PHILOSOPHICAL TRANSACTIONS.

XV.—*Researches in Physical Astronomy.* By J. W. Lubbock, *Esq. V.P. and Treas. R.S.*

Read May 19, 1831.

On the Theory of the Moon.

THE method pursued by Clairaut in the solution of this important problem of Physical Astronomy, consists in the integration of the differential equations furnished by the principles of dynamics, upon the hypothesis that in the gravitation of the celestial bodies the force varies inversely as the square of the distance, and in which the true longitude of the moon is the independent variable; the time is thus obtained in terms of the true longitude, and by the reversion of series the longitude is afterwards obtained in terms of the time, which is necessary for the purpose of forming astronomical tables. But while on the one hand this method possesses the advantage, that the disturbing function can be developed with somewhat greater facility in terms of the true longitude of the moon than in terms of the mean longitude, yet on the other hand, the differential equations in which the true longitude is the independent variable are far more complicated than those in which the time is the independent variable. The latter equations are used in the planetary theory; so that the method of Clairaut has the additional inconvenience, that while the lunar theory is a particular case of the problem of the three bodies, one system of equations is used in this case, and another in the case of the planets.

The method of Clairaut has been adopted, however, by Mayer, by Laplace, and by M. Damoiseau. The last-mentioned author has arranged his results with remarkable clearness, so that any part of his processes may be easily verified by any one who does not shrink from this gigantic undertaking; and the immense labour which this method requires, when all sensible quantities

are retained, may be seen in his invaluable memoir. Mr. Brice Bronwin has recently communicated to the Society a lunar theory, in which the same method is adopted.

Having reflected much upon the difficulties of this problem, I am led to believe that the integration of the differential equations in which the time is the independent variable, is at least as easy as the method hitherto, I think, solely employed, and I now have the honour to submit to the Society a lunar theory founded upon this integration, which is in fact merely an extension of the equations given in my Researches in Physical Astronomy, already printed, by embracing those terms which, in consequence of the magnitude of the eccentricity of the moon's orbit, are sensible; and the suppression of those, on the other hand, which are insensible on account of the great distance of the sun, the disturbing body. By means of the Table which I have given (Table II.), the developments may all be effected at once with the greatest facility.

The first column contains the indices, which I have employed to distinguish the inequalities. The numbers in the second column are the indices affixed by M. Damoiseau, in the Mém. sur la Théor. de la Lune, p. 547. to the inequalities of longitude.

$$t^* = nt - n_{\prime} t, \quad x = cnt - \varpi, \quad z = n_{\prime} t - \varpi_{\prime}, \quad y = gnt - \nu.$$

0	..	0	21	45	$2t-3x$	42	73	$2t-3x-z$
1	30	$2t$ †	22	46	$2t+3x$	43	..	$2t+3x+z$
2	1	x	23	21	$2x+z$	44	26	$3x-z$
3	31	$2t-x$ ‡	24	53	$2t-2x-z$	45	..	$2t-3x+z$
4	32	$2t+x$	25	54	$2t+2x+z$	46	..	$2t+3x-z$
5	16	z §	26	20	$2x-z$	47	..	$2x+2z$
6	33	$2t-z$	27	51	$2t-2x+z$	48	75	$2t-2x-2z$
7	34	$2t+z$	28	52	$2t+2x-z$	49	..	$2t+2x+2z$
8	2	$2x$	29	23	$x+2z$	50	..	$2x-2z$
9	35	$2t-2x$	30	59	$2t-x-2z$	51	..	$2t-2x+2z$
10	36	$2t+2x$	31	..	$2t+x+2z$	52	..	$2t+2x-2z$
11	19	$x+z$	32	22	$x-2z$	53	..	$x+3z$
12	41	$2t-x-z$	33	61	$2t-x+2z$	54	..	$2t-x-3z$
13	42	$2t+x+z$	34	60	$2t+x-2z$	55	..	$2t+x+3z$
14	18	$x-z$	35	..	$3z$	56	..	$x-3z$
15	39	$2t-x+z$	36	..	$2t-3z$	57	..	$2t-x+3z$
16	40	$2t+x-z$	37	..	$2t+3z$	58	..	$2t+x-3z$
17	17	$2z$	38	9	$4x$	59	..	$4z$
18	43	$2t-2z$	39	67	$2t-4x$	60	..	$2t-4z$
19	44	$2t+2z$	40	..	$2t+4x$	61	..	$2t+4z$
20	4	$3x$	41	27	$3x+z$	62	3	$2y$

* Inconvenience arises from using the letter t in this acceptation. I have done so in order to conform to the notation of M. Damoiseau. † Variation. ‡ Evection. § Annual Equation.

63	37	$2t-2y$	105	84	$t+z$	146	....	y
64	38	$2t+2y$	106	85	$t-2x$	147	....	$2t-y$
65	5	$x-2y$	107	86	$t+2x$	148	....	$2t+y$
66	6	$x+2y$	108	91	$t-x-z$	149	....	$x-y$
67	49	$2t-x-2y$	109	92	$t+x+z$	150	....	$x+y$
68	47	$2t-x+2y$	110	89	$t-x+z$	151	....	$2t-x-y$
69	48	$2t+x-2y$	111	....	$t+x-z$	152	....	$2t-x+y$
70	50	$2t+x+2y$	112	....	$t-2z$	153	....	$2t+x-y$
71	24	$z-2y$	113	....	$t+2z$	154	....	$2t+x+y$
72	25	$z+2y$	114	....	$t-2y$	155	....	$z-y$
73	57	$2t-z-2y$	115	....	$t+2y$	156	....	$z+y$
74	56	$2t-z+2y$	116	100	$3t$	157	....	$2t-z-y$
75	55	$2t+z-2y$	117	101	$3t-x$	158	....	$2t-z+y$
76	58	$2t+z+2y$	118	102	$3t+x$	159	....	$2t+z-y$
77	7	$2x-2y$	119	103	$3t-z$	160	....	$2t+z+y$
78	8	$2x+2y$	120	104	$3t+z$	161	....	$2x\ \ y$
79	65	$2t-2x-2y$	121	....	$3t-2x$	162	....	$2x+y$
80	63	$2t-2x+2y$	122	....	$3t+2x$	163	....	$2t-2x-y$
81	64	$2t+2x-2y$	123	....	$3t-x-z$	164	....	$2t-2x+y$
82	..	$2t+2x+2y$	124	....	$3t+x+z$	165	....	$2t+2x-y$
83	..	$x+z-2y$	125	....	$3t-x+z$	166	....	$2t+2x+y$
84	..	$x+z+2y$	126	....	$3t+x-z$	167	....	$x+z-y$
85	..	$2t-x-z-2y$	127	....	$3t-2z$	168	....	$x+z+y$
86	..	$2t-x-z+2y$	128	....	$3t+2z$	169	....	$2t-x-z-y$
87	..	$2t+x+z-2y$	129	....	$3t-2y$	170	....	$2t-x-z+y$
88	..	$2t+x+z+2y$	130	....	$3t+2y$	171	....	$2t+x+z-y$
89	..	$x-z-2y$	131	120	$4t$	172	....	$2t+x+z+y$
90	..	$x-z+2y$	132	121	$4t-x$	173	....	$x-z-y$
91	..	$2t-x+z-2y$	133	122	$4t+x$	174	....	$x-z+y$
92	..	$2t-x+z+2y$	134	123	$4t-z$	175	....	$2t-x+z-y$
93	..	$2t+x-z-2y$	135	124	$4t+z$	176	....	$2t-x+z+y$
94	..	$2t+x-z+2y$	136	125	$4t-2x$	177	....	$2t+x-z-y$
95	..	$2z-2y$	137	126	$4t+2x$	178	....	$2t+x-z+y$
96	..	$2z+2y$	138	131	$4t-x-z$	179	....	$2z-y$
97	..	$2t-2z-2y$	139	....	$4t+x+z$	180	....	$2z+y$
98	..	$2t-2z+2y$	140	129	$4t-x+z$	181	....	$2t-2z-y$
99	..	$2t+2z-2y$	141	....	$4t+x-z$	182	....	$2t-2z+y$
100	..	$2t+2z+2y$	142	....	$4t-2z$	183	....	$2t+2z-y$
101	80	t*	143	....	$4t+2z$	184	....	$2t+2z+y$
102	81	$t-x$	144	127	$4t-2y$	185	....	$t-y$
103	82	$t+x$	145	....	$4t+2y$	186	....	$t+y$
104	83	$t-z$						

$$\cos 2t \cos 2t = \frac{1}{2} \cos 4t + \frac{1}{2}$$

[131] [0]

$$\cos 2t \cos x = \frac{1}{2} \cos (2t + x) + \frac{1}{2} \cos (-2t + x)$$

[4] [-3]

Hence the multiplication of $\cos 2t$ by $\cos 2t$ produces the arguments 131 and 0, similarly the multiplication of $\cos x$ by $\cos 2t$ produces the arguments 4 and -3; proceeding in this way the following Table was formed, by writing down the indices instead of the arguments themselves.

* Parallactic inequality.

TABLE I.

Showing the arguments which result from the combination of the arguments 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 17, 20, 35, 62, 101, 146 and 147, with the arguments 1, 2, 3, &c. by addition and subtraction.

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
1	131 0	4 3	132 2	133 − 2	7 6	134 5	135 − 5	10 9	136 8	137 − 8	13 12	138 11	16 15	19 18	22 21	37 36	64 63	116 101	148 147	 146	1
2	4 − 3	8 0	1 −9	10 − 1	11 14	16 − 12	13 − 15	20 − 2	3 − 21	22 − 4	23 − 5	6 − 24	26 5	29 32	38 − 8	53 56	66 65	103 −102	150 149	153 −151	2
3	132 − 2	1 9	136 0	131 − 8	15 12	138 − 14	140 − 11	4 21	 2	133 − 20	7 24	 5	6 27	33 30	10 39	57 54	68 67	117 102	152 151	...... −149	3
4	133 2	10 1	131 8	137 0	13 16	141 11	139 14	22 3	132 20	...... − 2	25 6	134 23	28 7	31 34	40 9	55 58	70 69	118 103	154 153	 150	4
5	7 − 6	11 −14	15 − 12	13 − 16	17 0	1 − 18	19 − 1	23 −26	27 − 24	25 − 28	29 − 2	3 − 30	2 −32	35 − 5	41 −44	59 −17	72 71	105 −104	156 155	159 −157	5
6	134 − 5	16 12	138 14	141 − 11	1 18	142 0	131 − 17	28 24	 26	...... − 23	4 30	 2	34 3	7 36	46 42	19 60	74 73	119 104	158 157	...... −155	6
7	135 5	13 15	140 11	139 − 14	19 1	131 17	143 0	25 27	 23	...... − 26	31 3	132 29	4 33	37 6	43 45	61 18	76 75	120 105	160 159	 156	7
8	10 − 9	20 2	4 − 21	22 − 3	23 26	28 − 24	25 27	38 0	1 − 39	...... − 1	41 14	16 − 42	44 11	47 50	....	....	78 77	107 −106	162 161	165 −163	8
9	136 − 8	3 21	...... − 2	132 − 20	27 24	...... − 26	...... − 23	1 39	 0	131 − 38	15 42	...... − 14	12 45	51 48	....	....	80 79	121 106	164 163	...... −161	9
10	137 8	22 4	133 20	 2	25 28	 23	 26	40 1	131 38	 0	43 16	141 41	46 13	49 52	....	....	82 81	122 107	166 165	 162	10
11	13 − 12	23 5	7 − 24	25 − 6	29 2	4 − 30	31 − 3	41 −14	15 − 42	43 − 16	47 0	1 − 48	8 17	53 14	....	....	84 83	109 −108	168 167	171 −169	11
12	138 − 11	6 24	...... − 5	134 − 23	3 30	...... − 2	132 − 29	16 42	 14	141 − 41	1 48	 0	18 9	15 54	....	....	86 85	123 108	170 169	...... −167	12
13	139 11	25 7	135 23	 5	31 4	133 29	 2	43 15	140 41	..... − 14	49 1	131 47	10 19	55 16	....	....	88 87	124 109	172 171	 168	13
14	16 − 15	26 − 5	6 − 27	28 − 7	2 32	34 − 3	4 − 33	44 −11	12 − 45	46 − 13	8 −17	18 − 9	50 0	11 56	....	....	90 89	111 −110	174 173	177 −175	14
15	140 − 14	7 27	 5	135 − 26	33 3	132 − 32	...... − 2	13 45	 11	139 − 44	19 9	136 17	1 51	57 12	....	....	92 91	125 110	176 175	...... −173	15
16	141 14	28 6	134 26	...... − 5	4 34	 2	133 32	46 12	138 44	...... − 11	10 18	142 8	52 1	13 58	....	....	94 93	126 111	178 177	 174	16
17	19 − 18	29 −32	33 − 30	31 34	35 5	7 − 36	37 − 6	47 −50	51 − 48	49 − 52	53 −14	15 − 54	11 −56	59 0	....	....	96 95	113 −112	180 179	183 −181	17
18	142 − 17	34 30	 32	...... − 29	6 36	...... − 5	134 − 35	52 48	 50	...... − 47	16 54	 14	58 12	1 60	....	....	92 97	127 112	182 181	...... −179	18
19	143 17	31 33	 29	...... − 32	37 7	135 35	 5	49 51	 47	...... − 50	55 15	140 53	13 57	61 1	....	....	100 99	128 113	184 183	 180	19
20	22 − 21	38 8	10 − 39	40 − 9	41 44	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	20
21	...... − 20	9 39	...... − 8	136 − 38	45 42	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	21

TABLE I. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
22	......	40	......	......	43	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	22
	20	10	38	8	46	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
23	25	41	13	43	47	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	23
	– 24	11	– 42	– 12	8	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
24	......	12	......	138	9	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	24
	– 23	42	– 11	– 41	48	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
25	......	43	139	......	49	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	25
	23	13	41	11	10	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
26	28	44	16	46	8	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	26
	– 27	14	– 45	– 15	50	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
27	......	15	......	140	51	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	27
	– 26	45	– 14	– 44	9	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
28	......	46	141	......	10	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	28
	26	16	44	14	52	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
29	31	47	19	49	53	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	29
	– 30	17	– 48	– 18	11	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
30	......	18	......	142	12	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	30
	– 29	48	– 17	– 47	54	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
31	......	49	143	......	55	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	31
	29	19	47	17	13	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
32	34	50	18	52	14	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	32
	– 33	– 17	– 51	– 19	56	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
33	......	19	......	143	57	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	33
	– 32	51	17	– 50	15	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
34	......	52	142	......	16	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	34
	32	18	50	– 17	58	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
35	37	53	57	58	59	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	35
	– 36	– 56	– 54	– 58	17	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
36	......	58	......	......	18	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	36
	– 35	54	56	– 53	60	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
37	......	55	......	......	61	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	37
	35	57	53	– 56	19	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
62	64	66	68	70	72	74	76	78	80	82	84	86	90	96	......	......	...	115	...	148	62
	– 63	– 65	– 67	– 69	– 71	– 73	– 75	– 77	– 79	– 81	– 83	– 85	–89	– 95	......	......	0	–114	146	......	
63	144	69	......	......	75	......	......	81	......	......	87	......	93	99	......	......	1	129	147	......	63
	– 62	67	65	– 66	73	71	– 72	79	77	– 78	85	83	91	97	......	......	...	114	...	–146	
64	145	70	......	......	76	......	......	82	......	......	88	......	94	100	......	......	...	130	...	......	64
	62	68	66	– 65	74	72	– 71	80	78	– 77	86	84	92	98	......	......	1	115	148	......	
65	69	77	63	81	83	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	65
	– 68	– 62	77	– 64	89	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
66	70	78	64	82	84	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	66
	– 67	62	– 79	– 63	90	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
67	......	63	......	144	91	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	67
	– 66	79	– 62	– 78	85	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
68	......	64	......	145	92	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	68
	– 65	80	62	– 77	86	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
69	......	81	144	......	87	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	69
	65	63	77	– 62	93	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	

Table I. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
70	 66	82 64	145 78	 62	88 94	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	70
71	75 − 74	83 − 90	91 − 86	87 − 94	95 − 62	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	71
72	76 − 73	84 − 89	92 − 85	88 − 93	96 62	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	72
73	...... − 72	93 85	 89	...... − 84	63 97	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	73
74	...... − 71	94 86	 90	...... − 83	64 98	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	74
75	 71	87 91	 83	...... − 90	99 63	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	75
76	 72	88 92	 84	...... − 89	100 64	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	76
101	116 −101	103 102	117 −102	118 −103	105 104	119 −104	120 −105	107 106	121 −106	122 −107	109 108	123 −108	111 110	113 112	... ...	... ...	115 114	1 0	186 185	...... −185	101
102	117 −103	101 106	121 −101	116 −107	110 108	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	102
103	118 −102	107 101	116 −106	122 −101	109 111	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	103
104	119 −105	111 108	123 −110	126 −109	101 112	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	104
105	120 −104	109 110	125 −108	124 −111	113 101	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	105
116	 101	118 117	 103	 102	120 119	 105	 104	122 121	 107	 106	124 123	 109	126 125	128 127	... ...	... ...	130 129	131 1	... ...	 186	116
117	 102	116 121	 101	 106	125 123	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	117
118	 103	122 116	 107	 101	124 126	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	118
119	 104	126 123	 111	 108	116 127	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	119
120	 105	124 125	 109	 110	128 116	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	120
131	 1	133 132	 4	 3	135 134	 7	 6	137 136	 10	 9	139 138	 13	141 140	143 142	... ...	... ...	145 144	 116	... ...	 148	131
132	 3	131 136	 1	 9	140 138	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	132
133	 4	137 131	 10	 1	139 141	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	133
134	 6	141 138	 16	 12	131 142	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	134
135	 7	139 140	 13	 15	143 131	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	135
146	148 −147	150 −149	152 −151	154 −153	156 −155	158 −157	160 −159	162 −161	164 163	166 −165	168 −167	170 −169	174 −173	180 −179	... ...	... ...	...... −146	186 −185	62 0	1 − 63	146
147	...... −146	153 151	 149	...... −150	159 157	 155	...... −156	164 163	 161	...... −162	171 169	 167	177 175	183 181	... ...	... ...	148	 185	1 63	144 0	147

Table I. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
148	......	154	......	......	160	......	......	166	......	......	172	......	178	184	...	...	......	......	64	131	148
	146	152	150	−149	158	156	−155	164	162	−161	170	168	176	182	...	...	147	186	1	62	
149	153	161	147	165	167	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	149
	−152	−146	−164	−148	173	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
150	154	162	148	166	168	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	150
	−151	146	−163	−147	174	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
151	......	147	......	......	175	......	......	......	......	......	......	......	......	......	...	..	......	......	...	......	151
	−150	163	−146	−162	169	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
152	......	148	......	......	176	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	152
	−149	164	146	−161	170	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
153	......	165	......	......	171	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	153
	149	147	161	−146	177	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
154	......	166	......	......	172	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	154
	150	148	162	146	178	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
155	159	167	175	......	179	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	155
	−158	−174	−170	−178	−146	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
156	160	168	176	172	180	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	156
	−157	−173	−169	−177	146	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
157	......	177	......	......	147	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	157
	−156	169	173	−168	181	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
158	......	178	......	......	148	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	158
	−155	170	174	−167	182	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
159	......	171	......	......	183	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	159
	155	175	167	−174	147	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
160	......	172	......	......	184	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	160
	156	176	168	−173	148	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	

	38	59	
1	40	61	1
	39	60	

Table II.

Showing the arguments which, by their combination with the arguments 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 17, 20, 35, 62, 101, 146, 147, produce the arguments, 12, 3, &c. in the left hand column. This Table is formed from the preceding, by making the numbers in the left hand column in that Table change places with the rest. A full stop is placed after the figure where it does not occupy the same *cell* as in the preceding Table.

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
1	0 131	3 4	2 132	133. − 2	6 7	5	...... − 5	9 10	8 ...	...... − 8	12 13	11	15 16	18 19	... ...	... ...	63 64	101 116	147 148	146 ...	1
2	4. − 3	0 8	− 9. 1	10. − 1	14 11	16. − 12	− 15. 13	...... − 2	... 3	...... − 4	...... − 5	 6	5 ...	....	... ...	... ...	... ...	... ...	... ...	... ...	2
3	132. − 2	9 1	0	131. − 8	12 15	...... − 14	...... − 11	 4	2 ...	....	 7	5	... 6	....	... ...	... ...	... ...	... ...	... ...	... ...	3
4	2 133	1 10	8 131	0	16 13	11	14	3	... ...	...... − 2	6	....	7 ...	....	... ...	... ...	... ...	... ...	... ...	... ...	4
5	7. − 6	− 14. 11	15. − 12	− 16. 13	0 17	− 18. 1	19. − 1	....	... ...	....	...... − 2	 3	... 2	...... − 5	... ...	... ...	... ...	... ...	... ...	... ...	5
6	134. − 5	12 16	14	...... − 11	18 1	....	131. − 17	....	... ...	....	 4	2	3 ...	 7	... ...	... ...	... ...	... ...	... ...	... ...	6
7	5 135	15 13	11	...... − 14	1 19	17 131	....	....	... ...	....	3	....	... 4	6	... ...	... ...	... ...	... ...	... ...	... ...	7
8	10. − 9	2 20	− 21. 4	22. − 3	26 23	....	....	....	... 1	...... − 1	14	 16	11 ...	....	... − 2	... ...	... ...	... ...	... ...	... ...	8
9	...... − 8	21 3	...... − 2	132. − 20	24 27	....	....	 1	... ...	 131	 15	...... −14	... 12	....	... 4	... ...	... ...	... ...	... ...	.. ...	9
10	8	4 22	20 133	2	28 25	....	....	1	... 131	....	16	....	13 ...	....	3 ...	... ...	... ...	... ...	... ...	... ...	10
11	13. − 12	5 23	− 24. 7	25. − 6	2 29	 4	...... − 3	...... − 14	... 15	...... − 16	....	 1	17 8	14	... ...	... ...	... ...	... ...	... ...	... ...	11
12	...... − 11	24 6	...... − 5	134. − 23	30 3	...... − 2	....	 16	14 ...	....	 1	....	9 18	 15	... ...	... ...	... ...	... ...	... ...	... ...	12
13	11	7 25	23 135	5	4 31	....	2	15	... ...	...... − 14	1	 131	19 10	16	... ...	... ...	... ...	... ...	... ...	... ...	13
14	16. − 15	26. − 5	− 27. 6	28. − 7	32 2	...... − 3	 4	...... − 11	... 12	...... − 13	− 17. 8	18. − 9	... ...	 11	... ...	... ...	... ...	... ...	... ...	... ...	14
15	...... − 14	27 7	5	135. − 26	3 33	....	...... − 2	 13	11 ...	....	9 19	17	... 1	12	... ...	... ...	... ...	... ...	... ...	... ...	15
16	14	6 28	26 134	...... − 5	34 4	2	....	12	... ...	...... − 11	18 10	8	1 ...	 13	... ...	... ..	... ...	. . ...	... ...	... ...	16
17	19. − 18	− 32. 29	33 − 30	− 34. 31	5 35	 7	...... − 6	....	... ...	....	...... − 14	 15	... 11	0	... ...	... − 5	... ...	... ...	.. ...	... ...	17
18	...... − 17	30 34	32	...... − 29	36 6	...... − 5	....	....	... ...	....	 16	14	12 ...	 1	... ...	... 7	... ...	... ...	... ...	... ...	18
19	17	33 31	29	...... − 32	7 37	....	5	....	... ...	....	15	....	... 13	1	... ...	6 ...	... ...	... ..	... ...	... ...	19
20	22. − 21	8	 10	...... − 9	... ...	....	....	2	... 4	...... − 3	....	....	... ...	....	0 ...	... ...	... ...	... ...	... ...	... ...	20

TABLE II. (Continued.)

	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
21	......	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	21
	− 20	9	− 8	......	...	......	......	3	− 2	......	...	......	......	...	1	...	...	...	...	...	
22	20	10	......	8	...	......	......	4	......	2	...	......	......	...	1	...	...	...	...	...	22
	......	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	
23	25.	11	......	......	8	......	......	5	......	......	2	......	......	...	...	...	...	...	...	...	23
	− 24	......	13	− 12	...	10	− 9	......	7	− 6	...	4	......	...	...	...	...	...	...	...	
24	......	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	24
	− 23	12	− 11	......	9	− 8	......	6	− 5	......	3	− 2	......	...	...	...	...	...	...	...	
25	23	13	......	11	10	......	8	7	......	5	4	......	......	...	...	...	...	...	...	..	25
	......	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	
26	28.	14	......	......	...	......	......	......	......	......	...	......	2	...	...	...	...	...	...	...	26
	− 27	......	16	− 15	8	− 9	10	− 5	6	− 7	...	......	......	...	...	...	...	...	...	...	
27	......	......	......	......	9	......	......	......	5	......	...	......	......	...	..	...	...	...	...	...	27
	− 26	15	− 14	......	...	......	8	7	......	......	...	......	3	...	...	...	...	...	...	...	
28	26	16	......	14	...	8	......	6	......	......	...	......	4	...	...	...	...	...	...	...	28
	......	......	......	......	10	......	......	......	......	− 5	...	......	......	...	...	...	...	...	...	...	
29	31.	17	......	......	11	......	......	......	......	......	5	......	......	2	...	...	...	...	...	...	29
	− 30	......	19	− 18	...	13	− 12	......	......	......	...	7	......	...	...	...	...	...	...	...	
30	......	......	18.	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	30
	− 29	18	− 17	......	12	− 11	......	......	......	......	6	− 5	......	3	...	...	...	...	...	...	
31	29	19	......	17	13	......	11	......	......	......	7	......	......	4	..	...	...	...	...	...	31
	......	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	
32	34.	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	32
	− 33	− 17	18	− 19	14	− 15	16	......	......	......	...	......	− 5	2	...	...	...	...	...	...	
33	......	......	17	......	15	......	......	......	......	......	...	......	......	3	...	...	...	...	...	...	33
	− 32	19	......	......	...	......	− 14	......	......	......	...	......	7	...	...	...	...	...	...	...	
34	32	18	......	......	...	14	......	......	......	......	...	......	6	...	...	...	...	...	...	...	34
	......	......	......	17	16	......	......	......	......	......	...	......	......	4	...	...	...	...	...	...	
35	37.	......	......	......	17	......	......	......	......	......	...	......	......	5	...	...	...	...	...	...	35
	− 36	......	......	......	...	19	− 18	......	......	......	...	......	......	...	...	...	...	...	...	...	
36	......	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	36
	− 35	......	......	......	18	− 17	......	......	....	......	...	......	......	6	...	1	...	...	...	...	
37	35	......	......	......	19	......	17	......	......	......	...	......	......	7	...	1	...	...	...	...	37
	......	......	......	......	...	......	......	......	.. ..	......	...	......	......	...	...	...	...	...	...	...	
38	......	20	......	......	...	......	......	8	......	......	...	......	......	...	2	...	...	...	...	...	38
	......	......	22	− 21	...	......	......	......	10	− 9	...	......	......	...	...	...	...	...	...	...	
39	......	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	39
	.....	21	− 20	......	...	......	......	9	− 8	......	...	......	......	...	3	...	...	...	...	...	
40	......	22	......	20	...	......	......	10	......	......	...	......	......	...	4	...	...	...	...	...	40
	......	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	
41	......	23	......	......	20	......	......	11	......	......	8	......	......	..	5	...	...	...	...	...	41
	......	......	25	− 24	...	......	......	......	13	− 12	...	10	......	...	...	...	...	...	...	...	
42	......	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	42
	......	24	− 23	......	21	......	......	12	− 11	......	9	− 8	......	...	6	...	...	...	...	...	
43	......	25	......	23	22	......	......	13	......	11	10	......	......	...	7	...	...	...	...	...	43
	......	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	
44	......	26	......	......	...	... ..	......	14	......	......	...	......	8	...	...	...	...	...	...	...	44
	.. ...	......	28	− 27	20	......	......	......	16	− 15	...	......	......	...	−5	...	...	...	...	...	

TABLE II. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
45	......	......	......	......	21	...	...	......	......	......	...	......	......	...	...	...	...	...	...	...	45
	......	27	– 26	......	......	...	...	15	– 14	......	...	......	9	...	7	...	...	...	...	...	
46	......	28	......	26	......	...	...	16	......	14	...	......	10	...	6	...	...	...	...	...	46
	......	......	......	......	22	...	...	......	......	......	...	......	......	...	...	...	...	..	...	...	
47	......	29	......	......	23	...	...	17	......	......	11	......	......	8	...	...	...	...	...	...	47
	......	......	31	– 30	......	...	...	......	19	– 18	...	13	......	...	...	...	...	...	...	...	
48	......	......	......	......	......	...	...	......	......	......	...	......	......	...	...	...	...	...	...	...	48
	.....	30	– 29	......	24	...	...	18	– 17	......	12	– 11	......	9	...	...	...	...	...	...	
49	......	31	......	29	25	...	...	19	......	17	13	......	......	10	...	...	...	...	...	...	49
	......	......	......	......	......	...	...	......	......	......	...	......	......	..	...	...	...	...	...	...	
50	......	32	......	......	......	...	...	... ..	......	......	...	......	14	...	...	...	...	...	...	...	50
	......	......	34	– 33	26	...	...	– 17	18	– 19	...	......	......	8	...	...	...	...	...	...	
51	....	......	......	......	27	...	...	.....	17	......	...	......	......	9	...	...	...	...	..	...	51
	......	33	– 32	......	......	...	...	19	......	......	...	......	15	...	...	...	...	...	...	...	
52	......	34	......	32	......	...	...	18	......	......	...	......	16	...	...	...	...	...	...	...	52
	......	... ..	......	......	28	...	...	......	......	– 17	...	......	......	10	...	...	...	...	...	...	
53	......	35	......	......	29	...	...	......	......	......	17	......	......	11	...	2	...	...	...	...	53
	......	......	37	– 36	......	...	...	......	......	......	...	19	......	...	...	...	...	...	...	...	
54	......	......	......	......	......	...	...	......	......	......	...	......	......	...	...	...	...	...	...	...	54
	......	36	– 35	......	30	...	...	......	......	......	18	– 17	......	12	...	3	...	...	...	...	
55	......	37	......	......	31	...	...	......	......	......	19	......	......	13	...	4	...	...	...	...	55
	......	......	......	......	......	...	...	......	......	......	...	......	......	...	...	...	...	...	...	...	
56	......	......	......	......	......	...	...	......	......	......	...	......	......	...	...	...	...	...	...	...	56
	......	– 35	36	– 37	32	...	...	......	......	......	...	......	– 17	14	...	2	...	...	...	...	
57	......	......	35	......	33	...	...	......	......	......	...	......	......	15	...	3	...	...	...	...	57
	......	37	......	......	......	...	...	......	......	......	...	......	19	...	...	...	...	...	...	...	
58	......	36	......	35.	......	...	...	......	......	......	...	......	18	...	...	...	...	...	...	...	58
	......	......	......	– 35	34	...	...	......	......	......	...	......	......	16	...	4	...	...	...	...	
59	......	......	......	......	35	...	...	......	......	......	...	......	......	17	...	5	...	...	...	...	59
	......	... ..	......	......	......	...	...	......	......	......	...	......	......	...	...	...	...	...	...	...	
60	......	......	......	......	......	...	...	......	......	......	...	......	......	...	...	...	...	...	...	...	60
	......	......	......	.. ...	36	...	...	......	......	.....	...	......	......	18	...	6	...	...	...	...	
61	......	......	......	......	37	...	...	......	......	......	...	......	......	19	...	7	...	...	...	...	61
	......	......	......	......	......	...	...	......	......	......	...	......	......	...	..	...	...	...	...	...	
62	64.	66.	68.	70.	72.	...	...	......	......	......	...	......	......	...	...	..	...	...	146	...	62
	– 63	– 65	– 67	– 69	– 71	...	...	......	......	......	...	......	......	...	...	...	...	...	...	148	
63	......	67	65	......	73	...	...	......	......	......	...	......	......	...	...	...	...	...	...	...	63
	– 62	69	......	– 66	75	...	...	......	......	......	...	......	......	...	...	...	1	...	147	–146	
64	62	68	66	......	74	...	...	......	......	......	...	......	......	...	...	...	1	...	148	...	64
	......	70	......	– 65	76	...	...	......	......	......	...	......	......	...	...	...	...	...	...	...	
65	69.	......	......	......	......	...	...	......	......	......	...	......	......	...	...	...	...	...	...	...	65
	– 68	– 62	63	– 64	......	...	...	......	......	......	...	... ..	......	...	...	...	2	...	...	...	
66	70.	62	......	......	......	...	...	......	......	......	...	......	......	...	...	...	2	...	...	...	66
	– 67	......	64	– 63	......	...	...	......	......	......	...	......	......	..	...	...	...	...	...	...	
67	76	......	......	......	......	...	...	......	......	......	...	......	......	...	...	...	...	...	...	...	67
	– 66	63	– 62	......	......	...	...	......	......	......	...	......	......	...	...	...	3	...	...	...	
68	......	......	62	......	......	...	...	......	......	......	...	......	......	...	..	...	3	...	...	...	68
	– 65	64	......	......	......	...	...	......	......	......	...	......	......	...	...	...	...	...	...	...	

TABLE II. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
69	65	63	...	......	......	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	69
	......	...	...	− 62	......	......	......	......	......	......	......	......	......	...	...	...	4	...	...	...	
70	66	64	...	62	......	......	......	......	......	......	......	......	......	...	...	...	4	...	...	...	70
	......	...	...	......	......	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	
71	75.	...	...	......	......	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	71
	− 74	...	...	......	− 62	63	− 64	......	......	......	......	......	......	...	...	...	5	...	...	...	
72	76.	...	...	......	62	......	......	......	......	......	......	......	......	...	...	...	5	...	...	...	72
	− 73	...	...	......	......	64	− 63	......	......	......	......	......	......	...	...	...	...	...	...	...	
73	......	...	...	......	......	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	73
	− 72	...	...	......	63	− 62	......	......	......	......	......	......	......	...	...	...	6	...	...	...	
74	......	...	...	......	......	62	......	......	......	......	......	......	......	...	...	...	6	...	...	...	74
	− 71	...	...	......	64	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	
75	71	...	...	......	63	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	75
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39	 1	....	39	60	....	 1	60
40	1	....	40	61	....	1	61

Table II. may be used in forming the developments required in the method employed by MM. Laplace and Damoiseau; for this purpose it is only necessary to make $t = \lambda' - \lambda_{\prime}$ instead of $n\,t - n_{\prime}t$

$$x = c\,\lambda' - \varpi \quad . \quad . \quad . \quad c\,n\,t - \varpi$$

$$z = c_{\prime}\lambda_{\prime} - \varpi_{\prime} \quad . \quad . \quad . \quad c_{\prime}n_{\prime}t - \varpi_{\prime}$$

$$\text{and } y = g\,\lambda' - \nu \quad . \quad . \quad . \quad g\,n\,t - \nu$$

The notation throughout is the same as that used Phil. Trans. 1830, p. 328, with the exception of the indices of the arguments.

In the elliptic movement;

$$a^5 r^{-5} = 1 + 5\,e^2\left(1 + \frac{21}{8}e^2\right) + 5\,e\left(1 + \frac{27}{8}e^2\right)\cos x + 10\,e^2\left(1 + \frac{31}{12}e^2\right)\cos 2\,x$$
$$+ \frac{145}{8}e^3\cos 3\,x + \frac{745}{48}e^4\cos 4\,x$$

$$a^4 r^{-4} = 1 + 3\,e^2 + 4\,e\cos x + 7\,e^2\cos 2\,x$$

$$a^3 r^{-3} = 1 + \frac{3}{2}e^2\left(1 + \frac{5}{4}e^2\right) + 3e\left(1 + \frac{9}{8}e^2\right)\cos x + \frac{9}{2}e^2\left(1 + \frac{7}{9}e^2\right)\cos 2\,x$$
$$+ \frac{53}{8}e^3\cos 3\,x + \frac{77}{8}e^4\cos 4\,x$$

$$a^2 r^{-2} = 1 + \frac{e^2}{2}\left(1 + \frac{3}{4}e^2\right) + 2\,e\left(1 + \frac{3}{8}e^2\right)\cos x + \frac{5}{2}e^2\left(1 + \frac{2}{15}e^2\right)\cos 2\,x$$
$$+ \frac{13}{4}e^3\cos 3\,x + \frac{103}{24}e^4\cos 4\,x$$

$$a\,r^{-1} = 1 + e\left(1 - \frac{e^2}{8}\right)\cos x + e^2\left(1 - \frac{e^2}{3}\right)\cos 2\,x + \frac{9}{8}e^3\cos 3\,x + \frac{4}{3}e^4\cos 4\,x$$

$$\frac{r}{a} = 1 + \frac{e^2}{2} - e\left(1 - \frac{3\,e^2}{8}\right)\cos x - \frac{e^2}{2}\left(1 - \frac{2\,e^2}{3}\right)\cos 2\,x - \frac{3\,e^3}{8}\cos 3\,x - \frac{e^4}{3}\cos 4\,x$$

$$\frac{r^2}{a^2} = 1 + \frac{3\,e^2}{2} - 2\,e\left(1 - \frac{e^2}{8}\right)\cos x - \frac{e^2}{2}\left(1 - \frac{e^2}{3}\right)\cos 2\,x - \frac{e^3}{4}\cos 3\,x - \frac{e^4}{6}\cos 4\,x$$

$$\frac{r^3}{a^3} = 1 + 3\,e^2\left(1 + \frac{e^2}{8}\right) - 3\,e\left(1 + \frac{3}{8}e^2\right)\cos x - \frac{5}{8}e^4\cos 2\,x + \frac{e^3}{8}\cos 3\,x + \frac{e^4}{8}\cos 4\,x$$

$$\frac{r^4}{a^4} = 1 + 5\,e^2 - 4\,e\cos x + e^2\cos 2\,x$$

$$\begin{aligned}\frac{a}{r} = {} & r_0 \\ & + r_1\cos 2\,t \\ & + e\,r_2\cos x \\ & + e\,r_3\cos(2\,t - x) \\ & + e\,r_4\cos(2\,t + x) \\ & + e_{\prime}\,r_5\cos z \\ & + e_{\prime}\,r_6\cos(2\,t - z) + \text{\&c. \&c.}\end{aligned}$$

$$\begin{aligned}\lambda &= n\,t\\ &+ \lambda_1 \cos 2\,t\\ &+ e\,\lambda_2 \cos x\\ &+ e\,\lambda_3 \cos (2\,t - x)\\ &+ e\,\lambda_4 \cos (2\,t + x)\\ &+ e_{\prime}\,\lambda_5 \cos z \text{ \&c. \&c.}\end{aligned}$$

The quantities λ correspond to the quantities b in M. DAMOISEAU's notation.

$$\begin{aligned}s &= \gamma\, s_{146} \sin y\\ &+ \gamma\, s_{147} \sin (2\,t - y)\\ &+ \gamma\, s_{148} \sin (2\,t + y)\\ &+ e\,\gamma\, s_{149} \sin (x - y) \text{ \&c. \&c.}\\ \gamma &= \tan \iota\end{aligned}$$

$$R = m_{\prime}\left\{\frac{r^{\backprime} r_{\prime} \cos (\lambda - \lambda_{\prime})}{r_{\prime}^3} - \frac{1}{\{r^2 - 2\,r^{\backprime}\, r_{\prime} \cos (\lambda^{\backprime} - \lambda_{\prime}) + r_{\prime}^2\}^{\frac{1}{2}}}\right\}$$

$$= m_{\prime}\left\{-\frac{1}{r_{\prime}} + \frac{r^2}{2\,r_{\prime}^3} - \frac{3}{8}\,\frac{\{2\,r^{\backprime}\, r_{\prime} \cos (\lambda^{\backprime} - \lambda_{\prime}) - r^2\}^2}{r_{\prime}^5} - \frac{15}{48}\,\frac{\{2\,r^{\backprime}\, r_{\prime} \cos (\lambda^{\backprime} - \lambda_{\prime}) - r^2\}^3}{r_{\prime}^7}\right\}$$

$$= m_{\prime}\left\{-\frac{1}{r_{\prime}} + \frac{r^2}{2\,r_{\prime}^3} - \frac{3}{2}\,\frac{r^{\backprime 2} r_{\prime}^2}{r_{\prime}^5} \cos (\lambda^{\backprime} - \lambda_{\prime})^2 + \frac{3}{2}\,\frac{r^2 r^{\backprime} r_{\prime}}{r_{\prime}^5} \cos (\lambda - \lambda_{\prime}) - \frac{5}{2}\,\frac{r^{\backprime 3} r_{\prime}^3}{r_{\prime}^7} \cos (\lambda^{\backprime} - \lambda_{\prime})^3\right\}$$

$$= m_{\prime}\left\{-\frac{1}{r_{\prime}} - \frac{r^{\backprime 2}}{4\,r_{\prime}^3}\left\{1 + 3 \cos (2\,\lambda^{\backprime} - 2\,\lambda_{\prime}) - 2\,s^2\right\}\right.$$
$$\left. - \frac{r^{\backprime 3}}{8\,r_{\prime}^4}\left\{3\,(1 - 4\,s^2) \cos (\lambda^{\backprime} - \lambda_{\prime}) + 5 \cos (3\,\lambda^{\backprime} - 3\,\lambda_{\prime})\right\}\right\}$$

$$r^{\backprime}\, r_{\prime} \tfrac{\cos}{\sin} (\lambda^{\backprime} - \lambda_{\prime}) = r\, r_{\prime}\left\{\cos^2 \frac{\iota}{2} \tfrac{\cos}{\sin} (\lambda - \lambda_{\prime}) + \sin^2 \frac{\iota}{2} \tfrac{\cos}{\sin} (\lambda + \lambda_{\prime} - 2\,\nu)\right\}$$

$$=^{*} a\, a_{\prime} \cos^2 \frac{\iota}{2}\left\{\left(1 - \frac{e^2}{2} - \frac{e^4}{64}\right)\left(1 - \frac{e_{\prime}^2}{2} - \frac{e_{\prime}^4}{64}\right) \tfrac{\cos}{\sin}\, t - \frac{3\,e}{2}\left(1 - \frac{e_{\prime}^2}{2}\right) \tfrac{\cos}{\sin} (t - x)\right.$$
$$+ \frac{e}{2}\left(1 - \frac{3}{4}\,e^2\right)\left(1 - \frac{e_{\prime}^2}{2}\right) \tfrac{\cos}{\sin} (t + x) + \frac{3}{8}\,e^2 (1 - e^2)\left(1 - \frac{e_{\prime}^2}{2}\right) \tfrac{\cos}{\sin} (t + 2\,x)$$
$$+ \frac{e^3}{3} \tfrac{\cos}{\sin} (t + 3\,x) + \frac{125}{384}\,e^4 \tfrac{\cos}{\sin} (t + 4\,x) + \frac{e^2}{8}\left(1 + \frac{e^2}{3}\right)\left(1 - \frac{e_{\prime}^2}{2}\right) \tfrac{\cos}{\sin} (t - 2\,x)$$
$$+ \frac{e^3}{24} \tfrac{\cos}{\sin} (t - 3\,x) + \frac{3}{128}\,e^4 \tfrac{\cos}{\sin} (t - 4\,x) - \frac{3}{2}\,e_{\prime}\left(1 - \frac{e^2}{2}\right) \tfrac{\cos}{\sin} (t + z)$$
$$+ \frac{9}{4}\,e\,e_{\prime} \tfrac{\cos}{\sin} (t - x + z) - \frac{3}{4}\,e\,e_{\prime}\left(1 - \frac{3}{4}\,e^2\right) \tfrac{\cos}{\sin} (t + x + z)$$
$$- \frac{9}{16}\,e^2 e_{\prime} \tfrac{\cos}{\sin} (t + 2\,x + z) - \frac{e^3 e_{\prime}}{2} \tfrac{\cos}{\sin} (t + 3\,x + z) - \frac{3}{16}\,e^2 e_{\prime} \tfrac{\cos}{\sin} (t - 2\,x + z)$$

* See Phil. Trans. 1830, p. 343.

$$
\begin{aligned}
& -\frac{e^3 e_{\prime}}{16} \frac{\cos}{\sin}(t-3x+z) + \frac{e_{\prime}}{2}\left(1-\frac{3}{4}e_{\prime}^2\right)\left(1-\frac{e^2}{2}\right)\frac{\cos}{\sin}(t-z) \\
& -\frac{3}{4}e e_{\prime}\left(1-\frac{3}{4}e_{\prime}^2\right)\frac{\cos}{\sin}(t-x-z) \\
& +\frac{e e_{\prime}}{4}\left(1-\frac{3}{4}e^2\right)\left(1-\frac{3}{4}e_{\prime}^2\right)\frac{\cos}{\sin}(t+x-z) + \frac{3}{16}e^2 e_{\prime}\frac{\cos}{\sin}(t+2x-z) \\
& +\frac{e^3 e_{\prime}}{6}\frac{\cos}{\sin}(t+3x-z) + \frac{e^2 e_{\prime}}{16}\frac{\cos}{\sin}(t-2x-z) + \frac{e^3 e_{\prime}}{48}\frac{\cos}{\sin}(t-3x-z) \\
& +\frac{3}{8}e_{\prime}^2(1-e_{\prime}^2)\left(1-\frac{e^2}{2}\right)\frac{\cos}{\sin}(t-2z) - \frac{9}{16}e e_{\prime}^2\frac{\cos}{\sin}(t-x-2z) \\
& +\frac{3}{16}e e_{\prime}^2\frac{\cos}{\sin}(t+x-2z) + \frac{9}{64}e^2 e_{\prime}^2\frac{\cos}{\sin}(t+2x-2z) \\
& +\frac{3}{64}e^2 e_{\prime}^2\frac{\cos}{\sin}(t-2x-2z) + \frac{e_{\prime}^3}{3}\frac{\cos}{\sin}(t-3z) - \frac{e e_{\prime}^3}{2}\frac{\cos}{\sin}(t-x-3z) \\
& +\frac{e e_{\prime}^3}{6}\frac{\cos}{\sin}(t+x-3z) + \frac{125}{384}e_{\prime}^4\frac{\cos}{\sin}(t-4z) \\
& +\frac{e_{\prime}^2}{8}\left(1+\frac{e_{\prime}^2}{3}\right)\left(1-\frac{e^2}{2}\right)\frac{\cos}{\sin}(t+2z) - \frac{3}{16}e e_{\prime}^2\frac{\cos}{\sin}(t-x+2z) \\
& +\frac{e e_{\prime}^2}{16}\frac{\cos}{\sin}(t+x+2z) + \frac{3}{64}e^2 e_{\prime}^2\frac{\cos}{\sin}(t+2x+2z) + \frac{e^2 e_{\prime}^2}{64}\frac{\cos}{\sin}(t-2x+2z) \\
& +\frac{e_{\prime}^3}{24}\frac{\cos}{\sin}(t+3z) - \frac{e e_{\prime}^3}{16}\frac{\cos}{\sin}(t-x+3z) + \frac{e e_{\prime}^3}{48}\frac{\cos}{\sin}(t+x+3z) \\
& +\frac{3}{128}e_{\prime}^4\frac{\cos}{\sin}(t+4z)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
+ a a_{\prime}\sin^2\frac{\iota}{2}\Big\{ & \left(1-\frac{e^2+e_{\prime}^2}{2}\right)\frac{\cos}{\sin}(t-2y) - \frac{3}{2}e\frac{\cos}{\sin}(t+x-2y) + \frac{e}{2}\frac{\cos}{\sin}(t-x-2y) \\
& +\frac{3}{8}e^2\frac{\cos}{\sin}(t-2x-2y) + \frac{e^2}{8}\frac{\cos}{\sin}(t+2x-2y) - \frac{3}{2}e_{\prime}\frac{\cos}{\sin}(t+z-2y) \\
& +\frac{9}{4}e e_{\prime}\frac{\cos}{\sin}(t+x+z-2y) - \frac{3}{4}e e_{\prime}\frac{\cos}{\sin}(t-x+z-2y) \\
& +\frac{e_{\prime}}{2}\frac{\cos}{\sin}(t-z-2y) - \frac{3}{4}e e_{\prime}\frac{\cos}{\sin}(t+x-z-2y) + \frac{e e_{\prime}}{4}\frac{\cos}{\sin}(t-x-z-2y)
\end{aligned}
$$

$r'^2 r_{\prime}^2 \cos(\lambda' - \lambda_{\prime})^2$

$$
\begin{aligned}
= a^2 a_{\prime}^2\cos^4\frac{\iota}{2}\Big\{ & \frac{1}{2} + \left\{-\frac{1}{2}+\frac{9}{8}+\frac{1}{8}\right\}(e^2+e_{\prime}^2) + \Big\{\frac{1}{2}-\frac{9}{8}-\frac{1}{8}-\frac{9}{8}+\frac{81}{32}+\frac{9}{32} \\
& -\frac{1}{8}+\frac{9}{32}+\frac{1}{32}\Big\}e^2 e_{\prime}^2 + \left\{\frac{7}{64}-\frac{3}{16}+\frac{9}{128}+\frac{1}{128}\right\}(e^4+e_{\prime}^4)
\end{aligned}
$$

[0]

$$+\left\{\frac{1}{2}+\left\{-\frac{1}{2}-\frac{3}{4}\right\}(e^2+e_{\prime}^2)+\left\{\frac{1}{2}+\frac{3}{4}+\frac{3}{4}+\frac{9}{16}+\frac{9}{16}\right\}e^2 e_{\prime}^2\right.$$

$$\left.+\left\{\frac{7}{64}+\frac{9}{16}+\frac{3}{64}\right\}(e^4+e_{\prime}^4)\right\}\cos 2t$$

[1]

$$+\left\{-\frac{3}{2}+\frac{1}{2}+\left\{\frac{3}{4}-\frac{5}{8}-\frac{3}{16}+\frac{3}{16}\right\}e^2\right.$$

$$\left.+\left\{\frac{3}{2}-\frac{1}{2}-\frac{27}{8}+\frac{9}{8}-\frac{3}{8}+\frac{1}{8}\right\}e_{\prime}^2\right\}e\cos x$$

[2] [5]*

$$+\left\{-\frac{3}{2}+\left\{\frac{3}{4}+\frac{1}{16}\right\}e^2+\left\{\frac{3}{2}+\frac{9}{8}+\frac{9}{8}\right\}e_{\prime}^2\right\}e\cos(2t-x)$$

[3] [7]

$$+\left\{-\frac{1}{2}+\left\{\frac{5}{8}-\frac{9}{16}\right\}e^2+\left\{-\frac{1}{2}-\frac{3}{8}-\frac{3}{8}\right\}e_{\prime}^2\right\}e\cos(2t+x)$$

[4] [6]

$$+\left\{\frac{3}{8}+\frac{1}{8}-\frac{3}{4}+\left\{-\frac{9}{16}-\frac{1}{48}+\frac{9}{16}-\frac{1}{16}+\frac{1}{6}\right\}e^2\right.$$

$$\left.+\left\{-\frac{3}{8}-\frac{1}{8}+\frac{3}{4}+\frac{27}{32}+\frac{9}{32}-\frac{27}{16}+\frac{3}{32}+\frac{1}{32}-\frac{3}{16}\right\}e_{\prime}^2\right\}e^2\cos 2x$$

[8] [17]

$$+\left\{\frac{9}{8}+\frac{1}{8}+\left\{-\frac{1}{48}+\frac{1}{48}\right\}e^2\right.$$

$$\left.+\left\{-\frac{9}{8}-\frac{1}{8}-\frac{3}{32}-\frac{27}{16}-\frac{3}{32}\right\}e_{\prime}^2\right\}e^2\cos(2t-2x)$$

[9] [19]

$$+\left\{\frac{1}{8}+\frac{3}{8}+\left\{-\frac{3}{16}-\frac{9}{16}-\frac{1}{2}\right\}e^2\right.$$

$$\left.+\left\{-\frac{1}{8}-\frac{3}{8}-\frac{9}{32}-\frac{3}{16}-\frac{9}{32}\right\}e_{\prime}^2\right\}e^2\cos(2t+2x)$$

[10] [18]

$$+\left\{-\frac{3}{4}-\frac{3}{4}+\frac{9}{4}+\frac{1}{4}+\left\{\frac{15}{16}+\frac{3}{8}-\frac{9}{8}-\frac{3}{32}-\frac{9}{32}\quad\frac{5}{16}\right.\right.$$

$$\left.\left.+\frac{3}{32}+\frac{9}{32}\right\}(e^2+e_{\prime}^2)\right\}e\,e_{\prime}\cos(x+z)$$

[11]

* The coefficient of argument 5 being the same, e and $e_{\prime}$ changing places, that coefficient is not written down, in order to avoid useless repetition.

$$+\left\{-\frac{3}{4}-\frac{3}{4}+\left\{\frac{3}{8}+\frac{3}{8}+\frac{1}{32}+\frac{1}{32}\right\}e^2 + \left\{\frac{15}{16}+\frac{15}{16}+\frac{27}{32}+\frac{27}{32}\right\}e_{,}^2\right\} e\,e_{,} \cos(2t-x-z)$$

[12] [13]

$$+\left\{\frac{9}{4}+\frac{1}{4}\quad\frac{3}{4}-\frac{3}{4}+\left\{-\frac{9}{8}-\frac{5}{16}+\frac{9}{32}+\frac{3}{8}+\frac{15}{16}+\frac{3}{32}-\frac{9}{32}-\frac{3}{32}\right\}(e^2+e_{,}^2)\right\} e\,e_{,}\cos(x-z)$$

[14]

$$+\left\{\frac{9}{4}+\frac{9}{4}+\left\{-\frac{9}{8}-\frac{9}{8}-\frac{3}{32}-\frac{3}{32}\right\}(e^2+e_{,}^2)\right\} e\,e_{,}\cos(2t-x+z)$$

[15]

$$+\left\{\frac{1}{4}+\frac{1}{4}+\left\{-\frac{5}{16}-\frac{9}{32}-\frac{5}{16}-\frac{9}{32}\right\}(e_{,}+e_{,}^2)\right\}\cos(2t+x-z)$$

[16]

$$+\left\{\frac{1}{3}+\frac{1}{24}-\frac{9}{16}+\frac{1}{16}\right\} e\cos 3x$$

[20] [35]

$$+\left\{\frac{1}{24}-\frac{3}{16}\right\} e^3\cos(2t-3x)$$

[21] [37]

$$+\left\{\frac{1}{3}+\frac{3}{16}\right\} e^3\cos(2t+3x)$$

[22] [36]

$$+\left\{-\frac{9}{16}+\frac{1}{16}+\frac{9}{8}-\frac{3}{8}+\frac{3}{16}-\frac{3}{16}\right\} e^2 e_{,}\cos(2x+z)$$

[23] [29]

$$+\left\{\frac{1}{16}+\frac{9}{8}+\frac{1}{16}\right\} e^2 e_{,}\cos(2t-2x-z)$$

[24] [31]

$$+\left\{-\frac{9}{16}-\frac{3}{8}-\frac{9}{16}\right\} e^2 e_{,}\cos(2t+2x+z)$$

[25] [30]

$$+\left\{-\frac{3}{16}+\frac{3}{16}-\frac{3}{8}+\frac{9}{8}-\frac{9}{16}+\frac{1}{16}\right\} e^2 e_{,}\cos(2x-z)$$

[26] [32]

$$+\left\{-\frac{3}{16}-\frac{27}{8}-\frac{3}{16}\right\} e^2 e_{,}\cos(2t-2x+z)$$

[27] [33]

$$+\left\{\frac{3}{16}+\frac{1}{8}+\frac{3}{16}\right\} e^2 e_{,}\cos(2t+2x-z)$$

[28] [34]

$$+\left\{\frac{125}{384}+\frac{3}{128}-\frac{1}{2}+\frac{1}{48}+\frac{3}{64}\right\} e^4\cos 4x$$

[38] [59]

$$+\left\{\frac{1}{128}+\frac{3}{128}-\frac{1}{16}\right\}e^4\cos(2t-4x)$$

[39] [61]

$$+\left\{\frac{9}{128}+\frac{125}{384}+\frac{1}{6}\right\}e^4\cos(2t+4x)$$

[40] [60]

$$+\left\{-\frac{1}{2}+\frac{1}{48}+\frac{27}{32}+\frac{1}{32}-\frac{9}{32}+\frac{1}{6}-\frac{3}{32}-\frac{1}{16}\right\}e^3\cos(3x+z)$$

[41] [53]

$$+\left\{\frac{1}{48}-\frac{3}{32}-\frac{3}{32}+\frac{1}{48}\right\}e^3e_{\prime}\cos(2t-3x-z)$$

[42] [55]

$$+\left\{-\frac{1}{2}-\frac{9}{32}-\frac{9}{32}-\frac{1}{2}\right\}e^3e_{\prime}\cos(2t+3x+z)$$

[43] [54]

$$+\left\{-\frac{1}{16}+\frac{1}{6}-\frac{9}{32}-\frac{3}{32}+\frac{27}{32}-\frac{1}{2}+\frac{1}{32}+\frac{1}{48}\right\}e^3e_{\prime}\cos(3x-z)$$

[44] [56]

$$+\left\{-\frac{1}{16}+\frac{9}{32}+\frac{9}{32}-\frac{1}{16}\right\}e^3e_{\prime}\cos(2t-3x+z)$$

[45] [57]

$$+\left\{\frac{1}{6}+\frac{3}{32}+\frac{3}{32}+\frac{1}{6}\right\}e^2e_{\prime}\cos(2t+3x-z)$$

[46] [58]

$$+\left\{\frac{3}{64}+\frac{3}{64}-\frac{3}{32}-\frac{9}{32}+\frac{9}{64}+\frac{1}{64}-\frac{3}{32}+\frac{9}{16}-\frac{9}{32}\right\}e^2e_{\prime}^2\cos(2x+2z)$$

[47]

$$+\left\{\frac{9}{32}+\frac{3}{64}+\frac{27}{32}+\frac{3}{64}+\frac{1}{32}\right\}e^2e_{\prime}^2\cos(2t-2x+2z)$$

[48]

$$+\left\{\frac{9}{32}+\frac{3}{64}+\frac{1}{32}+\frac{3}{64}+\frac{27}{32}\right\}e^2e_{\prime}^2\cos(2t+2x+2z)$$

[49]

$$+\left\{\frac{9}{64}+\frac{1}{64}-\frac{9}{32}-\frac{3}{32}+\frac{3}{64}+\frac{3}{64}-\frac{9}{32}+\frac{9}{16}-\frac{3}{32}\right\}e^2e_{\prime}^2\cos(2x-2z)$$

[50]

$$+\left\{\frac{81}{32}+\frac{1}{64}+\frac{9}{32}+\frac{1}{64}+\frac{9}{32}\right\}e^2e_{\prime}^2\cos(2t-2x+2z)$$

[51]

$$+\left\{\frac{1}{32}+\frac{9}{64}+\frac{3}{32}+\frac{9}{64}+\frac{3}{32}\right\}e^2e_{\prime}^2\cos(2t+2x-2z)$$

[52]

$$+a^2a_{\prime}^2\sin^2\frac{\iota}{2}\cos^2\frac{\iota}{2}\left\{\left\{1+\left\{-1-\frac{3}{4}-\frac{3}{4}\right\}e^2+\left\{-1+\frac{9}{4}+\frac{1}{4}\right\}e_{\prime}^2\right\}\cos 2y\right.$$

[62]

$$\left.+\left\{1+\left\{-1+\frac{9}{4}+\frac{1}{4}\right\}e^2+\left\{-1-\frac{3}{4}-\frac{3}{4}\right\}e_{\prime}^2\right\}\cos(2t-2y)\right.$$

[63]

$$+\left\{-\frac{3}{2}-\frac{3}{2}\right\} e \cos(x-2y) + \left\{\frac{1}{2}+\frac{1}{2}\right\} e \cos(x+2y)$$
[65] [66]

$$+\left\{\frac{1}{2}-\frac{3}{2}\right\} e \cos(2t-x-2y)$$
[67]

$$+\left\{-\frac{3}{2}+\frac{1}{2}\right\} e \cos(2t+x-2y) + \left\{-\frac{3}{2}+\frac{1}{2}\right\} e_{,} \cos(z-2y)$$
[69] [71]

$$+\left\{\frac{1}{2}-\frac{3}{2}\right\} e_{,} \cos(z+2y) + \left\{\frac{1}{2}+\frac{1}{2}\right\} e_{,} \cos(2t-z-2y)$$
[72] [73]

$$+\left\{-\frac{3}{2}-\frac{3}{2}\right\} e_{,} \cos(2t+z-2y) + \left\{\frac{1}{8}+\frac{9}{4}+\frac{1}{8}\right\} e^2 \cos(2x-2\ \ )$$
[75] [77]

$$+\left\{\frac{3}{8}+\frac{1}{4}+\frac{3}{8}\right\} e^2 \cos(2x+2y)$$
[78]

$$+\left\{\frac{3}{8}-\frac{3}{4}+\frac{1}{8}\right\} e^2 \cos(2t-2x-2y) + \left\{\frac{1}{8}-\frac{3}{4}+\frac{3}{8}\right\} e^2 \cos(2t+2x-2y)$$
[79] [81]

$$+\left\{\frac{9}{4}+\frac{9}{4}-\frac{3}{4}-\frac{3}{4}\right\} e e_{,} \cos(x+z-2y)$$
[83]

$$+\left\{\frac{1}{4}+\frac{1}{4}-\frac{3}{4}-\frac{3}{4}\right\} e e_{,} \cos(x+z+2y)$$
[84]

$$+\left\{\frac{1}{4}-\frac{3}{4}+\frac{1}{4}-\frac{3}{4}\right\} e e_{,} \cos(2t-x-z-2y)$$
[85]

$$+\left\{\frac{9}{4}-\frac{3}{4}+\frac{9}{4}-\frac{3}{4}\right\} e e_{,} \cos(2t+x+z+2y)$$
[87]

$$+\left\{-\frac{3}{4}-\frac{3}{4}+\frac{9}{4}+\frac{9}{4}\right\} e e_{,} \cos(x-z-2y)$$
[89]

$$+\left\{-\frac{3}{4}-\frac{3}{4}+\frac{1}{4}+\frac{1}{4}\right\} e e_{,} \cos(x-z+2y)$$
[90]

$$+\left\{-\frac{3}{4}+\frac{9}{4}-\frac{3}{4}+\frac{9}{4}\right\} e e_{,} \cos(2t-x+z-2y$$
[91]

$$+\left\{-\frac{3}{4}+\frac{1}{4}-\frac{3}{4}+\frac{1}{4}\right\} e e_{,} \cos(2t+x-z-2y$$
[93]

$$+ \left\{ -\frac{3}{4} + \frac{3}{8} \right\} e_{\prime}^2 \cos(2z - 2y) + \left\{ -\frac{3}{4} + \frac{1}{8} \right\} e_{\prime}^2 \cos(2z + 2y)$$

[95] [96]

$$+ \left\{ \frac{1}{4} + \frac{3}{8} \right\} e_{\prime}^2 \cos(2t - 2z - 2y) + \left\{ \frac{9}{4} + \frac{1}{8} \right\} e_{\prime}^2 \cos(2t + 2z - 2y)$$

[97] [99]

$$+ a^2 a_{\prime}^2 \sin^4 \frac{\iota}{2} \left\{ \frac{1}{2} + \frac{1}{2} \cos(2t - 2y) \right\}$$

[63]

$$r'^2 r^2 \cos(\lambda' - \lambda)^2$$

$$= a^2 a_{\prime}^2 \cos^4 \frac{\iota}{2} \left\{ \frac{1}{2} + \frac{3}{4}(e^2 + e_{\prime}^2) + \frac{9}{8} e^2 e_{\prime}^2 + \left\{ \frac{1}{2} - \frac{5}{4}(e^2 + e_{\prime}^2) \right.\right.$$

$$\left. + \frac{23}{32}(e^4 + e_{\prime}^4) + \frac{25}{8} e^2 e_{\prime}^2 \right\} \cos 2t + \left\{ -1 + \frac{e^2}{8} - \frac{3}{2} e_{\prime}^2 \right\} e \cos x$$

[1] [2] [5]

$$+ \left\{ -\frac{3}{2} + \frac{13}{16} e^2 + \frac{15}{4} e_{\prime}^2 \right\} e \cos(2t - x) + \left\{ \frac{1}{2} - \frac{19}{16} e^2 - \frac{5}{4} e_{\prime}^2 \right\} e \cos(2t + x)$$

[3] [7] [4] [6]

$$+ \left\{ -\frac{1}{4} + \frac{1}{12} e^2 - \frac{3}{8} e_{\prime}^2 \right\} e^2 \cos 2x$$

[8] [17]

$$+ \left\{ \frac{5}{4} - \frac{25}{8} e_{\prime}^2 \right\} e^2 \cos(2t - 2x) + \left\{ \frac{1}{2} - \frac{5}{4} e^2 - \frac{5}{4} e_{\prime}^2 \right\} e \cos(2t + 2x)$$

[9] [19] [10] [18]

$$+ \left\{ 1 - \frac{1}{8}(e^2 + e_{\prime}^2) \right\} e e_{\prime} \cos(x + z)$$

[11]

$$+ \left\{ -\frac{3}{2} + \frac{13}{16} e^2 + \frac{57}{16} e_{\prime}^2 \right\} e e_{\prime} \cos(2t - x - z)$$

[12] [13]

$$+ \left\{ 1 - \frac{(e^2 + e_{\prime}^2)}{8} \right\} e e_{\prime} \cos(x - z)$$

[14]

$$+ \left\{ \frac{9}{2} - \frac{39}{16}(e^2 + e_{\prime}^2) \right\} e e_{\prime} \cos(2t - x + z)$$

[15]

$$+ \left\{ \frac{1}{2} - \frac{19}{16}(e^2 + e_{\prime}^2) \right\} e e_{\prime} \cos(2t + x - z) - \frac{e^3}{8} \cos 3x$$

[16 [20] [35]

$$- \frac{7e^3}{48} \cos(2t - 3x) + \frac{25e^3}{48} \cos(2t + 3x) + \frac{e^2 e_{\prime}}{4} \cos(2x + z)$$

[21] [37] [22] [36] [23] [29]

$$+ \frac{5}{4} e^2 e_i \cos(2t - 2x - z) - \frac{25}{16} e^2 e_i \cos(2t + 2x + z) + \frac{e^2 e_i}{4} \cos(2x - z)$$
[24] [31] [25] [30] [26] [32]

$$- \frac{15}{4} e^2 e_i \cos(2t - 2x + z) + \frac{e^2 e_i}{2} \cos(2t + 2x - z)$$
[27] [33] [28] [34]

$$- \frac{e^4}{12} \cos 4x - \frac{e^4}{32} \cos(2t - 4x) + \frac{9}{16} e^4 \cos(2t + 4x) + \frac{e^3 e_i}{8} \cos(3x + z)$$
[38] [59] [39] [61] [40] [60] [41] [53]

$$- \frac{7}{48} e^3 e_i \cos(2t - 3x - z) - \frac{25}{16} e^2 e_i \cos(2t + 3x + z)$$
[42] [55] [43] [54]

$$+ \frac{e^3 e_i}{8} \cos(3x - z) + \frac{7}{16} e^3 e_i \cos(2t - 3x + z) + \frac{25}{48} e^3 e_i \cos(2t + 3x - z)$$
[44] [56] [45] [57] [46] [58]

$$+ \frac{e^2 e_i}{16} \cos(2x + 2z) + \frac{5}{4} e^2 e_i^2 \cos(2t - 2x - 2z)$$
[47] [48]

$$+ \frac{5}{4} e^2 e_i \cos(2t + 2x + 2z) + \frac{e^2 e_i^2}{16} \cos(2x - 2z)$$
[49] [50]

$$+ \frac{25}{8} e^2 e_i^2 \cos(2t - 2x + 2z) + \frac{e^2 e_i^2}{2} \cos(2t + 2x - 2z)$$
[51] [52]

$$+ a^2 a_i^2 \cos^2 \frac{\iota}{2} \sin^2 \frac{\iota}{2} \left\{ \left\{ 1 - \frac{5}{2} e^2 + \frac{3}{2} e_i^2 \right\} \cos 2y + \left\{ 1 + \frac{3}{2} e^2 - \frac{5}{2} e_i^2 \right\} \cos(2t - 2y) \right.$$
[62] [63]

$$- 3e \cos(x - 2y) + e \cos(x + 2y) - e \cos(2t - x - 2y)$$
[65] [66] [67]

$$- e \cos(2t + x - 2y) - e_i \cos(z - 2y) - e_i \cos(z + 2y)$$
[69] [71] [72]

$$+ e_i \cos(2t - z - 2y) - 3 e_i \cos(2t + z - 2y)$$
[73] [75]

$$+ \frac{5}{2} e^2 \cos(2x - 2y) + e^2 \cos(2x + 2y)$$
[77] [78]

$$- \frac{e^2}{4} \cos(2t - 2x - 2y) - \frac{e^2}{4} \cos(2t + 2x - 2y)$$
[79] [81]

$$+ 3\,e\,e_{\prime} \cos(x + z - 2\,y) - e\,e_{\prime} \cos(x + z + 2\,y)$$
[83] [84]

$$- e\,e_{\prime} \cos(2\,t - x - z - 2\,y) + 3\,e\,e_{\prime} \cos(2\,t + x + z - 2\,y)$$
[85] [87]

$$+ 3\,e\,e_{\prime} \cos(x - z - 2\,y) - e\,e_{\prime} \cos(x - z + 2\,y)$$
[89] [90]

$$+ 3\,e\,e_{\prime} \cos(2\,t - x + z - 2\,y) - e\,e_{\prime} \cos(2\,t + x - z - 2\,y)$$
[91] [93]

$$- \frac{3}{8} e_{\prime}^2 \cos(2\,z - 2\,y) - \frac{5}{8} e_{\prime}^2 \cos(2\,z + 2\,y)$$
[95] [96]

$$\left. + \frac{5\,e_{\prime}^2}{8} \cos(2\,t - 2\,z - 2\,y) + \frac{19}{8} e_{\prime}^2 \cos(2\,t + 2\,z - 2\,y) \right\}$$
[97] [99]

$$+ a^2 a_{\prime}^2 \sin^4 \frac{\iota}{2} \left\{ \frac{1}{2} + \frac{1}{2} \cos(2\,t - 2\,y) \right\}$$
[63]

$$\frac{r^2}{2\,r_{\prime}^3} = \frac{a^2}{a_{\prime}^3} \left\{ \frac{1}{2} + \frac{3}{4} e^2 + \frac{3}{4} e_{\prime}^2 + \frac{9}{8} e^2 e_{\prime}^2 + \frac{15}{16} e_{\prime}^4 - e \left\{ 1 - \frac{e^2}{8} + \frac{3}{2} e_{\prime}^2 \right\} \cos x \right.$$
[2]

$$+ \frac{3}{2} e_{\prime} \left\{ 1 + \frac{3}{2} e^2 + \frac{9}{8} e_{\prime}^2 \right\} \cos z - \frac{e^2}{4} \left\{ 1 - \frac{e^2}{3} + \frac{3\,e_{\prime}^2}{2} \right\} \cos 2\,x$$
[5] [8]

$$- \frac{3}{2} e\,e_{\prime} \left\{ 1 - \frac{e^2}{8} + \frac{9}{8} e_{\prime}^2 \right\} \cos(x + z)$$
[11]

$$- \frac{3}{2} e\,e_{\prime} \left\{ 1 - \frac{e^2}{8} + \frac{9}{8} e_{\prime}^2 \right\} \cos(x - z)$$
[14]

$$+ \frac{9}{4} e_{\prime}^2 \left\{ 1 + \frac{7}{9} e_{\prime}^2 + \frac{3}{2} e^2 \right\} \cos 2\,z - \frac{e^3}{8} \cos 3\,x - \frac{3}{8} e^2 e_{\prime} \cos(2\,x + z)$$
[17] [20] [23]

$$- \frac{3}{8} e^2 e_{\prime} \cos(2\,x - z) - \frac{9}{4} e\,e_{\prime}^2 \cos(x + 2\,z) - \frac{9}{4} e\,e_{\prime}^2 \cos(x - 2\,z)$$
[26] [29] [32]

$$+ \frac{53}{16} e_{\prime}^3 \cos 3\,z - \frac{e^4}{12} \cos 4\,x - \frac{3\,e^3 e_{\prime}}{16} \cos(3\,x + z)$$
[35] [38] [41]

$$-\frac{3}{16}e^3 e_{,}\cos(3x-z)-\frac{9}{16}e^2 e_{,}^2\cos(2x+2z)-\frac{9}{16}e^2 e_{,}\cos(2x-2z)$$
[44] [47] [50]

$$-\frac{53}{16}e\,e_{,}^3\cos(x+3z)-\frac{53}{16}e\,e_{,}^3\cos(x-3z)+\frac{77}{16}e_{,}^4\cos 4z$$
[53] [56] [59]

Terms in R multiplied by $-\frac{3}{2}\cos^4\frac{\iota}{2}\frac{a^2}{a_{,}^3}$

$$=^*\left\{\frac{1}{2}+\frac{3}{4}(e^2+e_{,}^2)+\frac{9}{8}e^2e_{,}^2\right\}\left\{1+5e_{,}^2+\frac{105}{8}e_{,}^4\right\}$$
[0]

$$+\left\{-1+\frac{e_{,}^2}{8}-\frac{3}{2}e\right\}\left\{\frac{5}{2}e_{,}^2+\frac{135}{16}e_{,}^4\right\}-\frac{5}{4}e_{,}^4$$

$$+\left\{\left\{\frac{1}{2}-\frac{5}{4}(e^2+e_{,}^2)+\frac{23}{32}(e^4+e_{,}^4)+\frac{25}{8}e^2e_{,}^2\right\}\left\{1+5e_{,}^2+\frac{105}{8}e_{,}^4\right\}\right.$$

$$+\left\{\frac{1}{2}-\frac{19}{16}e_{,}^2-\frac{5}{4}e^2-\frac{3}{2}+\frac{13}{16}e_{,}^2+\frac{15}{4}e^2\right\}\left\{\frac{5}{2}e_{,}^2+\frac{135}{16}e_{,}^4\right\}$$

$$\left.+\left\{\frac{5}{2}+\frac{25}{4}\right\}e_{,}^4\right\}\cos 2t$$
[1]

$$+\left\{-1+\frac{e^2}{8}-\frac{3}{2}e_{,}^2-5e_{,}^2+\frac{5}{2}e_{,}^2+\frac{5}{2}e_{,}^2\right\}e\cos x$$
[2]

$$+\left\{-\frac{3}{2}+\frac{13}{16}e^2+\frac{15}{4}e_{,}^2-\frac{15}{2}e_{,}^2-\frac{15}{4}e_{,}^2+\frac{45}{4}e_{,}^2\right\}e\cos(2t-x)$$
[3]

$$+\left\{\frac{1}{2}-\frac{19}{16}e^2-\frac{5}{4}e_{,}^2+\frac{5}{2}e_{,}^2+\frac{5}{4}e_{,}^2-\frac{15}{4}e_{,}^2\right\}e\cos(2t+x)$$
[4]

$$+\left\{-1+\frac{e_{,}^2}{8}-\frac{3}{2}e^2-5e_{,}^2+\frac{5}{2}+\frac{15}{4}e^2+\frac{15}{4}e_{,}^2+\frac{135}{16}e_{,}^2-\frac{5}{8}e_{,}^2-5e_{,}^2\right\}e_{,}\cos z$$
[5]

$$+\left\{\frac{1}{2}-\frac{19}{16}e_{,}^2-\frac{5}{4}e^2+\frac{5}{2}e_{,}^2+\frac{5}{4}e_{,}^2+\frac{5}{4}-\frac{25}{8}e^2-\frac{25}{8}e_{,}^2\right.$$

$$\left.+\frac{135}{32}e_{,}^2-\frac{15}{2}e_{,}^2\right\}e_{,}\cos(2t-z)$$
[6]

* This multiplication of $r^2 r_{,}^2\cos(\lambda'-\lambda_{,})^2$ by $r_{,}^5$ may be effected at once by means of Table II.

$$+\left\{-\frac{3}{2}+\frac{13}{16}e_{\prime}^2+\frac{15}{4}e^2-\frac{15}{2}e_{\prime}^2+\frac{5}{4}-\frac{25}{8}e^2-\frac{25}{8}e_{\prime}^2+\frac{135}{32}e_{\prime}^2+\frac{25}{8}e_{\prime}^2+\frac{5}{2}e_{\prime}^2\right\}e_{\prime}\cos(2t+z)$$
[7]

$$+\left\{-\frac{1}{4}+\frac{e^2}{12}-\frac{3}{8}e_{\prime}^2-\frac{5}{4}e_{\prime}^2+\frac{5}{8}e_{\prime}^2+\frac{5}{8}e_{\prime}^2\right\}e^2\cos 2x$$
[8]

$$+\left\{\frac{5}{4}-\frac{25}{8}e_{\prime}^2+\frac{25}{4}e_{\prime}^2+\frac{25}{8}e_{\prime}^2-\frac{75}{8}e_{\prime}^2\right\}e^2\cos(2t-2x)$$
[9]

$$+\left\{\frac{1}{2}-\frac{5}{4}e^2-\frac{5}{4}e_{\prime}^2+\frac{5}{2}e_{\prime}^2+\frac{5}{4}e_{\prime}^2-\frac{15}{4}e_{\prime}^2\right\}e^2\cos(2t+2x)$$
[10]

$$+\left\{1-\frac{e^2}{8}-\frac{e_{\prime}^2}{8}+5e_{\prime}^2-\frac{5}{2}+\frac{5e^2}{16}-\frac{15}{4}e_{\prime}^2-\frac{135}{16}e_{\prime}^2+\frac{5e_{\prime}^2}{8}+5e_{\prime}^2\right\}e\,e_{\prime}\cos(x+z)$$
[11]

$$+\left\{-\frac{3}{2}+\frac{13}{16}e^2+\frac{57}{16}e_{\prime}^2-\frac{15}{2}e_{\prime}^2-\frac{15}{4}e_{\prime}^2-\frac{15}{4}+\frac{65}{32}e^2+\frac{75}{8}e_{\prime}^2\right.$$

$$\left.-\frac{405}{32}e_{\prime}^2+\frac{45}{2}e_{\prime}^2\right\}e\,e_{\prime}\cos(2t-x-z)+\left\{-\frac{3}{2}+\frac{13}{16}e_{\prime}^2+\frac{57}{16}e^2-\frac{15}{2}e_{\prime}^2+\frac{5}{4}\right.$$
[12]

$$\left.-\frac{95}{32}e^2-\frac{25}{8}e_{\prime}^2+\frac{135}{32}e_{\prime}^2+\frac{25}{8}e_{\prime}^2+\frac{5}{2}e_{\prime}^2\right\}e\,e_{\prime}\cos(2t+x+z)$$
[13]

$$+\left\{1-\frac{e^2}{8}-\frac{e_{\prime}^2}{8}+5e_{\prime}^2+\frac{5e_{\prime}^2}{8}-\frac{5}{2}+\frac{5}{16}e^2-\frac{15}{4}e_{\prime}^2-\frac{135}{16}e_{\prime}^2+5e_{\prime}^2\right\}e\,e_{\prime}\cos(x-z)$$
[14]

$$+\left\{\frac{9}{2}-\frac{39}{16}e^2-\frac{39}{16}e_{\prime}^2+\frac{45}{2}e_{\prime}^2-\frac{15}{4}+\frac{65}{32}e^2+\frac{75}{8}e_{\prime}^2-\frac{405}{32}e_{\prime}^2\right.$$

$$\left.-\frac{75}{8}e_{\prime}^2-\frac{15}{2}e_{\prime}^2\right\}e\,e_{\prime}\cos(2t-x+z)$$
[15]

$$+\left\{\frac{1}{2}-\frac{19}{16}e^2-\frac{19}{16}e_{\prime}^2+\frac{5}{2}e_{\prime}^2+\frac{5}{4}e_{\prime}^2+\frac{5}{4}-\frac{95}{32}e^2-\frac{25}{8}e_{\prime}^2+\frac{135}{32}e_{\prime}^2-\frac{15}{2}e_{\prime}^2\right\}e\,e_{\prime}\cos(2t+x-z)$$
[16]

$$+\left\{-\frac{1}{4}+\frac{e_{\prime}^2}{12}-\frac{3}{8}e^2-\frac{5}{4}e_{\prime}^2-\frac{5}{2}+\frac{5}{16}e_{\prime}^2-\frac{15}{4}e^2-\frac{135}{16}e_{\prime}^2-\frac{5}{16}e_{\prime}^2+5\right.$$

$$\left.+\frac{15}{2}e^2+\frac{15}{2}e_{\prime}^2+\frac{155}{12}e_{\prime}^2-\frac{145}{16}e_{\prime}^2\right\}e_{\prime}^2\cos 2z$$
[17]

$$+\left\{\frac{1}{2}-\frac{5}{4}e_{\prime}^2-\frac{5}{4}e^2+\frac{5}{2}e_{\prime}^2+\frac{125}{96}e_{\prime}^2+\frac{5}{4}-\frac{95}{32}e_{\prime}^2-\frac{25}{8}e^2+\frac{135}{32}e_{\prime}^2+\frac{5}{2}-\frac{25}{4}e^2-\frac{25}{4}e_{\prime}^2\right.$$

$$\left.+\frac{155}{24}e_{\prime}^2-\frac{435}{32}e_{\prime}^2\right\}e_{\prime}^2\cos(2t-2z)$$
[18]

$$+\left\{\frac{5}{4}-\frac{25}{8}e^2+\frac{25}{4}e_{\prime}^2-\frac{15}{4}+\frac{65}{32}e_{\prime}^2+\frac{75}{8}e^2-\frac{405}{32}e_{\prime}^2-\frac{35}{96}e_{\prime}^2+\frac{5}{2}-\frac{25}{4}e^2-\frac{25}{4}e_{\prime}^2\right.$$

$$\left.+\frac{155}{24}e_{\prime}^2+\frac{145}{32}e_{\prime}^2\right\}e_{\prime}^2\cos(2t+2z)$$

[19]

$$-\frac{e^3}{8}\cos 3x-\frac{7}{48}e^3\cos(2t-3x)+\frac{25}{48}e^3\cos(2t+3x)+\left\{\frac{1}{4}-\frac{5}{8}\right\}e^2e_{\prime}\cos(2x+z)$$

[20] [21] [22] [23]

$$+\left\{\frac{5}{4}+\frac{25}{8}\right\}e^2e_{\prime}\cos(2t-2x-z)-\left\{-\frac{3}{2}+\frac{5}{4}\right\}e^2e_{\prime}\cos(2t+2x+z)$$

[24] [25]

$$+\left\{\frac{1}{4}\quad\frac{5}{8}\right\}e^2e_{\prime}\cos(2x-z)+\left\{-\frac{15}{4}+\frac{25}{8}\right\}e^2e_{\prime}\cos(2t-2x+z)$$

[26] [27]

$$+\left\{\frac{1}{2}+\frac{5}{4}\right\}e^2e_{\prime}\cos(2t+2x-z)+\left\{\frac{1}{4}+\frac{5}{2}-5\right\}e\,e_{\prime}^2\cos(x+2z)$$

[28] [29]

$$+\left\{-\frac{3}{2}-\frac{15}{4}-\frac{15}{2}\right\}e\,e_{\prime}^2\cos(2t-x-2z)$$

[30]

$$+\left\{\frac{5}{4}-\frac{15}{4}+\frac{5}{2}\right\}e\,e_{\prime}^2\cos(2t+x+2z)+\left\{\frac{1}{4}+\frac{5}{2}-5\right\}e\,e_{\prime}^2\cos(x-2z)$$

[31] [32]

$$+\left\{-\frac{15}{4}+\frac{45}{4}-\frac{15}{2}\right\}e\,e_{\prime}^2\cos(2t-x+2z)+\left\{\frac{1}{2}+\frac{5}{4}+\frac{5}{2}\right\}e\,e_{\prime}^2\cos(2t+x-2z)$$

[33] [34]

$$+\left\{-\frac{1}{8}-\frac{5}{8}-5+\frac{145}{16}\right\}e_{\prime}^3\cos 3z+\left\{\frac{25}{48}+\frac{5}{4}+\frac{5}{2}+\frac{145}{32}\right\}e_{\prime}^3\cos(2t-3z)$$

[35] [36]

$$+\left\{-\frac{7}{48}+\frac{25}{8}-\frac{15}{2}+\frac{145}{32}\right\}e_{\prime}^3\cos(2t+3z)$$

[37]

$$-\frac{e^4}{12}\cos 4x-\frac{e^4}{32}\cos(2t-4x)+\frac{9}{16}e^4\cos(2t+4x)$$

[38] [39] [40]

$$+\left\{\frac{1}{8}-\frac{5}{16}\right\}e^3e_{\prime}\cos(3x+z)+\left\{-\frac{7}{48}-\frac{35}{96}\right\}e^3e_{\prime}\cos(2t-3x-z)$$

[41] [42]

$$+\left\{-\frac{25}{16}+\frac{125}{96}\right\}e^3e_{\prime}\cos(2t-3x-z)+\left\{\frac{1}{8}-\frac{5}{16}\right\}e^3e_{\prime}\cos(3x-z)$$

[43] [44]

$$+\left\{\frac{7}{16}-\frac{35}{96}\right\}e^3 e_{\prime}\cos(2t-3x+z)$$
[45]

$$+\left\{\frac{25}{48}+\frac{125}{96}\right\}e^3 e_{\prime}\cos(2t+3x-z)+\left\{\frac{1}{16}+\frac{5}{8}-\frac{5}{4}\right\}e^2 e_{\prime}^2\cos(2x+2z)$$
[46] [47]

$$+\left\{\frac{5}{4}+\frac{25}{8}+\frac{25}{4}\right\}e^2 e_{\prime}^2\cos(2t-2x-2z)+\left\{\frac{5}{4}-\frac{15}{4}+\frac{5}{2}\right\}e^2 e_{\prime}^2\cos(2t+2x+2z)$$
[48] [49]

$$+\left\{\frac{1}{16}+\frac{5}{8}-\frac{5}{4}\right\}e^2 e_{\prime}^2\cos(2x-2z)+\left\{\frac{25}{8}-\frac{75}{8}+\frac{25}{4}\right\}e^2 e_{\prime}^2\cos(2t-2x+2z)$$
[50] [51]

$$+\left\{\frac{1}{2}+\frac{5}{4}+\frac{5}{2}\right\}e^2 e_{\prime}^2\cos(2t+2x-2z)+\left\{\frac{1}{8}+\frac{5}{8}+5-\frac{145}{16}\right\}e\,e_{\prime}^3\cos(x+3z)$$
[52] [53]

$$+\left\{-\frac{25}{16}-\frac{15}{4}-\frac{15}{2}-\frac{435}{32}\right\}e\,e_{\prime}^3\cos(2t-x-3z)$$
[54]

$$+\left\{-\frac{7}{48}+\frac{25}{8}-\frac{15}{2}+\frac{145}{32}\right\}e\,e_{\prime}^3\cos(2t+x+3z)$$
[55]

$$+\left\{\frac{1}{8}+\frac{5}{8}+5-\frac{145}{16}\right\}e\,e_{\prime}^3\cos(x-3z)+\left\{\frac{7}{16}-\frac{75}{8}+\frac{45}{2}-\frac{435}{32}\right\}e\,e_{\prime}^3\cos(2t-x+3z)$$
[56] [57]

$$+\left\{\frac{25}{48}+\frac{5}{4}+\frac{5}{2}-\frac{145}{32}\right\}e\,e_{\prime}^3\cos(2t+x-3z)-\left\{-\frac{1}{12}-\frac{5}{16}-\frac{5}{4}-\frac{145}{16}+\frac{745}{96}\right\}e_{\prime}^4\cos 4z$$
[58] [59]

$$+\left\{\frac{9}{16}+\frac{125}{96}+\frac{5}{2}+\frac{145}{32}+\frac{745}{192}\right\}e_{\prime}^4\cos(2t-4z)$$
[60]

$$+\left\{-\frac{1}{32}-\frac{35}{96}+\frac{25}{4}-\frac{435}{32}+\frac{745}{192}\right\}e_{\prime}^4\cos(2t+4z)$$
[61]

Terms in R multiplied by $-\frac{3}{2}\sin^2\frac{\iota}{2}\cos^2\frac{\iota}{2}\frac{a^2}{a_{\prime}^3}$

$$=\left\{1-\frac{5}{2}e^2+\frac{3}{2}e_{\prime}^2+5e_{\prime}^2-\frac{5}{2}e_{\prime}^2-\frac{5}{2}e_{\prime}^2\right\}\cos 2y$$
[62]

$$+\left\{1+\frac{3}{2}e^2-\frac{5}{2}e_{\prime}^2+5e_{\prime}^2+\frac{5}{2}e_{\prime}^2-\frac{15}{2}e_{\prime}^2\right\}\cos(2t-2y)$$
[63]

$$-3e\cos(x-2y)+e\cos(x+2y)-e\cos(2t-x-2y)-e\cos(2t+x-2y)$$
[65] [66] [67] [69]

$$+\left\{-1+\frac{5}{2}\right\}e_{\prime}\cos(z-2y)+\left\{-1+\frac{5}{2}\right\}e_{\prime}\cos(z+2y)+\left\{1+\frac{5}{2}\right\}e_{\prime}\cos(2t-z-2y)$$

[71] [72] [73]

$$+\left\{-3+\frac{5}{2}\right\}e_{\prime}\cos(2t+z-2y)+\frac{5}{2}e^2\cos(2x-2y)+e^2\cos(2x+2y)$$

[75] [77] [78]

$$-\frac{e^2}{4}\cos(2t-2x-2y)-\frac{e^2}{4}\cos(2t+2x-2y)$$

[79] [81]

$$+\left\{3-\frac{15}{2}\right\}e\,e_{\prime}\cos(x+z-2y)+\left\{-1+\frac{5}{2}\right\}e\,e_{\prime}\cos(x+z+2y)$$

[83] [84]

$$+\left\{-1-\frac{5}{2}\right\}e\,e_{\prime}\cos(2t-x-z-2y)$$

[85]

$$+\left\{3-\frac{5}{2}\right\}e\,e_{\prime}\cos(2t+x+z-2y)+\left\{3-\frac{15}{2}\right\}e\,e_{\prime}\cos(2t-z-2y)$$

[87] [89]

$$+\left\{-1+\frac{5}{2}\right\}e\,e_{\prime}\cos(x-z+2y)+\left\{3-\frac{5}{2}\right\}e\,e_{\prime}\cos(2t-x+z-2y)$$

[90] [91]

$$+\left\{-1-\frac{5}{2}\right\}e\,e_{\prime}\cos(2t+x-z-2y)+\left\{-\frac{3}{8}-\frac{5}{2}+5\right\}e_{\prime}^2\cos(2z-2y)$$

[93] [95]

$$+\left\{-\frac{5}{8}-\frac{5}{2}+5\right\}e_{\prime}^2\cos(2z+2y)+\left\{\frac{5}{8}+\frac{5}{2}+5\right\}e_{\prime}^2\cos(2t-2z-2y)$$

[96] [97]

$$+\left\{\frac{19}{8}-\frac{15}{2}+5\right\}e_{\prime}^2\cos(2t+2z-2y)$$

[99]

Terms in R multiplied by $-\frac{3}{2}\cos^4\frac{\iota}{2}\frac{a^2}{a_{\prime}^3}$

$$=\frac{1}{2}+\frac{3}{4}e^2+\frac{3}{4}e_{\prime}^2+\frac{15}{16}e_{\prime}^4+\frac{9}{8}e^2e_{\prime}^2+\left\{\frac{1}{2}-\frac{5}{4}e^2-\frac{5}{4}e_{\prime}^2+\frac{23}{32}e^4+\frac{13}{32}e_{\prime}^4+\frac{25}{8}e^2e_{\prime}^2\right\}\cos 2t$$

[1]

$$+\left\{1+\frac{e^2}{8}-\frac{3}{2}e_{\prime}^2\right\}e\cos x+\left\{-\frac{3}{2}+\frac{13}{16}e^2+\frac{15}{4}e_{\prime}^2\right\}e\cos(2t-x)$$

[2] [3]

$$+\left\{\frac{1}{2}-\frac{19}{16}e^2-\frac{5}{4}e_{\prime}^2\right\}e\cos(2t+x)+\left\{\frac{3}{2}+\frac{9}{4}e^2+\frac{27}{16}e_{\prime}^2\right\}e_{\prime}\cos z$$

[4] [5]

$$+\left\{\frac{7}{4}-\frac{35}{8}e^2-\frac{123}{32}e_i^2\right\}e_i\cos(2t-z)$$
[6]

$$+\left\{-\frac{1}{4}+\frac{5}{8}e^2+e_i^2\right\}e_i\cos(2t+z)+\left\{-\frac{1}{4}+\frac{e^2}{12}-\frac{3}{8}e_i^2\right\}e^2\cos 2x$$
[7] [8]

$$+\left\{\frac{5}{4}-\frac{25}{8}e_i^2\right\}e^2\cos(2t-2x)+\left\{\frac{1}{2}-\frac{5}{4}e^2-\frac{5}{4}e_i^2\right\}e^2\cos(2t+2x)$$
[9] [10]

$$+\left\{-\frac{3}{2}+\frac{3}{16}e^2-\frac{27}{16}e_i^2\right\}e\,e_i\cos(x+z)+\left\{-\frac{21}{4}+\frac{91}{32}e^2+\frac{369}{32}e_i^2\right\}e\,e_i\cos(2t-x-z)$$
[11] [12]

$$+\left\{-\frac{1}{4}+\frac{19}{32}e^2+\frac{e_i^2}{32}\right\}e\,e_i\cos(2t+x+z)+\left\{-\frac{3}{2}+\frac{3}{16}e^2-\frac{27}{16}e_i^2\right\}e\,e_i\cos(x-z)$$
[13] [14]

$$+\left\{\frac{3}{4}-\frac{13}{32}e^2-\frac{3}{32}e_i^2\right\}e\,e_i\cos(2t-x+z)+\left\{\frac{7}{4}-\frac{133}{32}e^2-\frac{123}{32}e_i^2\right\}e\,e_i\cos(2t+x-z)$$
[15] [16]

$$+\left\{\frac{9}{4}+\frac{27}{8}e^2+\frac{21}{12}e_i^2\right\}e_i^2\cos 2z+\left\{\frac{17}{4}-\frac{85}{8}e^2-\frac{115}{12}e_i^2\right\}e_i^2\cos(2t-2z)^{*}-\frac{e^3}{8}\cos 3x$$
[17] [18] [20]

$$-\frac{7}{48}e^3\cos(2t-3x)+\frac{25}{48}e^3\cos(2t+3x)-\frac{3}{8}e^2e_i\cos(2x+z)$$
[21] [22] [23]

$$+\frac{35}{8}e^2e_i\cos(2t-2x-z)-\frac{e^2e_i}{4}\cos(2t+2x+z)-\frac{3}{8}e^2e_i\cos(2x-z)$$
[24] [25] [26]

$$-\frac{5}{8}e^2e_i\cos(2t-2x+z)+\frac{7}{4}e^2e_i\cos(2t+2x-z)$$
[27] [28]

$$-\frac{9}{4}e\,e_i^2\cos(x+2z)-\frac{51}{4}e\,e_i^2\cos(2t-x-2z)$$
[29] [30]

$$-\frac{9}{4}e\,e_i^2\cos(x-2z)+\frac{17}{4}e\,e_i^2\cos(2t+x-2z)$$
[32] [34]

$$+\frac{53}{16}e_i^3\cos 3z+\frac{845}{96}e_i^3\cos(2t-3z)+\frac{e^3e_i}{96}\cos(2t+3z)-\frac{e^4}{12}\cos 4x-\frac{e^4}{32}\cos(2t-4x)$$
[35] [36] [37] [38] [39]

* It is remarkable that the coefficient of argument 19 equals zero.

$$+\underset{[40]}{\frac{9}{16}e^4\cos(2t+4x)}-\underset{[41]}{\frac{3}{16}e^3e_{,}\cos(3x+z)}$$

$$-\underset{[42]}{\frac{49}{96}e^3e_{,}\cos(2t+3x-z)}-\underset{[43]}{\frac{25}{96}e^3e_{,}\cos(2t-3x-z)}-\underset{[44]}{\frac{3}{16}e^3e_{,}\cos(3x-z)}$$

$$+\underset{[45]}{\frac{7}{96}e^3e_{,}\cos(2t-3x+z)}+\underset{[46]}{\frac{175}{96}e^3e_{,}\cos(2t+3x-z)}-\underset{[47]}{\frac{9}{16}e^2e_{,}^2\cos(2x+2z)}$$

$$+\underset{[48]}{\frac{85}{8}e^2e_{,}^2\cos(2t-2x-2z)}-\underset{[50]}{\frac{9}{16}e^2e_{,}^2\cos(2x-2z)}+\underset{[52]}{\frac{17}{4}e^2e_{,}^2\cos(2t+2x-2z)}$$

$$-\underset{[53]}{\frac{53}{16}ee_{,}^3\cos(x+3z)}-\underset{[54]}{\frac{845}{32}ee_{,}^3\cos(2t\quad x-3z)}+\underset{[55]}{\frac{ee_{,}^3}{96}\cos(2t+x-3z)}$$

$$-\underset{[56]}{\frac{53}{16}ee_{,}^3\cos(x-3z)}-\underset{[57]}{\frac{ee_{,}^3}{32}\cos(2t-x+3z)}-\underset{[58]}{\frac{25}{96}ee_{,}^3\cos(2t+x-3z)}$$

$$-\underset{[59]}{\frac{283}{96}e_{,}^4\cos 4z}+\underset{[60]}{\frac{2453}{192}e_{,}^4\cos(2t-4z)}-\underset{[61]}{\frac{741}{192}e_{,}^4\cos(2t+4z)}$$

Terms in R multiplied by $-\frac{3}{2}\sin^2\frac{\iota}{2}\cos^2\frac{\iota}{2}\frac{a^2}{a_{,}^3}$ or $-\frac{3}{8}\sin^2\iota\,\frac{a^2}{a_{,}^3}$

$$=\underset{[62]}{\left\{1-\frac{5}{2}e^2+\frac{3}{2}e_{,}^2\right\}\cos 2y}+\underset{[63]}{\left\{1+\frac{3}{2}e^2-\frac{5}{2}e_{,}^2\right\}\cos(2t-2y)}-\underset{[65]}{3e\cos(x-2y)}$$

$$+\underset{[66]}{e\cos(x+2y)}-\underset{[67]}{e\cos(2t-x-2y)}-\underset{[69]}{e\cos(2t+x-2y)}+\underset{[71]}{\frac{3}{2}e_{,}\cos(z-2y)}+\underset{[72]}{\frac{3}{2}e_{,}\cos(z+2y)}$$

$$+\underset{[73]}{\frac{7}{2}e_{,}\cos(2t-z-2y)}-\underset{[75]}{\frac{e_{,}}{2}\cos(2t+z-2y)}+\underset{[77]}{\frac{5}{2}e^2\cos(2x-2y)}+\underset{[78]}{e^2\cos(2x+2y)}$$

$$-\underset{[79]}{\frac{e^2}{4}\cos(2t-2x-2y)}-\underset{[81]}{\frac{e^2}{4}\cos(2t+2x-2y)}-\underset{[83]}{\frac{9}{2}ee_{,}\cos(x+z-2y)}$$

$$+\underset{[84]}{\frac{3}{2}ee_{,}\cos(x+z+2y)}-\underset{[85]}{\frac{7}{2}ee_{,}\cos(2t-x-z-2y)}+\underset{[87]}{\frac{ee_{,}}{2}\cos(2t+x+z-2y)}$$

$$-\frac{9}{2}ee_{\prime}\cos(x-z-2y)+\frac{3}{2}ee_{\prime}\cos(x-z+2y)+\frac{ee_{\prime}}{2}\cos(2t-x+z-2y)$$
[89] [90] [91]

$$-\frac{7}{2}ee_{\prime}\cos(2t+x-z-2y)+\frac{17}{8}e_{\prime}^{2}\cos(2z-2y)+\frac{15}{8}e_{\prime}^{2}\cos(2z+2y)$$
[93] [95] [96]

$$+\frac{65}{8}e_{\prime}^{2}\cos(2t-2z-2y)-\frac{e_{\prime}^{2}}{8}\cos(2t+2z-2y)$$
[97] [99]

$$R=m_{\prime}\left\{-\frac{1}{r_{\prime}}-\frac{1}{4}\left\{1+\frac{3}{2}e^{2}+\frac{3}{2}e_{\prime}^{2}+\frac{9}{4}e^{2}e_{\prime}^{2}+\frac{15}{8}e_{\prime}^{4}-\frac{3}{2}\gamma^{2}-\frac{9}{4}\gamma^{2}e^{2}-\frac{9}{4}\gamma^{2}e_{\prime}^{2}+\frac{39}{8}\gamma^{4}\right\}\frac{a^{2}}{a_{\prime}^{3}}\right.$$
[0]

$$-\frac{3}{4}\left\{1-\frac{5}{2}e^{2}-\frac{5}{2}e_{\prime}^{2}+\frac{23}{16}e^{4}+\frac{25}{4}e^{2}e_{\prime}^{2}+\frac{13}{16}e_{\prime}^{4}\right\}\cos^{4}\frac{\iota}{2}\,\frac{a^{2}}{a_{\prime}^{3}}\cos 2t$$
[1]

$$+\frac{1}{2}\left\{1-\frac{e^{2}}{8}-\frac{3}{2}e_{\prime}^{2}-\frac{3}{2}\gamma^{2}\right\}\frac{a^{2}}{a_{\prime}^{3}}e\cos x$$
[2]

$$+\frac{9}{4}\left\{1-\frac{13}{24}e^{2}-\frac{5}{2}e_{\prime}^{2}\right\}\cos^{4}\frac{\iota}{2}\,\frac{a^{2}}{a_{\prime}^{3}}\,e\cos(2t-x)$$
[3]

$$-\frac{3}{4}\left\{1-\frac{19}{8}e^{2}-\frac{5}{2}e_{\prime}^{2}\right\}\cos^{4}\frac{\iota}{2}\,\frac{a^{2}}{a_{\prime}^{3}}e\cos(2t+x)$$
[4]

$$-\frac{3}{4}\left\{1+\frac{3}{2}e^{2}+\frac{9}{8}e_{\prime}^{2}-\frac{3}{2}\gamma^{2}\right\}\frac{a^{2}}{a_{\prime}^{3}}\,e_{\prime}\cos z$$
[5]

$$-\frac{21}{8}\left\{1-\frac{5}{2}e^{2}-\frac{123}{56}e_{\prime}^{2}\right\}\cos^{4}\frac{\iota}{2}\,\frac{a^{2}}{a_{\prime}^{3}}\,e_{\prime}\cos(2t-z)$$
[6]

$$+\frac{3}{8}\left\{1-\frac{5}{2}e^{2}-4e_{\prime}^{2}\right\}\cos^{4}\frac{\iota}{2}\,\frac{a^{2}}{a_{\prime}^{3}}\,e_{\prime}\cos(2t+z)$$
[7]

$$+\frac{1}{8}\left\{1-\frac{e^{2}}{3}+\frac{3}{2}e_{\prime}^{2}-\frac{3}{2}\gamma^{2}\right\}\frac{a^{2}}{a_{\prime}^{3}}\,e^{2}\cos 2x$$
[8]

$$-\frac{15}{8}\left\{1-\frac{5}{2}e_{\prime}^{2}\right\}\cos^{4}\frac{\iota}{2}\,\frac{a^{2}}{a_{\prime}^{3}}e^{2}\cos(2t-2x)$$
[9]

$$-\frac{3}{4}\left\{1-\frac{5}{2}e^{2}-\frac{5}{2}e_{\prime}^{2}\right\}\cos^{4}\frac{\iota}{2}\,\frac{a^{2}}{a_{\prime}^{3}}e^{2}\cos(2t+2x)$$
[10]

Development of R.

$$+\frac{3}{4}\left\{1-\frac{e^2}{8}+\frac{9}{8}e_{,}^2-\frac{3}{2}\gamma^2\right\}\frac{a^2}{a_{,}^3}e\,e_{,}\cos(x+z) \quad [11]$$

$$+\frac{63}{8}\left\{1-\frac{91}{168}e^2-\frac{123}{56}e_{,}^2\right\}\cos^4\frac{\iota}{2}\frac{a^2}{a_{,}^3}e\,e_{,}\cos(2t-x-z) \quad [12]$$

$$+\frac{3}{8}\left\{1-\frac{19}{8}e^2-\frac{e_{,}^2}{8}\right\}\cos^4\frac{\iota}{2}\frac{a^2}{a_{,}^3}e\,e_{,}\cos(2t+x+z) \quad [13]$$

$$+\frac{3}{4}\left\{1-\frac{e^2}{8}+\frac{9}{8}e_{,}^2-\frac{3}{2}\gamma^2\right\}\frac{a^2}{a_{,}^2}e\,e_{,}\cos(x-z) \quad [14]$$

$$-\frac{9}{8}\left\{1-\frac{13}{24}e^2-\frac{e_{,}^2}{8}\right\}\cos^4\frac{\iota}{2}\frac{a^2}{a_{,}^3}e\,e_{,}\cos(2t-x+z) \quad [15]$$

$$-\frac{21}{8}\left\{1-\frac{19}{8}e^2-\frac{123}{56}e_{,}^2\right\}\cos^4\frac{\iota}{2}\frac{a^2}{a_{,}^3}e\,e_{,}\cos(2t+x-z) \quad [16]$$

$$-\frac{9}{8}\left\{1+\frac{3}{2}e^2+\frac{7}{9}e_{,}^2-\frac{3}{2}\gamma^2\right\}\frac{a^2}{a_{,}^3}e_{,}^2\cos 2z \quad [17]$$

$$-\frac{51}{8}\left\{1-\frac{5}{2}e^2-\frac{115}{51}e_{,}^2\right\}\cos^4\frac{\iota}{2}\frac{a^2}{a_{,}^3}e_{,}^2\cos(2t-2z) \quad [18]$$

$$+\frac{1}{16}\frac{a^2}{a_{,}^3}e^3\cos 3x \;[20]\; +\frac{7\,a^2}{32\,a_{,}^3}e^3\cos(2t-3x) \;[21]\; -\frac{25\,a^2}{32\,a_{,}^3}e^3\cos(2t+3x) \;[22]$$

$$+\frac{3\,a^2}{16\,a_{,}^3}e^2e_{,}\cos(2x+z) \;[23]\; -\frac{105}{16}\frac{a^2}{a_{,}^3}e^2e_{,}\cos(2t-2x-z) \;[24]\; +\frac{3}{8}\frac{a^2}{a_{,}^3}e^2e_{,}\cos(2t+2x+z) \;[25]$$

$$+\frac{3}{16}\frac{a^2}{a_{,}^3}e^2e_{,}\cos(2x-z) \;[26]\; +\frac{15}{16}\frac{a^2}{a_{,}^3}e^2e_{,}\cos(2t-2x+z) \;[27]\; -\frac{21}{8}\frac{a^2}{a_{,}^3}e^2e_{,}\cos(2t+2x-z) \;[28]$$

$$+\frac{9}{8}\frac{a^2}{a_{,}^3}e\,e_{,}^2\cos(x+2z) \;[29]\; +\frac{153}{8}\frac{a^2}{a_{,}^3}e\,e_{,}^2\cos(2t-x-2z) \;[30]$$

$$+\frac{9}{8}\frac{a^2}{a_{,}^3}e\,e_{,}^2\cos(x-2z) \;[32]\; -\frac{51}{8}\frac{a^2}{a_{,}^3}e\,e_{,}^2\cos(2t+x-2z) \;[34]\; -\frac{53}{32}\frac{a^2}{a_{,}^3}e_{,}^3\cos 3z \;[35]$$

$$-\frac{845\,a^2}{64\,a_{,}^3}e_{,}^3\cos(2t-3z) \;[36]\; -\frac{1}{64}\frac{a^2}{a_{,}^3}e_{,}^3\cos(2t+3z) \;[37]\; +\frac{1}{24}\frac{a^2}{a_{,}^3}e^4\cos 4x \;[38]$$

$$+\frac{3}{64}\frac{a^2}{a_{,}^3}e^4\cos(2t-4x) \;[39]\; -\frac{27}{32}\frac{a^2}{a_{,}^3}e^4\cos(2t+4x) \;[40]\; +\frac{3}{32}\frac{a^2}{a_{,}^3}e^3e_{,}\cos(3x+z) \;[41]$$

Development of R.

$$+\frac{49}{64}\frac{a^2}{a_{\prime}^3}e^3 e_{\prime}\cos(2t-3x-z) + \frac{25}{64}\frac{a^2}{a_{\prime}^3}e^3 e_{\prime}\cos(2t-3x-z) + \frac{3}{32}\frac{a^2}{a_{\prime}^3}e^3 e_{\prime}\cos(3x-z)$$
[42] [44]

$$-\frac{7}{64}\frac{a^2}{a_{\prime}^3}e^3 e_{\prime}\cos(2t-3x+z) - \frac{175}{64}\frac{a^2}{a_{\prime}^3}e^3 e_{\prime}\cos(2t+3x-z)$$
[45] [46]

$$+\frac{9}{32}\frac{a^2}{a_{\prime}^3}e^2 e_{\prime}^2\cos(2x+2z) - \frac{255}{16}e^2 e_{\prime}^2\cos(2t-2x-2z)$$
[47] [48]

$$+\frac{9}{32}\frac{a^2}{a_{\prime}^3}e^2 e_{\prime}^2\cos(2x-2z) - \frac{51}{8}\frac{a^2}{a_{\prime}^3}e^2 e_{\prime}^2\cos(2t+2x-2z) + \frac{53}{32}\frac{a^2}{a_{\prime}^3}e\,e_{\prime}^3\cos(x+3z)$$
[50] [52] [53]

$$+\frac{2535}{64}\frac{a^2}{a_{\prime}^3}e\,e_{\prime}^3\cos(2t-x-3z) - \frac{1}{64}\frac{a^2}{a_{\prime}^3}e\,e_{\prime}^3\cos(2t+x+3z)$$
[54] [55]

$$+\frac{53}{32}\frac{a^2}{a_{\prime}^3}e\,e_{\prime}^3\cos(x-3z) + \frac{3}{64}\frac{a^2}{a_{\prime}^3}e\,e_{\prime}^3\cos(2t-x+3z)$$
[56] [57]

$$+\frac{45}{64}\frac{a^2}{a_{\prime}^3}e\,e_{\prime}^3\cos(2t+x-3z) + \frac{591}{64}\frac{a^2}{a_{\prime}^3}e_{\prime}^4\cos 4z$$
[58] [59]

$$-\frac{2453}{128}\frac{a^2}{a_{\prime}^3}e_{\prime}^4\cos(2t-4z) + \frac{741}{128}\frac{a^2}{a_{\prime}^3}e_{\prime}^4\cos(2t+4z)$$
[60] [61]

$$-\frac{3}{8}\left\{1-\frac{5}{2}e^2+\frac{3}{2}e_{\prime}^2\right\}\frac{a^2}{a_{\prime}^3}\gamma^2\cos 2y - \frac{3}{8}\left\{1+\frac{3}{2}e^2-\frac{5}{2}e_{\prime}^2+\frac{\gamma^2}{8}\right\}\frac{a^2}{a_{\prime}^3}\gamma^2\cos(2t-2y)$$
[62] [63]

$$+\frac{9}{8}\frac{a^2}{a_{\prime}^3}\gamma^2 e\cos(x-2y)$$
[65]

$$-\frac{3}{8}\frac{a^2}{a_{\prime}^3}\gamma^2 e\cos(x+2y) + \frac{3}{8}\frac{a^2}{a_{\prime}^3}\gamma^2 e\cos(2t-x-2y)$$
[66] [67]

$$+\frac{3}{8}\frac{a^2}{a_{\prime}^3}\gamma^2 e\cos(2t+x-2y) - \frac{9}{16}\frac{a^2}{a_{\prime}^3}\gamma^2 e_{\prime}\cos(z-2y)$$
[69] [71]

$$-\frac{9}{16}\frac{a^2}{a_{\prime}^3}\gamma^2 e_{\prime}\cos(z+2y) - \frac{21}{16}\frac{a^2}{a_{\prime}^3}\gamma^2 e_{\prime}\cos(2t-z-2y)$$
[72] [73]

$$+\frac{3}{16}\frac{a^2}{a_{\prime}^3}\gamma^2 e_{\prime}\cos(2t+z-2y) - \frac{15}{16}\frac{a^2}{a_{\prime}^3}\gamma^2 e^2\cos(2x-2y)$$
[75] [77]

Development of R.

$$-\frac{3}{8}\frac{a^2}{a_{,}^3}\gamma^2 e^2 \cos(2x+2y) + \frac{3}{32}\frac{a^2}{a_{,}^3}\gamma^2 e^2 \cos(2t-2x-2y)$$
[78] [79]

$$+\frac{3}{32}\frac{a^2}{a_{,}^3}\gamma^2 e^2 \cos(2t+2x-2y) + \frac{27}{16}\frac{a^2}{a_{,}^3}\gamma^2 e\, e_{,} \cos(x+z-2y)$$
[81] [83]

$$-\frac{9}{16}\frac{a^2}{a_{,}^3}\gamma^2 e\, e_{,} \cos(x+z+2y) + \frac{21}{16}\frac{a^2}{a_{,}^3}\gamma^2 e\, e_{,} \cos(2t-x-z-2y)$$
[84] [85]

$$-\frac{3}{16}\frac{a^2}{a_{,}^3}\gamma^2 e\, e_{,} \cos(2t+x+z-2y) + \frac{27}{16}\frac{a^2}{a_{,}^3}\gamma^2 e\, e_{,} \cos(x-z-2y)$$
[87] [89]

$$-\frac{9}{16}\frac{a^2}{a_{,}^3}\gamma^2 e\, e_{,} \cos(x-z+2y) - \frac{3}{16}\frac{a^2}{a_{,}^2}\gamma^2 e\, e_{,} \cos(2t-x+z-2y)$$
[90] [91]

$$+\frac{21}{16}\frac{a^2}{a_{,}^3}\gamma^2 e\, e_{,} \cos(2t+x-z-2y)$$
[93]

$$-\frac{51}{64}\frac{a^2}{a_{,}^3}\gamma^2 e_{,}^2 \cos(2z-2y) - \frac{45}{64}\frac{a^2}{a_{,}^3}\gamma^2 e_{,}^2 \cos(2z+2y)$$
[95] [96]

$$-\frac{195}{64}\frac{a^2}{a_{,}^3}\gamma^2 e_{,}^2 \cos(2t-2z-2y) + \frac{3}{64}\frac{a^2}{a_{,}^3}\gamma^2 e_{,}^2 \cos(2t+2z-2y)$$
[97] [99]

$$-\frac{3}{8}\left\{1+3e^2+3e_{,}^2-\frac{11}{4}\gamma^2\right\}\frac{a^3 *}{a_{,}^4}\cos t + \frac{15}{16}\frac{a^3}{a_{,}^4} e \cos(t-x)$$
[101] [102]

$$+\frac{3}{16}\frac{a^3}{a_{,}^4} e \cos(t+x) - \frac{9}{8}\frac{a^3}{a_{,}^4} e_{,} \cos(t-z) - \frac{3}{8}\frac{a^3}{a_{,}^4} e_{,} \cos(t+x)$$
[103] [104] [105]

$$-\frac{33}{64}\frac{a^3}{a_{,}^4} e^2 \cos(t-2x) + \frac{9}{64}\frac{a^3}{a_{,}^4} e^2 \cos(t+2x) + \frac{45}{16}\frac{a^3}{a_{,}^4} e\, e_{,} \cos(t-x-z)$$
[106] [107] [108]

$$+\frac{3}{16}\frac{a^3}{a_{,}^4} e\, e_{,} \cos(t+x+z) + \frac{15}{16}\frac{a^3}{a_{,}^4} e\, e_{,} \cos(t-x+z) + \frac{9}{16}\frac{a^3}{a_{,}^4} e\, e_{,} \cos(t+x-z)$$
[109] [110] [111]

$$-\frac{159}{64}\frac{a^3}{a_{,}^4} e_{,}^2 \cos(t-2z) - \frac{33}{64}\frac{a^3}{a_{,}^4} e_{,}^2 \cos(t+2z) - \frac{9}{16}\frac{a^3}{a_{,}^4}\gamma^2 \cos(t-2y)$$
[112] [113] [114]

$$-\frac{15}{32}\frac{a^3}{a_{,}^4}\sin^2\frac{\iota}{2}\cos(t+2y) - \frac{5}{8}\left\{1-6e^2-6e_{,}^2-\frac{3}{4}\gamma^2\right\}\frac{a^3}{a_{,}^4}\cos 3t$$
[115] [116]

* For the coefficients of the terms multiplied by $\frac{a^3}{a_{,}^4}$ see p. 39.

$$+\frac{45}{16}\frac{a^3}{a_i^4}e\cos(3t-x)\underset{[117]}{} - \frac{15}{16}\frac{a^3}{a_i^4}e\cos(3t+x)\underset{[118]}{} - \frac{25}{8}\frac{a^3}{a_i^4}e_i\cos(3t-z)\underset{[119]}{}$$

$$+\frac{5}{8}\frac{a^3}{a_i^4}e_i\cos(3t+z)\underset{[120]}{} - \frac{285}{64}\frac{a^3}{a_i^4}e^2\cos(3t-2x)\underset{[121]}{} - \frac{75}{64}\frac{a^3}{a_i^4}e^2\cos(3t+2x)\underset{[122]}{}$$

$$-\frac{225}{16}\frac{a^3}{a_i^4}e e_i\cos(3t-x-z)\underset{[123]}{} + \frac{15}{16}\frac{a^3}{a_i^4}e e_i\cos(3t+x+z)\underset{[124]}{}$$

$$-\frac{45}{16}\frac{a^3}{a_i^4}e e_i\cos(3t-x+z)\underset{[125]}{} - \frac{75}{16}\frac{a^3}{a_i^4}e e_i\cos(3t+x-z)\underset{[126]}{}$$

$$-\frac{635}{64}\frac{a^3}{a_i^4}e_i^2\cos(3t-2z)\underset{[127]}{} - \frac{5}{64}\frac{a^3}{a_i^4}e_i^2\cos(3t+2z)\underset{[128]}{} - \frac{15}{32}\frac{a^3}{a_i^4}\gamma^2\cos(3t-2y)\underset{[129]}{}$$

In the elliptic movement ;

$$s=\gamma\sin(g\lambda-\nu)$$

$$\lambda=nt+2e\sin x+\frac{5}{4}e^2\sin 2x$$

$$s=\gamma(1-e^2)\sin y\underset{[146]}{} + \gamma e\sin(x-y)\underset{[149]}{} + \gamma e\sin(x+y)\underset{[150]}{} + \gamma\frac{e^2}{8}\sin(2x-y)\underset{[161]}{} + \frac{9}{8}\gamma e^2\sin(2x+y)\underset{[162]}{}$$

$$s^2=\frac{\gamma^2}{2}-\frac{\gamma^2}{2}(1-4e^2)\cos 2y\underset{[62]}{} + \gamma^2 e\cos(x-2y)\underset{[65]}{} - \gamma^2 e\cos(x+2y)\underset{[66]}{}$$

$$+\frac{5}{8}\gamma^2e^2\cos(2x-2y)\underset{[77]}{} - \frac{5}{8}\gamma^2e^2\cos(2x+2y)\underset{[78]}{}$$

$$z^*=a\gamma\left(1-\frac{e^2}{2}\right)\sin y\underset{[146]}{} + \frac{3a\gamma e}{2}\sin(x-y)\underset{[149]}{} + \frac{a\gamma e}{2}\sin(x+y)\underset{[150]}{}$$

$$-\frac{a\gamma e^2}{8}\sin(2x-y)\underset{[161]}{} + \frac{3a\gamma e^2}{8}\sin(2x+y)\underset{[162]}{}$$

$$\frac{s}{r}=\frac{\gamma}{a}(1-e^2)\sin y\underset{[146]}{} + \frac{\gamma e}{2a}\sin(x-y)\underset{[149]}{} + \frac{3\gamma e}{2a}\sin(x+y)\underset{[150]}{}$$

* This quantity z, which is one of the rectangular coordinates of the moon, must not be confounded with $z=n_i t-\varpi_i$; this latter quantity should rather be x_i, but I think it better to conform as far as possible to the notation of M. Damoiseau.

$$+ \frac{\gamma e^2}{8a} \sin(2x - y) + \frac{17}{8} \frac{\gamma e^2}{a} \sin(2x + y)$$
[161] [162]

$$\frac{s}{r} \delta . \frac{1}{r} = \left\{ (1 - e^2) r_0 + \frac{e^2}{2} r_2 \right\} \frac{\gamma}{a^2} \sin y - \left\{ (1 - e^2) \frac{r_1}{2} + \frac{e^2}{4} r_3 - \frac{3e^2}{4} r_4 \right\} \frac{\gamma}{a^2} \sin(2t - y)$$
[146] [147]

$$+ \left\{ (1 - e^2) \frac{r_1}{2} - \frac{e^2}{4} r_4 + \frac{3e^2}{4} r_3 \right\} \frac{e\gamma}{a^2} \sin(2t + y) + \frac{e\gamma r_0}{2a^2} \sin(x - y)$$
[148] [149]

$$+ \frac{3 r_0}{2a^2} e\gamma \sin(x + y) + \left\{ - \frac{r_3}{2} - \frac{3 r_1}{4} \right\} \frac{e\gamma}{a^2} \sin(2t - x - y)$$
[150] [151]

$$+ \left\{ \frac{r_3}{2} - \frac{r_1}{4} \right\} \frac{e\gamma}{a^2} \sin(2t - x + y) + \left\{ - \frac{r_4}{2} - \frac{r_1}{4} \right\} \frac{e\gamma}{a^2} \sin(2t + x - y)$$
[152] [153]

$$+ \left\{ \frac{r_4}{2} + \frac{3}{4} r_1 \right\} \frac{e\gamma}{a^2} \sin(2t + x + y) + \frac{r_5 e_{\prime} \gamma}{2a^2} \sin(z - y) + \frac{r_5 e_{\prime} \gamma}{2a^2} \sin(z + y)$$
[154] [155] [156]

$$- \frac{r_6 e_{\prime} \gamma}{2a^2} \sin(2t - z - y) + \frac{r_6 e_{\prime} \gamma}{2a^2} \sin(2t - z + y) - \frac{r_7 e_{\prime} \gamma}{2a^2} \sin(2t + z - y)$$
[157] [158] [159]

$$+ \left\{ - \frac{r_9}{2} - \frac{3}{4} r_3 - \frac{17}{16} r_1 \right\} \frac{e^2 \gamma}{a^2} \sin(2t - 2x - y) + \left\{ \frac{r_9}{2} - \frac{r_3}{4} - \frac{r_1}{16} \right\} \frac{e^2 \gamma}{a^2} \sin(2t - 2x + y)$$
[163] [164)

$$+ \left\{ - \frac{r_{10}}{2} + \frac{r_4}{4} + \frac{r_1}{16} \right\} \frac{e^2 \gamma}{a^2} \sin(2t + 2x - y) + \left\{ \frac{r_{10}}{2} + \frac{3 r_4}{4} + \frac{17}{16} r_1 \right\} \frac{e^2 \gamma}{a^2} \sin(2t + 2x + y)$$
[165] [166]

$$+ \left\{ - \frac{r_{11}}{2} + \frac{r_5}{4} \right\} \frac{e e_{\prime} \gamma}{a^2} \sin(x + z - y) + \left\{ \frac{r_{11}}{2} + \frac{3 r_5}{4} \right\} \frac{e e_{\prime} \gamma}{a^2} \sin(x + z + y)$$
[167] [168]

$$+ \left\{ - \frac{r_{12}}{2} + \frac{r_6}{4} - \frac{3}{4} r_6 \right\} \frac{e e_{\prime} \gamma}{a^2} \sin(2t - x - z - y) + \left\{ \frac{r_{12}}{2} + \frac{r_6}{4} \right\} \frac{e e_{\prime} \gamma}{a^2} \sin(2t - x - z + y)$$
[169] [170]

$$+ \left\{ - \frac{r_{13}}{2} + \frac{r_7}{4} \right\} \frac{e e_{\prime} \gamma}{a^2} \sin(2t + x + z - y) + \left\{ \frac{r_{13}}{2} + \frac{3}{4} r_7 \right\} \frac{e e_{\prime} \gamma}{a^2} \sin(2t + x + z + y)$$
(171) (172)

$$+ \left\{ - \frac{r_{14}}{2} + \frac{r_5}{2} \right\} \frac{e e_{\prime} \gamma}{a^2} \sin(x - z - y) + \left\{ \frac{r_{14}}{2} + \frac{3}{4} r_5 \right\} \frac{e e_{\prime} \gamma}{a^2} \sin(x - z + y)$$
(173) (174)

$$+ \left\{ - \frac{r_{15}}{2} - \frac{3}{4} r_7 \right\} \frac{e e_{\prime} \gamma}{a^2} \sin(2t - x + z - y) + \left\{ \frac{r_{15}}{2} - \frac{r_7}{2} \right\} \frac{e e_{\prime} \gamma}{a^2} \sin(2t - x + z + y)$$
[175] [176]

$$-\frac{r_{16}\, e\, e_{,}\gamma}{2\, a^2} \sin(2t + x - z - y) + \left\{\frac{r_{16}}{2} + \frac{3}{4} r_6\right\} \frac{e\, e_{,}\gamma}{a^2} \sin(2t + x - z + y)$$
[177] [178]

$$-\frac{r_{17}}{2}\frac{e_{,}^2\gamma}{a^2} \sin(2z - y) + \frac{r_{17}}{2}\frac{e_{,}^2\gamma}{a^2} \sin(2z + y) - \frac{r_{18}}{2}\frac{e_{,}^2\gamma}{a^2} \sin(2t - 2z - y)$$
[179] [180] [181]

$$+\frac{r_{18}}{2}\frac{e_{,}^2\gamma}{a^2} \sin(2t - 2z + y) - \frac{r_{19}}{2}\frac{e_{,}^2\gamma}{a^2} \sin(2t + 2z - y) + \frac{r_{19}}{2}\frac{e_{,}^2\gamma}{a^2} \sin(2t + 2z + y)$$
[182] [183] [184]

$$\frac{m_{,} z}{r^3} = \frac{m_{,} a \gamma}{a_{,}^3}\left(1 + \frac{3}{2} e_{,}^2 - \frac{e^2}{2}\right) \sin y + \frac{3\, m_{,} a \gamma e}{2\, a_{,}^3} \sin(x - y) + \frac{m_{,} a \gamma}{2\, a_{,}^3} \sin(x + y)$$
[146] [149] [150]

$$-\frac{3\, m_{,} a \gamma e_{,}}{2\, a_{,}^3} \sin(z - y) + \frac{3\, m_{,} a \gamma e_{,}}{2\, a_{,}^3} \sin(z + y) - \frac{m_{,} a \gamma e^2}{8\, a_{,}^3} \sin(2x - y)$$
[155] [156] [161]

$$+\frac{3\, m_{,} a \gamma e^2}{8\, a_{,}^3} \sin(2x + y) + \frac{9\, m_{,} a \gamma e e_{,}}{4\, a_{,}^3} \sin(x + z - y) + \frac{3\, m_{,} a \gamma e e_{,}}{4\, a_{,}^3} \sin(x + z + y)$$
[162] [167] [168]

$$+\frac{9\, m_{,} a \gamma e e_{,}}{4\, a_{,}^3} \sin(x - z - y) + \frac{3\, m_{,} a \gamma e e_{,}}{4\, a_{,}^3} \sin(x - z + y) - \frac{9\, m_{,} a \gamma e_{,}^2}{4\, a_{,}^3} \sin(2z - y)$$
[173] [174] [179]

$$+\frac{9\, m_{,} a \gamma e_{,}^2}{4\, a_{,}^3} \sin(2z + y)$$
[180]

$$\frac{a^3}{\mathrm{r}^3} = 1 + \frac{3}{2} e^2 + 3 e \cos x + \frac{9}{2} e^2 \cos 2x$$

r being the elliptic value of r.

If $z = a\gamma z_{146} \sin y + a\gamma z_{147} \sin(2t - y) + a\gamma z_{148} \sin(2t + y)$ &c.

$$\frac{z}{r^3}^{*} = \left\{\left(1 + \frac{3 e^2}{2}\right) z_{146} + \frac{3}{2} e^2 z_{150} - \frac{3 e^2}{2} z_{149}\right\} \frac{\gamma}{a^2} \sin y$$
[146]

$$+\left\{\left(1 + \frac{3 e^2}{2}\right) z_{147} + \frac{3 e^2}{2} z_{151} + \frac{3 e^2}{2} z_{153}\right\} \frac{\gamma}{a^2} \sin(2t - y)$$
[147]

$$+\left\{\left(1 + \frac{3 e^2}{2}\right) z_{148} + \frac{3 e^2}{2} z_{152} + \frac{3 e^2}{2} z_{154}\right\} \frac{\gamma}{a^2} \sin(2t + y)$$
[148]

* This multiplication of z by r^{-3} may be effected at once by means of Table II.

$$+\left\{z_{149}-\frac{3}{2}z_{146}\right\}\frac{e\gamma}{a^2}\sin(x-y)+\left\{z_{150}+\frac{3}{2}z_{146}\right\}\frac{e\gamma}{a^2}\sin(x+y)$$
[149] [150]

$$+\left\{z_{151}+\frac{3}{2}z_{147}\right\}\frac{e\gamma}{a^2}\sin(2t-x-y)+\left\{z_{152}+\frac{3}{2}z_{148}\right\}\frac{e\gamma}{a^2}\sin(2t-x+y)$$
[151] [152]

$$+\left\{z_{153}+\frac{3}{2}z_{147}\right\}\frac{e\gamma}{a^2}\sin(2t+x-y)$$
[153]

$$+\left\{z_{154}+\frac{3}{2}z_{148}\right\}\frac{e\gamma}{a^2}\sin(2t+x+y)+z_{155}\frac{e_{\prime}\gamma}{a^2}\sin(z-y)+z_{156}\frac{e_{\prime}\gamma}{a^2}\sin(z+y)$$
[154] [155] [156]

$$+\left\{z_{161}+\frac{3}{2}z_{149}-\frac{9}{4}z_{146}\right\}\frac{e^2\gamma}{a^2}\sin(2x-y)+\left\{z_{162}+\frac{3}{2}z_{150}+\frac{9}{4}z_{146}\right\}\frac{e^2\gamma}{a^2}\sin(2x+y)$$
[161] [162]

$$+\left\{z_{163}+\frac{3}{2}z_{151}+\frac{9}{4}z_{147}\right\}\frac{e^2\gamma}{a^2}\sin(2t-2x-y)$$
[163]

$$+\left\{z_{164}+\frac{3}{2}z_{152}+\frac{9}{4}z_{148}\right\}\frac{e^2\gamma}{a^2}\sin(2t-2x+y)$$
[164]

$$+\left\{z_{165}+\frac{3}{2}z_{153}+\frac{9}{4}z_{147}\right\}\frac{e^2\gamma}{a^2}\sin(2t+2x-y)$$
[165]

$$+\left\{z_{166}+\frac{3}{2}z_{154}+\frac{9}{4}z_{148}\right\}\frac{e^2\gamma}{a^2}\sin(2t+2x+y)+\&c.$$
[166]

$$+\left\{z_{167}+\frac{3}{2}z_{155}\right\}\frac{ee_{\prime}\gamma}{a^2}\sin(x+z-y)+\left\{z_{168}+\frac{3}{2}z_{156}\right\}\frac{ee_{\prime}\gamma}{a^2}\sin(x+z+y)$$
[167] [168]

$$+\left\{z_{169}+\frac{3}{2}z_{157}\right\}\frac{ee_{\prime}\gamma}{a^2}\sin(2t-x-z-y)+\left\{z_{170}+\frac{3}{2}z_{158}\right\}\frac{ee_{\prime}\gamma}{a^2}\sin(2t-x-z+y)$$
[169] [170]

$$+\left\{z_{171}+\frac{3}{2}z_{159}\right\}\frac{ee_{\prime}\gamma}{a^2}\sin(2t+x+z-y)+\left\{z_{172}+\frac{3}{2}z_{160}\right\}\frac{ee_{\prime}\gamma}{a^2}\sin(2t+x+z+y)$$
[171] [172]

$$+\left\{z_{173}-\frac{3}{2}z_{156}\right\}\frac{ee_{\prime}\gamma}{a^2}\sin(x-z-y)+\left\{z_{174}-\frac{3}{2}z_{155}\right\}\frac{ee_{\prime}\gamma}{a^2}\sin(x-z+y)$$
[173] [174]

$$+\left\{z_{175}+\frac{3}{2}z_{159}\right\}\frac{ee_{\prime}\gamma}{a^2}\sin(2t-x+z-y)+\left\{z_{176}+\frac{3}{2}z_{160}\right\}\frac{ee_{\prime}\gamma}{a^2}\sin(2t-x+z+y)$$
[175] [176]

$$+\left\{z_{177}+\frac{3}{2}z_{157}\right\}\frac{ee_{\prime}\gamma}{a^2}\sin(2t+x-z-y)+\left\{z_{178}+\frac{3}{2}z_{158}\right\}\frac{ee_{\prime}\gamma}{a^2}\sin(2t+x-z+y)$$
[177] [178]

$$s = \frac{z}{r} \text{ nearly,}$$

$$= \left\{z_{146} + \frac{e^2}{2} z_{150} + \frac{e^2}{2} z_{149}\right\} \gamma \sin y$$

$$+ \left\{z_{147} + \frac{e^2}{2} z_{151} + \frac{e^2}{2} z_{153}\right\} \gamma \sin (2t - y)$$

$$+ \left\{z_{148} + \frac{e^2}{2} z_{152} + \frac{e^2}{2} z_{154}\right\} \gamma \sin (2t + y) + \&c.$$

$$\frac{d^2 . r^2}{2 . d t^2} - \frac{\mu}{r} + \frac{\mu}{a} + 2 \int d R + r \left(\frac{d R}{d r}\right) = 0$$

$$\frac{d^2 z}{d t^2} + \frac{\mu z}{r^3} + \frac{m_{\prime} z}{\{r^2 - 2 r r' \cos (\lambda - \lambda) + r_{\prime}^2\}^{\frac{3}{2}}}$$

$$r'^4 . \frac{d \lambda'^2}{d t^2} = h^2 - 2 \int r'^2 \left(\frac{d R}{d \lambda'}\right) d \lambda'$$

Neglecting the square of the disturbing force

$$\frac{- d^2 . r^3 \delta . \frac{1}{r}}{d t^2} - \mu \delta . \frac{1}{r} + 2 \int d R + r \left(\frac{d R}{d r}\right) = 0$$

$$\frac{d^2 z}{d t^2} + \frac{\mu z}{r^3} + \frac{m_{\prime} z}{r_{\prime}^3} + \frac{3 m_{\prime} z r' r \cos (\lambda' - \lambda)}{r_{\prime}^5} = 0$$

$$\frac{d^2 . \delta z}{d t^2} + \frac{3 \mu s \delta . \frac{1}{r}}{\mathrm{r}} + \frac{\mu \delta . z}{\mathrm{r}^3} + \frac{m_{\prime} z}{r_{\prime}^3} + \frac{3 \mu_{\prime} z r' r \cos (\lambda' - \lambda)}{r_{\prime}^5} = 0$$

$$\frac{d \lambda'}{d t} = \frac{h (1 + s^2)}{r^2} - \frac{(1 + \mathrm{s}^2)}{\mathrm{r}^2} \int \left(\frac{d R}{d \lambda'}\right) d t$$

$$r \left(\frac{d R}{d r}\right) = a \left(\frac{d R}{d a}\right), \ \frac{d R}{d \lambda'} = \frac{d R}{d t}, \ (t \text{ being used for } n t - n_{\prime} t).$$

Integrating the equation of p. 270, line 9, by the method of indeterminate coefficients, neglecting the cubes and higher powers of e in order to obtain a first approximation, m being equal to $\frac{n_{\prime}}{n}$ as in the notation of M. DAMOISEAU;

$$- r_0 - \frac{m_{\prime} a^3}{2 \mu a_{\prime}^3} \left\{1 + \frac{3}{2} e^2 + \frac{3}{2} e_{\prime}^2 - \frac{3}{2} \gamma^2\right\} = 0$$

$$4 (1 - m)^2 \left\{(1 + 3 e^2) r_1 - \frac{3 e^2}{2} \{r_3 + r_4\}\right\} - r_1$$

$$-\frac{3\,m_{\prime}\,a^3}{2\,\mu\,a_{\prime}^3}\left\{1-\frac{5}{2}\,e^2-\frac{5}{2}\,e_{\prime}^2-\frac{\gamma^2}{2}\right\}\left\{\frac{1}{1-m}+1\right\}=0$$

$$c^2 * \{1-3\,r_0\}-1+\frac{2\,m_{\prime}\,a^3}{\mu\,a^3}=0$$

$$(2-2\,m-c)^2\left\{r_3-\frac{3}{2}\,r_1\right\}-r_3+\frac{9}{2}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^3}\left\{\frac{2-c}{2-2\,m-c}+1\right\}=0$$

$$(2-2\,m+c)^2\left\{r_4-\frac{3}{2}\,r_1\right\}-r_4-\frac{3}{2}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^3}\left\{\frac{2+c}{2-2\,m+c}+1\right\}=0$$

$$m^2\,r_5-r_5-\frac{3}{2}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^3}=0$$

$$(2-3\,m)^2\,r_6-r_6-\frac{21}{4}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^3}\left\{\frac{2}{2-3\,m}+1\right\}=0$$

$$(2-m)^2\,r_7-r_7+\frac{3}{4}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^3}\left\{\frac{2}{2-m}+1\right\}=0$$

$$4\,c^2\left\{(1-3\,r_0)\,r_8-\frac{3}{4}+3\,r_0\right\}-r_8+\frac{m_{\prime}}{2\,\mu}\,\frac{a^3}{a_{\prime}^3}=0$$

$$(2-2\,m-2\,c)^2\left\{r_9-\frac{3}{2}\,r_3\right\}-r_9-\frac{15}{4}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^3}\left\{\frac{2-2\,c}{2-2\,m-2\,c}+1\right\}=0$$

$$(2-2\,m+2\,c)^2\left\{r_{10}-\frac{3}{2}\,r_4\right\}-r_{10}-\frac{3}{2}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^3}\left\{\frac{2+2\,c}{2-2\,m+2\,c}+1\right\}=0$$

$$(c+m)^2\left\{r_{11}-\frac{3}{2}\,r_5\right\}-r_{11}+\frac{3}{2}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^3}\left\{\frac{c}{c+m}+1\right\}=0$$

$$(2-3\,m-c)^2\left\{r_{12}-\frac{3}{2}\,r_6\right\}-r_{12}+\frac{63}{4}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^3}\left\{\frac{c}{2-3\,m-c}+1\right\}=0$$

$$(2-m+c)^2\left\{r_{13}-\frac{3}{2}\,r_7\right\}-r_{13}+\frac{3}{4}\,\frac{m_{\prime}}{\mu}\,\frac{a^2}{a_{\prime}^3}\left\{\frac{2+c}{2-m+c}+1\right\}=0$$

$$(c-m)^2\left\{r_{14}-\frac{3}{2}\,r_5\right\}-r_{14}+\frac{3}{2}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^3}\left\{\frac{c}{c-m}+1\right\}=0$$

$$(2-m-c)^2\left\{r_{15}-\frac{3}{2}\,r_7\right\}-r_{15}-\frac{9}{4}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^3}\left\{\frac{2-c}{2-m-c}+1\right\}=0$$

* The letter c does not strictly denote the same quantity as in the notation of M. DAMOISEAU, or in that of the Mathematical Tracts, p. 33.

$$(2-3m+c)^2\left\{r_{16}-\frac{3}{2}r_6\right\}-r_{16}-\frac{21}{4}\frac{m_{,}}{\mu}\frac{a^3}{a_{,}^3}\left\{\frac{2+c}{2-3m+c}+1\right\}=0$$

$$4m^2r_{17}-r_{17}-\frac{9}{4}\frac{m_{,}}{\mu}\frac{a^3}{a_{,}^3}=0$$

$$(2-4m)^2r_{18}-r_{18}-\frac{51}{4}\frac{m_{,}}{\mu}\frac{a^3}{a_{,}^3}\left\{\frac{2}{2-4m}+1\right\}$$

$$4r_{19}-r_{19}=0$$

The equation for determining z may be integrated in the same way.

$$-g^2z_{146}+3r_0+z_{146}+\frac{m_{,}}{\mu}\frac{a^3}{a_{,}^3}=0$$

$$-\left\{2(1-m)-g\right\}^2z_{147}-\frac{3r_1}{2}+z_{147}=0$$

$$-\left\{2(1-m)+g\right\}^2z_{148}+\frac{3r_1}{2}+z_{148}=0$$

$$-\left\{c-g\right\}^2z_{149}+\frac{3}{2}r_6+z_{149}-\frac{3}{2}z_{146}+\frac{3m_{,}}{2\mu}\frac{a^3}{a_{,}^3}=0$$

$$-\left\{c+g\right\}^2z_{150}+\frac{9}{2}r_6+z_{150}+\frac{3}{2}z_{146}+\frac{m_{,}}{\mu}\frac{a^3}{a_{,}^3}=0$$

$$-\left\{2(1-m)-c-g\right\}^2z_{147}+3\left\{-\frac{3r_1}{4}-\frac{r_3}{2}\right\}+z_{151}+\frac{3}{2}z_{147}=0$$

$$-\left\{2(1-m)-c+g\right\}^2z_{148}+3\left\{-\frac{r_1}{4}+\frac{r_3}{2}\right\}+z_{152}+\frac{3}{2}z_{148}=0$$

$$-\left\{2(1-m)+c-g\right\}^2z_{149}+3\left\{\frac{r_1}{4}-\frac{r_4}{2}\right\}+z_{153}+\frac{3}{2}z_{147}=0$$

$$-\left\{2(1-m)+c+g\right\}^2z_{150}+3\left\{\frac{3}{4}r_1+\frac{r_4}{2}\right\}+z_{154}+\frac{3}{2}z_{148}=0$$

$$-\left\{m-g\right\}^2z_{151}+\frac{3}{2}r_5+z_{155}-\frac{3m_{,}}{2\mu}\frac{a^3}{a_{,}^3}=0$$

$$-\left\{m+g\right\}^2z_{152}+\frac{3}{2}r_5+z_{156}+\frac{3m_{,}}{2\mu}\frac{a^3}{a_{,}^3}=0$$

$$-\left\{2(1-m)-m-g\right\}^2z_{153}-\frac{3}{2}r_6+z_{157}=0$$

$$-\left\{2(1-m)-m+g\right\}^2z_{154}+\frac{3}{2}r_6+z_{158}=0$$

$$-\left\{2(1-m)+m-g\right\}^2z_{155}-\frac{3}{2}r_7+z_{159}=0$$

$$-\left\{2(1-m)+m+g\right\}^2 z_{156}+\frac{3}{2}r_7+z_{160}=0$$

$$\frac{d\lambda'}{dt}=\frac{h}{\mathrm{r}^2}+\frac{2h}{\mathrm{r}}\delta.\frac{1}{r}+\frac{hz^2}{r^4}-\frac{(1+s^2)}{\mathrm{r}^2}\int\left(\frac{dR}{d\lambda'}\right)dt$$

$$\lambda'=\frac{h}{a^2}\left\{1+\frac{e^2}{2}+\frac{\gamma^2}{2}+2r_0\right\}t+\frac{2e(1+r_0)}{c}\sin x+\frac{5e^2(1+r_0)}{4c}\sin 2x$$

$$+\left\{2r_1+e^2(r_3+r_4)-\left\{-\left(1-\frac{5}{2}e^2-\frac{5}{2}e_{,}^2-\frac{\gamma^2}{2}\right)\frac{3}{4(1-m)}+\frac{9e^2}{2(2-2m-c)}\right.\right.$$

$$\left.\left.-\frac{3e^2}{2(2-2m+c)}\right\}\frac{m_{,}}{\mu}\frac{a^3}{a_{,}^3}\right\}\frac{1}{2(1-m)}\sin 2t$$

$$+\left\{2r_3+e^2r_1-\left\{\frac{9}{2(2-m-c)}-\frac{3}{2(2-m)}\right\}\frac{m_{,}a^3}{\mu a_{,}^3}\right\}\frac{e}{(2-2m-c)}\sin(2t-x)$$

$$+\left\{2r_4+e^2r_1-\left\{-\frac{3}{2(2-m+c)}-\frac{3}{2(2-m)}\right\}\frac{m_{,}}{\mu}\frac{a^3}{a_{,}^3}\right\}\frac{e}{(2-m+c)}\sin(2t+x)$$

$$+\frac{2r_5}{m}\sin z$$

$$+\left\{2r_6+\frac{21}{4(2-3m)}\frac{m_{,}}{\mu}\frac{a^3}{a_{,}^3}\right\}\frac{e_{,}}{(2-3m)}\sin(2t-z)$$

$$+\left\{2r_7-\frac{3}{4(2-m)}\frac{m_{,}}{\mu}\frac{a^3}{a_{,}^3}\right\}\frac{e_{,}}{(2-m)}\sin(2t+z)$$

$$+\left\{2r_9+r_3-\left\{-\frac{15}{4(2-2m-2c}+\frac{9}{2(2-2m-c}\right\}\frac{m_{,}}{\mu}\frac{a^3}{a_{,}^3}\right\}\frac{e^2}{2(1-m-c)}\sin(2t-2x)$$

$$+\left\{2r_{10}+r_4-\left\{-\frac{3}{2(2-2m+2c)}-\frac{3}{2(2-m+c)}\right\}\frac{m_{,}}{\mu}\frac{a^3}{a_{,}^3}\right\}\frac{e^2}{2(1-m+c)}\sin(2t+2x)$$

$$+\left\{2r_{11}+r_5\right\}\frac{ee_{,}}{(c+m)}\sin(x+z)$$

$$+\left\{2r_{12}+r_6-\left\{\frac{63}{4(2-3m-c)}-\frac{21}{4(2-3m)}\right\}\frac{m_{,}}{\mu}\frac{a^3}{a_{,}^3}\right\}\frac{ee_{,}}{(2-3m-c)}\sin(2t-x-z)$$

$$+\left\{2r_{13}+r_7-\left\{\frac{3}{4(2-m+c)}+\frac{3}{4(2-m)}\right\}\frac{m_{,}}{\mu}\frac{a^3}{a_{,}^3}\right\}\frac{ee_{,}}{(2-m+c)}\sin(2t+x+z)$$

Considering the terms which depend on the square of the disturbing force

$$\frac{d^2.r^2}{2dt^2}-\frac{\mu}{r}+\frac{\mu}{a}+2\int dR+r\left(\frac{dR}{dr}\right)=0$$

$$\frac{d^2 . r^2}{2\,d\,t^2} - \frac{d^2 . r^3 \,\delta . \frac{1}{r}}{d\,t^2} + \frac{3\,d^2 . r^4 \left(\delta . \frac{1}{r}\right)^2}{2\,d\,t^2} - \frac{\mu}{r} + \frac{\mu}{a} + 2\int d\,R + r\left(\frac{d\,R}{d\,r}\right) = 0$$

$$\frac{d^2 z}{d\,t^2} + \frac{\mu\,z}{r^3} + \frac{m_{,}\,z}{r_{,}^3} - \frac{3\,m_{,}\,z\,r\,r\cos(\lambda' - \lambda)}{r_{,}^5} = 0.$$

$$\frac{d\,\lambda'}{d\,t} = \frac{h}{r'^2}\left\{1 - \frac{1}{h}\int\left(\frac{d\,R}{d\,\lambda'}\right)d\,t\left\{1 - \frac{1}{h^2}\int\left(\frac{d\,R}{d\,\lambda'}\right)d\,t\right\} - \frac{1}{2\,h^2}\left\{\int\left(\frac{d\,R}{d\,\lambda'}\right)d\,t\right\}^2\right.$$

$$= \frac{h\,(1+s^2)}{r^2} - \frac{(1+s^2)}{r^2}\int\left(\frac{d\,R}{d\,\lambda'}\right)d\,t + \frac{(1+s^2)}{2\,r^2\,h}\left\{\int\left(\frac{d\,R}{d\,\lambda'}\right)d\,t\right\}^2$$

d R = the differential of R, supposing $n\,t$ only variable + the differential of R, with regard to $n_{,}t$ only in as much as it is contained in the terms in r, λ and s due to the perturbations ; hence

d R = the differential of R, supposing only $n\,t$ variable $+ \frac{d\,R}{d\,r'} . d . \delta r' + \frac{d\,R}{d\,r} d . \delta\lambda' + \frac{d\,R}{d\,r} d . \delta s$; $d . \delta r'$, $d . \delta\lambda$, and $d . \delta s$, being restrained to mean the differentials of those quantities with regard to $n_{,}t$ only.

$$\delta\,R = \left(\frac{d\,R}{d\,r'}\right)\delta\,r' + \left(\frac{d\,R}{d\,\lambda'}\right)\delta\,\lambda' + \left(\frac{d\,R}{d\,s}\right)\delta\,s = -\,a\left(\frac{d\,R}{d\,a}\right)r'\,\delta . \frac{1}{r'} + \left(\frac{d\,R}{d\,t}\right)\delta\,\lambda' + \left(\frac{d\,R}{d\,s}\right)\delta\,s,$$

(t being used in the sense $n\,t - n_{,}t$.) $\left(\frac{d\,R}{d\,s}\right)\delta\,s = \frac{r^2}{2\,r_{,}^3}\,s\,\delta\,s$ nearly.

$$\left(\frac{d\,R}{d\,r}\right)d . \delta\,r + \left(\frac{d\,R}{d\,\lambda'}\right)d . \delta\,\lambda' + \left(\frac{d\,R}{d\,s}\right)d . \delta\,s = -\,a\left(\frac{d\,R}{d\,a}\right)d . r\,\delta . \frac{1}{r} + \left(\frac{d\,R}{d\,t}\right)d . \delta\,\lambda' + \left(\frac{d\,R}{d\,s}\right)d . \delta\,s$$

$d . r'\delta\frac{1}{r'}$, $d . \delta\lambda'$ and $d . \delta s$ being restrained to mean the differentials of those quantities with regard to $n_{,}t$ only.

$$\delta . r\left(\frac{d\,R}{d\,r}\right) = d . \frac{r\left(\frac{d\,R}{d\,r}\right)}{d\,r'} . \delta\,r' + d . \frac{r\left(\frac{d\,R}{d\,r}\right)}{d\,\lambda'}\,\delta\,\lambda' + d . \frac{r\left(\frac{d\,R}{d\,r}\right)}{d\,s}\,\delta\,s$$

$$= -\,a\,d . \frac{r\left(\frac{d\,R}{d\,r}\right)}{d\,a}\,r' . \delta . \frac{1}{r'} + d . \frac{r\left(\frac{d\,R}{d\,r}\right)}{d\,t} . \delta\,\lambda' + d . \frac{r\left(\frac{d\,R}{d\,r}\right)}{d\,s}\,\delta\,s$$

$$\delta . \left(\frac{d\,R}{d\,\lambda'}\right) = -\,a . d . \frac{\left(\frac{d\,R}{d\,\lambda'}\right)}{d\,a}\,r'\,\delta . \frac{1}{r'} + d . \frac{\left(\frac{d\,R}{d\,\lambda}\right)}{d\,t}\,\delta\,\lambda' + \frac{\left(\frac{d\,R}{d\,\lambda'}\right)}{d\,s}\,\delta\,s$$

A similar theorem exists with the quantity $\delta \cdot \frac{\mathrm{d} R}{\mathrm{d} z}$, and it will readily be seen that all the developments δR, $\delta . r \left(\frac{\mathrm{d} R}{\mathrm{d} r}\right)$, $\delta . \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda}\right)$ and $\delta . \left(\frac{\mathrm{d} R}{\mathrm{d} z}\right)$ may be effected very easily by means of Table II.

Similarly, if δ' denote the variation due to the disturbance of the earth by the moon,

$$\delta' R = - a_{,} \mathrm{d} . \left(\frac{\mathrm{d} R}{\mathrm{d} a_{,}}\right) r_{,} \delta . \frac{1}{r_{,}} - \mathrm{d} . \left(\frac{\mathrm{d} R}{\mathrm{d} t}\right) \delta \lambda_{,}$$

In $\mathrm{d} R$ the terms which arise from

$$- a \left(\frac{\mathrm{d} R}{\mathrm{d} a}\right) \mathrm{d} . r \delta . \frac{1}{r} + \left(\frac{\mathrm{d} R}{\mathrm{d} t}\right) \mathrm{d} . \delta \lambda + \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) \mathrm{d} . \delta s$$

are multiplied by the small quantity m.

Considering in $r \left(\frac{\mathrm{d} R}{\mathrm{d} r}\right)$ and R the terms multiplied by $\frac{a^2}{a_{,}^3}$,

$$r \left(\frac{\mathrm{d} R}{\mathrm{d} r}\right) = 2 R, \qquad \delta . r \left(\frac{\mathrm{d} R}{\mathrm{d} r}\right) = 2 \delta R;$$

considering the terms multiplied by $\frac{a^3}{a_{,}^4}$,

$$r \left(\frac{\mathrm{d} R}{\mathrm{d} r}\right) = 3 R, \qquad \delta . r \left(\frac{\mathrm{d} R}{\mathrm{d} r}\right) = 3 \delta R$$

Hence the value of $r \left(\frac{\mathrm{d} R}{\mathrm{d} r}\right)$ and $\delta . r \left(\frac{\mathrm{d} R}{\mathrm{d} r}\right)$ may at once be inferred from R and δR.

I reserve the formation of these developments and of the final equations for determining the coefficients of the different inequalities to another opportunity. These equations are voluminous when all sensible quantities are taken into account; but they are formed with so much facility by means of Table II., that error is not likely to arise in this part of the process. Error is more, I think, to be apprehended in the terms of R multiplied by the cubes and fourth powers of the eccentricities; the rest have been verified by an independent method. See p. 39.

Addition to Table I.

	146	149	150			146	149	150			146	149	150	
1	148 147	153 152	154 151	1	147	1 63	69 3	4 67	147	155	5 71	83 − 14	11 − 90	155
2	150 149	161 162	162 −146	2	148	64 1	4 68	70 3	148	156	72 5	11 − 89	84 − 14	156
3	152 151	147 164	148 163	3	149	2 65	77 0	8 − 62	149	157	6 73	93 12	16 85	157
4	154 153	165 148	166 147	4	150	66 2	8 62	78 0	150	158	74 6	16 86	94 12	158
5	156 155	167 −173	168 −174	5	151	3 67	63 9	1 79	151	159	7 75	87 15	13 91	159
6	158 157	169 170	178 169	6	152	68 3	1 80	64 9	152	160	76 7	13 92	88 15	160
7	160 159	171 176	172 175	7	153	4 69	81 1	10 63	153					
146	62 0	2 − 65	66 − 2	146	154	70 4	10 64	82 1	154					

	161	162			161	162	
	165 164	166 163	1	147	81 9	10 79	147
146	8 − 77	78 − 8	146	148	10 80	82 9	148

Addition to Table II.

	146	149	150			146	149	150			146	149	150	
1	147 148	152 153	151 154	1	10	165 166	154	153	10	64	148	 154	152	64
2	149 150	146	...... −146	2	11	167 168	156	155	11	65	 149	...... −146	....	65
3	151 152	 147	 148	3	12	169 170	 157	 158	12	66	150	....	146	66
4	153 154	148	147	4	13	171 172	160	159	13	67	 151	....	 147	67
5	155 156	....	....	5	14	173 174	...... −155	...... −156	14	68	152	 148	....	68
6	157 158	....	....	6	15	175 176	 159	 160	15	69	 153	147	....	69
7	159 160	....	....	7	16	177 178	158	157	16	70	154	....	148	70
8	161 162	150	149	8	62	146	 150	...... −149	62	71	 155	....	....	71
9	163 164	 151	 152	9	63	 147	151	 153	63	72	156	....	....	72

	146	149	150	
73	 157	....	....	73
74	158	....	....	74
75	 159	....	....	75
76	160	....	....	76
77	 161	149	....	77
78	162	....	150	78
79	 163	....	 151	79
80	164	 152	....	80
81	 165	153	....	81
82	166	....	154	82
83	 167	155	....	83
84	168	....	156	84
85	 169	....	 157	85
86	170	 158	....	86
87	 171	159	....	87
88	172	....	160	88
89	 173	...... −156	....	89
90	174	....	...... −155	90
91	 175	....	 159	91
92	176	 160	....	92
93	 177	157	....	93

	146	149	150	
94	178	....	158	94
146	 62	....	...... − 2	146
147	63 1	3	 4	147
148	1 64	 4	3	148
149	 2	....	....	149
150	2	....	....	150
151	 3	....	 1	151
152	3	 1	....	152
153	 4	1	....	153
154	4	....	1	154
155	 5	....	....	155
156	5	....	....	156
157	 6	....	....	157
158	6	....	....	158
159	 7	....	....	159
160	7	....	....	160
161	 8	2	....	161
162	8	 2	2	162
163	 9	....	 3	163
164	9	 3	....	164
165	 10	4	....	165

	146	149	150	
166	10	....	4	166
167	 11	5	....	167
168	11	....	5	168
169	 12	6	 6	169
170	12	 6	....	170
171	 13	7	....	171
172	13	....	7	172
173	 14	...... − 5	....	173
174	14	....	...... − 5	174
175	 15	....	 7	175
176	15	 7	....	176
177	 16	....	....	177
178	16	....	6	178
179	 17	....	....	179
180	17	....	....	180
181	 18	....	....	181
182	18	....	....	182
183	 19	....	....	183
184	19	....	....	184

	161	162	
8	146	...... −146	8
9	 147	 148	9
10	148	147	10
77	...... −146	....	77

	161	162	
78	....	146	78
79	....	 147	79
80	 148	....	80

	161	162	
81	147	....	81
82	....	148	82
163	....	 1	163

	161	162	
164	 1	....	164
165	1	....	165
166	....	1	166

On the Precession of the Equinoxes, supposing the Earth to revolve in a resisting medium.

In my last paper on Physical Astronomy, I gave expressions for the variations of the six constants which enter into the solution of this problem, upon the hypothesis that the body revolves in a medium devoid of resistance.

If we suppose a plane to revolve in a resisting medium, about an axis perpendicular to itself, the resistance of the medium can produce no effect, and the phenomena will only be modified in a slight degree by the friction of the plane surface against the medium. If, however, the inclination of the plane on the axis of rotation differs from 90°, the effect of the resistance of the medium becomes sensible, tending to retard the motion of the plane; the effect being greatest when the axis of rotation is parallel to the plane.

This principle is used in some machines, as in self-playing organs, to regulate the motion by means of a vane, of which the inclination to its axis of rotation can be varied at pleasure.

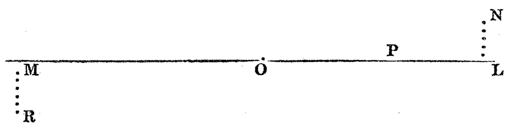
In the case of a sphere, whatever be the direction of the axis of rotation, this effect of the resistance is insensible; and also in the case of a solid of revolution when the axis of rotation coincides with the axis of the figure, but not otherwise. If the difference of the latitude of the axis of rotation from 90° (supposing the equator from which the latitudes are reckoned to coincide with the equator of the figure) be at any time small, the mathematical investigation appears to show, that the effect of the resistance of the medium is to diminish continually this difference. In the case of the earth, this quantity is now insensible; but as the probability is small that this was the case in the first instance, may this circumstance arise from the resistance of a medium of small density acting for a great length of time? and can the change of climate on the surface of the earth, a change of which the probability is indicated by many geological phenomena, be explained in the same manner? It may be remarked, however, that the effect of a resisting medium in diminishing the eccentricities of the orbits of the planets is of the same order, and that these, although for the most part small, are far from having reached zero. The tendency of a resisting medium is also to diminish the major axes of the orbits of the planets; these effects, if they exist, will probably be most sensible

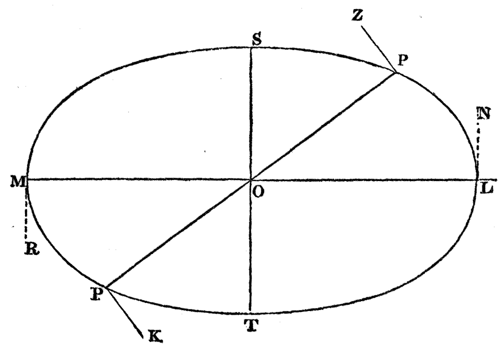
in the case of comets, not only on account of their great eccentricity, but also on account of their small density, in the same manner as a flock of any light substance is wafted by the gentlest wind and prevented from reaching the ground. The eccentricity of the orbit of the comet of HALLEY in 1759 is known with great accuracy, and as its perturbations have been calculated with great care by MM. DAMOISEAU and DE PONTÉCOULANT, the eccentricity which it should have in 1835, when it will again visit this part of space, unless it be affected by a resisting medium, is also known with great precision. It is scarcely probable, however, that any change will be perceptible in one revolution, even if the cause exists; but the succeeding revolutions of this body will no doubt throw light upon this question. The ratio of the change of the semi-major axis to the change of the eccentricity, due to the action of the resisting medium, is known, being a function of the eccentricity, and independent of the constant, which depends upon the density of the medium; this ratio therefore may also tend to elucidate the question, if it can be determined by observation with sufficient accuracy.

Let x', y', z' be the co-ordinates of any point P corresponding to the elementary portion of the surface ds, and referred to axes passing through the centre of gravity and revolving with the body in motion.

Let P be the point of which the co-ordinates are x', y', z', A P the direction of the normal at the point P, B P perpendicular to the axis of instantaneous rotation, and cutting it in B, and C P the direction of motion of the point P. I suppose the resistance of the medium to create a force proportional to $v^2 \cos APC\, ds$, acting in the direction of the normal A P upon the point P, v being the velocity of the point P.

Suppose the straight line M O P L to revolve about an axis passing through O, and perpendicular to it, and in the direction L N, the action of the resisting medium will be in the direction N L, on one side only of the line O L, upon all the points P between O and L, and upon all the points between M P it will be in the contrary direction R M, and on the other side of the line.





Now, let L S M T be the section of a cylinder revolving about an axis, passing through O perpendicular to the plane L S M T, and let the cylinder revolve in the direction L N. The action of the resisting medium will be in the direction Z P, perpendicular to O P upon all the points P between L S; and in the contrary direction K P upon all the points, P between T M. These remarks show that in what follows, the integrations must not be made throughout the whole surface of the body revolving: this consideration however does not affect the nature of the results.

The equation to a plane perpendicular to the axis of rotation, and passing through the centre of gravity of the body, is $p x + q y + r z = 0$.

Let the body revolving be a spheroid of which the equation is

$$x^2 + y^2 + z^2 (1 + e^2) = a^2 (1 + e^2)$$

The equation to the tangent plane to the spheroid at the point x, y, z is

$$x x' + y y' + z z' (1 + e^2) = a^2 (1 + e^2)$$

The equations to the planes from whose intersection the line P B results, are

$$* \; x (q z' - r y') + y (r x' - p z') + z (p y' - q x') = 0$$
$$p x + q y + r z = D$$

D being a constant. The equations to the line P C are

$$x\{r (q z' - r y') - p (p y' - q x')\} + y \{r (r x' - p z') - q (p y' - q x')\} = 0$$
$$x\{q (q z' - r y') - p (r x' - p z')\} + z \{q (p y' - q x') - r (r x' - p z')\} = 0$$

and neglecting p^2, q^2, $p q$,

$$x (q z' - r y') = y (p z' - r x')$$
$$x (q y' + p x') = z (p z' - r x')$$

The equations to the direction of motion of the point P are

$$x (p z' - r x') = y (r y' - q r')$$
$$x (q x' - p y') = z (r y' - q z')$$

Cos . angle, which the direction of motion of P makes with the normal to the surface or cos A P C

$$= \frac{x' (r y' - q z') + y' (p z' - r x') + z' (1 + e^2) (q x' - p y')}{\sqrt{\{(r y' - q z')^2 + (p z' - r x')^2 + (q y - p x')^2\}} \sqrt{\{x'^2 + y'^2 + z'^2 (1 + 2 e^2)\}}}$$

* The notation is the same as p. 20, except that the accents at foot of $x_{\prime}$, $y_{\prime}$, $z_{\prime}$ are omitted.

$$= \frac{e^2 z' (q x' - p y')}{r \sqrt{x'^2 + y'^2} \sqrt{x'^2 + y'^2 + z'^2}} \text{ nearly.}$$

The resistance acting in the direction of the normal, and since the velocity $= \sqrt{x'^2 + y'^2} \sqrt{(p^2 + q^2 + r^2)}$ nearly;

$$C \mathrm{d} r = 0$$

$$B \mathrm{d} q + (A - C) r p \mathrm{d} t = \mathrm{d} t \int \frac{\{z' x' - x' z' (1 + e^2)\} e^2 z' (q x' - p y') \sqrt{x'^2 + y'^2} \mathrm{d} s (p^2 + q^2 + r^2)}{r \{x'^2 + y'^2 + z'^2\}}$$

$$A \mathrm{d} p + (C - B) q r \mathrm{d} t = \mathrm{d} t \int \frac{\{y' z' (1 + e^2) - z' y'\} e^2 z' (q x' - p y') \sqrt{x'^2 + y'^2} \mathrm{d} s (p^2 + q^2 + r^2)}{r \{x'^2 + y'^2 + z'^2\}}$$

$$\sin \frac{C - A}{A} (n t + \gamma) \mathrm{d} c + c \frac{(C - A)}{A} \cos \frac{C - A}{A} (n t + \gamma) \mathrm{d} \gamma$$

$$= - \frac{n \mathrm{d} t e^4}{A} \int \frac{x' z'^2 (q x' - p y') \sqrt{x'^2 + y'^2} \mathrm{d} s}{\{x'^2 + y'^2 + z'^2\}}$$

$$\cos \frac{C - A}{A} (n t + \gamma) \mathrm{d} c - c \frac{(C - A)}{A} \sin \frac{C - A}{A} (n t + \gamma) \mathrm{d} \gamma$$

$$= \frac{n \mathrm{d} t e^4}{A} \int \frac{y' z'^2 (q x' - p y') \sqrt{x'^2 + y'^2} \mathrm{d} s}{\{x'^2 + y'^2 + z'^2\}}$$

since $\int x'^2 z'^2 \mathrm{d} s = \int y'^2 z'^2 \mathrm{d} s$

$$\mathrm{d} c = - \frac{n \mathrm{d} t e^4 c}{A} \int \frac{x'^2 z'^2 \sqrt{x'^2 + y'^2} \mathrm{d} s}{\{x'^2 + y'^2 + z'^2\}} + \frac{n \mathrm{d} t e^4}{2 A} \sin 2 \frac{(C - A)}{A} (n t + \gamma) \int \frac{x' y' z'^2 \sqrt{x'^2 + y'^2} \mathrm{d} s}{\{x'^2 + y'^2 + z'^2\}}$$

neglecting the term which is periodic,

$$\mathrm{d} c = - n c \frac{e^4 \mathrm{d} t}{A} \int \frac{x'^2 z'^2 \sqrt{x'^2 + y'^2} \mathrm{d} s}{\{x'^2 + y'^2 + z'^2\}}$$

Let $\displaystyle\int \frac{x'^2 z'^2 \sqrt{x'^2 + y'^2} \mathrm{d} s}{\{x'^2 + y'^2 + z'^2\}} = D$

D being a positive quantity.

$$\mathrm{d} c = - \frac{n D c e^4 \mathrm{d} t}{A} \qquad e^{\frac{1}{c}} = \frac{n D e^4 t}{A},$$ e being the base of Naperian logarithms.

When t is infinite $c = 0$; hence the latitude of the axis of instantaneous rotation increases until it reaches 90°, which is its limit.

Having determined the variations of c, γ and n by means of the above equations, the variations of the other constants ω, ψ_0 and φ_0 may be determined from the equations

$$p \mathrm{d} t = \sin \varphi \sin \theta \mathrm{d} \psi - \cos \varphi \mathrm{d} \theta$$
$$q \mathrm{d} t = \cos \varphi \sin \theta \mathrm{d} \psi + \sin \varphi \mathrm{d} \theta$$
$$r \mathrm{d} t = \mathrm{d} \varphi - \cos \theta \mathrm{d} \psi$$