

XI. *Observations made with the invariable pendulum (No. 4. JONES), at the Royal Observatory, Cape of Good Hope, for the purpose of determining the compression of the earth. By the REV. FEARON FALLOWS, F.R.S. Astronomer of the Cape Observatory. Communicated by the Lords Commissioners of the Admiralty.*

Read February 18, 1830.

THE important problem of ascertaining the ellipticity of the earth has, for a long time past, drawn forth the talents and labours of some of the most scientific men in Europe; and it still continues to be deemed an object of especial regard by all who feel an interest in the promotion of natural knowledge. To attempt to depict the strenuous exertions, the innumerable fatigues, the ardour for the improvement of science, which actuated so many illustrious persons in endeavouring to discover the true figure of the earth, would only be a waste of time; as learned Societies have always recognised and stamped the due meed of merit to each, and invariably appreciated and published to the world the valuable results which have with so many difficulties been obtained.

The nations of Europe, emulous of each other in a work which particularly distinguishes the acquirements of modern times, have encouraged and laudably given their protection and sanction to eminent individuals engaged in undertakings so conducive to the honour of an enlightened community.

Whatever difference of opinion may exist as to the respective values to be attached to observations in the admeasurement of an arc of the meridian, or by the variation in the lengths of the seconds pendulum made in different latitudes, there can be no doubt that the former has this paramount advantage; that certain fixed points are determined, by which the geography of a country is considerably promoted, and at length brought as nearly as possible to perfection: whilst the latter method possesses a superiority in being able to concentrate under the immediate eye of the observer the results

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of his inquiries, which are readily obtained, and easily performed and accomplished.

The Lords Commissioners of the Admiralty having furnished this observatory with the invariable pendulum (No. 4. JONES), which had for several years been most strictly examined by Captain SABINE at Mr. BROWNE's, in Portland Place, London, and subsequently by the same learned gentleman in the course of his inquiries in different parts of the earth, I wished at the earliest opportunity to have a series of experiments made upon it, which might unite every possible degree of accuracy on the part of the observers, as well as the utmost stability in the structure intended for supporting the pendulum. From many unforeseen circumstances, I found at length that the completion of this observatory would be delayed for a considerable time, though previously I had, too fondly perhaps, anticipated finishing it at a much earlier period; and I therefore took advantage of a small out-house (then of no further use) within a short distance of the building, to erect a stout brick pier, well bonded, for the support of the small transit instrument; the same which I used in forming my catalogue of 273 southern stars reduced to the beginning of 1824.

Temporary chases were made for meridian observations; and the recess for the clock (MOLYNEUX's*) and pendulum was soon constructed in a most substantial manner within a few feet of the transit. The top of this recess consisted of a thick strong Robben-Island stone, perforated in the middle, and chiselled out at the upper part, for the reception of the brass plate containing the agate planes upon which the knife-edges of the pendulum rest. This plate was securely fastened to the stone with cement; and I found that when the agates were once truly adjusted to level, they remained (with one exception only, and I think this might be attributed to some error in one of the small levels,) during the whole of the observations that succeeded. It is hardly worthy of remark that at the end of each day's work the knife-edges were screwed up from the agates, and remained so till the commencement of another series on the first favourable opportunity. The clock (adjusted to sidereal time and corrected for rate by the stars,) was placed in the recess, having the bottom of the case resting upon a large block of stone embedded in well-wrought clay, and the back of the case was tightly screwed to a strong piece of well seasoned

* The new clock, not the old one.

wood let into the wall. I have little doubt but that the good going of the clock was by these means in a great measure secured. On each side of the pendulum, somewhat above the middle of it, were suspended two of JONES's thermometers. The disk was formed of a small circle of card paper about three-tenths of an inch in diameter, this measure being found to be the best and most convenient. Instead of the wooden stand for supporting the small telescope, through which the coincidences, or rather the disappearances and re-appearances of the pendulum-rod and the disk are noted, as well as the magnitude of the arc of vibration, a small thick brick pier was raised so as to receive the plate to which the telescope is attached. The usual adjustments of the clock, pendulum and telescope, being completed, we were now ready to commence.

I here deem it right to notice the very able assistance which I received from Captain RONALD: his exertions were truly praiseworthy; I cannot recommend them too highly. I likewise avail myself of paying a just tribute to the aid which was afforded by Lieutenant JOHNSON, of the Honourable East India Company's Service, now superintendant of the observatory at St. Helena. This gentleman being on a visit to me, very kindly at my request took an active part in all the observations.

The sheets accompanying this short paper must be considered as the united labours of Captain RONALD, Mr. JOHNSON, and myself, and the responsibility of each as to accuracy must depend upon the papers signed with the observer's name. As far as I am able to judge of these things, the near agreement of three independent series of observations is no small argument of their accuracy.

It must not for a moment be conceived that I reckon our results as in any way final in the determination of the compression of the earth, inasmuch as it would be advisable to begin a new series of observations (say) in the midst of winter, or what would be better, in different seasons of the year, lest the coefficient for temperature might require some correction; though even this test can hardly be supposed, from the experiments made by Captain SABINE on this very pendulum in London, to be required. I have ever been of opinion, (how far correct or not, I leave others to judge,) that the invariable pendulum ought

to be a standard instrument in an observatory ; that it should be swung at all seasons of the year ; that it would be proper to forward it on authorized occasions to the various fixed observatories now situated in the northern and southern hemispheres ; that the instrument should return again to the same stations as before, and the observations be renewed ; that, finally, after each circuit it should undergo a strict examination at the spot where it was first tried, in order that it might proceed again as before. Should any harm take place from improper packing or accidental circumstances, the evil would be soon discovered, and the instrument repaired. The pendulum is of a very delicate construction, and consequently it is the more necessary that it should be as often as possible compared at those points where it has previously been used.

Formulæ used in computing the Observations.

a = greater, and b the lesser arc observed at the beginning and end of each set of coincidences.

t = mean of the thermometers immediately adjacent to the pendulum.

H = height of the barometer.

τ = height of the attached thermometer.

It must be remarked that the sidereal day is reduced to a mean solar day for comparison in London.

$$\text{Log (reduct. for arc)} = 9.55132 + \log(a+b) + \log(a-b) - \log\{\log a - \log b\}$$

$$\text{Reduct. for temp.} = (t - 62^\circ) \times 0.421.$$

$$\text{Log (reduct. for vacuum)} = 9.31083 + \log H.$$

$$- \left\{ \log(1 + .002083 \cdot t - 53^\circ) + \log(1 + .0001 \cdot \tau - 53^\circ) \right\}$$

The specific gravity of the pendulum is assumed . . . 8.6 SABINE.

The expansion for 1° of the pendulum 0.421 —

The temperature assumed as the standard for spec. grav. = 53° —

The temperature assumed as the standard for pendulum = 62° —

Specific gravity of the air (water = 1) $\frac{1}{8.36}$ —

The height of barometer assumed as standard for spec. grav. = 29.27 inches.

The height of the pendulum above the mean level of the sea in Table Bay was found to be 32 feet nearly, and above the low-water mark 34 feet, on the day when it was determined. Hence

	Vibrations.
For 34 feet at the Cape	Reduction = 0.14
For 83 feet in London	Reduction = 0.34
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	Difference = 0.20

This quantity 0.20 vibration is additive to the difference between the number of vibrations (in a mean solar day) at Mr. BROWNE's and the Cape.

Brief statement of the final results.

1st,—Captain RONALD.

Observations made in London.—Pendulum (No. 4. JONES).

	Vibrations in a Mean Solar Day.
Mean of 100 coincidences	= 86164.62

Observations made at the Cape of Good Hope.

	Vibrations in a Mean Solar Day.
Mean of 270 coincidences	= 86097.64
Mean of 160 ———	= 86097.88
Mean of 200 ———	= 86097.74
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True Mean of <u>630</u> ———	= <u>86097.73</u>

2nd,—Mr. JOHNSON.

	Vibrations in a Mean Solar Day.
Mean of 240 coincidences	= 86097.792
Mean of 180 ———	= 86097.898
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True Mean of <u>420</u> ———	= <u>86097.83</u>

3rd,—Mr. FALLOWS.

	Vibrations in a Mean Solar Day.
Mean of 401 coincidences	= 86097.67
Mean of 419 ———	= 86097.66
Mean of 507 ———	= 86097.70
Mean of 81 ———	= 86097.69
<hr/> True Mean of 1408 ———	<hr/> = 86097.68
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Summary at the Cape.

	Vibrations in a Mean Solar Day.
Captain RONALD. . True Mean of 630 coincidences	= 86097.73
Mr. JOHNSON . . . True Mean of 420 ———	= 86097.83
Mr. FALLOWS . . . True Mean of 1408 ———	= 86097.68
<hr/> True Mean of 2458 ———	<hr/> = 86097.72
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From this last conclusion, compared with that obtained by Captain SABINE at Mr. BROWNE's, in Portland Place, and which only differs $\frac{1}{56}$ th of a vibration in a day from Captain RONALD's subsequent determination, the compression of the earth is very readily ascertained.

Let n = the difference in the number of vibrations of the pendulum at London and the Cape. Then for

$$\text{Compression} \left\{ \begin{array}{l} \frac{1}{280} \quad . \quad . \quad . \quad . \quad . \quad n = 65.752 \\ \frac{1}{290} \quad . \quad . \quad . \quad . \quad . \quad n = 67.344 \\ \frac{1}{300} \quad . \quad . \quad . \quad . \quad . \quad n = 68.828 \end{array} \right.$$

London. No. of vibrations in a mean solar day . = 86164.64 SABINE.

Cape. No. of vibrations in a mean solar day . = 86097.72

66.92

Reduction due to difference of level from low-water mark = 0.20

True difference in the No. of Vib^{ns} in a mean solar day . = 67.12

This quantity gives the compression of the earth . . = $\frac{1}{283.5}$

I am aware that this compression somewhat differs from that obtained in the Southern Hemisphere by Sir THOMAS BRISBANE, at Paramatta, though it is nearly in accordance with M. FREYCINET's observations in Cape Town. The documents, however, are here given, and I have every hope that they may meet the public eye, and undergo the usual test of candid and liberal criticism.

FEARON FALLOWS.

Royal Observatory, Cape of Good Hope,

May 19th, 1829.

P. S. The latitude of the Observatory is not yet ascertained by actual observation with the new mural circle. This instrument has been attached to its pier for several months past; but from some discrepancies in reading, the cause of which is not yet discovered, I am under the necessity of assuming $33^{\circ} 55' 56''$ as a very near approximate latitude*. The examination of the mural will go on, and the conclusions be forwarded home when some definite opinion may be formed of it.

* I connected the Observatory with my former temporary one in Cape Town, by a survey over the intermediate ground.

APPENDIX.

Observations made by Captain RONALD with the invariable pendulum (No. 4. of SABINE), in Portland Place, London. Lat. $51^{\circ} 31' 8''$.4.

1825.	Barom.	Therm.	No. of Coin.	Disapp.	Re-app.	Time of Coincidence.	Arc.
	inches.	$^{\circ}$		m s	m s	h m s	$^{\circ}$
July 31	29.950	70.48	1	31 21	31 28	1 31 24.5	1.185
	29.934	70.50	11	28 03	28 18	3 28 10.5	.610
Aug. 1	29.944	69.75	1	13 35	13 45	11 13 40.0	1.110
	29.942	70.25	11	10 34	10 49	1 10 41.5	.575
	29.942	70.35	1	33 55	34 01	1 33 58.0	1.320
"	29.941	70.65	11	30 31	30 45	3 30 38.0	.670
	29.950	71.00	1	49 29	49 36	10 49 32.5	1.225
2	29.944	71.90	11	45 59	46 15	0 46 7.0	.640
	29.944	71.90	1	58 01	58 08	0 58 4.5	1.305
"	29.944	71.80	11	54 26	54 41	2 54 33.5	.665
	29.952	71.00	1	9 39	9 44	11 9 41.5	1.370
3	29.952	71.80	11	6 02	6 14	1 6 8.0	.695
	29.953	71.80	1	18 47	18 55	1 18 51.0	1.180
"	29.924	72.00	11	15 16	15 29	3 15 22.5	.605
	29.584	72.20	1	52 51	52 57	0 52 54.0	1.310
4	29.574	72.00	11	49 11	49 24	2 49 17.5	.665
	29.500	70.30	1	33 38	33 44	10 33 41.0	1.290
5	29.520	70.75	11	30 18	30 30	0 30 24.0	.640
	29.522	70.80	1	36 24	36 31	0 36 27.5	1.370
"	29.530	71.05	11	32 51	33 03	2 33 57.0	.685
	29.628	67.20	1	46 11	46 16	11 46 13.5	1.315
6	29.616	68.60	11	43 21	43 34	1 43 27.5	.650
Mean..	29.804						

Observations computed.

Observations.	Mean Temp.	Interval.	Vib ^{ns} in 24 Mean Solar Hours.	Corr. for Arc.	Reduct. to $70^{\circ} 82' \text{FAHR.}$	Reduced Vib ^{ns} in 24 Mean Solar Hours at 62°FAHR.
1	70.49	h m s 1 56 46.0	86153.354	+1.277	-0.139	86154.492
2	70.00	1 57 01.5	86153.899	1.124	0.345	86154.678
3	70.50	1 56 40.0	86153.143	1.568	0.135	86154.576
4	71.45	1 56 34.5	86152.949	1.378	+0.265	86154.592
5	71.85	1 56 29.0	86152.754	1.534	0.434	86154.722
6	71.40	1 56 26.5	86152.666	1.685	0.244	86154.595
7	71.90	1 56 31.5	86152.843	1.260	0.455	86154.558
8	72.10	1 56 23.5	86152.560	1.541	0.539	86154.640
9	70.52	1 56 43.0	86153.249	1.467	-0.126	86154.590
10	70.92	1 56 29.5	86152.772	1.665	+0.042	86154.479
11	67.90	1 57 14.0	86154.336	+1.521	-1.229	86154.629
Mean =	70.82					86154.595
						Correction for rate +0.441
						Correction for buoyancy +5.873
						Reduction to 62°FAHR. +3.713
						86164.622*

* NOTE.—The accordance of this result with that obtained by Captain SABINE, is sufficient to show that the instrument remained unaltered, and warrants the comparison of any future observations with those obtained by former observers; subject, of course, to any change, which it may have undergone since that period, or that it may yet meet with, which can only be satisfactorily ascertained, when another series shall have been made with it, in London, where its rate is probably better known, than at any other of Captain SABINE'S stations.

PENDULUM AT THE ROYAL OBSERVATORY, CAPE OF GOOD HOPE. 161

Mr. BROWNE, at whose house these observations were made, favoured me with the rate of the clock, as follows :

Observations made with the pendulum (No. 4, of SABINE). Observatory,
Cape of Good Hope. Lat. 33° 55' 56". By Captain RONALD.

1828.	Barometer.	Thermometers.		No. of Coin.	Disapp.	Re-app.	Time of Coincidence.	Arc.
		A.	B.					
Nov. 23	inches.	^o 72.2	^o 73.2	1	m s 4 53	m s 5 9	h m s 12 5 1.0	^o 1.03
	29.906	72.2	73.2	31	42 5	42 17	14 42 11.0	.48
	29.917	74.7	75.7	1	50 2	50 15	14 50 8.5	1.11
	29.917	75.7	74.6	31	27 13	27 26	17 27 19.5	.51
24	29.947	71.3	72.3	1	56 22	56 35	12 56 28.5	1.215
	29.956	67.8	68.4	31	33 57	34 9	15 34 3.0	.520
	29.983	71.2	71.8	1	14 13	14 27	16 14 20.0	1.27
	29.986	70.2	70.9	31	51 50	52 2	18 15 56.0	.52
25	29.951	69.3	69.9	1	57 45	57 59	18 57 52.0	1.27
	29.951	69.3	69.9	31	35 23	35 36	21 35 29.5	.53
	29.964	68.1	68.9	1	55 49	56 1	9 55 55.0	1.175
	29.966	66.3	67.2	31	33 44	33 59	12 33 51.5	.495
26	29.998	67.5	68.2	1	41 20	41 33	12 41 26.5	1.32
	29.998	67.5	68.1	31	19 5	19 17	15 19 11.0	.54
	30.006	70.6	71.4	1	25 52	26 8	16 26 0.0	1.200
	29.997	72.7	73.4	31	3 8	3 22	19 3 15.0	.515
27	29.966	72.3	73.0	1	19 7	19 21	19 19 14.0	1.20
	29.963	71.7	72.6	31	56 31	56 44	21 56 37.5	.50
Mean..	29.963							

Observations computed. The clock making on the 23rd, 86636.826*, on the 24th, 86636.916*, and on the 25th, 86637.024* sidereal vibrations in a mean solar day.

Number of Observations.	Mean Temp.	Interval.	Vib ^{ns} in 24 Mean Solar Hours.	Correc. for Arc.	Reduc. to 62° FAHR.	Reduced Vib ^{ns} in 24 Mean Solar Hours at 62° F.
1	73.95	h m s 2 37 10.0	86085.584	+ 0.891	+ 5.031	86091.506
2	73.47	2 37 11.0	86085.643	1.024	4.829	86091.496
3	69.80	2 37 34.5	86087.102	1.164	3.284	86091.550
4	70.02	2 37 36.0	86087.189	1.232	3	

Rate of the Sidereal Clock.

Nov. 22nd.			Nov. 24th.			Nov. 25th.		
Stars.	Rates.	Compared with Observations of the 19th and 21st.	Stars.	Rates.	Compared with Observations of the 22nd.	Stars.	Rates.	Compared with Observations of the 22nd and 24th.
	^s			^s			^s	
α Pegasi	+0.10		α Arietis ..	+0.10		α Pegasi	+0.35	
α Andromedæ	.07		α Ceti29		α App. Sc. ..	.40	
γ Pegasi04		γ Eridani..	.42		β Arietis45	
β Arietis12		Aldebaran .	.35		α ————....	.24	
α ————....	.13		Mean..	+ .290		ρ Ceti49	
γ Ceti17					δ ————....	.36	
α ————....	.28					γ ————....	.43	
γ Eridani....	.37					α ————....	.69	
Aldebaran ..	+ .23					γ Eridani....	.55	
Mean..	+ .168					Aldebaran ..	+ .63	
						Mean..	+ .459	

Here the rate appears to have been increasing, and therefore a proportion was made for each day's observations, thus :

$$\begin{array}{rcl}
 \text{For the observations of the 23d, Rate } & + & \overset{s}{0.270} \\
 24\text{th, . . . } & + & .360 \\
 25\text{th, . . . } & + & .468
 \end{array}$$

* NOTE.—In these numbers, both in this and the two following series, the rate of the clock is included, being = 86636.556 + rate.

Observations with the pendulum (Continued).

1829.	Attached Therm.	Barom.	Thermometers.		No. of Coin.	Disapp.	Re-app.	Time of Coincidence.	Arc.
			A.	B.					
Jan. 7	72.0	30.035	72.4	73.0	1	m s	m s	h m s	1.12
	73.5	30.020	73.0	73.6	21	36 20	36 34	17 36 27.0	.62
"	77.5	29.964	76.2	76.8	1	34 22	34 36	21 34 29.0	1.18
	77.0	29.919	74.8	75.2	21	19 10	19 25	23 19 17.5	.66
"	77.5	29.919	75.6	76.0	1	30 4	30 18	23 30 11.0	1.37
	76.5	29.900	74.5	75.1	21	14 46	15 2	1 14 54.0	.73
8	69.0	29.952	69.0	69.6	1	45 56	46 11	15 46 3.5	1.30
	69.6	29.951	70.6	71.1	21	31 3	31 18	17 31 10.5	.71
"	72.0	29.951	70.6	71.2	1	36 41	36 56	17 36 48.5	1.26
	72.5	29.932	72.0	72.6	21	21 45	21 58	19 21 51.5	.69
"	72.5	29.932	72.0	72.7	1	27 44	27 58	19 27 51.0	1.23
	74.0	29.925	72.6	73.4	21	12 43	12 58	21 12 50.5	.66
"	74.0	29.925	72.3	73.0	1	24 10	24 22	21 24 16.0	1.49
	73.0	29.926	72.4	73.1	21	8 59	9 15	23 9 7.0	.83
"	73.0	29.926	72.3	72.9	1	14 46	15 2	23 14 54.0	1.19
	73.0	29.918	70.9	71.5	21	59 45	0 0	0 59 52.5	.66
Mean..	73.54	29.943							

On the morning of the 9th the clock was found stopped, which prevented the continuance of this series.

Observations computed. The clock making on the 7th, 86636.306, and on the 8th, 86636.215 sidereal vibrations in a mean solar day.

Rate of the Sidereal Clock.

Jan. 7th.			Jan. 8th.		
Stars.	Rates.	Compared with Observations of the 6th.	Stars.	Rates.	Compared with Observations of the 6th & 7th.
	^s			^s	
α Ceti	— 0.32		α Ceti	— 0.40	
α Persei10		γ Eridani . .	.43	
γ Eridani . .	.30		γ Cali Sc. . .	.10	
σ ——— . .	.28		Capella50	
X ——— . .	.04		Rigel30	
43d ——— . .	.40		γ Orionis . .	.20	
ψ ——— . .	.30		ζ ——— . .	.30	
γ Cali Sc. . .	.47		β Aurigæ . .	— .50	
γ Orionis . .	.52		Mean	— .341	
δ ——— . .	.08				
ϵ ——— . .	.28				
ζ ——— . .	.12				
β Aurigæ . .	— .04				
Mean	— .250				

Observations with the pendulum (Continued).

1829.	Attached Therm.	Barom.	Thermometers.		No. of Coin.	Disapp.	Re-app.	Time of Coincidence.	Arc.
			A.	B.					
Jan. 16		inches.	\circ	\circ		m s	m s	h m s	
	78.0	29.986	80.1	80.8	1	57 22	57 34	20 57 28.0	1.41
"	78.0	29.998	73.2	73.7	21	41 55	42 9	22 42 2.0	.77
	78.0	29.998	74.0	74.6	1	47 16	47 30	22 47 23.0	1.30
"	76.0	29.990	72.7	73.2	21	32 2	32 16	0 32 9.0	.71
	76.0	29.990	72.8	73.3	1	37 44	37 58	0 37 51.0	1.11
"	76.0	29.990	72.4	72.9	21	22 40	22 55	2 22 47.5	.59
	70.0	29.988	71.0	71.7	1	32 32	32 44	13 32 38.0	1.46
17	72.0	30.014	71.3	71.8	21	17 32	17 47	15 17 39.5	.73
	72.0	30.014	71.3	71.8	1	20 39.5	20 54	15 20 46.25	1.42
"	74.0	30.026	71.8	72.5	21	5 39	5 55	17 5 47.0	.75
	74.0	30.026	72.0	72.7	1	9 4	9 18	17 9 11.0	1.22
"	75.5	30.025	72.6	73.1	21	54 6	54 21	18 54 13.5	.64
	75.6	30.025	73.0	73.5	1	57 1	57 14	18 57 7.5	1.24
"	75.6	30.023	72.9	73.4	21	41 59	42 13	20 42 6.0	.70
	75.6	30.023	73.6	74.1	1	45 54	46 8	20 46 1.0	1.35
"	76.0	30.006	73.1	73.7	21	30 47	31 0	22 30 53.5	.72
	76.0	30.006	74.0	74.6	1	35 49	36 2	22 35 55.5	1.30
"	75.0	29.990	73.3	73.8	21	20 43	20 57	0 20 50.0	.70
	75.0	29.990	73.6	74.8	1	24 34	24 48	0 24 41.0	1.44
"	72.0	29.988	73.0	73.6	21	9 25	9 40	2 9 32.5	.74
Mean..	75.01	30.005							

Observations computed. The clock making on the 16th, 86636.057, and on the 17th, 86636.027 sidereal vibrations in a mean solar day.

Number of Observations.	Mean Temp.	Interval.	Vib ^{ns} in 24 Mean Solar Hours.	Corr. for Arc.	Reduc. to 62° FAHR.	Reduced Vib ^{ns} in 24 Mean Solar Hours at 62° F.
1	76.95	<div>h m s</div> <div>1 44 34.0</div>	86083.707	+1.890	+6.294	86091.891
2	73.62	1 44 46.0	86084.762	1.607	4.892	86091.261
3	72.85	1 44 56.5	86085.681	1.146	4.568	86091.395
4	71.45	1 45 1.5	86086.088	1.890		

Rate of the Sidereal Clock.

Jan. 16th.			Jan. 17th.		
Stars.	Rates.	Compared with Observations of the 15th.	Stars.	Rates.	Compared with Observations of the 15th and 16th.
12 Eridani..	^s -0.67		α Persei ..	^s -0.60	
17 ———..	.40		17 Eridani..	.50	
δ ———..	.52		19 ———..	.50	
ζ Orionis ..	.33		δ ———..	.42	
β Aurigæ ..	.70		γ Orionis ..	.65	
δ Geminor..	.28		δ ———..	.59	
β Can. Min.	.38		ϵ ———..	.58	
Castor56		ζ ———..	.74	
Procyon....	— .65		μ Columbæ	.60	
Mean..	— .499		β Aurigæ ..	.33	
			θ Leporis ..	.50	
			ϵ Can. Maj.	.50	
			δ ———..	.48	
			δ Geminor.	.57	
			β Can. Min.	.38	
			Procyon....	.47	
			Pollux	— .58	
			Mean..	— .529	

Abstract of Captain RONALD's observations.

London. Mean of 110 coincidences . = 86164.62

Cape. . Mean of 270 coincidences . . . = 86097.64

— . ——— 160 = 86097.88

— . ——— 200 = 86097.74

True Mean of 630 coincidences at the Cape = 86097.73

Rate of the Sidereal Clock.

Dec. 14th.		Dec. 15th.	
By α Arietis.....	^s -0.53	By α Ceti	^s -0.74
Aldebaran	0.26	Aldebaran	0.28
Capella	0.50	Capella	0.50
Rigel	0.12	Rigel	0.14
β Tauri	0.34	β Tauri	0.44
ζ Orionis	0.60	δ Orionis	0.30
β Aurigæ	0.74	ζ ———	0.39
Mean Rate..	-0.44	Mean Rate..	-0.39

Mean Rate on the 18th by Mr. FALLOWS's Observations 0.03

Remark.—On the 16th and 17th the weather was unfavourable for observation; the quantity assumed for the rate of the clock on those days is a mean of its rate on the 15th and 18th: probably on the 17th the rate was less than that which has been assigned.

Observations with the pendulum (Continued).

1829.	Attached Therm.	Barom.	Therm.		No. of Coin.	Disapp.	Re-app.	Time of Coin.	Arc.	Remarks.
			A.	B.						
Jan.		inches.	68.6	69.0		m s	m s	h m s		
20.	68.5	29.972	68.6	69.0	1	9 46	9 55	17 9 50.5	1.67	The Pendulum was not stopped between these Observations.
	73.7	30.017	73.0	73.6	21	54 19	34 35	18 54 27.0	1.12	
	73.7	30.017	72.8	73.4	1	59 34	59 49	18 59 41.5	1.12	
"	71.7	30.010	71.7	72.0	21	44 22	44 37	20 44 29.5	0.83	The Pendulum was not stopped between these Observations.
	71.7	30.010	71.8	72.0	1	1 55	2 5	21 2 0.0	1.60	
	73.5	30.003	73.0	73.5	21	46 18	46 33	22 46 25.5	1.31	
"	73.5	30.003	73.2	73.7	1	51 31	51 45	22 51 38.0	1.31	The Pendulum was not stopped between these Observations.
	74.5	30.003	73.5	74.0	21	36 6	36 22	24 36 14.0	0.90	
	67.0	30.137	68.8	69.0	1	14 28	14 37	15 14 32.5	1.68	
21.	70.0	30.145	69.4	69.9	21	59 25	59 41	16 59 30.0	0.91	The Pendulum was not stopped between these Observations.
	70.0	30.145	69.4	69.9	1	4 41	4 57	17 4 49.0	0.90	
	72.0	30.145	70.0	70.4	21	50 1	50 17	18 50 9.0	0.50	
"	72.0	30.145	70.6	71.0	1	7 6	7 17	19 7 11.5	1.60	The Pendulum was not stopped between these Observations.
	73.0	30.137	70.2	70.6	21	52 3	52 17	20 52 10.0	0.87	
	73.0	30.137	70.5	70.8	1	57 18	57 34	20 57 26.0	0.86	
"	72.0	30.116	70.3	70.6	21	43 34	43 49	22 43 41.5	0.48	The Pendulum was not stopped between these Observations.
	72.0	30.116	70.7	71.3	1	59 40	59 49	22 59 44.5	1.70	
"	71.0	30.114	69.9	70.2	22	49 51	50 9	0 50 0.0	0.87	

Observations computed.

No. of Observ.	Mean Temp.	Interval.	Uncorrected No. of Vibrations.	Clock's Rate.	Correc. for Arc.	Reduc. to 62° FAHR.	Reduced No. of Vib ^{ns} in 24 Mean Solar Hours at 62° FAHR.
		h m s					
1	71.05	1 44 36.5	86084.431	+0.54	+3.148	+3.810	86091.929
2	72.50	1 44 48.0	86085.440	+0.54	1.547	4.421	86091.948
3	72.57	1 44 25.5	86083.460	+0.54	3.458	4.454	86091.912
4	73.60	1 44 36.0	86084.392	+0.54	1.978	4.883	86091.793
5	69.27	1 45 0.5	86086.529	-0.410	2.666	3.061	86091.846
6	69.93	1 45 20.0	86088.216	-0.410	0.781	3.338	86091.924
7	70.60	1 44 58.5	86086.373	-0.410	2.425	3.620	86092.008
8	70.55	1 45 15.5	86087.823	-0.410	0.716	3.600	86091.729
9	70.52	1 50 15.5	86086.529	-0.410	+2.609	+3.587	86092.315
Mean.... Correction for buoyancy							86091.934 +5.944
Number of Vibrations in vacuo, in 24 Mean Solar Hours ..							86097.878

Rate of the Sidereal Clock.

Jan. 20th.		Jan. 21st.	
By β Tauri	^s +0.44	By β Tauri	^s -0.44
δ Orionis	0.47	δ Orionis	0.40
ζ ———	0.50	ζ ———	0.34
α ———	0.82	α ———	0.48
Procyon	0.64	Mean Rate.. -0.41	
Pollux	0.88		
Mean Rate..	+0.54		

Remark.—From the above result, it appears that the rate varied nearly a second in the space of 24 hours: there is reason to suppose that the change was owing to some sudden derangement which the clock might have suffered on the night of the 20th. From the tolerable accordance with each other of the pendulum observations of the 21st, it may be presumed that the rate was equable during that day.

Abstract of Mr. JOHNSON'S observations.

Mean of 240 coincidences = 86097.792

Mean of 180 coincidences = 86097.878

True mean of 420 coincidences = 86097.829

General Remark.—During both series of experiments, the observatory was kept in the same state, the North shutter being open during the day, and the door of the recess, in which the clock and pendulum stood, constantly closed, with the exception of the two small openings made in it for the purpose of showing the dial plate, and the graduated arc of the pendulum. During the second series, the weather was particularly favourable, both for observations of the transit instrument and pendulum.

Cape of Good Hope ; Observer Mr. FALLOWS.

First Series.

1829.	Attached Therm.	Barom.	Therm.		No. of Coin.	Disapp.	Re-app.	Time of Coin. by the Sidereal Clock.	Arc.	Remarks.
			A.	B.						
Dec.		inches.				m s	m s	h m s		
2	66.5	30.205	68.2	69.1	1	50 41	50 56 $\frac{1}{2}$	16 50 48 $\frac{3}{4}$	1.38	This day very favourable for observation ; little wind—the Observatory chase shut, except when observing. The Thermometers read off before the North chase was opened.
	67.0	30.205	68.0	68.9	31	28 31	28 51	19 28 41	0.51	
	67.1	30.205	68.0	68.9	1	33 50	34 4 $\frac{1}{2}$	19 33 57 $\frac{1}{4}$	1.48	
"	67.5	30.207	67.1	68.0	31	11 44	12 1	22 11 52 $\frac{1}{2}$	0.61	
3	66.0	30.205	67.1	68.0	1	38 1	38 16	14 38 8 $\frac{1}{2}$	1.38	The weather fine during the whole of the day.
	68.5	30.205	69.6	70.5	31	15 56	16 13	17 16 4 $\frac{1}{2}$	0.57	
	68.7	30.205	70.1	71.2	1	42 43 $\frac{1}{2}$	42 59	17 42 51 $\frac{1}{4}$	1.20	
"	68.5	30.175	70.2	71.2	31	20 24	20 44 $\frac{1}{2}$	20 20 34 $\frac{1}{4}$	0.50	
	68.6	30.174	70.2	71.3	1	30 6	30 22	20 30 14	1.22	The South-east wind springing up. All the doors closed. What is called at Cape Town a South-easter, ought to be designated South-by-east.
"	68.0	30.151	68.0	69.0	31	7 57	8 15	23 8 6	0.49	
4	65.0	30.102	64.2	65.0	1	3 18	3 32	12 3 25	1.38	
	67.5	30.139	66.9	67.6	31	41 31	41 49	14 41 40	0.57	
	67.6	30.140	66.9	67.6	1	47 24	47 40	14 47 32	1.20	The weather rainy; wind due North ; no stars last night, from clouds.
"	68.5	30.165	69.1	70.0	31	25 26 $\frac{1}{2}$	25 41 $\frac{1}{2}$	17 25 34	0.51	
	68.6	30.166	69.1	70.0	1	31 10	31 26	17 31 18	1.35	
"	68.5	30.120	68.3	69.1	31	9 0	9 18	20 9 9	0.58	
	68.5	30.120	67.0	67.8	1	19 40	19 54	20 19 47	1.41	The sky clearing up.
"	68.0	30.108	67.3	68.0	31	57 33	57 52	22 57 42 $\frac{1}{2}$	0.58	
5	64.0	30.125	63.0	63.6	1	40 18	40 34	10 40 26	1.42	
	64.5	30.127	64.0	64.8	31	18 32	18 49	13 18 40 $\frac{1}{2}$	0.68	
	64.5	30.127	64.1	64.8	1	24 13	24 28	13 24 20 $\frac{1}{2}$	1.29	
"	66.4	30.127	65.8	66.4	31	2 31	2 46	16 2 38 $\frac{1}{2}$	0.56	
	66.3	30.127	66.3	67.0	1	13 24	13 32	16 13 28	1.72	
"	68.0	30.200	68.4	69.2	42	49 5 $\frac{1}{2}$	49 22 $\frac{1}{2}$	19 49 14	0.50	
	68.0	30.200	68.1	68.9	1	54 25	54 41	19 54 33	1.13	
"	68.0	30.115	68.0	68.5	31	32 20	32 36	22 32 28	0.49	
Mean	67.23	30.1594								

First Series computed.

1829.	Mean Temp.	Interval.	Uncorrected No. of Vibrations.	Clock's Rate.	Correc. for Arc.	Reduc. to 62° FAHR.	Reduced Vib ^{ns} in 24 Mean Solar Hours.
	68.550	h m s 2 37 52 $\frac{1}{4}$	86087.78	-0.52	+1.354	+2.758	86091.37
	68.000	2 37 55 $\frac{1}{4}$	86087.94	0.52	1.682	2.526	86091.63
	68.800	2 37 56	86087.98	0.41	1.464	2.863	86091.50
	70.540	2 37 43	86087.24	0.41	1.114	3.595	86091.54
	69.625	2 37 52	86087.76	0.41	1.121	3.210	86091.68
	65.925	2 38 15	86089.08	0.26	1.464	1.652	86091.94
	68.400	2 38 2	86088.34	0.26	1.362	2.694	86092.14
	69.125	2 37 51	86087.70	0.26	1.442	3.000	86091.88
	67.525	2 37 55 $\frac{1}{2}$	86087.98	0.26	1.564	2.326	86091.61
	63.850	2 38 14 $\frac{1}{2}$	86089.06	0.26	1.730	0.779	86091.31
	65.275	2 38 18	86089.26	0.26	1.326	1.379	86091.69
	67.725	3 35 46	86088.24	0.26	1.797	2.410	86092.15
	68.375	2 37 55	86087.94	-0.26	+1.017	+2.684	86091.38
Mean..	67.824	Mean Buoyancy					86091.69 + 5.98
(No. of Coincidences = 401) Number of Vibrations in vacuo, } in 24 Mean Solar Hours.. }							860 97.67

Rate of the Sidereal Clock.

Dec. 2nd.		Dec. 3rd.		Dec. 4th to Dec. 9th.	
By α Arietis.....	^s -0.38	By α Ceti	^s -0.24	By α Ceti	^s -0.33
α Ceti	0.52	Capella	0.30	Capella	0.26
Rigel	0.50	β Tauri	0.50	Rigel	0.22
β Tauri	0.70	δ Orionis.....	0.58	β Tauri	0.23
δ Orionis.	0.50	ζ ———	0.36	δ Orionis.....	0.23
ζ ———	0.55	β Aurigæ	-0.50	β Aurigæ	0.27
β Aurigæ	-0.46			Sirius	-0.28
Mean Rate..	-0.52	Mean Rate ..	-0.41	Mean Rate..	-0.26

Second Series.

1829.	Attached Therm.	Barom.	Thermometers.		No. of Coin.	Disapp.	Re-app.	Time of Coincid. by the Sidereal Clock.	Arc.
			A.	B.					
Dec. 18	{	inches.				m s	m s	h m s	
		66.1 30.022	65.2	65.7	1	15 26	15 38	11 15 32	1.78
		69.5 30.053	68.1	68.7	53	49 17	49 35	15 49 26	0.39
		70.0 30.053	71.2	71.7	1	0 9	0 13	16 0 11	1.87
"		72.0 30.016	72.2	72.4	59	4 21	4 37	21 4 29	0.40
"		72.1 30.015	72.2	72.4	1	25 47	26 4	21 25 55 $\frac{1}{2}$	1.12
"		71.0 30.024	71.0	71.5	16	44 30	44 47	22 44 38 $\frac{1}{2}$	0.76
"		71.0 30.025	71.0	71.4	1	55 5	55 22	22 55 13 $\frac{1}{2}$	0.89
"		71.5 30.054	70.1	70.5	18	24 26	24 42	0 24 34	0.55
19	{	67.5 30.165	68.0	68.3	1	56 39	56 47	12 56 43	1.72
		72.0 30.185	71.5	72.0	58	16 5	16 18	17 56 11 $\frac{1}{2}$	0.37
"		72.0 30.185	71.6	72.2	1	1 37	1 45	18 1 41	1.79
"		72.5 30.102	70.6	71.2	54	39 43	39 57	22 39 50	0.39
"		72.5 30.101	71.2	71.7	1	44 54	45 11	22 45 2 $\frac{1}{2}$	0.92
"		71.0 30.086	69.8	70.2	20	24 48	25 7	0 24 57 $\frac{1}{2}$	0.58
20	{	67.5 30.023	67.2	67.8	1	19 24	19 31	11 19 27 $\frac{1}{2}$	1.77
		72.0 29.995	73.8	74.2	63	44 56	45 7	16 45 1 $\frac{1}{2}$	0.34
"		72.5 29.994	74.9	74.9	1	51 39	51 47	16 51 43	1.74
"		75.0 29.909	76.8	77.1	52	18 26	18 44	21 18 35	0.44
"	{	75.0 29.908	77.0	77.0	1	30 29	30 46	21 30 37 $\frac{1}{2}$	1.31
		74.5 29.901	74.8	75.3	36	33 38	33 55	0 33 46 $\frac{1}{2}$	0.50
Mean..		71.360 30.036							

Second Series computed.

1829.	Mean Temp.	Interval.	Uncorrected No. of Vibrations.	Clock's Rate.	Correc. for Arc.	Reduc. to 62° F _{AHR} .	Reduced Vib ^{ns} in 24 Mean Solar Hours.
		h m s					
	66.925	4 33 54	86088.34	-0.03	+1.628	+2.073	86092.04
	71.875	5 4 18	86086.14	0.03	1.757	4.157	86092.02
	71.775	1 18 43	86086.28	0.03	1.427	4.115	86091.79
	70.600	1 29 20 $\frac{1}{2}$	86087.04	0.03	0.849	3.621	86091.48
	69.950	4 59 28 $\frac{1}{2}$	86086.90	0.17	1.505	3.347	86091.54
	71.400	4 38 9	86086.30	0.17	1.641	3.957	86091.73
	70.725	1 39 55	86087.40	0.17	0.906	3.673	86091.81
	70.750	5 25 34	86086.59	0.08	1.499	3.684	86091.69
	75.925	4 26 52	86084.68	0.08	1.689	5.862	86092.15
	76.025	3 3 9	86084.68	-0.08	+1.248	+5.905	86091.75
Mean..	71.595				Mean....		86091.76
					Buoyancy.....		+5.90
(No. of Coincidences = 419) Number of Vibrations in vacuo, } ..							86097.66
in 24 Mean Solar Hours .. }							

Third Series computed.

1829.	Mean Temp.	Interval.	Uncorrected No. of Vibrations.	Clock's Rate.	Correc. for Arc.	Reduc. to 62° FAHR.	Reduced Vib ^{as} in 24 Mean Solar Hours.
		^{h m s}					
	71.100	2 21 46	86086.56	—0.01	+1.361	+3.831	86091.74
	67.300	5 21 2	86087.82	+0.14	1.587	2.231	86091.78
	73.400	1 59 39	86085.38	+0.14	1.567	4.799	86091.89
	75.825	2 37 17	86085.72	+0.14	0.675	5.820	86092.35
	74.350	2 47 37	86085.22	+0.14	1.336	5.199	86091.89
	73.325	5 3 12	86085.12	+0.14	1.640	4.768	86091.67
	74.150	5 35 20	86085.36	—0.02	1.167	5.115	86091.62
	80.825	1 44 27	86083.60	—0.02	0.974	7.925	86092.45
	81.250	2 41 40	86082.82	—0.02	1.098	8.104	86092.00
	79.675	2 5 11	86082.88	—0.02	1.241	7.441	86091.54
	76.975	5 34 40	86084.30	—0.02	1.428	6.304	86092.01
	80.375	2 20 50	86082.92	—0.02	1.348	7.736	86091.98
	80.600	2 26 0	86082.72	—0.02	1.464	7.831	86091.99
	79.175	1 33 53	86082.86	—0.02	+1.302	+7.231	86091.37
Mean..	76.301	Mean Buoyancy					86091.85 +5.85
(No. of Coincidences = 507) Number of Vibrations in vacuo, } in 24 Mean Solar Hours. } ..							86097.70

Rate of the Sidereal Clock.

Dec 23rd.		Dec. 24th.		Dec. 25th, 26th, & 27th.	
By α Ceti	^s —0.02	By α Ceti	^s +0.02	By Rigel	^s +0.09
Aldebaran	—0.01	Aldebaran	0.26	β Tauri	+0.01
Rigel	—0.05	Rigel	0.17	δ Orionis.....	—0.10
β Tauri	—0.01	β Tauri	0.02	ζ —.....	—0.08
δ Orionis.....	+0.01	δ Orionis.....	0.00	Mean Rate..	—0.02
ζ —.....	—0.08	ζ —.....	+0.35		
Mean Rate ..	—0.01	Mean Rate..	+0.14		

Fourth Series.

1830.	Attached Therm.	Barom.	Therm.		No. of Coin.	Disapp.	Re-app.	Time of Coin. by the Sidereal Clock.	Arc.	Remarks.
			A.	B.						
Jan. 22 {	67.5	30.088	67.8	68.2	1	16 11	16 29	16 16 20	1.40	Shut the clock door, and kept the Northern chase open.
	68.0	30.085	67.2	67.4	11	8 47	9 5	17 8 56	1.03	
	68.0	30.085	67.8	68.1	1	14 4	14 20	17 14 12	1.01	
	69.0	30.085	68.2	68.6	11	6 47	7 3	18 6 55	0.76	
	69.0	30.085	69.2	69.7	1	12 4	12 20	18 12 12	0.72	
	70.0	30.087	69.8	70.2	11	4 48	5 4	19 4 56	0.52	
	72.0	30.087	72.0	72.3	1	18 30½	18 36½	19 18 33½	1.91	
	72.0	30.084	71.5	71.7	11	10 47	11 2	20 10 54½	1.36	
	72.0	30.084	72.6	73.2	1	16 1	16 17	20 16 9	1.34	Closed the clockdoor, the Northern chase still open.
	72.0	30.062	71.2	71.5	11	8 31	8 47	21 8 39	0.97	
	72.0	30.062	71.7	71.9	1	13 45½	14 2½	21 13 54	0.93	
	72.0	30.055	70.5	70.8	11	6 20	6 36	22 6 28	0.71	
	72.5	30.037	71.2	71.5	1	10 21	10 37	23 10 29	1.47	
	72.5	30.035	70.2	70.4	12	8 1½	8 18½	0 8 10	1.06	
	72.5	30.035	71.2	71.5	1	13 17	13 33	0 13 25	1.02	
	72.5	30.043	70.0	70.3	11	5 51½	6 8½	1 6 0	0.78	
Mean..	70.844	30.063								

Fourth Series computed.

1830.	Mean Temp.	Interval.	Uncorrected No. of Vibrations.	Clock's Rate.	Correc. for Arc.	Reduc. to 62° FAHR.	Reduced Vib ^{ns} in 24 Mean Solar Hours.
		h m s					
	67.650	0 52 36	86087.52	-0.93	+2.481	+2.379	86091.45
	68.175	0 52 43	86088.76	0.93	1.275	2.600	86091.70
	69.725	0 52 44	86088.90	0.93	0.624	3.252	86091.85
	71.875	0 52 21	86084.94	0.93	4.340	4.157	86092.51
	72.125	0 52 30	86086.48	0.93	2.168	4.263	86091.98
	71.225	0 52 34	86087.18	0.93	1.095	3.984	86091.33
	70.825	0 57 41	86085.86	0.93	2.600	3.715	86091.24
	70.750	0 52 35	86087.34	-0.93	+2.053	+3.684	86092.15
Mean..	70.294	Mean.... Buoyancy.....					86091.78 +5.91
(No. of Coincidences = 81) Number of Vibrations in vacuo, } in 24 Mean Solar Hours.. }							86097.69

Rate of the Sidereal Clock.

Jan. 22nd.	
By Castor.....	^s -0.90
ε Geminor.....	0.96
Sirius	0.82
δ Geminor.....	0.90
Castor.....	0.80
Procyon	1.02
Pollux.....	-1.08
Mean Rate ..	-0.93

Abstract of Mr. FALLOWS's observations.

Mean of 401 coincidences . . . = 86097.67

Mean of 419 coincidences . . . = 86097.66

Mean of 507 coincidences . . . = 86097.70

Mean of 81 coincidences . . . = 86097.69

True Mean of 1408 coincidences . . . = 86097.68

NOTE BY CAPTAIN SABINE.

The observations which Mr. FALLOWS has communicated to the Society in this memoir, having been corrected for buoyancy and expansion, before the volume of the Phil. Trans. had reached the Cape, in which the true value of those corrections is assigned from experiments with a pendulum of the same materials and figure as that employed by Mr. FALLOWS, I have re-computed his results with the correct elements of reduction, and find the retardation of the vibrations at the Cape, compared with those in London, to be 67.15 per diem, instead of 67.12, the difference between Mr. FALLOWS's calculation and mine amounting only to three hundredths of a vibration per diem.

The small amount of the difference, on the employment of the more correct elements of reduction, is an illustration of the remark, with which I concluded the paper on the reduction to a vacuum of an invariable pendulum (Phil. Trans. 1829, page 236.) ; that in relative experiments, computed before the true reduction to a vacuum was known, and in which a correction for expansion was employed, derived directly from the vibration of the pendulum at the same spot in different temperatures, (as is the case in Mr. FALLOWS's calculation,) a compensation takes place of the errors of the respective reductions for expansion and resistance, leaving the only uncompensated error in the final result, that arising from barometric differences, which in all cases of comparison between stations not far removed from the level of the sea, cannot be otherwise than extremely small.

In Mr. FALLOWS's calculation he has taken the rate in London of the invariable pendulum which Captain RONALD took out to the Cape, solely from my observations with it : if, however, Captain RONALD's observations with the same pendulum in London be added to mine, and a true mean be taken corresponding to the number of observations of each observer, the retardation is precisely that stated by Mr. FALLOWS ; namely, 67.12 vibrations per diem.