

IX. *On the Use of the Barometric Thermometer for the Determination of Relative Heights.* By JAMES R. CHRISTIE, Esq., of the Royal Military Academy. Communicated by S. HUNTER CHRISTIE, Esq., Sec. R.S., &c.

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ALTHOUGH the observation of the temperature of boiling water has been for some time, but not extensively, employed for the determination of relative heights, yet the only means which experiment has confirmed of reducing it to a measure of the atmospheric pressure as usually estimated by the height of an equiponderate column of mercury has, till very recently, been overlooked; and it may perhaps be owing to this circumstance that the instrument for making the requisite observations remains to have fully developed in it the advantages it undoubtedly possesses, in portability and strength of construction, over the fragile and easily deranged barometer.

My attention having been called to this subject by a remark made by Professor FORBES in his interesting work on the Alps, to the effect that he had found the temperature of boiling water to decrease uniformly with the increase in height of the place of observation, and at the rate of one degree of FAHRENHEIT for every 550 feet of vertical ascent, I considered that it would be highly satisfactory to verify this result during an excursion over the Alps of Savoy and Piedmont which I then had in contemplation, and in the course of which I proposed to visit some localities at very considerable elevations above the sea level: and I was induced also to seek for some foundation for this very simple law. In prosecuting the latter inquiry, I soon found that, by assuming the truth of DE LUC's formula for the determination of the boiling-point from the barometric pressure, at all accessible heights, a corroboration of the law in question is at once arrived at. I have since found, by reference to a paper in Vol. xv. of the Transactions of the Royal Society of Edinburgh, that Professor FORBES had himself verified his original conjecture in the same manner.

The formula alluded to, when reduced to the common English units of measure, becomes

$$b = \frac{99}{.899} \cdot \log 10 \beta - 60.804 \quad \text{I.}$$

where b is the variable boiling-point on FAHRENHEIT's scale, and β the corresponding barometric pressure in inches of mercury. From this we obtain at once

$$\log 10 \beta = \frac{.899}{99} (b + 60.804)$$

R 2

mine whether the instrumental graduations could be relied on; and not having had an opportunity of comparing the instrument with any standard barometer in England, I was under the necessity of employing my Geneva observations for that purpose, comparing them with the corresponding indications of the barometer at the Geneva observatory, the height of which I estimated at seventy feet above the spot at which my observations were made. This difference of level will amount to

$$\frac{70}{548} = \cdot 128^{\circ} \text{ FAHR.}$$

to be added to the boiling-point as deduced from the height of the barometer. These corresponding observations were

| <i>Geneva.</i> | | |
|-------------------------|----------------------------|---------------------------|
| Observed boiling-point. | Reduced barometer. | Calculated boiling-point. |
| 208 ^o 820 | ·72129 metre. | 209 ^o 357 |
| 209·115 | ·72345 metre. | 209·500 |
| We have then | 209·357 + ·128 — 208·820 = | ·665 error, |
| and | 209·500 + ·128 — 209·115 = | ·513 error |
| | <hr/> 2)1·178 | |
| | <hr/> Mean error = | —·589 |

The non-accordance of the errors I am inclined to attribute to the latter pair of observations not being simultaneous, mine being made at 6 A.M. and the barometric indication being deduced from that recorded at 9 A.M., which is the earliest.

It was at once evident to me that the above error was not likely to afford a constant correction to be applied to all observations made at different points of the scale, because the lower point of graduation, being necessarily above the freezing-point, must have been determined by reference to a standard thermometer, and it was therefore not at all probable that it would differ from the truth by so large a quantity. An excellent opportunity of proving the truth of this conjecture offered itself at the Convent of the Great St. Bernard, where a register is kept of the variations of the barometric column. Arranging these as before, I obtain the results:—

| <i>Great St. Bernard.</i> | | | |
|---------------------------|--------------------|---------------------------|--------|
| Observed boiling-point. | Reduced barometer. | Calculated boiling-point. | Error. |
| 197 ^o 640 | ·56538 metre. | 197 ^o 709 | —·069 |
| 197·680 | ·56630 metre. | 197·787 | —·107 |

There was a difference of level of ten or twelve feet between the two places of observation, the first having been made in the *salle-à-manger* of the Convent, and the other in one of the dormitories on the next floor where the barometer is fixed. The correction for this change of level is

$$-\frac{12}{548} = -\cdot 022$$

to be added to the error of the first observation, which becomes then $-.091$; and the mean error derived from the two observations is $-.099$.

The instrumental error thus diminishing appears to indicate that there is a point on the scale where it becomes 0, and beyond which, as a correction, it must be applied in a negative form; and this point can readily be determined on the principle that the differences of the errors are proportional to the differences of the corresponding boiling-points; for, calling B the point on the scale at which the error vanishes, b , b' , e and e' two other points and their corresponding errors, we have the equation

$$\frac{b-B}{b-b'} = \frac{e \pm 0}{e-e'};$$

whence

$$B = b - \frac{e(b-b')}{e-e'};$$

and, substituting in this the values above found, viz.

$$b = 208.967, \quad b' = 197.649,$$

$$e = -.589, \quad e' = -.099,$$

we find

$$B = 195.363.$$

In order, by means of this result, to determine the correction to be applied as a multiplier to the number of degrees above or below B, it remains to determine the amount of error due to each degree, which, in fact, when its sign is changed, will be the required factor; calling this correction C, we have

$$C = \frac{-e}{b-B} = \frac{.589}{13.604} = .0433.$$

It may be remarked, that the principle on which the determination of this correction is founded remains unaffected by the supposition that the errors from which it is derived are partly attributable to the diameter of the bore of the thermometer increasing uniformly in descending; this I have satisfied myself by the usual means to be the case, though to a comparatively trifling amount.

The observations recorded at the Geneva Observatory and those at the Great St. Bernard afford an excellent means of applying my own to the determination of the height of my several stations, since those places present the advantage of being, the one my starting-point, and the other a position central to my series of stations. The readiest way of thus applying them appears to be, to reduce the height of the barometric column as given in millimètres to the form of the corresponding temperature of boiling water, by means of DE LUC's formula above given, and then, for the determination of the difference of altitude between each station and Geneva, and the Great St. Bernard, to employ the formula

$$H = \{548(b-b') \pm 24D\} \{1 + (t-32) \cdot 00222\},$$

where D is the difference in degrees between B (=195.4) and the observed boiling-point; and the factor 24 is the product $548 \times .0433$; the sign + being used when my own station is *lowest* and *vice versa*, because the instrumental error is always one of defect.

Thus, taking the observations made on the Bonhomme and the corresponding ones at Geneva and the Great St. Bernard, we have,

| | |
|---|--|
| Barom. Geneva. | Boiling-point. |
| 720·97 mm. gives | 209·335 |
| Obs ^d boilg-p ^t 197·700 | 197·700 |
| <u>195·4</u> | <u>11·635 × 548 = 6376</u> |
| 2·3 × 24 = 55 | 55 |
| | <u>6321 × 1·059 = 6694 above Geneva Obs^y.</u> |
| | 1334 Observ ^y above sea. |

Croix du Bonhomme above the sea 8028 feet.

| | |
|-------------------|--|
| Barom. G.S.B. | Boiling-point. |
| 563·94 mm. gives | 197·587 |
| <u>197·700</u> | |
| ·113 × 548 = 61·9 | |
| 55 | |
| | <u>116·9 × 1·0357 = 121 below Great St. Bernard.</u> |
| | 8170 Gt. St. Bernard above the sea. |

Croix du Bonhomme above the sea 8049 feet.

The following Table shows the simultaneous observations at the two fixed stations and at the variable one, and the results derived from them by the above method.

TABLE I.

| Station. | Observed boiling-point. | Temperature of the air, FAHRENHEIT. | Geneva barometer (reduced to 32° F.) in millimetres. | Geneva external thermometer centigrade. | Great St. Bernard barometer (reduced to 32° F.) in millimetres. | Great St. Bernard external thermometer centigrade. | Calculated height in feet above Geneva observatory. | Calculated depression in feet below the Great St. Bernard. | Sum of the two preceding (6836 true). | Height in feet above the sea by Geneva observation. | Height in feet above the sea by G.S.B. observatory. | Height in feet previously determined. | Authority for the preceding column. | |
|---------------------------|-------------------------|-------------------------------------|--|---|---|--|---|--|---------------------------------------|---|---|---------------------------------------|-------------------------------------|-----------|
| Les Rousses | 204.48 | 57 | 722.01 | 19.4 | 566.66 | 5.9 | 2644 | 4024 | 6668 | 3978 | 4146 | 3919 | WOERL. | |
| Geneva | 208.82 | 71 | 721.29 | 19.5 | 563.98 | 8.2 | (-70) | 6862 | 6792 | | 1308 | 1334 | Bib. Univ. de Genève. | |
| Bonneville | 208.715 | 66 | 722.69 | 21.7 | 564.14 | 6.4 | 90 | 6730 | 6820 | 1424 | 1440 | 1452 | SAUSSURE. | |
| Môle (summit) | 199.855 | 53 | 723.03 | 19.7 | 565.00 | 4.0 | 5519 | 1300 | 6819 | 6853 | 6870 | 6221 | PICOT. | |
| St. Jeoire | 208.40 | 62 | 724.54 | 17.9 | 565.73 | 5.3 | 352 | 6424 | 6776 | 1686 | 1746 | 1778 | KELLER. | |
| Samoens..... | 207.94 | 55.5 | 727.61 | 17.0 | 566.91 | 2.5 | 747 | 6017 | 6764 | 2085 | 2153 | 2257 | WOERL. | |
| Col Dichaud | 198.24 | 52 | 727.73 | 23.9 | 568.46 | 7.3 | 6695 | 224 | 6919 | 8029 | 7946 | | | |
| Servoz | 207.31 | 58 | 729.06 | 15.5 | 569.25 | 4.0 | 1171 | 5571 | 6742 | 2505 | 2599 | | | |
| Chamounix—1..... | 205.70 | 66.5 | 728.50 | 20.3 | 569.44 | 10.0 | 2174 | 4695 | 6869 | 3508 | 3475 | 3364 | SAUSSURE. | |
| Chamounix—2..... | 205.50 | 50.5 | 723.91 | 17.6 | 568.38 | 5.1 | 2076 | 4663 | 6739 | 3410 | 3507 | 3425 | FORBES. | |
| Chamounix—3..... | 205.52 | 68 | 721.40 | 15.4 | 566.11 | 9.2 | 1996 | 4751 | 6747 | 3330 | 3419 | 3408 | WOERL. | |
| Montanvert | 200.27 | 62 | 722.57 | 22.1 | 568.19 | 9.6 | 5286 | 1464 | 6750 | 6620 | 6706 | 6300 | FORBES. | |
| Contamines—1..... | 204.71 | 63 | 722.61 | 19.3 | 565.80 | 5.5 | 2544 | 4223 | 6767 | 3877 | 3947 | 4260 | WOERL. | |
| Contamines—2..... | 204.72 | 62 | 722.41 | 15.4 | 564.33 | 6.5 | 2561 | 4305 | 6866 | 3896 | 3865 | 3259 | KELLER. | |
| Croix du Bonhomme..... | 197.70 | 49 | 720.97 | 19.9 | 563.94 | 8.5 | 6694 | 131 | 6825 | 8028 | 8049 | 8026 | SAUSSURE. | |
| Chapiu—1..... | 202.35 | 54 | 721.67 | 15.5 | 563.55 | 3.4 | 3889 | 2883 | 6772 | 5223 | 5287 | 5071 | SAUSSURE. | |
| Chapiu—2..... | 202.655 | 52 | 724.52 | 17.5 | 563.02 | 1.2 | 3821 | 3069 | 6880 | 5155 | 5101 | 5006 | WOERL. | |
| Bourg St. Maurice..... | 207.00 | 59 | 726.97 | 16.5 | 565.59 | 1.2 | 1295 | 5534 | 6829 | 2629 | 2636 | | | |
| Hospice Pt. St. Bernard | 199.24 | 50 | 724.47 | 22.9 | 566.64 | 9.0 | 5870 | 905 | 6775 | 7203 | 7265 | 7194 | SAUSSURE. | |
| Cormayeur—1..... | 204.75 | 57 | 723.74 | 16.0 | 565.69 | 4.3 | 2441 | 4336 | 6777 | 3775 | 3834 | 3997 | SAUSSURE. | |
| Cormayeur—2..... | 204.60 | 55.5 | 721.98 | 12.8 | 562.32 | 4.2 | 2535 | 4280 | 6815 | 3869 | 3890 | 4167 | FORBES. | |
| Cormayeur—3..... | 204.36 | 54 | 724.81 | 17.3 | 565.61 | 7.3 | 2806 | 3990 | 6796 | 4140 | 4180 | 4046 | WOERL. | |
| Aosta—1..... | 208.59 | 73 | 724.19 | 19.4 | 565.72 | 8.7 | 225 | 6651 | 6876 | 1561 | 1519 | 1938 | SAUSSURE. | |
| Aosta—2..... | 208.46 | 61 | 721.74 | 14.2 | 561.61 | + | 2 | 6581 | 6786 | 1539 | 1589 | | | |
| Aosta—3..... | 208.35 | 68 | 722.23 | 15.9 | 561.26 | + | 1.8 | 6611 | 6904 | 1627 | 1559 | | | |
| Aosta—4..... | 208.59 | 57 | 726.08 | 15.3 | 563.75 | + | 2.1 | 6505 | 6798 | 1627 | 1665 | | | |
| Aosta—5..... | 208.56 | 72 | 726.41 | 20.4 | 567.18 | + | 6.6 | 6542 | 6903 | 1695 | 1628 | | | |
| St. Remy | 202.545 | 52 | 725.55 | 17.4 | 564.82 | + | 0.6 | 3928 | 2913 | 6841 | 5262 | 5257 | SAUSSURE. | |
| Great St. Bernard—1. | 197.64 | 39 | 725.23 | 17.4 | 565.27 | + | 2.7 | 6791 | | 6791 | 8125 | | 8170 Bib. Univ. de Genève. | |
| Great St. Bernard—2. ... | 197.68 | 38 | 726.42 | 17.1 | 566.67 | 6.5 | 6753 | | 6753 | 8087 | | 8038 | SAUSSURE. | |
| St. Vincent—1..... | 209.28 | 63 | 727.96 | 15.9 | 568.84 | 3.6 | — 54 | 6785 | 6731 | 1280 | 1385 | | | |
| St. Vincent—2..... | 209.64 | 55 | 730.25 | 20.0 | 569.04 | 2.8 | — 184 | 6799 | 6615 | 1150 | 1371 | | | |
| Col de Jou | 202.82 | 56 | 730.25 | 21.0 | 569.99 | 5.8 | 3984 | 2871 | 6855 | 5318 | 5299 | | | |
| Brussone | 204.65 | 66.5 | 730.25 | 22.8 | 570.41 | 7.3 | 2904 | 3740 | 6644 | 4238 | 4430 | 4431 | FORBES. | |
| Col Ranzola | 199.45 | 53 | 728.99 | 24.3 | 570.59 | 7.9 | 6012 | 843 | 6855 | 7346 | 7327 | 7136 | FORBES. | |
| Gressonay—1..... | 204.06 | 58.5 | 730.52 | 18.0 | 570.94 | 4.3 | 3224 | 3681 | 6905 | 4558 | 4489 | 4208 | SAUSSURE. | |
| Gressonay—2..... | 204.33 | 58.5 | 730.15 | 18.4 | 570.29 | 6.1 | 3050 | 3766 | 6816 | 4384 | 4404 | 4526 | WOERL. | |
| Gressonay—3..... | 203.92 | 57 | 730.37 | 16.9 | 568.27 | 2.6 | 3294 | 3752 | 7046 | 4628 | 4418 | | | |
| Col Val Dobbia..... | 197.44 | 52 | 730.40 | 19.4 | 568.75 | 2.8 | 7226 | — 262 | 6964 | 8560 | 8432 | 7904 | SAUSSURE. | |
| La Riva—1..... | 205.54 | 61 | 729.94 | 15.8 | 569.74 | 2.5 | 2299 | 4494 | 6793 | 3633 | 3676 | 3568 | SAUSSURE. | |
| La Riva—2..... | 205.55 | 55 | 727.70 | 18.0 | 569.40 | 7.0 | 2200 | 4527 | 6727 | 3534 | 3643 | 3636 | WOERL. | |
| Varallo—1..... | 209.36 | 67 | 728.18 | 20.0 | 570.86 | 7.4 | — 96 | 6814 | 6718 | 1238 | 1356 | | | |
| Varallo—2..... | 209.26 | 63 | 727.82 | 20.0 | 570.52 | 6.2 | — 47 | 6727 | 6680 | 1287 | 1443 | 1491 | WOERL. | |
| La Colma | 206.72 | 70 | 728.20 | 27.1 | 571.67 | 13.8 | 1560 | 5263 | 6823 | 2894 | 2907 | | | |
| Orta—1..... | 210.70 | 74 | 729.11 | 21.5 | 572.34 | 8.5 | — 895 | 7621 | 6726 | 439 | 549 | 1108 | WOERL. | |
| Orta—2..... | 210.83 | 70 | 729.79 | 22.9 | 571.80 | 11.2 | — 921 | 7628 | 6707 | 387 | 518 | | | |
| Orta—3..... | 210.35 | 66 | 728.80 | 18.0 | 569.67 | 5.6 | — 681 | 7452 | 6771 | 653 | 718 | | | |
| Mogadino—1..... | 211.04 | 77 | 727.80 | 19.4 | 568.60 | 5.2 | — 1146 | 7985 | 6839 | 188 | 185 | 678 | SAUSSURE. | |
| Mogadino—2..... | 211.20 | 68 | 724.55 | 18.0 | 566.74 | 6.0 | — 1368 | 8110 | 6742 | — 34 | + | 60 | | |
| Domo d'Ossola—1. | 210.68 | 78 | 723.38 | 17.4 | 565.64 | 5.9 | — 1104 | 7933 | 6829 | 230 | 237 | 1004 | SAUSSURE. | |
| Domo d'Ossola—2. | 210.44 | 66 | 723.27 | 16.0 | 563.89 | 4.5 | — 943 | 7751 | 6808 | 391 | 419 | | | |
| Simplon Village..... | 203.28 | 57 | 724.03 | 17.0 | 564.12 | 7.4 | 3440 | 3441 | 6881 | 4729 | 4774 | 4792 | WOERL. | |
| Simplon Hospice | 201.07 | 48 | 724.00 | 19.0 | 564.13 | 5.3 | 4753 | 2084 | 6837 | 6087 | 6086 | 6580 | SAUSSURE. | |
| Brieg—1..... | 208.69 | 61 | 726.01 | 11.4 | 564.45 | + | 1.6 | 229 | 6606 | 6835 | 1563 | 1564 | 2328 | SAUSSURE. |
| Brieg—2..... | 208.35 | 56 | 729.28 | 15.0 | 566.00 | — 1.3 | 552 | 6194 | 6746 | 1886 | 1976 | 2183 | WOERL. | |
| Leukerbad—1..... | 204.38 | 47.5 | 733.42 | 14.6 | 570.43 | — 1.2 | 3104 | 3673 | 6777 | 4438 | 4497 | 4699 | WOERL. | |
| Leukerbad—2..... | 204.545 | 45 | 734.75 | 13.7 | 571.54 | + | 2.0 | 3025 | 3749 | 6774 | 4359 | 4421 | | |
| S. Crest of the Gemmi ... | 199.235 | 41 | 734.44 | 19.7 | 572.06 | 4.7 | 6235 | 620 | 6855 | 7569 | 7550 | 7626 | WOERL. | |
| Kandersteg—1..... | 206.34 | 55.5 | 733.06 | 16.3 | 572.45 | 3.0 | 1922 | 4816 | 6738 | 3256 | 3354 | 3493 | WOERL. | |
| Kandersteg—2..... | 206.17 | 48 | 732.09 | 15.8 | 571.74 | 9.3 | 1972 | 4765 | 6737 | 3306 | 3405 | | | |
| Lauterbrunnen—1. | 208.28 | 55 | 728.81 | 16.6 | 570.66 | 8.8 | 585 | 6112 | 6697 | 1918 | 1958 | | | |
| Lauterbrunnen—2..... | 208.37 | 49 | 727.68 | 17.6 | 569.18 | 10.3 | 484 | 6215 | 6699 | 1818 | 1955 | | | |
| Bern | 209.26 | 72 | 726.97 | 22.4 | 569.91 | 11.0 | — 82 | 6883 | 6801 | 1252 | 1287 | 1661 | WOERL. | |

The discrepancies in many of the above results as compared with the column of previously determined heights are considerable, but it is to be observed that in most cases where the height of the station may be considered as particularly well-established, the observed height accords with it in a very remarkable manner. This is particularly the case at Geneva (where the height is of course deduced only from that of the Great St. Bernard), at Bonneville, at Chamounix (taking a mean of the six observations, which gives 3442), at the Croix du Bonhomme*, at St. Remy, at the Great St. Bernard and at Brusson: the chief exceptions being at the Môle†, the Montanvert, the Col di Val Dobbia, Orta, Mogadino, Domo d'Ossola, the Simplon Hospice and Brieg, the great differences at which places are to me quite unaccountable, unless on the supposition, in the case of the five last-named, that their distances from the fixed stations were too great to allow of trustworthy results being obtained from single observations. In confirmation of this view, it will be seen presently that these observations give far more satisfactory results when employed without any reference to the simultaneous observations at Geneva and the Great St. Bernard; and also in the above Table we may notice that, after crossing the Bonhomme, in those cases where the heights determined from the two fixed stations differ from one another to any considerable amount, that obtained by reference to the Great St. Bernard generally approximates more nearly to the height as previously determined than the other, obtained by a comparison with the more distant station.

The numbers in the tenth column which are the sums of the two values, the height above Geneva and the depression below the Great St. Bernard, as determined by observation, should, supposing both theory and observations to be perfect, give the constant value 6836, which is the difference of height of those two stations; but this difference will vary when determined by means of each pair of barometric observations, the standard value 6836 being the mean of a very considerable number. Now it is very evident that were it not for that term in the formula which involves the correction for the expansion of the air, the sum of the above values would give the difference of altitude, however erroneous the intermediate observation might be: for calling b' , b'' and b''' the three boiling-points, we should have

$$548(b' - b'') - 24D + 548(b'' - b''') + 24D = 548(b' - b''').$$

The observation, therefore, of the external temperature at the intermediate station becomes the means of rendering the accordance of the sums of the component altitudes with the whole height between Geneva and the Great St. Bernard, as determined in each case, a test of the correctness of the observation of the boiling temperature. We may in fact, by equating the formula for the whole height with the sum

* All who have traversed the Pass of the Bonhomme are aware that in crossing to the south, upon Chapiu, the tract rises considerably after passing what ought to be and, I believe, is considered the "*Col*," viz. the point from which a view of the mountains of the Tarentaise is first obtained; this point is marked by a cross, larger than usual on most passes, and than any of the numerous others on this pass. This cross I believe to be the "*Croix du Bonhomme*" of DE SAUSSURE, and it was at its base the observation above referred to was made.

† See note, p. 131.

of those expressing the two component heights, arrive at an expression for the intermediate boiling-point in terms of the atmospheric temperatures at the three stations and the upper and lower boiling-points; such an expression is

$$b'' = \frac{23}{24} \cdot \left\{ \frac{t-T'}{T-T'} \cdot b' + \frac{T-t}{T-T'} b''' + 8.4 \right\}$$

where T , t and T' are the temperatures at the stations where the boiling-points are respectively b' , b'' and b''' ; but this, when put in the form

$$b'' = \frac{23}{24} \cdot \left\{ \frac{t \cdot (b' - b''') + T b''' - T' b'}{T - T'} + 8.4 \right\},$$

shows us that an error in the value of t , of trifling moment where that value is to be employed in the determination of the difference of level, will cause an error in the resulting boiling temperature of very serious amount when used for the same purpose. Though this test cannot therefore be made use of in its full rigour, it will be highly satisfactory if an accordance can be shown between the results above referred to: the following, Table II., exhibits a comparison of these values. The numbers in the second column are the differences of the boiling temperatures, in degrees of FAHRENHEIT, deduced from the barometric observations at Geneva and the Great St. Bernard, which were simultaneous with those of my own made at the places named in the first column.

TABLE II.

| Station. | Difference of boiling-points derived from the observations at Geneva and the Great St. Bernard. | Value of $(t-32^\circ)$ in formula II. | Derived difference of height. | Difference of height from Table I. | Station. | Difference of boiling-points derived from the observations at Geneva and the Great St. Bernard. | Value of $(t-32^\circ)$ in formula II. | Derived difference of height. | Difference of height from Table I. |
|-------------------------------|---|--|-------------------------------|------------------------------------|------------------------------|---|--|-------------------------------|------------------------------------|
| Les Rousses | 11.583 | 22.8 | 6669 | 6668 | Col de Jou | 11.860 | 24.1 | 6847 | 6855 |
| Geneva | | | | 6792 | Brussone | 11.372 | 27.0 | 6605 | 6644 |
| Bonneville | 11.856 | 25.3 | 6862 | 6820 | Col Ranzola | 11.716 | 29.0 | 6833 | 6855 |
| Môle | 11.794 | 21.4 | 6768 | 6810 | Gressonay—1. | 11.789 | 20.0 | 6719 | 6905 |
| St. Jeoire | 11.832 | 20.9 | 6784 | 6776 | Gressonay—2. | 11.818 | 22.0 | 6763 | 6816 |
| Samoens..... | 11.936 | 17.6 | 6796 | 6764 | Gressonay—3. | 12.003 | 17.5 | 6833 | 7046 |
| Col Dichaud | 11.813 | 28.1 | 6877 | 6919 | Col di Val Dobbia..... | 11.964 | 20.0 | 6847 | 6964 |
| Servoz | 11.834 | 12.6 | 6666 | 6742 | La Riva—1. | 11.851 | 16.5 | 6735 | 6793 |
| Chamounix—1. | 11.781 | 27.3 | 6846 | 6869 | La Riva—2. | 11.729 | 22.5 | 6749 | 6727 |
| Chamounix—2. | 11.568 | 20.5 | 6627 | 6739 | Varallo—1. | 11.640 | 24.7 | 6728 | 6718 |
| Chamounix—3. | 11.593 | 22.2 | 6664 | 6747 | Varallo—2. | 11.646 | 23.6 | 6716 | 6680 |
| Montanvert | 11.491 | 28.6 | 6696 | 6750 | La Colma | 11.574 | 26.8 | 6860 | 6823 |
| Contamines—1. | 11.699 | 22.3 | 6728 | 6767 | Orta—1..... | 11.578 | 27.0 | 6725 | 6726 |
| Contamines—2. | 11.811 | 19.7 | 6755 | 6866 | Orta—2..... | 11.661 | 25.7 | 6758 | 6707 |
| Croix du Bonhomme..... | 11.748 | 25.6 | 6803 | 6825 | Orta—3..... | 11.782 | 21.3 | 6761 | 6771 |
| Chapiu—1..... | 11.828 | 17.0 | 6726 | 6772 | Mogadino—1. | 11.805 | 22.1 | 6786 | 6839 |
| Chapiu—2..... | 12.061 | 16.9 | 6857 | 6880 | Mogadino—2. | 11.748 | 21.6 | 6746 | 6742 |
| Bourg St. Maurice | 12.005 | 16.0 | 6811 | 6829 | Domo d'Ossola—1. | 11.764 | 20.9 | 6746 | 6829 |
| Hospice Pt. St. Bernard | 11.642 | 28.7 | 6786 | 6775 | Domo d'Ossola—2. | 11.865 | 18.5 | 6762 | 6808 |
| Cormayeur—1. | 11.748 | 25.6 | 6803 | 6777 | Simplon Village..... | 11.936 | 22.0 | 6859 | 6881 |
| Cormayeur—2. | 11.953 | 15.4 | 6775 | 6815 | Simplon Hospice | 11.933 | 20.9 | 6842 | 6837 |
| Cormayeur—3. | 11.861 | 27.1 | 6852 | 6796 | Brieg—1. | 12.038 | 11.7 | 6768 | 6835 |
| Aosta—1..... | 11.809 | 25.3 | 6835 | 6876 | Brieg—2. | 12.121 | 12.4 | 6826 | 6746 |
| Aosta—2. | 11.997 | 13.0 | 6785 | 6786 | Leukerbad—1. | 12.020 | 12.1 | 6763 | 6777 |
| Aosta—3..... | 12.060 | 15.9 | 6842 | 6904 | Leukerbad—2. | 12.014 | 14.2 | 6790 | 6774 |
| Aosta—4..... | 12.102 | 13.2 | 6806 | 6798 | South Crest of the Gemmi ... | 11.950 | 22.0 | 6868 | 6855 |
| Aosta—5..... | 11.895 | 24.3 | 6835 | 6903 | Kandersteg—1..... | 11.828 | 17.4 | 6731 | 6738 |
| St. Remy | 11.976 | 16.2 | 6798 | 6841 | Kandersteg—2..... | 11.823 | 22.6 | 6804 | 6737 |
| Great St. Bernard—1. | 11.911 | 18.1 | 6790 | 6791 | Lauterbrunnen—1. | 11.700 | 22.9 | 6737 | 6697 |
| Great St. Bernard—2. | 11.873 | 21.3 | 6816 | 6753 | Lauterbrunnen—2. | 11.749 | 25.1 | 6797 | 6699 |
| St. Vincent—1..... | 11.795 | 16.5 | 6715 | 6731 | Bern | 11.641 | 30.1 | 6805 | 6801 |
| St. Vincent—2..... | 11.930 | 20.5 | 6835 | 6615 | | | | | |

It is to be observed, that the differences of height determined by the two methods do not differ from one another in any single case by so large a quantity as do the greatest and least difference of level in the fourth column, determined from the barometric observations at the two places; while in many the accordance is almost perfect. One of the most satisfactory results in this respect is, however, that derived from the observation No. 2. at Mogadino, which appears from Table I. to be one of those least likely to be correct, since it places the surface of the Lago Maggiore much below its possible level. This alone sufficiently shows that the accordance above remarked can only be taken as a general indication of the accuracy of the instrument.

Having thus shown that the instrument is perfectly capable of furnishing correct data for the determination of difference of level, though apparently its indications cannot in all cases be relied on, it becomes further interesting to inquire how far it may be depended on when made use of without reference to corresponding observations at any fixed station. It is obvious that the accuracy of a series of heights thus obtained must, to a certain extent, be vitiated by the diurnal changes in the atmospheric pressure, but it will be seen that this source of error does not operate to the extent that might, *à priori*, be supposed probable, and that, whether from a fortuitous balancing of errors, or from some other cause, the discrepancies in those cases where the previously determined height is probably most near the truth, are less considerable than might be expected. In order thus to obtain the results given in the following Table III., it became necessary again to modify the standard formula on account of the instrumental error. By multiplying the correction, $C = .0433$, by 548 we obtain it in the form of a difference of altitude for each degree of observed difference of boiling temperature at two successive places of observation, to be added to the factor 548 to form the constant multiplier for this particular instrument. We thus obtain

$$548 \times 1.0433 = 571.72,$$

or, with sufficient accuracy, 572.

The working formula then becomes

$$H = 572(b - b') \cdot \{1 + (t - 32^\circ) \cdot 00222\}.$$

In the following Table, in which the results have been calculated with this formula, the differences of level are determined from station to station in the order in which the observations were made, and, in case of two or more observations having been made at the same station, those most approximate in time have been employed. Taking the station at Geneva as a starting-point, and assuming its level, derived from that of the surface of the lake given in the Bibliothèque Universelle de Genève, to be 1264 English feet above the sea level, the calculated differences have been added to this in succession, and the altitudes by observation obtained.

TABLE III.

| Station. | Time of observation, 1844. | External tempera- ture of the air. | Observed boiling- point. | Dif. of alti- tude in ft. | Alti- tude in feet. | Altitude as previ- ously de- termined. | Authority for the altitudes in the prece- ding column. |
|-------------------------------|-------------------------------|---|--------------------------------|------------------------------------|------------------------------|---|--|
| Les Rousses (Jura) | June 25. 5 30 A.M. | 57 | 204.48 | 2767 | 4031 | 3919 | WOERL. |
| Geneva—1. | 25. 0 30 P.M. | 71 | 208.82 | 2767 | | 1264 | Bib. Univ. de Genève. |
| Geneva—2. | 26. 6 15 A.M. | 62 | 209.115 | | | | |
| Bonneville | 26. 11 45 A.M. | 66 | 208.715 | 247 | 1511 | 1452 | SAUSSURE. |
| Môle | 26. 6 15 P.M. | 53 | 199.855 | 5388 | 6899 | { 6094 6221 | SAUSSURE. PICTET. |
| St. Jeoire | 27. 8 0 A.M. | 62 | 208.40 | 5166 | 1733 | 1893 | DE LUC. |
| Samoens. | 28. 5 30 A.M. | 55.5 | 207.94 | 278 | 2011 | 2270 | DE LUC. |
| Col Dichaud (Buet) | 28. 3 45 P.M. | 52 | 198.24 | 5816 | 7827 | | |
| Servoz | 29. 7 30 A.M. | 58 | 207.31 | 5453 | 2374 | | |
| Chamounix—1. | 29. 1 45 P.M. | 66.5 | 205.70 | 983 | 3357 | 3364 | SAUSSURE. |
| Chamounix—2. | 30. 4 30 A.M. | 50.5 | 205.50 | | | 3425 | FORBES. |
| Montanvert | 30. 0 15 P.M. | 62 | 200.27 | 3152 | 6509 | { 6101 6300 | SAUSSURE. FORBES. |
| Chamounix—3. | 30. 4 20 P.M. | 68 | 205.52 | 3223 | 3286 | 3356 | PICTET. |
| Contamines | July 1. 7 15 P.M. | 63 | 204.71 | 500 | 3786 | 3328 | KELLER. |
| | 2. 9 15 A.M. | 62 | 204.72 | | | 4260 | WOERL. |
| Croix du Bonhomme. | 2. 3 30 P.M. | 49 | 197.70 | 4269 | 8055 | 8026 | SAUSSURE. |
| Chapiu—1. | 2. 7 30 P.M. | 54 | 202.35 | 2775 | 5280 | 5071 | SAUSSURE. |
| Chapiu—2. | 3. 9 20 A.M. | 52 | 202.655 | | | 4974 | PICTET. |
| Bourg St. Maurice | 3. 8 0 P.M. | 59 | 207.00 | 2566 | 2714 | | |
| Petit St. Bernard | 4. 11 45 A.M. | 50 | 199.24 | 4660 | 7374 | 7194 | SAUSSURE. |
| Cormayeur—1. | 4. 9 30 P.M. | 57 | 204.75 | 3302 | 4072 | 3997 | SAUSSURE. |
| Cormayeur—2. | 7. 7 0 A.M. | 54 | 204.36 | | | 4167 | FORBES. |
| Aosta—1. | 7. 1 45 P.M. | 73 | 208.59 | 2589 | 1483 | 1938 | SAUSSURE. |
| Aosta—2. | 9. 5 30 A.M. | 57 | 208.59 | | | 1980 | WOERL. |
| St. Remy | 9. 11 45 A.M. | 52 | 202.545 | 3631 | 5114 | 5263 | SAUSSURE. |
| Grand St. Bernard—1. | 9. 3 30 P.M. | 39 | 197.640 | 2891 | 8003 | 8038 | SAUSSURE. |
| Grand St. Bernard—2. | 10. 6 30 A.M. | 38 | 197.680 | | | 8170 | Bib. Univ. de Genève. |
| Aosta—3. | 10. 0 30 P.M. | 72 | 208.560 | 6541 | 1462 | | |
| St. Vincent—1. | 10. 9 0 P.M. | 63 | 209.280 | 444 | 1018 | | |
| St. Vincent—2. | 11. 5 30 A.M. | 55 | 209.64 | | | | |
| Col de Jou | 11. 10 30 A.M. | 56 | 202.82 | 4148 | 5166 | | |
| Brussone. | 11. 0 0 noon. | 66.5 | 204.65 | 1109 | 4057 | 4431 | FORBES. |
| Col de Ranzola | 11. 5 30 P.M. | 53 | 199.45 | 3157 | 7214 | 7136 | FORBES. |
| Gressonay—1. | 11. 7 45 P.M. | 58.5 | 204.06 | 2776 | 4438 | 4208 | SAUSSURE. |
| Gressonay—2. | 13. 8 15 A.M. | 57 | 203.92 | | | 4526 | WOERL. |
| Col di Val Dobbia. | 13. 1 0 P.M. | 52 | 197.44 | 3892 | 8330 | 7904 | SAUSSURE. |
| La Riva—1. | 13. 7 30 P.M. | 61 | 205.54 | 4885 | 3445 | 3568 | SAUSSURE. |
| La Riva—2. | 14. 6 0 A.M. | 55 | 205.55 | | | 3712 | WOERL. |
| Varallo—1. | 14. 10 0 P.M. | 67 | 209.36 | 2319 | 1126 | 1491 | WOERL. |
| Varallo—2. | 15. 6 45 A.M. | 63 | 209.26 | | | | |
| La Colma | 15. 2 15 P.M. | 70 | 206.72 | 1566 | 2692 | | |
| Orta—1. | 15. 6 45 P.M. | 74 | 209.70 | 1856 | 836 | 1108 | WOERL. |
| Orta—2. | 17. 4 30 A.M. | 66 | 210.35 | | | | |
| Mogadino—1. | 17. 6 30 P.M. | 77 | 211.04 | 430 | 406 | 678 | SAUSSURE. |
| Mogadino—2. | 18. 6 0 A.M. | 68 | 211.20 | | | | |
| Domo d'Ossola—1. | 18. 7 0 P.M. | 78 | 210.68 | 324 | 735 | 1004 | SAUSSURE. |
| Domo d'Ossola—2. | 19. 4 30 A.M. | 66 | 210.44 | | | | |
| Simplon (Village) | 19. 0 30 P.M. | 57 | 203.28 | 4368 | 5098 | 4792 | WOERL. |
| Simplon Hospice | 19. 4 50 P.M. | 48 | 201.07 | 1323 | 6421 | 6580 | SAUSSURE. |
| Brieg—1. | 19. 10 30 P.M. | 61 | 208.69 | 4581 | 1840 | 2328 | SAUSSURE. |
| Brieg—2. | 20. 7 15 A.M. | 56 | 208.35 | | | 2183 | WOERL. |
| Leukerbad—1. | 20. 8 0 P.M. | 47.5 | 204.38 | 2371 | 4211 | 4697 | WOERL. |
| Leukerbad—2. | 21. 7 0 A.M. | 45 | 204.545 | | | | |
| Gemmi (châlet on south crest) | 21. 0 30 P.M. | 41 | 199.235 | 3112 | 7323 | 7626 | WOERL. |
| Kandersteg—1. | 21. 6 30 P.M. | 55.5 | 206.34 | 4208 | 3115 | 3493 | WOERL. |
| Kandersteg—2. | 22. 6 45 A.M. | 48 | 206.17 | | | | |
| Lauterbrunnen—1. | 22. 9 0 P.M. | 55 | 208.28 | 1251 | 1864 | | |
| Lauterbrunnen—2. | 23. 6 45 A.M. | 49 | 208.37 | | | | |
| Bern | 24. 10 20 A.M. | 72 | 209.26 | 536 | 1328 | 1661 | WOERL. |

Though the discrepancies in these results, when compared with the previously determined altitudes, appear very considerable, they are probably in most cases not more so than would have resulted from the employment of the usual form of barometer. I am led to this conclusion from the inspection of DE LUC's series of barometric measurements, wherein the determinations of the height of the same station at different times frequently differ from one another 250 feet, and, in one case, as much as 360 feet. The greatest error in the above table, viz. that at Aosta*, 400 feet, certainly exceeds either of these, but there is, I think, good reason to suppose that the observed height at Aosta is not so erroneous as the comparison with DE SAUSSURE's result makes it appear to be, when we remark that the height of the Great St. Bernard determined from it is one of the most accurate of the series, and also that the observations made *the next day* at the Great St. Bernard and at Aosta give the same difference of level, within twenty feet.

The only other instance in which I was enabled to repeat an observation after the ascent of any considerable height occurred at Chamounix, and in it a tolerably close accordance in the observations before and after the ascent of the Montanvert is also to be remarked, though the height derived from the intermediate observation differs very considerably even from that given by PROFESSOR FORBES, which is however equally in excess above that by DE SAUSSURE.

Since the calculation of the above results I have had an opportunity of comparing the instrument with a standard barometer, and have been surprised to find the instrumental correction, under the pressure 30 inches, very different from that obtained above from the Geneva observations. I cannot assign any reason for supposing an error in making these observations; indeed, every consideration induces me to believe that no such error could have occurred, but at the same time I find it very difficult to account for this change in the correction, unless it be partly attributable to a difference in construction of the two barometers and partly to a difference in the degree of purity of the water, or to a change in the instrument itself; all of which suppositions are highly improbable. To whichever of the above causes this remarkable change is referred, I do not think it should be considered as vitiating the results derived from the recorded observations, since they have been connected with those made with the barometers by means of which the correction made use of has been determined; and moreover the time elapsed between the first and last recorded observations was only one month, and the sources from which the water was derived were in all cases so far similar as apparently to ensure its chemical identity with respect to boiling temperature.

The utility of an instrument to be applied to purposes which involve the necessity

* The circumstances of danger under which the observation on the Môle was made, make me think that an erroneous reading is not unlikely to have occurred there, and I accordingly exclude it from consideration in these remarks.

of its removal from place to place under difficult circumstances, is not *exclusively* to be measured by the degree of delicacy or even of accuracy of its indications; since however perfect it may be theoretically, when stationary, it can never be of practical benefit unless it be of such a construction as to bear the concussions and shaking it must be necessarily exposed to when conveyed in the manner in which it can alone arrive at the point where its agency is required. This construction has been attained in the instruments by means of which are determined the two principal coordinates of geographical position, under all circumstances involving difficulty of access to the place of observation, while the third, *height above the sea level*, has had almost exclusively applied to its determination an instrument more liable than almost any other to suffer during removal, and, from its very nature, incapable of any great improvement in this respect. With regard to the instrument under consideration, I can, in reference to the above points, speak most positively; not only were the concussions which it constantly suffered very considerable, but on two or three occasions it escaped uninjured by blows which must have shivered any common barometer, however externally protected; and it has only at length been destroyed through having been inadvertently left boiling;—the water having evaporated, the increase of temperature of the empty boiler proved too great for the capacity of the tube of the thermometer, and the bulb burst.

Since writing the above my attention has been directed to the researches of HOLTZMANN and MAGNUS, given in Vol. iv. Part 14. of the “Scientific Memoirs,” and I have applied the formula for the expansive force of steam deduced by the former, to the determination of an expression for the difference of level in terms of the difference of boiling temperature. Reduced to English measures, HOLTZMANN’s result gives

$$b - 32^\circ = 425 \cdot 2 \times \frac{\log \beta + \cdot 74883}{6 \cdot 73157 - \log \beta}, \quad \dots \dots \dots \text{III.}$$

b and β being the same as in (I.).

Applying this as before, in the case of DE LUC’s, I obtain

$$H = \frac{191939000}{(393 \cdot 2 + b)(393 \cdot 2 + b')} \cdot (b - b') \{1 + (t - 32^\circ) \cdot 00222\} \quad \dots \dots \text{IV.}$$

The results derived from (III.), within the limits of my observations, differ but little from those obtained from (I.), but have a tendency to diminish the boiling temperature more rapidly as the pressure decreases, as the following results sufficiently show:—

| Pressure. | Temperature. | | Diff. |
|------------|--------------|------------|-------|
| | DE LUC. | HOLTZMANN. | |
| 721·74 mm. | 209·386 | 209·425 | +·039 |
| 561·61 mm. | 197·389 | 197·237 | —·152 |

Even were this last difference greater, we must still rely upon DE LUC’s until it can be shown that the results obtained with (III.) are more nearly in accordance with observations made under natural atmospheric pressures than those derived from (I.) have been found to be.

The practical disadvantages of (IV.) are evident.