XXII. *Researches in physical astronomy.* *By* JOHN WILLIAM LUBBOCK, *Esq. Fellow of the Royal Society.*

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IN the first volume of the Mécanique Céleste, LAPLACE has given expressions for the variations of the elliptic constants, which are true when the square and higher powers of the disturbing force are neglected; and he has proved, upon the supposition that the planets move in the same direction, in orbits nearly circular and little inclined one to another, that the eccentricities and inclinations vary within small limits, thereby demonstrating within these conditions the stability of the planetary system. But these conditions are not necessary to the stability of a system of bodies, subject to the law of attraction, which obtains in our system. I have given in the following investigation the expressions for the variations of the elliptic constants, which are rigorously true whatever power of the disturbing force be retained; and it is easy to conclude from the form of their expressions, that however far the approximation be carried, the eccentricity, the major axis, and the tangent of the inclination of the orbit to a fixed plane, contain no term which varies with the time; their variations are all periodic, and they oscillate therefore within certain limits. This theorem is no longer true if the planet moves in a resisting medium.

I have also given some equations which obtain when an angle is taken for the independent variable, which in the elliptic movement is the eccentric anomaly, which are of remarkable simplicity, and which, as far as I know, have never been noticed, and the development of the disturbing function R to the quantities involving the squares and products of the eccentricities inclusive.

Let x, y, z denote the rectangular co-ordinates
r ...... distance from the sun
r' ...... distance from the sun projected upon the plane x, y
*λ ...... longitude reckoned upon the plane of its orbit
λ' ...... longitude reckoned upon the plane $x\,y$
s ...... tangent of the latitude
v .... a variable, which in the elliptic theory is the eccentric anomaly
m .... the mass
a ...... semiaxis major
e ...... eccentricity
ϖ ...... longitude of the perihelion
ε ...... longitude of the epoch
ν ...... longitude of the ascending node
ι ...... inclination of the orbit to the plane $x\,y$
α .... a constant quantity which accompanies v

of the planet m.

M .... the mass of the sun.

$$M + m = \mu, \quad \sqrt{\frac{\mu}{a^3}} = n.$$

$$x = r'\cos\lambda', \; y = r'\sin\lambda', \; z = r's, \text{ and in the elliptic motion } r'^2 d\lambda' = h\,d\,t.$$

$$R = m_{\prime}\left\{\frac{x\,x_{\prime} + y\,y_{\prime} + z\,z_{\prime}}{\{x_{\prime}^2 + y_{\prime}^2 + z_{\prime}^2\}^{\frac{3}{2}}} - \frac{1}{\{(x - x_{\prime})^2 + (y - y_{\prime})^2 + (z - z_{\prime})^2\}^{\frac{1}{2}}}\right\}$$

$$= m_{\prime}\left\{\frac{r'\{\cos(\lambda' - \lambda'_{\prime}) + s\,s_{\prime}\}}{r'^2_{\prime}(1 + s_{\prime}^2)^{\frac{3}{2}}} - \frac{1}{\left\{r'^2(1+s^2) - 2r'r'_{\prime}\{\cos(\lambda' - \lambda'_{\prime}) + s\,s_{\prime}\} + r'^2_{\prime}(1 + s_{\prime}^2)\right\}^{\frac{1}{2}}}\right\}$$

* LAPLACE uses the letter v to denote longitude, u the eccentric anomaly, and φ the inclination of the orbit to a fixed plane; but as v is very frequently used to signify velocity, and φ geographical latitude, and as the letters of the Greek alphabet are generally used for angles, I have taken the letters λ, v, and ι for these quantities.

$$= - m_{,} \left\{ \begin{array}{l} \dfrac{1}{r'_{,} \sqrt{1 + s_{,}^2}} + \dfrac{r'^2}{4 r_{,}'^3} \{1 + 3 \cos (2 \lambda' - 2 \lambda_{,}') + 12 s s_{,} \cos (\lambda' - \lambda_{,}') - 2 s^2\} \\ \qquad + \dfrac{r'^3}{8 r_{,}'^4} \{3 (1 - 4 s^2) \cos (\lambda' - \lambda_{,}') + 5 \cos (3 \lambda' - 3 \lambda_{,}')\} \\ \qquad + \dfrac{r'^4}{64 r_{,}'^5} \{9 + 20 \cos (2 \lambda' - 2 \lambda_{,}') + 35 \cos (4 \lambda' - 4 \lambda_{,}')\} \end{array} \right\}$$

$$\left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) = m_{,} \left\{ \frac{\cos (\lambda' - \lambda_{,}') + s s_{,}}{r_{,}'^2 (1 + s_{,}^2)^{\frac{3}{2}}} + \frac{r' (1 + s^2) - r_{,}' \{\cos (\lambda' - \lambda_{,}') + s s_{,}\}}{\{r'^2 (1 + s^2) - 2 r' r_{,}' \{\cos (\lambda' - \lambda_{,}') + s s_{,}\} + r_{,}'^2 (1 + s_{,}^2)\}^{\frac{3}{2}}} \right\}$$

$$\left(\frac{\mathrm{d} R}{\mathrm{d} \lambda}\right) = - m_{,} \left\{ \frac{r' \sin \lambda' - \lambda_{,}'}{r_{,}'^2 (1 + s'^2)^{\frac{3}{2}}} - \frac{r' r_{,}' \sin (\lambda' - \lambda_{,}')}{\left\{r'^2 (1 + s^2) - 2 r' r_{,}' \{\cos (\lambda' - \lambda_{,}') + s s_{,}\} + r_{,}'^2 (1 + s_{,}^2)\right\}^{\frac{3}{2}}} \right\}$$

$$\left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) = m_{,} \left\{ \frac{r' s_{,}}{r_{,}'^2 (1 + s_{,}^2)^{\frac{3}{2}}} + \frac{r'^2 s - r' r_{,}' s_{,}}{\left\{r'^2 (1 + s^2) - 2 r' r_{,}' \{\cos (\lambda' - \lambda_{,}') + s s_{,}\} + r_{,}'^2 (1 + s_{,}^2)\right\}^{\frac{3}{2}}} \right\}$$

Let P be the place of the planet m, S the place of the sun, S N the intersection of the orbits of m and $m_{,}$, S L the line from which longitudes are reckoned, $P_{,}'$, the projection of $P_{,}$ upon the plane of the orbit of P; then if the plane $x y$ coincide with the orbit of m, $SP = r$, $PSL = \lambda$, $NSL = \nu$, $P_{,} SN + NSL = \lambda_{,}$, $P_{,}' SL = \lambda_{,}'$, $SP_{,}' = r_{,}'$, $SP_{,} = r_{,} = r_{,}' (1 + s_{,}^2)^{\frac{1}{2}}$

$$R = m_{,} \left\{ \frac{SP \times SP_{,} \cos PSP_{,}}{SP_{,}^3} - \frac{1}{\{SP^2 - 2 SP \times SP_{,} \cos PSP_{,} + SP_{,}^2\}^{\frac{1}{2}}} \right\}$$

$$\left(\frac{\mathrm{d} R}{\mathrm{d} r}\right) = m_{,} \left\{ \frac{SP_{,}' \cos PSP_{,}'}{SP_{,}^3} + \frac{SP - SP_{,}' \cos PSP_{,}'}{\{SP^2 - 2 SP \times SP_{,}' \cos PSP_{,}' + SP_{,}^2\}^{\frac{3}{2}}} \right\}$$

$$r \left(\frac{\mathrm{d} R}{\mathrm{d} r}\right) = m_{,} \left\{ \frac{SP \times SP_{,} \cos PSP_{,}}{SP_{,}^3} + \frac{SP^2 - SP \times SP_{,} \cos PSP_{,}}{\{SP^2 - 2 SP \times SP_{,}' \cos PSP_{,}' + SP_{,}^2\}^{\frac{3}{2}}} \right\}$$

$$\left(\frac{\mathrm{d} R}{\mathrm{d} \lambda}\right) = - m_{,} \left\{ \frac{SP \times SP_{,}' \sin PSP_{,}'}{SP_{,}^3} - \frac{SP \times SP_{,}' \sin PSP_{,}'}{\{SP^2 - 2 SP \times SP_{,}' \cos PSP_{,}' + SP_{,}^2\}^{\frac{3}{2}}} \right\}$$

$$\left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) = m_{,} \left\{ \frac{SP \times P_{,} P_{,}'}{SP_{,}^3} - \frac{SP \times P_{,} P_{,}'}{\{SP^2 - 2 SP \times SP_{,}' \cos PSP_{,}' + SP_{,}^2\}^{\frac{3}{2}}} \right\}$$

$$\cos (\lambda - \lambda_{,}) = \cos (\lambda - \nu) \cos (\lambda_{,} - \nu) + \cos \iota \sin (\lambda - \nu) \sin (\lambda_{,} - \nu)$$

$$= \cos^2 \frac{\iota}{2} \cos (\lambda - \lambda_{,}) + \sin^2 \frac{\iota}{2} \cos (\lambda + \lambda_{,} - 2 \nu)$$

$$\tan (\lambda_{,}' - \nu) = \cos \iota \tan (\lambda_{,} - \nu)$$

$$\sin (\lambda_{,}' - \nu) = \frac{\cos \iota \tan (\lambda_{,} - \nu)}{(1 + \cos^2 \iota \tan^2 (\lambda_{,} - \nu))^{\frac{1}{2}}}$$

$$\cos(\lambda'_{,} - \nu) = \frac{1}{(1 + \cos^2 \iota \tan^2 (\lambda_{,} - \nu))^{\frac{1}{2}}}$$

$$r\, r'_{,} \sin(\lambda - \lambda'_{,}) = r\, r'_{,} \{\sin \lambda \cos \lambda'_{,} - \cos \lambda \sin \lambda_{,}\}$$

$$\cos \lambda'_{,} = \cos(\lambda'_{,} - \nu) \cos \nu - \sin(\lambda'_{,} - \nu) \sin \nu$$

$$= \frac{\cos(\lambda_{,} - \nu)\cos\nu - \sin(\lambda_{,} - \nu)\sin\nu\left(1 - 2\sin^2\frac{\iota}{2}\right)}{(1 - \sin^2\iota \sin^2(\lambda_{,} - \nu))^{\frac{1}{2}}}$$

$$= \frac{\cos\lambda_{,} + 2\sin^2\frac{\iota}{2}\sin(\lambda_{,} - \nu)\sin\nu}{(1 - \sin^2\iota\sin^2(\lambda_{,} - \nu))^{\frac{1}{2}}}$$

$$\sin \lambda'_{,} = \sin(\lambda'_{,} - \nu)\cos\nu + \cos(\lambda'_{,} - \nu)\sin\nu$$

$$= \frac{\sin(\lambda_{,} - \nu)\cos\nu\left(1 - 2\sin^2\frac{\iota}{2}\right) + \cos(\lambda_{,} - \nu)\sin\nu}{(1 - \sin^2\iota\sin^2(\lambda_{,} - \nu))^{\frac{1}{2}}}$$

$$\sin \mathrm{P}_{,}\mathrm{S}\,\mathrm{P}'_{,} = \sin\iota\sin(\lambda_{,} - \nu), \quad r'_{,} = r_{,}\cos \mathrm{P}_{,}\mathrm{S}\,\mathrm{P}'_{,} = r_{,}\left(1 - \sin^2\iota\sin^2(\lambda_{,} - \nu)\right)^{\frac{1}{2}}$$

therefore,

$$r\, r'_{,}\sin(\lambda - \lambda'_{,}) = r\, r_{,}\left\{\sin(\lambda - \lambda_{,}) + 2\sin^2\frac{\iota}{2}\sin(\lambda_{,} - \nu)\cos(\lambda - \nu)\right\}$$

$$= r\, r_{,}\left\{\cos^2\frac{\iota}{2}\sin(\lambda - \lambda_{,}) + \sin^2\frac{\iota}{2}\sin(\lambda + \lambda_{,} - 2\nu)\right\}$$

similarly it may be shown that

$$r\, r'_{,}\cos(\lambda - \lambda'_{,}) = r\, r_{,}\left\{\cos^2\frac{\iota}{2}\cos(\lambda - \lambda_{,}) + \sin^2\frac{\iota}{2}\cos(\lambda + \lambda_{,} - 2\nu)\right\}$$

$$\mathrm{S\,P}_{,} = \frac{a_{,}(1 - e_{,}^2)}{1 + e_{,}\cos(\mathrm{P}_{,}\mathrm{S\,N} + \mathrm{N\,S\,L} - \varpi_{,})} = \frac{a_{,}(1 - e_{,}^2)}{1 + e_{,}\cos(\lambda_{,} - \varpi_{,})}$$

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \frac{\mu x}{r^3} + \left(\frac{\mathrm{d}R}{\mathrm{d}x}\right) = 0, \quad \frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + \frac{\mu y}{r^3} + \left(\frac{\mathrm{d}R}{\mathrm{d}y}\right) = 0, \quad \frac{\mathrm{d}^2 z}{\mathrm{d}t^2} + \frac{\mu z}{r^3} + \left(\frac{\mathrm{d}R}{\mathrm{d}z}\right) = 0$$

$$\frac{r'^2\mathrm{d}\lambda'^2 + \mathrm{d}r'^2(1+s^2) + 2r's\,\mathrm{d}r'\mathrm{d}s + r'^2\mathrm{d}s^2}{\mathrm{d}t^2} - \frac{2\mu}{r'(1+s^2)^{\frac{1}{2}}} + \frac{\mu}{a} + 2\int \mathrm{d}R = 0$$

d R being the differential of R with regard only to the co-ordinates of the planet m.

$$\frac{\mathrm{d}^2 r'(1+s^2) - r'\mathrm{d}\lambda'^2 + 2s\,\mathrm{d}r'\mathrm{d}s + r's\,\mathrm{d}^2 s}{\mathrm{d}t^2} + \frac{\mu}{r'(1+s^2)^{\frac{1}{2}}} + \left(\frac{\mathrm{d}R}{\mathrm{d}r'}\right) = 0$$

$$\frac{\mathrm{d}.\frac{r'^2\mathrm{d}\lambda'}{\mathrm{d}t}}{\mathrm{d}t} + \left(\frac{\mathrm{d}R}{\mathrm{d}\lambda'}\right) = 0$$

$$\frac{r's\,\mathrm{d}^2 r' + 2r'\mathrm{d}r'\mathrm{d}s + r'^2\mathrm{d}^2 s}{\mathrm{d}t^2} + \frac{s}{r'(1+s^2)^{\frac{3}{2}}} + \left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) = 0$$

$$\frac{r' d^2 r' - r'^2 d\lambda'^2}{dt^2} + \frac{\mu}{r'(1+s^2)^{\frac{1}{2}}} + r'\left(\frac{dR}{dr'}\right) - s\left(\frac{dR}{ds}\right) = 0$$

$$\frac{r'^2 d^2 s + 2 r' dr' ds - r'^2 s\, d\lambda'^2}{dt^2} + (1+s^2)\left(\frac{dR}{ds}\right) - r' s\left(\frac{dR}{dr'}\right) = 0$$

$$\frac{d^2 . r'^2 (1+s^2)}{2 dt^2} - \frac{\mu}{r'(1+s^2)^{\frac{1}{2}}} + \frac{\mu}{a} + 2\int dR + r'\left(\frac{dR}{dr'}\right) = 0$$

Making λ the independent variable instead of t,

$$\frac{d^2 r'}{dt^2} - 2\frac{dr' d^2 t}{dt^3} - r'^2 \frac{d\lambda'^2}{dt^2} + \frac{\mu}{r'(1+s^2)^{\frac{1}{2}}} + r'\left(\frac{dR}{dr'}\right) - s\left(\frac{dR}{ds}\right) = 0$$

$$r'^4 \frac{d\lambda'^2}{dt^2} = h^2 - 2\int r'^2\left(\frac{dR}{d\lambda'}\right) d\lambda',\ h \text{ being a constant,}$$

$$4 r'^3 dr' \frac{d\lambda'^2}{dt^2} - 2 r'^4 \frac{d\lambda'^2 d^2 t}{dt^3} = -2 r'^2 \left(\frac{dR}{d\lambda'}\right) d\lambda'$$

$$\left\{\frac{d^2 . \frac{1}{r'}}{d\lambda'^2} + \frac{1}{r'}\right\}\left\{1 - \frac{2}{h^2}\int r'^2\left(\frac{dR}{d\lambda'}\right) d\lambda'\right\} - \frac{\mu}{h^2(1+s^2)^{\frac{3}{2}}}$$

$$- \frac{r'}{h^2}\left\{r'\left(\frac{dR}{dr'}\right) - s\left(\frac{dR}{ds}\right) - \frac{1}{r'}\left(\frac{dR}{d\lambda'}\right)\frac{dr'}{d\lambda'}\right\} = 0$$

$$\left\{\frac{d^2 s}{d\lambda'^2} + s\right\}\left\{1 - \frac{2}{h^2}\int r'^2\left(\frac{dR}{d\lambda'}\right) d\lambda'\right\}$$

$$+ \frac{r'^2}{h^2}\left\{(1+s^2)\left(\frac{dR}{ds}\right) - r'\left(\frac{dR}{dr'}\right) - \left(\frac{dR}{d\lambda'}\right)\frac{ds}{d\lambda'}\right\} = 0$$

When the disturbing force is neglected

$$\frac{d . r'^2 d\lambda'}{dt'} = 0, \quad \frac{d^2 . \frac{1}{r'}}{d\lambda'^2} + \frac{1}{r'} - \frac{\mu}{h^2(1+s^2)^{\frac{3}{2}}} = 0, \quad \frac{d^2 s}{d\lambda'^2} + s = 0$$

of which equations the integrals are,

$$r'^2 d\lambda' = h\, dt, \quad \frac{1}{r'} = \frac{\mu \cos \iota^2}{h^2}\left\{(1+s^2)^{\frac{1}{2}} + e\cos(\lambda' - \varpi)\right\}$$

$$s = \tan \iota \sin(\lambda' - \nu).$$

If $dt = \sqrt{\frac{a}{\mu}}\, r'\, dv$, and v be taken for the independent variable

$$r' \frac{d^2 r'}{dt^2} - r' dr' \frac{d^2 t}{dt^3} - r'^2 \frac{d\lambda'^2}{dt^2} + \frac{\mu}{r'(1+s^2)^{\frac{1}{2}}} + r'\left(\frac{dR}{dr'}\right) - s\left(\frac{dR}{ds}\right) = 0$$

$$\mathrm{d}^2 t = \left(\frac{a}{\mu}\right)^{\frac{1}{2}} \mathrm{d} r' \, \mathrm{d} v$$

$$\frac{\mathrm{d}^2 r'}{\mathrm{d} v^2} + \left(\frac{\mathrm{d}.r' s}{\mathrm{d} v}\right)^2 - \frac{a}{(1+s^2)^{\frac{1}{2}}} + r' + \frac{a r'}{\mu}\left\{2\int \mathrm{d} R + r'\left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - s\left(\frac{\mathrm{d} R}{\mathrm{d} s}\right)\right\} = 0.$$

If the orbit of the planet m coincide with the plane $x\,y$, s is of the order of the disturbing force, of which therefore neglecting the square, $r' = r$, $\lambda' = \lambda$

$$\frac{\mathrm{d}^2 r}{\mathrm{d} v^2} - a + r + \frac{a r}{\mu}\left\{2\int \mathrm{d} R + r\left(\frac{\mathrm{d} R}{\mathrm{d} r}\right)\right\} = 0.$$

When the disturbing force is neglected, the integral of this equation is $r = a\{1 - e\cos(v - \alpha)\}$, v being the eccentric anomaly.

$$n t + \varepsilon - \varpi = v - \alpha - e \sin(v - \alpha)$$

$$\tan\frac{\lambda - \varpi}{2} = \left\{\frac{1+e}{1-e}\right\}^{\frac{1}{2}} \tan\frac{v - \alpha}{2}.$$

If Q be put for the quantity $\frac{a r}{\mu}\left\{2\int \mathrm{d} R + r\left(\frac{\mathrm{d} R}{\mathrm{d} r}\right)\right\}$ and the constant α, which may afterwards be replaced, be omitted for the present,

$$r = a\{1 - e\cos v\} - \sin v \int Q \cos v \, \mathrm{d} v + \cos v \int Q \sin v \, \mathrm{d} v$$

$$\mathrm{d} t = \sqrt{\frac{a}{\mu}}\left\{a\{1 - e\cos v\} - \sin v \int Q \cos v \, \mathrm{d} v + \cos v \int Q \sin v \, \mathrm{d} v\right\} \mathrm{d} v$$

$$n t + \varepsilon - \varpi = v - e \sin v - \frac{1}{a}\left\{\int Q \, \mathrm{d} v - \cos v \int Q \cos v \, \mathrm{d} v - \sin v \int Q \sin v \, \mathrm{d} v\right\}$$

If $v = \mathrm{f}(n\,t + \varepsilon - \varpi)$ in the elliptic theory, then neglecting the square of the disturbing force,

$$v = \mathrm{f}(n t + \varepsilon - \varpi) + \frac{\mathrm{d}.\mathrm{f}(n t + \varepsilon - \varpi)}{\mathrm{d} t}\frac{1}{a}\left\{\int Q \, \mathrm{d} v - \cos v \int Q \cos v \, \mathrm{d} v - \sin v \int Q \sin v\right\} \mathrm{d} v$$

If $\delta\, v$, $\delta\, r$ denote the values of those parts of r and v which are due to the first power of the disturbing force,

$$\delta\, v = \frac{\mathrm{d} v}{a n \mathrm{d} t}\left\{\int Q \, \mathrm{d} v - \cos v \int Q \cos v \, \mathrm{d} v - \sin v \int Q \sin v \, \mathrm{d} v\right\}$$

$$\delta\, r = a\, e \sin v \,\delta\, v - \sin v \int Q \cos v \, \mathrm{d} v + \cos v \int Q \sin v \, \mathrm{d} v$$

$$= \frac{\sin v}{1 - e\cos v}\int Q\,\{e - \cos v\}\, \mathrm{d} v - \frac{e - \cos v}{1 - e\cos v}\int Q \sin v \, \mathrm{d} v.$$

In the elliptic theory,

$$\mathrm{d}\, v = \frac{n \,\mathrm{d} t}{1 - e\cos v}, \qquad \frac{\cos v - e}{1 - e\cos v} = \cos\lambda, \qquad \frac{\sin v\,(1 - e^2)^{\frac{1}{2}}}{1 - e\cos v} = \sin\lambda$$

therefore,

$$\delta r = \frac{a\,n\cos\lambda\int r\sin\lambda\left\{2\int \mathrm{d}R + r\left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)\right\}\mathrm{d}t - a\,n\sin\lambda\int r\cos\lambda\left\{2\int\mathrm{d}R + r\left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)\right\}\mathrm{d}t\}}{\mu(1-e^2)^{\frac{1}{2}}}$$

which is the equation X of the Mécanique Céleste, vol. i. p. 258.

Multiplying the equation of p. 114, l. 2, by $\frac{2r'^2\mathrm{d}\lambda'}{\mathrm{d}t}$, and integrating, $\frac{r'^4\mathrm{d}\lambda'^2}{\mathrm{d}t^2} + 2\int r'^2\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda'}\right)\mathrm{d}\lambda' = h_0^2$, if $h^2 = h_0^2 - 2\int r'^2\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda'}\right)\mathrm{d}\lambda'$, h being variable, and h_0 the value of h at a given epoch, $h\,\mathrm{d}h = -r'^2\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda'}\right)\mathrm{d}\lambda'$, $r^2\,\mathrm{d}\lambda' = h\,\mathrm{d}t$, and making λ' the independent variable instead of t, $2\,r'\,\mathrm{d}r'\,\mathrm{d}\lambda' = \mathrm{d}h\,\mathrm{d}t + h\,\mathrm{d}^2t$

$$r'\frac{\mathrm{d}^2r'}{\mathrm{d}t^2} - \frac{r'\mathrm{d}r'\mathrm{d}^2t}{\mathrm{d}t^3} - \frac{r^2\mathrm{d}\lambda'^2}{\mathrm{d}t^2} + \frac{\mu}{r'(1+s^2)^{\frac{3}{2}}} + r'\left(\frac{\mathrm{d}R}{\mathrm{d}r'}\right) - s\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) = 0$$

$$r'\left(\frac{\mathrm{d}^2r'}{\mathrm{d}t^2}\right) + \frac{r'\mathrm{d}r'\mathrm{d}t}{h\,\mathrm{d}t^2} - \frac{2r'^2\mathrm{d}r'^2\mathrm{d}\lambda}{h\,\mathrm{d}t^3} - \frac{r'^2\mathrm{d}\lambda'^2}{\mathrm{d}t^2} + \frac{\mu}{r'(1+3^2)^{\frac{3}{2}}} + r'\left(\frac{\mathrm{d}R}{\mathrm{d}r'}\right) - s\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) = 0$$

$$\frac{\mathrm{d}^2.\frac{1}{r'}}{\mathrm{d}\lambda'^2} + \frac{1}{r'} - \frac{\mu}{h^2(1+s^2)^{\frac{3}{2}}} - \frac{r'}{h^2}\left\{r'\left(\frac{\mathrm{d}R}{\mathrm{d}r'}\right) - s\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) - \frac{1}{r'}\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda'}\right)\frac{\mathrm{d}r'}{\mathrm{d}\lambda'}\right\} = 0$$

$$\frac{\mathrm{d}^2s}{\mathrm{d}\lambda'^2} + s + \frac{r'^2}{h^2}\left\{(1+s^2)\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) - s\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) - \left(\frac{\mathrm{d}R}{\mathrm{d}\lambda'}\right)\frac{\mathrm{d}s'}{\mathrm{d}\lambda'}\right\} = 0$$

If all the constants in the elliptic integrals are supposed to vary, subject to the condition that they still satisfy these differential equations, and that the form of the first differential coefficients $\frac{\mathrm{d}r'}{\mathrm{d}\lambda'}$, $\frac{\mathrm{d}s}{\mathrm{d}\lambda'}$, remains unaltered,

$$\frac{\mathrm{d}.\frac{1}{r'}}{\mathrm{d}\lambda'} = \frac{\mu\cos\iota^2}{h^2}\left\{\frac{s}{(1+s^2)^{\frac{1}{2}}}\frac{\mathrm{d}s}{\mathrm{d}\lambda'} - e\sin(\lambda'-\varpi)\right\}, \qquad \frac{\mathrm{d}s}{\mathrm{d}\lambda'} = \tan\iota\cos(\lambda'-\nu)$$

$$-2\{(1+s^2)^{\frac{1}{2}} + e\cos(\lambda'-\varpi)\}\{h^2\sin\iota\,\mathrm{d}\iota + \cos\iota\,h\,\mathrm{d}h\}$$

$$+h^2\cos\iota\cos(\lambda'-\varpi)\,\mathrm{d}e + h^2\cos\iota\,e\sin(\lambda'-\varpi)\,\mathrm{d}\varpi = 0$$

$$-2\left\{\frac{s\tan\iota\cos(\lambda'-\nu)}{(1+s^2)^{\frac{1}{2}}} - e\sin(\lambda'-\varpi)\right\}\{h^2\sin\iota\,\mathrm{d}\iota + \cos\iota\,h\,\mathrm{d}h\}$$

$$-h^2\cos\iota\sin(\lambda'-\varpi)\,\mathrm{d}e + h^2\cos\iota\,e\cos(\lambda'-\varpi)\,\mathrm{d}\varpi$$

$$-\frac{\cos\iota\,s\,r'^2}{(1+s^2)^{\frac{1}{2}}}\left\{(1+s^2)\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) - r'\left(\frac{\mathrm{d}R}{\mathrm{d}r'}\right) - \left(\frac{\mathrm{d}R}{\mathrm{d}\lambda'}\right)\left(\frac{\mathrm{d}s}{\mathrm{d}\lambda'}\right)\right\}\mathrm{d}\lambda'$$

$$- \frac{h^2 r'}{\mu \cos \iota} \left\{ r' \left(\frac{d R}{d r'}\right) - s \left(\frac{d R}{d s}\right) - \frac{1}{r'} \left(\frac{d R}{d \lambda'}\right) \left(\frac{d r'}{d \lambda'}\right) \right\} d \lambda' = 0$$

$$\sin (\lambda' - \nu) \frac{d \iota}{\cos \iota^2} - \tan \iota \cos (\lambda' - \nu) d \nu = 0$$

$$\cos (\lambda' - \nu) \frac{d \iota}{\cos \iota^2} + \tan \iota \sin (\lambda' - \nu) d \nu + \frac{r'^2}{h^2} \left\{ (1 + s^2) \left(\frac{d R}{d s}\right) - s \left(\frac{d R}{d r'}\right) - \left(\frac{d R}{d \lambda'}\right) \left(\frac{d s}{d \lambda'}\right) \right\} d \lambda' = 0$$

$$h \, d h = - r'^2 \left(\frac{d R}{d \lambda'}\right) d \lambda'$$

Whence by elimination,

$$h^2 \cos \iota \, d e + 2 \left\{ (1 + s^2)^{\frac{1}{2}} \cos (\lambda' - \varpi) + e - \frac{\tan \iota \, s \cos (\lambda' - \nu) \sin (\lambda' - \varpi)}{(1 + s^2)^{\frac{1}{2}}} \right\}$$

$$\left\{ \sin \iota \cos^2 \iota \, r'^2 \cos (\lambda' - \nu) \left\{ (1 + s^2) \left(\frac{d R}{d s}\right) - s \left(\frac{d R}{d r'}\right) - \left(\frac{d R}{d \lambda'}\right) \left(\frac{d s}{d \lambda'}\right) \right\} + \cos \iota \, r'^2 \left(\frac{d R}{d \lambda'}\right) \right\} d \lambda'$$

$$+ \frac{s \cos \iota \, r'^2 \sin (\lambda' - \varpi)}{(1 + s^2)^{\frac{1}{2}}} \left\{ (1 + s^2) \left(\frac{d R}{d s}\right) - r' \left(\frac{d R}{d r'}\right) - \left(\frac{d R}{d \lambda'}\right) \left(\frac{d s}{d \lambda'}\right) \right\} d \lambda'$$

$$+ \frac{h^2 r'}{\mu \cos \iota} \sin (\lambda' - \varpi) \left\{ r' \left(\frac{d R}{d r'}\right) - s \left(\frac{d R}{d s}\right) - \frac{1}{r'} \left(\frac{d R}{d \lambda'}\right) \left(\frac{d r'}{d \lambda'}\right) \right\} d \lambda' = 0$$

$$h^2 \cos \iota \, e \, d \varpi + 2 \left\{ (1 + s^2)^{\frac{1}{2}} \sin (\lambda' - \varpi) + \frac{s \tan \iota \cos (\lambda' - \nu) \cos (\lambda' - \varpi)}{(1 + s^2)^{\frac{1}{2}}} \right\}$$

$$\left\{ \sin \iota \cos^2 \iota \, r'^2 \cos (\lambda' - \nu) \left\{ (1 + s^2) \left(\frac{d R}{d s}\right) - s \left(\frac{d R}{d r'}\right) - \left(\frac{d R}{d \lambda'}\right) \left(\frac{d s}{d \lambda'}\right) \right\} + \cos \iota \, r'^2 \left(\frac{d R}{d \lambda'}\right) \right\} d \lambda'$$

$$- \frac{s \cos \iota \, r'^2 \cos (\lambda - \varpi)}{(1 + s^2)^{\frac{1}{2}}} \left\{ (1 + s^2) \left(\frac{d R}{d s}\right) - r' \left(\frac{d R}{d r'}\right) - \left(\frac{d R}{d \lambda'}\right) \left(\frac{d s}{d \lambda'}\right) \right\} d \lambda'$$

$$- \frac{h^2 r'}{\mu \cos \iota} \cos (\lambda' - \varpi) \left\{ r' \left(\frac{d R}{d r'}\right) - s \left(\frac{d R}{d s}\right) - \frac{1}{r'} \left(\frac{d R}{d \lambda'}\right) \left(\frac{d r'}{d \lambda'}\right) \right\} d \lambda' = 0$$

$$d \iota + \frac{r'^2 \cos \iota^2 \cos (\lambda' - \nu)}{h^2} \left\{ (1 + s^2) \left(\frac{d R}{d s}\right) - s \left(\frac{d R}{d r'}\right) - \left(\frac{d R}{d \lambda'}\right) \left(\frac{d r'}{d \lambda'}\right) \right\} d \lambda' = 0$$

$$d \nu + \frac{r'^2 \sin (\lambda' - \nu)}{h^2 \tan \iota} \left\{ (1 + s^2) \left(\frac{d R}{d s}\right) - s \left(\frac{d R}{d r'}\right) - \left(\frac{d R}{d \lambda'}\right) \left(\frac{d s}{d \lambda'}\right) \right\} d \lambda' = 0$$

If $x = \frac{r \cos \lambda'}{(1 + s^2)^{\frac{1}{2}}}$, $\quad y = \frac{r \sin \lambda'}{(1 + s^2)^{\frac{1}{2}}}$, $\quad z = \frac{r s}{(1 + s^2)^{\frac{1}{2}}}$,

$$\frac{\mathrm{d}\, r^2 + \frac{r^2}{1 + s^2}\left(\frac{\mathrm{d}\, s^2}{1 + s^2} + \mathrm{d}\, \lambda'^2\right)}{\mathrm{d}\, t^2} - \frac{2\mu}{r} + \frac{\mu}{a} + 2\int \mathrm{d}\, R = 0$$

$$\frac{\mathrm{d}^2 r}{\mathrm{d}\, t^2} - \frac{r}{1 + s^2} \frac{\left(\frac{\mathrm{d}\, s^2}{1 + s^2} + \mathrm{d}\, \lambda'^2\right)}{\mathrm{d}\, t^2} + \frac{\mu}{r^2} + \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, r}\right) = 0$$

$$\frac{\mathrm{d} \, . \, \frac{r^2}{1 + s^2} \, . \, \frac{\mathrm{d}\, \lambda'}{\mathrm{d}\, t}}{\mathrm{d}\, t} + \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, \lambda'}\right) = 0$$

$$\frac{r^2 \,\mathrm{d}^2 s + 2\, r\, \mathrm{d}\, r\, \mathrm{d}\, s - \frac{2\, r^2 s\, \mathrm{d}\, s^2}{(1 + s^2)} + r^2 s\, \mathrm{d}\, \lambda'^2}{\mathrm{d}\, t^2} + (1 + s^2)^2 \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, s}\right) = 0$$

Of which equations the integrals are

$$\frac{r^2}{1 + s^2} \, . \, \mathrm{d}\, \lambda' = h\, \mathrm{d}\, t, \quad \frac{1}{r} = \frac{\mu \cos \iota^2}{h^2 (1 + s^2)^{\frac{1}{2}}} \left\{ (1 + s^2)^{\frac{1}{2}} + e \cos (\lambda' - \varpi) \right\}$$

$$s = \tan \iota \sin (\lambda' - \nu)$$

If $\mathrm{d}\, t = \sqrt{\frac{a}{\mu}}\, r\, \mathrm{d}\, v$, and v be taken for the independent variable

$$\frac{\mathrm{d}^2 r}{\mathrm{d}\, v^2} - a + r + \frac{a\, r}{\mu} \left\{ 2\int \mathrm{d}\, R + r \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, r}\right) \right\} = 0$$

$r = a\, \{1 - e' \cos (v - \alpha)\}$ in the elliptic motion.
e' is accented for the present in order to distinguish it from e.

If the constants in the elliptic integrals are supposed to vary, subject to the condition that they still satisfy these differential equations, and that the form of the first differential coefficient $\frac{\mathrm{d}\, r}{\mathrm{d}\, v}$ remains unaltered,

$$\mathrm{d}^2 t = \frac{r\, \mathrm{d}\, v\, \mathrm{d}\, a}{2 \sqrt{a \mu}} + \sqrt{\frac{a}{\mu}}\, \mathrm{d}\, r\, \mathrm{d}\, v, \quad \frac{r\, \mathrm{d}^2 r}{\mathrm{d}\, t^2} = \frac{r\, \mathrm{d}\, r^2}{\mathrm{d}\, t^3} - \frac{r\, \mathrm{d}\, r\, \mathrm{d}^2 t}{\mathrm{d}\, t^3}$$

$$\frac{\mathrm{d}^2 r}{\mathrm{d}\, v^2} - a + r + \frac{a\, r^2}{\mu} \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, r}\right) - \frac{\mathrm{d}\, r\, \mathrm{d}\, a}{2\, a\, \mathrm{d}\, v^2} = 0$$

$$\big(1 - e' \cos (v - \alpha)\big)\, \mathrm{d}\, a - a \cos (v - \alpha)\, \mathrm{d}\, e' - a\, e' \sin (v - \alpha)\, \mathrm{d}\, \alpha = 0$$

$$e' \sin (v - \alpha)\, \mathrm{d}\, a + a \sin (v - \alpha)\, \mathrm{d}\, e' - a\, e' \cos (v - \alpha)\, \mathrm{d}\, \alpha$$

$$+ \frac{a r^2}{\mu} \left(\frac{\mathrm{d} R}{\mathrm{d} r}\right) \mathrm{d}\, v + \frac{a^2 e}{\mu} \sin (v - \alpha) \,\mathrm{d} R = 0$$

$$\mathrm{d}\, a = - \frac{2 a^2}{\mu} \mathrm{d} R$$

$$\mathrm{d}\, e' = \frac{a}{\mu} \{2 e' - 2 \cos (v - \alpha) - e' \sin (v - \alpha)^2\} \,\mathrm{d} R - \frac{r^2}{\mu} \sin (v - \alpha) \left(\frac{\mathrm{d} R}{\mathrm{d} r}\right) \mathrm{d}\, v$$

$$e\, \mathrm{d}\, \alpha = \frac{a}{\mu} \sin (v - \alpha) \{e' \cos (v - \alpha) - 2\} \,\mathrm{d} R + \frac{r^2}{\mu} \cos (v - \alpha) \left(\frac{\mathrm{d} R}{\mathrm{d} r}\right) \mathrm{d}\, v$$

$$\int n\, \mathrm{d}\, t + \varepsilon - \varpi = v - \alpha - e' \sin (v - \alpha)$$

$$\mathrm{d}\, \varepsilon - \mathrm{d}\, \varpi = - \mathrm{d}\, \alpha - \sin (v - \alpha)\, \mathrm{d}\, e' + e' \cos (v - \alpha)\, \mathrm{d}\, \alpha.$$

The equation of condition which obtains between the constants a, e, ϖ, ν, ι and h may be found from the equation

$$\frac{\mathrm{d}\, r^2 + \frac{r^2}{1 + s^2} \left(\frac{\mathrm{d}\, s^2}{1 + s^2} + \mathrm{d}\, \lambda'^2\right)}{\mathrm{d}\, t^2} - \frac{2 \mu}{r} + \frac{\mu}{a} = 0$$

$$\frac{\mathrm{d}\, r^2}{\mathrm{d}\, t^2} + \frac{h^2}{r^2 \cos \iota^2} - \frac{2 \mu}{r} + \frac{\mu}{a} = 0$$

which gives

$$\frac{\mu \cos \iota^2}{h^2} \left\{ e^2 \cos^2 \iota \sin^2 (\nu - \varpi) - \{1 + e \cos (\nu - \varpi)\} \{1 - e \cos (\nu - \varpi)\} \right\} + \frac{1}{a} = 0$$

Equating the values of r which have been found,

$$\frac{h^2 \sqrt{1 + s^2}}{\mu \cos \iota^2 \{ \sqrt{1 + s^2} + e \cos (\lambda - \varpi)\}} = a \{1 - e' \cos (v - \alpha)\}$$

since the origin of the angle v is arbitrary, we may suppose $\lambda - \varpi = 0$, and $v - \alpha = 0$ at the same time,

so that $$\frac{h^2 \sqrt{1 + \tan \iota^2 \sin (\varpi - \nu)^2}}{\mu \cos \iota^2 \{ \sqrt{1 + \tan \iota^2 \sin (\varpi - \nu)^2} + e\}} = a\, (1 - e')$$

All the equations which have hitherto been proved are rigorously true, whatever powers of the disturbing function be retained. They are susceptible of simplification when the square and higher powers of the disturbing function are neglected: in this case, if the orbit be supposed to coincide with the plane $x\, y$, $\tan \iota = 0$, and if the longitude be reckoned from the perihelion of the planet P,

$$h^2\, \mathrm{d}\, e + \{2 \cos \lambda + e + e \cos \lambda^2\}\, r^2 \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda}\right) \mathrm{d}\, \lambda + \frac{h^2 r^2 \sin \lambda}{\mu} \left(\frac{\mathrm{d} R}{\mathrm{d} r}\right) \mathrm{d}\, \lambda = 0$$

$$h^2\, e\, \mathrm{d}\,\varpi + \{2 + \cos\lambda\} \sin\lambda\, r^2 \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda}\right) \mathrm{d}\,\lambda - h^2\, r^2 \cos\lambda \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right) \mathrm{d}\,\lambda = 0$$

$$\mathrm{d}\,\iota + r^2 \frac{\cos(\lambda - \nu)}{\mu} \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,s}\right) \mathrm{d}\,\lambda = 0$$

$$\mathrm{d}\,\nu + r^2 \frac{\sin(\lambda - \nu)}{\mu} \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,s}\right) \mathrm{d}\,\lambda = 0$$

by equation, p. 336, line 12, $\mu\, a\,(1 - e^2) = h^2$, and by equation, line 17, $e = e'$.

$$\mathrm{d}\,e = \frac{a}{\mu}\left\{2\,e - 2\cos(\upsilon - \alpha) - e\sin(\upsilon - \alpha)^2\right\}\mathrm{d}\,R - \frac{r^2}{\mu}\sin(\upsilon - \alpha)\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right)\mathrm{d}\,\upsilon$$

$$e\,\mathrm{d}\,\alpha = \frac{a}{\mu}\left\{e\cos(\upsilon - \alpha) - 2\right\}\sin(\upsilon - \alpha)\,\mathrm{d}\,R + \frac{r^2}{\mu}\cos(\upsilon - \alpha)\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right)\mathrm{d}\,\upsilon$$

$$\int n\,\mathrm{d}\,t + \varepsilon - \varpi = \upsilon - \alpha - e\sin(\upsilon - \alpha)$$

$$\mathrm{d}\,\varepsilon - \mathrm{d}\,\varpi = -\{1 - e\cos(\upsilon - \alpha)\}\,\mathrm{d}\,\alpha - \sin(\upsilon - \alpha)\,\mathrm{d}\,e$$

$$\mathrm{d}\,\varepsilon - \mathrm{d}\,\varpi = -\frac{r^2}{a^2\sqrt{1 - e^2}}\,\mathrm{d}\,\varpi - \left\{\frac{r}{a(1 - e^2)} + 1\right\}\sin(\upsilon - \alpha)\,\mathrm{d}\,e$$

$$\frac{\frac{h^2}{\mu}}{1 + e\cos(\lambda - \varpi)} = a\,\{1 - e\cos(\upsilon - \alpha)\}$$

$$\mathrm{d}\,e = -\frac{a\,n\,\mathrm{d}\,t}{\mu\sqrt{1 - e^2}}\left\{2\cos(\lambda - \varpi) + e + e\cos(\lambda - \varpi)^2\right\}\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda}\right)$$
$$- \frac{a^2\,n\,\mathrm{d}\,t\sqrt{1 - e^2}}{\mu}\sin(\lambda - \varpi)\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right)$$

$$e\,\mathrm{d}\,\varpi = -\frac{a\,n\,\mathrm{d}\,t}{\mu\sqrt{1 - e^2}}\left\{2 + e\cos(\lambda - \varpi)\right\}\sin(\lambda - \varpi)\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda}\right)$$
$$+ \frac{a^2\,n\,\mathrm{d}\,t\sqrt{1 - e^2}}{\mu}\cos(\lambda - \varpi)\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right)$$

$$\mathrm{d}\,\varepsilon - \mathrm{d}\,\varpi = -\sqrt{1 - e^2}\,\mathrm{d}\,\varpi + \frac{2\,a^2\,n\,(1 - e^2)\,\mathrm{d}\,t}{\mu\{1 + e\cos(\lambda - \varpi)\}}\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right)$$

If the longitudes be reckoned from the perihelion of the planet P,

$$\frac{\mathrm{d}\,e}{\mathrm{d}\,t} = -\frac{a\,n\,(\cos\lambda + e)}{\mu\sqrt{1 - e^2}}\left(\frac{\mathrm{d}\,\mathrm{R}}{\mathrm{d}\,\lambda}\right) - \frac{m_{,}\,a^2\,n\sqrt{1 - e^2}}{\mu}\left\{\frac{r_{,}'\sin\lambda_{,}'}{r_{,}^3} + \frac{r\sin\lambda - r_{,}'\sin\lambda_{,}'}{\{r^2 - 2\,r\,r_{,}'\cos(\lambda - \lambda_{,}') + r_{,}'^2\}^{\frac{3}{2}}}\right\}$$

$$\frac{d\varpi}{dt} = -\frac{a\,n\sin\lambda}{\mu\sqrt{1-e^2}}\left(\frac{dR}{d\lambda}\right) - \frac{m_{\prime}a^2 n\sqrt{1-e^2}}{\mu}\left\{\frac{r_{\prime}'\cos\lambda_{\prime}'}{r_{\prime}^3} + \frac{r\cos\lambda - r_{\prime}'\cos\lambda_{\prime}'}{\{r^2 - 2rr_{\prime}'\cos(\lambda-\lambda_{\prime}') + r_{\prime}^2\}^{\frac{3}{2}}}\right\}$$

and since $n\,dt = \frac{r}{a}\,d\,v$,

$$\frac{de}{dv} = -\frac{r(\cos\lambda + e)}{\mu\sqrt{1-e^2}}\left(\frac{dR}{d\lambda}\right) - \frac{m_{\prime}ar\sqrt{1-e^2}}{\mu}\left\{\frac{r_{\prime}'\sin\lambda_{\prime}'}{r_{\prime}^3} + \frac{r\sin\lambda - r_{\prime}'\sin\lambda_{\prime}'}{\{r^2 - 2rr_{\prime}'\cos(\lambda-\lambda_{\prime}') + r_{\prime}^2\}^{\frac{3}{2}}}\right\}$$

$$\frac{e\,d\varpi}{dv} = -\frac{r\sin\lambda}{\mu\sqrt{1-e^2}}\left(\frac{dR}{d\lambda}\right) + \frac{m_{\prime}ar\sqrt{1-e^2}}{\mu}\left\{\frac{r_{\prime}'\cos\lambda_{\prime}'}{r_{\prime}^3} + \frac{r\cos\lambda - r_{\prime}'\cos\lambda_{\prime}'}{\{r^2 - 2rr_{\prime}'\cos(\lambda-\lambda_{\prime}') + r_{\prime}^2\}^{\frac{3}{2}}}\right\}$$

$$\frac{d\varepsilon - d\varpi}{dv} = -\frac{r^2}{a^2\sqrt{1-e^2}}\frac{d\varpi}{dv} - \left\{\frac{r}{a(1-e^2)} + 1\right\}\sin v\,\frac{de}{dv}$$

$$d\,a = -\frac{2a^2}{\mu}\,d\,R$$

and since $r^2\,d\,\lambda = \sqrt{\mu\,a\,(1-e^2)}\,d\,t = a\sqrt{1-e^2}\,r\,d\,v$

$$\frac{d\iota}{dv} + \frac{a\sqrt{1-e^2}\,r\cos\lambda}{\mu}\left(\frac{dR}{ds}\right) = 0$$

$$\frac{d\nu}{dv} + \frac{a\sqrt{1-e^2}\,r\sin\lambda}{\mu}\left(\frac{dR}{ds}\right) = 0.$$

The last six equations serve to determine the perturbations of a comet.

Let $(\Delta\,e)_n$ be the variation of any element e during the variation $\Delta\,v$ of v at any given epoch n, neglecting the square and higher powers of $\Delta\,v$,

$$(\Delta\,e)_n = \left(\frac{de}{dv}\right)_n \Delta\,v$$

If the values of $(\Delta\,e)_n$ be calculated for the epochs $0, 1, 2 \ldots m$ corresponding to the values v, $v + i\,\Delta\,v$, $v + 2\,i\,\Delta\,v$, &c., differing from each other by $i\,\Delta\,v$, then the whole variation $(\delta\,e)$ of v corresponding to the variation $i\,\Delta\,v$ of v

$$= i\left\{(\Delta\,e)_0 + (\Delta\,e)_1 \ldots\ldots\ldots + (\Delta\,e)_{m-1}\right\}$$

$$+ \frac{i-1}{2}\left\{(\Delta\,e)_m - (\Delta\,e)_0\right\} - \frac{(i-1)(i+1)}{12\,i}\left\{(\Delta^2\,e)_m - (\Delta^2\,e)_0\right\} + \text{\&c.}$$

$$= i\left\{\left(\frac{de}{dv}\right)_0 + \left(\frac{de}{dv}\right)_1 \ldots\ldots\ldots + \left(\frac{de}{dv}\right)_{m-1}\right\}\Delta\,v$$

$$+ \frac{i-1}{2}\left\{\left(\frac{de}{dv}\right)_m - \left(\frac{de}{dv}\right)_0\right\}\Delta\,v - \frac{(i-1)(i+1)}{12\,i}\left\{\Delta\left(\frac{de}{dv}\right)_m - \Delta\left(\frac{de}{dv}\right)_0\right\}\Delta\,v.$$

When the interval $\Delta\, v$ is indefinitely diminished, $i\, \Delta\, v$ is still equal to the variation of v between the epochs for which the quantities $\left(\frac{d\,e}{d\,v}\right)_0$, $\left(\frac{d\,e}{d\,v}\right)_1$ &c. are calculated, and

$$\delta\, e = i\, \Delta\, v \left\{ \left(\frac{d\,e}{d\,v}\right)_0 + \left(\frac{d\,e}{d\,v}\right)_1 \ldots\ldots + \left(\frac{d\,e}{d\,v}\right)_{m-1} + \frac{1}{2}\left\{\left(\frac{d\,e}{d\,v}\right)_m - \left(\frac{d\,e}{d\,v}\right)_0\right\} - \frac{1}{12}\left\{\Delta\left(\frac{d\,e}{d\,v}\right)_m - \Delta\left(\frac{d\,e}{d\,v}\right)_0\right\} + \&c. \right\}$$

If the radius be taken for unity, and $i\, \Delta\, v$ is the mth part of the circumference; $i\, \Delta\, v = \frac{2\,\times\,3{\cdot}14159}{m}$, or, in other words, the resulting values of $\delta\, e$ and $\delta\, a$ in the equations given above must be multiplied by 2×3.14159, and divided by $360°$ expressed in the same unit as $i\, \Delta\, v$.

n is equal to the angular circumference divided by the periodic time expressed in the same unit as t; so that if a degree be taken as the unity of angular circumference, $n = 360°$ divided by the periodic time expressed in the same unit as t.

In the elliptic movement or first approximation

$\lambda = n\,t + \varepsilon +$ a series of sines of arcs multiples of $n\,t$ &c.

$\lambda_{,} = n_{,}\,t + \varepsilon_{,} +$ a series of sines of arcs multiples of $n_{,}\,t$ &c.

$\frac{a}{r} =$ constant $+$ a series of cosines.

$s =$ a series of sines.

$s_{,} =$ a series of sines.

These values being substituted in the equations of p. 334 give $\frac{d\,e}{d\,\lambda}$, $\frac{d\,h}{d\,\lambda}$ and $\frac{d\,\iota}{d\,\lambda}$ each equal to a series of sines without any constant quantity, and $\frac{d\,\varpi}{d\,\lambda}$, $\frac{d\,\nu}{d\,\lambda}$ and $\frac{d\,\varepsilon}{d\,\lambda}$ each equal to a series of cosines $+$ a constant quantity.

In the second approximation the values of λ, r and s retain the same form; and it is easy to see from the form of the expressions for $\frac{d\,e}{d\,\lambda}$, $\frac{d\,h}{d\,\lambda}$, &c. p. 334, that the form of the values of these quantities is not altered however far the approximation be carried.

If the sun or primary be a spheroid, ω the angle which the plane of the sun's equator makes with the plane of the orbit of the planet; and if the longitude be reckoned from the line of intersection of the sun's equator with the orbit of the planet; R is increased by the quantity $c \left\{ \frac{3 \sin^2 \omega \sin^2 \lambda - 1}{r^3} \right\}$, c being a constant dependent upon the figure of the sun; but the partial differential coefficients of this quantity, which are introduced into the values of $\mathrm{d}\, e$, $\mathrm{d}\, \varpi$, &c. do not change the form of the expressions for those quantities.

If the planet move in a medium which resists according to any power n of the velocity, if c be a constant and v the velocity, the term $2\, c \int v^{n+1}\, \mathrm{d}\, t$ must be added to $2 \int \mathrm{d}\, R$,

$$c\, v^{n-1} \left\{ (1 + s^2) \frac{\mathrm{d}\, r'}{\mathrm{d}\, t} + r'\, s \frac{\mathrm{d}\, s}{\mathrm{d}\, t} \right\} \text{ to } \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, r'} \right),$$

$$c\, v^{n-1}\, r'^2 \frac{\mathrm{d}\, \lambda'}{\mathrm{d}\, t} \text{ to } \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, \lambda'} \right), \quad \text{and } c\, v^{n-1}\, r' \frac{\mathrm{d}\, .\, r'\, s}{\mathrm{d}\, t} \text{ to } \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, s} \right)$$

in the equations of p. 330.

If the orbit of the planet be supposed to coincide with the plane $x\, y$, so that $s = 0$, then by the equations of p. 337 after reductions

$$\mathrm{d}\, a = -2\, c\, a^2 \left(\frac{\mu}{a} \right)^{\frac{n+1}{2}} \left\{ \frac{1 + e \cos v}{1 - e \cos v} \right\}^{\frac{n+1}{2}} \frac{(1 - e \cos v)}{n}\, \mathrm{d}\, v$$

$$\mathrm{d}\, e = -2\, c \left(\frac{\mu}{a} \right)^{\frac{n-1}{2}} \left\{ \frac{1 + e \cos v}{1 - e \cos v} \right\}^{\frac{n-1}{2}} \left(\frac{1 - e^2}{n} \right) \cos v\, \mathrm{d}\, v$$

$$e\, \mathrm{d}\, \varpi = -2\, c \left(\frac{\mu}{a} \right)^{\frac{n-1}{2}} \left\{ \frac{1 + e \cos v}{1 - e \cos v} \right\}^{\frac{n-1}{2}} \sqrt{\frac{1 - e^2}{n}} \sin v\, \mathrm{d}\, v$$

$$\mathrm{d}\, \varepsilon - \mathrm{d}\, \varpi = 2\, c \left(\frac{\mu}{a} \right)^{\frac{n-1}{2}} \left\{ \frac{1 + e \cos v}{1 - e \cos v} \right\}^{\frac{n-1}{2}} \frac{\sin v}{n} \left\{ \frac{1 - e^3 \cos v}{e} \right\} \mathrm{d}\, v$$

The form of these equations differs from that which obtained before, now the variations of e, ϖ and ε are periodical, while that of a has a term which varies with the time. $\frac{\mathrm{d}\, e}{\mathrm{d}\, v}$ contains only odd powers of $\cos v$ and for that reason has no constant term. The periods of the periodic inequalities of all the elliptic constants due to the action of the resisting medium are fractional parts of the periodic time of the planet.

If the origin of t coincides with the instant of the perihelion passage, by LAGRANGE's theorem

$$\cos v = \cos n\,t - e \sin n\,t^2 - \frac{e^2}{2} \cdot \frac{\mathrm{d}\,.\sin n\,t^3}{\mathrm{d}\,.n\,t} - \frac{e^3}{2\,.\,3} \cdot \frac{\mathrm{d}^2\,.\sin n\,t^4}{(\mathrm{d}\,.n\,t)^2} - \frac{e^4}{2\,.\,3\,.\,4} \frac{\mathrm{d}^3 \sin n\,t^5}{(\mathrm{d}\,.n\,t^3} - \&c.$$

$$\sin v = \sin n\,t + e \sin n\,t \cos n\,t + \frac{e^2}{2} \frac{\mathrm{d}\,.\sin n\,t^2 \cos n\,t}{\mathrm{d}\,.n\,t} + \frac{e^3}{2\,.\,3} \frac{\mathrm{d}^2 .\sin n\,t^3 \cos n\,t}{(\mathrm{d}\,.n\,t)^2} + \frac{e^4}{2\,.\,3\,.\,4} \frac{\mathrm{d}^3 .\sin n\,t^4 \cos n\,t}{(\mathrm{d}\,.n\,t)^3}$$

$$\sin n\,t^2 = \frac{1 - \cos 2\,n\,t}{2}, \quad \sin n\,t^3 = \frac{3 \sin n\,t - \sin 3\,n\,t}{4}$$

$$\sin n\,t^4 = \frac{3 - 4 \cos 2\,n\,t + \cos 4\,n\,t}{8}, \quad \sin n\,t^5 = \frac{10 \sin n\,t - 5 \sin 3\,n\,t + \sin 5\,n\,t}{16}$$

$$\frac{\mathrm{d}\,.\sin n\,t^3}{\mathrm{d}\,.n\,t} = \frac{3 \cos n\,t - 3 \cos 3\,n\,t}{4}$$

$$\frac{\mathrm{d}\,.\sin n\,t^4}{\mathrm{d}\,.n\,t} = \frac{2 \sin 2\,n\,t - \sin 4\,n\,t}{2}, \quad \frac{\mathrm{d}^2\,.\sin n\,t^4}{(\mathrm{d}\,.n\,t)^2} = 2 \cos 2\,n\,t - 2 \cos 4\,n\,t$$

$$\frac{\mathrm{d}\,.\sin n\,t^5}{\mathrm{d}\,.n\,t} = \frac{10 \cos n\,t - 15 \cos 3\,n\,t + 5 \cos 5\,n\,t}{16}$$

$$\frac{\mathrm{d}^2 \sin n\,t^5}{(\mathrm{d}\,.n\,t)^2} = \frac{-10 \sin n\,t + 45 \sin 3\,n\,t - 25 \sin 5\,n\,t}{16}$$

$$\frac{\mathrm{d}^3\,.\sin n\,t^5}{(\mathrm{d}\,.n\,t)} = \frac{-10 \cos n\,t + 135 \cos 3\,n\,t - 125 \cos 5\,n\,t}{16}$$

$$\cos v = \cos n\,t - \frac{e}{2} + \frac{e}{2} \cos 2\,n\,t - \frac{e^2}{2} \left\{ \frac{3 \cos n\,t - 3 \cos 3\,n\,t}{4} \right\} - \frac{e^3}{2\,.\,3} \left\{ 2 \cos 2\,n\,t - 2 \cos 4\,n\,t \right\}$$

$$- \frac{e^4}{2\,.\,3\,.\,4} \left\{ \frac{-10 \cos n\,t + 135 \cos 3\,n\,t - 125 \cos 5\,n\,t}{16} \right\}$$

If the origin of the time does not coincide with the perihelion passage, $n\,t + \varepsilon - \varpi$ must be substituted for $n\,t$, but as ε always accompanies $n\,t$, it may be suppressed at present for convenience, and afterwards replaced.

$$\cos v = \left\{ 1 - \frac{3}{8} e^2 + \frac{5\,e^4}{192} \right\} \cos (n\,t - \varpi) - \frac{e}{2} + \frac{e}{2} \left\{ 1 - \frac{2\,e^2}{3} \right\} \cos (2nt - 2\,\varpi)$$

$$+ \frac{3}{8}\, e^2 \left\{ 1 - \frac{15}{16} e^2 \right\} \cos (3\,n\,t - 3\varpi) + \frac{e^3}{3} \cos (4\,nt - 4\varpi) + \frac{125}{384} e^4 \cos (5nt - 5\,\varpi)$$

$$\sin n\,t^2 \cos n\,t = \frac{\mathrm{d}\,.\sin n\,t^3}{3\,\mathrm{d}\,.n\,t} = \frac{\cos n\,t - \cos 3\,n\,t}{4}, \quad \frac{\mathrm{d}\,.\sin n\,t^2 \cos n\,t}{\mathrm{d}\,.n\,t} = \frac{-\sin n\,t + 3 \sin 3\,n\,t}{4}$$

$$\sin n\,t^3 \cos n\,t = \frac{\mathrm{d}\,.\sin n\,t^4}{4\,\mathrm{d}\,.\,n\,t} = \frac{2\sin 2\,n\,t - \sin 4\,n\,t}{8}, \qquad \frac{\mathrm{d}\,.\sin n\,t^3 \cos n\,t}{\mathrm{d}\,.\,n\,t} = \frac{\cos 2\,n\,t - \cos 4\,n\,t}{2}$$

$$\frac{\mathrm{d}^2\,.\sin n\,t^3 \cos n\,t}{(\mathrm{d}\,.\,n\,t)^2} = -\sin 2\,n\,t + 2\sin 4\,n\,t$$

$$\sin n\,t^4 \cos n\,t = \frac{\mathrm{d}\,.\sin n\,t^5}{5\,\mathrm{d}\,.\,n\,t} = \frac{2\cos n\,t - 3\cos 3\,n\,t + \cos 5\,n\,t}{16}$$

$$\frac{\mathrm{d}\,.\sin n\,t^4 \cos n\,t}{\mathrm{d}\,.\,n\,t} = \frac{-2\sin n\,t + 9\sin 3\,n\,t - 5\sin 5\,n\,t}{16}$$

$$\frac{\mathrm{d}^2\,.\sin n\,t^4 \cos n\,t}{(\mathrm{d}\,.\,n\,t)^2} = \frac{-2\cos n\,t + 27\cos 3\,n\,t - 25\cos 5\,n\,t}{16}$$

$$\frac{\mathrm{d}^3\,.\sin n\,t^4 \cos n\,t}{(\mathrm{d}\,.\,n\,t)^3} = \frac{2\sin n\,t - 81\sin 3\,n\,t + 125\sin 5\,n\,t}{16}$$

$$\sin v = \sin n t + \frac{e}{2}\sin 2\,n t + \frac{e^2}{2}\left\{\frac{-\sin n\,t + 3\sin 3\,n\,t}{4}\right\} + \frac{e^3}{2\,.\,3}\{-\sin 2\,n\,t + 2\sin 4 n t\}$$
$$+ \frac{e^4}{2\,.\,3\,.\,4}\left\{\frac{2\sin n t - 81\sin 3\,n t + 125\sin 5\,n t}{16}\right\}$$

and replacing $n\,t$ by $n\,t - \varpi$

$$\sin v = \left\{1 - \frac{3\,e^2}{8} + \frac{5}{192}\,e^4\right\}\sin(n\,t - \varpi) + \frac{e}{2}\left\{1 - 2\,\frac{e^2}{3}\right\}\sin(2\,n\,t - 2\,\varpi)$$
$$+ \left\{\frac{2\,e^2}{8} - \frac{4}{192}\,e^4\right\}\sin(n\,t - \varpi) \qquad + \frac{e^3}{6}\quad \sin(2\,n\,t - 2\,\varpi)$$
$$+ \frac{3\,e^2}{8}\left\{1 - \frac{15}{16}\,e^2\right\}\sin(3\,n\,t - 3\,\varpi) + \frac{e^3}{3}\sin(4\,n\,t - 4\,\varpi) + \frac{125}{384}\,e^4\sin(5 n t - 5\varpi)$$
$$+ \frac{3}{8}\cdot\frac{6}{16}\,e^4\sin(3\,n\,t - 3\,\varpi)$$

$$\cos(v+\varpi) = \left\{1 - \frac{e^2}{4} + \frac{e^4}{64}\right\}\cos n\,t - \frac{e}{2}\cos\varpi + \frac{e}{2}\left\{1 - \frac{e^2}{2}\right\}\cos(2 n t - \varpi)$$
$$+ \frac{3}{8}\,e^2\left\{1 - \frac{3}{4}\,e^2\right\}\cos(3\,n\,t - 2\,\varpi) + \frac{e^3}{3}\cos(4\,n\,t - 3\,\varpi) + \frac{125}{384}\,e^4\cos(5\,n t - 4\,\varpi)$$
$$- \frac{e^2}{8}\left\{1 - \frac{e^2}{12}\right\}\cos(n\,t - 2\,\varpi) - \frac{e^3}{12}\cos(2\,n\,t - 3\,\varpi) - \frac{9}{128}\,e^4\cos(3\,n\,t - 4\,\varpi)$$

$$\sin(v+\varpi) = \left\{1 - \frac{e^2}{4} + \frac{e^4}{64}\right\}\sin n\,t - \frac{e}{2}\sin\varpi + \frac{e}{2}\left\{1 - \frac{e^2}{2}\right\}\sin(2 n t - \varpi)$$
$$+ \frac{3}{8}\,e^2\left\{1 - \frac{3}{4}\,e^2\right\}\sin(3\,n\,t - 2\,\varpi) + \frac{e^3}{3}\sin(4\,n\,t - 3\,\varpi) + \frac{125}{384}\,e^4\sin(5\,n\,t - 4\varpi)$$

$$+\frac{e^2}{8}\left(1-\frac{e^2}{12}\right)\sin(nt-2\varpi)+\frac{e^3}{12}\sin(2nt-3\varpi)+\frac{9}{128}e^4\sin(3nt-4\varpi)$$

$$\cos(v-\varpi)=\left(1-\frac{e^2}{4}\right)\cos(nt-2\varpi)-\frac{e}{2}\cos\varpi+\frac{e}{2}\cos(2nt-3\varpi)+\frac{3}{8}e^2\cos(3nt-4\varpi)-\frac{e^2}{8}\cos nt$$

$$\sin(v-\varpi)=\left(1-\frac{e^2}{4}\right)\sin(nt-2\varpi)+\frac{e}{2}\sin\varpi+\frac{e}{2}\sin(2nt-3\varpi)+\frac{3}{8}e^2\sin(3nt-4\varpi)+\frac{e^2}{8}\sin nt$$

$$r\cos\lambda=r\cos(\lambda-\varpi)\cos\varpi-r\sin(\lambda-\varpi)\sin\varpi=a\{(\cos v-e)\cos\varpi-(1-e^2)^{\frac{1}{2}}\sin v\sin\varpi\}$$

$$r\sin\lambda=r\sin(\lambda-\varpi)\cos\varpi+r\cos(\lambda-\varpi)\sin\varpi=a\{(1-e^2)^{\frac{1}{2}}\sin v\cos\varpi+(\cos v-e)\sin\varpi\}$$

$$r\cos\lambda=a\left\{\left(1-\frac{e^2}{4}-\frac{e^4}{16}\right)\cos(v+\varpi)-e\cos\varpi+\frac{e^2}{4}\left(1+\frac{e^2}{4}\right)\cos(v-\varpi)\right\}$$

$$r\sin\lambda=a\left\{\left(1-\frac{e^2}{4}-\frac{e^4}{16}\right)\sin(v+\varpi)-e\sin\varpi-\frac{e^2}{4}\left(1+\frac{e^2}{4}\right)\sin(v-\varpi)\right\}$$

$$\begin{matrix} r\cos\lambda \\ r\sin\lambda\end{matrix}=a\left\{\left(1-\frac{e^2}{2}-\frac{e^4}{64}\right)\begin{matrix}\cos\\ \sin\end{matrix}\,nt-\frac{3\,e}{2}\begin{matrix}\cos\\ \sin\end{matrix}\,\varpi+\frac{e}{2}\left(1-\frac{3}{4}e^2\right)\begin{matrix}\cos\\ \sin\end{matrix}(2nt-\varpi)\right.$$

$$+\frac{3}{8}e^2\left(1-e^2\right)\begin{matrix}\cos\\ \sin\end{matrix}(3nt-2\varpi)+\frac{e^3}{3}\begin{matrix}\cos\\ \sin\end{matrix}(4nt-3\varpi)+\frac{125}{384}e^4\begin{matrix}\cos\\ \sin\end{matrix}(5nt-4\varpi)$$

$$\left.\begin{matrix}+\\-\end{matrix}\frac{e^2}{8}\left(1+\frac{e^2}{3}\right)\begin{matrix}\cos\\ \sin\end{matrix}(nt-2\varpi)\begin{matrix}+\\-\end{matrix}\frac{e^3}{24}\begin{matrix}\cos\\ \sin\end{matrix}(2nt-3\varpi)\begin{matrix}+\\-\end{matrix}\frac{3}{128}e^4\begin{matrix}\cos\\ \sin\end{matrix}(3nt-4\varpi)\,\&c.\right\}$$

$$rr_{\prime}\begin{matrix}\cos\\ \sin\end{matrix}\{\lambda-\lambda_{\prime}\}=aa_{\prime}\left\{\left(1-\frac{e^2}{2}-\frac{e^4}{64}\right)\left(1-\frac{e_{\prime}^2}{2}-\frac{e_{\prime}^4}{64}\right)\begin{matrix}\cos\\ \sin\end{matrix}(nt-n_{\prime}t)\right.$$

$$\begin{matrix}-\\+\end{matrix}\frac{3}{2}e\left(1-\frac{e_{\prime}^2}{2}\right)\begin{matrix}\cos\\ \sin\end{matrix}(n_{\prime}t-\varpi)+\frac{e}{2}\left(1-\frac{3}{4}e^2\right)\left(1-\frac{e_{\prime}^2}{2}\right)\begin{matrix}\cos\\ \sin\end{matrix}(2nt-n_{\prime}t-\varpi)$$

$$+\frac{3}{8}e^2(1-e^2)\left(1-\frac{e_{\prime}^2}{2}\right)\begin{matrix}\cos\\ \sin\end{matrix}(3nt-n_{\prime}t-2\varpi)+\frac{e^3}{3}\begin{matrix}\cos\\ \sin\end{matrix}(4nt-n_{\prime}t-3\varpi)$$

$$+\frac{125}{384}e^4\begin{matrix}\cos\\ \sin\end{matrix}(5nt-n_{\prime}t-4\varpi)\begin{matrix}+\\-\end{matrix}\frac{e^2}{8}\left(1+\frac{e^2}{3}\right)\left(1-\frac{e_{\prime}^2}{2}\right)\begin{matrix}\cos\\ \sin\end{matrix}(nt+n_{\prime}t-2\varpi)$$

$$\begin{matrix}+\\-\end{matrix}\frac{e^3}{24}\begin{matrix}\cos\\ \sin\end{matrix}(2nt+n_{\prime}t-3\varpi)\begin{matrix}+\\-\end{matrix}\frac{3}{128}e^4\begin{matrix}\cos\\ \sin\end{matrix}(3nt+n_{\prime}t-4\varpi)$$

$$-\frac{3}{2}e_{\prime}\left(1-\frac{e^2}{2}\right)\begin{matrix}\cos\\ \sin\end{matrix}(nt-\varpi_{\prime})+\frac{9}{4}ee_{\prime}\begin{matrix}\cos\\ \sin\end{matrix}(\varpi-\varpi_{\prime})$$

$$-\frac{3}{4}e\,e_{\prime}\left(1-\frac{3}{4}e^2\right)\begin{matrix}\cos\\ \sin\end{matrix}(2nt-\varpi-\varpi_{\prime})-\frac{9}{16}e^2e_{\prime}\begin{matrix}\cos\\ \sin\end{matrix}(3nt-2\varpi-\varpi_{\prime})$$

$$-\frac{e^3e_{\prime}}{2}\begin{matrix}\cos\\ \sin\end{matrix}(4nt-3\varpi-\varpi_{\prime})\begin{matrix}-\\+\end{matrix}\frac{3}{16}e^2e_{\prime}\begin{matrix}\cos\\ \sin\end{matrix}(nt-2\varpi+\varpi_{\prime})\begin{matrix}-\\+\end{matrix}\frac{e^3e_{\prime}}{16}\begin{matrix}\cos\\ \sin\end{matrix}(2nt-3\varpi+\varpi_{\prime})$$

$$+\frac{e_{,}}{2}\left(1-\frac{3}{4}e_{,}^{2}\right)\left(1-\frac{e^{2}}{2}\right)\frac{\cos}{\sin}(nt-2n_{,}t+\varpi_{,})\mp\frac{3}{4}ee_{,}\left(1-\frac{3}{4}e_{,}^{2}\right)\frac{\cos}{\sin}(2n_{,}t-\varpi-\varpi_{,})$$

$$+\frac{ee_{,}}{4}\left(1-\frac{3}{4}e^{2}\right)\left(1-\frac{3}{4}e_{,}^{2}\right)\frac{\cos}{\sin}(2nt-2n_{,}t-\varpi+\varpi_{,})+\frac{3}{16}e^{2}e_{,}\frac{\cos}{\sin}(3nt-2n_{,}t-2\varpi+\varpi_{,})$$

$$+\frac{e^{3}e_{,}}{6}\frac{\cos}{\sin}(4nt-2n_{,}t-3\varpi+\varpi_{,})\pm\frac{e^{2}e_{,}}{16}\frac{\cos}{\sin}(nt+2n_{,}t-2\varpi-\varpi_{,})$$

$$\pm\frac{e^{3}e_{,}}{48}\frac{\cos}{\sin}(2nt+2n_{,}t-3\varpi-\varpi_{,})+\frac{3}{8}e_{,}^{2}(1-e_{,}^{2})\left(1-\frac{e^{2}}{2}\right)\frac{\cos}{\sin}(nt-3n_{,}t+2\varpi_{,})$$

$$\mp\frac{9}{16}ee_{,}^{2}\frac{\cos}{\sin}(3n_{,}t-\varpi-2\varpi_{,})+\frac{3}{16}ee_{,}^{2}\frac{\cos}{\sin}(2nt-3n_{,}t-\varpi+2\varpi_{,})$$

$$+\frac{9}{64}e^{2}e_{,}^{2}\frac{\cos}{\sin}(3nt-3n_{,}t-2\varpi+2\varpi_{,})\pm\frac{3}{64}e^{2}e_{,}^{2}\frac{\cos}{\sin}(nt+3n_{,}t-2\varpi-2\varpi_{,})$$

$$+\frac{e^{3}}{3}\frac{\cos}{\sin}(nt-4n_{,}t+3\varpi_{,})\mp\frac{e\,e_{,}^{3}}{2}\frac{\cos}{\sin}(4n_{,}t-\varpi-3\varpi_{,})$$

$$+\frac{e\,e_{,}^{3}}{6}\frac{\cos}{\sin}(2nt-4n_{,}t-\varpi+3\varpi_{,})+\frac{125}{384}e_{,}^{4}\frac{\cos}{\sin}(nt-5n_{,}t+4\varpi_{,})$$

$$+\frac{e_{,}^{2}}{8}\left(1+\frac{e^{2}}{3}e_{,}^{2}\right)\left(1-\frac{e^{2}}{2}\right)\frac{\cos}{\sin}(nt+n_{,}t-2\varpi_{,})-\frac{3}{16}ee_{,}^{2}\frac{\cos}{\sin}(n_{,}t+\varpi-2\varpi_{,})$$

$$+\frac{e\,e_{,}^{2}}{16}\frac{\cos}{\sin}(2nt+n_{,}t-\varpi-2\varpi_{,})+\frac{3}{64}e^{2}e_{,}^{2}\frac{\cos}{\sin}(3nt+n_{,}t-2\varpi-2\varpi_{,})$$

$$\pm\frac{e^{2}e_{,}^{2}}{64}\frac{\cos}{\sin}(nt-n_{,}t-2\varpi+2\varpi_{,})+\frac{e_{,}^{3}}{24}\frac{\cos}{\sin}(nt+2n_{,}t-3\varpi_{,})$$

$$\mp\frac{e\,e_{,}^{3}}{16}\frac{\cos}{\sin}(2n_{,}t+\varpi-3\varpi_{,})+\frac{e\,e_{,}^{3}}{48}\frac{\cos}{\sin}(2nt+2n_{,}t-\varpi-3\varpi_{,})$$

$$+\frac{\text{[illegible]}}{128}e_{,}^{4}\frac{\cos}{\sin}(nt+3n_{,}t-4\varpi_{,})+\&c.$$

$$rr_{,}\frac{\cos}{\sin}\{\lambda+\lambda_{,}\}=aa_{,}\left\{\left(1-\frac{e^{2}+e_{,}^{2}}{2}\right)\frac{\cos}{\sin}(nt+n_{,}t)\right.$$

$$-\frac{3}{2}e\frac{\cos}{\sin}(n_{,}t+\varpi)+\frac{e}{2}\frac{\cos}{\sin}(2nt+n_{,}t-\varpi)$$

$$+\frac{3}{8}e^{2}\frac{\cos}{\sin}(3nt+n_{,}t-2\varpi)\pm\frac{e^{2}}{8}\frac{\cos}{\sin}(nt-n_{,}t-2\varpi)$$

$$-\frac{3}{2}e_{,}\frac{\cos}{\sin}(nt+\varpi_{,})+\frac{9}{4}ee_{,}\frac{\cos}{\sin}(\varpi+\varpi_{,})-\frac{3}{4}ee_{,}\frac{\cos}{\sin}(2nt-\varpi+\varpi_{,})$$

$$+\frac{e_{\prime}}{2}\,{}^{\cos}_{\sin}\,(nt+2n_{\prime}t-\varpi_{\prime})-\frac{3}{4}\,ee_{\prime}\,{}^{\cos}_{\sin}\,(2n_{\prime}t+\varpi-\varpi_{\prime})+\frac{ee_{\prime}}{4}\,{}^{\cos}_{\sin}\,(2nt+2nt-\varpi-\varpi_{\prime})+\&c.$$

$$rr_{\prime}\,{}^{\cos}_{\sin}\,\{\lambda+\lambda_{\prime}-2\nu\}=aa_{\prime}\left\{\left(1-\frac{e^2+e_{\prime}^2}{2}\right){}^{\cos}_{\sin}\,(nt+n_{\prime}t-2\nu)\right.$$

$$-\frac{3}{2}\,e\,{}^{\cos}_{\sin}\,(n_{\prime}t+\varpi-2\nu)+\frac{e}{2}\,{}^{\cos}_{\sin}\,(2nt+n_{\prime}t-\varpi-2\nu)$$

$$+\frac{3}{8}\,e^2\,{}^{\cos}_{\sin}\,(3nt+n_{\prime}t-2\varpi-2\nu)\,{}^{+}_{-}\,\frac{e^2}{8}\,{}^{\cos}_{\sin}\,(nt-n_{\prime}t-2\varpi+2\nu)$$

$$-\frac{3}{2}\,e_{\prime}\,{}^{\cos}_{\sin}\,(nt+\varpi_{\prime}-2\nu)+\frac{9}{4}\,e\,e_{\prime}\,{}^{\cos}_{\sin}\,(\varpi+\varpi_{\prime}-2\nu)$$

$$-\frac{3}{4}\,ee_{\prime}\,{}^{\cos}_{\sin}\,(2nt-\varpi+\varpi_{\prime}-2\nu)+\frac{e_{\prime}}{2}\,{}^{\cos}_{\sin}\,(nt+2n_{\prime}t-\varpi_{\prime}-2\nu)$$

$$-\frac{3}{4}\,ee_{\prime}\,{}^{\cos}_{\sin}\,(2n_{\prime}t+\varpi-\varpi_{\prime}-2\nu)+\frac{ee_{\prime}}{4}\,{}^{\cos}_{\sin}\,(2nt+2n_{\prime}t-\varpi-\varpi_{\prime}-2\nu)+\&c.$$

$$r=a\left\{1+\frac{e^2}{2}-e\left(1-\frac{3}{8}\,e^2\right)\cos\,(nt-\varpi)-\frac{e^2}{2}\left(1-\frac{2e^2}{3}\right)\cos\,(2nt-2\varpi)\right.$$

$$-\frac{3}{8}\,e^3\cos\,(3nt-3\varpi)-\frac{e^4}{3}\cos\,(4nt-4\varpi)+\&c.$$

$$r^2=a^2\left\{1+\frac{3\,e^2}{2}-2\,e\left(1-\frac{e^2}{8}\right)\cos\,(nt-\varpi)-\frac{e^2}{2}\left(1-\frac{e^2}{3}\right)\cos\,(2nt-2\varpi)\right.$$

$$-\frac{e^3}{4}\cos\,(3nt-3\varpi)-\frac{e^4}{6}\cos\,(4nt-4\varpi)+\&c.$$

$$r^{-3}=a^{-3}\left\{1+\frac{3}{2}\,e^2\left(1+\frac{e^2}{2}\right)+3\,e\left(1+\frac{9}{8}\,e^2\right)\cos\,(nt-\varpi)\right.$$

$$+\frac{9}{2}\,e^2\left(1-\frac{e^2}{18}\right)\cos\,2nt-2\varpi+\frac{53}{8}\,e^3\cos\,(3nt-3\varpi)$$

$$+\frac{31}{4}\,e^4\cos\,(4nt-4\varpi)+\&c.$$

If the coefficient of $\cos n\,\theta$ in the development of $\left\{1-\frac{2\,a}{a_{\prime}}\cos\theta+\frac{a^2}{a_{\prime}^2}\right\}^{-m}$ according to cosines of arcs mulitples of θ is called $b_{2m,n}$

$$b_{1,0}=1+\left(\frac{1}{2}\right)^2\frac{a^2}{a_{\prime}^2}+\left(\frac{1.3}{2.4}\right)^2\frac{a^4}{a_{\prime}^4}+\left(\frac{1.3.5}{2.4.6}\right)^2\frac{a^6}{a_{\prime}^6}+\&c.$$

$$b_{1,1} = \frac{a}{a_{\prime}} + \frac{1.3}{2.4}\frac{a^3}{a_{\prime}^3} + \frac{1.3.3.5}{2.4.4.6}\frac{a^5}{a_{\prime}^5} + \&c.$$

$$b_{1,2} = \frac{3}{4}\frac{a^2}{a_{\prime}^2} + \frac{1.3.5}{2.4.6}\frac{a^4}{a_{\prime}^4} + \frac{1.3.3.5.7}{2.4.4.6.8}\frac{a^6}{a_{\prime}^6} + \&c.$$

$$b_{1,3} = \frac{3.5}{4.6}\frac{a^3}{a_{\prime}^3} + \frac{1.3.5.7}{2.4.6.8}\frac{a^5}{a_{\prime}^5} + \frac{1.3.3.5.7.9}{2.4.4.6.8.10}\frac{a^7}{a_{\prime}^7} + \&c.$$

$$b_{3,0} = 1 + \left(\frac{3}{2}\right)^2\frac{a^2}{a_{\prime}^2} + \left(\frac{3.5}{2.4}\right)^2\frac{a^4}{a_{\prime}^4} + \left(\frac{3.5.7}{2.4.6}\right)^2\frac{a^6}{a_{\prime}^6} + \&c.$$

$$b_{3,1} = \frac{3\,a}{a_{\prime}} + \frac{3.3.5}{2.4}\frac{a^3}{a_{\prime}^3} + \frac{3.5.3.5.7}{2.4.4.6}\frac{a^5}{a_{\prime}^5} + \&c.$$

$$b_{3,2} = \frac{3.5}{4}\frac{a^2}{a_{\prime}^2} + \frac{3.3.5.7}{2.4.6}\frac{a^4}{a_{\prime}^4} + \frac{3.5.3.5.7}{2.4.4.6.8}\frac{a^6}{a_{\prime}^6} + \&c.$$

$$b_{3,3} = \frac{3.5.7}{4.6}\frac{a^3}{a_{\prime}^3} + \frac{3.3.5.7.9}{2.4.6.8}\frac{a^5}{a_{\prime}^5} + \frac{3.5.3.5.7.9.11}{2.4.4.6.8.10}\frac{a^7}{a_{\prime}^7} + \&c.$$

$$b_{5,0} = 1 + \left(\frac{5}{2}\right)^2\frac{a^2}{a_{\prime}^2} + \left(\frac{5.7}{2.4}\right)^2\frac{a^4}{a_{\prime}^4} + \left(\frac{5.7.9}{2.4.6}\right)^2\frac{a^6}{a_{\prime}^6} + \&c.$$

$$b_{5,1} = 5\,\frac{a}{a_{\prime}} + \frac{5.5.7}{2.4}\frac{a^3}{a_{\prime}^3} + \frac{5.7.5.7.9}{2.4.4.6}\frac{a^5}{a_{\prime}^5} + \&c.$$

$$b_{5,2} = \frac{5.7}{4}\frac{a^2}{a_{\prime}^2} + \frac{5.5.7.9}{2.4.6}\frac{a^4}{a_{\prime}^4} + \frac{5.7.5.7.9.11}{2.4.4.6.8}\frac{a^6}{a_{\prime}^6} + \&c.$$

$$b_{5,3} = \frac{5.7.9}{4.6}\frac{a^3}{a_{\prime}^3} + \frac{5.5.7.9.11}{2.4.6.8}\frac{a^5}{a_{\prime}^5} + \frac{5.7.5.7.9.11.13}{2.4.4.6.8.10}\frac{a^7}{a_{\prime}^7}$$

$$b_{3,1} = 3\,\frac{a}{a_{\prime}}\left\{b_{5,0} - \frac{1}{2}\,b_{5,2}\right\} \qquad b_{1,1} = \frac{a}{a_{\prime}}\left\{b_{3,0} - \frac{1}{2}\,b_{3,2}\right\}$$

$$2\,b_{3,2} = \frac{3}{2}\,\frac{a}{a_{\prime}}\left\{b_{5,1} - b_{5,3}\right\} \qquad 2\,b_{1,2} = \frac{1}{2}\,\frac{a}{a_{\prime}}\left\{b_{3,1} - b_{3,3}\right\}$$

$$3\,b_{3,3} = \frac{3}{2}\,\frac{a}{a_{\prime}}\left\{b_{5,2} - b_{5,4}\right\} \qquad 3\,b_{1,3} = \frac{1}{2}\,\frac{a}{a_{\prime}}\left\{b_{3,2} - b_{3,4}\right\}$$

$$R = m_{\prime}\left\{\frac{r\,r_{\prime}'\cos(\lambda - \lambda_{\prime}')}{r_{\prime}^3} - \frac{1}{\{r^2 - 2\,r\,r_{\prime}'\cos(\lambda - \lambda_{\prime}') + r_{\prime}^2\}^{\frac{1}{2}}}\right\}$$

and since

$$r r_{\prime}'\cos(\lambda - \lambda_{\prime}') = r r_{\prime}\left\{\cos^2\frac{\iota_{\prime}}{2}\cos(\lambda - \lambda_{\prime}) + \sin^2\frac{\iota_{\prime}}{2}\cos(\lambda + \lambda_{\prime} - 2\,\nu_{\prime})\right\},$$ p. 330;

$$R = m_{\prime}\left\{\frac{r r_{\prime}\left\{\cos^2\frac{\iota_{\prime}}{2}\cos(\lambda-\lambda_{\prime})+\sin^2\frac{\iota_{\prime}}{2}\cos(\lambda+\lambda_{\prime}-2\nu_{\prime})\right\}}{r_{\prime}^3}\right.$$
$$\left. - \frac{1}{\left\{r^2 - 2 r r_{\prime}\left\{\cos^2\frac{\iota_{\prime}}{2}\cos(\lambda-\lambda_{\prime})+\sin^2\frac{\iota_{\prime}}{2}\cos(\lambda+\lambda_{\prime}-2\nu_{\prime})\right\}+r_{\prime}^2\right\}^{\frac{1}{2}}}\right\}$$

Neglecting the cubes and higher powers of e

$$R = m_{\prime}\frac{a}{a_{\prime}^2}\left\{\left(1-\sin^2\frac{\iota_{\prime}}{2}-\frac{e^2+e_{\prime}^2}{2}\right)\cos(nt-n_{\prime}t)-\frac{3}{2}e\cos(n_{\prime}t-\varpi)-\frac{3}{2}e_{\prime}\cos(nt-\varpi_{\prime})\right.$$
$$+\frac{e}{2}\cos(2nt-n_{\prime}t-\varpi)+\frac{e_{\prime}}{2}\cos(nt-2n_{\prime}t+\varpi_{\prime})+\frac{3}{8}e^2\cos(3nt-n_{\prime}t-2\varpi)$$
$$+\frac{3}{8}e_{\prime}^2\cos(nt-3n_{\prime}t+2\varpi_{\prime})+\frac{e^2}{8}\cos(nt+n_{\prime}t-2\varpi)+\frac{e_{\prime}^2}{8}\cos(nt+n_{\prime}t-2\varpi_{\prime})$$
$$+\frac{9}{4}ee_{\prime}\cos(\varpi-\varpi_{\prime})-\frac{3}{4}ee_{\prime}\cos(2nt-\varpi-\varpi_{\prime})-\frac{3}{4}ee_{\prime}\cos(2n_{\prime}t-\varpi-\varpi_{\prime})$$
$$\left.+\frac{ee_{\prime}}{4}\cos(2nt-2n_{\prime}t-\varpi+\varpi_{\prime})+\sin^2\frac{\iota}{2}\cos(nt+n_{\prime}t-2\nu)\right\}\left\{1+\frac{3}{2}e^2_{\prime}\right.$$
$$\left.+3e_{\prime}\cos(n_{\prime}t-\varpi_{\prime})+\frac{9}{2}e_{\prime}^2\cos(2n_{\prime}t-2\varpi_{\prime})+\&c.\right\}$$
$$-m_{\prime}\left\{a^2\left\{1+\frac{3}{2}e^2-2e\cos(nt-\varpi)-\frac{e^2}{2}\cos(2nt-2\varpi)\right\}\right.$$
$$-2aa_{\prime}\left\{\left(1-\sin^2\frac{\iota_{\prime}}{2}-\frac{e^2+e_{\prime}^2}{2}\right)\cos(nt-n_{\prime}t)-\frac{3}{2}e\cos(n_{\prime}t-\varpi)\right.$$
$$-\frac{3}{2}e_{\prime}\cos(nt-\varpi_{\prime})+\frac{e}{2}\cos(2nt-n_{\prime}t-\varpi)+\frac{e_{\prime}}{2}\cos(nt-2n_{\prime}t+\varpi_{\prime})$$
$$+\frac{3}{8}e^2\cos(3nt-n_{\prime}t-2\varpi)+\frac{3}{8}e_{\prime}^2\cos(nt-3n_{\prime}t+2\varpi_{\prime})+\frac{e^2}{8}\cos(nt+n_{\prime}t-2\varpi)$$
$$+\frac{e_{\prime}^2}{8}\cos(nt+n_{\prime}t-2\varpi_{\prime})+\frac{9}{4}ee_{\prime}\cos(\varpi-\varpi_{\prime})-\frac{3}{4}ee_{\prime}\cos(2nt-\varpi-\varpi_{\prime})$$
$$-\frac{3}{4}ee_{\prime}\cos(2n_{\prime}t-\varpi-\varpi_{\prime})+\frac{ee_{\prime}}{4}\cos(2nt-2n_{\prime}t-\varpi+\varpi_{\prime})$$
$$\left.+\sin^2\frac{\iota}{2}\cos(nt+n_{\prime}t-2\nu)\right\}+a_{\prime}^2\left\{1+\frac{3}{2}e_{\prime}^2\right.$$
$$\left.\left.-2e_{\prime}\cos(n_{\prime}t-\varpi_{\prime})-\frac{e_{\prime}^2}{2}\cos(2n_{\prime}t-2\varpi_{\prime})+\&c.\right\}\right\}^{\frac{1}{2}}$$

$= \text{terms independent of } b$

$$- \frac{m_{\prime}}{a_{\prime}} \left\{ b_{1,0} + b_{1,1} \cos(nt - n_{\prime}t) + b_{1,2} \cos(2nt - 2n_{\prime}t) + \&c. \right\}$$

$$+ \frac{m_{\prime}}{2 a_{\prime}^3} \left\{ b_{3,0} + b_{3,1} \cos(nt - n_{\prime}t) + b_{3,2} \cos(2nt - 2n_{\prime}t) \;\&c. \right\} \left\{ a^2 \left\{ \frac{3}{2} e^2 \right.\right.$$

$$\left. - 2 e \cos(nt - \varpi) - \frac{e^2}{2} \cos(2nt - 2\varpi) \right\} - 2 a a_{\prime} \left\{ \left(- \sin^2 \frac{\iota_{\prime}}{2} - \frac{e^2 + e_{\prime}^2}{2} \right) \cos(nt - n_{\prime}t) \right.$$

$$- \frac{3}{2} e \cos(n_{\prime}t - \varpi) - \frac{3}{2} e_{\prime} \cos(nt - \varpi_{\prime}) + \frac{e}{2} \cos(2nt - n_{\prime}t - \varpi)$$

$$+ \frac{e_{\prime}}{2} \cos(nt - 2n_{\prime}t + \varpi_{\prime}) + \frac{3}{8} e^2 \cos(3nt - n_{\prime}t - 2\varpi) + \frac{3}{8} e_{\prime}^2 \cos(nt - 3n_{\prime}t + 2\varpi_{\prime})$$

$$+ \frac{e^2}{8} \cos(nt + n_{\prime}t - 2\varpi) + \frac{e_{\prime}^2}{8} \cos(nt + n_{\prime}t - 2\varpi_{\prime}) + \frac{9}{4} e e_{\prime} \cos(\varpi - \varpi_{\prime})$$

$$- \frac{3}{4} e e_{\prime} \cos(2nt - \varpi - \varpi_{\prime}) - \frac{3}{4} e e_{\prime} \cos(2n_{\prime}t - \varpi - \varpi_{\prime}) + \frac{e e_{\prime}}{4} \cos(2nt - 2n_{\prime}t - \varpi + \varpi_{\prime})$$

$$\left.\left. + \sin^2 \frac{\iota_{\prime}}{2} \cos(nt + n_{\prime}t - 2\nu_{\prime}) \right\} + a_{\prime}^2 \left\{ \frac{3}{2} e_{\prime}^2 - 2 e_{\prime} \cos(n_{\prime}t - \varpi_{\prime}) - \frac{e_{\prime}^2}{2} \cos(2n_{\prime}t - 2\varpi_{\prime}) \&c. \right\} \right\}$$

$$- \frac{1 \cdot 3\, m_{\prime}}{2 \cdot 4\, a_{\prime}^5} \left\{ b_{5,0} + b_{5,1} \cos(nt - n_{\prime}t) + b_{5,2} \cos(2nt - 2n_{\prime}t) + \&c. \right\} \left\{ 2 a^2 e \cos(nt - \varpi) \right.$$

$$- 3 a a_{\prime} e \cos(n_{\prime}t - \varpi) - 3 a a_{\prime} e_{\prime} \cos(nt - \varpi_{\prime}) + a a_{\prime} e \cos(2nt - n_{\prime}t - \varpi)$$

$$\left. + a a_{\prime} e_{\prime} \cos(nt - 2n_{\prime}t + \varpi_{\prime}) + 2 a_{\prime}^2 e_{\prime} \cos(n_{\prime}t - \varpi_{\prime}) \right\}^2 + \&c.$$

$$\left\{ 2 a^2 e \cos(nt - \varpi) - 3 a a_{\prime} e \cos(n_{\prime}t - \varpi) - 3 a a_{\prime} e_{\prime} \cos(nt - \varpi_{\prime}) \right.$$

$$\left. + a a_{\prime} e \cos(2nt - n_{\prime}t - \varpi) + a a_{\prime} e_{\prime} \cos(nt - 2n_{\prime}t + \varpi_{\prime}) + 2 a_{\prime}^2 e_{\prime} \cos(n_{\prime}t - \varpi_{\prime}) \right\}^2$$

$$= 2 a^4 e^2 + 5 a^2 a_{\prime}^2 (e^2 + e_{\prime}^2) + 2 a_{\prime}^4 e_{\prime}^2 + a^2 (2 a^2 - 3 a_{\prime}^2) e^2 \cos(2nt - 2\varpi)$$

$$+ \frac{9}{2} a^2 a_{\prime}^2 e^2 \cos(2n_{\prime}t - 2\varpi) + \frac{9}{2} a^2 a_{\prime}^2 e_{\prime}^2 \cos(2nt - 2\varpi_{\prime}) + \frac{a^2 a_{\prime}^2}{2} e^2 \cos(4nt - 2n_{\prime}t - 2\varpi)$$

$$+ \frac{a^2 a_{\prime}^2}{2} e_{\prime}^2 \cos(2nt - 4n_{\prime}t + 2\varpi_{\prime}) + a_{\prime}^2 (2 a_{\prime}^2 - 3 a^2) e_{\prime}^2 \cos(2n_{\prime}t - 2\varpi_{\prime})$$

$$- 4 (a^2 e^2 + a_{\prime}^2 e_{\prime}^2) a a_{\prime} \cos(nt - n_{\prime}t) - 6 a^3 a_{\prime} e^2 \cos(nt + n_{\prime}t - 2\varpi)$$

$$- 6\, a a_{\prime}^3 e_{\prime}^2 \cos(nt + n_{\prime}t - 2\varpi_{\prime}) - 6\,(a^2 + a_{\prime}^2)\, a a_{\prime} e e_{\prime} \cos(\varpi - \varpi_{\prime})$$

$$- 2(3a^2 - a_{\prime}^2)\, a a_{\prime} e e_{\prime} \cos(2nt - \varpi - \varpi_{\prime}) - 2(3\, a_{\prime}^2 - a^2)\, a a_{\prime} e e_{\prime} \cos(2n_{\prime}t - \varpi - \varpi_{\prime})$$

$$+ 2\, a^3 a_{\prime} e^2 \cos(3nt - n_{\prime}t - 2\varpi) + 2\, a\, a_{\prime}^3 e_{\prime}^2 \cos(nt - 3n_{\prime}t + 2\varpi_{\prime})$$

$$+ 2(a^2 + a_{\prime}^2)\, a a_{\prime} e e_{\prime} \cos(2\,nt - 2\,n_{\prime}t - \varpi + \varpi_{\prime}) + 9\, a^2 a_{\prime}^2 e e_{\prime} \cos(nt - n_{\prime}t + \varpi - \varpi_{\prime})$$

$$+ 14\, a^2 a_{\prime}^2\, e\, e_{\prime} \cos(nt + n_{\prime}t - \varpi - \varpi_{\prime}) - 3\, a^2 a_{\prime}^2 (e^2 + e_{\prime}^2) \cos(2nt - 2n_{\prime}t)$$

$$- 3\, a^2 a_{\prime}^2 e e_{\prime} \cos(nt - 3n_{\prime}t + \varpi + \varpi_{\prime}) - 3\, a^2 a_{\prime}^2 e e_{\prime} \cos(3nt - n_{\prime}t - \varpi - \varpi_{\prime})$$

$$- 2\, a^2 a_{\prime}^2 e e_{\prime} \cos(nt - n_{\prime}t - \varpi + \varpi_{\prime}) + a^2 a_{\prime}^2 e e_{\prime} \cos(3nt - 3n_{\prime}t - \varpi + \varpi_{\prime})$$

Replacing ε which accompanies nt,

$$R = m_{\prime} \Big\{ -\frac{b_{1,0}}{a_{\prime}} + 3\,\frac{(a^2 e^2 + a_{\prime}^2 e_{\prime}^2)}{2\,.\,2\, a_{\prime}^3} b_{3,0} + \frac{a\, a_{\prime}}{2\, a_{\prime}^3}\Big(\sin^2 \frac{\iota_{\prime}}{2} + \frac{e^2 + e_{\prime}^2}{2}\Big) b_{3,1}$$

$$- \frac{1\,.\,3}{2\,.\,4\,.\,a_{\prime}^5}\Big(2a^4 e^2 + 5a^2 a_{\prime}^2 (e^2 + e_{\prime}^2) + 2a_{\prime}^4 e_{\prime}^2\Big) b_{5,0} + \frac{1\,.\,3\,.\,2}{2\,.\,4\, a_{\prime}^5}(a^2 e^2 + a_{\prime}^2 e_{\prime}^2)\, a a_{\prime} b_{5,1}$$

$$+ \frac{1\,.\,3\,.\,3}{2\,.\,4\,.\,2}\,\frac{a^2 a_{\prime}^2}{a_{\prime}^5}\,(e^2 + e_{\prime}^2) b_{5,2} \qquad [0^*]$$

$$+ m_{\prime} \Big\{ \frac{a}{a_{\prime}^2}\Big\{\cos^2 \frac{\iota_{\prime}}{2} - \frac{e^2 + e_{\prime}^2}{2}\Big\} - \frac{b_{1,1}}{a_{\prime}} + \frac{a\, a_{\prime}}{a_{\prime}^3}\Big\{\sin^2 \frac{\iota_{\prime}}{2} + \frac{e^2 + e_{\prime}^2}{2}\Big\} \Big\{ b_{3,0} + \tfrac{1}{2}\, b_{3,2} \Big\}$$

$$+ \frac{3\,(a^2 e^2 + a_{\prime}^2 e_{\prime}^2)}{2\,.\,2\, a_{\prime}^3}\, b_{3,1} + \frac{1\,.\,3\,.\,4}{2\,.\,4\, a_{\prime}^5}\,(a^2 e^2 + a_{\prime}^2 e_{\prime}^2)\, a a_{\prime} b_{5,0}$$

$$- \frac{1\,.\,3}{2\,.\,4\, a_{\prime}^5}\Big\{2a^4 e^2 + \frac{7}{2}\, a^2 a_{\prime}^2 (e^2 + e_{\prime}) + 2a_{\prime}^4 e_{\prime}^2\Big\} b_{5,1} + \frac{1\,.\,3\,.\,2}{2\,.\,4\, a_{\prime}^5}(a^2 e^2 + a_{\prime}^2 e_{\prime}^2)\, a a_{\prime} b_{5,2}$$

$$+ \frac{1\,.\,3\,.\,3}{2\,.\,4\,.\,2}\,\frac{a^2 a_{\prime}^2}{a_{\prime}^5}\,(e^2 + e_{\prime}^2) b_{5,3} \Big\} \cos(nt - n_{\prime}t + \varepsilon - \varepsilon_{\prime}) \qquad [1]$$

$$+ m_{\prime} \Big\{ -\frac{b_{1,2}}{a_{\prime}} + \frac{a\, a_{\prime}}{2\, a_{\prime}^3}\Big\{\sin^2 \frac{\iota_{\prime}}{2} + \frac{e^2 + e_{\prime}^2}{2}\Big\} \Big\{ b_{3,1} + b_{3,3} \Big\} + \frac{3\,(a^2 e^2 + a_{\prime}^2 e_{\prime}^2)}{2\,.\,2 a_{\prime}^3}\, b_{3,2}$$

$$+ \frac{1\,.\,3\,.\,3}{2\,.\,4}\,\frac{a^2 a_{\prime}^2 (e^2 + e_{\prime}^2)}{a_{\prime}^5}\, b_{5,0} + \frac{1\,.\,3\,.\,2}{2\,.\,4}\,\frac{(a^2 e^2 + a_{\prime}^2 e_{\prime}^2)}{a_{\prime}^5}\, a a_{\prime} b_{5,1}$$

* The numbers at the side serve to distinguish the arguments.

$$-\frac{1.3}{2.4\,a_i^5}\left\{2a^4e^2+5a^2a_i^2(e^2+e_i^2)+2a_i^4e_i^2\right\}b_{5,2}$$

$$+\frac{1.3.2}{2.4\,a_i^5}(a^2e^2+a_i^2e_i^2)\,b_{5,3}\Big\}\cos(2nt-2n_it+2\varepsilon-2\varepsilon_i) \qquad [2]$$

$$+m_i\Big\{-\frac{b_{1,3}}{a_i}+\frac{a\,a_i}{2\,a_i^3}\left\{\sin^2\frac{\iota_i}{2}+\frac{e^2+e_i^2}{2}\right\}b_{3,2}+\frac{3(a^2e^2+a_i^2e_i^2)}{2.2\,a_i^3}b_{3,3}$$

$$+\frac{1.3.3}{2.4.2}\frac{a^2a_i^2(e^2+e_i^2)}{a_i^5}b_{5,1}+\frac{1.3.2}{2.4}\frac{(a^2e^2+a_i^2e_i^2)}{4a_i^5}aa_ib_{5,2}$$

$$-\frac{1.3}{2.4}\left\{2a^4e^2+5a^2a_i^2(e^2+e_i^2)+2a_i^4e_i^2\right\}b_{5,3}\Big\}\cos(3nt-3n_it+3\varepsilon-3\varepsilon_i) \qquad [3]$$

$$+m_i\Big\{-\frac{b_{1,4}}{a_i}+\frac{a\,a_i}{2\,a_i^3}\left\{\sin^2\frac{\iota_i}{2}+\frac{e^2+e_i^2}{2}\right\}b_{3,3}+\frac{1.3.3}{2.4.2}\frac{a^2a_i^2(e^2+e_i^2)}{a_i^5}b_{5,2}$$

$$+\frac{1.3.2}{2.4}\frac{(a^2e^2+a_i^2e_i^2)}{a_i^5}aa_ib_{5,3}\Big\}\cos(4nt-4n_it+4\varepsilon-4\varepsilon_i) \qquad [4]$$

$$+m_i\Big\{-\frac{b_{1,5}}{a_i}+\frac{1.3.3}{2.4.2}\frac{a^2a_i^2(e^2+e_i^2)}{a_i^5}b_{5,3}\Big\}\cos(5nt-5n_it+5\varepsilon-5\varepsilon_i) \qquad [5]$$

$$+m_i\Big\{-\frac{3\,a}{2\,a_i^2}+\frac{3\,a\,a'}{2\,a_i^3}b_{3,0}-\frac{a^2}{2\,a_i^3}b_{3,1}-\frac{a\,a_i}{2.2\,a_i^3}b_{3,2}\Big\}e\cos(n_it+\varepsilon_i-\varpi) \qquad [6]$$

$$+m_i\Big\{-\frac{a^2}{a_i^3}b_{3,0}+\frac{a\,a_i}{2\,a_i^3}b_{3,1}\Big\}e\cos(nt+\varepsilon-\varpi) \qquad [7]$$

$$+m_i\Big\{\frac{1}{2}\frac{a}{a_i^2}-\frac{a\,a_i}{2\,a_i^3}b_{3,0}-\frac{a^2}{2\,a_i^3}b_{3,1}+\frac{3}{2.2}\frac{a\,a_i}{a_i^3}b_{3,2}\Big\}e\cos(2nt-n_it+2\varepsilon-\varepsilon_i-\varpi) \qquad [8]$$

$$+m_i\Big\{-\frac{a\,a_i}{2.2\,a_i^3}b_{3,1}-\frac{a^2}{2\,a_i^3}b_{3,2}+\frac{3}{2.2}\frac{a\,a_i}{a_i^3}b_{3,3}\Big\}e\cos(3nt-2n_it+3\varepsilon-2\varepsilon_i-\varpi) \qquad [9]$$

$$+m_i\Big\{-\frac{a\,a_i}{2.2\,a_i^3}b_{3,2}-\frac{a^2}{2\,a_i^3}b_{3,3}\Big\}e\cos(4nt-3n_it+4\varepsilon-3\varepsilon_i-\varpi) \qquad [10]$$

$$-m_i\frac{a\,a_i}{2.2\,a_i^3}b_{3,3}\,e\cos(5nt-4n_it+5\varepsilon-4\varepsilon_i-\varpi) \qquad [11]$$

$$+m_i\Big\{\frac{3}{2.2}\frac{a\,a_i}{a_i^3}b_{3,1}-\frac{a^2}{2\,a_i^3}b_{3,2}-\frac{a\,a_i}{2.2\,a_i^3}b_{3,3}\Big\}e\cos(nt-2n_it+\varepsilon-2\varepsilon_i+\varpi) \qquad [12]$$

$$+m_i\Big\{\frac{3}{2.2}\frac{a\,a_i}{a_i^3}b_{3,2}-\frac{a^2}{2\,a_i^3}b_{3,3}\Big\}e\cos(2nt-3n_it+2\varepsilon-3\varepsilon_i+\varpi) \qquad [13]$$

$$+m_i\frac{3\,a\,a_i}{2.2\,a_i^3}b_{3,3}\,e\cos(3nt-4n_it+3\varepsilon-4\varepsilon_i+\varpi) \qquad [14]$$

$$+ m_{\prime}\left\{ - \frac{a_{\prime}^{2}}{a_{\prime}^{3}} b_{3,0} - \frac{a a_{\prime}}{2 a_{\prime}^{3}} b_{3,1} \right\} e_{\prime} \cos(n_{\prime}t + \varepsilon_{\prime} - \varpi_{\prime}) \qquad [15]$$

$$+ m_{\prime}\left\{ \frac{3 a a_{\prime}}{2.2 a_{\prime}^{3}} b_{3,1} - \frac{a_{\prime}^{2}}{2 a_{\prime}^{3}} b_{3,2} - \frac{a a_{\prime}}{2.2 a_{\prime}^{3}} b_{3,3} \right\} e_{\prime} \cos(2nt - n_{\prime}t + 2\varepsilon - \varepsilon_{\prime} - \varpi_{\prime}) \qquad [17]$$

$$+ m_{\prime}\left\{ \frac{3 a a_{\prime}}{2.2 a_{\prime}^{3}} b_{3,2} - \frac{a_{\prime}^{2}}{2 a_{\prime}^{3}} b_{3,3} \right\} e_{\prime} \cos(3nt - 2n_{\prime}t + 3\varepsilon - 2\varepsilon_{\prime} - \varpi_{\prime}) \qquad [18]$$

$$+ m_{\prime} \frac{3 a a_{\prime}}{2.2 a_{\prime}^{3}} b_{3,3}\, e_{\prime} \cos(4nt - 3n_{\prime}t + 4\varepsilon - 3\varepsilon_{\prime} - \varpi_{\prime}) \qquad [19]$$

$$+ m_{\prime}\left\{ \frac{2a}{a_{\prime}^{2}} - \frac{a a_{\prime}}{2 a_{\prime}^{3}} b_{3,0} - \frac{a_{\prime}^{2}}{2 a_{\prime}^{3}} b_{3,1} + \frac{3}{2.2} \frac{a a_{\prime}}{a_{\prime}^{3}} b_{3,2} \right\} e_{\prime} \cos(nt - 2n_{\prime}t + \varepsilon - 2\varepsilon_{\prime} + \varpi_{\prime}) \qquad [20]$$

$$+ m_{\prime}\left\{ - \frac{a a_{\prime}}{2.2 a_{\prime}^{3}} b_{3,1} - \frac{a_{\prime}^{2}}{2 a_{\prime}^{3}} b_{3,2} + \frac{3}{2.2} \frac{a a_{\prime}}{a_{\prime}^{3}} b_{3,3} \right\} e_{\prime} \cos(2nt - 3n_{\prime}t + 2\varepsilon - 3\varepsilon_{\prime} + \varpi_{\prime}) \qquad [21]$$

$$+ m_{\prime}\left\{ - \frac{a a_{\prime}}{2.2 a_{\prime}^{3}} b_{3,2} - \frac{a_{\prime}^{2}}{2 a_{\prime}^{3}} b_{3,3} \right\} e_{\prime} \cos(3nt - 4n_{\prime}t + 3\varepsilon - 4\varepsilon_{\prime} + \varpi_{\prime}) \qquad [22]$$

$$- m_{\prime} \frac{a a_{\prime}}{2.2 a_{\prime}^{3}} b_{3,3}\, e_{\prime} \cos(4nt - 5n_{\prime}t + 4\varepsilon - 5\varepsilon_{\prime} + \varpi_{\prime}) \qquad [23]$$

$$+ m_{\prime}\left\{ - \frac{a a_{\prime}}{2.8 a_{\prime}^{3}} b_{3,1} - \frac{a^{2}}{2.4 a_{\prime}^{3}} b_{3,2} - \frac{3}{2.8} \frac{a a_{\prime}}{a_{\prime}^{3}} b_{3,3} - \frac{1.3.9}{2.4.2} \frac{a^{2} a_{\prime}^{2}}{a_{\prime}^{5}} b_{5,0} + \frac{1.3.3}{2.4} \frac{a^{3} a_{\prime}}{a_{\prime}^{5}} b_{5,1} \right.$$

$$\left. - \frac{1.3}{2.4.2} \frac{(2a^{2} - 3a_{\prime}^{2})}{a_{\prime}^{5}} a^{2} b_{5,2} - \frac{1.3}{2.4} \frac{a^{3} a_{\prime}}{a_{\prime}^{5}} b_{5,3} \right\} e^{2} \cos(2n_{\prime}t + 2\varepsilon_{\prime} - 2\varpi) \qquad [24]$$

$$+ m_{\prime}\left\{ \frac{a}{8 a_{\prime}^{2}} - \frac{a a_{\prime}}{2.4 a_{\prime}^{3}} b_{3,0} - \frac{a^{2}}{2.4 a_{\prime}^{3}} b_{3,1} - \frac{3 a a_{\prime}}{2.8 a_{\prime}^{3}} b_{3,2} + \frac{1.3.6}{2.4} \frac{a^{3} a_{\prime}}{a_{\prime}^{5}} b_{5,0} \right.$$

$$- \frac{1.3 a^{2}}{2.4.4} \frac{(4a^{2} + 3a_{\prime}^{2})}{a_{\prime}^{5}} b_{5,1} - \frac{1.3}{2.4} \frac{a^{3} a_{\prime}}{a_{\prime}^{5}} b_{5,2}$$

$$\left. \cdot \frac{1.3}{2.4.4} \frac{a^{2} a_{\prime}^{2}}{a_{\prime}^{5}} b_{5,3} \right\} e^{2} \cos(nt + n_{\prime}t + \varepsilon + \varepsilon_{\prime} - 2\varpi) \qquad [25]$$

$$+ m_{\prime}\left\{ - \frac{a^{2}}{2.2 a_{\prime}^{3}} b_{3,0} - \frac{a a_{\prime}}{2.2 a_{\prime}^{3}} b_{3,1} - \frac{1.3}{2.4} \frac{a^{2}(2a^{2} - 3a_{\prime}^{2})}{a_{\prime}^{5}} b_{5,0} \right.$$

$$\left. + \frac{1.3.2}{2.4} \frac{a^{3} a_{\prime}}{a_{\prime}^{5}} b_{5,1} - \frac{1.3.5}{2.4.2} \frac{a^{2} a_{\prime}^{2}}{a_{\prime}^{5}} b_{5,2} \right\} e^{2} \cos(2nt + 2\varepsilon - 2\varpi) \qquad [26]$$

$$+ m_{\prime}\left\{ \frac{3}{8} \frac{a}{a_{\prime}^{2}} - \frac{3}{8} \frac{a a_{\prime}}{a_{\prime}^{3}} b_{3,0} - \frac{a a_{\prime}}{2.4 a_{\prime}^{3}} b_{3,1} - \frac{a^{2}}{2.8 a_{\prime}^{3}} b_{3,2} - \frac{1.3.2}{2.4} \frac{a^{3} a_{\prime}}{a_{\prime}^{5}} b_{5,0} \right.$$

$$- \frac{1.3 a^{2}}{2.4.4} \frac{(4a^{2} - 5a_{\prime}^{2})}{a_{\prime}^{5}} b_{5,1} + \frac{1.3.3}{2.4} \frac{a^{3} a_{\prime}}{a_{\prime}^{5}} b_{5,2}$$

$$- \frac{1.3.9}{2.4.4} \frac{a^2 a_{\prime}^2}{a_{\prime}^5} b_{5,3} \Big\} e^2 \cos(3nt - n_{\prime}t + 3\varepsilon - \varepsilon_{\prime} - 2\varpi) \qquad [27]$$

$$+ m_{\prime} \Big\{ - \frac{3 a a_{\prime}}{2.8 a_{\prime}^3} b_{3,1} - \frac{a^2}{2.4 a_{\prime}^3} b_{3,2} - \frac{a a_{\prime}}{2.8 a_{\prime}^3} b_{3,3} - \frac{1.3 a^2 a_{\prime}^2}{2.4.2 a_{\prime}^5} b_{5,0}$$

$$- \frac{1.3}{2.4} \frac{a^3 a_{\prime}}{a_{\prime}^5} b_{5,1} - \frac{1.3 a^2}{2.4.2} \frac{(2 a^2 - 3 a_{\prime}^2)}{a_{\prime}^5} b_{5,2}$$

$$+ \frac{1.3.3}{2.4} \frac{a^3 a_{\prime}}{a_{\prime}^5} b_{5,3} \Big\} e^2 \cos(4nt - 2n_{\prime}t + 4\varepsilon - 2\varepsilon_{\prime} - 2\varpi) \qquad [28]$$

$$+ m_{\prime} \Big\{ - \frac{3 a a_{\prime}}{2.8 a_{\prime}^3} b_{3,2} - \frac{a^2}{2.4 a_{\prime}^3} b_{3,3} - \frac{1.3}{2.4.4} \frac{a^2 a_{\prime}^2}{a_{\prime}^5} b_{5,1} - \frac{1.3}{2.4} \frac{a^3 a_{\prime}}{a_{\prime}^5} b_{5,2}$$

$$- \frac{1.3 a^2}{2.4.2} \frac{(2 a^2 - 3 a_{\prime}^2)}{a_{\prime}^5} b_{5,3} \Big\} e^2 \cos(5nt - 3n_{\prime}t + 5\varepsilon - 3\varepsilon_{\prime} - 2\varpi) \qquad [29]$$

$$+ m_{\prime} \Big\{ - \frac{3 a a_{\prime}}{2.8 a_{\prime}^3} b_{3,3} - \frac{1.3}{2.4.4} \frac{a^2 a_{\prime}^2}{a_{\prime}^5} b_{5,2}$$

$$- \frac{1.3}{2.4} \frac{a^3 a_{\prime}}{a_{\prime}^5} b_{5,3} \Big\} e^2 \cos(6nt - 4n_{\prime}t + 6\varepsilon - 4\varepsilon_{\prime} - 2\varpi) \qquad [30]$$

$$- m_{\prime} \frac{1.3}{2.4.4} \frac{a^2 a_{\prime}^2}{a_{\prime}^5} b_{5,3} e^2 \cos(7nt - 5n_{\prime}t + 7\varepsilon - 5\varepsilon_{\prime} - 2\varpi) \qquad [31]$$

$$+ m_{\prime} \Big\{ - \frac{a a_{\prime}}{2.8 a_{\prime}^3} b_{3,2} - \frac{a^2}{2.4 a_{\prime}^3} b_{3,3} - \frac{1.3.9}{2.4} \frac{a^2 a_{\prime}^2}{a_{\prime}^5} b_{5,1} + \frac{1.3.3}{2.4} \frac{a^3 a_{\prime}}{a_{\prime}^5} b_{5,2}$$

$$- \frac{1.3}{2.4} \frac{a^2}{2} \frac{(2 a^2 - 3 a_{\prime}^2)}{a_{\prime}^5} b_{5,3} \Big\} e^2 \cos(nt - 3n_{\prime}t + \varepsilon - 3\varepsilon_{\prime} + 2\varpi) \qquad [32]$$

$$- m_{\prime} \Big\{ - \frac{a a_{\prime}}{2.8 a_{\prime}^3} b_{3,3} - \frac{1.3.9}{2.4.4} \frac{a^2 a_{\prime}^2}{a_{\prime}^5} b_{5,2}$$

$$+ \frac{1.3.3}{2.4} \frac{a^3 a_{\prime}}{a_{\prime}^5} b_{5,3} \Big\} e^2 \cos(2nt - 4n_{\prime}t + 2\varepsilon - 4\varepsilon_{\prime} + 2\varpi) \qquad [33]$$

$$- m_{\prime} \frac{1.3.9}{2.4.4} \frac{a^2 a_{\prime}^2}{a_{\prime}^5} b_{5,3} e^2 \cos(3nt - 5n_{\prime}t + 3\varepsilon - 5\varepsilon_{\prime} + 2\varpi) \qquad [34]$$

$$+ m_{\prime} \Big\{ - \frac{5}{2.2} \frac{a a_{\prime}}{a_{\prime}^3} b_{3,1} + \frac{1.3.2}{2.4} \frac{a^2 a_{\prime}^2}{a_{\prime}^5} b_{5,0} + \frac{1.3.2}{2.4} \frac{(a^2 + a_{\prime}^2)}{a_{\prime}^5} a a_{\prime} b_{5,1}$$

$$- \frac{1.3.5}{2.4} \frac{a^2 a_{\prime}^2}{a_{\prime}^5} b_{5,2} \Big\} e e_{\prime} \cos(nt - n_{\prime}t + \varepsilon - \varepsilon_{\prime} - \varpi + \varpi_{\prime}) \qquad [35]$$

$$+ m_{\prime} \Big\{ \frac{a}{a_{\prime}^2} - \frac{a a_{\prime}}{2.2 a_{\prime}^3} b_{3,0} - \frac{9}{2.4} \frac{a a_{\prime}}{a_{\prime}^3} b_{3,2} - \frac{1.3.2}{2.4} \frac{(a^2 + a_{\prime}^2)}{a_{\prime}^5} a a_{\prime} b_{5,0}$$

$$+\frac{1.3}{2.4.2}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,1}+\frac{1.3.3}{2.4}\frac{a^2+a_{\prime}^2}{a_{\prime}^5}aa_{\prime}b_{5,2}$$

$$\left.-\frac{1.3.9}{2.4.2}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,3}\right\}ee_{\prime}\cos(2nt-2n_{\prime}t+2\varepsilon-2\varepsilon_{\prime}-\varpi+\varpi_{\prime}) \quad [36]$$

$$+m_{\prime}\left\{-\frac{aa_{\prime}}{2.4a_{\prime}^3}b_{3,1}-\frac{9}{2.4}\frac{aa_{\prime}}{a_{\prime}^3}b_{3,3}-\frac{1.3}{2.4}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,0}-\frac{1.3}{2.4}\frac{(a^2+a_{\prime}^2)}{a_{\prime}^5}aa_{\prime}b_{5,1}+\frac{1.3}{2.4}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,2}\right.$$

$$\left.+\frac{1.3.3}{2.4}\frac{a^2+a_{\prime}^2}{a_{\prime}^5}aa_{\prime}b_{5,3}\right\}ee_{\prime}\cos(3nt-3n_{\prime}t+3\varepsilon-3\varepsilon_{\prime}-\varpi+\varpi_{\prime}) \quad [37]$$

$$+m_{\prime}\left\{-\frac{aa_{\prime}}{2.4a_{\prime}^3}b_{3,2}-\frac{1.3}{2.4.2}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,1}-\frac{1.3}{2.4}\frac{(a^2+a_{\prime}^2)}{a_{\prime}^5}aa_{\prime}b_{5,2}\right.$$

$$\left.+\frac{1.3}{2.4}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,3}\right\}ee_{\prime}\cos(4nt-4n_{\prime}t+4\varepsilon-4\varepsilon_{\prime}-\varpi+\varpi_{\prime}) \quad [38]$$

$$+m_{\prime}\left\{-\frac{aa_{\prime}}{2.4a_{\prime}^3}b_{3,3}-\frac{1.3}{2.4.2}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,2}\right.$$

$$\left.-\frac{1.3}{2.4}\frac{(a^2+a_{\prime}^2)}{a_{\prime}^5}aa_{\prime}b_{5,3}\right\}ee_{\prime}\cos(5nt-5n_{\prime}t+5\varepsilon-5\varepsilon_{\prime}-\varpi+\varpi_{\prime}) \quad [39]$$

$$-m_{\prime}\frac{1.3}{2.4}\frac{a^2a_{\prime}^2}{2a_{\prime}^5}b_{5,3}\cos(6nt-6n_{\prime}t-\varpi+\varpi_{\prime}) \quad [40]$$

$$+m_{\prime}\left\{-\frac{9}{2.2}\frac{aa_{\prime}}{a_{\prime}^3}b_{3,0}-\frac{aa_{\prime}}{2.4a_{\prime}^3}b_{3,2}+\frac{1.3.6}{2.4}\frac{(a^2+a_{\prime}^2)}{a_{\prime}^5}aa_{\prime}b_{5,0}-\frac{1.3.7}{2.4.2}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,1}\right.$$

$$\left.-\frac{1.3}{2.4}\frac{(a^2+a_{\prime}^2)}{a_{\prime}^5}a_{\prime}a\,b_{5,2}-\frac{1.3}{2.4.2}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,3}\right\}ee_{\prime}\cos(\varpi-\varpi_{\prime}) \quad [41]$$

$$+m_{\prime}\left\{-\frac{9}{2.4}\frac{aa_{\prime}}{a_{\prime}^3}b_{3,1}-\frac{aa_{\prime}}{2.4a_{\prime}^3}b_{3,3}-\frac{1.3.9}{2.4}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,0}+\frac{1.3.3}{2.4}\frac{(a^2+a_{\prime}^2)}{a_{\prime}^5}aa_{\prime}b_{5,1}\right.$$

$$\left.+\frac{1.3}{2.4}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,2}-\frac{1.3}{2.4}\frac{(a^2+a_{\prime}^2)}{a_{\prime}^5}aa_{\prime}b_{5,3}\right\}ee_{\prime}\cos(nt-n_{\prime}t+\varepsilon-\varepsilon_{\prime}+\varpi-\varpi_{\prime}) \quad [42]$$

$$+m_{\prime}\left\{-\frac{9}{2.4}\frac{aa_{\prime}}{a_{\prime}^3}b_{3,2}-\frac{1.3.9}{2.4.2}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,1}+\frac{1.3.3}{2.4}\frac{(a^2+a_{\prime}^2)}{a_{\prime}^5}aa_{\prime}b_{5,2}\right.$$

$$\left.+\frac{1.3}{2.4}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,3}\right\}ee_{\prime}\cos(2nt-2n_{\prime}t+2\varepsilon-2\varepsilon_{\prime}+\varpi-\varpi_{\prime}) \quad [43]$$

$$+m_{\prime}\left\{-\frac{9}{2.4}\frac{aa_{\prime}}{a_{\prime}^3}b_{3,3}-\frac{1.3.9}{2.4.2}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,2}\right.$$

$$\left.+\frac{1.3.3}{2.4}\frac{(a^2+a_{\prime}^2)}{a_{\prime}^5}aa_{\prime}b_{5,3}\right\}ee_{\prime}\cos(3nt-3n_{\prime}t+3\varepsilon-3\varepsilon_{\prime}+\varpi-\varpi_{\prime}) \quad [44]$$

$$-m_{,}\frac{1.3.9}{2.4.2}\frac{a^2 a_{,}^2}{a_{,}^5}b_{5,3}\,ee_{,}\cos(4nt-4n_{,}t+4\varepsilon-4\varepsilon_{,}+\varpi-\varpi_{,}) \qquad [45]$$

$$+m_{,}\left\{-\frac{3a}{a_{,}^2}+\frac{3aa_{,}}{2.2a_{,}^3}b_{3,0}+\frac{3aa_{,}}{2.4a_{,}^3}b_{3,2}+\frac{1.3.2}{1.4}\frac{(3a_{,}^2-a^2)}{a_{,}^5}aa_{,}b_{5,0}-\frac{1.3.11}{2.4.2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,1}\right.$$

$$\left.+\frac{1.3}{2.4}\frac{(3a_{,}^2-a^2)}{a_{,}^5}aa_{,}b_{5,2}+\frac{1.3.3}{2.4.2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,3}\right\}ee_{,}\cos(2n_{,}t+2\varepsilon_{,}-\varpi-\varpi_{,}) \qquad [46]$$

$$+m_{,}\left\{\frac{3}{2.2}\frac{aa_{,}}{a_{,}^3}b_{3,1}-\frac{1.3.14}{2.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,0}+\frac{1.3.2}{2.4}\frac{(a^2+a_{,}^2)}{a_{,}^5}aa_{,}b_{5,1}\right.$$

$$\left.+\frac{1.3.3}{2.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,2}\right\}ee_{,}\cos(nt+n_{,}t+\varepsilon+\varepsilon_{,}-\varpi-\varpi_{,}) \qquad [47]$$

$$+m_{,}\left\{\frac{3aa_{,}}{2.2a_{,}^3}b_{3,0}+\frac{3}{2.4}\frac{aa_{,}}{a_{,}^3}b_{3,2}+\frac{1.3.2}{2.4}\frac{(3a^2-a_{,}^2)}{a_{,}^5}aa_{,}b_{5,0}-\frac{1.3.11}{2.4.2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,1}\right.$$

$$\left.+\frac{1.3}{2.4}\frac{(3a_{,}^2-a^2)}{a_{,}^5}aa_{,}b_{5,2}+\frac{1.3.3}{2.4.2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,3}\right\}ee_{,}\cos(2nt+2\varepsilon-\varpi-\varpi_{,}) \qquad [48]$$

$$+m_{,}\left\{\frac{3}{2.4}\frac{aa_{,}}{a_{,}^3}b_{3,1}+\frac{3.aa_{,}}{2.4a_{,}^3}b_{3,3}+\frac{1.3.3}{2.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,0}\right.$$

$$+\frac{1.3}{2.4}\frac{(3a^2-a_{,}^2)}{a_{,}^5}aa_{,}b_{5,1}-\frac{1.3.7}{2.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,2}$$

$$\left.+\frac{1.3}{2.4}\frac{(3a_{,}^2-a^2)}{a_{,}^5}aa_{,}b_{5,3}\right\}ee_{,}\cos(3nt-n_{,}t+3\varepsilon-\varepsilon_{,}-\varpi-\varpi_{,}) \qquad [49]$$

$$+m_{,}\left\{\frac{3}{2.4}\frac{aa_{,}}{a_{,}^3}b_{3,2}+\frac{1.3.3}{2.4.2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,1}+\frac{1.3}{2.4}\frac{(3a^2-a_{,}^2)}{a_{,}^5}aa_{,}b_{5,2}\right.$$

$$\left.-\frac{1.3.7}{2.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,3}\right\}ee_{,}\cos(4nt-2n_{,}t+4\varepsilon-2\varepsilon_{,}-\varpi-\varpi_{,}) \qquad [50]$$

$$+m_{,}\left\{\frac{3}{2.4}\frac{aa_{,}}{a_{,}^3}b_{3,3}+\frac{1.3.3}{2.4.2}\frac{a^2a_{,}^2}{a_{,}^5}aa_{,}b_{5,2}\right.$$

$$\left.+\frac{1.3}{2.4}\frac{(3a^2-a_{,}^2)}{a_{,}^5}aa_{,}b_{5,3}\right\}ee_{,}\cos(5nt-3n_{,}t+5\varepsilon-3\varepsilon_{,}-\varpi-\varpi_{,}) \qquad [51]$$

$$+m_{,}\frac{1.3.3}{2.4.2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,3}\,ee_{,}\cos(6nt-4n_{,}t+6\varepsilon-4\varepsilon_{,}-\varpi-\varpi_{,}) \qquad [52]$$

$$+m_{,}\left\{\frac{3}{2.4}\frac{aa_{,}}{a_{,}^3}b_{3,1}+\frac{3}{2.4}\frac{aa_{,}}{a_{,}^3}b_{3,3}+\frac{1.3.3}{2.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,0}\right.$$

$$+\frac{1.3}{2.4}\frac{(3a_{,}^2-a^2)}{a_{,}^5}aa_{,}b_{5,1}-\frac{1.3.7}{2.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,2}$$

$$+\frac{1.3}{2.4}\frac{(3a^2-a_i^2)}{a_i^5}aa_ib_{5,3}\Big\}ee_i\cos(nt-3n_it+\varepsilon-3\varepsilon_i+\varpi+\varpi_i) \qquad [53]$$

$$+m_i\Big\{\frac{3}{2.4}\frac{aa_i}{a_i^3}b_{3,1}+\frac{3}{2.4}\frac{aa_i}{a_i^3}b_{3,2}+\frac{1.3.3}{2.4}\frac{a^2a_i^2}{a_i^5}b_{5,0}$$

$$+\frac{1.3}{2.4}\frac{(3a_i^2-a^2)}{a_i^5}aa_ib_{5,1}+\frac{1.3}{2.4}\frac{(3a^2-a_i^2)}{a_i^5}aa_ib_{5,2}$$

$$-\frac{1.3.7}{2.4}\frac{a^2a_i^2}{a_i^5}b_{5,3}\Big\}ee_i\cos(2nt-4n_it+2\varepsilon-4\varepsilon_i+\varpi+\varpi_i) \qquad [54]$$

$$+m_i\Big\{\frac{3}{2.4}\frac{aa_i}{a_i^3}b_{3,3}+\frac{1.3.3}{2.4.2}\frac{a^2a_i^2}{a_i^5}b_{5,2}$$

$$+\frac{1.3}{2.4}\frac{(3a_i^2-a^2)}{a_i^5}aa_ib_{5,3}\Big\}ee_i\cos(3nt-5n_it+3\varepsilon-5\varepsilon_i+\varpi+\varpi_i) \qquad [55]$$

$$+m_i\frac{1.3.3}{2.4.2}\frac{a^2a_i^2}{a_i^5}b_{5,3}ee_i\cos(4nt-6n_it+4\varepsilon-6\varepsilon_i+\varpi+\varpi_i) \qquad [56]$$

$$+m_i\Big\{-\frac{a_i^2}{2.2a_i^3}b_{3,0}-\frac{aa_i}{2.2a_i^3}b_{3,1}-\frac{1.3}{2.4}\frac{(2a_i^2-3a^2)}{a_i^5}a_i^2b_{5,0}$$

$$+\frac{1.3.2}{2.4}\frac{aa_i^3}{a_i^5}b_{5,1}-\frac{1.3.5}{2.4.2}\frac{a^2a_i^2}{a_i^5}b_{5,2}\Big\}e_i^2\cos(2n_it+2\varepsilon_i-2\varpi_i) \qquad [57]$$

$$+m_i\Big\{\frac{a}{8a_i^2}-\frac{aa_i}{2.4a_i^3}b_{3,0}-\frac{a_i^2}{2.4a_i^3}b_{3,1}-\frac{3aa_i}{2.8a_i^3}b_{3,2}+\frac{1.3.6}{2.4}\frac{a_i^3a}{a_i^5}b_{5,0}$$

$$-\frac{1.3}{2.4.4}\frac{(4a^2+3a_i^2)}{a_i^5}a_i^2b_{5,1}-\frac{1.3}{2.4}\frac{aa_i^3}{a_i^5}b_{5,2}$$

$$-\frac{1.3}{2.4.4}\frac{a^2a_i^2}{a_i^5}b_{5,3}\Big\}e_i^2\cos(nt+n_it+\varepsilon+\varepsilon_i-2\varpi_i) \qquad [58]$$

$$+m_i\Big\{-\frac{aa_i}{2.8a_i^3}b_{3,1}-\frac{a_i^2}{2.4a_i^3}b_{3,2}-\frac{3}{2.8}\frac{aa_i}{a_i^3}b_{3,3}-\frac{1.3.9}{2.4.2}\frac{a^2a_i^2}{a_i^5}b_{5,0}+\frac{1.3.3}{2.4}\frac{aa_i^3}{a_i^5}b_{5,1}$$

$$-\frac{1.3}{2.4.2}\frac{(2a_i^2-3a^2)}{a_i^5}a_i^2b_{5,2}-\frac{1.3}{2.4}\frac{aa_i^3}{a_i^5}b_{5,3}\Big\}e_i^2\cos(2nt+2\varepsilon-2\varpi_i) \qquad [59]$$

$$+m_i\Big\{-\frac{aa_i}{2.8a_i^3}b_{3,2}-\frac{a_i^2}{2.4a_i^3}b_{3,3}-\frac{1.3.9}{2.4.4}\frac{a^2a_i^2}{a_i^5}b_{5,1}+\frac{1.3.3}{2.4}\frac{a_i^3a}{a_i^5}b_{5,2}$$

$$-\frac{1.3}{2.4.2}\frac{(2a_i^2-3a^2)}{a_i^5}a_i^2b_{5,3}\Big\}e_i^2\cos(3nt-n_it+3\varepsilon-\varepsilon_i-2\varpi_i) \qquad [60]$$

$$+m_i\Big\{-\frac{aa_i}{2.8a_i^3}b_{3,3}-\frac{1.3.9}{2.4.4}\frac{a^2a_i^2}{a_i^5}b_{5,2}$$

$$+\frac{1.3.3}{2.4}\frac{a_{\prime}^{3}a}{a_{\prime}^{5}}b_{5,3}\Big\} e_{\prime}^{2}\cos(4nt-2n_{\prime}t+4\varepsilon-2\varepsilon_{\prime}-2\varpi_{\prime}) \qquad [61]$$

$$-m_{\prime}\frac{1.3.9}{2.4.4}\frac{a^{2}a_{\prime}^{2}}{a_{\prime}^{5}}b_{5,3}e_{\prime}^{2}\cos(5nt-3n_{\prime}t+5\varepsilon-3\varepsilon_{\prime}-2\varpi_{\prime}) \qquad [62]$$

$$+m_{\prime}\Big\{\frac{27}{8}\frac{a}{a_{\prime}^{2}}-\frac{3\,a\,a_{\prime}}{2.4\,a_{\prime}^{3}}b_{3,0}-\frac{a_{\prime}^{2}}{2.4\,a_{\prime}^{3}}b_{3,1}-\frac{aa_{\prime}}{2.8\,a_{\prime}^{3}}b_{3,2}-\frac{1.3.2}{2.4}\frac{a_{\prime}^{3}a}{a_{\prime}^{5}}b_{5,0}$$

$$-\frac{1.3}{2.4.4}\frac{(4\,a_{\prime}^{2}-5\,a^{2})}{a_{\prime}^{5}}a_{\prime}^{2}b_{5,1}+\frac{1.3.3}{2.4}\frac{aa_{\prime}^{3}}{a_{\prime}^{5}}b_{5,2}$$

$$-\frac{1.3.9}{2.4.4}\frac{a^{2}a_{\prime}^{2}}{a_{\prime}^{5}}b_{5,3}\Big\} e_{\prime}^{2}\cos(nt-3n_{\prime}t+\varepsilon-3\varepsilon_{\prime}+2\varpi_{\prime}) \qquad [63]$$

$$+m_{\prime}\Big\{-\frac{3\,aa_{\prime}}{2.8\,a_{\prime}^{3}}b_{3,1}-\frac{a_{\prime}^{2}}{2.4\,a_{\prime}^{3}}b_{3,2}-\frac{aa_{\prime}}{2.8\,a_{\prime}^{3}}b_{3,3}-\frac{1.3}{2.4.2}\frac{a^{2}a_{\prime}^{2}}{a_{\prime}^{5}}b_{5,0}$$

$$-\frac{1.3}{2.4}\frac{aa_{\prime}^{3}}{a_{\prime}^{5}}b_{5,1}-\frac{1.3}{2.4.2}\frac{(2\,a_{\prime}^{2}-3\,a^{2})}{a_{\prime}^{5}}a_{\prime}^{2}b_{5,2}$$

$$+\frac{1.3.3}{2.4}\frac{aa_{\prime}^{3}}{a_{\prime}^{5}}b_{5,3}\Big\} e_{\prime}^{2}\cos(2nt-4n_{\prime}t+2\varepsilon-4\varepsilon_{\prime}+2\varpi_{\prime}) \qquad [64]$$

$$+m_{\prime}\Big\{-\frac{3\,aa_{\prime}}{2.8\,a_{\prime}^{3}}b_{3,2}-\frac{a_{\prime}^{2}}{2.4\,a_{\prime}^{3}}b_{3,3}-\frac{1.3}{2.4.4}\frac{a^{2}a_{\prime}^{2}}{a_{\prime}^{5}}b_{5,1}-\frac{1.3}{2.4}\frac{aa_{\prime}^{3}}{a_{\prime}^{5}}b_{5,2}$$

$$-\frac{1.3}{2.4.2}\frac{(2\,a_{\prime}^{2}-3\,a^{2})}{a_{\prime}^{5}}a_{\prime}^{2}b_{5,3}\Big\} e_{\prime}^{2}\cos(3nt-5n_{\prime}t+3\varepsilon-3\varepsilon_{\prime}+2\varpi_{\prime}) \qquad [65]$$

$$+m_{\prime}\Big\{-\frac{3}{2.8}\frac{aa_{\prime}}{a_{\prime}^{3}}b_{3,3}-\frac{1.3}{2.4.4}\frac{a^{2}a_{\prime}^{2}}{a_{\prime}^{5}}b_{5,2}$$

$$-\frac{1.3}{2.4}\frac{aa_{\prime}^{3}}{a_{\prime}^{5}}b_{5,3}\Big\} e_{\prime}^{2}\cos(4nt-6n_{\prime}t+4\varepsilon-6\varepsilon_{\prime}+2\varpi_{\prime}) \qquad [66]$$

$$-m_{\prime}\frac{1.3}{2.4.4}\frac{a^{2}a_{\prime}^{2}}{a_{\prime}^{5}}b_{5,3}e_{\prime}^{2}\cos(5nt-7n_{\prime}t+5\varepsilon-7\varepsilon_{\prime}+2\varpi_{\prime}) \qquad [67]$$

$$+m_{\prime}\Big\{\frac{a}{a_{\prime}^{2}}\Big(1+\frac{3}{2}e_{\prime}^{2}\Big)-\frac{aa_{\prime}}{a_{\prime}^{3}}b_{3,0}\Big\}\sin^{2}\frac{\iota_{\prime}}{2}\cos(nt+n_{\prime}t+\varepsilon+\varepsilon_{\prime}-2\nu_{\prime}) \qquad [68]$$

$$-m_{\prime}\frac{a\,a_{\prime}}{2\,a_{\prime}^{3}}b_{3,1}\sin^{2}\frac{\iota_{\prime}}{2}\cos(2n_{\prime}t+2\varepsilon_{\prime}-2\nu_{\prime}) \qquad [69]$$

$$m\,\frac{a\,a_{\prime}}{2\,a_{\prime}^{3}}b_{3,1}\sin^{2}\frac{\iota_{\prime}}{2}\cos(2nt+2\varepsilon-2\nu_{\prime}) \qquad [70]$$

$$-m_{\prime}\frac{a\,a_{\prime}}{2\,a_{\prime}^{3}}b_{3,2}\sin^{2}\frac{\iota_{\prime}}{2}\cos(nt+3n_{\prime}t+\varepsilon-3\varepsilon_{\prime}+2\nu_{\prime}) \qquad [71]$$

$$-m_{,}\frac{a\,a_{,}}{2\,a_{,}^{3}}b_{3,2}\sin^{2}\frac{\iota_{,}}{2}\cos(3nt-n_{,}t+3\varepsilon-\varepsilon_{,}-2\nu_{,}) \qquad [72]$$

$$-m_{,}\frac{a\,a_{,}}{2\,a_{,}^{3}}b_{3,3}\sin^{2}\frac{\iota_{,}}{2}\cos(2nt-4n_{,}t+2\varepsilon-4\varepsilon_{,}+2\nu_{,}) \qquad [73]$$

$$-m_{,}\frac{a\,a_{,}}{2\,a_{,}^{3}}b_{3,3}\sin^{2}\frac{\iota_{,}}{2}\cos(4nt-2n_{,}t+4\varepsilon-2\varepsilon_{,}-2\nu_{,}) \qquad [74]$$

In the development of R, I have supposed $\iota = 0$, so that $\iota_{,}$ is the angle contained between the orbits of the planets P and $P_{,}$, or P_1 and P_2; in the general case, when ι_1, and ι_2, are retained, $\cos\iota_{,} = \cos\iota_1\cos\iota_2 + \sin\iota_1\sin\iota_2\cos(\iota_1 - \iota_2)$, ι_1 and ι_2, being the inclinations of the orbits of the planets P_1, P_2 to any plane $x\,y$, of which the direction is arbitrary,

$$r^{\backprime}r_{,}^{\backprime}\sin(\lambda^{\backprime}-\lambda_{,}^{\backprime}) = r_1 r_2\Big\{\cos^2\frac{\iota_1}{2}\cos^2\frac{\iota_2}{2}\sin(\lambda_1-\lambda_2)$$

$$-\sin^2\frac{\iota_1}{2}\cos^2\frac{\iota_2}{2}\sin(\lambda_1+\lambda_2-2\,\nu_1) + \sin^2\frac{\iota_2}{2}\cos^2\frac{\iota_1}{2}\sin(\lambda_1+\lambda_2-2\,\nu_2)$$

$$-\sin^2\frac{\iota_1}{2}\cos^2\frac{\iota_2}{2}\sin(\lambda_1-\lambda_2-2\,\nu_1+2\,\nu_2)\Big\}$$

$$r^{\backprime}r_{,}^{\backprime}\Big\{\cos(\lambda^{\backprime}-\lambda_{,}^{\backprime}) + s\,s_{,}\Big\} = r_1 r_2\Big\{\cos^2\frac{\iota_1}{2}\cos^2\frac{\iota_2}{2}\cos(\lambda_1-\lambda_2)$$

$$+\sin^2\frac{\iota_1}{2}\cos^2\frac{\iota_2}{2}\cos(\lambda_1+\lambda_2-2\,\nu_1) + \sin^2\frac{\iota_2}{2}\cos^2\frac{\iota_1}{2}\cos(\lambda_1+\lambda_2-2\,\nu_2)$$

$$+\sin^2\frac{\iota_1}{2}\sin^2\frac{\iota_2}{2}\cos(\lambda_1-\lambda_2-2\,\nu_1+2\,\nu_2)$$

$$+\sin\iota_1\sin\iota_2\cos(\lambda_1-\lambda_2-\nu_1+\nu_2) - \sin\iota_1\sin\iota_2\cos(\lambda_1+\lambda_2-\nu_1-\nu_2)\Big\}$$

ERRATA.

In page 330, *for* ι and ν, *read* $\iota_{,}$ and $\nu_{,}$.