once under our view, is an advantage not to be despised. Nor ought we, lastly, to omit, in our estimate of advantages, the means thus afforded us of subjecting the minutest fragments of a crystal to a scrutiny as

severe as the most splendid specimen, and thus extending our researches to an infinitely greater variety of natural bodies, than we could otherwise hope to examine.

In order, however, to render observations on the tints developed by polarised light available, they must be comparable to each other ; and it therefore becomes an object of the first importance, to ascertain the existence, and discover the laws of any causes which may operate to disturb their regularity. Ever since I first engaged in experimental enquiries on the polarisation of light, I was struck by the very considerable deviation from the succession of colours in thin laminæ, as observed by NEWTON, which many crystals exhibit when cut into plates perpendicular to one of their axes. I at first attributed this to a want of perfect regularity in their structure, or to inequalities in their thickness, arising from my own inexpertness in grinding and polishing their surfaces ; and it was not till habit had rendered me familiar with all the usual causes of deception, that, finding the same phenomena uniformly repeated in different and perfect specimens, my curiosity became excited to enquire into their cause, the more so, as they now began to assume the form of a radical and unanswerable objection to the theory of M. BIOT, above alluded to, which affords so perfect an explanation of the tints in crystals with one axis.

These phenomena have not escaped the vigilance of Dr. BREWSTER. In his paper of 1818, he distinctly notices the fact of a deviation from NEWTON's scale, in crystals with two axes, and promises a more detailed account of it, which however has not yet appeared. But the object of the present communication is not thereby anticipated, as in the only passage in that paper

in which he expresses himself otherwise than obscurely on its cause, he appears to regard the deviated tints as analogous to those developed along the axis of rock crystal and by certain liquids; an analogy which, in the present state of our knowledge on that perplexing subject, it seems not easy to admit. In a paper too, which has lately appeared, containing the interesting observations of the same excellent philosopher on the optical structure of the apophyllite, he remarks the very striking deviation of the colours of this crystal from NEWTON'S scale "in the first orders" of its rings; and while he remarks that such deviations are common enough, and indeed universal in crystals "in which the rings are formed by the joint action of two axes," seems to think this analogy close enough to authorize the substitution of two rectangular axes of a negative character for the single positive axis actually observed, according to his own peculiar and ingenious views on this subject. I lost no time in endeavouring to procure a specimen of this mineral, and by the kindness of my friend, Sir SAMUEL YOUNG, (to whom I owe more than one obligation of this nature) was favoured with one sufficiently transparent for optical examination. From my observations on this body, I think I shall be able to demonstrate satisfactorily, that the phenomena of the apophyllite depend on a principle distinct from that which produces the chief part of the deviation of tints in most crystals with two axes.

The course I propose to pursue is, first, to describe the phenomena themselves. I shall then show how these phenomena, complicated as they are in appearance, are all reducible to one very simple and general fact; viz. that the axes

of double refraction differ in their position in the same crystal for the differently coloured rays of the spectrum, being dispersed in one plane over an angle more or less considerable, according to the nature of the substance. In many bodies, the magnitude of this dispersion of the axes is comparatively trifling, while in some, not otherwise remarkable for a high ordinary or extraordinary dispersive power, it is enormous, and must render all computation of the tints in which it is not taken into consideration, completely erroneous; and indeed obliterating almost every trace of the Newtonian scale of colour. We have here, then, a new element, which for the future must enter into all formulæ of double refraction pretending to rigour, and at the same time are presented with another very striking instance of the inherent distinction between the differently coloured molecules of light, which, since the time of NEWTON, every new step in optical science has tended to place in a stronger point of view. At the same time, by the easy and complete explanation this principle affords of all the more perplexing anomalies in the tints, the theory of alternate polarisation to which they were hitherto so palpable and formidable an objection, stands relieved from every difficulty, and may now be received as fully adequate to the representation of all the phenomena of the polarised rings, and entitled to rank with the fits of easy transmission and reflection, as a general and simple physical law. In fact, if we investigate by this theory a general analytical expression of the tint developed for any position and thickness of the plate, taking this element into consideration, it will be found to include all the phenomena, as far as they can be computed,

while the *law* of dispersion remains unknown. But we may go yet farther. The nature of the formula furnishes an equation by which the actual quantity of the separation of the extreme red and violet axes may be deduced from observations of the tints of a very simple and accurate nature, being perfectly analogous in principle to the “method of coincidences,” which has of late been applied with such success to the most delicate investigations in every department of physical science. The comparison of the results afforded by that equation, with those deduced by direct observation on homogeneous light, while it leaves nothing to desire in point of accuracy, leads to another important result, viz. that the proportionality of the minimum lengths of the periods performed by differently coloured molecules, in a doubly refracting crystal to the lengths of their fits of easy reflection and transmission, supported as it is by an induction of no ordinary extent and accuracy, is yet not universal, admitting a deviation to a very large amount. Hence must of course arise a kind of secondary deviation in the scale of tints. In crystals with two axes, however, this is masked by the much more powerful effect of the separation of the coloured axes; yet even there, is not altogether insensible in an extreme case. In the apophyllite, however, the agency of this secondary cause is placed in the fullest evidence. The application of our general formula to the anomalous tints of that body, while it proves incontestably the exact coincidence of the axes for all the coloured rays, points out at the same time a peculiarity in its action on the more refrangible extremity of the spectrum, of a nature so singular, so entirely without example in all the multitude of

natural and artificial bodies hitherto examined, as to render me extremely desirous of prosecuting the research, with the aid of more perfect specimens, and improved methods of observation.

Having arrived at the general result of a dispersion of the axes by the sole consideration of the gradation of tints in plates of various thicknesses, it becomes interesting to verify it by direct and independent observation. This I have accordingly done; and the fortunate discovery of a substance in which it is of enormous magnitude, puts it in our power to render the fact sensible to the eye of the most unpractised observer, by an exceedingly simple experiment, to be described in its place.

II. *Of the general phenomena of crystals which develop tints deviating from NEWTON'S scale, by exposure to polarised light.*

In describing the phenomena, I shall at present confine myself to the tints developed along the principal section of the crystal, which is supposed placed in an azimuth 45° with the plane of primitive polarisation. The observations of the tints in this position are most easily made, and least liable to error, and we shall see presently that it would be superfluous as well as embarrassing to examine other situations, the law of the phenomena being completely deducible from this. In this series of observations, then, we traverse the polarised rings (Pl. V. fig. 1.) in the direction of their axis of symmetry AA', passing through their poles P, P' and centre O. Now if we subject to this examination any one of the following substances :

Sulphate of soda ?	Arragonite,
Sulphate of baryta,	Sugar,
Nitrate of potash,	Hyposulphite of strontia,

it will be seen that the tints between the poles PP' correspond to lower orders of colour, than would result from assuming P, P' , as the origin of the scale, and agree much better with the assumption of certain points p, p' without the poles, as their zero, or commencement of the scale. The poles themselves, too, instead of being absolutely black, are tinged with colour; and the tints beyond them, instead of descending in the scale from the poles outwards, continue to rise till they reach their maximum (which is a white, more or less brilliant, or an absolute black) at the points p, p' ; after which they descend again to infinity. Not that in any case they coincide precisely with the scale of NEWTON, even with this correction, but, except in extreme cases, approximate to it within some moderate limit of error.

If, on the other hand, we examine in the same manner one of the following bodies:

Tartrate of soda and potash,	Sulphate of magnesia,
Borax,	Topaz,
Mica,	

it will be found that the imaginary points p, p' (which we shall call the *virtual poles*) from which the tints must be reckoned inwards and outwards, to produce the nearest possible agreement with NEWTON's scale, lie between the poles P, P' .

In all these crystals, as the thickness of the plate examined increases, the virtual poles pp' recede from the actual ones PP' , at least in respect of the number of alternations of colour which intervene between them: in other words, the tint deve-

loped in the poles, or along the apparent axes of the crystal, descends in the scale of colour, as the thickness of the plate increases, and *vice versa*. In very small thicknesses, the tints approximate pretty closely to NEWTON'S scale, or wholly coincide with it, while in very great ones, the tint developed in the poles is the composite white of the extremity of the scale. The angular distance, however, of the virtual poles from each other and from the axes, remains absolutely unchanged for all thicknesses; and this striking fact, which I have proved by numerous and satisfactory experiments, was first suggested for examination as a result of theory, and would equally hold good, as will presently be proved, for every conceivable law of double refraction.

The substances which I have examined most attentively, are sulphate of baryta, nitre, mica, and Rochelle salt, and the subjoined tables of tints developed for different inclinations in plates of the first and last of these, may serve as examples of the mode of action of the respective classes to which they belong on light, and will afford data for some calculations to follow. The first two columns contain the inclinations corresponding to similar tints of the incident ray on the moveable plate which carries the crystal, in the general apparatus imagined by M. BIOT, for observations of this kind. Were the plate cut in a direction precisely perpendicular to the optic axis, (or line bisecting the angle between those of double refraction) and adjusted with perfect accuracy on the instrument, the excesses or defects of these angles above or below 90° would represent the angles of incidence. Neither of these conditions were, of course, exactly fulfilled. But it is obvious that the small errors in these particulars (which

were ascertained not to exceed 1° or 2°), must affect the computed angles of refraction on both sides of the perpendicular with equal and opposite errors. The same may be said of any error arising from a slight prismaticity of the plate, which, however, must have been extremely small, the plate having always been rendered parallel by the delicate test of the sphærometer, within a very few divisions.* Consequently, in calculating on these data, the mean angle of refraction determined by the simultaneous use of both observations, (their semi-difference being taken for the angle of incidence) may be expected to differ from the truth by an extremely minute quantity. The third column contains the tint developed in the ordinary pencil, and the fourth in the extraordinary. The last notices the remarkable points in the system of rings to which the tints and angles in the other columns correspond. The positions of the poles were determined by interposing a red glass between the crystal examined and the reflector used to polarise the incident light. The glass used for this purpose, was of that kind occasionally found in old church windows, and whose manufacture seems to be numbered among arts now forgotten. It transmits almost the whole of the red rays, and part of the orange, while it completely stops all the more refrangible colours. I have endeavoured in vain to procure a specimen, whose limits of transmission are more confined. Such are said to exist, though very rare, and in the absence of such, the indications of that employed may be taken to correspond to the mean red rays.

* Each equal to the 23809th part of an inch.

TABLE I. *Sulphate of baryta.* Thickness of plate = 0.11964^{in.}

Corresponding inclinations.		Ordinary pencil.	Extraordinary.	Remarks, &c.
43	55	134 37	Pink	Tints beyond the poles.
45	0	133 42	Bluish green	
46	0	132 45	Rich pink	
		132 0	Whitish	
46	58	131 37	Splendid green	
		131 0	Blue	
47	53	130 45	Crimson	
			Yellow inclining to orange	
			Yellow	
48	51	129 30	Blue green	
			Blue	Virtual poles pp', or points of coincidence.
		128 58	Purple	
			Rich crimson red	
			Fine orange	
			Pale orange yellow	
50	58	127 45	White	
			Bluish white	
			Light blue	
			Sombre greenish blue	
52	0	126 50	Dirty and very sombre purple	
			Sombre and narrow pinkish red	The poles P, P' Tints between the axes.
			Very pale violet or pinkish white	
			White	
53	1	125 47	White	
		125 30	White slightly yellowish	
		125 0	Pale and dirty olive green	
			Very narrow violet	
54	8	124 30	Blue very sombre and narrow	
			Light blue	
			Bluish or greenish white	
55	25	123 30	Yellowish white	
			Light yellow	
			Dull orange pink	
56	33	122 15	Sombre purple	
			Blue	
			Pale green	
57	30		Pale yellow	
57	50	121 0	Light yellowish pink	
58	15	120 15	Rich pink	
			Pale purple	
59	12	119 35	Blue green	
			White	
60	45	118 20	Pink	
		117 40	Pale purple	
62	0	117 5	Greenish blue	
62	58	116 25	White	
63	25	115 35	Pink	
64	28	114 47	Dull purple	
65	40	113 20	Bluish green	
67	30	111 30	Pink	
69	15	110 0	Pale greenish blue	
			Pale pink	

In this plate the virtual poles correspond very nearly to the second minimum of the extraordinary pencil beyond the poles PP'. The same plate was now reduced by grinding to the thickness 0.08816 inches. In this operation, care was taken to grind away the side of the plate most distant from the eye only, leaving the other perfectly untouched and unimpaired in its polish. The plate being reduced to exact parallelism by the sphærometer, was again examined, the same side still remaining next the eye. By this arrangement the same angles of emergence from the posterior surface correspond rigorously to the same directions of the ray in the interior of the crystal, with respect to the axes of its molecules; and thus we avoid completely any errors which might arise from using plates cut at different angles, it being almost impossible to cut two plates precisely alike in this respect.

TABLE II. *Sulphate of baryta.* Thickness = $0.08816^{\text{in.}}$

Corresponding inclinations.				Ordinary pencil.	Extraordinary.	Remarks, &c.
58	0	120	0	Pink, verging to orange yellow .	Blue somewhat greenish .	Poles for the mean red rays.
				Yellow	Dark blue	
57	30	120	47	Pale yellow	Purple	
				Greenish white	Pink	Tints beyond the poles.
				Light blue	Yellow verging to orange	
				Dark blue	Bright yellow	
55	57	122	15	Sombre purple	Yellow white	Virtual poles.
				Very indifferent sombre pink .	Bluish white	
				Pale yellow	Dark indigo blue	
54	5	124	0	White	Sombre violet	
				Very pale violet white	Dusky greenish yellow	
52	57	125	33	Very sombre violet, almost black	Pure brilliant white	
				Sombre and dirty olive green .	White rather ruddy	
				Very pale blue	Orange white	
				White	Sombre orange or brick red	
51	30	127	0	White	Narrow purple	
				Orange white	Blue	
				Bright orange	Pale blue	
				Bright scarlet	Bluish white	
				Narrow crimson	White	
50	0	128	30	Purple	White	
				Blue	Yellow	
				Bluish white	Rich crimson	
48	40	130	0	Yellowish or greenish white .	Purple	
				Orange yellow	Bright blue	
47	35	131	10	Rich crimson	Green	
47	10	131	28	Purple	Good yellow	
				Bright blue	Pink yellow	
46	7	132	33	Good green	Rich pink or crimson	
45	3	133	45	Rich pink	Splendid green	
44	33	134	30	Greenish purple	Pinkish white	
43	35	135	15	Good green, but pale	Fine pink	
42	23	136	30	Pink red	Greenish blue	
40	55	138	0	Pale bluish green	Pale pink	
39	30	139	50	Pale pink	Very pale greenish blue	

In this plate the virtual poles correspond to the second maximum of the extraordinary pencil. It is needless to detail the tints between the poles. The same plate once more reduced, with the same precaution to leave the posterior surface untouched, developed the following series of colours beyond the poles.

TABLE III. *Sulphate of baryta.* Thickness = $0.05758^{\text{in.}}$

Corresponding inclinations.		Ordinary pencil.	Extraordinary.	Remarks, &c.
60° 57'	122° 50'	Fine yellow	Indigo	Poles, for the mean red rays. Tints beyond the poles.
60° 20'	123° 27'	Pale yellow	Purple	
		White inclining to yellow . .	Dull crimson red . .	
		Bluish white	Dull orange	
		Indigo	Yellow	
57° 55'	126° 3'	Sombre purple	White	Virtual poles.
57° 50'	126° 7'	Sombre reddish violet	White	
57° 20'	126° 33'	Dirty violet yellow	White	
	127° 30'	Pale yellow	Violet white	
55° 40'	128° 2'	White	Sombre violet	
55° 33'	128° 20'	Pure brilliant white	Black	
	128° 30'	White	Sombre dirty green	
		Pale orange	Pale dirty bluish green	
53° 40'	130° 10'	Sombre orange or brick red . .	White	
53° 27'	130° 30'	Sombre and narrow purple . .	White	
		Blue	Ruddy white	
		Pale blue	Orange	
		Bluish white	Orange red	
	132° 20'	White	Narrow crimson	
51° 3'	132° 40'	Pale yellow, &c. &c. . . .	Purple, &c. &c. . . .	

Here the virtual poles p, p' correspond precisely to the first minimum of the extraordinary pencil.

In a plate of Rochelle salt, cut nearly, but not quite perpendicular to the optic axis, and whose thickness was 0.194425 inches, the rings beyond the poles were almost entirely obliterated, while those between them exhibited the following singular succession of colours, which will show to what an extent the deviation from NEWTON'S scale is carried in this substance.

TABLE IV. *Rochelle salt, perpendicular to the optic axis. Thickness* $\frac{\text{in.}}{=0.194425}$.

Corresponding inclinations.				Ordinary pencil.	Extraordinary.	Remarks, &c.
201	30	330	10	White	White	Poles PP' for red rays.
207	10			Exceeding pale blue	Exceeding pale pink	
209	0			Exceeding pale pink	Exceeding pale blue	
210	30			Very pale blue	Very pale pink	
212	30			Very pale pink	Very pale blue	
213	20			Very pale blue	Very pale pink	
215	25			Pale pink	Pale blue green	
216	40			Pale blue	Pale yellow pink	
218	0			Pink	Blue green	
219	29			Pale blue	Pale yellow pink	
220	58			Pink	Greenish blue	
				White	Very pale purple	
222	30			Blue	Yellow pink	
223	45	309	30	Yellowish pink	Greenish blue	
224	50	308	15	Pale greenish blue	Yellowish pink	
				Blue	Pale pinkish yellow	
226	10	307	0	Pale pink	Pale greenish blue	
		306	10	Pale yellow	Dark blue	
				White	Pale purple	
227	25			Bluish or greenish white	Very pale violet pink	
228	30			Blue	Very pale yellow	Virtual poles p, p'.
				Violet almost imperceptible	White	
229	3	304	10	Pure white	Pure white	
				Exceeding pale yellow	A little violet	
229	50			Pale yellow	Very narrow dark blue	
				Very pale tawny orange	Blue, sombre and pale	
231	20	302	20	Fine purplish crimson	Pale yellow green	
				Very pale purple	White	
232	25			Very pale green	Fine crimson	
				White	Pale purple	
234	0	299	20	Splendid crimson	Very pale green	
				Pale purple	White	
235	40			Very pale green	Rich crimson	
				White	Pale purple	
236	23	296	15	Rich crimson	Very pale green	
				Pale purple	White	
238	50	294	35	Pale blue green	Good pink, almost crimson	
				White	Very pale purple	
240	12	292	55	Pink rather pale	Pale greenish blue	
				White	White	
242	10	291	30	Pale blue	Pale yellowish pink	
243	40	289	50	Pale pink yellow	Pale blue	
245	10	287	45	Very pale blue	Very pale yellow	
246	50	286	45	Very pale yellow green	Very pale lilac blue	
248	25	284	55	Pale lilac	Yellow green	
				White	White	
250	40	282	30	Fine yellow green	Fine lilac	
				White	White	
252	57	280	12	Lilac	Fine yellow green	
				White	White	
256	5	277	25	Fine yellow green	Fine lilac	
				White	White	
259	20	274	40	Pale lilac	Fine yellow green	
261	40	271	40	White	White	
266	40	266	40	Green yellow	Pale lilac	The middle tint

In order however to avoid the effect of the dispersive power, which at such considerable obliquities would render the observations liable to some uncertainty, I cut another plate, in such a manner that the perpendicular to its surface, instead of coinciding nearly with the optic axis, was directed very nearly to one of the virtual poles. Its thickness was then gradually reduced in the manner above described for sulphate of baryta, though, owing to the nature of the body, it was found impossible to avoid the necessity of re-polishing the posterior surface at each operation; but as this was done with all possible care, only a very slight error can have arisen from this cause.

TABLE V. *Rochelle salt.* Thickness = 0.11518 in.

Inclinations.	Ordinary pencil.	Extraordinary.	Remarks, &c.
277 8	Very pale pink	Very pale bluish green . . .	Pole P for mean red rays. Perpendicular incidence.
261 8	
261 0	White tinged with orange .	Very fine intense indigo	
260 25	Yellowish or greenish white .	Purple rather sombre	The virtual pole p.
259 45	Very pale greenish blue .	Indifferent purplish pink	
	Dull blue	Yellowish pink white	
258 50	Fine deep indigo	White inclining to orange	
258 30	Violet purple	Yellowish white	
257 35	White, a little tinged with violet .	White, not very brilliant .	
	Yellowish white	Pale violet blue	
256 30	Pale yellow	{ Sombre indigo inclining to violet, narrow & well defined.	
	Pale pink yellow	Sombre violet white	
255 10	Pinkish purple	Very pale greenish yellow	
254 30	Rich sombre purple somewhat fiery .	White tinged with greenish yellow	
	Pale green	Fine crimson [low	
252 30	Extremely pale green . . .	The richest deep damask crimson	
	White	Livid imperfect purple [son	
250 27	The richest damask crimson .	Fine pale green	
	Livid imperfect purple . . .	White	
248 5	Pale bluish green	Fine rich crimson	
245 45	Pink approaching to red . . .	Pale blue green	
243 30	Sky blue	Light pink strongly inclining to	
240 25	Pink orange	Pale greenish blue [orange red	
	Sky blue inclining to lilac .	Fine yellow	
237 30	Pale bluish green	Fine pink a little purple	
237 0	Splendid yellow green . . .	Lilac	
234 20	Rich lilac	Splendid green	
230 10	Splendid green	Rich lilac	
	White	White	
225 45	Lilac blue	Splendid yellow green	
	White	White	
220 40	Splendid green yellow . . .	Pale lilac blue	
	White	White	
211 40	Pale lilac	Pale greenish yellow	
197 30	Fine yellow green	Fine lilac	

Here the virtual pole was coincident with the 5th maximum (or thereabouts) of the extraordinary ray from the pole P; the succession of tints, however, unless close to the virtual pole, is omitted, in order to shorten the table.

TABLE VI. *Rochelle salt.* Thickness = $0.08557^{\text{in.}}$.

Inclinations.	Ordinary pencil.	Extraordinary.	Remarks, &c.
262 0	Good light pink . . .	Light blue green . . .	Pole P for mean red. Perpendicular incidence.
278 0	
278 20	Very pale yellow green . . .	Bluish purple	
278 55	Very pale yellow green . . .	Violet	Virtual pole.
280 0	Very pale bluish green . . .	Very light pink, or pinkish white	
	Indigo . . .	Very pale yellow, almost white	
	Pale violet . . .	Very pale yellow	
281 30	White . . .	White, perfectly equal and alike	
	Very pale greenish yellow . . .	Pale lilac	
282 35	Pale greenish yellow . . .	Sombre lilac purple	
	Pale pinkish yellow . . .	Dull and impure blue	
	Pale pink . . .	Pale yellow green	
285 15	Crimson . . .	Yellow green	
285 45	Rich fiery damask crimson . . .	Very pale yellow green	
	Livid imperfect purple . . .	Very pale pink yellow	
287 50	Fine light green . . .	Fine pink	
288 45	Very pale green . . .	Splendid crimson	
	Good crimson . . .	Blue green	
291 40	Crimson, almost scarlet, &c. . .	Pale blue green, &c.	

In this plate the virtual pole fell about half way between the 4th maximum and the 5th minimum of the extraordinary ray from the apparent pole P for the mean red rays. When once more ground down, it gave as follows:

TABLE VII. *Rochelle salt.* Thickness = $0.05437^{\text{in.}}$.

Inclina- tions.	Ordinary pencil.	Extraordinary.	Remarks, &c.
262 25	Fine pink	Fine light blue green	Pole P for mean red rays.
	Indifferent purple	Yellowish white	
277 20	Indifferent lilac pink	Very pale greenish yellow	
278 25	Pale yellow inclining to orange	Blue rather pale	Perpendicular incidence.
	Fine pale yellow	Beautiful sombre indigo	
280 50	Yellowish white or pale yellow	Violet	
282 0	White	White with an almost imper- ceptible tinge between yellow	} Virtual pole
	Very pale blue	Yellow white [and violet]	
284 50	Sombre indigo	Pale yellow white	
	Very pale blue	Extremely pale pink white	
	Yellow green	Lilac pink	
	Pale yellow green	Deep lilac pink	
288 25	Greenish white	Rich but sombre purplish crim- son	
	White	Dull purple [son	
	Very pale pink	Good blue green	
293 33	Deep fiery crimson	White	
	Very dull purple (greenish)	White	
	Blue	Pink yellow	[&c.
294 50	Very pale blue, &c.	Rich orange, bordering on red,	

III. *On the causes of these phenomena.*

The developement of colour along the axis of double refraction, is at first sight analogous to the production of the secondary tints along the axis of rock crystal, discovered by M. ARAGO, and recently explained by M. BIOT, in a masterly memoir communicated to the Academy of Sciences, on the hypothesis of a force inherent in its molecules independent of their state of aggregation, by which they communicate a rotation in an invariable direction to the axes of polarisation of the luminous rays. And this analogy is partially supported by the fact, that the tint developed along the axis, descends in the scale of colour as the thickness increases.

A more scrupulous examination however will show, that its origin must not be sought in any cause of this nature, for (not to mention the impossibility of explaining the phenomena of the virtual poles by this hypothesis) if we place the principal section of the crystal in the azimuth zero, the extraordinary image will be found to vanish completely for every angle of incidence, and whatever be the thickness of the plate. I may add too, that I have in my possession a crystal of quartz, which exhibits with tolerable distinctness in some parts the phenomena of two axes, and the appearances produced by the interference of the secondary tints in this specimen, while they agree completely with M. BIOT's explanation, differ entirely from those which form the subject of this Paper.

Neither are the phenomena above described explicable on any supposition of a peculiar action of the crystal on the differently coloured rays, analogous to its ordinary or extraordinary dispersive power, by which the periods of alternate polarisation of the molecules of some colours, should be lengthened, and of others contracted, so as to disturb that exact proportionality to their periods of easy reflection and transmission, which M. BIOT has proved to be a necessary condition for the production of the tints of NEWTON's scale. It is true, such laws of action may be imagined, and I shall presently show must really exist; in all crystals probably to a small extent, but in two instances at least, to a surprising degree. But this alone will avail us nothing. To show this, and at the same time obtain a general analytical expression for the tint developed at any inclination, and for every hypothesis of the action of the crystal on the differently coloured molecules, let us denote by c , the length of a complete period of easy

transmission and reflection, or the extent of one pulse, on the undulatory hypothesis, in vacuo, and at a perpendicular incidence for any homogeneous ray, and let C denote its colour and proportional intensity or illuminating power in the prismatic spectrum. Then will the formula representing a beam of white light intromitted into the crystal, be

$$C + C' + C'' + \&c.$$

from one end of the spectrum to the other.

Let n be the number of periods (each consisting of a double alternation) and parts of a period performed by the elementary pencil C , in its passage through the medium: then, according to the theory of M. BIOT, when n is 0, 1, 2, 3, &c. ad inf. the pencil will wholly pass into the ordinary image; but when the values of n are $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, &c. it will wholly* be thrown into the extraordinary one, and in the intermediate states of n , partly into one and partly into the other. These conditions are satisfied if we represent by $\sin.^2 (n \pi)$ the intensity of the ray in the ordinary image, taking unity for its original intensity; and it will I believe be found, that the gradation of intensity given by this formula for the inter-

* The amplitude or total extent of each oscillation of the plane of polarisation is here supposed 90° , in which case the contrast of colour in the two pencils is at its maximum. This is the case in the situation we are considering, but in general the intensity of the extraordinary ray, instead of being represented by $\sin.^2 n \pi$, will have an additional factor, a function of the azimuth A of the principal section of the crystallized plate and the position of the refracted ray, and which becomes unity when $A=45^\circ$, and the plane of incidence is that of the principal section. It is on this factor that the gradation of *brightness* in the isochromatic lines, and the black cross or hyperbolic branches which intersect them, depend. But it is not my intention at present to enter on this part of the subject for reasons to be explained farther on. See note in page 84.

mediate values of n , will agree sufficiently with the judgment of the eye to warrant its adoption.* The part of the elementary pencil C then, which enters into the extraordinary image, will be $C. \sin^2 (n \pi)$. Let us denote by $S \{ C. \sin^2 (n \pi) \}$ the aggregate of all such elements from one extremity of the spectrum to the other, or take

$$S \{ C. \sin^2 (n \pi) \} = C. \sin^2 (n \pi) + C'. \sin^2 (n' \pi) + \&c.$$

Then will this expression represent the tint developed in the extraordinary image, and consequently, $S \{ C. \cos^2 (n \pi) \}$ that in the ordinary one.

Now, n , the number of periods performed depends, 1st. on the nature of the ray, or on c ; 2dly, on the intrinsic energy of the action of the medium on that ray; and 3dly, on the direction of its course, the thickness of the plate, and whatever other cause or limit of periodicity may happen to prevail. Hence we may take $n = M \times k$, k being a function of c , dependent only on the nature of the body through which the ray C passes, and M being a certain multiplier whose form we shall consider presently. This substitution made, the expression for the tint becomes $S \{ C. \sin^2 (M k. \pi) \}$

In the theory of the Newtonian colours of thin plates and the polarised rings in crystals with one axis, the multiplier M is independent on c , varying only with the direction of the ray and the thickness of the plate. It is therefore the same for all the coloured rays, and the tint, for any value of M , will be

* No part of our subsequent reasoning depends on the *form* of this function. It is sufficient to know that it must be a periodical, and even function of n . It is only in the computation of numerical values that it is necessary to make any more precise assumption.

$$C. \sin^2 (M k \pi) + C'. \sin^2 (M k' \pi) + \&c. \quad (a)$$

Now, suppose M to begin from zero, and to pass, by a variation either in the direction of the ray or thickness of the medium, or both, through all gradations of value, to infinity, or to its maximum, if not susceptible of infinite increase : then we see that for every value of M a certain peculiar tint will arise, and that, provided M commence at zero and continue increasing, the same succession of tints will invariably be developed in the same order. Consequently, if we fix upon any two tints in this scale of colour, or any two values of M , the same succession and the same number of alternations of colour must invariably intervene between them, however we pass from one to the other.

In a crystal with two or more axes, the value of M for any ray C must of course be zero in the direction of the axis, and therefore if the same supposition of the independence of M on c be made, the same conclusions should follow ; namely, 1st. that the extraordinary ray must always vanish in the pole, whatever be the thickness of the plate ; and 2dly, that the same succession and number of alternations of colour should intervene between the pole and any assigned unequivocal tint, such as black, or the pure brilliant green of the 3d order of NEWTON'S scale. Both these conclusions are totally at variance with the facts above detailed, as to the developement of colour in the poles, and the situation in the order of the rings of what we have called the virtual poles. Hence we are necessitated either to give up the theory of alternate polarisation altogether, or to admit the dependence of the multiplier M on c , or on the nature of the ray. Let us see to what this will lead us.

According to the theory of the polarised rings if extended to crystals with two axes, the number of periods performed in a given space ($= 1$) by a molecule of a given colour, transmitted in a direction making angles θ, θ' with the axes, can only be a function of the form $k \cdot \psi(\theta, \theta')$, k depending on the intensity of the polarising force; or as before, being a function of c , the nature of the ray, and of the intrinsic energy of the molecules of the crystal. Now if we call t the thickness of the plate, and ϕ the angle of refraction, $\frac{t}{\cos \phi}$ is the length of the path described, and therefore we must have for the number of periods

$$n = \frac{k t}{\cos \phi} \cdot \psi(\theta, \theta') ;$$

so that the value of M must be $\frac{t \cdot \psi(\theta, \theta')}{\cos \phi}$, which must be a function of c . Now t is obviously independent of it; and if we neglect at present the very trifling effect at moderate incidences of the ordinary dispersive powers of the media examined,* ϕ is so also. It is therefore in the form of the function $\psi(\theta, \theta')$ that we must look for the cause of the phenomena; and since, we have $\theta' = \theta + 2a$, $2a$ being the angle between the axes (because the observations are made in the principal section) we see that $\psi(\theta, \theta + 2a)$ must involve c , and consequently, θ being arbitrary and independent, a must be a function of c . In order then to render the theory of alternations applicable, we must admit the angle between the

* It is easy to see that in the two classes of crystals above described, the effects of the dispersive powers will be opposite to each other, in one opposing, and in the other conspiring with the causes which produce the deviation of tints. In the tables, Nos. V, VI, VII, where the virtual poles were observed almost at a perpendicular incidence, the influence of the dispersive power is quite insensible.

axes of double refraction to differ in the same crystal for the differently coloured rays. We must now show that this supposition is sufficient to represent the phenomena correctly.

The symmetry of the rings and total evanescence of colour in the principal section at an azimuth zero, requires that the axes of all the different colours shall be symmetrically arranged, on either side of a fixed line (which may be called the optic axis) in this plane, or in one perpendicular to it. At present we need only consider the former case. Let a represent the angular distance of the axis for any one standard species of ray C (the extreme red, for instance) from this line, $a + \delta a$, the same distance for any other ray. Then the distance of the transmitted ray C, from the axes of rays of that colour being θ, θ' , the corresponding distances from *their* respective axes for rays of any other colour C' emerging in the same direction will be $\theta - \delta a + \delta \phi$ and $\theta' + \delta a + \delta \phi$, $\delta \phi$ being the difference ($= \phi' - \phi$) of the angles of refraction, corresponding to the same incidence, for the colours C, C'. The positive values of θ here reckon outwards from the pole; δa is negative for crystals of the second class, and $\delta \phi$ is negative or positive according as C or C' is the less refrangible colour.

Let us for a moment consider rays of only these two colours. The portion of the extraordinary pencil due to them will be

$$C. \sin^2 \left(\frac{k t}{\cos \phi} \psi(\theta, \theta'). \pi \right) + C'. \sin^2 \left(\frac{k' t}{\cos \phi'} \psi(\theta - \delta a + \delta \phi, \theta' + \delta a + \delta \phi). \pi \right).$$

The rays of these colours of the same order in their respective series of rings will therefore coincide, and that in the proper

degree of proportional intensity for the production of a white image, provided we suppose

$$\frac{k}{\cos \phi} \cdot \psi(\theta, \theta') = \frac{k'}{\cos \phi'} \cdot \psi(\theta - \delta a + \delta \phi, \theta' + \delta a + \delta \phi); \quad (b)$$

which, since $k, k', a, \delta a$, are constant elements, ϕ, ϕ' determinate functions of θ , and $\theta' = \theta + 2a$, suffices to determine θ .

If we suppose C and C' to represent the extreme red and violet rays, it is evident that the coincidence of the extraordinary pencils of the same order for these two extremes, will ensure that of the intermediate ones, at least very nearly. It would do so precisely, were the value of δa for any intermediate ray, such a function of k as would result from making θ constant in the preceding equation, because the two laws, that of the dispersion of the axes, and that of the magnitude of the rings of different colours, would then act in exact opposition to each other throughout their whole extent. It is, in fact, a case precisely analogous to that of the compound achromatic prism, where if the law of dispersion in the one medium were identical with that in the other, a perfectly colourless pencil would emerge, and when these laws differ, the coincidence of the red and violet rays ensures an approximate coincidence of all the rest. Should these laws however differ very considerably, an uncorrected colour will appear at the point so determined, and a nearer approximation will be obtained by uniting two of the more powerful intermediate rays, such as for instance the mean red and the blue, or limit of the green and blue.

This then is the origin of the virtual poles or points beyond or between the axes where the tint rises to a white of the first order, more or less feeble, or even to an absolute black; and we may now see the reason why the tints in

reckoning from these points approximate in a general way to the Newtonian scale. In fact, the periods of the more refrangible rays being performed more rapidly than those of the less, if we suppose the coincidence above spoken of to take place at any point (the minimum for instance), of the n^{th} ring, the intervals between the n^{th} and $(n \pm 1)^{\text{th}}$ minimum will be greatest for the red and least for the violet, &c. Consequently, when the violet next disappears totally from the extraordinary pencil, there will remain yet a little of the red, less of the orange, and so on, and this difference increasing at every succeeding minimum on either side, will produce a succession of colours approximating in a general way to NEWTON'S scale. This approximation will however be much less close on the side of the virtual pole towards the nearest axis, because the disturbing influence of the separation of the axes on the figure of the rings and the law of their successive intervals, is much more sensible than at a distance from the pole. This will be evident if we consider that in the interval between the extreme coloured axes, the tints will be regulated entirely by the law of their distribution. Now this is perfectly corroborated by the succession of tints in the foregoing tables, as well as by numerous experiments made on other bodies.

Our equation (b) gives room for a remark of some consequence, as it affords a striking verification of the theory here delivered. It will be observed that this equation does not involve t , and in consequence, the angle θ determined from it, at which the coincidence takes place, is the same for all values of t , or for all thicknesses of the plate. The observations of the tints in the tables given above, afford us ample

means of putting this remarkable consequence to the test of experiment. In fact, in the three series of tints observed in sulphate of baryta, the apparent angles between the axes for the mean red rays are respectively $62^{\circ} 0'$, $62^{\circ} 2'$, and $61^{\circ} 53'$, the mean of which is $61^{\circ} 58'$, while the apparent angles between the virtual poles in the same series are $72^{\circ} 46'$, $72^{\circ} 36'$, and $72^{\circ} 47'$. The semi-excesses of the latter angles over the mean value of the former, are the apparent angular distances of the virtual poles from the axes of mean red rays, and are respectively $5^{\circ} 24'$, $5^{\circ} 19'$, $5^{\circ} 25'$, neither of which differs more than $4'$ from the mean.

To ascertain the real value of θ , we have only to compute the angles of refraction. In the specimen employed, I found 1.6475 for the index of ordinary refraction, and the angles of incidence (the halves of the above angles) being $30^{\circ} 59'$ and $36^{\circ} 23'$, $36^{\circ} 18'$, $36^{\circ} 24'$; the corresponding angles of refraction are $18^{\circ} 12' 30'' (= a)$, $21^{\circ} 6' 10''$, $21^{\circ} 3' 40''$, $21^{\circ} 6' 40''$; and since $\theta = \phi - a$ in this case, we find for the values of θ , $2^{\circ} 53' 40''$, $2^{\circ} 51' 10''$, $2^{\circ} 54' 10''$, the mean of which gives $2^{\circ} 53' 0''$ for the real angular distance of the virtual pole from the axis of mean red rays in sulphate of baryta.

Again, in the series of tints, tables V, VI, VII, for Rochelle salt, the apparent angular distances of the mean red axis from the virtual pole were

$$277^{\circ} 8' - 257^{\circ} 35' = 19^{\circ} 33'; \quad 281^{\circ} 30' - 262^{\circ} 0' = 19^{\circ} 30', \\ \text{and } 282^{\circ} 0' - 262^{\circ} 25' = 19^{\circ} 35'$$

of which neither differs more than $3'$ from the mean $19^{\circ} 33'$.

Dr. BREWSTER (Phil. Trans. 1814, p. 216) has stated the refractive index of this salt at 1.515 ; but this is certainly a

little too large. In four experiments made at distant intervals of time, and by different modes of observation, I have found

1.49640

1.50220

1.49670

1.49853

for the index for the mean yellow-green rays, of which determinations the last is to be preferred, having been made with great care. The same experiment gave 1.49293 for the index for mean red rays. The apparent angular distance of the axis for red rays from the perpendicular was 16° , which leaves $3^\circ 33'$ for the angular distance of the virtual pole from the perpendicular. These angles of incidence correspond to the respective angles of refraction $10^\circ 38' 20''$ and $2^\circ 22' 40''$, of which the sum $13^\circ 1'$ is the real angle between the virtual pole and mean red axis in Rochelle salt. The series in table IV, gives $13^\circ 2' 25''$ for the value of θ , which agrees completely with the foregoing determination.

I took seven plates of nitre of various thicknesses, and cut from different crystals, and by a mode of observation to be described hereafter, found as follows :

TABLE VIII.

Designation of the plate.	Mean distance of the virtual poles from the axis of mean yellow rays (apparent.)	Excess above the mean.	Order of the coincidence.
1	$1^\circ 50'$	$-4'$	Between the 4th minimum and 5th maximum.
2	$1^\circ 51'$	$-3'$	A little beyond the 3d maximum.
3	$1^\circ 57'$	$+3'$	Different at the two extremities.
4	$1^\circ 53'$	$0'$	2d maximum.
5	$1^\circ 55'$	$+2'$	2d minimum.
6	$1^\circ 59'$	$+5'$	9th minimum.
7	$1^\circ 53'$	$0'$	A little beyond the 8th minimum.
Mean	$1^\circ 53'$		

Although the constancy of the position of the virtual pole for different thicknesses is sufficiently made out here, the small differences which exist are certainly not attributable to errors of observation, which, in the method I employed, are usually confined within much narrower limits. They are due to minute irregularities in the crystals themselves, consisting, probably, in a state of imperfect equilibrium of the molecular forces of aggregation, to which this salt is so subject, that it is rather rare to find a specimen in which the rings beyond both poles have exactly the same breadth or tints.

ART. IV. *Of the tints developed by crystals with two axes out of the principal section.*

If we place a crystallized plate at an azimuth zero in a tourmaline apparatus, having the axes of the tourmalines at right angles, we shall observe, if its thickness be at all considerable, that the two oval spots on either side the axis of symmetry (which is now perfectly black) instead of being exactly regular in their figure, as in Fig. 2. Pl. V, and tinged with colours symmetrically disposed on either side of a line mn perpendicular to the principal section, are invariably coloured at one extremity r with a strong prismatic red hue, and drawn out at the other v into more or less elongated and tapering spectra or tails of blue and violet light. The extremities r, r of the rings too have a large excess of the red rays, and the opposite v, v of the violet rays. In crystals of the first class above described, the red extremity is turned towards the other pole, while in those of the second it is directed from it. If we subject a plate of Rochelle salt to

this examination, the ovals a, a , are drawn out to a surprising length, and the whole prismatic spectrum is displayed in them with great vividness of colour, while the violet portions of the rings are greatly elongated also, and appear to run into one another. If the plate be turned round in azimuth, the phenomena assume the most singular appearances of distortion; and as the rotation approaches to 45° , the rings in the vicinity of the pole are gradually obliterated by their mutual overlapping, which is the greater the thicker the plate. In all situations, however, the interposition of a red or dark green glass immediately restores the perfect symmetry and regularity of the rings, which are then seen in much greater number, and completely well defined.

All this is the necessary consequence of the want of coincidence of the axes for different colours. The lateral spots, for example, are formed for each homogeneous colour with perfect regularity close to their corresponding pole, and regularly decreasing in size from the red to the violet. Their arrangement will therefore be as represented in Fig. 3, Pl. V, R, O, Y, &c. being the poles corresponding to the several colours red, orange, &c. The oval spots composed of red rays being represented by r, r , those of the other colours will be super-imposed on them in their order, overlapping, as represented by the dotted ovals oo, yy , &c. like the circular coloured images of the sun in the spectrum of an ordinary prism, giving rise to the long prismatic tails above described. Similar considerations will apply to the anomalous appearances presented by the rings of all the other orders and in every situation.

This suggests a very simple and pleasing experiment, which affords an ocular demonstration of the adequacy of the explanation I have advanced. Let a plate of Rochelle salt be placed in a tourmaline apparatus in any azimuth (45° is the most convenient) and firmly fixed on a proper stand in a dark room. The eye being now applied, let an assistant illuminate the emiered glass or lens of short focus* which disperses the light previous to its incidence on the first tourmaline, with the several colours of the prismatic spectrum in succession, beginning with the red. The rings will then be seen formed successively of each of the colours, perfectly regular in their figure, but contracting rapidly in dimension as they become illuminated with the more refrangible rays.† At the same time the pole about which they form will be seen to move regularly in the direction of the other axis of the crystal, and if we pass alternately from a red to a violet illumination, will shift its position accordingly, backwards and forwards through a very considerable angle. If rays of two colours be thrown at once on the apparatus, their two corresponding sets of rings will be seen at once, crossing, but not obliterating one another, and the distance between their respective centres will be observed to increase with the difference of their colours.

By measuring the interval occupied by the projections of

* See the description of an apparatus of this kind, subjoined.

† See *Lectiones Opticæ*, lib. ii. Pars. i. Obs. xiii. from which the idea of this experiment is taken. “*Magnâque voluptate perfusus*” says NEWTON, with the enthusiasm of the true philosopher who loves the field he labours in, “*videbam eos dilatare aut contrahere se gradatim pro eo ac color luminis immutabatur.*” It is impossible to witness the very beautiful phenomenon described in the text without entering into the same train of feeling.

the centres of the last visible red and violet rings, as well as those of the intermediate colours, on a screen at a known distance seen with the other eye, I found the following values of the apparent and real separation of the several coloured axes :

Between	Apparent interval.	Real Interval.
Red and orange	0 37 very uncertain	0 25
Yellow	1 50 rather uncertain	1 13
Green	3 43 Do.	2 29
Blue	6 5	4 3
Indigo	8 19	5 33
Violet	9 46	6 31

As a mode of measurement this method is very inaccurate, especially in the extreme red and violet rays, both of which would be copiously, and indeed almost entirely absorbed in their passage through two plates of tourmaline of a yellowish-green colour. Much more exact and unexceptionable measures will be presently given, but these are quite sufficient to establish the *reality* of the phenomenon described.

V. *Of a secondary cause of the deviation of tints, subsisting in certain crystals, and of the anomalous tints of the apophyllite.*

To determine the dispersive power of any medium, and obtain some rough knowledge of its law, we make a prism of it act in opposition to one of a standard substance. To ascertain the total dispersion of the axes of a crystal, or the angle by which the extreme red and violet axes differ, we may make it act against itself. Since the violet rings are more elevated by refraction than the red, from the situation in which they

would appear to an eye immersed in the medium, a plate may be conceived cut in such a direction as to make their apparent centres coincide, in which case the tints immediately about the poles will coincide with NEWTON's scale, and the extraordinary image will totally disappear in the pole at an azimuth 45° . This condition gives $\theta = 0$, $\theta - \delta a + \delta \phi = 0$, whence (supposing R, R' the indices of refraction for extreme red and violet rays and $\delta R = R' - R$) we find

$$\delta a = \delta \phi = -\frac{\delta R}{R} \cdot \tan \phi$$

The angle ϕ however becomes imaginary, and this method, in consequence, inapplicable when the separation of the extreme axes (δa) is greater than the maximum dispersion of the colours of an intromitted white ray, that is, when

$$\delta a > \frac{\delta R}{R \cdot \sqrt{R^2 - 1}}$$

Let us resume our equation (b), and supposing the form of the function ψ , and the constants a, k, k', R and δR ascertained, let the angle θ , at which the coincidence takes place be observed, and the value of δa will then become known. If we suppose it small, which it is in the generality of crystals, we get

$$\delta a = \frac{\frac{k-k'}{k'} \cdot \psi - \delta \phi \cdot \left\{ \frac{k}{k'} \cdot \sin \phi \cdot \psi + \frac{d\psi}{d\theta'} + \frac{d\psi}{d\theta} \right\}}{\frac{d\psi}{d\theta'} - \frac{d\psi}{d\theta}}; \quad (c)$$

(ψ being put for $\psi(\theta, \theta')$ for the sake of brevity). At incidences nearly perpendicular, $\delta \phi$ may be neglected, and the expression reduces itself to

$$\delta a = \frac{k-k'}{k'} \cdot \frac{\psi}{\frac{d\psi}{d\theta'} - \frac{d\psi}{d\theta}} \quad (d)$$

The comparison of these formulæ with observation, which

will lead to some very remarkable consequences, requires us to know the form of the function ψ and the values of k, k' . We will begin with the former, and in this investigation the first step is to determine the general equation of the isochromatic lines. In order to this, we must separate in all cases the law of the tint from that of its intensity. The latter depends entirely on the greater or less facility with which the emergent ray finds in penetrating the prism of Iceland spar employed for its analysis, and will not enter into the present investigation. When we examine a crystallized plate in a convenient graduated apparatus, between tourmaline plates crossed at right angles, turning it slowly round between them in its own plane, the form of the coloured bands, if illuminated with homogeneous light, will remain perfectly unchanged during the rotation, but the two black hyperbolic branches passing through the poles, will obliterate in succession every part of their periphery; and the space over which the darkness extends, as well as the degree of illumination of what remains visible, varying at every instant, give rise to so great a variety of appearances, that some little attention is required to recognize this perfect identity of figure. When the tourmaline next the eye is made to revolve, the crystallized plate remaining fixed, the complicated changes which take place, are perfectly reconcilable with the superposition of the primary on its complementary set of rings, the relative intensities of the two sets at any point being regulated by laws we have no occasion to consider at present, but the figure of the isochromatic lines, where visible, remains absolutely unchanged by any rotation in this part of the apparatus.

To form a first hypothesis on the nature of the function

which determines the equation of any one of these curves, we must select a crystal, where the proximity of the axes and intensity of the polarising forces are such, as to bring the whole system of rings within a very small angular compass; as by this means we avoid almost entirely the disturbing effect of the variation in thickness, arising from obliquity of incidence. Dr. BREWSTER, in his Paper of 1818, has chosen nitre, as affording the best general view of the phenomena, and it is admirably adapted for this purpose; the whole system of rings being comprised at a very moderate thickness within a space of 10° , allowing us to regard their projection on a plane perpendicular to the optic axis as a true representation of their figure, undistorted by refraction at the surface, &c. If we examine the rings in this crystal (illuminated with homogeneous light, or by the intervention of a red glass in common day-light) it will be evident that the general form of any one of them is a re-entering symmetrical oval, which no straight line can cut in more than four points, and which, by a variation of some constant parameter, is in one state wholly concave, as 1 (Fig. 4. Plate V.) then becomes flattened, as 2; then acquires a minimum ordinate and points of contrary flexure, as 3; then a node, as 4; after which it separates into two conjugate ovals, as 5; which ultimately contract themselves into the poles P, P' as conjugate points. The general idea bears a striking resemblance to the variation in form of the curve of the fourth order, so well known to geometers under the name of the lemniscate, whose equation is

$$(x^2 + y^2 + a^2)^2 = a^2 (b^2 + 4x^2)$$

when the parameter b gradually diminishes from infinity to

zero, $2a$ being the constant distance between the poles. In order, however, to put this to a satisfactory examination, accurate measures must be taken, which, in the case of nitre, from the minuteness of the system of rings, presented at first some difficulties. These I obviated, after many fruitless trials, by a mode of observation which I have found extremely convenient and accurate, and which applies particularly well to the present purpose. It consists in projecting the rings by solar light on a screen in a darkened room, by which means they may be magnified to any required extent; examined at perfect leisure, and in all their phases, and measured or traced with a pencil with the utmost exactness and facility. They may be thus exhibited too to a number of spectators at once; a thing which may prove serviceable to the Lecturer, for which reason I have subjoined to this paper a brief description of the apparatus I employ.

Having cut a very perfect crystal of nitre at right angles to its axis of crystallization, and adjusted it properly on this apparatus, the rings were projected on a large sheet of paper, stretched, while moist, on a drawing board, by which means it assumes a truly plane surface by the contraction it undergoes while drying. The poles were then marked, and the loci of the successive maxima for red rays carefully outlined. The screen being then removed, a series of lemniscates were laid down by points, having the same poles and one common point in each, chosen where the tint was most decided. It is unnecessary to give any comparative statement of measures in the observed and constructed curves, as the points, graphically laid down, uniformly fell on the pencilled outlines, or, in the few instances to the contrary,

within limits less than the very trifling irregularities of the outlines themselves.

The graphical construction of these curves is rendered extremely easy by the elegant and well-known property of the lemniscate, in which the rectangle under two lines drawn from the foci (or poles) to any point in the periphery, is invariable throughout the whole extent of the curve. This is easily shown from its equation, and the value of this constant rectangle in any one curve is expressed by $a b$.

We must next enquire how the constant parameter b varies in passing from ring to ring. To this end I projected the rings, illuminated by red light only, on a screen as before, and having outlined the successive loci of the minima of illumination, and laid down the poles, found the values of $a b$ in the several lemniscates, as in the following table :

Order of the minimum.	Observed values of $a b$ in square inches.	Differences.	Values of $a b$ computed from formula $a b = 1.59 \times n$	Excess of computed above observed values of $a b$
$n = 0$	0.00		0.00	0.00
1	1.62	1.62	1.59	— 0.03
2	3.165	1.545	3.18	+ 0.02
3	4.69	1.525	4.77	+ 0.08
4	6.27	1.58	6.36	+ 0.09
5	7.87	1.60	7.95	+ 0.08
6	9.56	1.69	9.54	— 0.02
7	11.09	1.53	11.13	+ 0.04
8	12.77	1.68	12.72	— 0.05
9	14.33	1.56	14.31	— 0.02
10	15.93	1.60	15.90	— 0.03
		Mean 1.59		

The nature of the illumination not allowing the delineation to be performed with the same freedom and precision as in a fuller light, the values of $a b$ in the second column are the means of a great number of measures, taken in every part

of their respective curves. The numbers in the 5th column exhibit the excesses of the terms of the arithmetical progression in the 4th (whose common difference is 1.59, the mean of all the differences in the third column) above the observed values of a b , and are so small as fully to authorize the conclusion, that these values, and of course those of the parameter b , increase in arithmetical progression with the order of the rings; or in other words, that the number of periods performed in a given space ($=1$) by a luminous molecule going to form any point M in the projection of any ring, is proportional to the rectangle of the distances PM , $P'M$ of that point from the two poles.

Now, if we extend our views to crystals in which the distance between the axes is considerable, we may reasonably expect that the usual transition which takes place in analytical formulæ from the arc to its sine, when we pass from a plane to a spherical surface, will hold good. If this be the case, we shall have at once, and in all cases

$$\psi(\theta, \theta') = \sin \theta \cdot \sin \theta'$$

and the nature of the isochromatic curve for the n^{th} complete period will be expressed by the equation

$$\sin \theta \cdot \sin \theta' = \frac{n}{k t} \cdot \cos \phi = n h \cdot \cos \phi \quad (e)$$

putting h for $\frac{1}{k t}$. If the plate be cut at right angles to the optic axis

$$\cos \phi = \frac{\cos \theta + \cos \theta'}{2 \cdot \cos a}$$

and consequently

$$\sin \theta \cdot \sin \theta' = \frac{n}{2 k t \cdot \cos a} (\cos \theta + \cos \theta'); \quad (f)$$

To put this to the trial, I took a plate of mica, whose thickness measured 0.023078 inch, and having adjusted it accurately on a divided apparatus, placed it in an azimuth 45° ,

and, by the intervention of the red glass above mentioned, observed the maxima and minima of the extraordinary pencil between the poles. As these observations, when repeated, seldom agreed unless within a few minutes, ten were taken of each maximum and minimum. The angles of incidence, deduced from a mean of similar observations on each side of the perpendicular, are set down in the 2d column of the following table, each number in which is therefore a mean result of 20 observations. The 1st column contains the values of n , or the order of the ring observed; the 3d, the angles of refraction, to obtain which I used the index 1.500, employed by M. BIOT.* The 4th and 5th columns contain the values of θ, θ' thence computed, and the 6th, values of the coefficient h , deduced from the formula $h = \frac{\sin \theta \cdot \sin \theta'}{n \cdot \cos \phi}$

TABLE X. *Mica.* Thickness = 0.023078 in.

Values of n	Angles of Incidence.	Angles of Refraction.	Values of θ .	Values of θ'	Values of h .	Excesses above mean.
0	35 3 30	22 31 0	0 0 0	45 2 0		
0.5	32 55 20	21 14 40	1 16 20	43 45 40	0.032952	-0.000195
1	30 34 40	19 49 30	2 41 30	42 20 30	0.033622	+0.000475
1.5	28 15 40	18 24 0	4 7 0	40 55 0	0.033035	-0.000112
2	25 34 20	16 43 30	5 47 30	39 14 30	0.033327	+0.000180
2.5	22 46 20	14 57 15	7 33 45	37 28 15	0.033148	+0.000001
3	19 35 40	12 55 10	9 35 50	35 26 10	0.033058	-0.000089
3.5	15 48 40	10 27 50	12 3 10	32 58 50	0.033026	-0.000121
4	10 48 50	7 11 10	15 19 50	29 42 10	0.033010	-0.000137
					0.033147	

* Recherches sur les Mouvements des Molecules de la lumiere, &c. p. 482. He takes it equal to that of glass—"ce qui ne doit pas etre fort éloigné de la vérité." I have attempted, without success, to measure its value. What has satisfied M. BIOT and Dr. BREWSTER (for the latter has evidently used this index, or one very near it, Phil. Trans. 1818, p. 230) ought to satisfy every one: yet it is fortunate that in the present instance, a slight variation in the refractive index will produce but a very trifling change in the *relative* values of b .

The last column of this table exhibits the deviations in excess or defect of the values of the quantity h , so computed from the mean of all of them. Their smallness, in comparison with the quantity itself, and their alternations of sign, are evident proofs of the constancy of this coefficient, and we are therefore authorized to take $\sin \theta \times \sin \theta'$ as the general value of $\psi(\theta, \theta')$. The observations on Rochelle salt, presently to be noticed, confirm this law.* If we denote by l the minimum length of a double oscillation, or the space passed over during one complete period by a ray transmitted at right angles to both axes, we have $h = \frac{t}{l}$; and consequently $h = \frac{l}{t}$, $l = h t$. If we substitute for h and t their values above found, we obtain

$$l = 0.00076497 \text{ inch}$$

for the minimum length of a period performed by a mean red ray in mica.

* When $\theta = \theta'$, as in crystals with one axis, we have $\psi(\theta, \theta') = \sin^2 \theta$, a result long since confirmed by the accurate experiments of BREWSTER and BIOT. The velocity of the extraordinary ray in such crystals is given by the formula $v^2 = V^2 + \alpha \cdot \sin^2 \theta$. Following this analogy, we may conclude that in crystals with two axes we should have $v^2 = V^2 + \alpha \cdot \sin \theta \cdot \sin \theta'$. Now this is precisely the expression at which M. BIOT has recently arrived. This very simple and elegant result was communicated to me by that eminent philosopher in the spring of this year, and subsequently in a letter of the 2d May. His Memoir on the subject, which appears (by the *Ann. de Chim.*) to have been read to the Institute in April, I have not seen, nor do I know by what precise steps he was led to it, but presume it must have been by some considerations of the nature above described. In the foregoing investigation of the law of periodicity, I beg leave therefore to disclaim all intention of arrogating to myself any share in this beautiful discovery, but have thought it necessary to state the steps in the text, in order to demonstrate a truth essential to the investigations to follow, which could not have been taken for granted, or deduced by any legitimate reasoning, independent of experiment, from the equation $v^2 = V^2 + \alpha \cdot \sin \theta \cdot \sin \theta'$, by reason of our ignorance of the nature and mode of action of the polarising forces; and, have purposely abstained from entering any farther into the general laws of double refraction and polarisation than I could possibly avoid.

Resuming our general equations (b) and (d) if we substitute the value now determined for ψ , and write $\frac{l}{l'}$ for $\frac{k}{k'}$, we have

$$l' \cdot \cos \phi' \cdot \sin \theta \cdot \sin \theta' = l \cdot \cos \phi \cdot \sin (\theta - \delta a + \delta \phi) \cdot \sin (\theta' + \delta a + \delta \phi); \quad (g)$$

whence it is easy to derive (independent of any approximation)

$$\cos 2(a + \delta a) = \cos 2\phi' + 2 \frac{l'}{l} \cdot \frac{\cos \phi'}{\cos \phi} \cdot \sin \theta \cdot \sin \theta'; \quad (h)$$

while our approximate equation (d) furnishes the following very convenient formula for incidences nearly perpendicular

$$\sin \delta a = \frac{l-l'}{l} \cdot \frac{\sin \theta \cdot \sin \theta'}{\sin 2a}. \quad (i)$$

The simplest supposition we can frame relative to the values of the constant elements l, l' is their proportionality to those of c, c' , or the lengths of the fits of easy reflection and transmission. This cannot certainly be far from the truth in crystals with one axis, in which the coincidence of the tints, with those of NEWTON'S scale, is for the most part exact. In sulphate of lime too, and mica, the only crystals with two axes which have been examined with sufficient exactness, and under the proper circumstances for ascertaining this important point, the law of proportionality seems to be sustained with great precision. This may seem to authorize the general conclusion, that in all cases, $\frac{c}{c'} = \frac{l}{l'}$. Let us see how this agrees with the measures given in the former part of this paper.

In sulphate of baryta, if we take Dr. BREWSTER'S measure of the dispersive power,* we have $\delta R = 0.019$, and consequently, calculating on the data determined in page 71, we must have, at the virtual pole,

$$\phi = 21^\circ 5' 30'' \quad \phi' = 20^\circ 50' 30'' \quad \delta \phi = -15'$$

Now, if we suppose $l = 6.3463$ $l' = 3.9982$, the values of c

* $\delta R = 0.019$. Treatise on new Philosophical Instruments.

and c' respectively for the extreme red and violet rays,* we shall find by substitution in our formula (h)

$$\delta a = 51' 10''$$

But a red ray penetrating the surface from within the crystal at an angle $a = 18^\circ 12' 30''$, and a violet one at an angle $a + \delta a = 19^\circ 3' 40''$, would emerge at the respective angles $30^\circ 59'$ and $32^\circ 58' 20''$, and would include between them an angle of $1^\circ 59' 20''$, which should be the *apparent* separation of the red and violet axes in the plate employed. Now, previous to the computation of this result, I had carefully measured this angle, by observing the incidences at which the extreme red and violet rays of the prismatic spectrum, received on the reflector of a graduated apparatus, respectively disappeared from the extraordinary image at the poles P, P'. I thus found

Interval of the poles P, P' for red rays - $62^\circ 2'$

Do. for violet - - $66^\circ 5'$

Semi-difference, or apparent separation of the axes $2^\circ 1' 30''$ which differs from its computed value only by $2' 10''$. We may therefore fairly conclude, that in the case of sulphate of baryta, the hypothesis $\frac{l}{l'} = \frac{c}{c'}$ does not deviate sensibly from the truth.

If we apply our formula (i) to the measures above given for Rochelle salt, the result will be widely different. The same supposition as to the values of l, l' being made, we get

$$\delta a = 4^\circ 2' 50''$$

The incidence being nearly perpendicular, and the angle small, we need only increase it in the proportion $1.499 : 1$, to have the apparent angle, which thus comes out $6^\circ 4'$. We have already found $9^\circ 46'$ for the same angle, by a method which

* Biot, Traité de Physique. Vol. IV.

must necessarily give a result much below the truth. This difference is by far too great to arise from any errors of observation ; but to obtain more exact measures, I took several times the apparent angular separation of the axis of each colour from that of the extreme red by the direct homogeneous light of a sunbeam, separated by the prism, and received on the reflector of a divided apparatus, when, after the proper reductions for refraction and dispersion, the results were as follow :

Colour.	Apparent separation of the axes.	Real separation — $\delta a =$	Values of $2 a.$	Number of obser- vations.
Extreme Red.	0 0	0 0	75 42	13
Mean Red.	1 33	1 2	73 38	45
Do. Orange.	2 37	1 44	72 14	18
Do. Yellow.	4 0	2 40	70 23	20
Green.	5 49	3 52	67 57	16
Blue.	8 2	5 21	65 0	13
Indigo.	10 21	6 54	61 54	33
Indigo Violet.	11 17	7 31	60 40	2
Mean Violet.	13 58	9 17	57 8	2
Extreme Violet.	15 23	10 14	55 14	8

Though the total separation of the red and violet axes in this table so far exceeds what we had before estimated it at, I am fully satisfied that it is no way exaggerated, but rather falls short of the truth. It is very practicable, by combinations of coloured glasses, liquids, &c. to insulate either extremity of the spectrum in a state of the most absolute purity. In this climate, the dispersed light of the sky in the neighbourhood of the sun, which always mixes with the prismatic beam, is so considerable as to obliterate the feeble rays which compose the two extremities of the spectrum, and it is only by interposing such combinations between the eye and the Iceland crystal used to analyze the polarised ray, that they

can be examined with any certainty. The combination I employed for the extreme red was such, that when the whole spectrum thrown on a white screen was viewed through it, it was seen reduced to a perfectly circular, well defined, deep red image, whose *centre* fell on the very farthest *termination* of the red as seen by the naked eye, and whose circumference attained, or perhaps surpassed the point where the maximum of the calorific rays has been supposed to be situated. In like manner, when the same spectrum was examined with the violet combination, a very slightly elongated violet image became perceptible, but every trace of the indigo, and the brighter portion of the violet rays was extinguished. For observations on the indigo, and all the more refrangible portion, I employed similar artifices, without which I found it perfectly impracticable to obtain any regular and comparable results.

The coefficient $\frac{l-l'}{l}$ in our formula being the only part not immediately deduced from observation, it is evident that the assumption $\frac{l}{l'} = \frac{c}{c'}$ must be widely erroneous in the present instance, and it therefore becomes necessary to ascertain the values of l by direct measures. This is rendered easy by the equation (*e*) which gives

$$l = t \cdot \frac{\sin \theta \cdot \sin \theta'}{n \cdot \cos \phi}.$$

We have only therefore to observe the inclinations of a plate of known thickness, properly cut and adjusted to 45° azimuth, which correspond to the alternate disappearances of the ordinary and extraordinary images, at which points the values of n are $\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \&c$; computing then the values of θ, θ' , and ϕ , and substituting, we get the values of l , without

detailing particular experiments. The following table expresses the final result of a great number of such measures.

Colour.	Values of l in inches.	Number of Observations.
Extreme Red.	0.0056158	64
Mean Red.	0.0050032	14
Mean Orange.	0.0045852	24
Mean Yellow.	0.0040583	52
Mean Green.	0.0036549	62
Mean Blue.	0.0032863	22
Mean Indigo.	0.0029868	52
Extreme Violet.	0.0025093	49

The observations from which this table was calculated, were made indiscriminately on the maxima and minima of all orders. Those of different orders were of course computed separately, and found to agree without exception in giving the same values of l within limits of error less than those to which the observations are liable; thus affording another proof of the exactness of the law of periodicity above employed. Now, if we compare these, one with another, and with those of c as deduced by M. BIOT from NEWTON'S observations, we shall have as follows:

Colour.	Values of $\frac{l'}{l}$	Values of $\frac{c'}{c}$
Extreme Red.	1.00000	1.00000
Mean Red.	0.89093	0.96215
Mean Orange.	0.81659	0.90490
Mean Yellow.	0.72266	0.85550
Mean Green.	0.65082	0.79433
Mean Blue.	0.58520	0.73725
Mean Indigo.	0.53156	0.69641
Extreme Violet.	0.44684	0.63000

It appears from this comparative statement, that the forces of polarisation and double refraction in the body now examined, act with much greater proportional energy on the more refrangible rays than in mica, sulphate of lime, and

other similar bodies, and consequently that, even were its axes coincident, its tints, though perfectly regular, would still differ very sensibly from the colours of thin plates. This secondary cause of deviation ought to become sensible in plates cut so as to contain both axes, if examined at a perpendicular incidence; but I have not yet had an opportunity of making the trial.

If we calculate on the numbers above given, it will soon appear that a perfect coincidence of all the colours in a single virtual pole is impossible. For this purpose we may employ our equation (i) which easily affords the following

$$\begin{aligned}\cos 2(a + \theta) &= \cos 2a \left\{ 1 + \frac{2l}{l-l'} \cdot \tan 2a \cdot \sin(-\delta a) \right\} \\ &= \frac{\cos 2a}{(\cos M)^2}\end{aligned}$$

taking M an auxiliary angle such that

$$\tan M = \sqrt{2 \cdot \tan 2a \cdot \frac{l}{l-l'} \cdot \sin(-\delta a)}$$

whence the value of θ or the position of the coincidence of any two coloured rays becomes known, the values of l, l' , and $-\delta a$ being given from the foregoing tables. If we unite the mean red with the mean green, these formulæ give $\theta = -11^\circ 29'$, and if with the mean blue, $\theta = -14^\circ 8'$, of which the one falls short of, and the other exceeds the angle $-13^\circ 1'$ given by observation. If we determine by interpolation the values of l' and $-\delta a$, which give $\theta = -13^\circ 1'$, we shall find very nearly

$$l' = 34581 \quad -\delta a = 3^\circ 37' \quad -\delta a + 1^\circ 2' = 4^\circ 39'$$

which correspond to a blue ray strongly inclining to green, and in the brightest part of the colour. Now it is evident that when a rigorous union of all the rays in the proportion in which they exist in white light, is impossible, that of the

strongest and brightest colours in opposition to each other will at least ensure the nearest approach to a virtual pole on the principles above demonstrated, and a white will thus be produced, not indeed mathematically perfect, but containing no marked excess of any of the more powerful colours.

The apophyllite is the only crystal with one axis whose tints exhibit a sensible deviation from the scale of NEWTON. Its phenomena, however, are entirely independent of the first and principal cause which produces the deviation in crystals with two axes, viz. the separation of the axes of differently coloured rays, and are referable solely to the secondary and subordinate cause, of which Rochelle salt has just afforded an example, viz. a peculiarity in the law which regulates the lengths of the minimum oscillations of the differently coloured rays within the medium.

1. The tints of the apophyllite commence at the centre of the rings and increase in regular progression outwards, following the same order, whatever be the thickness of the plate.

It follows from this, that the multiplier M in our general formula, (a) is the same for all the coloured rays, being zero at the commencement of the scale; and hence it follows, as a necessary consequence, that the axes of all the colours are united in one, and the virtual and actual poles coincide with each other and with the centre. Did any sensible separation of the axes exist, it must become perceptible by the ellipticity of the rings when examined with homogeneous light of that colour from which they are farthest asunder; but with the greatest attention, in plates of considerable thickness, I have not been able to observe the slightest shifting of the axis, or deviation from the circular figure, in passing from a red to a

violet illumination. Moreover, it is evident from the preceding theory, that any difference which may exist in their position, if too small to be sensible to the eye, can produce only an imperceptible deviation of tints. In fact, if we suppose $a = 0$ for any colour, we get, for the position of the virtual pole,

$$\sin \theta = \sqrt{\frac{1-l'}{l}} \cdot \sin \delta a$$

θ being the angular distance of the point of coincidence from the single axis of that colour. It is, consequently, insensible when δa is so. Now, the polarising force of the apophyllite being very feeble, the diameters of the rings in any plate of moderate thickness must so far exceed this very minute quantity, that the virtual poles, did any exist, must fall within the limit of the central blackness; the Newtonian scale would still appear to commence from the centre, nor could any sensible deviation from it arise from this cause.

2. When the prismatic spectrum is passed over an apparatus containing a plate of this mineral, no perceptible change in the *magnitudes* of the rings for different colours takes place. Hence it appears that the value of the function l for all the coloured rays is nearly alike. By measures taken on a divided apparatus, a slight difference is observed. Taking the mean refractive index R at 1.5431 (by a very careful measure) and the dispersion δR at 0.017, the formula

$$l = t \cdot \frac{\sin \theta^2}{n \cdot \cos \phi} = \frac{t}{n} \cdot \sin \theta \cdot \tan \phi$$

gave as follows :

Extreme Red.	$l = 0.0093066$
Mean Red.	0.0092810
Mean Orange.	0.0092337
Mean Yellow.	0.0091503
Mean Green.	0.0090643
Mean Blue.	0.0092059
Mean Indigo.	0.0093964
Extreme Violet.	0.0100660 ?

This table, though not given as exact, owing to imperfections in the specimen examined, agrees with the succession of tints which, as far as the fourth order, were as follows :

Apophyllite. Thickness = $0.0829^{\text{in.}}$

Incidence.	Ordinary pencil.	Extraordinary.	
0 0	Bright white	Black	The axis.
13 50	White with a trace of purple	White slightly greenish	1st. maximum of illumination.
21 50	Exceeding sombre violet	Pure bright white	
25 12	Pale greenish yellow	Purplish white	
29 45	White	Sombre violet blue	1st. minimum.
30 50	White	Extremely sombre violet	
33 3	White with a strong tinge of violet	Pale yellow green	
35 50	Blue strongly inclining to purple	Greenish white	2d. maximum.
37 20	Sombre indigo inclining to violet	White	
38 5	Sombre violet	White	
40 10	Tolerably good yellow green	Purplish white	2d. minimum.
43 55	White with a trace of yellow	Obscure indigo inclining to purple.	
44 27	White	Sombre violet	
46 45	Pale purple	Tolerable yellow green	3d. maximum.
49 57	Sombre purple blue	Yellowish white	
50 30	Sombre violet	White	
53 40	Green yellow	Pale purple	3d. minimum.
56 0	Yellowish white	Sombre indigo blue	
56 40	Yellowish white	Sombre violet	
	White	Livid grey	4th. maximum.
59 35	Pale purple	Yellow green	
61 45	Sombre indigo	Pale yellowish white	
62 10	Sombre violet	Yellowish white	4th minimum.
63 0	Faint violet white	White	
	Livid grey	White	
66 0	Tolerable green yellow	Purple	4th minimum.
69 30	Yellow white	Very sombre indigo	

In the colours of thin plates and others of the like composition, the difference in the lengths of the periods of the different rays is so considerable, that after seven or eight alternations of colour the rings confound one another, and are blended into a uniform whiteness. Were the periods more nearly equal, a greater number of rings should be visible, and were they strictly so, the succession of alternate whiteness and blackness should be continued to infinity. As the values of

$l, l', \&c.$ in the apophyllite approximate pretty closely to this limit, we should expect to see a much greater number of rings, and this I find to be really the case. By enclosing a thick plate in balsam of copaiba in a proper apparatus to increase the range of incidence, I have counted as far as the 35th order, when I desisted; not from any want of alternate colours, but owing to their extreme closeness, which rendered it impossible to number them distinctly. Indeed I have no doubt, that could a very thick and limpid specimen be procured, hundreds might be seen without artificial aid.

In two instances then, at least, and probably in many more, or perhaps to a certain small degree in all cases, the minimum lengths of the periods deviate in their respective proportions from those of the fits of easy transmission and reflection; a circumstance which of itself is sufficient to prove the independence of the causes of these laws of periodicity. If we take $Rr = RA$, Fig. 5, Pl. V. and construct a curve whose abscissas AP are the values of $c, c', \&c.$ and ordinates those of $l, l', \&c.$ the straight line $roygbiv$ inclined at 45° to AR will represent the locus for crystals, such as carbonate of lime, in which the periods follow the Newtonian law, $r'o'y'g'b'i'v'$ will represent the same locus for tartrate of soda and potash, while $r''o''y''g''b''i''v''$ is the curve similarly traced for apophyllite.*

* Having communicated to Dr. BREWSTER my observations on the deviation of tints, and the conclusion I had thence deduced as to the separation of the axes of the differently coloured rays, I received in answer a letter, from which, in justice to that indefatigable observer, I subjoin the following extracts.

"MY DEAR SIR,

Esk Hill, by Roslin Laswade, Sept. 18, 1819.

"In consequence of having been some time from home, I have only now received your letter, and hasten to reply to that part of it in which you request me to state

what results I had obtained respecting the deviation of the tints from NEWTON'S scale. The following general points will enable you to judge of the progress which I had made in the enquiry.

" 1. In almost all crystals with two axes there is a deviation from the tints of NEWTON'S scale.

" 2. This deviation is greater in some crystals than in others, being a maximum in acetate of lead and tartrate of potash and soda.

" 3. That all these crystals may be divided into two classes, viz. those which have the *red* ends of the rings inwards and the *blue* ends outwards, and those which have the red ends outwards and the blue ends inwards.

" 4. That in all crystals with two axes, the doubly refracting force of *one axis* in general acts differently upon the differently coloured rays from the doubly refracting force of the *second axis*.

" 5. That as the polarising force is always proportional to the force of double refraction, the polarising force of one axis will act differently on the differently coloured rays from the polarising force of the other axis.

" 6.

" 7. The consequence of this is, that there will be different resultant axes, or different points of compensation for the differently coloured rays.

" 8. All these effects may be calculated with the utmost accuracy, if the ratio of the dispersive powers of the two extraordinary refractive forces is given, or *vice versa*, the dispersive powers may be obtained from the angles of the resultant axes for the red and violet rays of the spectrum.

" 9. I have found crystals in which these phenomena *are decidedly connected* with the rotatory phenomena; and from this highly important fact I am led to conclude, that both have the same origin, and that all the rotatory phenomena are, as I have stated in my paper, the result of the uncompensated tints of two axes, equal for the *mean ray*, but unequal for all the rest. (*Here follows an illustration by a diaphragm.*)

" 10. The division into two classes in § iii. as founded merely on observation, is converted into another division into two classes, viz. 1. That in which the doubly refracting force of the principal axis acts more powerfully on the blue rays than the other axis does; and 2. That in which it acts less powerfully. The first class comprehends those crystals in which the blue ends are inwards, and the second those in which the red ends are inwards, or nearer the principal axis."

In a subsequent letter (Oct. 4), he adds,

" The *virtual poles*, which you mention, I discovered in the year 1815, and I have two accounts of them in my Journal, the one signed on the 24th January, 1816, and

the other 6th January, 1817, by Sir G. MACKENZIE, President of the Physical Class of the Royal Society."

No comments on the above extracts are necessary. They establish at once the priority of Dr. BREWSTER's observations, and the independence of mine. With regard to the division of crystals into two classes, which observation has alike suggested to both of us, it is unnecessary, if we regard either of the two classes as having the angle between the resultant axes greater than a right angle. In Dr. BREWSTER's table, Phil. Trans. 1818, p. 230, succinic acid and sulphate of iron are stated as having this angle 90° . If this determination corresponds, as in all probability it does, to the yellow rays, they belong at once to both classes, and are, in fact, instances of the limit where one class passes into the other. Bi-carbonate of ammonia, in which I can perceive no separation of the axes of different colours, nor of course, any virtual poles, belongs in like manner to both classes, or to neither.

JOHN F. W. HERSCHEL.

APPENDIX.

Description of an instrument employed in the foregoing experiments on the polarised rings.

The singular property possessed by the tourmaline, by which a plate of it of any moderate thickness cut in a direction parallel to its axis of double refraction, is enabled to absorb the whole, or nearly the whole, of an incident pencil polarised in a plane parallel to that axis,* was pointed out by

* The same property is observable in the epidote, the axinite, and all other natural and artificial crystals which exhibit any degree of dichroism when examined by unpolarised light. Muriate of palladium and potash possesses it in the highest perfection. This remarkable effect is easily explained by a reference to the general principles laid down by Dr. BREWSTER in his paper on absorption, Phil. Trans. 1819, p. 11. The incident pencil is separated by the doubly refractive force into two, oppositely polarised, one of which is partly absorbed, the other emerges (polarised in its proper plane) of nearly its original intensity.

M. Biot, in the fourth vol. of his *Traité de Physique*, and he has availed himself of it with his accustomed ingenuity, as affording an extremely ready and convenient mode of viewing the phenomena of polarisation, much more so than by the use of plates of agate, prisms of Iceland spar, or a second reflection. It follows, from the above mentioned property, that if a beam of ordinary light be made to traverse such a plate, the whole of the emergent pencil, or nearly so, will be polarised in a plane at right angles to the axis; for the incident ray being divided by the doubly refracting force into two pencils, polarised in planes, the one parallel, the other perpendicular to the axis, the former is extinguished in its passage, while the latter emerges with nearly its full intensity.

Hence, if two such plates are crossed at right angles, though separately very transparent, their combination will be opaque. There is a great difference, however, in the degree in which tourmalines of different colours possess this power. Those of a light green, pink, or bluish colour, are quite improper, allowing a considerable portion of light to pass when so crossed, while, on the other hand, those whose colour verges strongly to the honey yellow, or to the hair brown, or purplish brown, effect nearly a complete absorption, and afford, when crossed, a combination almost impervious to light. In ignorance of this distinction, I sacrificed several fine and valuable specimens before I could obtain proper plates.

When a crystallized lamina, cut in a proper direction, is interposed between such a combination of plates, it disturbs the polarisation which the light has received in traversing the first plate, and renders a certain portion of it capable of

traversing the second : the colour and intensity of this portion varying with the direction of the ray, give rise to the phenomena of the polarised rings, which may accordingly be seen by applying the eye, and receiving on it the dispersed light of the clouds, &c.

In order, however, to equalize as well as disperse the light, which is of great importance to obtaining a perfect view of the phenomena, an emiered glass may be cemented on the anterior plate, or the first surface of the plate itself roughened ; but it will be found more convenient in practice to employ a double convex lens of short focus for this purpose, by which, if necessary, a very strong illumination may be obtained, and an extremely minute portion of a crystal subjected to examination. I have thus, occasionally, examined the rings in a portion not exceeding the hundredth of an inch in diameter, and thus detected irregularities of crystallization of a very singular nature, in many bodies, which would have eluded any other mode of observation. For this purpose, the crystal must be cemented over a small aperture in a thin sheet of brass, on which the focus of the lens must be exactly adjusted to fall.*

If, instead of applying the eye to receive the light so dispersed, we place a screen at some distance in a darkened room, the apparatus is converted into a solar microscope, and the rings will be seen projected on the screen. The con-

* I have now an apparatus preparing, in which the first plate of tourmaline itself is formed into a double convex lens, by which the loss of light at two surfaces will be suppressed. It is easy to adapt such a lens to a double microscope, for the purpose of detecting microscopic irregularities ; and I have reason to suppose a variety of curious results will be brought to light by these means.

struction of the apparatus I employed is as follows. AB is a brass tube, within which are fitted, 1st, a fixed diaphragm, *aabb*, carrying the first plate of tourmaline in its centre; 2dly, a diaphragm, *ccdd*, moveable freely in its own plane by means of the pin *g* passing through a slit in the side of the cylinder AB, which occupies an arc of about 120° of its circumference. This is destined to receive the crystallized plate *dd*, while a cylinder, *hheeff*, made to slide and turn smoothly within AB, carries the second tourmaline, *ff*. It is essential that the tourmalines employed for this purpose, and especially the posterior one, should be perfectly free from all flaws and blemishes; but large plates not being required, this condition is easily satisfied. The plates so arranged, and brought as near together as possible, the extremity A, of the cylinder AB, is fitted to slide somewhat stiffly on the brass tube PQ, furnished with a lens L, of about two inches focus, and a screw PP, by which it can be adapted to the apparatus usually employed for reflecting a sunbeam into a darkened chamber. The sliding motion of the cylinder AB allows the focus of the lens to be adjusted so as to fall exactly on the first surface of the posterior tourmaline *f*, while its rotation suffers the axis of the anterior one to be placed perpendicular to the plane of reflection. By this arrangement two advantages are gained. The reflector employed (though metallic) always polarises a more or less considerable portion of the reflected beam, which in any other position is partially, or totally extinguished by the first tourmaline, and a great loss of light ensues, which it is of the utmost consequence to avoid: moreover, by this disposition, the action

of the reflector is brought to conspire with that of the tourmaline, and the polarisation of the light which traverses it (which is never rigorously exact) is thereby rendered more complete.

It is convenient to have sliding tubes containing lenses of different focal lengths according to the crystal examined, for the intensity of illumination of any point in the screen being, *cæteris paribus*, as the square of the focal length, consequently, when the rings lie within a very small angular compass, a greater illumination of every part of them may be obtained by using a lens of a longer focus.

The dimensions of the figure, Fig. 6. Pl. V. are nearly of the actual size.

