

XII. *Researches on the Tides.—Fifth Series. On the Solar Inequality and on the Diurnal Inequality of the Tides at Liverpool. By the Rev. WILLIAM WHEWELL, F.R.S., Fellow of Trinity College, Cambridge.*

Received February 23,—Read March 3, 1836.

SECT. 1. *Present State of the subject.*

1. **THE** great success with which recent researches on the Tides have been attended, has encouraged me to attempt some further advances in this subject. The laws of the semimenstrual inequality of the times were shown by Mr. LUBBOCK, from the London observations, to agree very closely with the equilibrium-theory: and this result has been confirmed by the examination of observations made at many other places. I have shown, from the Liverpool observations, that the semimenstrual inequality of the heights presents a still more complete agreement with the equilibrium-theory*; and by the help of Mr. LUBBOCK's discussions of the London and of the Liverpool observations, I have shown, in the Second and Fourth Series of these Researches, that the inequalities depending on the changes of lunar parallax and declination may be very well represented by the equilibrium-theory, with certain modifications, which are far from inconsistent with the best mechanical views we can at present form of the laws of the motion of fluids.

The most obvious points which now remain requiring still to be made out and explained, are the diurnal inequality, and the solar inequalities of the time and height of high water.

2. The *Diurnal Inequality* of the tides is that which makes the tide of the morning and evening of the same day at the same place, differ both in height and time of high water, according to a law depending on the time of the year. This is called the diurnal inequality, because its cycle is a day.

The existence of such an inequality in the heights of high water has often been noticed by seamen and other observers, as I have stated in the First Series of these Researches†. But its reality has only recently been confirmed by regular and measured observations, and its laws have never been correctly laid down. Its existence appeared very palpably in the curves which I constructed in order to examine the results of the tide observations made by the coast-guard in June 1834, and also in the curves drawn by the self-registering tide-gauge erected near Bristol; although the Sheerness machine did not exhibit it, in consequence of the tide at that place

* Fourth Series: Philosophical Transactions, 1836, page 1.

† Philosophical Transactions, 1833, page 221.

being a compound of two others. But this inequality had never been obtained in numbers till the recent discussion of the Liverpool tides. Under Mr. LUBBOCK's direction, Mr. DESSIOU has obtained it from the observations of Mr. HUTCHINSON. Mr. BYWATER, who has calculated Tide-Tables for Liverpool for the present year, has also obtained this inequality from his own calculations, (suggested, as he states, by the remarks made in the First Series of these Researches,) and has introduced it, for the first time, into published tables; and I have just learnt from Mr. BUNT, who is at present employed in forming Tide-Tables for Bristol, that he has also obtained the general form of this inequality, agreeing, on the whole, with the other results, although with some discrepancies. These arise, I conceive, principally from the shortness of the series of observations which he had at his command (less than two years). The tide observations now going on at these ports, when hereafter discussed as the preceding ones have been, will, I have not any doubt, lead to the production of tide-tables possessing a degree of accuracy which, a little while ago, would have been considered unattainable.

3. It is natural for us to wish to refer the effects of the diurnal inequality, so far as they have yet been obtained, to the equilibrium-theory. I will say a few words on this subject.

The general relation of these results to the equilibrium-theory it is not difficult to see. When the moon is south of the equator, the equilibrium-tide corresponding to her upper transit at any place having southern latitude, would be greater than the tide corresponding to her lower transit: when she is north of the equator the contrary would be the case. When she is in the equator the two tides are equal. Now in one lunation she moves in an orbit inclined to the equator. Hence, while she moves from the sun to the sun again, (that is, while the time of her upper transit passes through the whole twenty-four hours from noon to noon again,) the tide which corresponds to her upper transit will, during one lunation, be greater during half the period, and less during the other half, than the tide of the next half-day; and at two particular times in the lunation the difference will vanish. The times of moon's transit for which the diurnal difference vanishes, correspond to the times when the moon is in the equator, and are therefore different at different seasons of the year. Now from the general correspondence of the phenomena of the tides with the equilibrium-theory, we may expect that the circumstances of the diurnal inequality will be the same as those which have been described; but that the time when the diurnal inequality vanishes will not be the time when the moon is in the equator, but some time afterwards.

4. By the statements above referred to on this subject it appears that the time at which the diurnal difference vanishes is when the moon's transit takes place about 9^h 30^m in January, and two hours earlier in each succeeding month, taking the general average of the facts. Now in the middle of January the sun is 4^h 30^m from the vernal equinox, and hence the moon is 5^h beyond the equinox when the diurnal inequality

vanishes. The same result would be obtained from the other months, since the sun's right ascension increases on an average two hours in each month*. Hence the evanescence of the diurnal inequality which, in the equilibrium-spheroid, would take place when the moon is in the equator, does not take place till she has described 5^h of right ascension after that time, and this requires six days and a quarter.

5. In the inequalities hitherto considered, which were the effects of the sun, and of changes of the moon's parallax and declination, the circumstances of the tide agree with those of the equilibrium-spheroid one day and a half or two days previous. It appears that such an interval suffices for the forces, when near their maximum, to accumulate and transmit their effects to Liverpool; and after this interval the diminution of the actual forces overbalances the increase arising from their continued action. But in this case of the diurnal inequality we find that above six days are required for this accumulation and transfer. The inequality vanishes six days after its cause vanishes, and in the same way it reaches its maximum six days after the producing force is greatest. On a little consideration we shall not be surprised at the great time required to bring this inequality to its full magnitude. The semi-diurnal tides, alternately greater and less, which are transmitted from the Southern Ocean to Liverpool, may be compared to oscillations of the ocean, and these are augmented by the action of the forces occurring at intervals equal to those of the oscillations. Hence the oscillations go on increasing for a considerable period after the forces have gone on diminishing, and reach their maximum almost a week after the forces have passed theirs.

6. In January the tide at Liverpool, which follows the moon's upper transit by about eleven hours, is *less* than the following tide, when the moon is near the sun, and consequently when the moon is south of the equator. This agrees with the equilibrium-tide depending upon the position of the moon at the time of transit. But it is clear that the tide which reaches Liverpool does in fact come from the Southern Ocean; and hence the equilibrium-tide for the time of the moon's upper transit at Liverpool will be *greater* than the following one, when the moon is near the sun in January. Hence the tide at Liverpool is not that which corresponds to the equilibrium-tide *at* the time of transit, but to the equilibrium-tide about either twelve or thirty-six hours *earlier*. The latter is probably the right conclusion, and agrees with the inference already obtained from the effects of the solar forces, that the tide agrees with an equilibrium-tide thirty-seven hours and a half previous to the moon's transit.

8. The succeeding sections of this paper will be devoted to the investigation of the *Solar Inequalities* at Liverpool. By carefully eliminating the lunar effects, which the preceding researches enable us to do, I have determined, as I conceive beyond dis-

* In order to detect the diurnal inequality, the observations have been classed by calendar months; but as this inequality depends mainly on the moon's declination, it would probably have been obtained more distinctly if the observations had been classed according to the moon's declination, distinguishing north and south declination, and also increasing and decreasing declination.

pute, the approximate circumstances of the solar correction for the *height*. I have also obtained, though by no means with the same certainty, evidence of the existence and laws of the solar inequality of the *times*. These inequalities, as thus discovered, exhibit the same general agreement with the equilibrium-theory which has been disclosed in all the inequalities hitherto detected*.

9. But though the equilibrium-theory thus seems to suggest and express the laws of the various inequalities of the tides, I would by no means be understood to rate this theory above its true value. It is not the true theory, but a very inaccurate and insufficient substitute for it, which we are compelled to adopt in consequence of the *extremely imperfect state* of the mathematical science of hydrodynamics. The tides are a problem of the motion, not of the *equilibrium* of fluids; and we can never fully explain the circumstances of the phenomena till the problem has been solved in its genuine form. This solution is perhaps not beyond the powers of modern mathematics, but it has certainly never yet been given. LAPLACE's solution, besides being obtained by means of a precarious assumption, rests upon several arbitrary hypotheses, fatal to it even as a first approximation; and, I believe it will be found, leaves out of consideration an essential portion of the forces. To obtain any useful result, the question must be taken up afresh and treated in another manner.

10. I hope some mathematician will be found able and willing to execute this task. But in the mean time I may be permitted to observe, that what has been already done in the discussion of tide observations, and in bringing to light the empirical laws of the phenomena, has entirely altered the position of this branch of science with respect to the mathematical theory. A little while ago the theory was in advance of observation; at present observation is in advance of theory. A very few years since, the equilibrium-theory and the Laplacian theory were in a condition to assign laws regulating the changes of the times and heights under given astronomical circumstances, and it had not been shown from observation whether these laws were obeyed. We can now state what the agreement and disagreement is between such theoretical laws and the facts; and we call upon the mathematician to substitute for these two theories, both confessedly false, some other, which shall come nearer to the true state of the case, and, by that means, nearer to the laws of the phenomena. The performance of this task is requisite for the completion of the Newtonian theory of the universe.

§ 2. *On the Effect of the Moon's Declination on the Tides at Liverpool†.*

11. In order to obtain the Solar Inequality, it was necessary to eliminate the effect of the variations of the lunar tidal forces; the results of the two sets of forces being combined in the tides when tabulated according to months, as is done in Mr. LUB-

* While I write this, I am informed by Mr. BUNT that he has obtained the law of the solar inequality of the times at Bristol. The agreement with my results is very remarkable.

† In the calculations of this Memoir, I have been assisted by Mr. NAYLER of Queen's College in this University.

BOCK's Tables I., II., III.* In these tables the parallax may be already considered as reduced to its mean value ; for each number is the result of about ninety observations, distributed through nineteen years, in which time the moon's perigee moves twice round the ecliptic ; and hence the deviations above and below the mean will balance each other. But it is otherwise with the declination ; for since the mean moon may be considered as moving in the mean elliptic, a certain time of the year, with a certain hour of moon's transit, implies a certain declination ; for the moon's time of transit added to the sun's right ascension is the moon's right ascension, which of course determines the declination. Hence the variations of the numbers in Tables II. and III. are those which arise from the varying forces of the mean moon and of the sun ; and the greater part of the variation arises from the change of the declination of the moon. This part must be eliminated, in order to bring into view the solar effect ; and we are able to perform this elimination by means of Table I., since that Table contains also the mean declination of the moon for each set of observations. In each set, the number of observations being large, and the limits of declination small, the mean correction for the lunar declination may be taken to be identical with the correction for the mean lunar declination, although the correction varies nearly as the square of the declination. But in order to apply this correction, we must have a table of the effect of every degree of declination ; whereas Mr. LUBBOCK's Tables XII. and XVI. only contain the effect for every 3° of declination. It is obvious also, by inspection of this table, that it is affected by many casual irregularities which must be got rid of by interpolation. Among these irregularities, however, I do not include the apparent difference of the effect of north and south declination, which is shown in Tables XI. and XV. ; since it will be seen by comparison with Table X. that the greater part of this difference arises from the effect of parallax ; for the changes of parallax and declination so nearly recur in the same cycle, that their effects are not insulated in a period of nineteen years. The remainder of the difference arises from the solar inequality of which we are in search.

12. I have therefore laid down Mr. LUBBOCK's Tables XII. and XVI., and interpolated them by means of curves, and have thus obtained two new declination tables for the *Times* and the *Heights*. These tables I shall designate as Declination Tables (W. T.) and (W. H.) respectively ; meaning by the former letter to distinguish them from Mr. LUBBOCK's Tables XII. and XVI. They may be used in calculating the time and height of high water at Liverpool (the table for the height being altered by a constant if the measures are not taken from Mr. HUTCHINSON's zero). When so applied, they take the place of Mr. LUBBOCK's Tables XXV. and XXVI., from which they differ—in including the semimenstrual inequality, in being interpolated independently, in being given for the middle of each hour instead of the beginning, and for every degree of declination instead of every three degrees. These Tables will be found at the end of this Memoir, namely,

* Philosophical Transactions, 1835, p. 282.

Declination Table (W. T.). *To be used in predicting the Time of high water at Liverpool, for the mean parallax.*

Declination Table (W. H.). *To be used in predicting the Height of high water at Liverpool, for the mean parallax.*

When these tables are used in predicting the tides, the times and heights found by these must be corrected for parallax by Mr. LUBBOCK's Tables XXIII. and XXIV., or by the formulæ given in my Memoir on the Empirical Laws of the Tides in the Port of Liverpool. I will insert, at the end of this paper, Tables of the correction for parallax given by these formulæ.

§ 3. *On the Solar Inequality of the Heights of High Water at Liverpool.*

13. By means of the Declination Tables (W. T.) and (W. H.) we can reject from Mr. LUBBOCK's Tables II. and III. the part which depends on the moon's declination, and we thus have the remainder, which is the solar correction as far as it can be obtained from the observations of Mr. HUTCHINSON. This is what I have done in the following calculations (Table A.). The third column of each table, marked (W. H.), contains the height due by the declination table; the fourth column, marked (L. III.), contains the observed height as given by Mr. LUBBOCK's Table III.; and the column (Diff. H.) is the excess of the latter, and is therefore the residual height due to the solar effect. In like manner the columns marked (W. T.), (L. II.) and (Diff. T.) contain the *Intervals* due to the moon's declination, the observed intervals, and the residual solar effect.

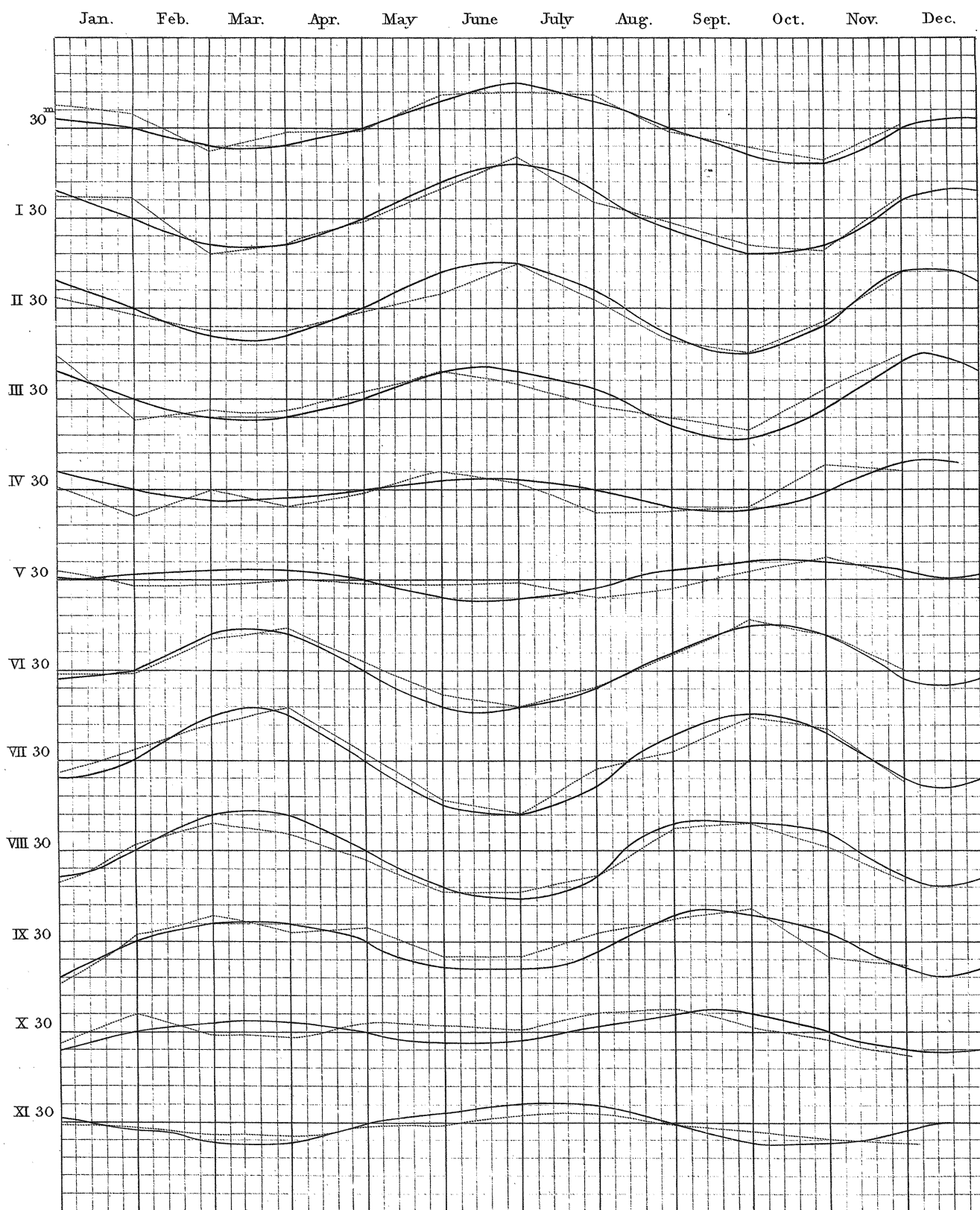
I shall consider the solar effect on the *Heights* in the first place, since its law is more manifest. It may be expected, like the other tidal inequalities, to be resolvable into a non-periodic and a periodic part. In order to effect this, I take the mean of each monthly column of differences, and subtract it from each number in the column. In this manner I obtain Table (B. H.).

In order to trace the law of the periodical part, which exists in the remainders, if at all, I lay down these remainders by coordinates, as in Plate XVII.

14. If we draw lines through the dots belonging to each hour of the moon's transit, it becomes manifest that these remainders really result from a solar inequality. For the curves (the dotted curves in Plate XVII.) have all the same general form, having a reference to an annual cycle. Thus there are, for all of them, a maximum or minimum in March, another in July, another in October, and another in December. Hence the effect of lunar declination has been nearly or altogether eliminated; for this effect is a maximum for different hours at different times of the year, as appears by the reason of the case, and as is shown in Mr. LUBBOCK's Tables IV. and V.

It appears also by the curves, that the effects of the changes of the solar force are the greatest for the hours of moon's transit, 1^h 30^m and 7^h 30^m, and are in these two cases the opposite of one another, which agrees with the nature of the case; for the

SOLAR CORRECTION OF HEIGHTS. PERIODIC PARTS. LIVERPOOL.



The ordinates are here six times too large. The positive ordinates are measured downwards.

height of the solar tide is added to that of the lunar in the former case (at spring-tides), and subtracted in the latter (at neap-tides).

15. Thus the general course of the phenomena shows the existence of the solar inequality; but we shall trace its law more distinctly by taking the suggestions of the equilibrium-theory.

If h , h' , be the height of the solar and lunar tides, ϕ the angular distance of the sun and moon, y the compound tide, we have

$$y = \sqrt{h^2 + h'^2 + 2 h h' \cos 2 (\phi - \alpha)},$$

as shown on former occasions; α being a quantity which is to be determined so as to accommodate the equilibrium-theory to the actual case.

Let h undergo any change so as to become $h + \Delta h$, Δh being small. Then y becomes approximately

$$y + \Delta y = y + \frac{dy}{dh} \Delta h = y + \frac{h + h' \cos 2 (\phi - \alpha)}{\sqrt{h^2 + h'^2 + 2 h h' \cos 2 (\phi - \alpha)}} \Delta h.$$

The quantity Δh varies according to the different season of the year, and depends principally on the sun's declination, the changes of solar parallax being too small to affect much the amount of solar tidal force. The curve which expresses the changes of Δh has, as appears in the figures, a maximum at the end of March, soon after the time when the sun is in the equator, and when, consequently, his tidal force is the greatest. It cuts the axis in May, soon after the sun has his mean effective declination: it has a minimum in July, soon after his greatest declination, at which time his force is least. The mean effect recurs in the end of August: there is another maximum in the end of September. About November, December, and January, there is another mean and another minimum, arising from the mean declination in November and the greatest declination in December. Thus the general course of the value of the correction for a given value of ϕ agrees with the equilibrium-theory. The want of perfect regularity in the form of the curves is due partly to the combination of the effects of solar parallax and solar declination. According to the theory, the greatest amount of the correction for solar declination is about one tenth, and the greatest amount of the correction for solar parallax about one twentieth, of the whole solar tide.

16. For a given season of the year, if we follow the changes of ϕ through twelve hours, we easily see from the formula that Δy has a continuous series of values, among which are a maximum and a minimum value. Hence the curves in Plate XVII. ought to be such that the ordinates for each month, taken in the successive hour-lines, form a continuous series. The subtraction of the non-periodical part, which we have performed, will not affect this continuity, since this part is constant for the month.

Hence in correcting the original curves of Plate XVII., so as to get rid of irregularities, we must endeavour to make them conform to these two conditions:—that the ordinates for the same month shall form a continuous series, with a maximum and minimum; and, that the curves for the different hours shall be similar to each other,

and have their maxima and minima at the same seasons. This latter condition may perhaps be slightly modified, so that α may be somewhat different for different values of Δh .

The curves drawn with full lines in Plate XVII., are drawn under these conditions, the ordinates being those contained in Table (C. H.). Their agreement with the original curves is such as to entitle us to consider them as correct interpolations, for the coincidence is almost complete in the cases where the corrections are the largest; as the hours $0^h 30^m$, $1^h 30^m$, $2^h 30^m$, $6^h 30^m$, $7^h 30^m$, $8^h 30^m$; and there are no material discrepancies except in the lines $4^h 30^m$ and $10^h 30^m$; and even in these the difference is only a displacement of a maximum of four inches by about a month. The observations, therefore, prove that there is a solar correction of the heights which follows, nearly, the law suggested by the equilibrium-theory. The greatest amount of the periodical part of this correction is about half a foot *plus* and *minus*; and to this must be added the non-periodical part, which at some seasons amounts to one fifth of a foot.

17. We may expand the formula above given for Δy into a non-periodical and a periodical part. We have

$$\begin{aligned}\Delta y &= \frac{h + h' \cos 2(\varphi - \alpha)}{\sqrt{\{h^2 + h'^2 + 2h h' \cos 2(\varphi - \alpha)\}}} \cdot \Delta h, \\ &= \frac{h'}{\sqrt{(h^2 + h'^2)}} \frac{c + \cos 2(\varphi - \alpha)}{\sqrt{\{1 + \frac{2c}{1+c^2} \cos 2(\varphi - \alpha)\}}} \cdot \Delta h,\end{aligned}$$

Making $c = \frac{h}{h'}$. Expanding and omitting c^2 , &c.

$$\begin{aligned}\Delta y &= \frac{h'}{\sqrt{(h^2 + h'^2)}} (c + \cos 2(\varphi - \alpha)) (1 - c \cos 2(\varphi - \alpha)) \Delta h \\ &= \frac{h'}{\sqrt{(h^2 + h'^2)}} \left\{ c + \cos 2(\varphi - \alpha) - c \cos^2 2(\varphi - \alpha) \right\} \Delta h \\ &= \frac{h'}{\sqrt{(h^2 + h'^2)}} \left\{ \frac{c}{2} + \cos 2(\varphi - \alpha) - \frac{c}{2} \cos 4(\varphi - \alpha) \right\} \Delta h\end{aligned}$$

Hence the periodical part is

$$\frac{h' \Delta h}{\sqrt{(h^2 + h'^2)}} \left\{ \cos 2(\varphi - \alpha) - \frac{c}{2} \cos 4(\varphi - \alpha) \right\}.$$

This vanishes for two values of φ , which are a little less than 90° or 6^h from each other when c is small. In Table (B. H.) it appears that the periodical solar correction vanishes for two values of the hour-angle which are nearly 6^h from each other in each month. Hence we may suppose that the periodical part involving $\cos 2(\varphi - \alpha)$ is alone sensible. This vanishes when $\cos 2(\varphi - \alpha) = 6^h$ or 18^h , $\varphi = 3^h + \alpha$ or $9^h + \alpha$. Hence we find that in January and September α is $2^h 30^m$; in March, April, June, July, October α is about 2^h ; in August and November it is $1^h 30^m$; in December it is 3^h .

These differences in the value of α may arise from the lunar declination not being completely eliminated in our previous calculation, the interpolation of the Table being slightly inexact. But it is by no means improbable that they are the discrepancies, arising from mechanical principles, which exist between the tides of water in motion and the results of the equilibrium-hypothesis. The general agreement of the equilibrium-theory with the facts (assuming $\alpha = 2^h$) is near enough to be very remarkable. It may be possible, by additional care and labour, to bring the solar interpolated curves nearer to the observations, preserving the requisite conditions: but I conceive that enough has been done to establish the general law.

§ 4. *On the Solar Inequality of the Time of High Water at Liverpool.*

18. The solar inequality of the time must be found in exactly the same manner as the solar inequality of the heights. By interpolation of Mr. LUBBOCK's Table XVI., I have obtained Declination Table (W. T.); and by comparing the time of high water due to lunar declination by this Table with the observed times as given in column (L. II.) of the calculations, I obtain the residual quantities in the columns (Diff. T.), which should exhibit the solar correction of the times.

I then find the mean of the column for each month, and subtract it from every number in the column, in order to separate the non-periodical from the periodical part of the residual quantities. The means and the remainders are exhibited in Table (B. T.). The remainders were laid down by coordinates, but I have not thought it necessary to give these curves.

The points thus found being joined by continuous lines, I had a series of curves which ought to exhibit the solar inequality. These lines were less obviously regular than those which we obtained by a similar treatment of the heights; but they were still apparently free from any lunar effect, which would have given a maximum passing successively from one month to another; and I could trace a solar cycle in them; namely, the curves for $1^h 30^m$, $2^h 30^m$, and $3^h 30^m$ had a maximum about April, and a minimum about August, while the curves for $5^h 30^m$, $6^h 30^m$, $7^h 30^m$ had these features inverted, and the rest of the curves had small ordinates only.

19. Let us compare the laws of the phenomena of which we thus catch a glimpse with the laws according to the equilibrium-theory. We have, on that hypothesis,

$$\tan(\theta' - \lambda') = -\frac{h \sin 2(\phi - \alpha)}{h' + h \cos 2(\phi - \alpha)} = t \text{ suppose,}$$

where θ' is the interval of the tide and moon's transit.

Now let h become $h + \Delta h$, and θ' become $\theta' + \Delta \theta'$, we have, approximately,

$$\tan(\theta' - \lambda') + \frac{d \cdot \tan(\theta' - \lambda')}{d \theta'} \Delta \theta' = t + \frac{d t}{d h} \Delta h$$

$$\sec^2(\theta' - \lambda') \cdot \Delta \theta' = \frac{d t}{d h} \Delta h$$

$$\begin{aligned}\Delta \theta' &= \frac{1}{1+t^2} \cdot \frac{d t}{d h} \Delta h \\ &= - \frac{h' \sin 2(\phi - \alpha)}{h^2 + h'^2 + 2 h h' \cos 2(\phi - \alpha)} \cdot \Delta h.\end{aligned}$$

Hence for a given value of Δh , that is, for a given month, $\Delta \theta'$ goes through a cycle which has a minimum and a maximum, when $\cos 2(\phi - \alpha) = -\frac{2 h h'}{h^2 + h'^2}$. If $\frac{h'}{h} = 3$, which has been found to be nearly the value by other phenomena, the two values of $2(\phi - \alpha)$ are 144° and 216° , or 10^h and 17^h nearly; and if in this case, α be $-1^h 30^m$, the maxima and minima will agree with $\phi = 3^h 30^m$, $\phi = 7^m 0^m$, which appears best to represent the phenomena.

20. After drawing the lines described in Art. 18, I drew interpolated curves for the hours $1^h 30^m$, $2^h 30^m$, $3^h 30^m$, and $5^h 30^m$, $6^h 30^m$, $7^h 30^m$; the general agreement of the course of the interpolated and the original curves appeared to me to be such as to show that the errors arose from a solar inequality, following in its general changes the law given by the equilibrium-theory. I have given the corrections upon this supposition in Table (C. T.). The displacement of the zero points and maxima, and the want of proportionality in the maximum values, which appear in some of the lines, I have admitted, because such modifications may arise from my not having got rid of the whole of the non-periodical effect. Such irregularities may also arise from there being still some vestige of the lunar declination correction not got rid of by the processes which I have employed: but this, if it exists, must be very small, and I do not think there can be any doubt as to the general form of the solar correction of the time. I have, however, abstained from filling up the Table, as not thinking my present materials sufficient to enable me to do so with any confidence.

DECLINATION TABLE (W. T.). To be used in predicting the *Time* of high water at Liverpool for the mean parallax.

Moon's Transit.	0° Decl.	1° Decl.	2° Decl.	3° Decl.	4° Decl.	5° Decl.	6° Decl.	7° Decl.	8° Decl.	9° Decl.	10° Decl.	11° Decl.	12° Decl.	13° Decl.
h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
0 30	11 20	11 20	11 20	11 20	11 20	11 20	11 20	11 20	11 20	11 19	11 19	11 19	11 18	11 18
1 30	11 5	11 5	11 5	11 5	11 5	11 5	11 5	11 5	11 4	11 4	11 4	11 3	11 3	11 2
2 30	10 51	10 51	10 51	10 51	10 51	10 51	10 50	10 50	10 50	10 49	10 49	10 49	10 47	10 47
3 30	10 39	10 39	10 39	10 39	10 39	10 39	10 39	10 39	10 38	10 38	10 38	10 37	10 36	10 36
4 30	10 31	10 31	10 31	10 31	10 31	10 31	10 31	10 31	10 30	10 29	10 29	10 28	10 27	10 27
5 30	10 35	10 35	10 35	10 35	10 34	10 34	10 34	10 34	10 34	10 33	10 32	10 31	10 30	10 28
6 30	10 54	10 54	10 54	10 53	10 53	10 53	10 52	10 51	10 51	10 50	10 49	10 48	10 47	10 46
7 30	11 24	11 24	11 24	11 24	11 24	11 24	11 23	11 23	11 23	11 22	11 22	11 20	11 20	11 19
8 30	11 47	11 47	11 47	11 47	11 47	11 47	11 47	11 47	11 47	11 46	11 46	11 45	11 44	11 44
9 30	11 53	11 53	11 53	11 53	11 53	11 53	11 53	11 53	11 53	11 52	11 52	11 52	11 51	11 51
10 30	11 46	11 46	11 46	11 46	11 46	11 46	11 46	11 46	11 46	11 46	11 46	11 46	11 46	11 46
11 30	11 35	11 35	11 35	11 35	11 35	11 35	11 35	11 35	11 35	11 34	11 34	11 34	11 34	11 34
	14° Decl.	15° Decl.	16° Decl.	17° Decl.	18° Decl.	19° Decl.	20° Decl.	21° Decl.	22° Decl.	23° Decl.	24° Decl.	25° Decl.	26° Decl.	27° Decl.
0 30	11 17	11 17	11 16	11 16	11 15	11 14	11 14	11 13	11 12	11 12	11 11	11 11	11 10	11 9
1 30	11 2	11 1	11 1	11 0	10 59	10 59	10 58	10 58	10 57	10 57	10 56	10 54	10 53	10 52
2 30	10 47	10 46	10 43	10 44	10 44	10 43	10 42	10 41	10 40	10 39	10 37	10 37	10 36	10 34
3 30	10 35	10 34	10 33	10 32	10 32	10 31	10 29	10 28	10 27	10 26	10 25	10 24	10 22	10 21
4 30	10 26	10 25	10 24	10 23	10 22	10 21	10 20	10 18	10 17	10 16	10 14	10 12	10 11	10 9
5 30	10 27	10 26	10 25	10 23	10 22	10 21	10 19	10 18	10 17	10 15	10 13	10 12	10 11	10 10
6 30	10 44	10 43	10 42	10 41	10 39	10 37	10 36	10 35	10 33	10 32	10 31	10 29	10 27	10 25
7 30	11 18	11 18	11 17	11 15	11 13	11 12	11 11	11 10	11 8	11 7	11 6	11 4	11 2	11 0
8 30	11 44	11 44	11 43	11 42	11 42	11 41	11 40	11 39	11 38	11 37	11 36	11 34	11 33	11 33
9 30	11 51	11 51	11 50	11 50	11 49	11 49	11 48	11 48	11 47	11 47	11 46	11 45	11 44	11 44
10 30	11 45	11 44	11 44	11 44	11 43	11 43	11 42	11 42	11 41	11 40	11 39	11 39	11 38	11 37
11 30	11 33	11 32	11 32	11 32	11 32	11 31	11 30	11 29	11 28	11 28	11 27	11 27	11 26	11 25

DECLINATION TABLE (W. H.). To be used in predicting the *Height* of high water at Liverpool for the mean parallax.

The heights in feet from Mr. HUTCHINSON'S zero.

Moon's Transit.	0° Decl.	1° Decl.	2° Decl.	3° Decl.	4° Decl.	5° Decl.	6° Decl.	7° Decl.	8° Decl.	9° Decl.	10° Decl.	11° Decl.	12° Decl.	13° Decl.
h m	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.
0 30	18.3	18.3	18.3	18.3	18.3	18.3	18.3	18.3	18.3	18.25	18.25	18.2	18.15	18.1
1 30	18.3	18.3	18.3	18.3	18.3	18.3	18.3	18.3	18.3	18.25	18.25	18.2	18.15	18.1
2 30	17.65	17.65	17.65	17.65	17.65	17.65	17.65	17.65	17.65	17.6	17.6	17.55	17.5	17.5
3 30	16.6	16.6	16.6	16.6	16.6	16.55	16.55	16.55	16.55	16.5	16.5	16.5	16.45	16.4
4 30	15.2	15.2	15.2	15.2	15.2	15.2	15.15	15.15	15.1	15.1	15.05	15.05	15.0	14.95
5 30	13.95	13.95	13.9	13.9	13.9	13.9	13.85	13.8	13.75	13.75	13.75	13.7	13.65	13.65
6 30	13.0	13.0	13.0	13.0	12.95	12.95	12.9	12.9	12.85	12.8	12.75	12.7	12.65	12.6
7 30	12.95	12.95	12.95	12.95	12.95	12.9	12.9	12.9	12.85	12.8	12.75	12.7	12.65	12.6
8 30	13.95	13.95	13.9	13.9	13.85	13.85	13.85	13.8	13.75	13.75	13.7	13.65	13.65	13.55
9 30	15.35	15.35	15.35	15.35	15.35	15.35	15.35	15.35	15.3	15.3	15.25	15.25	15.2	15.15
10 30	16.8	16.8	16.8	16.8	16.75	16.75	16.75	16.75	16.7	16.7	16.65	16.6	16.55	16.5
11 30	17.7	17.7	17.7	17.7	17.65	17.65	17.65	17.65	17.65	17.65	17.6	17.55	17.55	17.45
	14° Decl.	15° Decl.	16° Decl.	17° Decl.	18° Decl.	19° Decl.	20° Decl.	21° Decl.	22° Decl.	23° Decl.	24° Decl.	25° Decl.	26° Decl.	27° Decl.
0 30	18.05	17.95	17.9	17.8	17.7	17.65	17.5	17.4	17.3	17.2	17.1	17.0	16.8	16.7
1 30	18.05	17.95	17.9	17.8	17.7	17.65	17.5	17.4	17.3	17.2	17.1	17.0	16.8	16.7
2 30	17.45	17.4	17.35	17.25	17.2	17.1	17.0	16.9	16.85	16.75	16.65	16.55	16.4	16.3
3 30	16.35	16.35	16.3	16.25	16.25	16.2	16.15	16.1	16.0	15.95	15.9	15.8	15.7	15.6
4 30	14.95	14.9	14.8	14.75	14.75	14.65	14.6	14.55	14.5	14.4	14.35	14.25	14.25	14.15
5 30	13.6	13.55	13.5	13.45	13.4	13.35	13.25	13.2	13.1	13.05	12.95	12.9	12.8	12.75
6 30	12.55	12.5	12.45	12.35	12.25	12.2	12.1	12.0	11.9	11.8	11.7	11.5	11.35	11.2
7 30	12.5	12.5	12.4	12.35	12.25	12.2	12.1	12.0	11.9	11.75	11.65	11.5	11.35	11.2
8 30	13.5	13.45	13.4	13.3	13.25	13.2	13.15	13.05	12.95	12.9	12.8	12.75	12.65	12.55
9 30	15.1	15.05	15.0	14.95	14.9	14.85	14.75	14.7	14.6	14.5	14.4	14.3	14.2	14.1
10 30	16.45	16.4	16.35	16.3	16.25	16.2	16.1	16.0	15.85	15.75	15.6	15.45	15.3	15.1
11 30	17.4	17.35	17.25	17.2	17.1	17.0	16.9	16.8	16.7	16.6	16.45	16.3	16.15	16.0

PARALLAX TABLE (W. T.). To be used in correcting the *Time* of high water at Liverpool, predicted for mean parallax 57'.

Moon's Transit.	54'.	55'.	56'.	57'.	58'.	59'.	60'.	61'.
h m	m	m	m	m	m	m	m	m
0 30	9	6	3	0	— 3	— 6	— 9	—12
1 30	7	5	2	0	— 2	— 5	— 7	— 9
2 30	5	3	2	0	— 2	— 3	— 5	— 6
3 30	3	2	1	0	— 1	— 2	— 3	— 3
4 30	1	1	0	0	0	— 1	— 1	— 1
5 30	1	1	0	0	0	— 1	— 1	— 2
6 30	4	3	1	0	— 1	— 3	— 4	— 5
7 30	9	6	3	0	— 3	— 6	— 9	—12
8 30	13	8	4	0	— 4	— 8	—13	—17
9 30	14	9	5	0	— 5	— 9	—14	—19
10 30	13	9	4	0	— 4	— 9	—13	—18
11 30	11	7	4	0	— 4	— 7	—11	—15

PARALLAX TABLE (W. H.). To be used in correcting the *Height* of high water at Liverpool, predicted for mean parallax 57'.

H. P. 54'.	H. P. 55'.	H. P. 56'.	H. P. 57'.	H. P. 58'.	H. P. 59'.	H. P. 60'.	H. P. 61'.
ft.	ft.	ft.	ft.	ft.	ft.	ft.	ft.
—1·2	—0·8	—0·4	0	+0·4	+0·8	+1·2	+1·6

TABLE (A).

Calculation of the Differences between the Time and Height of High Water at Liverpool, as due to the Moon's Declination, and as shown by Mr. HUTCHINSON's Observations, for every month of the year.

January.							
Moon's Transit.	Declination.	(W. H.) Height due to Declination.	(L. III.) Observed Height.	Diff. H.	(W. T.) Time due to Declination.	(L. II.) Observed Time.	Diff. T.
h m	°	feet.	feet.		h m	h m	m
0 30	18	17·70	17·47	—·23	11 15	11 13·7	—1·3
1 30	15	17·95	17·74	—·21	11 1	10 59·5	—1·5
2 30	10	17·60	17·52	—·08	10 48	10 47·0	—1·0
3 30	5	16·55	16·08	—·47	10 40	10 37·1	—2·9
4 30	5	15·15	15·14	—·01	10 31	10 31·1	+·1
5 30	8	13·75	13·19	—·06	10 33	10 33·8	+·8
6 30	14	12·55	12·61	+·06	10 44	10 49·3	+5·3
7 30	19	12·20	12·38	+·18	11 12	11 16·1	+4·1
8 30	19	13·20	13·56	+·36	11 41	11 37·8	—3·2
9 30	23	14·50	14·95	+·45	11 47	11 45·0	—2·0
10 30	22	15·85	15·98	+·13	11 41	11 38·5	—2·5
11 30	22	16·70	16·73	+·03	11 28	11 34·2	+6·2

TABLE (A). Continued.

February.							
Moon's Transit.	Declina- tion.	(W. H.) Height due to Declination.	(L. III.) Observed Height.	Diff. H.	(W. T.) Time due to Declination.	(L. II.) Observed Time.	Diff. T.
h m	°	feet.	feet.		h m	h m	m
0 30	10	18·25	18·05	—·20	11 19	11 18·3	—·7
1 30	5	18·30	18·04	—·26	11 5	11 8·2	+3·2
2 30	5	17·65	17·69	+·04	10 50	10 49·7	—·3
3 30	8	16·55	16·73	+·18	10 39	10 37·3	—1·7
4 30	14	14·95	15·20	+·25	10 26	10 27·3	+1·3
5 30	18	13·40	13·39	—·01	10 22	10 26·3	+4·3
6 30	21	12·00	11·96	—·04	10 35	10 37·9	+2·9
7 30	21	11·95	11·78	—·17	11 10	11 8·0	—2·0
8 30	23	12·90	12·79	—·11	11 38	11 37·4	—·6
9 30	22	14·60	14·47	—·13	11 47	11 51·0	+4·0
10 30	19	16·20	15·96	—·24	11 43	11 44·0	+1·0
11 30	15	17·35	17·39	+·04	11 32	11 31·5	—·5
March.							
0 30	5	18·3	18·38	+·08	11 20	11 19·1	—·9
1 30	8	18·25	18·49	+·24	11 4	11 3·9	—·1
2 30	13	17·5	17·63	+·13	10 47·5	10 49·4	+1·9
3 30	16	16·3	16·31	+·01	10 33	10 33·3	+·3
4 30	21	14·6	14·47	—·13	10 19	10 20·3	+1·3
5 30	22	13·1	13·01	—·09	10 17	10 16·2	—·8
6 30	22	11·9	11·42	—·48	10 33	10 30·2	—2·8
7 30	22	11·85	11·31	—·54	11 8·5	11 5·8	—2·7
8 30	20	13·1	12·68	—·42	11 40	11 40·7	+·7
9 30	15	15·05	14·62	—·43	11 50·5	11 50·3	—·2
10 30	10	16·65	16·54	—·11	11 46	11 44·9	—1·1
11 30	6	17·65	17·67	+·02	11 35	11 33·4	—1·6
April.							
0 30	12	18·15	18·01	—·14	11 19	11 19·4	+·4
1 30	17	17·8	17·88	+·08	11	11 1·1	+1·1
2 30	20	17	17·01	+·01	10 42	10 43·4	+1·4
3 30	22	16·05	15·98	—·07	10 27	10 30·4	+3·4
4 30	23	14·45	14·44	—·01	10 15	10 14·2	—·8
5 30	22	13·1	12·89	—·21	10 17	10 12·8	—4·2
6 30	20	12·1	11·44	—·66	10 36	10 31·8	—4·2
7 30	16	12·4	11·71	—·69	11 16·5	11 8·5	—8·0
8 30	11	13·65	13·28	—·37	11 45	11 44·0	—1·0
9 30	6	15·35	15·07	—·28	11 53	11 54·3	+1·3
10 30	5	16·75	16·61	—·14	11 46	11 48·2	+2·2
11 30	7	17·65	17·60	—·05	11 35	11 36·0	+1·0
May.							
0 30	20	17·5	17·33	—·17	11 14	11 16·3	+2·3
1 30	22	17·3	17·15	—·15	10 57	10 57·9	+·9
2 30	23	16·75	16·56	—·19	10 39	10 38·6	—·4
3 30	22	16	15·75	—·25	10 27	10 25·1	—1·9
4 30	20	14·65	14·48	—·17	10 20	10 16·0	—4·0
5 30	16	13·5	13·33	—·17	10 25	10 18·8	—6·2
6 30	12	12·65	12·35	—·30	10 47	10 41·9	—5·1
7 30	7	12·85	12·58	—·27	11 23	11 20·6	—2·4
8 30	5	13·85	13·75	—·10	11 46·5	11 46·9	+·4
9 30	7	15·35	15·01	—·34	11 53	11 52·5	—·5
10 30	12	16·55	16·28	—·27	11 46	11 46·9	+·9
11 30	17	17·2	17·01	—·19	11 32	11 31·2	—·8

Table (A). Continued.

June.							
Moon's Transit.	Declination.	(W. H.) Height due to Declination.	(L. III.) Observed Height.	Diff. H.	(W. T.) Time due to Declination.	(L. II.) Observed Time.	Diff. T.
h m	°	feet.	feet.		h m	h m	m
0 30	23	17.2	16.73	— .47	11 12	11 13.4	+ 1.4
1 30	22	17.3	16.84	— .46	10 57	10 55.5	— 1.5
2 30	20	17	16.74	— .26	10 42	10 39.8	— 2.2
3 30	16	16.3	15.81	— .49	10 33	10 29.6	— 3.4
4 30	11	15.05	14.69	— .36	10 28	10 24.9	— 3.1
5 30	6	13.85	13.75	— .10	10 34	10 31.6	— 2.4
6 30	5	12.95	13.07	+ .12	10 52.5	10 52.9	+ .4
7 30	8	12.8	13.09	+ .29	11 22.5	11 24.6	+ 2.1
8 30	12	13.6	13.86	+ .26	11 45	11 44.0	— 1.0
9 30	17	14.95	14.93	— .02	11 50	11 49.2	— .8
10 30	20	16.1	15.91	— .19	11 42	11 41.8	— .2
11 30	22	16.7	16.57	— .13	11 28	11 28.9	+ .9
July.							
0 30	19	17.6	17.08	— .52	11 14	11 13.5	— .5
1 30	16	17.9	17.14	— .76	11 1	10 57.8	— 3.2
2 30	11	17.55	16.84	— .61	10 49	10 45.2	— 3.8
3 30	6	16.55	16.29	— .26	10 39	10 37.6	— 1.4
4 30	5	15.15	14.96	— .19	10 31	10 33.1	+ 2.1
5 30	8	13.75	13.83	+ .08	10 33	10 34.2	+ 1.2
6 30	13	12.6	12.87	+ .27	10 45	10 51.6	+ 6.6
7 30	18	12.25	12.72	+ .47	11 13	11 12.2	— 0.8
8 30	21	13.05	13.39	+ .34	11 40	11 38.9	— 1.1
9 30	22	14.6	14.64	+ .04	11 45.5	11 48.5	+ 3.0
10 30	23	15.75	15.61	— .14	11 40	11 39.6	— .4
11 30	22	16.7	16.52	— .18	11 28	11 27.2	— .8
August.							
0 30	11	18.2	17.68	— .52	11 19	11 17.5	— 1.5
1 30	6	18.3	17.96	— .34	11 5	11 4.5	— .
2 30	5	17.65	17.37	— .28	10 51	10 51.6	+ .
3 30	7	16.55	16.46	— .09	10 39	10 35.4	— 3.5
4 30	13	14.95	15.03	+ .08	10 27	10 30.2	+ 3.2
5 30	17	13.45	13.46	+ .01	10 23	10 28.7	+ 5.7
6 30	21	12	12.00	.00	10 35	10 38.4	+ 3.4
7 30	22	11.85	11.80	— .05	11 8	11 8.6	+ .6
8 30	25	12.7	12.82	+ .12	11 36	11 37.4	+ 1.4
9 30	22	14.6	14.34	— .26	11 47.5	11 46.9	— .6
10 30	19	16.2	15.84	— .36	11 43	11 42.4	— .8
11 30	16	17.25	16.99	— .26	11 32	11 30.1	— 1.9
September.							
0 30	4	18.3	18.30	.00	11 20	11 20.4	+ .4
1 30	7	18.3	18.39	+ .09	11 10	11 3.0	— 7.0
2 30	12	17.5	17.89	+ .39	10 48	10 48.8	+ .8
3 30	17	16.25	16.50	+ .25	10 32.5	10 34.0	+ 1.5
4 30	20	14.65	14.94	+ .29	10 20	10 21.3	+ 1.3
5 30	22	13.1	13.19	+ .09	10 17	10 16.9	— .1
6 30	23	11.8	11.63	— .17	10 32	10 32.1	+ .1
7 30	22	11.85	11.78	— .07	11 8	11 4.8	— 3.2
8 30	20	13.1	12.86	— .24	11 40	11 39.3	— .7
9 30	16	15	14.78	— .22	11 50	11 50.3	+ .3
10 30	11	16.6	16.38	— .22	11 46	11 46.4	+ .4
11 30	6	17.65	17.69	+ .04	11 35	11 35.2	+ .2

TABLE (A). Continued.

October.							
Moon's Transit.	Declina- tion.	(W. H.) Height due to Declination.	(L. III.) Observed Height.	Diff. H.	(W. T.) Time due to Declination.	(L. II.) Observed Time.	Diff. T.
h m	°	feet.	feet.		h m	h m	h m
0 30	11	18.2	18.52	+ .32	11 19	11 19.1	+ .1
1 30	16	17.85	18.28	+ .43	11 1	11 1.7	+ .7
2 30	20	17	17.62	+ .62	10 42	10 42.3	+ .3
3 30	22	16	16.48	+ .46	10 27	10 26.7	- .3
4 30	23	14.45	14.76	+ .31	10 15	10 14.7	- .3
5 30	22	13.1	13.15	+ .05	10 16	10 12.2	- 3.8
6 30	20	12.1	11.70	- .40	10 36	10 29.1	- 6.9
7 30	17	12.3	11.95	- .35	11 15	11 11.0	- 4.0
8 30	13	13.55	13.37	- .18	11 45	11 45.1	+ .1
9 30	7	15.35	15.13	- .22	11 53	11 54.0	+ 1.0
10 30	4	16.75	16.83	+ .08	11 46	11 48.7	+ 2.7
11 30	7	17.65	17.92	+ .27	11 35	11 35.3	+ .3
November.							
0 30	20	17.5	17.95	+ .45	11 14	11 18.4	+ 4.4
1 30	22	17.3	17.73	+ .43	10 57.5	10 58.7	+ 1.2
2 30	23	16.75	16.99	+ .24	10 38	10 39.1	+ 1.1
3 30	22	16	16.01	+ .01	10 27	10 24.1	- 2.9
4 30	20	14.65	14.50	- .15	10 20	10 14.7	- 5.3
5 30	16	13.5	13.33	- .17	10 24.5	10 17.1	- 7.4
6 30	13	12.6	12.28	- .32	10 45	10 40.4	- 4.6
7 30	7	12.85	12.62	- .23	11 23	11 21.3	- 1.7
8 30	5	13.85	13.95	+ .10	11 47	11 47.9	+ .9
9 30	6	15.35	15.61	+ .26	11 53	11 53.8	+ .8
10 30	12	16.55	16.73	+ .18	11 46	11 46.7	+ .7
11 30	17	17.2	17.50	+ .30	11 32	11 32.9	+ .9
December.							
0 30	23	17.2	17.25	+ .05	11 12.5	11 12.9	+ .4
1 30	21	17.4	17.28	- .12	10 58	10 54.8	- 3.2
2 30	19	17.1	16.80	- .30	10 43	10 40.8	- 2.2
3 30	16	16.3	15.89	- .41	10 33	10 28.7	- 4.3
4 30	11	15.05	14.95	- .10	10 28	10 23.9	- 4.1
5 30	8	13.75	13.82	+ .07	10 33	10 30.6	- 2.4
6 30	5	12.95	13.06	+ .11	10 52.5	10 52.5	- .0
7 30	7	12.85	13.21	+ .36	11 23	11 23.4	+ .4
8 30	12	13.6	14.07	+ .47	11 45	11 45.7	+ .7
9 30	17	14.95	15.35	+ .40	11 50	11 48.2	- 1.8
10 30	20	16.05	16.38	+ .33	11 42.5	11 42.2	- .3
11 30	21	16.8	17.14	+ .34	11 29	11 27.7	- 1.3

TABLE (B. H.). The Lunar Effect rejected.

Residual quantities. *Heights.*

D's Transit.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
Means	+·01	−·05	−·14	−·21	−·20	−·15	−·12	−·16	+·02	+·12	+·09	+·10
Remainders.												
h m												
0 30	−·24	−·15	+·22	+·07	+·03	−·32	−·40	−·36	−·02	+·20	+·36	−·05
1 30	−·22	−·21	+·38	+·29	+·05	−·31	−·64	−·18	+·07	+·31	+·34	−·22
2 30	−·09	+·09	+·27	+·22	+·01	−·11	−·49	−·12	+·37	+·50	+·15	−·40
3 30	−·48	+·23	+·15	+·14	−·05	−·34	−·14	+·07	+·23	+·36	−·08	−·51
4 30	−·02	+·30	+·01	+·20	+·03	−·21	−·07	+·24	+·27	+·19	−·24	−·20
5 30	−·07	+·04	+·05	−·00	+·03	+·05	+·20	+·17	+·07	−·07	−·26	−·03
6 30	+·05	+·01	−·34	−·45	−·10	+·27	+·39	+·16	−·19	−·52	−·41	+·01
7 30	+·17	−·12	−·40	−·48	−·10	+·44	+·59	+·11	−·09	−·47	−·32	+·26
8 30	+·35	−·06	−·28	−·16	+·10	+·41	+·46	+·28	−·26	−·30	+·01	+·37
9 30	+·44	−·08	−·29	−·07	−·14	+·13	+·16	−·10	−·24	−·34	+·17	+·30
10 30	+·12	−·19	+·03	+·07	−·07	−·04	−·02	−·20	−·24	−·04	+·09	+·23
11 30	+·02	+·09	+·16	+·16	+·01	+·02	−·06	−·10	+·02	+·15	+·21	+·24

TABLE (C. H.). The Remainders in Table (B. H.) corrected by Interpolation.

Periodical and non-periodical part of the Solar Effect, in *tenths of feet.*

D's Transit.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
h m												
0 30	− 1	0	+ 2	+ 2	0	− 3	− 5	− 3	0	+ 3	+ 4	0
1 30	− 3	0	+ 3	+ 3	0	− 4	− 6	− 3	+ 1	+ 4	+ 3	− 2
2 30	− 3	0	+ 3	+ 3	0	− 4	− 5	− 2	+ 3	+ 5	+ 2	− 4
3 30	− 3	0	+ 2	+ 2	0	− 3	− 3	− 1	+ 3	+ 4	+ 1	− 5
4 30	− 2	0	+ 1	+ 1	0	− 1	− 1	0	+ 2	+ 2	0	− 3
5 30	0	0	− 1	− 1	0	+ 2	+ 2	+ 1	− 1	− 2	− 2	− 1
6 30	+ 1	0	− 4	− 4	0	+ 4	+ 4	+ 2	− 2	− 5	− 4	+ 1
7 30	+ 2	0	− 5	− 5	0	+ 5	+ 6	+ 3	− 3	− 5	− 3	+ 2
8 30	+ 3	0	− 4	− 4	0	+ 4	+ 5	+ 3	− 3	− 3	− 2	+ 3
9 30	+ 4	0	− 2	− 2	0	+ 3	+ 3	+ 2	− 3	− 3	− 1	+ 3
10 30	+ 2	0	− 1	− 1	0	+ 1	0	0	− 2	− 2	+ 0	+ 2
11 30	0	0	+ 2	+ 2	0	− 1	− 2	− 2	0	+ 2	+ 2	+ 1
Non-periodical	0	− 1	− 2	− 2	− 2	− 2	− 1	− 1	0	+ 1	+ 1	+ 1

SOLAR TABLE (W. H.). Correction of the Heights for the Sun's Effect, in *tenths of feet.*

D's Transit.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
h m												
0 30	− 1	− 1	0	0	− 2	− 5	− 6	− 4	0	+ 4	+ 5	+ 1
1 30	− 3	− 1	+ 1	+ 1	− 2	− 6	− 7	− 4	+ 1	+ 5	+ 4	− 1
2 30	− 3	− 1	+ 1	+ 1	− 2	− 6	− 6	− 3	+ 3	+ 6	+ 3	− 3
3 30	− 3	− 1	+ 0	0	− 2	− 5	− 4	− 2	+ 3	+ 5	+ 2	− 4
4 30	− 2	− 1	− 1	− 1	− 2	− 3	− 2	− 1	+ 2	+ 3	+ 1	− 2
5 30	0	− 1	− 3	− 3	− 2	0	+ 1	0	− 1	− 1	− 1	0
6 30	+ 1	− 1	− 6	− 6	− 2	+ 2	+ 3	+ 1	− 2	− 4	− 3	+ 2
7 30	+ 2	− 1	− 7	− 7	− 2	+ 3	+ 5	+ 2	− 3	− 4	− 2	+ 3
8 30	+ 3	− 1	− 6	− 6	− 2	+ 2	+ 4	+ 2	− 3	− 2	− 1	+ 4
9 30	+ 4	− 1	− 4	− 4	− 2	+ 1	+ 2	+ 1	− 3	− 2	0	+ 4
10 30	+ 2	− 1	− 3	− 3	− 2	− 1	− 1	− 1	− 2	− 1	+ 1	+ 3
11 30	+ 0	− 1	0	0	− 2	− 3	− 3	− 3	0	+ 1	+ 3	+ 2

TABLE (B. T.). The Lunar Effects rejected.
Residual Quantities.—*Times.*

☾'s Transit.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
Means.....	+·2	+·9	—·5	—·6	—1·4	—·8	+·1	+·5	—·5	—·8	—1·0	—1·5
Remainders.												
h m	m	m	m	m	m	m	m	m	m	m	m	m
0 30	—1·5	—1·6	—·4	+1·0	+3·7	+2·2	—·6	—2·0	+·9	+·9	+5·4	+1·9
1 30	—1·7	+2·3	+·4	+1·7	+2·3	—·7	—3·3	—1·0	—6·5	+1·5	+2·2	—1·7
2 30	—1·2	—1·2	+2·4	+2·0	+1·0	—1·4	—3·9	+·1	+1·3	+1·1	+2·1	—·7
3 30	—3·1	—2·6	+·8	+4·0	—·5	—2·6	—1·5	—4·1	+2·0	+·5	—1·9	—2·8
4 30	—·1	+·4	+1·8	—·2	—2·6	—2·3	+2·0	+2·7	+1·8	+·5	—4·3	—2·6
5 30	+·6	+3·4	—·3	—3·6	—4·8	—1·6	+1·1	+5·2	+·4	—3·0	—6·4	—·9
6 30	+5·1	+2·0	—2·3	—3·6	—3·7	+1·2	+6·5	+2·9	+·6	—6·1	—3·6	+1·5
7 30	+3·9	—2·9	—2·2	—7·4	—1·0	+2·9	—·9	+·1	—2·7	—3·2	—·7	+1·9
8 30	—3·4	—1·5	+1·2	—·4	+1·8	—·2	—1·2	+·9	—·2	+·9	+1·9	+2·2
9 30	—2·2	+3·1	+·3	+1·9	+·9	—·0	—2·9	—1·1	+·8	+1·8	+1·8	·3
10 30	—2·7	+·1	—·6	+2·8	+2·3	+·6	—·5	—1·1	+·9	+3·5	+1·7	+1·2
11 30	+6·0	—1·4	—1·1	+1·6	+·6	+1·7	—·9	—2·4	+·7	+1·1	+1·9	+·2

TABLE (C. T.). Periodical part of the Solar Effect.
The remainders in Table (B. T.) corrected by interpolation.

☾'s Transit.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
h m												
0 30												
1 30	—2	0	+2	+3	+2	0	—3	—4	—2	0	+2	0
2 30	—2	0	+3	+3	+2	0	—3	—2	0	+2	+2	0
3 30	—3	0	+3	+4	+3	0	—4	—4	0	+1	+1	0
4 30												
5 30	+3	0	—3	—4	—3	0	+4	+4	0	—4	—4	0
6 30	+4	0	—4	—5	—4	0	+5	+5	0	—5	—5	0
7 30	+4	0	—4	—5	—4	0	+2	+2	0	—3	—3	0
8 30												
9 30												
10 30												
11 30												

SOLAR TABLE (W. T.). Correction of the Times for the Sun's Effect.

For this correction we may use Table (C. T.), omitting the correction for the numbers there omitted till further investigations have made the Table more correct : and introducing also the interpolated nonperiodical correction, viz.

Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
0	m +1	0	m —1	m —1	m —1	0	m +1	0	m —1	m —1	m —1