XVI. *An Investigation of the Laws which govern the Motion of Steam Vessels, deduced from Experiments. By* PETER W. BARLOW, *Esq., Civil Engineer. Communicated by* PETER BARLOW, *Esq. F.R.S.*

Received May 8,—Read May 29, 1834.

THE increasing extent of steam navigation, and its importance to the welfare of this country, demand a strict attention, not only to the construction of the vessels, but to the application of the power of steam, in order that the greatest possible useful effect may be produced with a given quantity of coals. The action of paddle-wheels, although a subject strictly mathematical, has hitherto but little engaged the attention of scientific men; in fact, the motion of the vessel being horizontal, while that of the wheel is rotary, there result a certain peculiarity and complication of action which almost defy the theorist to unravel without the aid of a complete set of experiments; for the great commotion of the water in the neighbourhood of a steam-vessel is such, that the results calculated from the usual laws of the resistance of fluids would scarcely be considered satisfactory without having the means of comparing them with practical results. In this respect I have been very fortunately circumstanced; all, or nearly all, HIS MAJESTY'S vessels are fitted out at Woolwich, and each vessel is submitted to an accurate experiment, to ascertain its speed before it leaves the river, sometimes light and sometimes laden. The exact amount of their cargoes is known, their registered and actual tonnage, area of paddle, and every other particular which can serve as a guide to such inquiry; and I have availed myself of these circumstances, and of my personal acquaintance with many of their officers, and the officers of the yard, to attend several of the experiments myself, and in other cases to obtain an exact record of them. I am in hopes, therefore, that some of the remarks in the following pages may not be without utility as a future guidance in the practice of steam navigation.

Within a few years, several of HIS MAJESTY'S vessels have been fitted out with wheels of a new construction, in which the floats are so contrived as, by the aid of machinery, to enter and leave the water nearly in a vertical position. This is found to reduce the shock on the engine produced in common wheels by the floats entering the water, and, in cases of deep immersion, to give an increased speed to the vessel.

A wheel in which the paddle should enter the water in a vertical position has long been considered a desideratum to remedy the supposed loss of power from the oblique action of the common wheel; and various methods of effecting this object have been

invented, which have, however, been so complicated in their construction, and attended with so much friction and liability to get out of order, that they have not hitherto been brought into general use. The method employed in the wheel above alluded to is certainly the most simple that has appeared: it is, however, attended with a considerable liability to derangement, and consequent expense in repair, which materially lessens its value.

As my object in the present paper is in some measure to make a comparison of the action of this with that of the common wheel, I have added a short description of its construction. It is represented in fig. 1. where *a a*, *a a*, &c., are paddles, which turn upon spindles having a bearing in the frame-work *c*, *c*, *c*, &c., of the wheel, which is of a polygonal figure, having as many sides as it is required to have paddles. The inside frame or polygon is alone attached to the shaft of the engine, which does not continue beyond the side of the vessel, and the outer one has an independent bearing on a centre attached to the paddle-box, so that it receives its motion entirely from the rim or angles of the polygon: by this means the space between the wheels is left quite free. A is a part of the shaft or centre upon which the outer polygon of the wheel revolves, projected in an inclined direction to the middle between the sides, but of course to a point considerably eccentric with the wheel. Each paddle has a crank *b* attached to it at an angle of about 70°, and rods *d*, *d*, &c., connect the extremities of the cranks, with a moveable boss which revolves upon the fixed point A.

It will thus be seen, that in consequence of the point A being situated out of the centre, the paddles will assume different positions during the revolution of the wheel, which positions can be so arranged as to differ very little from the vertical while passing the lower part of the revolution, or that part where the action of the paddle takes place.

Description of the Experiments.

In making the experiments above referred to, the time chosen was generally as nearly as possible that of high water, when there is but little tide; but the effect of this, whatever it may be, was always eliminated by the following equations:

Let t = time in seconds in performing the mile against the tide.

t' = the time with the tide.

v = velocity in miles per hour of the boat.

v' = velocity of the tide water.

n = number of seconds in an hour.

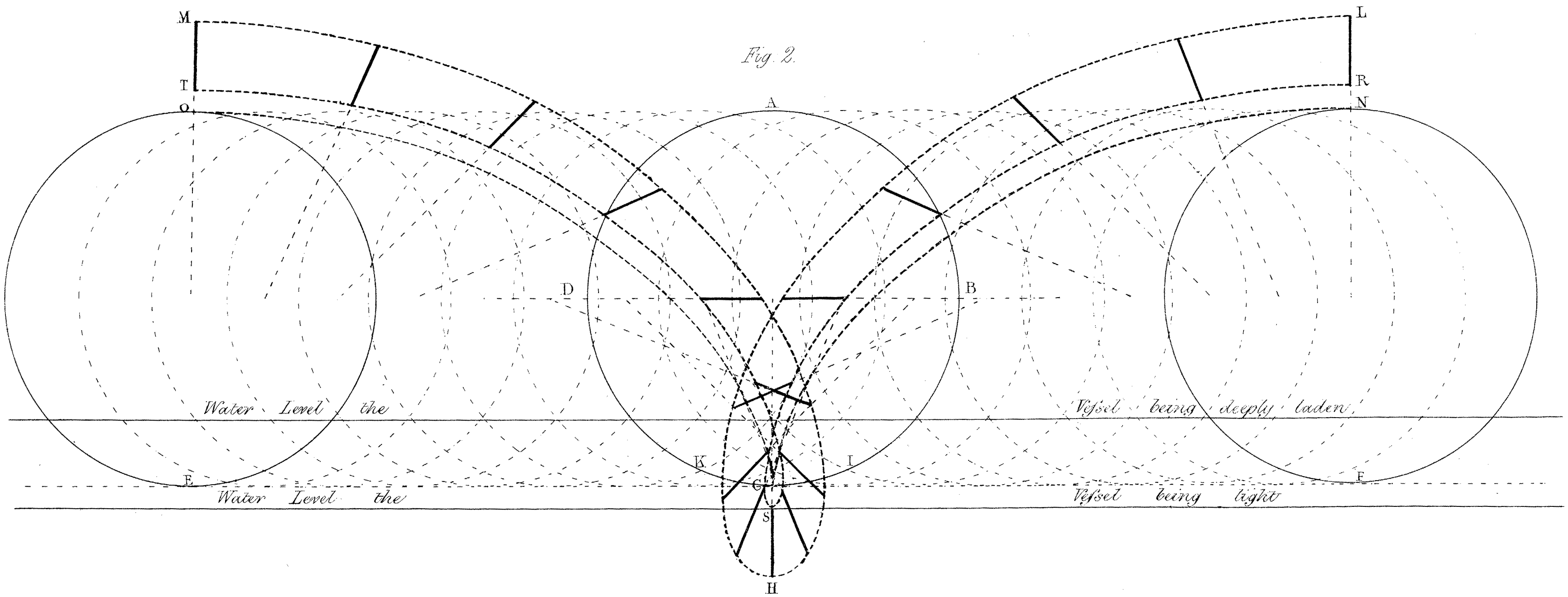
Then $\frac{n}{t'} = v + v'$.

$$\frac{n}{t} = v - v'.$$

Therefore $\frac{n(t+t')}{2tt'} = v$, the velocity independently of the tide.

Position of the Floats of a Paddle Wheel when in motion.

Fig. 2.



Morgan's Paddle Wheel.

Fig. 3.

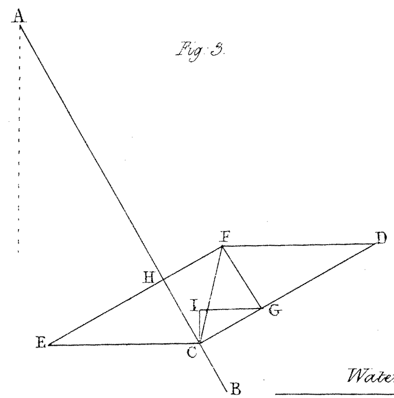


Fig. 1.

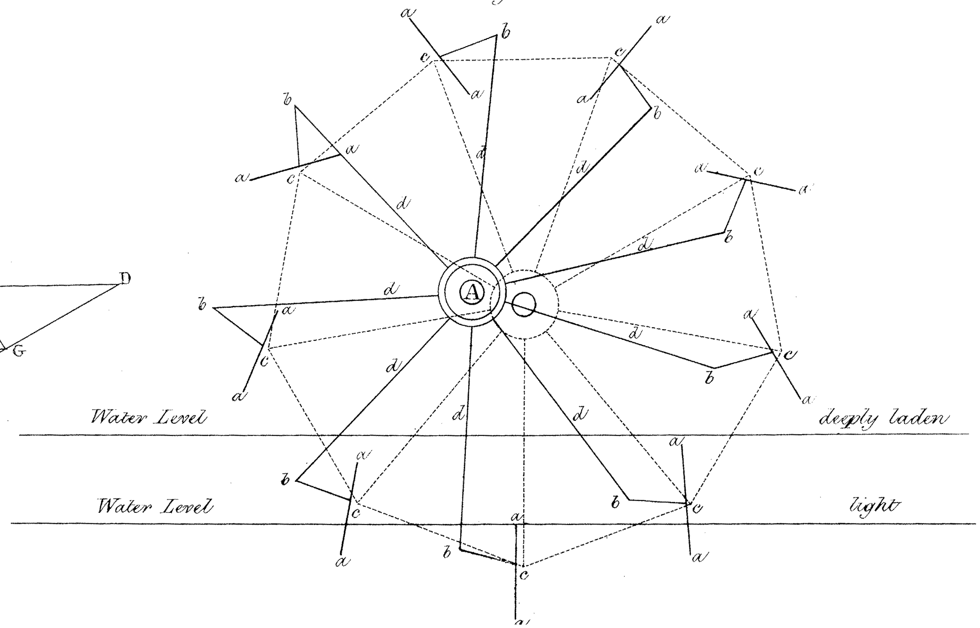
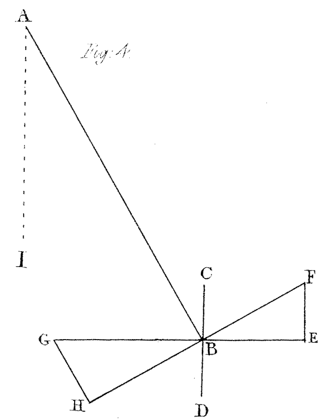


Fig. 4.



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The following experiments on His Majesty's steam-vessel Dee will illustrate the formula:

Power, 200 horses. 23 strokes per minute.

		m	s		s
1st Exp.	Time of running a mile against the tide . .	5	51	or	351
	———— with the tide . . .	5	33	or	333
2nd Exp.	Time of running a mile with the tide . . .	4	23	or	263
	———— against the tide . .	7	58	or	478
3rd Exp.	Time of running a mile with the tide . . .	4	50	or	290
	———— against the tide . .	6	35	or	395

1st Exp. $v = \left(\frac{351 + 333}{2 . 351 . 333}\right) 3600'' = 10{\cdot}52$ miles per hour.

2nd Exp. $v = \left(\frac{263 + 478}{2 . 263 . 478}\right) 3600 = 10{\cdot}58$ ————

3rd Exp. $v = \left(\frac{290 + 395}{2 . 290 . 395}\right) 3600 = 10{\cdot}76$ ————

Mean $= 10{\cdot}62$

In this way the actual speeds of the vessels in the annexed Table have been determined.

The remaining columns, not being calculated results, will be sufficiently understood by the heads.

TABLE I. Experiments.

Name of the vessel.	Tonnage.	Horse power.	Quantity of coals in chaldrons.	Quantity of stores.	Diameter of wheel.	Length of paddle-board.	Depth of paddle-board.	Dip of paddle-board.	Strokes per minute.	Speed in English miles.	Diameter of piston.	Length of double strokes.	Number of strokes per minute, full power.	Remarks.
					ft. in.	ft. in.	ft. in.	ft. in.			inches.			
Alban	294	100	14	None.	13 0	9 0	1 6	not known	27	8·84	40	7	30	Government vessel.
Messenger............	730	200	60	Channel service.	19 4	10 0	2 0	....	$20\frac{1}{2}$	9·75	$53\frac{3}{4}$	10	22	Ditto.
Messenger............	730	200	130	Ditto.	19 4	10 0	2 0	....	18	8·0	....	....	....	Ditto.
Pluto..................	365	100	14	None.	14 4	9 0	1 10	1 9	$26\frac{1}{2}$	10·15	40	7	30	Ditto.
Hermes...............	730	140	130	Channel service.	17 6	9 0	2 0	not known	18	6·3	44	9	24	Ditto.
Meteor	296	100	8	Ditto.	13 0	9 0	1 6	1 6	32	9·0	40	7	30	
Firebrand	494	140	10	None.	17 0	9 0	2 0	2 4	24	10·15	44	9	24	
Firebrand Morgan's wheel	494	120	12	Channel service.	14 6 (Polygon)	....	....	2 $11\frac{1}{2}$	28	10·55	42	8	$27\frac{1}{2}$	Ditto.
Flamer	494	120	15	None.	13 0 (Polygon)	....	....	3 $11\frac{1}{2}$	27	10·9	42	8	$27\frac{1}{2}$	Ditto.
Flamer Morgan's wheel	494	120	112	Channel service.	13 0 (Polygon)	....	....	5 6	24	9·57	....	....	....	Ditto.
Carron	294	100	8	None.	13 0	9 0	1 6	1 4	28	9·15	40	7	30	Ditto.
Dee *..................	710	200	30	Ditto.	19 4	10 0	2 0	1 6	23	10·62	$53\frac{3}{4}$	10	22	Ditto.
Rhadamanthus......	820	220	46	Ditto.	20 4	9 0	2 6	not known	20	10·39	$55\frac{1}{2}$	10	22	Ditto.
Salamander*.........	820	220	210	Channel service.	20 4	9 0	2 6	5 6	15	8·15	$55\frac{1}{2}$	10	22	Ditto.
Firefly	550	140	152	Ditto.	17 6	9 0	2 0	3 4	20	8·3	44	9	24	Ditto.
Magnet...............	360	140	6	None.	16 0	10 0	1 6	1 8	$29\frac{1}{2}$	11·75	44	9	24	Private.
Phœnix	820	220	12	Ditto.	20 4	9 0	2 6	2 6	21	11·7	$55\frac{1}{2}$	10	22	Government vessel.
Medea * Morgan's wheel	825	220	15	Ditto.	21 0 (Polygon)	in the Basin		upper edge 4 inches above water line.	$12\frac{1}{2}$	one wheel	$55\frac{1}{2}$	10	22	Ditto.
Columbia Morgan's wheel	360	100	80	Channel service.	14 0 (Polygon)	....	....	4 10	24	8·5	40	7	30	Ditto.
Firebrand Morgan's wheel	494	120	40	Ditto.	14 6 (Polygon)	....	....	3 7	27	10·1	42	8	$27\frac{1}{2}$	Ditto.
Medea Morgan's wheel	825	220	2	None.	21 0 (Polygon)	....	....	3 11	$22\frac{1}{2}$	11·33	$55\frac{1}{2}$	10	22	Ditto.
Monarch	872	220	....	....	21	10 0	2 0	3 0	$20\frac{1}{2}$	10·72	....	10	22	Private.
Monarch	......	......	....	....	....	....	....	....	$20\frac{1}{3}$	10·50	....	....	....	Ditto.
Monarch	......	......	....	....	....	....	....	....	21	11·02	....	....	....	Additional weight on the safety-valve.

* Since the above experiments, a comparison of the speed of the Medea, Dee, and Salamander was made in the River Medway, at which the Lords of the Admiralty attended: each vessel was laden with nearly her full cargo of coals, stores, &c., amounting in the Medea and Salamander to 200 tons, and in the Dee to a quantity proportional to the tonnage. The exact speed of each was not ascertained for want of a measured mile on the banks of the river; but the result was entirely in favour of the Medea, whose speed amounted to nearly three quarters of a mile per hour beyond that of the other two vessels, their speeds being as nearly as possible the same.

Additional Experiments.

The Medea being moored in the basin at Woolwich, the throttle-valve of her engines quite open, and one wheel only in action, the number of strokes was found to be $12\frac{1}{2}$ per minute. Both wheels being now put in gear, the number of strokes per minute was $8\frac{1}{2}$; so that the resistance on a double surface, at a velocity of $8\frac{1}{2}$, is equal to that on a single surface at a velocity of $12\frac{1}{2}$: from which it follows, (with a very little allowance for the friction attending the working of the paddles,) that the resistance, notwithstanding the violent commotion of the water, is very nearly as the square of the velocity; which law is therefore adopted in the following investigations.

It should be observed, that the engine was scarcely in perfect order when these experiments were made; but being in the same state in both cases, the results are quite comparable.

In an experiment on the Phœnix made subsequently to that in the Table, she was, after being laden, lashed to the Warrior hulk, and her engines started. When her wheels acted with the tide, the number of revolutions per minute was $7\frac{1}{2}$, against the tide $6\frac{1}{2}$, and when free $16\frac{1}{2}$, her speed then being 9·01 miles per hour.

Illustrations of the action of Paddle-wheels in a Vessel in motion.

In order to dispose of the power of an engine to the best advantage, it becomes first necessary to know the manner in which it is at present consumed, and to calculate accurately that portion which is effective in propelling the vessel. When this is fairly understood, the arrangement and proportion of the parts which will effect an improvement will be readily seen.

When a steam-vessel is in motion, the force which opposes the engine is the resistance produced by the paddles moving through the water at a velocity equal to the difference of that of the centre of pressure of the wheel and that of the vessel. The part of this resistance which, when resolved, is in a horizontal direction, is that which is effective: the remaining part of the power is consumed by the resistance opposed to the paddles in a vertical direction, the back water, and other circumstances attending this mode of exerting the power of an engine. Some additional velocity may be given by the tendency of the paddle in its descent to raise the vessel in the water, by diminishing the sectional area of resistance: this, however, if any, is so small as not to be worth consideration; and it may therefore be fairly assumed that the horizontal resistance above mentioned is equal to that opposed to the motion of the vessel.

To make a calculation of these resistances, it becomes necessary to find, with some degree of accuracy, the position of the centre of pressure in the float or paddle, as the calculation is built upon the difference of the velocities of the boat and this centre, which are in some cases so nearly equal that the top of the paddle has no motion through the water.

To find the exact position of this point is, however, a question of very intricate calculation; and as it varies according to the depth of immersion of the paddle or float, the diameter of the wheel, and other circumstances, which vary in different boats, I have contented myself with assuming a point which will meet the ordinary cases, and which I have decided upon from the following considerations.

It is very evident that in every case the resistance upon different parts of the paddle is as the square of the distance from the centre of motion, because the resistance of a fluid varies as the square of the velocity: this ratio is, however, always increased more or less, in consequence of the extremity acting for a greater length of time than the inner part.

In the case of a wheel in motion, in a vessel at rest, if the length of the arc described by the outer extremity of the paddle exceed that described by the inner edge, in the ratio of the large radius to the smaller, the resistance upon any part of the paddle will vary exactly as the square of the radius; but this can only occur when the wheel is either totally immersed or up to the centre of motion: in every other circumstance it is evident that the arc described by the extremity will exceed that of the inner edge in a greater ratio, depending upon the degree of immersion, radius of wheel, &c. Consequently, the resistance upon any part of the paddle will increase in a greater ratio than the square of the distance from the centre of motion. It is, moreover, evident that the position of the centre of pressure will not only vary with every change of immersion, but will continue to ascend from the moment the paddle enters the water until it is immersed below the surface, when it becomes constant, and continues so until the upper part of the paddle again leaves the water.

As these experiments are made entirely with vessels in motion, it is not necessary to enter into a calculation of this precise point. I have merely spoken of the above case with a view to facilitate the investigation of the more complicated question of the centre of pressure of the paddle when the vessel is in motion.

In this case it will be seen, that as the revolution of the paddle resembles a circle rolling on a plane, every part of it will describe a cycloid. That point whose velocity is equal to that of the vessel will move through a simple cycloid, points within that circle in prolate cycloids, and every point without in curtate or contracted cycloids. In fig. 2. is represented the position of the float of a paddle-wheel in different parts of its revolution. The circumference, whose velocity is equal to that of the vessel, is here equal to two thirds of that which passes through the extremity of the paddle, which is about a medium case. It will be readily seen that the effect of the vessel being in motion will be to roll the circle A B C D on the line E F, so that the inner edge of every paddle will move through the cycloid R S T, whilst the extremity moves through the cycloid L K H I M, as shown by the dotted lines in the figure. As the centre of pressure varies at every angle of the paddle, in order to come at the true position it becomes necessary to find the relative velocity of the two extremes of the floats, or the distance moved in the two cycloids, at every instant of time. This

would, however, lead to a calculation of greater labour than the nature of the present investigation demands: and as the circumstances upon which such calculations would be founded vary in every experiment, according to the diameter of the wheel, depth of immersion, &c., I have contented myself with assuming two points, one of which is intended to meet the ordinary cases of slightly immersed, and the other that of deeply immersed, paddles. It appears, again, referring to the figure, that whilst the extremity of the paddle is moving through the part of the curtate cycloid below the level of the water, a point, C, in the radius of the wheel, which is situated in the circumference of the rolling circle, has scarcely moved in the simple cycloid N C O. The difference of the curves during the lower part of the motion amounts nearly to what is due to an arc described with a radius equal to the difference of the extreme radius of the wheel and that of the circle of equal velocity with the ship.

I have considered from this cause that the resistance on any part of the float varies nearly as the square of its distance from the rolling circle; and having at the same time taken into consideration the greater length of time of the action of the extremity than of the inner edge of the paddle, I find, from the examination of several experiments, that in the case of slight immersions the assumption of the resistance on any point varying as the cube of the distance from the rolling circle, and in deep immersions as the 2·5 power, will be a sufficiently near approximation for the present purpose.

Having thus assumed the ratio of resistance with respect to the radius, we readily find the position of the centre of pressure by the following equation.

Let r be the difference of the radius of the rolling circle and that of the wheel, n the power of the resistance in relation to the radius, b the depth of the paddle, x any variable distance from its upper edge, and y the distance of the mean centre of pressure, also from the upper edge; then the integral of $(r+x)^n\, d\,x$, will be the sum of all the resistances, and $(r+y)^n\, b$ the expression to which it is to be equal. We have therefore, when $x = b$,

$$\frac{1}{n+1}\,(r+b)^{n+1} = (r+y)^n\; b,$$

which, when $n = 3$, gives

$$y = \left(\frac{(r+b)^4}{4\,b}\right)^{\frac{1}{3}} - r.$$

And when $r = 2{\cdot}5$,

$$y = \left(\frac{2\,(r+b)^{\frac{7}{2}}}{7\,b}\right)^{\frac{2}{5}} - r.$$

From these equations, the diameters to the centre of pressure of the common wheel (given in column 16 of the following Table) have been calculated.

In the new wheel, the centre of pressure will be nearly in the centre of gravity when the paddle is totally immersed, the motion of the paddle being nearly vertical;

but in consequence of the lower part coming sooner into and continuing longer in action, it must be taken some distance below the centre of gravity.

It is not easy to determine this by calculation; but by a comparison of all circumstances bearing upon this question, I have been induced to make an allowance of one eighth of the paddle on this account.

It may be proper to observe, that in these wheels there is no relation between the diameter of the polygon and the diameter to the centre of pressure, the paddles being differently hung and differently shaped in the several vessels, particulars it has not been thought necessary to introduce into the Table. The remaining calculated columns will, I believe, be sufficiently understood by the heads, except 17 and 18, the former of which exhibits the actual pressure in pounds upon the lower or vertical paddle, as due to the velocity given by the experiment, and the latter the portion of the whole power of the engine which is exerted upon it. The formula for column 17 (V being the velocity of the centre of pressure, v that of the vessel, a the area of the paddle, $62\frac{1}{2}$ the weight of a cubic foot of water in pounds, and $64\frac{1}{3} = 4\,g$, g denoting the force of gravity,) will be $\left(\frac{V - v}{64\frac{1}{3}}\right)^2 \times 62\frac{1}{2} \times a$, the pressure upon a surface moving in a fluid at the velocity $(V - v)$, being equal to the weight of a column of water whose base is the area of the surface, and altitude the height, through which a body must fall to acquire that velocity. This pressure being overcome at a velocity V, the above result, when multiplied by V, will express the power expended upon the vertical paddles; and this number, divided by the whole power of the engine, gives the decimal part of the whole power consumed by the vertical paddle given in column 18. In estimating the part of the power of the engine exerted in any case, the number of strokes made in a minute is compared with the actual number of strokes which ought to be made for the engine to perform its full duty, assuming, as usual, 33000 lbs. raised one foot high per minute to denote the power of one horse.

TABLE II.

Exhibiting the ratio of the velocity of the Wheel and Vessel, the whole pressure upon the vertical Paddle, and other results calculated from the preceding experiments.

	Name of the vessel.	Tonnage.	Horse power.	Quantity of coals in chaldrons.	Diameter of the wheel.	Length of the paddle-board.	Depth of the paddle-board.	Number of paddles.	Dip of the extremity of the paddle.	Strokes of the engine per minute.	Number of strokes per minute, for full power.	Speed of the vessel in English miles.	Area of paddle per horse power.	Tons burden per horse power.	Velocity of the vessel, that of the wheel being 1.	Diameter of rolling circle.	Diameter to centre of pressure.	Pounds pressure upon the vertical paddle.	Proportion of the power of the engine expended on the vertical paddles.
					ft. in.	ft. in.	ft. in.		ft. in.							ft.	ft.	lbs.	
1.	Messenger......	730	200	60	19 4	10 0	2 0	16	....	20½	22	9·75	·20	3·65	·754	13·31	17·65	844	·154
2.	Messenger......	730	200	130	19 4	10 0	2 0	16	....	18	22	8·00	·20	3·65	·730	12·80	17·53	886	·166
3.	Dee	710	200	30	19 4	10 0	2 0	16	1 6	23	22	10·61	·20	3·55	·732	12·91	18·00	1101	·207
4.	Rhadamanthus	820	220	46	20 4	9 0	2 0	16	....	20	22	10·39	·204	3·66	·791	14·54	18·36	720	·123
5.	Salamander ...	820	220	210	20 4	9 0	2 6	16	5 6	15	22	8·15	·204	3·66	·828	15·21	18·36	268	·047 mean ·157
6.	Phœnix	820	220	12	20 4	9 0	2 6	16	2 6	26	22	11·7	·204	3·66	·840	15·60	18·57	468	·080
7.	Monarch	872	200	....	21 0	10 0	2 0	18	3 6	20½	22	10·72	·200	4·36	·748	14·62	19·55	1086	·220
8.	Monarch	......	......	....	....	....	....	......	....	20⅛	......	10·50	....	....	·746	14·60	....	1057	·217
9.	Monarch	......	......	....	....	....	....	......	....	21	......	11·02	....	....	·756	14·69	....	1061	·208
10.	Alban	294	100	14	13 0	9 0	1 6	14	....	27	30	8·84	·27	2·94	·777	9·15	11·77	354	·126
11.	Pluto	365	100	14	14 4	9 0	1 10	14	1 9	26½	30	10·15	·34	3·65	·823	10·71	13·01	308	·126
12.	Hermes	730	140	130	17 6	9 0	2 0	......	....	18	24	6·3	·25	5·21	·626	9·80	15·66	1070	·277
13.	Meteor	296	100	8	13 0	9 0	1 6	......	1 6	32	30	9·0	·27	2·96	·671	7·87	11·70	1083	·370
14.	Firebrand	494	140	10	17 0	9 0	2 0	14	2 4	24	24	10·15	·25	3·51	·772	11·88	15·38	691	·178 ·193
15.	Firefly	550	140	152	17 6	9 0	2 0	14	3 4	20	24	8·3	·245	3·93	·733	11·60	15·81	684	·178
16.	Magnet	360	140	6	16 0	10 0	1 6	......	1 8	29½	24	11·75	·214	2·57	·763	11·16	14·62	840	·149
17.	Carron	294	100	8	13 0	9 0	1 6	......	1 4	28	30	9·15	·27	2·94	·777	9·15	11·77	378	·137
18.	Medea	835	220	20	21 0*	4 10†	3 11	11	3 11	22½	22	11·33	·172	3·79	·627	13·79	22·03	3024	·666
19.	Flamer	494	120	13	13 0	5 9	2 9	9	3 11½	27	27½	10·9	·266	4·11	·683	11·30	16·55	1715	·625
20.	Flamer	......	......	112	....	....	....	......	5 6	24	27½	9·57	....	....	·674	11·16	16·55	1441	·526
21.	Firebrand	494	120	12	14 6	4 6⅓	2 10	9	2 11½	28	27½	10·55	·212	4·11	·667	10·50	15·73	1508	·526 ·546
22.	Firebrand	......	......	40	....	....	....	......	3 7	27	27½	10·10	....	....	·666	10·48	15·73	1404	·476
23.	Columbia	360	100	80	14 0	3 11	3 0	9	4 10	24	30	8·5	·237	3·6	·654	9·91	15·15	1008	·454
		1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.

* Polygon. † Mean length.

Deductions from the tabular numbers.

The results obtained in Columns 17 and 18 will be seen to differ so much in many cases from each other, as to throw an appearance of doubt upon the accuracy of the experiments. These discrepances are attributable, in a great measure, to the result depending upon the square of the difference of two velocities, so that the slightest error in either of the observations is greatly magnified. As the velocity of the wheel is derived from the number of strokes of the engine, the fraction of a stroke makes a very sensible difference in the relative velocities; and it is to this source that these discrepances may be attributed, for the experiments not having been made with a view to the present investigation, the importance of minute attention to this point was not foreseen.

There is no doubt, however, with so many experiments, made at different times and under different circumstances, that the means obtained in the Table are sufficiently near the truth for practical purposes.

As the Table affords us several observations upon vessels of various tonnage and horse power, I have considered it preferable to make a separate class of the larger and smaller ones, because as the proportions of the wheel, and floats, and other circumstances are different, a slight difference in the laws may exist which would be lost sight of by making a general mean.

The experiments are therefore divided into three classes:—

1st Class. Vessels having the common wheel of greater diameter than nineteen feet.

2nd Class. Vessels having the common wheel of a diameter less than nineteen feet.

3rd Class. Vessels having the new wheels.

To show what is stated above, that the differences in the results in Columns 17 and 18 of the Table may arise from the want of minute attention to the number of strokes of the engine, and the approximation of the general means to the truth, I have calculated what the number of strokes per minute would require to be to give the mean number of each class. And it will be seen how much a fractional part of a stroke per minute affects the numbers, and how little correction is necessary to produce the most accurate agreement in the result.

1st Class. Mean = ·157.

	Number of strokes observed.	Number required to give the Mean.
Messenger	$20\frac{1}{2}$	20·5
Messenger	18	17·8
Dee	23	21·8
Rhadamanthus	20	20·5
Salamander	15	17·2
Phœnix	21	22·2
Monarch	$20\frac{1}{2}$	19·6
Monarch	$20\frac{1}{8}$	19·2
Monarch	21	20·1

2nd Class. Mean = ·193.

	Number of strokes observed.	Number required to give the Mean.
Alban	27	28·3
Pluto	$26\frac{1}{2}$	27·5
Hermes	18	16·2
Meteor	32	29·0
Firebrand	24	23·6
Carron	28	29·1
Firefly	20	20·2
Magnet	$29\frac{1}{2}$	29·1

3rd Class. Mean = ·546.

Medea	$22\frac{1}{2}$	21·9
Firebrand	28	28·2
Firebrand	27	27·1
Flamer	27	26·7
Flamer	24	24
Columbia	24	24·4

It thus appears, that the number of strokes required to give the mean in each class, differs generally but a fraction of a stroke from the registered observations; except in a very few cases, and these can be accounted for by the vessel being particularly light or deeply immersed. At all events, there is no doubt that the mean of each set will approximate very nearly to the truth, the immersion of the paddle being also a mean.

A striking difference is observable between the ratio of the resistance of the paddle in a vertical position to the power of the engine in the common wheels, and in the new wheels; the former being ·157 and ·193 with the large and small boats, and the latter ·546. This difference arises from the nature of their action. In the new wheels the vertical position is the most effective in propelling the vessel, and in the common wheels it is the least so, as may be proved in the following manner.

Let A B, fig. 3, be the position of the paddle-rod of a vessel in motion, V being the velocity of the wheel, and v that of the ship, and ϕ the angle of inclination of the paddle-rod with a vertical line: let C D represent the velocity V at right angles to the paddle, and E C that of the vessel in a horizontal direction. Then it is evident that C F, which is the resultant of these velocities, will represent the velocity and direction of motion of the paddle with respect to still water.

Resolve F C into the two velocities F G, C G, one at right angles to, and the other in the direction of, the paddle, of which the latter is lost, while the former will represent the velocity with which the paddle meets the water in a direction at right angles to its face; then E G or H F = E F − E H = $V - v \cos \phi$. Consequently $(V - v \cos \phi)^2$ will represent the whole resistance which the paddle opposes to the engine at any angle ϕ.

In order to get an expression for the resistance in a horizontal direction, or that part of the power which is effective in propelling the vessel, C G must be resolved into the two resistances G I, C I, of which the former is $(V - v \cos \phi)^2 \cos \phi$; and it is to be shown that a mean resistance which would act uniformly through the arc ϕ, so as to be equal to this variable action, will exceed that of the mean action of the lower paddle; while in the new wheel, the mean resistance is less than that of the lower paddle, and hence the great difference in the mean numbers in the Table.

In the new wheel it has been already stated, that the paddle enters the water nearly in a vertical position; and in order to simplify the investigation, I consider it to be truly vertical in every position, which is so near the truth, in that part of the revolution where the action of the paddle takes place, that the results will be but slightly affected. Let C D, figure 4, be any position of a vertical paddle moving at a velocity V, in the direction F B of a tangent to the circumference. Then by resolving this velocity into two, one at right angles to, and one in the direction of, the paddle, we find the velocity with which it meets the water at right angles to its face, to be $V \cos \phi$, ϕ being as before the angle of inclination of the radius A B with a vertical.

The resistance opposed to the vertical paddle when the ship is in motion with a velocity V, will therefore be $(V \cos \phi - v)^2$, so that in the vertical paddle, when $V \cos \phi$ is equal to v, no resistance is opposed to the engine, and when it is less the paddle opposes a resistance in a contrary direction; and it is sufficiently obvious that the resistance in every position in this case is less than when in its lowest position, while in the old wheel it is everywhere greater, at least within practical limits, which fully accounts for the difference in question.

It is observed above, that the horizontal resistance of the oblique paddle is always greater than in its vertical position within the limits prescribed by practice. Let us examine what the actual limits are, by finding, when with given velocities V and v, $(V - v \cos \phi)^2 \cos \phi$ is a maximum, or when

$$V^2\, d \cos \phi - 4\, V\, v \cos \phi \,.\, d \cos \phi + 3\, V^2 \cos v^2\, d \cos \phi = 0.$$

Whence

$$\cos \phi^2 - \frac{4\, V \cos \phi}{3\, v} = - \frac{V^2}{3\, v^2},$$

and

$$\cos \phi = \frac{V}{3\, v}.$$

It depends, therefore, on the relative velocities of the wheel and vessel.

When $V = 5, v = 4,$ then $\phi = 65° \, 33'$
$V = 4, v = 3,$ $\phi = 63 \;\; 37$
$V = 3, v = 2,$ $\phi = 60 \;\;\; 0.$

These results at once account for the ratio of the power of the engine to that of the resistance on the vertical paddle being greater in the old than in the new wheel. For it appears, contrary to the usual opinion, that not only the total resistance to the

paddle increases as it deviates from the vertical, but that the effective horizontal force also increases up to all angles within the limits of the immersion of paddle-wheels.

It should be stated, however, that although an increased propelling power is obtained from the vertical paddle upwards as far as these limits, it is not to be understood that so great an angle is practically advantageous; for the vertical resistance becomes very great, and the shock on the engine by the paddles striking the water at so great an angle is tremendous.

Comparative efficiency of oblique and vertically acting Paddle-wheels.

In order to make a comparison of the efficiency of different wheels, it is necessary to estimate what part of the whole power of an engine is transmitted through them upon the boat, and what part is actually lost. In oblique-acting paddles, a loss of power is sustained in two ways: first, by the resolution of the power of the engine, in which one part is referred to a vertical line, which is wholly inefficient as a propelling power; this part therefore is lost: and of the resolved horizontal force, another part is lost by the motion of the wheel backwards in the water. This may perhaps be best illustrated by the case of a locomotive engine. If the friction between the wheel and the rail is such that the former does not slip, the motion of the carriage is the same as that of the circumference of the wheel; the whole power of the engine is employed in propelling the carriage, and consequently there is no lost power. But if the friction be not sufficient, the wheel will slip back some quantity, the same steam will be consumed in a revolution of the wheel, but the carriage will not be advanced as before, and there will therefore be a loss of power proportional to the skidding or receding of the wheel. So in a steam vessel, all that the centre of pressure actually goes back in the water, or all that its circumferential velocity exceeds that of the vessel (the expense of steam being proportional to the former and the effect to the latter,) may be esteemed lost power; for although the nature of the medium requires some back motion of the wheel to get up the necessary resistance, yet the less there is of this retrogradation, within the limits of practical convenience, the better; because the less power will thus be absorbed by the paddle and the more will be left to act upon the vessel; hence the superiority of one wheel or another will depend upon the quantity of lost power it gives rise to, that wheel being of course to be preferred in which the loss is the least.

The lost power of an engine with the common wheel, or, which is better, the effective part of the power, may be found as follows: First, reduce the variable tangential resistance on the paddle to a mean constant resistance; find also the mean resolved horizontal resistance; then if this mean resolved resistance be supposed to be applied to the circumference of the wheel, the case will exactly resemble that of a locomotive engine, and the part of this force which is to the whole as v to V, will be that part which is employed in propelling the vessel, all the rest will be lost power. The general expression for the tangential resistance on an oblique paddle has been shown to be

$(V - v \cos \phi)^2$ and the integral of $(V - v \cos \phi)^2 d . \phi$, divided by ϕ, will be the mean resistance; or this may be obtained sufficiently near for practical purposes arithmetically. Let this be m, then $V m$ will express the whole power of the engine. Again, the resolved horizontal resistance is expressed generally by $(V - v \cos \phi)^2 \cos \phi$, and the mean value of this is found by dividing the integral of $(V - v \cos \phi) \cos \phi \, d \phi$ by ϕ. Or find the same arithmetically: let this be m', then $V m'$ will be the whole resolved horizontal force, which may be supposed to be applied at the circumference, as in the locomotive engine, and the part of this force expressed by $m' v$, will be the effective force exerted on the vessel; but the whole force is $m V$, therefore $\frac{m' v}{m V}$ will be the proportional part of the power saved, the original engine power being 1. These numbers are computed and arranged within all practical limits in the following little Table.

In the vertical paddle, the mean resistance applied tangentially is the integral of $(V \cos \phi - v)^2 d \phi$ divided by ϕ, which may be all supposed to be applied horizontally as in the locomotive carriage; and the lost power is therefore simply the difference of velocity; that is, the effective horizontal force is $\frac{v}{V}$, the power of the engine being 1; and as in this case the mean velocities are generally as 3 to 2, the part of the whole power which becomes effective is ·666.

In the manner above described, the following Table, exhibiting a comparison of the lost power of the common wheel and that of the new wheel at several states of immersion, has been calculated.

TABLE III.

Angle at which the centre of pressure of the paddle entered the water.	Proportion immersed of the radius of the wheel.	Effective power, that of the engine being 1.		Lost power, that of the engine being 1.		Mean effective resistance, the resistance of the vertical paddle being 1.		Mean resistance opposed to the engine, that of the vertical paddle being 1.		Remarks.
		Common wheel.	MORGAN's wheel.	Common wheel.	MORGAN's wheel.	Common wheel.	MORGAN's wheel.	Common wheel.	MORGAN's wheel.	
35°	·252	·660	·666	·340	·333	1·298	7·02	1·457	·674	Vessel very light, the immersion to the top of the paddle.
44	·350	·645	·666	·355	·333	1·510	5·47	1·750	·522	Mean immersiou of experiments.
50	·430	·620	·666	·380	·333	1·628	5·04	1·971	·482	Mean expressing the ordinary data.
60	·550	·553	·666	·447	·333	1·85	4·25	2·510	·404	Very deep immersion.

Explanation of the manner in which the Power of the Engine is expended in the two Wheels.

Having obtained an expression for the whole resistance opposed to the engine at any angle of the paddle, we may, as before stated, find such a mean resistance, which continuing the same throughout the whole arc, would produce the same effect as the variable resistances expressed in the above formula; and this multiplied by the num-

ber of paddles and tangential velocity should be equal to the power of the engine. The depth of immersion not being given in every experiment, the exact angle at which the paddles entered the water is not known; but as the experiments were generally made immediately after the engines were fitted, when the vessel had not taken in her cargo of coals, the paddles were but slightly immersed.

From the preceding data, and inquiries I have made, I am led to assume a dip of three feet six inches, or the water level to be twelve inches above the top of the paddle, as a mean for the first class, which will make the centre of pressure enter the water at 44°.

Then calling $V = 4$ and $v = 3$, which is nearly the mean ratio of the velocities of the common wheel and vessel, I find the mean resistance of the paddle passing through the whole arc to be to the resistance of the vertical one as 1·75 : 1. Now as the whole circumference contains sixteen paddles, and the arc passed through is 88°, we may consider three paddles and a half to be acting; this will make the whole resistance to the engine equal to 6·12 times that opposed by the vertical paddle, or the power of the engine exerted on the vertical paddle = ·163, the whole power being 1; while the mean obtained from the experiments is ·157. In the second class, the paddles, though smaller, (being proportionably immersed,) may be considered to enter the water at the same angle of inclination, so that the same mean resistance will result from it, viz. 1·75. The number of paddles, however, being less in the small wheels, there are not more than three of them effective, which gives the proportion of the power of the engine exerted on the lower or vertical paddle ·190; the mean obtained from the experiments being ·193. We are thus able in the common wheels to account for the power of the engine, which not only proves quite satisfactorily the accuracy of the principles adopted in the preceding calculations, but that the supposed lost power from back-water is very trifling.

We have in the above investigations considered the paddles to be in the direction of the radii from the centre. It is necessary, however, to mention, that in some of the wheels the radii of the paddles are made to proceed from a point on one side of the centre, with a view of reducing the shock produced by the paddle striking the water at too great an angle. But this deviation is not sufficient to make any sensible difference in the amount of the resistance opposed to the engine; for although it is decreased at the commencement of its action by the angle being smaller, it is increased after passing the centre, which resistance observes the same law until the column of water above is less than that due to the square of the velocity.

It now remains to account for the power of the engine in the new wheel, where we have found the horizontal resistance to the paddle to be $(V \cos \phi - v)^2$. The power of the engine necessary to overcome this resistance will be $(V \cos \phi - v)^2 \cos \phi$, as will be readily seen from the following resolutions.

Referring again to figure 4, let G B represent the horizontal resistance or force on the paddle: it is to be ascertained what force in the direction F B will overcome it. Resolve G B into two forces H G, H B, one at right angles to, and one in the direction

of, the radius rod. The effect to turn the line A B about the point A will be the force H B alone; the force G H, which is in the direction of the line A B, having no power to turn it, its whole action being on the axle of the wheel. It therefore follows, that the force F B at right angles to the radius rod required to retain the point B in equilibrio, or to exert a force in a horizontal direction equal to G B, is G B $\cos \varphi$, because the angle G B H = B A I, and consequently equal to $(V \cos \varphi - v)^2 \cos \varphi$, as already stated in a preceding page.

Having assumed in this case the same angle of 44° when the paddles begin to act, I find the mean of the horizontal resistances on the paddle from the equation $(V \cos \varphi - v)^2$ to be ·547, and the mean of the forces necessary to balance these resistances from the equation $(V \cos \varphi - v)^2 \cos \varphi$ to be ·522 (the force on the lower paddle being 1), which multiplied by $2\frac{3}{4}$, the number of paddles acting, makes the whole power of the engine employed on the paddles to be 1·436 times that exerted on the vertical paddle, or the proportion of the power of the engine employed on the lower paddle to be ·696; the mean given by the experiments being ·546. There is, therefore, a deficiency of ·150 of the power of the engine to account for, which I suppose partly due to the friction of the wheel, and partly to the deviations of the paddle from the vertical position, which, as before observed, results from the construction.

Consumption of Coals at different speeds.

It may be seen by referring to our first Table of experiments, that in deeply laden vessels the engines make little more than two thirds of the number of strokes due to their full power. Now if this number of strokes required only two thirds the steam necessary to keep the engine in full action, the loss sustained in deeply laden vessels would be simply that due to the oblique action of the wheel; but unfortunately nearly as much steam and as much fuel are required in these cases to procure fifteen strokes, as to make the engine perform its full number of twenty-two strokes under other circumstances. To verify this assertion, which is perhaps a very unexpected one, to those who are not intimately acquainted with the navigation of steam vessels in long voyages, I give the following Table, kindly supplied me by Captain Austin, R.N., the observations having been made by his order and under his own superintendence whilst in command of His Majesty's steamer Salamander. By a reference to this Table it will be seen that there is no proportional relation between the speed of the vessel, or even the speed of the piston, and the consumption of fuel, which may be accounted for in a great measure by the loss of heat from the radiation being constant at all velocities; but from whatever cause it proceeds, it is obviously an object of the greatest importance to the progress of improvement of steam navigation, that some means should be found of enabling an engine to perform its full duty under all degrees of immersion. When a vessel commences a long voyage she is necessarily deeply immersed, and at the end of it, her fuel being consumed, her paddles are not perhaps immersed so deeply by nearly three feet. In the latter case the effective part of the power exerted is ·660, and the power exerted is the whole power of the engine;

while in the former the effective part of the power exerted is only ·553, and this power is itself only two thirds of the whole power; so that when a vessel is deeply laden, not above ·368, or three eighths of the whole power of the engine, is employed effectively, while the fuel expended is nearly the same as when light. It is not proposed in this place to speak of a remedy for this evil, but to point it out, in order, if possible, to obtain some means of improvement.

TABLE IV.

No. of revolutions in one minute.	Distance run in two hours by log.				Consumption of coals in two hours.			
	Max.	Min.	Mean.	No. of hours' trial.	Max.	Min.	Mean.	No. of hours' trial.
	K. F.	K. F.	K. F.		Bushels.	Bushels.	Bushels.	
6	5 2	3 4	4 03	14	16	9	10¾	28
7	6 6	3 4	4 76	38	23	9	14	64
8	13 0	4 4	9 63	14	30	9½	18	38
9	13 0	4 0	10 58	22	32	24	28¼	36
10	17 0	4 0	9 80	46	35	14	26¼	62
11	16 0	5 4	5 60	4	37	24	34½	16
12	15 4	7 0	11 13	30	39	26	37¾	52
13	16 0	5 0	14 06	74	41	34	36¼	102
14	18 6	9 0	14 47	120	46	25	36¼	172
15	17 4	7 2	14 12	110	46	28	37¾	162
16	19 4	8 0	15 14	68	46	28	37¾	98
17	18 0	9 4	15 14	80	41	32	37¾	140
18	19 2	10 0	15 24	68	48	31	38½	96
19	21 0	14 2	17 68	68	45	29	39	92
20	20 0	14 6	19 80	48	50	39	41	58
21	21 0	20 0	20 50	4	41	32	38½	14

On the relation between the Diameter of the Wheel, Area of the Paddle, and the Velocity of the Vessel.

When the area of the float of a paddle-wheel is so adjusted to any given diameter that the engine is capable of performing its whole duty, it is evident that the same duty might also be performed with a less paddle and larger wheel, or with a smaller wheel and larger paddle; but the velocity of the vessel will not be the same in the two cases, and the question therefore is to determine what change must be made in the area of the paddle, and what change would take place in the speed of the vessel with a given change in the diameter of the wheel, so that the engine in both cases may perform its whole duty.

Let $d =$ diameter of the first wheel.

$V =$ its circumferential velocity.

$a =$ the area of paddle.

$v =$ the velocity of the vessel.

$r\,d =$ the diameter of the second wheel.

$r\,V =$ the circumferential velocity.

Let a' = the required area of paddle.
v' = the new resulting velocity of the vessel.

All of which quantities are given except a' and v', which may be determined from the following considerations, viz.

1st. That the whole duty of the engine is exerted in both cases; consequently

$$(V - v)^2 V a = (r V - v')^3 r V a'.$$

2nd. That the resistance on the paddle in each case is equal to that of the vessel, and therefore proportional to the squares of the two velocities v' and v, that is,

$$(V - v)^2 a : (r V - v')^2 a' : v^2 : v'^2.$$

From these two equations we find

$$v' = \frac{v}{\sqrt{r}}, \qquad \text{and} \qquad a' = \frac{(V - v)^2}{(r^{\frac{3}{2}} V - v)^2} \times a.$$

From the first it appears that the two velocities are to each other inversely as the square roots of the radii. And by the second the new area of paddle will be found to increase and decrease so rapidly, that generally little practical advantage can be taken of the condition of the first equation.

It appears from the above that there are two different diameters of wheel, with dependent area of paddles, that will allow the full power of the engine to be developed. And when from circumstances of loading, &c. the whole power of the engine cannot develop itself, there are two ways in which this effect can be insured; the one by reducing the paddle, and the other by reducing the diameter of the wheel; by the former it will be seen that the speed of the vessel will remain the same, but by the latter it will be increased inversely as the cube root of the power developed in the two cases.

We have seen that $(V - v)^2 V a$ expresses the whole amount of the power exerted, which in the case we are now supposing, is less than the engine is capable of exerting. Let the amount of power, or, which is the same thing, the number of strokes made in the two cases be as 1 to m.

Now supposing, in the first place, the diameter to remain the same, the velocity V will become m V; and we may find a' and the resulting velocity v' from the equations

$$(V - v)^2 V a : (m V - v')^2 m V a' :: 1 : m,$$

and

$$(V - v)^2 a : (m V - v')^2 a' :: v^2 : v'^2;$$

that is, by making the whole power in the two cases as 1 to m, and the resistances on the paddles as v^2 to v'^2.

From these equations it appears that $v' = v$, or that no increase of velocity will be given to the vessel by reducing the paddle, so as to bring out the full power of the engine.

But if the diameter of the wheel be changed, the paddle remaining the same, both

the velocities V and v will be changed. Let the former become p V, and the latter $n\,v$; our equations are therefore

$$(\mathrm{V} - v)^2\,\mathrm{V}\,a : (p\,\mathrm{V} - n\,v)^2\,p\,\mathrm{V}\,a : 1 : m,$$

$$(\mathrm{V} - v)^2\,a : (p\,\mathrm{V} - n\,v)^2\,a : 1 : n^2,$$

which reduced, give $p = n$, and each equal to the $\sqrt[3]{m}$; that is, the velocity of the vessel will be increased in the ratio of the cube root of the powers expended.

We see therefore that when an engine is not able to perform its whole duty, the diameter of the wheel ought to be reduced, and not, as is usually done, the area of the paddle; for in the former case the velocity is increased in the ratio of the cube roots of the number of strokes, while in the former it remains the same as when the less power was developed.

To find the change in the diameter required to produce this effect, we know the circumferential velocities are as $\mathrm{V} : \mathrm{V}\sqrt[3]{m}$, or as $1 : \sqrt[3]{m}$; we know also that these velocities are as the number of strokes multiplied by the radii of the wheels; putting therefore r and r' for the two radii, the velocities are as $r : m\,r'$, or $r : m\,r' : 1 : \sqrt[3]{m}$, whence

$$r' = \frac{r}{m^{\frac{2}{3}}},$$

the required radius of paddle.

In the case of the Salamander, from the great immersion of the paddles the engine could only make fifteen strokes instead of twenty, its full duty. We may now find what increase of speed would have been given to the vessel by reducing the wheel so as to allow the engine to perform its whole duty.

We have $m = 1{\cdot}25$, whence $r' = {\cdot}8617\,r$, and $n\,v = 1{\cdot}077\,v$; if therefore the diameter of the wheel of the Salamander had been reduced in the ratio of 1 to ·8617, the speed of the vessel would have been increased in the ratio of 1 to 1·075; that is, by reefing each paddle about fifteen inches, the speed of the vessel would have been increased about two thirds of a mile, and at the same time the consumption of fuel would be increased only in a very small degree, as has been demonstrated by the experiments given in the preceding article.

In these calculations I have assumed a similar action of the paddles with every variation of diameter, which in reality is not strictly true, as every change of the position of the floats will vary the angle at which the centre of pressure enters the water. I find, however, in the greatest extent of reefing ever required, this variation to be so small, that it is not necessary to introduce it into the calculation. As far as its effect extends it is favourable to the reefing, as thereby the obliquity of action is diminished, and consequently the loss of power.

Comparison of the Resistance of a Steam Vessel with that of a Plane Surface.

The resistance of vessels being a subject which has of late much engaged the attention of engineers, I have been induced to add the following comparison of the re-

sistance of a steam vessel with that of the paddles, a calculation which the preceding investigations and experiments have enabled me to arrive at with considerable accuracy. Having obtained, through the kindness of O. Lang, jun. Esq. (to whom I have been highly indebted for many of the preceding data), the sectional immersed area of several of the vessels, I have made the calculations, and obtained the results given in the annexed Table. These have been made in the following manner.

Let V = the velocity of the wheel, v that of the vessel, s its sectional area immersed, and a the area of a paddle whose action is horizontal, and effect equal to that of all the paddles. The resistance being as the square of the velocity, $(V - v)^2 a$, will express the resistance on the paddle, and $v^2 s$ would be the resistance of the vessel if it were a plane surface; but the real resistance being $(V - v)^2 a$, the fraction of the resistance compared with a plane will be $\frac{(V-v)^2 a}{v^2 s}$.

The value of a has been obtained by knowing the depth of immersion, so as to ascertain the angle at which the centre of pressure entered the water, and thence the number of times the whole effective action exceeds that of the vertical paddle; this multiplied into the area of the paddle, gives the whole surface above denoted by a.

In the following Table is given the effective pressure exerted by the engine in every experiment where the dip or immersion of the paddle was known; but the comparison of the resistance of the vessel with a plane, is of course limited to those experiments only in which the area of the immersed section could be ascertained.

Table V.

Name of the vessel.	Tonnage.	Horse power.	Effective pressure exerted by the engine.	Velocity of the vessel, that of the wheel being 1.	Velocity of the vertical paddles through the water, that of the wheel being 1.	Area of the paddle-board.	Area of a vertical paddle equal in effect to all the paddles.	Immersed sectional area of the vessel.	Ratio of the resistance of the vessel to that of a plane surface of the same section.
			lbs.			ft. in.			
Medea	835	220	4536	·627	·373	19 0	54·00	263	$\frac{1}{15}$
Flamer	494	120	2814	·683	·317	16 0	52·44	174	$\frac{1}{15}$
Flamer	494	120	2593	·674	·326	16 0	57·60	218	$\frac{1}{16}$
Firebrand..	494	120	2472	·667	·333	12 9	38·56	200	$\frac{1}{19}$
Firebrand..	494	12	2527	·666	·334	12 9	42·00	214	$\frac{1}{18}$
Columbia ..	360	100	1807	·654	·346	12 0	43·10	202	$\frac{1}{18}$ } $\frac{1}{17}$
Salamander	820	220	2180	·833	·167	22 6	398·70	359	$\frac{1}{21}$
Dee	710	200	2531	·732	·268	20 0	69·00	209	$\frac{1}{19}$
Firefly	550	140	3808	·733	·267	18 0	201·00	275	$\frac{1}{11}$
Firebrand..	494	140	2474	·772	·228	18 0	128·61	200	$\frac{1}{17}$
Pluto	365	100	985	·823	·117	16 6	105·23	116	$\frac{1}{24}$
Monarch ..	872	200	7167	·748	·252	20 0			
Monarch ..	872	200	6976	·746	·254	20 0			
Monarch ..	872	200	7002	·756	·244	20 0			
Magnet....	360	140	3672	·763	·237	15 0			
Meteor	296	100	4320	·671	·229	13 6			
Carron	294	100	1731	·777	·323	13 6			
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.

It thus appears, contrary to the results of all experiments hitherto made upon a small scale, that the resistance of a well-shaped vessel does not exceed $\frac{1}{17}$th part that of a plane of the same sectional area.

The above mean being founded on several experiments, I have no doubt is very near the truth, although in each so much error may exist from the want of minute attention to the number of strokes of the engine, as to afford no test of the best-shaped vessel.

As, however, the results are very extraordinary, it may be well to submit them to a totally independent mode of estimation. In the above investigation the mean number of acting paddles with their corresponding velocities and areas, are compared with the sectional area of the vessel and its velocity: but we might have made the calculation in another way, that is, by comparing the force necessary to urge a plane section equal to that of the vessel, with the velocity at which it passes through the water, with the actual power of the engine employed to propel the vessel, which ought to give nearly the same fraction as the other method.

Of the whole power of the engine we have seen that with the vertically acting paddle one third is lost by the retrograding of the wheel. In the Medea therefore the power employed in propelling the vessel is two thirds of $220 = 146$ horse powers. Now the velocity of the vessel having been 11·33 English miles per hour, or 16·62 feet per second, the resistance in feet of water is $\frac{(16{\cdot}62)^2}{64\frac{1}{3}}$ and in pounds $\frac{(16{\cdot}62)^2}{64\frac{1}{3}} \times 62\frac{1}{2}$ on each square foot. The number of feet in the section is 263, and the velocity in feet per minute is 997. The whole force therefore expended in a minute is 70796970, which divided by 33000 gives 2150 horse power for the force necessary to urge a plane section of 263 feet through the water at the rate of 11·33 miles per hour. But the vessel itself is urged with that velocity by the power of 146 horses. The resistance of a vessel is therefore to that of a plane section of the same area as 146 to 2150, or as 1 to 15 very nearly, which agrees exactly with the number given in the Table. The agreement is equally close in the Flamer; and I find the mean obtained this way from the whole set of experiments, is very nearly the same as that given in the above Table.

Since this Paper was read at the Royal Society, Henry Beaufoy, Esq. has, with a noble generosity, presented to his scientific countrymen one of the most valuable collections of resistances ever published, made by his father, the late celebrated Colonel Beaufoy; and it is very satisfactory to be able to confirm the above extraordinary results on the authority of his tables.

I have found above that to urge a plane section of 263 feet area at the rate of 11·33 English miles, or 9·84 nautical miles per hour through still water, would require 2150 horse powers. According to Colonel Beaufoy's results it would require a power of

2444 horses, which would give a still less fraction than a fifteenth; but compared with a cylinder with flat ends, the number of horse powers is 2275, and the fraction greater than $\frac{1}{16}$th, but less than $\frac{1}{15}$th, a confirmation which I could have scarcely hoped to have obtained.

These results are deduced as below. According to Colonel BEAUFOY's experiments, Table I. Part III., it requires a force of 203·79 pounds to urge a plane of one square foot through still water at the rate of eight nautical miles per hour, or 810 feet per minute: now the Medea moved with a velocity of 11·33 miles per hour, or 966 feet per minute; it would therefore require, according to Colonel BEAUFOY, (the resistance being as the square of the velocity,)

$$810^2 \ : \ 996^2 \ : \ 203{\cdot}79 \text{ lbs.} \ : \ 308 \text{ lbs.}$$

Now the section being 263 feet, the resistance per foot 308 pounds, and the velocity 996 feet per minute,

$$\frac{308 \times 996 \times 263}{33000} = 2444,$$

the number of horse power requisite to urge a plane section of this area at the given rate. But if instead of a mere plane we take Colonel BEAUFOY's experiments for a cylinder with flat ends, which required only 190·78 pounds, we obtain the number of horse power 2275 as above stated.

If I had made use of the results of Colonel BEAUFOY's experiments throughout the preceding investigations, the numbers in Column 17 of Table II. and in Column 4 of Table V. would have been increased by about one seventh; and in estimating the power exerted on the paddles, it would have been found to exceed the nominal power of the engine, which proves that engines work above their nominal power.

General Deductions from the preceding Investigations.

On a general examination of the preceding results, I am led to the following conclusions.

1st. That when the vessels are so laden that the wheel is but slightly immersed, there is little advantage in the vertically acting paddle.

2nd. That in cases of deep immersion it has considerable advantage over the common wheel as at present constructed. It has an advantage, in consequence, in a sea where the degree of immersion is continually varying.

3rd. That in the common wheel, while the paddle passes the lower part of the arc, or when its position is vertical, it not only affords less resistance to the engine, but is less effective in propelling the vessel than in any part of its revolution.

4th. That in the new wheel the paddle while passing the lower part of the arc affords more resistance to the engine, and is more effective in propelling the vessel, than in any part of its revolution.

This property of the vertical paddle is a serious deduction from the value of the

wheel; for in consequence of the total resistance to all the paddles being so much less than in the common wheel, much greater velocity is required to obtain the requisite pressure, which is attended with the consumption of an additional quantity of steam, and of course of a proportionate loss of power.

This loss of power is most sensible when the wheel is slightly immersed, as may be seen from the Table; whereas the lost power from the oblique action of the common wheel is then scarcely perceptible. When the vessel is more immersed, and the angle of inclination at which the paddle enters is greater, the proportion of lost power in the common wheel is much increased, while that of the vertical paddle remains nearly constant, so that in cases of deep immersion the vertical paddle has considerably the advantage.

5th. That in any wheel the larger the paddles the less is the loss of power, because the velocity of the wheel is not required to exceed that of the vessel in so high a degree in order to acquire the resistance necessary to propel the vessel.

6th. That with the same boat and the same wheel no advantage is gained by reducing the paddle, so as to bring out, as it is called, the full power of the engine; the effect produced being simply that of increasing the speed of the wheel, and consuming steam to no purpose.

7th. That with the same boat and the same wheel an increase of speed will be obtained by reducing the diameter, or by reefing the paddles, at least within certain limits, viz. as long as the floats remain immersed in the water, and the velocity of the engine does not exceed that at which it can perform its work properly. The increase of speed is in the ratio of the square roots of the radii, or the cube roots of the powers employed.

This result is very important to vessels intended for long voyages, where the great quantity of coals with which they are required to be laden, so much increases the immersion of the paddles, that the engine is not able to exert more than two thirds or three fourths of its full power. In such cases an increase of speed will be given, amounting to nearly one mile an hour, by reducing the diameter of the wheel so as to allow the engine to perform its full duty; and at the same time the consumption of fuel is but little increased, as is shown by Captain Austin's experiments on the Salamander.

8th. That an advantage would be derived from a wheel of large diameter, as far as the immersion of the paddle produced by loading the vessel is concerned, as it would not so sensibly affect the angle of inclination at which it entered the water. This, however, cannot be attained advantageously with an engine of the same length of stroke, because to allow it to make its full number of strokes with the large wheel the size of the paddles must be diminished, which is a much greater evil than a wheel of small diameter with large paddles. To have larger wheels, it is therefore either necessary to have the engines made with longer strokes, or to have the paddle-wheel on a different shaft, in order to diminish their speed. These are both practical incon-

veniences in sea boats, and I therefore consider the wheels to have gained their greatest limit in point of diameter. In the navigation of rivers, where a much greater speed can be attained, wheels of larger diameter may probably be required.

The ill effect of making the wheels of too large diameter, and the paddles too small, is very sensibly exhibited in the experiments on the Medea; her engines have the same length of stroke as those of the Salamander, Phœnix, and Rhadamanthus; and the wheel is twenty-one feet in diameter to the centre of pressure, while those of the latter vessels are not above eighteen feet five inches, or eighteen feet six inches. The consequence is a considerable loss of power, from the greater velocity of the wheel than of the ship. This loss of power is of course still small compared with that of the common wheel when deeply immersed, so that in the experiment at Sheerness her superiority of speed is perfectly consistent with the preceding calculations; at the same time I have no hesitation in saying, that an increase of speed of half a mile per hour, at least, might be obtained by a smaller wheel and a greater surface of paddle-board.

This view of the case is satisfactorily confirmed by experiments on the Monarch steam vessel. A former experiment on this vessel, not reported in the Table, with a wheel of less diameter and larger floats, gave a speed above eleven miles per hour. In the experiment given in the Table the average velocity is 10·8, so that a sensible diminution of speed was produced by enlarging the diameter of the wheel.