

V. *On the Air-Engine.* By JAMES PRESCOTT JOULE, F.R.S., F.C.S., *Corr. Mem.*
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IT has long been suspected that important advantages might be derived from the substitution of air for steam as a prime mover of machinery. It has been alleged that the air-engine would be safer, lighter, and more economical in the expenditure of fuel than the steam-engine. Until comparatively recent times, however, experimental science was hardly in the state of advancement requisite to enable the physicist, in his investigation of this important subject, to arrive at conclusions sufficiently certain to give confidence to the practical machinist. Professor THOMSON, Mr. RANKINE, and M. CLAUSIUS have of late, however, published papers of great value on the mechanical action of gases, and particularly of steam, founded on tolerably correct experimental data. I hope that the following remarks founded on the same general principles, but applied to a particular kind of air-engine, may be interesting to the Royal Society.

The air-engine, the performance of which I propose to discuss, consists of two parts, in one of which the air is compressed into a receiver, where its elasticity is increased by the application of heat, and in the other it is allowed to escape again from the receiver into the atmosphere. By the former work is absorbed, by the latter it is evolved in a larger quantity, the excess constituting the work evolved by the engine on the whole. The simple question, therefore, is to determine the quantity of work so evolved, together with the heat applied to increase the elasticity of the air in the receiver.

In Plate VI. fig. 1 let A be the pump by which air is forced into the receiver C, where heat may be communicated to it from an external source, and B the cylinder, by which the same quantity is allowed to escape again into the atmosphere. Moreover, let the material of which the apparatus is made, with the exception of that part through which heat may be communicated to the air in C, be impervious to, and destitute of capacity for heat. Such a machine may be conceived to work in the following manner.

The cylinder of the pump A being filled with air of the atmospheric temperature and pressure, the piston compresses the air until, at a point *n*, its pressure is rendered equal to that of the air in the receiver C, which has been previously filled with air of an elevated temperature and pressure. The work absorbed by this action will be that communicated to the air in the cylinder, minus the work due to the atmospheric

MDCCCLII.

K

pressure through $m n$. The moment the piston has passed the point n the valve will open, admitting the air into the receiver C; and as this receiver may be conceived to be of indefinite magnitude, the alteration of pressure in it, consequent upon the introduction of fresh air, may be neglected. Heat is then communicated to the air in the receiver, in order to restore its temperature to the intensity which existed before the admission of air at a lower temperature. The air is then allowed to escape from the receiver into the base of the cylinder B, evolving work until, on the arrival of the piston at n' , the same quantity has been removed from the receiver as was forced into it by the pump. The further supply of air from the receiver is then cut off, and that which has entered the cylinder expands, evolving work until, on the arrival of the piston at m' , its pressure is reduced to that of the atmosphere. By opening valves at the bases of A and B, the pistons are then brought to their first positions.

The problem which must be solved in order to estimate the power and consumption of fuel in an engine similar to that just described, is as follows:—To determine the pressure and temperature for any point of the stroke of a piston which compresses a given volume of air, and the quantity of work absorbed in forcing the piston to that point. For the temperature and pressure Poisson has furnished the following formulæ,—

$$\frac{T'}{T} = \left(\frac{V}{V'}\right)^{k-1},$$

and

$$\frac{P'}{P} = \left(\frac{V}{V'}\right)^k,$$

where T , P , and V are the temperature from absolute zero (estimated at 491°FAHR. below the freezing-point of water), pressure, and volume of the air before compression; T' , P' , and V' the temperature from absolute zero, pressure, and volume of air after compression; and k is the ratio of the specific heat of air at constant pressure to that at constant volume. Professor W. THOMSON has deduced, as a consequence of the above, the following formula for the work absorbed,

$$W = PV \frac{1}{k-1} \left\{ \left(\frac{V}{V'}\right)^{k-1} - 1 \right\}^*.$$

From the foregoing formulæ I have calculated the work absorbed by compressing air in a cylinder 1 foot long, and of the capacity of 12 cubic inches, the absolute temperature of the air, and its pressure at each tenth of an inch of the piston's progress. The following data were employed in the computation:—Weight of 100 cubic inches of atmospheric air of 15 lbs. pressure on the square inch, and 491°

* The above formula was kindly communicated to the author by Professor THOMSON, in a letter dated January 15, 1851, from which the following is an extract:—"It is required to find the work necessary to compress a given mass of air to a given fraction of its volume, when no heat is permitted to leave the air. Let P , V , T be the primitive pressure, volume, and temperature, respectively; let p , v , and t be the pressure, volume, and temperature at any instant during the compression; and let P' , V' , and T' be what they become

FAHR. from the absolute zero, 33·2237 grs.; specific heat of air at constant volume, 0·19742. Ratio of the specific heat of air at constant pressure to that at constant volume, as determined from the experiments of DELAROCHE and BERARD, and the mechanical equivalent of heat, 1·3519325*. The results are shown in Table I.

I now proceed to give some estimates of the performance of an air-engine similar in principle to that already described, worked at various pressures and temperatures, those of the atmospheric air being 15 lbs. on the square inch, and 32° FAHR. or 491° FAHR. from the absolute zero. In order to render the results easily available in calculating the duty of engines of greater size, I shall assume that the condensing pump is 12 inches long, and has a sectional area equal to 1 square inch, and that the cylinder, also of 1 inch section, has a length which may be made to vary according to the pressure and temperature employed.

I take as the first example, a case in which the receiver C contains air of the atmospheric density, and of which the absolute temperature is 849°·464 FAHR. or 390°·464 of the scale of FAHRENHEIT's thermometer. The pressure in the receiver will then be 25·95104 lbs. on the square inch, as given in the third column of Table II. The air in the pump A will be brought to the same pressure, and to the absolute temperature 566°·3094 after the piston has traversed 4 inches. The work absorbed by the air will be 6·537154 foot-pounds, from which, by subtracting 5 foot-pounds, the work communicated by the pressure of the atmosphere following the piston, we obtain 1·537154 foot-pounds as the work of the engine absorbed by the first part of the stroke. This result is consigned to column 6. Immediately after the piston has passed the fourth inch of the pump, the valve will be opened admitting the compressed air into the receiver C. The work of the engine absorbed by the re-

when the compression is concluded. Then if k denote the ratio of the specific heat of air at constant pressure to the specific heat of air kept in a space of constant volume, and if, as appears to be nearly, if not rigorously true, k be constant for varying temperatures and pressures, we shall have by the investigation in MILLER's 'Hydrostatics' (Edit. 1835, p. 22)—

$$\frac{1 + Et}{1 + ET} = \left(\frac{V}{v}\right)^{k-1}.$$

But

$$\frac{pv}{PV} = \frac{1 + Et}{1 + ET},$$

therefore

$$pv = PV \left(\frac{V}{v}\right)^{k-1}.$$

Now the work done in compressing the mass from volume v to volume $v - dv$ will be $p dv$, or by what precedes,

$$PV \cdot V^{k-1} \frac{dv}{v^k}.$$

Hence by the integral calculus we readily find, for the work, W , necessary to compress from V to V' ,

$$W = PV \cdot \frac{1}{k-1} \left\{ \left(\frac{V}{V'}\right)^{k-1} - 1 \right\}."$$

* The experiments of DESORMES and CLEMENT give 1·354; those of GAY-LUSSAC and WELTER 1·375; and those described under the article 'Hygrometry' (Enc. Brit.), 1·333. See Art. 'Sound,' Enc. Brit., 7th Edit.

maintaining 8 inches of the piston's stroke will be $\frac{8}{12} (25.95104 - 15) = 7.300693$ foot-pounds, as given in the seventh column. The air thus forced into the receiver at the absolute temperature $566^{\circ}.3094$ FAHR. must then be raised to $849^{\circ}.464$ FAHR., the constant absolute temperature of the receiver. The heat necessary for this purpose, being that due to the capacity for heat of air at constant pressure, will be that which is able to raise the temperature of 1 lb. of water $0^{\circ}.04304312$ FAHR., as given in column 15. On leaving the receiver, the air enters the cylinder of expansion B, and having propelled the piston through 12 inches, the same quantity of air will have passed out of the receiver as was pumped into it by A. The further supply of air is then cut off, and the air after expanding through the remaining 6 inches of the cylinder (which in this case must be 18 inches long), will be reduced to the pressure of 15 lbs. on the square inch, and the absolute temperature $\frac{3}{2} (491^{\circ}) = 736^{\circ}.5$.

The work evolved by the piston will also be to that absorbed in the condensing pump, as the volume of the cylinder B is to that of the pump A; from which we find $\frac{3}{2} (7.300693) = 10.95104$ foot-pounds, and $\frac{3}{2} (1.537154) = 2.305731$ foot-pounds, the work evolved by the first and second parts of the piston's stroke, as given in columns 11 and 12. The work evolved by the engine on the whole, being the difference between the work evolved by B, and the work absorbed by A, will be equal to one-third of the former, or one-half of the latter, or 4.418924 foot-pounds, as given in column 14. Dividing this by $0^{\circ}.04304312$, we obtain 102.66276 foot-pounds as the work evolved by the engine out of each 1° FAHR. per lb. of water communicated to the receiver. This result, which is consigned to the sixteenth column, informs us of the economical value of the engine, which is of course great in proportion to its approach to 772 foot-pounds, the theoretical maximum. The seventeenth column contains the theoretical duty according to Professor THOMSON's law, viz. that the range of temperature divided by the maximum absolute temperature is equal to the fraction of heat converted into force by any perfect engine*.

It will be observed that the numbers in column 16, representing the work evolved out of each unit of heat, increase with the temperature and pressure of the air in the receiver. In every example given, with the exception of the first, the economical value of the air-engine in question is greater than that of the steam-engine calculated by Mr. RANKINE in his paper on the Mechanical Action of Heat†. In considering the relative merits of the engines, we must not, however, lose sight of a most important fact discovered by RANKINE and CLAUSIUS, viz. that a portion of the heat

* See Professor THOMSON's "Investigation of the Duty of a perfect Thermo-Dynamic Engine," at the end of this paper.

† Transactions of the Royal Society of Edinburgh, vol. xx. part 1. Professor THOMSON, in a paper "On the Dynamical Theory of Heat," recently read before the Royal Society, Edinburgh, gives 209 foot-pounds as the duty of an absolutely perfect steam-engine, with a range of temperature between 30° and 140° Centigrade.

TABLE I.

Distance traversed by piston, in inches.	Work absorbed, in foot-pounds.	Temperature from absolute zero, in degrees FAHR.	Pressure on the piston, in lbs.	Distance traversed by piston, in inches.	Work absorbed, in foot-pounds.	Temperature from absolute zero, in degrees FAHR.	Pressure on the piston, in lbs.
0	0	491	15	6.0	11.77479	626.6480	38.28805
0.1	0.1257008	492.4481	15.17066	6.1	12.09828	630.3747	39.16857
0.2	0.2528426	493.9128	15.34473	6.2	12.42768	634.1694	40.08375
0.3	0.3814514	495.3944	15.52230	6.3	12.76566	638.0630	41.03738
0.4	0.5113882	496.8913	15.70343	6.4	13.11172	642.0498	42.03120
0.5	0.6432696	498.4106	15.88841	6.5	13.46626	646.1341	43.06763
0.6	0.7763749	499.9440	16.07709	6.6	13.82962	650.3201	44.14936
0.7	0.9111464	501.4966	16.26974	6.7	14.20221	654.6124	45.27926
0.8	1.047533	503.0678	16.46643	6.8	14.58441	659.0155	46.46043
0.9	1.185586	504.6582	16.66731	6.9	14.97668	663.5345	47.69625
1.0	1.325341	506.2682	16.87248	7.0	15.37948	668.1749	48.99042
1.1	1.466805	507.8979	17.08209	7.1	15.79334	672.9426	50.34692
1.2	1.610032	509.5479	17.29627	7.2	16.21879	677.8438	51.77015
1.3	1.755090	511.2190	17.51516	7.3	16.65633	682.8845	53.26481
1.4	1.901962	512.9110	17.73892	7.4	17.10672	688.0730	54.83624
1.5	2.050727	514.6248	17.96770	7.5	17.57047	693.4156	56.49008
1.6	2.201444	516.3611	18.20167	7.6	18.04842	698.9216	58.23268
1.7	2.354089	518.1196	18.44097	7.7	18.54128	704.5995	60.07098
1.8	2.508791	519.9018	18.68582	7.8	19.04988	710.4585	62.01267
1.9	2.665542	521.7076	18.93638	7.9	19.57510	716.5093	64.06620
2.0	2.824402	523.5377	19.19283	8.0	20.11797	722.7632	66.24102
2.1	2.985424	525.3927	19.45538	8.1	20.67947	729.2318	68.54755
2.2	3.148667	527.2733	19.72426	8.2	21.26078	735.9287	70.99750
2.3	3.314184	529.1801	19.99966	8.3	21.86317	742.8683	73.60395
2.4	3.481993	531.1133	20.28182	8.4	22.48795	750.0659	76.38145
2.5	3.652224	533.0744	20.57099	8.5	23.13667	757.5393	79.34657
2.6	3.824870	535.0633	20.86740	8.6	23.81096	765.3072	82.51786
2.7	4.000022	537.0811	21.17132	8.7	24.51263	773.3907	85.91642
2.8	4.177719	539.1282	21.48302	8.8	25.24353	781.8109	89.56590
2.9	4.358080	541.2060	21.80279	8.9	26.00607	790.5954	93.49398
3.0	4.541123	543.3147	22.13095	9.0	26.80260	799.7716	97.73172
3.1	4.726945	545.4554	22.46778	9.1	27.63584	809.3707	102.3153
3.2	4.915606	547.6288	22.81364	9.2	28.50889	819.4284	107.2862
3.3	5.107206	549.8361	23.16887	9.3	29.42512	829.9836	112.6929
3.4	5.301779	552.0776	23.53383	9.4	30.38842	841.0810	118.5920
3.5	5.499456	554.3549	23.90892	9.5	31.40316	852.7710	125.0499
3.6	5.700287	556.6685	24.29452	9.6	32.47432	865.1110	132.1452
3.7	5.904370	559.0196	24.69107	9.7	33.60755	878.1660	139.9716
3.8	6.111816	561.4094	25.09902	9.8	34.80946	892.0122	148.6412
3.9	6.322706	563.8389	25.51884	9.9	36.08755	906.7362	158.2897
4.0	6.537154	566.3094	25.95104	10.0	37.45073	922.4402	169.0827
4.1	6.755242	568.8218	26.39613	10.1	38.90927	939.2430	181.2239
4.2	6.977122	571.3779	26.85467	10.2	40.47547	957.2860	194.9666
4.3	7.202863	573.9785	27.32725	10.3	42.16396	976.7377	210.6300
4.4	7.432590	576.6250	27.81448	10.4	43.99234	997.8010	228.6204
4.5	7.666465	579.3193	28.31703	10.5	45.98215	1020.724	249.4640
4.6	7.904577	582.0624	28.83559	10.6	48.15980	1045.811	273.8522
4.7	8.147090	584.8562	29.37090	10.7	50.55854	1073.445	302.7106
4.8	8.394129	587.7021	29.92373	10.8	53.22073	1104.114	337.3058
4.9	8.645876	590.6023	30.49494	10.9	56.20106	1138.448	379.4122
5.0	8.902414	593.5577	31.08536	11.0	59.57200	1177.282	431.5900
5.1	9.164006	596.5713	31.69598	11.1	63.43234	1221.754	497.6597
5.2	9.430732	599.6440	32.32776	11.2	67.92089	1273.463	583.5623
5.3	9.702832	602.7787	32.98178	11.3	73.23965	1334.736	699.0180
5.4	9.980470	605.9771	33.65917	11.4	79.69880	1409.147	860.9854
5.5	10.26387	609.2420	34.36112	11.5	87.80466	1502.528	1101.650
5.6	10.55323	612.5754	35.08897	11.6	98.46010	1625.281	1489.565
5.7	10.84876	615.9800	35.84405	11.7	113.4919	1798.450	2197.699
5.8	11.15067	619.4581	36.62784	11.8	137.4363	2074.295	3802.170
5.9	11.45928	623.0133	37.44196	11.9	187.1806	2647.359	9705.187

TABLE II.

No. of Example.	Receiver C.			Pump of Compression A. Length 12 inches. Sectional Area = 1 square inch.						Cylinder of Expansion B. Sectional Area = 1 square inch.					Work evolved out of each degree Fahr. in the capacity of a lb. of water, in foot-pounds.	Difference between the numbers in columns 4 and 13, divided by the numbers in column 4, and multiplied by the mechanical equivalent of heat.
	Density of the air, that of the atmosphere being called unity.	Pressure of the air in lbs. on the square inch.	Absolute temperature of the air, in degrees Fahr. from the absolute zero.	Length of the first part of the stroke.	Work of the engine absorbed by the first part of the stroke of the piston, in foot-pounds.	Work of the engine absorbed by the second part of the stroke of the piston, in foot-pounds.	Absolute temperature of the air forced into receiver C, in degrees Fahr. from the absolute zero.	Length of cylinder B, in inches.	Length of the first part of the piston's stroke, in inches.	Work communicated to the engine by the first part of the stroke of the piston, in foot-pounds.	Work communicated to the engine by the second part of the stroke, in foot-pounds.	Absolute temperature of the air escaping into the atmosphere, in degrees Fahr. from absolute zero.	Work evolved by the engine by each stroke of the piston, in foot-pounds.	Heat communicated to the air in receiver C, in degrees Fahr. per capacity of a lb. of water.	Work evolved out of each degree Fahr. in the capacity of a lb. of water, in foot-pounds.	
1	1	25.95104	849.464	4	1.537154	7.300693	566.3094	18	12	10.95104	2.305731	736.5	4.418924	0.04304312	102.6628	102.6626
2	1	66.24102	2168.289	8	10.11797	17.08034	722.7632	36	12	51.24102	30.35391	1473.0	54.39662	0.2197384	247.5517	247.5515
3	2	66.24102	1084.145	8	10.11797	17.08034	722.7632	18	6	25.62051	15.17695	736.5	13.59915	0.0549346	247.5517	247.5515
4	2	169.0827	2767.321	10	24.95073	25.68045	922.4402	36	6	77.04135	74.85219	1473.0	101.2624	0.2804454	361.0769	361.0770
5	4	169.0827	1383.660	10	24.95073	25.68045	922.4402	18	3	38.52067	37.42610	736.5	25.31569	0.07011137	361.0769	361.0770
6	8	431.59	1765.923	11	45.82200	34.71583	1177.282	18	1.5	52.07375	68.73300	736.5	40.26892	0.08948094	450.0278	450.0279
7	20	1101.65	1803.034	11.5	73.42966	45.27708	1502.528	14.4	0.6	54.33250	88.11559	589.2	23.74135	0.04563073	519.7236	519.7238
8	100	9705.187	3176.831	11.9	172.3056	80.75156	2647.359	14.4	0.12	96.90187	206.7667	589.2	50.61143	0.0804865	628.8189	628.8189
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

employed to evaporate water in the boiler is afterwards evolved in the form of work, in consequence of the liquefaction, in the cylinder, of a portion of the expanding vapour. This fact would induce the hope that a great portion of the latent heat of evaporation, which is at present almost entirely lost, might by an increase of temperature, and by extending the principle of expansion, be converted into mechanical effect.

If, as would appear from the experiments of DE LA RIVE and MARCET, HAYCRAFT and DULONG, the capacity for heat of a given volume is the same in all gases taken at the same pressure and temperature, the results of the above Tables will be equally true whatever elastic fluid be employed.

It now only remains to offer a few observations, with a view to facilitate the labours of those who may be desirous of constructing a good practical air-engine.

It may be remarked, in the first place, that the receiver C need not be of much greater capacity than the cylinder B. For in the reciprocating engine, the air could be introduced from the pump A, at the same time that an equal amount would be expelled into the cylinder B. It would therefore be only requisite to pass the air through tubes heated by a proper furnace, as in NEILSON'S *hot-blast*, the tubes themselves constituting the receiver C. For a temperature under the red heat, these tubes might be constructed of wrought or cast iron. They might be either straight, like the tubes of a locomotive boiler, or arranged in the form of a coil, as represented by fig. 2, in which *a* is the pipe which conveys the air from the pump, *c, c, c*, &c. is the coil of wrought or cast-iron tubing, and *b* is the pipe which conveys the heated air to the cylinder. The coil is surrounded by a massive arch of brickwork, which serves at once to support the pipes, and to prevent waste of heat. To prevent the temperature exceeding the proper limits, the pipe *b* might, as it expands by the heat of the inclosed air, move a piece of mechanism in connection with the damper of the flue. I may remark that, on the scales adopted, fig. 2 represents the size of receiver which would be required for an engine the cylinder of which is 3 feet in diameter.

I would here venture to suggest whether the combustion of the fuel could not, by suitable mechanical arrangements, be carried on within the receiver C; if this could be accomplished, the heat, which in the form of receiver already described is lost up the chimney, would be economized, and a great saving of weight and space would be effected. An engine furnished with a receiver of this kind would be strikingly analogous to the electro-magnetic engine, and present a beautiful illustration of the evolution of mechanical effect from chemical forces.

In both of the above forms of receiver, it would be desirable, as already hinted, that the introduction of the air into the receiver should be simultaneous with the expulsion of the same quantity into the cylinder. This is necessary in order both to keep the pressure in the receiver uniform and to promote the smooth action of the engine. For this purpose the piston-rods of the pump and cylinder, *a* and *b* (fig. 3), must be attached to cranks on different parts of the circumference of the revolving shaft *c c*, so contrived that the piston shall arrive at the top or bottom of the cylinder

the moment that the pump-valve opens admitting a fresh supply of air into the receiver. The cylinder should of course be provided with proper expansion gear to cut off the air at the required part of the stroke, which must be a constant quantity for each engine. The valves of the pump would of course be self-acting.

In an engine similar to that described, it will be obvious that if the temperature of the receiver be kept constant, the pressure of air in it will also remain constant. For whilst the same quantity of air is always introduced into the receiver by each stroke of the pump, the quantity expelled out of it would increase with an augmentation and decrease with a diminution of pressure.

In conclusion, I would recommend the examples No. 3 and No. 5 of Table II. to the attention of those who may be willing to construct an air-engine. In both of these cases the capacity of the pump is two-thirds of that of the cylinder. In the cylinder of No. 3 the air is to be cut off at one-third of the stroke; and in that of No. 5 at one-sixth of the stroke. The temperature of the air in the receiver (supposing that of the atmosphere to be 32° FAHR.) is $625^{\circ}145$ FAHR. in No. 3, and $924^{\circ}66$ FAHR. in No. 5. The consumption of fuel in No. 3 need not exceed one-half, nor that in No. 5 one-third of that in the most perfect steam-engines at present constructed.

Acton Square, Salford, Manchester,

May 6, 1851.

Note to the foregoing Paper, with a New Experimental Determination of the Specific Heat of Atmospheric Air.

Received March 23, 1852.

Since the above was written, Professor W. H. MILLER has directed my attention to the probable incorrectness of the value of k , as deduced from the experiments of DELAROCHE and BERARD on the specific heat of air, and my own determination of the mechanical equivalent of heat; in comparison with the value deduced from the numerous and excellent experiments on the velocity of sound. Mr. RANKINE considers that the discrepancy between the two values arises from the incorrectness of DELAROCHE and BERARD's result, an opinion which would seem to be justified by the entire want of accordance between the determination of these philosophers, and those of SUERMANN, and CLEMENT and DESORMES. I have therefore been induced to make the following careful experiments in order to obtain a fresh and, if possible, more correct value of the specific heat of air at constant pressure.

The apparatus I employed is represented by fig. 4, in which a and b are two vessels, each of which contains a coil of leaden piping, eight yards long and one quarter of an inch in internal diameter. The coil of the upper vessel passes three-eighths of an inch through the bottom, to which it is soldered at c , and is thence connected with the coil of the lower vessel by a piece of vulcanized india rubber tubing. This part

of the apparatus will be better understood by a reference to fig. 5, in which a section of it is represented, *a* being the upper, *b* the lower vessel, and *w* the surface of the water in the latter. *xx* are a pair of wooden pincers by means of which the india rubber tube could be compressed so as to prevent, when desired, any communication between the air in the two coils of piping. Referring again to fig. 4, *g* is a gas-lamp to maintain the water in the upper vessel at a constant high temperature, and *j* is a tall jar filled with coarsely pounded chloride of calcium, in passing through which the air was entirely deprived of aqueous vapour; a length of vulcanized india rubber tubing, *p*, connects the coil of the lower vessel with a good air-pump, each barrel of which was found to have a capacity of 12·77 cubic inches. The temperature of the pump could be ascertained by means of a small thermometer, the bulb of which was kept in contact with one of the barrels.

The method of experimenting was as follows:—The lower vessel being filled with cold water, and the upper with water raised to about 190°, their exact temperatures were read off, with the usual precautions, from the scales of delicate and accurate thermometers. The pump was then worked at a uniform velocity for twenty-six minutes, the water in the lower vessel being agitated from time to time by a stirrer. The examination of the barometer and thermometers a second time occupied four minutes more; so that the whole time occupied by each experiment was exactly half an hour. The pincers were now applied so as to cut off all communication between the air in the two coils, and the effect of the various causes of a change of temperature in the lower vessel, unconnected with the current of heated air, was observed during another half-hour. Experiments of both the above kinds were repeated several times with the results tabulated below.

I may remark in this place that I had ascertained, by preliminary experiments, that the air passed from the coils of the vessels sensibly at the temperatures registered by the thermometers plunged into the surrounding water.

SERIES I.—Pump worked 26', at the rate of twenty-four strokes per minute.

No. of Experiment.	Source of calorific effect.	Height of Barometer.	Temperature of Barometer.	Temperature of Air-pump.	Temperature of upper vessel.	Temperature of the room.	Temperature of the lower vessel.		Increase of temperature.
							Commencement of Experiment.	Termination of Experiment.	
1	Radiation.....		°	°	°	46·081	41·270	41·814	0·544
1	Heated air and radiation ...	30·195	46	49·3	189·28	46·188	41·814	42·802	0·988
2	Radiation.....					46·497	42·802	43·304	0·502
2	Heated air and radiation ...	30·205	46·75	50·3	189·43	46·785	43·304	44·246	0·942
3	Radiation.....					46·948	44·246	44·694	0·448
3	Heated air and radiation ...	30·22	47·5	51·1	189·89	47·068	44·694	45·590	0·896
4	Radiation.....					47·197	45·590	45·983	0·393
4	Heated air and radiation ...	30·235	48·	51·7	194·85	47·283	45·983	46·856	0·873
5	Radiation.....					47·455	46·856	47·211	0·355
Mean.	Heated air and radiation ...	30·214	47·06	50·6	190·862	46·831	43·949	44·874	0·925
Mean.	Radiation.....					46·836	44·153	44·601	0·448

It will be observed that the excess of the temperature of the room above the mean temperature of the water in the lower vessel, was, in the experiments with heated air, $2^{\circ}42$, but in the experiments on the effect of radiation $2^{\circ}459$. A comparison of the several experiments with one another, furnished the means of determining the amount of the small correction due to this circumstance. Hence $0^{\circ}925 + 0^{\circ}002 - 0^{\circ}448 = 0^{\circ}479$ will be the corrected mean increase of temperature due to the current of heated air. The material in which this increase took place consisted of 175500 grs. of water, 15635 grs. of copper, and 53370 grs. of lead, the whole having a capacity for heat equivalent to that of 178535 grs. of water. The volume of air passed through the pump was $12\cdot77 \times 26 \times 24 = 7968\cdot48$ cubic inches, which, at the observed barometric pressure and the temperature $50^{\circ}6$, would weigh 2537·94 grs. We have therefore for the specific heat of atmospheric air at constant pressure—

$$\frac{178535 \times 0\cdot479}{2537\cdot94 \times 146\cdot45} = 0\cdot23008.$$

SERIES II.—Pump worked $26'$, at the rate of forty strokes per minute.

No. of Experiment.	Source of calorific effect.	Height of Barometer.	Temperature of Barometer.	Temperature of Air-pump.	Temperature of upper vessel.	Temperature of the room.	Temperature of the lower vessel.		Increase of temperature.
							Commencement of Experiment.	Termination of Experiment.	
1	Radiation.....		°	°	°	$47^{\circ}223$	$44^{\circ}200$	$44^{\circ}648$	$0^{\circ}448$
1	Heated air and radiation ...	30·6	$47^{\circ}75$	52	$197^{\circ}71$	$47^{\circ}558$	$44^{\circ}648$	$45^{\circ}902$	$1^{\circ}254$
2	Radiation.....					$47^{\circ}841$	$45^{\circ}902$	$46^{\circ}319$	$0^{\circ}417$
2	Heated air and radiation ...	30·602	$48^{\circ}25$	$53^{\circ}5$	$198^{\circ}63$	$48^{\circ}099$	$46^{\circ}319$	$47^{\circ}516$	$1^{\circ}197$
3	Radiation.....					$48^{\circ}339$	$47^{\circ}516$	$47^{\circ}860$	$0^{\circ}344$
3	Heated air and radiation ...	30·61	$49^{\circ}5$	$55^{\circ}4$	$202^{\circ}42$	$49^{\circ}107$	$49^{\circ}327$	$50^{\circ}443$	$1^{\circ}116$
4	Radiation.....					$49^{\circ}524$	$50^{\circ}443$	$50^{\circ}728$	$0^{\circ}285$
4	Heated air and radiation ...	30·607	$50^{\circ}25$	$56^{\circ}4$	$203^{\circ}13$	$49^{\circ}850$	$50^{\circ}728$	$51^{\circ}809$	$1^{\circ}081$
5	Radiation.....					$50^{\circ}030$	$51^{\circ}809$	$52^{\circ}037$	$0^{\circ}228$
Mean.	Heated air and radiation ...	30·605	$48^{\circ}94$	$54^{\circ}32$	$200^{\circ}472$	$48^{\circ}653$	$47^{\circ}755$	$48^{\circ}917$	$1^{\circ}162$
Mean.	Radiation.....					$48^{\circ}591$	$47^{\circ}974$	$48^{\circ}318$	$0^{\circ}344$

In the above series $1^{\circ}162 + 0^{\circ}006 - 0^{\circ}344 = 0^{\circ}824$ will be the corrected mean increase of temperature due to the current of heated air. The material in which this increase took place consisted of 175000 grs. of water, 15635 grs. of copper, and 53370 grs. of lead, the whole having a capacity for heat equivalent to that of 178035 grs. of water. The volume of air passed through the pump was $12\cdot77 \times 26 \times 40 = 13280\cdot8$ cubic inches, which, at the observed barometric pressure and the temperature $54^{\circ}32$, would weigh 4252·7 grs. Hence we have for the specific heat—

$$\frac{178035 \times 0\cdot824}{4252\cdot7 \times 152\cdot136} = 0\cdot22674.$$

By another series of experiments, in which the air-pump was worked at the velocity of twenty strokes per minute for twenty minutes, I obtained the value 0·2325. The mean of the three results is 0·22977, or nearly 0·23, which we may take as the specific heat of air at constant pressure determined by the above experiments.

Professor W. H. MILLER has remarked that MOLL's experiments, when correctly reduced, give a velocity of sound equal to 332·475 metres per second in dry air at 32°. Hence he deduces 1·41029 as the value of k . Calling it in round numbers 1·41, and the mechanical equivalent of heat 772, we obtain 0·238944 as the value of the specific heat of air at constant pressure, a result sufficiently near the experimental determination to show that the value of k , as deduced by Professor MILLER, is much nearer the truth than that upon which the tables of the foregoing paper are founded.

The values of k , as determined by the experiments of DESORMES and CLEMENT, GAY-LUSSAC and WELTER, and Mr. MEIKLE, referred to in the note to page 67, are respectively only 1·354, 1·375, and 1·333. In these experiments a small portion of air having been withdrawn from a large receiver, the equilibrium was re-established by opening for an instant a large aperture communicating with the external air, and then, after the receiver and its contents had regained their original temperature, the alteration of pressure, indicating the sudden rise of temperature which had taken place on the admission of the air, was noted. But it is obvious that the sudden admission of the air would cause the development of *sound*, and that, a portion of the *vis viva* escaping in this form, the increase of temperature and the deduced ratio of the specific heats would be diminished accordingly.

I subjoin Tables, similar to Tables I. and II., calculated from the data $k=1·41$, and the specific heat of air at constant volume $=0·169464$, or at constant pressure $=0·238944$.

In Table IV., the examples 9, 10 and 11 may be suggested to the notice of the practical engineer, the temperature of the receiver being in all those cases below that of redness. I may remind the reader that the Table is founded on the supposition that the air which enters the pump has 491° of temperature from the absolute zero, and that its pressure is 15 lbs. on the square inch. If this initial temperature be altered, the whole of the other temperatures in the Table must be altered in the same proportion, but the pressure, work and economical duty will remain unchanged. If the initial pressure be altered, all the other pressures and work will suffer a proportionate change, but the temperatures and economical duty will remain the same. The above are obvious deductions from the formulæ on which the Tables are founded.

Acton Square, Salford,

March 20, 1852.

TABLE III.

Distance traversed by piston, in inches.	Work absorbed, in foot-pounds.	Temperature from absolute zero, in degrees FAHR.	Pressure on the piston, in pounds avoirdupois.	Distance traversed by piston, in inches.	Work absorbed, in foot-pounds.	Temperature from absolute zero, in degrees FAHR.	Pressure on the piston, in pounds avoirdupois.
0	0	491	15	6.0	12.025096	652.3847	39.86055
0.1	0.1257463	492.6876	15.17803	6.1	12.36122	656.8958	40.81647
0.2	0.2529680	494.3950	15.35970	6.2	12.70547	661.5159	41.81223
0.3	0.3817395	496.1232	15.54513	6.3	13.05820	666.2498	42.85023
0.4	0.5120683	497.8723	15.73438	6.4	13.41977	671.1023	43.93309
0.5	0.6439993	499.6429	15.92769	6.5	13.79055	676.0785	45.06354
0.6	0.7775396	501.4351	16.12503	6.6	14.17096	681.1838	46.24464
0.7	0.9127567	503.2498	16.32661	6.7	14.56146	686.4245	47.47966
0.8	1.049665	505.0872	16.53253	6.8	14.96245	691.8060	48.77217
0.9	1.188301	506.9478	16.74292	6.9	15.37448	697.3358	50.12596
1.0	1.328719	508.8323	16.95793	7.0	15.79807	703.0207	51.54529
1.1	1.471827	510.7529	17.17773	7.1	16.23377	708.8680	53.03472
1.2	1.615031	512.6748	17.40240	7.2	16.68220	714.8862	54.59923
1.3	1.761007	514.6339	17.63216	7.3	17.14397	721.0835	56.24433
1.4	1.908922	516.6190	17.86716	7.4	17.61984	727.4700	57.97600
1.5	2.058809	518.6306	18.10755	7.5	18.11050	734.0550	59.80080
1.6	2.210724	520.6694	18.35353	7.6	18.61680	740.8498	61.72605
1.7	2.364719	522.7361	18.60529	7.7	19.13958	747.8659	63.75969
1.8	2.520828	524.8312	18.86299	7.8	19.67980	755.1160	65.91058
1.9	2.679114	526.9555	19.12686	7.9	20.23844	762.6133	68.18854
2.0	2.839636	529.1098	19.39710	8.0	20.81664	770.3732	70.60445
2.1	3.002421	531.2945	19.67392	8.1	21.41559	778.4115	73.17045
2.2	3.167547	533.5106	19.95757	8.2	22.03660	786.7459	75.90002
2.3	3.335073	535.7589	20.24830	8.3	22.68110	795.3954	78.80838
2.4	3.505041	538.0400	20.54632	8.4	23.35061	804.3808	81.91249
2.5	3.677529	540.3549	20.85193	8.5	24.04690	813.7254	85.23164
2.6	3.852596	542.7044	21.16540	8.6	24.77180	823.4542	88.78743
2.7	4.030240	545.0885	21.48700	8.7	25.52742	833.5950	92.60451
2.8	4.210744	547.5110	21.81705	8.8	26.31602	844.1786	96.71086
2.9	4.393947	549.9697	22.15585	8.9	27.14016	855.2390	101.1385
3.0	4.580211	552.4695	22.50375	9.0	28.00265	866.8144	105.9244
3.1	4.769047	555.0038	22.86105	9.1	28.90667	878.9468	111.1106
3.2	4.961064	557.5808	23.22823	9.2	29.85575	891.6840	116.7460
3.3	5.156197	560.1996	23.60557	9.3	30.85385	905.0792	122.8892
3.4	5.354524	562.8613	23.99352	9.4	31.90550	919.1930	129.6058
3.5	5.556132	565.5670	24.39246	9.5	33.01577	934.0936	136.9750
3.6	5.761092	568.3177	24.80292	9.6	34.19049	949.8591	145.0905
3.7	5.969523	571.1150	25.22533	9.7	35.43632	966.5791	154.0638
3.8	6.181561	573.9607	25.66015	9.8	36.76094	984.3564	164.0289
3.9	6.397243	576.8553	26.10795	9.9	38.17335	1003.312	175.1490
4.0	6.616735	579.8010	26.56929	10.0	39.68387	1023.584	187.6224
4.1	6.840106	582.7988	27.04473	10.1	41.30480	1045.338	201.6945
4.2	7.067466	585.8501	27.53490	10.2	43.05077	1068.770	217.6720
4.3	7.299071	588.9584	28.04045	10.3	44.93904	1094.112	235.9413
4.4	7.534902	592.1234	28.56207	10.4	46.99081	1121.648	256.9966
4.5	7.775119	595.3475	29.10049	10.5	49.23177	1151.723	281.4802
4.6	8.019950	598.6331	29.65651	10.6	51.69402	1184.768	310.2390
4.7	8.269468	601.9818	30.23092	10.7	54.41741	1221.318	344.4105
4.8	8.523854	605.3958	30.82462	10.8	57.45355	1262.065	385.5594
4.9	8.783274	608.8774	31.43856	10.9	60.86887	1307.901	435.8856
5.0	9.047890	612.4287	32.07366	11.0	64.75245	1360.021	498.5821
5.1	9.317890	616.0523	32.73102	11.1	69.22596	1420.059	578.4345
5.2	9.593477	619.7509	33.41175	11.2	74.46110	1490.318	682.9350
5.3	9.874827	623.5267	34.11703	11.3	80.71017	1574.184	824.4195
5.4	10.16216	627.3830	34.84815	11.4	88.36276	1676.887	1024.575
5.5	10.45581	631.3240	35.60646	11.5	98.06077	1807.041	1324.918
5.6	10.75570	635.3486	36.39342	11.6	110.9605	1980.164	1814.815
5.7	11.06235	639.4641	37.21060	11.7	129.4314	2228.056	2722.671
5.8	11.37594	643.6727	38.05962	11.8	159.4567	2631.016	4822.635
5.9	11.69678	647.9786	38.94231	11.9	223.8930	3495.794	12815.505

TABLE IV.

No. of Example.	Receiver C.			Pump of Compression A. Length 12 inches. Sectional Area = 1 square inch.				Cylinder of Expansion B. Sectional Area = 1 square inch.					Work evolved by the engine by each stroke of the piston, in foot-pounds.	Heat communicated to the air in receiver C, in degrees Fahr., per capacity of a lb. of water.	Work evolved out of each degree Fahr., in the capacity of a lb. of water, in foot-pounds.	Difference between the numbers in columns 4 and 13, divided by the numbers in column 4, and multiplied by the me- chanical equivalent of heat.
	Density of the air, that of the at- mosphere being called unity.	Pressure of the air in pounds on the square inch.	Absolute temperature of the air, in degrees Fahr., from the absolute zero.	Length of the first part of the stroke.	Work of the engine absorbed by the first part of the stroke of the piston, in foot-pounds.	Work of the engine absorbed by the second part of the stroke of the piston, in foot-pounds.	Absolute temperature of the air forced into the receiver C, in de- grees Fahr., from the absolute zero.	Length of the cylinder B, in inches.	Length of the first part of the piston's stroke, in inches.	Work communicated to the engine by the first part of the stroke, in foot-pounds.	Work communicated to the engine by the second part of the stroke, in foot-pounds.	Absolute temperature of the air escaping into the atmosphere, in degrees Fahr., from the absolute zero.				
1	1	26.56929	869.7014	4	1.616735	7.71286	579.801	18	1.2	11.56929	2.425102	736.5	4.664797	0.03945268	118.2378	118.2377
2	1	70.60445	2311.119	8	10.81664	18.53482	770.3732	36	1.2	55.60446	32.44992	1473.0	58.70292	0.2096809	279.9632	279.9630
3	2	70.60445	1155.559	8	10.81664	18.53482	770.3732	18	6	27.80223	16.22496	736.5	14.67573	0.05242022	279.9632	279.9628
4	2	187.6224	3070.753	10	27.18387	28.7704	1023.584	36	6	86.3112	81.55161	1473.0	111.90854	0.2786002	401.6815	401.6817
5	4	187.6224	1535.3765	10	27.18387	28.7704	1023.584	18	3	43.1556	40.7758	736.5	27.97713	0.06965	401.6815	401.6815
6	8	498.5821	2040.032	11	51.00245	40.29851	1360.021	18	1.5	60.44776	76.50367	736.5	45.65047	0.092343	493.2893	493.2897
7	20	1324.918	2168.449	11.5	83.68577	54.57992	1807.041	14.4	0.6	65.4959	100.4229	589.2	27.65313	0.04918419	562.2364	562.2361
8	100	12815.505	4194.953	11.9	209.018	106.6709	3495.794	14.4	0.12	128.0051	250.8216	589.2	63.1378	0.0951489	663.5685	663.5692
9	3	105.92437	1155.7525	9	16.75265	22.73109	866.8144	16	4	30.30812	22.33687	654.666	13.16125	0.03932171	334.7069	334.7069
10	2.5	105.92437	1386.903	9	16.75265	22.73109	866.8144	19.2	4.8	36.36974	26.80424	785.6	23.69024	0.0707791	334.7068	334.7068
11	4.5	187.6224	1364.779	10	27.18387	28.7704	1023.584	16	2.3	38.36053	36.24516	654.666	18.65142	0.0464334	401.6814	401.6816
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

Additional Note on the preceding Paper. By WILLIAM THOMSON, M.A., F.R.S., F.R.S.E., Fellow of St. Peter's College, Cambridge, and Professor of Natural Philosophy in the University of Glasgow.

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1. *Synthetical Investigation of the Duty of a Perfect Thermo-Dynamic Engine founded on the Expansions and Condensations of a Fluid, for which the gaseous laws hold and the ratio (k) of the specific heat under constant pressure to the specific heat in constant volume is constant; and modification of the result by the assumption of MAYER'S hypothesis.*

Let the source from which the heat is supplied be at the temperature S , and let T denote the temperature of the coldest body that can be obtained as a refrigerator. A cycle of the following four operations, *being reversible in every respect*, gives, according to CARNOT'S principle, first demonstrated for the Dynamical Theory by CLAUDIUS, the greatest possible statical mechanical effect that can be obtained in these circumstances from a quantity of heat supplied from the source.

(1.) Let a quantity of air contained in a cylinder and piston, at the temperature S , be allowed to expand to any extent, and let heat be supplied to it to keep its temperature constantly S .

(2.) Let the air expand farther, without being allowed to take heat from or to part with heat to surrounding matter, until its temperature sinks to T .

(3.) Let the air be allowed to part with heat so as to keep its temperature constantly T , while it is compressed to such an extent that at the end of the fourth operation the temperature may be S .

(4.) Let the air be farther compressed, and prevented from either gaining or parting with heat, till the piston reaches its primitive position.

The amount of mechanical effect gained on the whole of this cycle of operations will be the excess of the mechanical effect obtained by the first and second above the work spent in the third and fourth. Now if P and V denote the primitive pressure and volume of the air, and if P_1 and V_1 , P_2 and V_2 , P_3 and V_3 , P_4 and V_4 denote the pressure and volume respectively, at the ends of the four successive operations, we have by the gaseous laws, and by POISSON'S formula and a conclusion from it quoted above, the following expressions:—

Mechanical effect obtained by the first operation $= PV \log \frac{V_1}{V}$.

Mechanical effect obtained by the second operation $= P_2 V_2 \cdot \frac{1}{k-1} \cdot \left\{ \left(\frac{V_2}{V_1} \right)^{k-1} - 1 \right\}$.

Work spent in the third operation $= P_3 V_3 \log \frac{V_2}{V_3}$.

$$\text{Work spent in the fourth operation} \quad . \quad . \quad . \quad . \quad = P_3 V_3 \cdot \frac{1}{k-1} \left\{ \left(\frac{V_3}{V_4} \right)^{k-1} - 1 \right\}.$$

Now, according to the gaseous laws, we have

$$P_1 V_1 = P V; \quad P_2 V_2 = P_1 V_1 \frac{1+ET}{1+ES};$$

$$P_3 V_3 = P_2 V_2; \quad \text{and (since } V_4 = V), P_4 = P.$$

Also by Poisson's formula,

$$\left(\frac{V_2}{V_1} \right)^{k-1} = \left(\frac{V_3}{V} \right)^{k-1} = \frac{1+ES}{1+ET}.$$

By means of these we perceive that the work spent in the fourth operation is equal to the mechanical effect gained in the second; and we find, for the whole gain of mechanical effect (denoted by M), the expressions

$$M = (P V - P_3 V_3) \log \frac{V_1}{V} = P V \log \frac{V_1}{V} \cdot \frac{E(S-T)}{1+ES}.$$

All the preceding formulæ are founded on the assumption of the gaseous laws and the constancy of the ratio (k) of the specific heat under constant pressure to the specific heat in constant volume, for the air contained in the cylinder and piston, and involve no other hypothesis*. If now we add the assumption of MAYER's hypothesis, which for the actual circumstance is $P V \log \frac{V_1}{V} = J H$, where H denotes the heat abstracted by the air from the surrounding matter in the first operation, and J the mechanical equivalent of a thermal unit, we have

$$M = J H \cdot \frac{E(S-T)}{1+ES}.$$

The investigation of this formula given in my paper on the Dynamical Theory of Heat, shows that it would be true for every perfect thermo-dynamic engine, if MAYER's hypothesis were true for a fluid subject to the gaseous laws of pressure and density, whether, for such a fluid (did it exist), k were constant or not.

It was first obtained by using, in the formula

$$M = J H e^{-\frac{1}{J} \int_T^S \mu dt},$$

* From the sole hypothesis that k is constant for a single fluid fulfilling the gaseous laws, and having E for its coefficient of expansion, I find it follows, as a necessary consequence, that CARNOT's function would have the form $\frac{J E}{1+Et+C}$; where C denotes an unknown absolute constant, and t the temperature measured by a thermometer founded on the equable expansions of that gas. From this it follows, that for such a gas subjected to the four operations described in the text, we must have $P V \log \frac{V_1}{V} = J H \frac{1+ES}{1+ES+C}$, and consequently, $M = J H \frac{E(S-T)}{1+ES+C}$, which is Mr. RANKINE's general formula.

which involves no hypothesis, the expression

$$\mu = \frac{J}{\frac{1}{E} + t}$$

for CARNOT's function, which Mr. JOULE had suggested to me in a letter dated December 9, 1848, as the expression of MAYER's hypothesis, in terms of the notation of my "Account of CARNOT's Theory.*" Mr. RANKINE† has arrived at a formula agreeing with it (with the exception of a constant term in the denominator, which, as its value is unknown, but probably small, he neglects in the actual use of the formula), as a consequence of the fundamental principles assumed in his Theory of Molecular Vortices, when applied to any fluid whatever, experiencing a cycle of four operations satisfying CARNOT's criterion of reversibility (being in fact precisely analogous to those described above, and originally invented by CARNOT); and he thus establishes CARNOT's law as a consequence of the equations of the mutual conversion of heat and expansive power, which had been given in the first section of his paper on the Mechanical Action of Heat‡.

2. *Note on the Specific Heats of Air.*

Let N be the specific heat of unity of weight of a fluid at the temperature t , kept within constant volume, v ; and let kN be the specific heat of the same fluid mass, under constant pressure, p . Without any other assumption than that of CARNOT's principle, the following equation is demonstrated in my paper§ on the Dynamical Theory of Heat, § 48,

$$kN - N = \frac{\left(\frac{dp}{dt}\right)^2}{\mu \times -\frac{dp}{dv}},$$

where μ denotes the value of CARNOT's function, for the temperature t , and the differentiations indicated are with reference to v and t considered as independent variables, of which p is a function. If the fluid be subject to BOYLE's and MARIOTTE's law of compression, we have

$$\frac{dp}{dv} = -\frac{p}{v};$$

and if it be subject also to GAY-LUSSAC's law of expansion,

$$\frac{dp}{dt} = \frac{Ep}{1 + Et}.$$

* Royal Society of Edinburgh, January 2, 1849, Transactions, vol. xvi. part 5.

† On the Economy of Heat in Expansive Engines. Royal Society of Edinburgh, April 21, 1851, Transactions, vol. xx. part 2.

‡ Royal Society of Edinburgh, February 4, 1850, Transactions, vol. xx. part 1.

§ Royal Society of Edinburgh, March 17, 1851, Transactions, vol. xx. part 2.

Hence, for such a fluid,

$$kN - N = \frac{E^2 p v}{\mu (1 + Et)^2} *.$$

In the case of dry air these laws are fulfilled to a very high degree of approximation, and, for it, according to REGNAULT's observations,

$$\frac{pv}{1+E_t} = 26215, \quad E = .00366$$

(a British foot being the unit of length, and the weight of a British pound at Paris, the unit of force).

We have consequently, for dry air,

$$kN - N = \frac{26215E^2}{\mu(1 + Et)} \quad \dots \dots \dots (1)$$

Now it is demonstrated, without any other assumption than that of CARNOT's principle, in my "Account of CARNOT's Theory" (Appendix III.), that

$$\frac{E}{\mu(1 + Et)} = \frac{H}{W},$$

if W denote the quantity of work that must be spent in compressing a fluid subject to the gaseous laws, to produce H units of heat when its temperature is kept at t . Hence

$$kN-N=26215E \times \frac{H}{W}=95.947 \times \frac{H}{W} (2)$$

If we adopt the values of μ shown in Table I. of the "Account of CARNOT'S Theory," depending on no uncertain data except the densities of saturated steam at different temperatures, which, for want of accurate experimental data, were derived from the value $\frac{1}{1693.5}$ for the density of saturated vapour at 100° , by the assumption of the "gaseous laws" of variation with temperature and pressure; we find 1357 and 1369 for the values of $\frac{E}{\mu(1 + Et)}$ at the temperatures 0 and 10° respectively; and hence, for these temperatures,

$$\left. \begin{aligned} (t=0) \quad kN-N &= \frac{95.947}{1357} = .07071 \\ (t=10^\circ) \quad kN-N &= \frac{95.947}{1369} = .07008 \end{aligned} \right\} \cdot \cdot \cdot \cdot \cdot \cdot \cdot (a).$$

Or, if we adopt MAYER'S hypothesis, according to which $\frac{W}{H}$ is equal to the mechanical equivalent of the thermal unit†, we have $\frac{W}{H}=1390$; and hence, for all temperatures,

[illegible]

* This equation expresses a proposition first demonstrated by CARNOT. See "Account of CARNOT's Theory," Appendix III. (Transactions Royal Society of Edinburgh, vol. xvi. part 5.)

† The number 1390, derived from Mr. JOULE's experiments on the friction of fluids, cannot differ by $\frac{1}{100}$, and probably does not differ by $\frac{1}{300}$, of its own value, from the true value of the mechanical equivalent of the thermal unit.

The very accurate observations which have been made on the velocity of sound in air, taken in connection with the results of REGNAULT'S observations on its density, &c., lead to the value 1·410 for k , which is probably true in three if not in four of its figures. Now, k being known, the preceding equations enable us to determine the absolute values of the two specific heats (kN , and N) according to the hypotheses used in (a) and in (a') respectively; and we thus find,

	Specific heat of air under constant pressure (kN).	Specific heat of air in constant volume (N).	
for $t=0$,	·2431	·1724,	} according to the tabulated values of CARNOT'S function.
for $t=10$,	·2410	·1709,	
Or, for all temperatures,	·2374	·1684,	} according to MAYER'S hy- pothesis.

By the adoption of hypotheses involving that of MAYER, and taking 1389·6 and 1·4 as the values of J and k , respectively, Mr. RANKINE finds ·2404 and ·1717 as the values of the two specific heats.

Hence it is probable that the values of the specific heat of air under constant pressure, found by SUERMANN (·3046), and by DE LA ROCHE and BERARD (·2669), are both considerably too great; and the true value, to two significant figures, is probably ·24.

Glasgow College,
February 19, 1852.

