

### III. *On the Geometrical Isomorphism of Crystals.*

By HENRY JAMES BROOKE, *F.R.S., Hon. M.C.P.S.*

Received June 11,—Read June 19, 1856.

CRYSTALS, as it is well known, are solids bounded by plane surfaces termed faces, which vary in shape and in their relative positions. They occur generally in small imperfect fragments, and all that we can be said to know about them, is the shapes and positions of their faces, and the angles which those faces make with each other.

All the crystals at present known have been classed in six groups or systems, each group consisting of a series of elementary solids, or as they will be termed primary forms, and of other forms, termed secondary, derived from the primary according to laws to be afterwards explained. These different groups or systems are distinguished from each other by the shapes and positions of some of their faces, and the crystals of the different minerals comprised in each group or system, except the cubic, are distinguished by the angles which particular faces make with each other.

The six systems, and the elementary solids proper to each, are as follows:—

The cubic, consisting of cubes which are bounded by six square faces.

The pyramidal, consisting of prisms having their terminal faces square, and their lateral faces rectangular.

The rhombohedral, consisting of solids bounded by six equal rhombs.

The prismatic, consisting of solids contained within three pairs of parallel rectangular faces at right angles to each other.

The oblique, consisting of solids, also contained within three pairs of parallel faces, of which only two intersect at right angles, the third being an oblique pair.

The anorthic. It is said that in the few solids comprised in this system, no two faces intersect each other at a right angle; and hence the name of the system.

The accompanying Tables of the pyramidal and rhombohedral systems contain lists of the crystallized minerals comprised in each, arranged in a manner new, as the writer believes, to crystallography; presenting that science under a new aspect, and affording some altogether unexpected results, which it is the object of this paper to communicate to the Society.

*Notation.*—Previously, however, to stating these results, it is necessary to explain briefly the crystallographic language and notation that will be employed in describing them.

The term *form* is used in crystallography in two different senses: in one of these it simply means shape; in the other—a sense peculiar to this science—it denotes particular sets of faces that may or may not enclose a finite solid. Thus, in some instances, single pairs of parallel faces constitute crystallographic forms.

MDCCCLVII.

F

The *angles* quoted in the accompanying Tables, and throughout this paper, are those *between normals* to the several faces, although, to avoid the frequent repetition of the words "between normals," they are expressed as angles between faces. The angle between any two faces, as given by most other authors, and the angle between normals to those faces, are supplements to each other, so that, if either is given, the other can be readily found.

A *zone* in crystallography is a series of faces on any crystal which intersect each other in parallel lines, or would do so if sufficiently enlarged.

It has been found that the secondary faces of crystals are not produced as it were at random, but that they cut the primary edges in definite proportions measured along those edges in particular directions; and it is by means of the differences of those proportions, and of the directions in which they are measured, that secondary faces can be denoted and distinguished from each other. This system of notation was first proposed by the Rev. Dr. WHEWELL, in a paper read before the Royal Society and published in the Philosophical Transactions for 1825, and has been employed by Professor MILLER of Cambridge in a work on Crystallography published in 1839, and in the new edition of the Treatise on Mineralogy, by the late WILLIAM PHILLIPS.

According to this system, the faces of a primary form, as they are usually shown in engraved figures, are expressed by the symbols 100, 010, 001, placed in the relative positions shown in fig. 1; and the faces respectively parallel to these are denoted by  $\bar{1}00$ ,  $0\bar{1}0$ ,  $00\bar{1}$ .

These numbers, taken separately, and regarding 0 or zero as a number, are termed *indices*, and those with a line over them are termed *negative* or *minus*.

It is seen in the skeleton (fig. 2) of the primary form, that the edge between the faces 001 and 010, and the three edges parallel to these, have an  $m$  at one end and  $\bar{m}$  at the other; that the edge between 001 and 100 and the three parallel edges have  $n$  and  $\bar{n}$  on each edge; and that the edge between 100 and 010, and the three parallel edges, have  $s$  and  $\bar{s}$  on each edge.

By means of these indices an appropriate symbol can be assigned to every possible face of every crystal, an advantage not afforded by any other system of notation.

Suppose a secondary face to occur, as in fig. 3, on the solid angle surrounded by the letters  $mns$ , and to be such as would cut away  $\frac{1}{3}$  of the edge  $m$ ,  $\frac{1}{2}$  of  $n$ , and  $\frac{1}{4}$  of  $s$ , measured along each respective edge from the point where  $m$ ,  $n$ , and  $s$  meet. This face is denoted by the symbol 324; the separate indices being *the reciprocals* of the proportions supposed to be cut away from the respective edges by this particular face.

If a face truncates that solid angle of fig. 3 which is in fig. 2

Fig. 1.

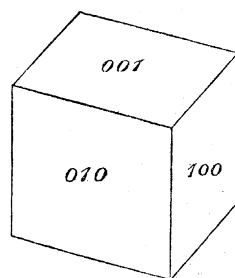


Fig. 2.

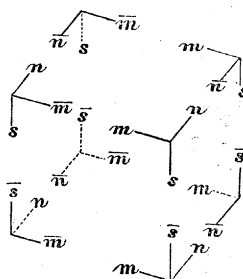
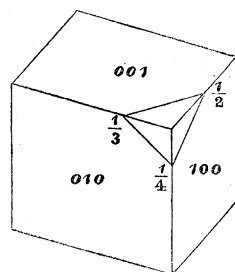


Fig. 3.



surrounded by  $\bar{m}$ ,  $n$ ,  $s$ , and intersects  $\bar{m}n$  at  $\frac{1}{3}$  of its length measured from the point where  $\bar{m}$ ,  $n$ ,  $s$  meet, and if it theoretically removes  $\frac{1}{2}$  of the edge  $n$ , and a  $\frac{1}{4}$  of  $s$ , measured from the same point, its symbol would be  $\bar{3}24$ .

An analogous face on the solid angle  $\bar{m}$ ,  $\bar{n}$ ,  $s$  would have for its symbol  $\bar{3}\bar{2}4$ , and another similar face on the solid angle  $m$ ,  $\bar{n}$ ,  $s$  would have the symbol  $3\bar{2}4$ ; and if we suppose a similar face to truncate the solid angle  $\bar{m}$ ,  $\bar{n}$ ,  $\bar{s}$ , its symbol would be  $\bar{3}\bar{2}\bar{4}$ .

A face that would cut away equal proportions of each of the edges  $m$ ,  $n$ ,  $s$ , would have for its symbol  $111$ , and the face parallel to it  $\bar{1}\bar{1}\bar{1}$ ; and the symbol of a face parallel to any given face is denoted by simply changing the positive indices of the given face to negative, and the negative indices to positive.

The faces which replace the edges of crystals being always parallel to those edges, do not anywhere intersect them, and in consequence the index relating to the replaced edge is always 0.

Thus in fig. 4 the edge  $m$  is entirely removed, while  $\frac{1}{2}$  of  $n$  and  $\frac{1}{4}$  of  $s$  are theoretically cut away by the truncating face, whence its symbol becomes  $(024)$ . The symbol of the face in fig. 5 is similarly  $304$ , and of that in fig. 6 is  $320$ .

Fig. 4.

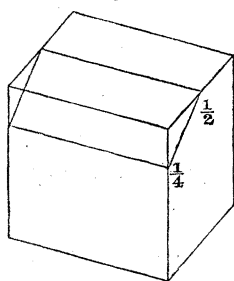


Fig. 5.

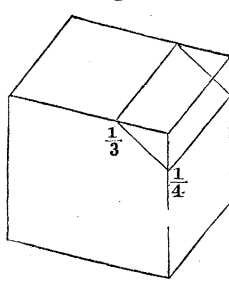
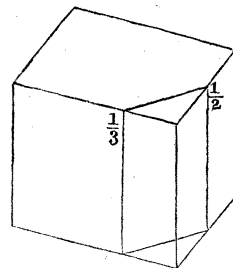


Fig. 6.



Thus recollecting that the symbols of primary faces have two zeros, and of faces on the edges one, the position of any face denoted by any symbol may be readily perceived and easily remembered.

It may be noticed, that the order in which the indices stand in each of the preceding symbols is that of the letters  $m$ ,  $n$ ,  $s$ ; and in the symbol of any face whatever, the first index always denotes the reciprocal of the proportion supposed to be cut away from one of the edges  $m$ ; the second index, the reciprocal of the proportion removed from one of the edges  $n$ ; and the third, the reciprocal of the proportion removed from one of the edges  $s$ .

Faces may be expressed by their symbols, as face  $100$  or  $111$ .

An angle between two faces may be expressed by means of their symbols, as the angle between  $100$  and  $111$ , or simply the angle  $100, 111$ .

A zone may be expressed by the symbols of any two faces in it, as  $(100, 111)$ .

There are in each of the systems, except the cubic, angles between particular faces by which the crystals belonging to the different minerals contained respectively in each are distinguished from each other. These will be termed *elementary angles*.

The crystals of the different minerals comprised in the cubic system constitute geometrically a single isomorphous group, and hence the elementary angle is the same throughout the group.

In the pyramidal system, the elementary angle is between a face  $001$  and some other face assumed to be denoted by the symbol  $101$ .

In the rhombohedral system, the elementary angle is the angle between the face  $111$  and some other face denoted by the symbol  $100$ .

It is thus that at the very outset of our investigations of the forms and angles of crystals an arbitrary assumption of certain elements, not indicated by the crystals themselves, is forced upon us. And hence it is not surprising that such an apparently capricious choice, as will be afterwards pointed out, of faces to be represented by particular symbols should have been exercised by different observers.

But, it may be asked, is the assumption of particular faces to represent those of the primary and other forms as arbitrary as is here supposed?

*Cleavage.*—It is well known that HAÜY regarded cleavage as a natural indication of primary form, and the same opinion is probably entertained by some crystallographers of the present day. It is therefore necessary, before we proceed, to show that cleavage does not afford any definite rule for the choice of a form to be regarded as the primary, but that it is properly only a physical character of the same nature as that of slaty structure generally. It seems to be only a separation of plates of crystalline matter parallel to particular faces of the crystal, and the tendency to separate parallel to such faces may be conceived to depend upon the relative degrees of cohesion of the crystalline particles in planes parallel to different surfaces.

This view appears to be supported by the directions of the cleavage in different crystals belonging to the cubic system. In some of these, as in galena, the particles cohere with less force in planes parallel to the cubic surfaces than to those of any other of its forms. In fluor the cubic surfaces cohere so firmly as to be inseparable when the crystals are broken, but the octahedral surfaces are held together with so small a force as to separate easily when the crystalline mass is broken. In blende the dodecahedral faces are those of easy separation, while the cohesion at the surfaces of the cubic and octahedral forms is so perfect as not to show a trace of cleavage in those directions. In many cases there are crystals without any perceptible cleavage, and in others there is no cleavage parallel to the assumed primary faces, although there are cleavages in other directions; and in other instances there are two or more sets of cleavages, either of which might be taken to indicate the primary form.

It does not appear that this greater or less degree of cohesive force at the surfaces of particular faces has received any satisfactory explanation, and it is unnecessary here to consider it further; the present object being only to show that the direction of the cleavage may be parallel to the faces of any one or more of the many forms that a crystal belonging to any system may present, and cannot therefore be a certain guide to the choice of an elementary or primary form.



*Magnitude of Indices.*—It has also been suggested that the numerical values of the indices of secondary faces should in some degree influence the choice of the form to be regarded as the primary. For as it has been observed that in the cubic system the highest numerical index is seldom greater than 6, it has been inferred as a sort of law that all indices ought to be expressible in low numbers, and that any form which allows the other forms of the crystal to be denoted by the lowest numbers, is on that account entitled to a preference as a primary form.

But on looking through the lists of indices in the new edition of WILLIAM PHILLIPS'S 'Mineralogy,' several instances occur of indices as high as 23, and there does not appear theoretically to be any reason why still higher numbers might not be so employed, as they occasionally are in the accompanying Tables, and in a recent paper on Quartz by M. DESCLOITZEAUX, in which indices of observed *direct* faces are as high as 32, and of *direct* faces corresponding to observed *inverse* faces as high as 80.

The terms *direct* and *inverse* faces relate only to rhombohedral crystals, and may be explained as follows:—

Let  $h k k$  be the symbol of any direct face in the quadrant  $111, 100$  of the zone  $111, 100$ , and let  $p q q$  be that of a face in the same zone lying on the opposite side of the principal axis of the crystal and making an equal angle with the face  $111$ . Then  $p q q$  is termed an inverse face.

It frequently happens that direct faces and the corresponding inverse faces occur in pairs upon the same crystal.

Sometimes, however, a face occurs in a direct position without a corresponding inverse face, and sometimes an inverse face occurs without any corresponding direct face. In the cases where the inverse faces occur alone, the symbols of the corresponding direct faces have been computed and entered in the accompanying Tables, R 1 and R 2, as those of direct faces.

But it is necessary to consider the subject of high indices rather more in detail.

The indices of faces do not express absolute numbers high or low, but only the ratios of the numbers so employed. Thus the symbol of  $102$  is equivalent to, and denotes geometrically, the same face as  $204, 306, 10020, 990198$ , or any other numbers in the same ratio.

It is usual to express indices in their lowest terms in order to avoid writing figures unnecessarily, and to save time and trouble in the calculations of indices and angles. But it has the disadvantage of masking, and thus keeping out of sight, the true relations among such indices as are now under consideration.

Let it, for example, be proposed to ascertain what faces can occur in the pyramidal system between the faces  $101$  and  $103$ . The face  $102$  first presents itself as the intermediate face. But let  $101$  be expressed by  $202$ , and  $103$  by  $206$ , and  $203, 204$ , and  $205$  appear as intermediate faces, and present to the eye a relation which is not so immediately perceived if we state the numbers as  $203, 102, 205$ . If the expressions  $101$  and  $103$  become  $303$  and  $309$ , the symbols  $304, 305, 306 (=102), 307, 308$

appear to be those of possible intermediate faces. And when higher multiples are employed, the number of possible intermediate faces is increased.

If the multiplier is 20 the original symbols become 20 0 20 and 20 0 60, showing thirty-nine possible intermediate faces between 1 0 1 and 1 0 3, as in the following list:—

20 0 20 ..... = 1 0 1	20 0 41
20 0 21	20 0 42 = 10 0 21
20 0 22 = 10 0 11	20 0 43
20 0 23	20 0 44 = 10 0 22 = 5 0 11
20 0 24 = 10 0 12 = 5 0 6	20 0 45 ..... = 4 0 9
20 0 25 ..... = 4 0 5	20 0 46 = 10 0 23
20 0 26 = 10 0 13	20 0 47
20 0 27	20 0 48 = 10 0 24 = 5 0 12
20 0 28 = 10 0 14 = 5 0 7	20 0 49
20 0 29	20 0 50 = 10 0 25 = 4 0 10 = 2 0 5
20 0 30 = 10 0 15 = 4 0 6 = 2 0 3	20 0 51
20 0 31	20 0 52 = 10 0 26 = 5 0 13
20 0 32 = 10 0 16 = 5 0 8	20 0 53
20 0 33	20 0 54 = 10 0 27
20 0 34 = 10 0 17	20 0 55 ..... = 4 0 11
20 0 35 ..... = 4 0 7	20 0 56 = 10 0 28 = 5 0 14
20 0 36 = 10 0 18 = 5 0 9	20 0 57
20 0 37	20 0 58 = 10 0 29
20 0 38 = 10 0 19	20 0 59
20 0 39	20 0 60 = 10 0 30 = 5 0 15 ..... = 1 0 3
20 0 40 = 10 0 20 = 4 0 8 = 2 0 4 = 1 0 2	

It is apparent from this list, that when the number of faces in a zone increases, one or more of the indices by which the additional faces can be expressed must become larger.

It is equally clear, that when the number of faces in a zone increases, the angles between adjacent faces must become less, and hence when the angle between any two adjacent faces in a zone becomes less, one or more of the indices required to denote one or both of such faces must become larger. To limit therefore the magnitude of indices would virtually be to limit the number of possible faces in a zone.

We have before shown that the indices of faces are the reciprocals of the proportions of the primary edges conceived to be cut away by such faces, and in order to provide an exact expression of these proportions, in the cases of high indices, we have only to imagine the primary edges divided into a sufficiently large number of equal parts to allow of such an exact expression.

It is evident that the symbols in the preceding list accurately denote separate faces. But it may be asked, are all these faces equally possible, and if so, are they equally probable; or is 20 0 40 more probable than 20 0 39, because the indices 20 0 40 can be divided by 20 without a remainder, and because those of 20 0 39 cannot? There does not appear to be any theoretical reason why any one of these faces should be more probable than any other.

But it may be said that the face 20 0 40 has been frequently observed and expressed by the symbol 1 0 2, while there is no published record of the faces 20 0 39 or 20 0 41 having ever been noticed, and that nature has therefore shown a preference of one of

these faces to either of the others, and has thus established a sort of probability that the face so preferred will occur more frequently than the others.

Is it, however, true that the others have not been observed, although the observations have not been recorded; or is it not equally probable that such faces have been frequently seen and measured, but from the nearness of their angles with other faces, to the angles of the face 1 0 2 with the same other faces, they have been taken as imperfect instances of the face 1 0 2?

If the three faces above referred to should occur on the same crystal they might doubtless be distinguished, but if they occur on different crystals there is only one natural test for ascertaining whether they are really different, or are only imperfect examples of the same face.

This test is, to find whether the face on one of the crystals is common to any two zones, and whether the face on the other crystal is common to the same two zones. If the faces are common to the same two zones on both crystals, the faces are similar, and if they are not, they are different faces. But from the occurrence of crystals generally in only small fragments, it does not often happen that any of these are sufficiently perfect to present clearly the faces of any two required zones, and hence the difficulty of ascertaining with certainty the identity or otherwise of such nearly corresponding faces.

But although there may be no theoretical limit to the magnitude of indices, there is a practical limit, beyond which faces denoted by high indices would not be distinguishable with certainty from each other. This limit occurs when the difference of the angles of any two faces with some other face is within the probable range of error of ordinary measurement, arising from imperfection of faces, or from imperfect observation. It is difficult to state the probable amount of error which may be ascribed to these causes, but perhaps about half a degree to a degree may not be an improbable quantity.

The following are the several angles between the face 0 0 1 and the first and last ten faces in the preceding list:—

20 0 21 = 43° 11'	20 0 26 = 37° 9'	20 0 51 = 21° 8'	20 0 56 = 19° 23'
20 0 22 = 41 51	20 0 27 = 36 7	20 0 52 = 20 45	20 0 57 = 19 4
20 0 23 = 40 35	20 0 28 = 35 8	20 0 53 = 20 24	20 0 58 = 18 46
20 0 24 = 39 23	20 0 29 = 34 12	20 0 54 = 20 3	20 0 59 = 18 28
20 0 25 = 38 15	20 0 30 = 33 18	20 0 55 = 19 42	20 0 60 = 18 11

It appears from the small differences in the angles in this statement, that some of the adjacent faces in the preceding list might not be distinguishable from each other by ordinary measurement, and that the zone test already alluded to might here become necessary.

But besides these possible sources of error in our crystallographic investigations, another element of disagreement occurs in practical crystallography.

The ordinary method of examining a crystal is, first, to determine, from the character and relative positions of its faces, the system to which it belongs.

With regard to the cubic, pyramidal, and rhombohedral systems, this process is

attended with little difficulty; but in the cases of the prismatic, oblique, and anorthic systems it is far from definite.

Having determined the system to which the crystal belongs, the next operation is to measure the angles between the several faces, and to select from among these the angle which is to be regarded as elementary. And as the faces themselves afford no assistance in fixing upon a proper angle, the selection is left to the arbitrary choice of the observer.

After having fixed upon the elementary angle, the symbols of the faces which at their intersection produce that angle are determined, and from these and the known angles between other faces, other symbols and angles may be found.

It is usual to measure as large a number of angles as the crystals in our possession will allow. And when the observed angles between apparently similar pairs of faces are slightly different, these slight differences have been hitherto ascribed to the imperfection of the measured faces, and a mean of the measured angles has in this case been assumed as the probable angle between the faces. But as it seldom if ever happens that the mean of any number of measured angles agrees exactly with the geometric angle corresponding to the assumed symbols of the faces yielding that angle, it is usual to *adjust*, as it is termed, the symbols and angles in question, by increasing or diminishing the value of the measured angles, and by altering, if necessary, the indices until both the angles and indices are in conformity with the elementary faces and angles of the crystals.

This process of adjustment is the element of disagreement above alluded to, and is really the adapting or fitting to a particular primary form the angles and symbols of the other faces of the crystal, so that if a new primary form has to be assumed, the new symbols may become expressible in only very high numbers, and a readjustment of angles and symbols has then to take place. This process will, however, be governed by the number and greater or less degree of perfection of the observed faces, and by the care with which the angles have been measured.

In nearly all the hitherto published works on mineralogy the faces of crystals have, for the convenience of writing and printing, been distinguished by letters. But it appears to the writer that the use of WHEWELL'S symbols for this purpose instead of letters, is so much to be preferred, on account of its presenting at once to the eye the geometrical relations of the several faces, as to more than counterbalance the inconvenience of having three or more figures to write instead of one letter; and particularly as these letters are frequently accompanied by one or more conventional marks of distinction which are troublesome both to write and to remember.

*Construction of Tables.*—We have now to explain the construction of the accompanying Tables, to consider the facts they present, and their results.

These Tables do not contain all the observed angles of the crystals belonging to the different minerals, but only such as have been, or might have been used in each case as elementary angles, these being sufficient for the purposes of this paper.

No table is given of the *cubic system*, as that system consists of only a single strictly isomorphous group.

In the Tables of the two other systems the names of the minerals at present comprised respectively in each, are given in an upper horizontal line in the order in which they occur in the new edition of the work of WILLIAM PHILLIPS.

In the *pyramidal system*, Table P 1 (Plate II.), the first vertical column below the name of each mineral contains the angles between the face  $001$  and the observed faces in the quadrant  $(001\dots100)$ .

The third column contains the angles between the face  $001$  and the observed faces in the quadrant  $(001\dots110)$ .

The second and fourth columns contain the symbols hitherto assigned to the several faces which make with the face  $001$  the angles shown in the first and third columns.

In the *rhombohedral system*, Table R 1 (Plate IV.), the first vertical column under each mineral contains the angles between the face  $111$  and the observed direct faces in the quadrant  $(111\dots100)$ , and also the angles between the face  $111$  and some observed inverse faces in the quadrant  $(111\dots\bar{2}11)$ .

*Facts presented by these Tables.*—The most important of these are the horizontal ranges of nearly equal angles, as shown in each system, and the general disagreement in the symbols of the faces which make with some other face those nearly agreeing angles.

With regard to these facts no difference of opinion can arise, unless the sources from which they have been derived (some of the most recent works on mineralogy) are incorrect. And it is possible that from inaccurate transcription, or error in printing, some of the angles may not be correctly given, and hence perhaps may have arisen some of the anomalies presented by the Tables.

But differences of opinion may be entertained relative to the interpretation of these facts.

The interpretation to which the writer inclines, is that the near agreement in angle between two corresponding faces is not simply accidental, but that it is the effect of some natural relation, not hitherto noticed, among all the crystals in each respective system; and hence that where the angles between particular faces nearly agree, there ought to be a corresponding agreement in the forms of their symbols.

It would seem from these disagreements between angles and symbols that a tacit impression has existed in the minds of crystallographers, that the crystals of different minerals are always to be regarded as individual and isolated cases of crystallization, except in the instances of isomorphous groups.

The idea that first suggested itself to the mind of the writer as to the nature of these newly observed relations was, that as in each system the angles between particular faces nearly agree, there might also be a near agreement in the elementary angles of all the crystals belonging to each system; and an examination of the crystals in the pyramidal and rhombohedral systems with a view of ascertaining how far this idea might be well founded, has led to the very unexpected result, that a geometrical isomorphism can be shown to exist throughout each of these two systems, and consequently that similar relations may be imagined to exist in the others.

The following remarks will further illustrate and explain the relations in question.

*Pyramidal tables*, P 1, P 2 (Plates II. and III.).—In the pyramidal system there are in the

case of each mineral two possible prisms,  $a$  and  $b$ , of equal dimensions, but related to each other in position in the manner shown in the annexed sketch, fig. 7, which is a projection of the two intersecting prisms on a plane parallel to their common terminal face  $001$ . When these prisms occur together in natural crystals, they present the outline shown in fig. 8. From which it is evident, that if the prism  $a$  be taken as the primary prism, the faces of  $b$  will appear to truncate the lateral edges of  $a$ ; and conversely, if  $b$  is taken as the primary. It is also clear that any face which truncates a terminal edge of  $a$  would be a truncation of a solid angle of  $b$ , and conversely.

It does not appear that natural crystals afford any indication to govern our choice of one or the other of these prisms as the primary form of any mineral; and as relates to the crystals of any single mineral, it is indifferent which prism we select. But it is not so with respect to the crystals of any group of two or more different minerals which have afterwards to be compared with each other.

In these cases the same prism,  $a$  or  $b$ , should be assumed as the primary prism throughout the group. And as the position of  $a$  is that in which pyramidal crystals are usually drawn, the faces of all the crystals exhibited in the Tables P 1 and P 2, are referred to elementary prisms assumed to be in the position of  $a$ .

It has hence been necessary where  $b$  has been given in previous works, and in Table P 1 (Plate II.), as the elementary prism of some of the minerals, to substitute  $a$  for  $b$  in Table P 2 (Plate III.), and to make the necessary changes in the symbols of the other faces consequent upon the change of the elementary prism.

It appears in Table P 1 (Plate II.), that the faces of different minerals making equal or nearly equal angles with the face  $001$  have been referred by different observers to a prism in the position of  $a$  in some cases, and to a prism  $b$  in others.

Thus a face which in Towanite makes an angle of  $54^\circ 20'$  with the face  $001$ , has for its symbol  $111$ , denoting a face replacing a solid angle; while in Braunite, a face making the same angle with  $001$ , has for its symbol  $011$ , denoting a face on an edge, and showing that if the prism  $a$  has been used for the primary of Towanite, the prism  $b$  has been used for that of Braunite.

Again, it will be seen in Table P 1 (Plate II.), that two of the faces of Scheelite make angles of  $36^\circ 34'$  and  $46^\circ 22'$  respectively with the face  $001$ , and that two faces making the same angles with  $001$  occur in Wulfenite. Yet, instead of regarding these as angles between corresponding faces in the two minerals, they are referred in Scheelite to the prism  $a$ , and to  $b$  in Wulfenite. And as if further to complicate and conceal the identity of these crystalline forms, the symbols assigned to their corresponding faces also disagree.

Having reduced the elementary prisms throughout the system to corresponding positions, the next point for consideration has been the angle which might be selected as a proximate elementary angle of the group.

Fig. 7.

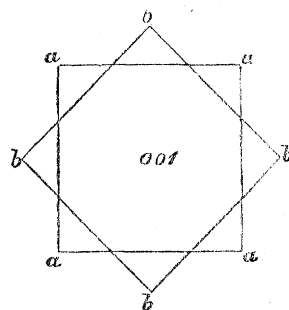
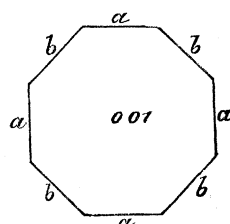


Fig. 8.



A guide to this selection presents itself in the crystals of Apophyllite, the angles of which scarcely differ from those of a cube, and therefore admit an elementary angle of exactly  $45^\circ$ , which angle has been assumed as a variable type of the elementary angles throughout the system.

Whether this angle of  $45^\circ$  is accidental, or is the effect of any at present unknown relation between the cube and the square prism, the writer is not aware. But it is a rather curious fact that there should also be in the rhombohedral system a series of elementary angles of about  $55^\circ$ , not differing much from angles between two corresponding faces of a cube.

On referring to Table P 1 (Plate II.), it will be seen that the crystals of Somervillite, Faujasite and Romeine are represented by single faces only, to each of which the symbol 1 0 1 has been hitherto applied. But as the occurrence of only those particular faces on the three crystals must be regarded as purely accidental, there is no reason for applying that particular symbol to either of them. And as the symbols of faces are not in any manner indicated by the faces themselves, we are at liberty to assign to those or to any other faces any symbols which are consistent with the geometrical relations of the faces in question. And hence have arisen some of the differences of symbols shown in the Tables P 1 and P 2 (Plates II. and III.).

The elementary angle of the crystals of Towanite being  $44^\circ 35'$ , and as those crystals present as great a number of observed faces as those of any other mineral in this group, and as the position of the primary form may be assumed to be that of the prism  $a$ , the symbols and angles of this mineral have been used as standards for comparing and regulating the symbols of the remainder of the group, as shown in Table P 2 (Plate III.).

The method of reducing the series of minerals in each of the pyramidal and rhombohedral systems to single groups, has been to form in the first instance in each case small groups, and then to associate these into larger ones.

Thus in Table P 1 (Plate II.), there appears in

Towanite . an angle of  $19^\circ 13'$ , corresponding to the symbol 1 1 4  
 Anatase . . an angle of  $19^\circ 34'$ , corresponding to the symbol 1 1 7  
 Apophyllite an angle of  $19^\circ 30'$ , corresponding to the symbol 1 1 5,

forming a small geometrical group, to each member of which in Table P 2 the symbol 1 1 4 of Towanite has been assigned as a common and connecting symbol of the group.

Again, in Table P 1 (Plate II.), in

Towanite . an angle of  $33^\circ 18'$  has the symbol 2 0 3  
 Rutile . . an angle of  $32^\circ 47'$  has the symbol 1 0 1  
 Ox. Tin . . an angle of  $33^\circ 55'$  has the symbol 1 0 1  
 Zircon . . an angle of  $32^\circ 38'$  has the symbol 1 0 1  
 Somervillite, an angle of  $32^\circ 51'$  has the symbol 1 0 1,

producing another analogous group under the connecting symbol 2 0 3 of Towanite, and thus directly associating four other minerals with Towanite.



But this direct method has not in all cases been practicable, and it has consequently been necessary in some instances to associate some small groups indirectly; that is, to form small groups not immediately connected with Towanite, and then to connect them with Towanite by means of some intermediate minerals.

Thus it is already shown, that Anatase and Apophyllite may be regarded as plesiomorphous with Towanite.

But Anatase may also be associated with Uranite by the angles—

Anatase  $39^{\circ} 57'$ ,      Uranite .  $39^{\circ} 53'$ ;

with Calomel and Matlockite by the angles—

Anatase  $60^{\circ} 38'$ ,      Calomel .  $60^{\circ} 9'$ ,  
Matlockite  $60 36$ ,

thus connecting indirectly Uranite, Calomel, and Matlockite, with Towanite. And by proceeding in this manner, it has been found that the entire series of pyramidal crystals can be brought to agree very nearly in their elements with those of Towanite.

In Table P 1 the series of elementary angles range from  $21^{\circ} 5'$  to  $61^{\circ} 38'$ . In P 2 the new series, chiefly of angles which, although probable to occur, have not yet been observed, appears as a horizontal line of nearly agreeing angles, the greater part of which have been computed from observed faces.

Table P 2 also exhibits the changes of symbols consequent upon the adoption of these angles as elementary, and upon the changes of the fundamental prism from  $b$  to  $a$  where required. The entire series of pyramidal crystals is thus presented as a single geometrical plesiomorphous group.

*Rhombohedral Tables*, R 1, R 2 (Plates IV. and V.).—After the explanations already given relative to the pyramidal tables, only a few additional remarks will be necessary concerning those of the rhombohedral system.

The construction of Table R 1 has been already explained, and the distinction between direct and inverse faces has been pointed out.

On referring to the Table R 1 (Plate IV.), we may perceive the same horizontal ranges of nearly equal angles, and the same disparity between angles and symbols, as in the pyramidal system; and we may also observe, that the rhombohedral elementary angles range in this Table from  $20^{\circ} 50'$  in Millerite to  $81^{\circ} 20'$  in Parisite. It will also be seen in this system, as in the pyramidal, that where a mineral is represented by a single face, the angle of that face with the face 1 1 1 is assumed to be the elementary angle of that mineral, disregarding altogether its apparent relation to the nearly equal angles between the corresponding faces of other minerals.

In the construction of Table R 2, the first point to be considered was the angle which should become the elementary type of the system.

It appears in R 1, that there is one principal horizontal range of nearly agreeing angles of about  $37^{\circ}$  or  $38^{\circ}$ ; another, of about  $43^{\circ}$  to  $46^{\circ}$ ; and another, of about  $55^{\circ}$  to  $58^{\circ}$ , either of which might supply this elementary type: but as there are more instances of

agreeing angles in the range between  $55^\circ$  and  $58^\circ$  than in the others, the elementary angle of  $57^\circ 30'$  of Hematite has been adopted as the elementary type.

Now Hematite may be associated with Quartz by the angles—

Hematite  $75^\circ 42'$ ,      Quartz  $75^\circ 18'$ ,

Hematite  $82^\circ 44'$ ,      Quartz  $82^\circ 31'$ ,

and with Calcite by the angles—

Hematite  $38^\circ 7'$ ,      Calcite  $38^\circ 17'$ ,

Hematite  $44^\circ 27'$ ,      Calcite  $44^\circ 37'$ ,

forming a small plesiomorphous group, with which other individuals and other groups have been successively connected, so as to reduce the entire system to a single plesiomorphous group of nearly equal angles, as shown in Table R 2.

To exhibit the effect of this grouping upon Hematite, Quartz, and Calcite, the Table T has been framed, by first assuming new primary forms for Quartz and Calcite, then transferring to each of the three minerals the symbols of the other two, and then calculating in each case the angles corresponding to the transferred symbols.

Symbols common to	T. Corresponding Angles of			Symbols common to	T. Corresponding Angles of		
	Hematite.	Quartz.	Calcite.		Hematite.	Quartz.	Calcite.
6 5 5	$5^\circ 36'$	$5^\circ 26'$	$5^\circ 15'$	$10 \bar{1} \bar{1}$	$65^\circ 8'$	$64^\circ 30'$	$63^\circ 50'$
10 7 7	$11^\circ 6'$	$10^\circ 47'$	$10^\circ 29'$	$68 \bar{7} \bar{7}$	$65^\circ 22'$	$64^\circ 43'$	$64^\circ 3'$
8 5 5	$14^\circ 40'$	$14^\circ 15'$	$13^\circ 52'$	$8 \bar{1} \bar{1}$	$66^\circ 59'$	$66^\circ 23'$	$65^\circ 45'$
7 4 4	$17^\circ 26'$	$16^\circ 57'$	$16^\circ 29'$	$13 \bar{2} \bar{2}$	$69^\circ 5'$	$68^\circ 31'$	$67^\circ 55'$
2 1 1	$21^\circ 25'$	$20^\circ 52'$	$20^\circ 18'$	$5 \bar{1} \bar{1}$	$72^\circ 20'$	$71^\circ 50'$	$71^\circ 20'$
11 5 5	$24^\circ 9'$	$23^\circ 32'$	$22^\circ 55'$	$17 \bar{4} \bar{4}$	$74^\circ 44'$	$74^\circ 18'$	$73^\circ 51'$
5 2 2	$27^\circ 37'$	$26^\circ 56'$	$26^\circ 15'$	$4 \bar{1} \bar{1}$	$75^\circ 42'$	$75^\circ 18'$	$74^\circ 53'$
3 1 1	$32^\circ 6'$	$31^\circ 22'$	$30^\circ 37'$	$19 \bar{5} \bar{5}$	$76^\circ 34'$	$76^\circ 11'$	$75^\circ 47'$
22 7 7	$33^\circ 11'$	$32^\circ 25'$	$31^\circ 40'$	$82 \bar{23} \bar{23}$	$77^\circ 41'$	$77^\circ 19'$	$76^\circ 57'$
17 5 5	$34^\circ 54'$	$34^\circ 7'$	$33^\circ 20'$	$7 \bar{2} \bar{2}$	$78^\circ 0'$	$77^\circ 40'$	$77^\circ 18'$
4 1 1	$38^\circ 7'$	$37^\circ 19'$	$36^\circ 30'$	$10 \bar{3} \bar{3}$	$78^\circ 54'$	$78^\circ 35'$	$78^\circ 15'$
13 3 3	$39^\circ 35'$	$38^\circ 44'$	$37^\circ 55'$	$23 \bar{7} \bar{7}$	$79^\circ 11'$	$78^\circ 52'$	$78^\circ 32'$
31 7 7	$39^\circ 56'$	$39^\circ 10'$	$38^\circ 17'$	$3 \bar{1} \bar{1}$	$80^\circ 57'$	$80^\circ 41'$	$80^\circ 25'$
6 1 1	$44^\circ 27'$	$43^\circ 37'$	$42^\circ 46'$	$122 \bar{43} \bar{43}$	$82^\circ 5'$	$81^\circ 51'$	$81^\circ 37'$
20 3 3	$45^\circ 45'$	$44^\circ 54'$	$44^\circ 3'$	$17 \bar{6} \bar{6}$	$82^\circ 7'$	$81^\circ 53'$	$81^\circ 38'$
7 1 1	$46^\circ 18'$	$45^\circ 28'$	$44^\circ 37'$	$11 \bar{4} \bar{4}$	$82^\circ 44'$	$82^\circ 31'$	$82^\circ 18'$
13 1 1	$51^\circ 28'$	$50^\circ 39'$	$49^\circ 49'$	$35 \bar{13} \bar{13}$	$83^\circ 11'$	$82^\circ 59'$	$82^\circ 47'$
14 1 1	$51^\circ 54'$	$51^\circ 5'$	$50^\circ 15'$	$8 \bar{3} \bar{3}$	$83^\circ 24'$	$83^\circ 12'$	$82^\circ 59'$
16 1 1	$52^\circ 36'$	$51^\circ 47'$	$50^\circ 58'$	$21 \bar{8} \bar{8}$	$83^\circ 44'$	$83^\circ 33'$	$83^\circ 19'$
20 1 1	$53^\circ 35'$	$52^\circ 47'$	$51^\circ 57'$	$76 \bar{29} \bar{29}$	$83^\circ 46'$	$83^\circ 35'$	$83^\circ 24'$
1 0 0	$57^\circ 30'$	$56^\circ 44'$	$55^\circ 57'$	$19 \bar{8} \bar{8}$	$85^\circ 53'$	$85^\circ 50'$	$85^\circ 42'$
$11 \bar{1} \bar{1}$	$64^\circ 27'$	$63^\circ 48'$	$63^\circ 7'$	$116 \bar{49} \bar{49}$	$86^\circ 1'$	$85^\circ 54'$	$85^\circ 47'$

The assumed new primary form of Quartz has been observed by M. DESCLOIZEAUX as an inverse and as a direct face, and gives  $56^{\circ} 44'$  as the angle between the new  $100$  and  $111$ .

The new primary form of Calcite corresponds to the old direct form  $8\bar{1}\bar{1}$ , deduced from the old inverse form  $\bar{4}55$ , and gives  $100, 111=55^{\circ} 57'$ .

Table T also contains many additional symbols with their corresponding angles, to show the relations between greatly disagreeing symbols and nearly agreeing angles.

Thus the angle of  $111$  with the face denoted by  $82\bar{2}\bar{3}\bar{2}\bar{3}$ , differs but  $19'$  from the nearly corresponding angle of  $111$  with the face denoted by  $7\bar{2}\bar{2}$ . The difference between the angle  $111, 122\bar{4}\bar{3}\bar{4}\bar{3}$ , and  $111, 17\bar{6}\bar{6}$  is only  $2'$ , rendering the distinction of these faces by ordinary measurement impracticable.

This Table is open to other remarks of the same nature, but after the previous explanations it appears unnecessary to pursue the subject further.

These new elementary angles, as well as those in the pyramidal system, will doubtless undergo alterations as more numerous and better crystals present themselves for examination.

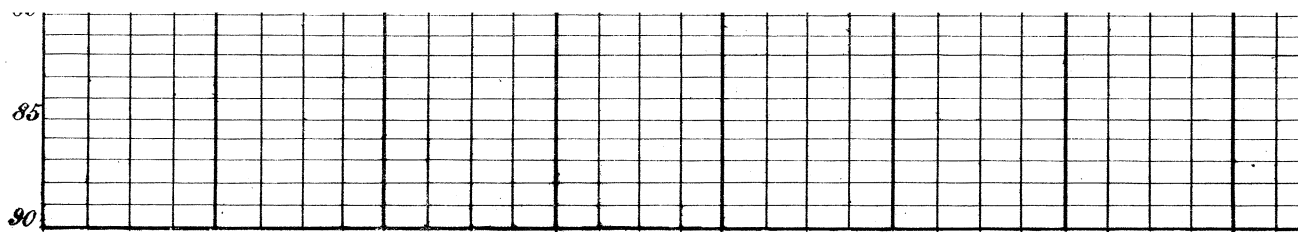
[illegible]

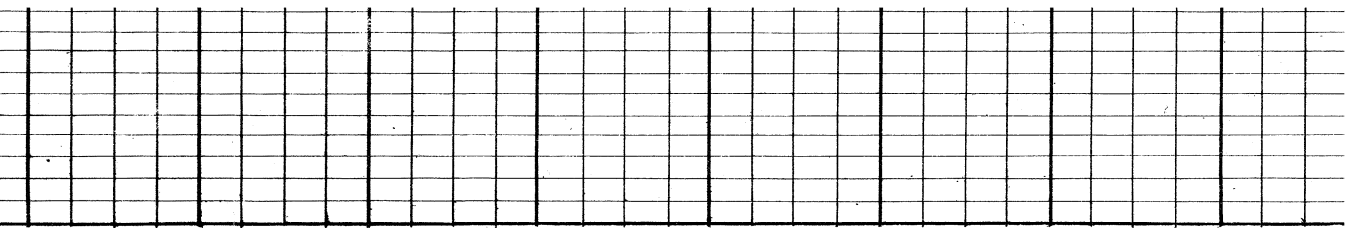
[illegible]

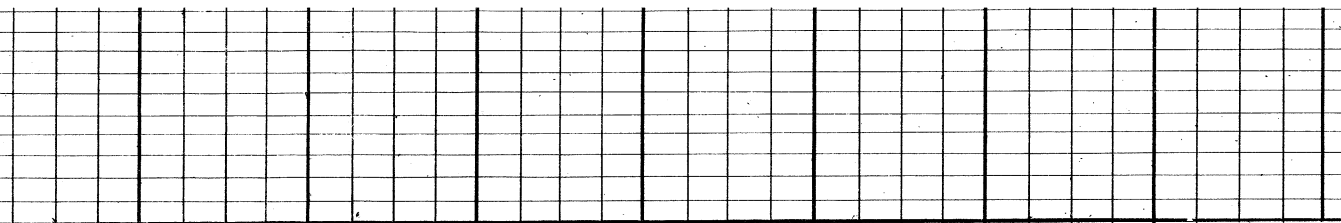
<i>Auvasite</i>	<i>Edingtonite</i>	<i>Scheelite</i>	<i>Stolzite</i>	<i>Wulfenite</i>	<i>Xenotime</i>	<i>Uranite</i>	<i>Chiolite</i>
1 0 1	0 1 0 1	0 1 0 1	0 1 0 1	0 1 0 1	0 1 0 1	0 1 0 1	0 1 0 1
				7 55 1116			
		76 32 105					
35 26 102				26 18 229			
				27 41 103			
						32 4 102	
		34 58 113					
	36 34 102			36 34 113			
				38 12 102		39 53 203	
					41 101		
43 40 101						43 25	
		46 22 112		46 22 203			46
			47 53 112				47 8 101
48 101						51 25 101	
	53 28 111						
		56 1 101					56 43 111
			57 21 101	57 34 101			
		64 31 111				64 12	
			65 43 111	65 48 111			
				67 2 302		68 15	
						71 8	
				73 19 332			
			77 17 221				











[illegible]

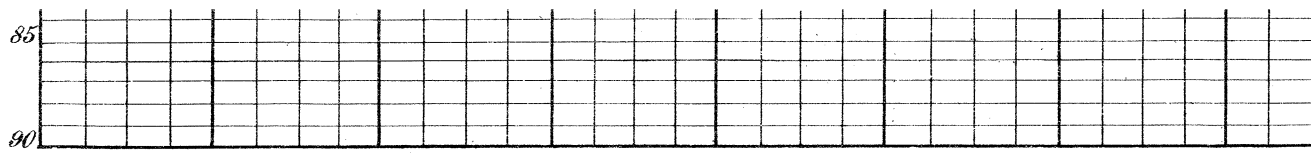
[illegible]

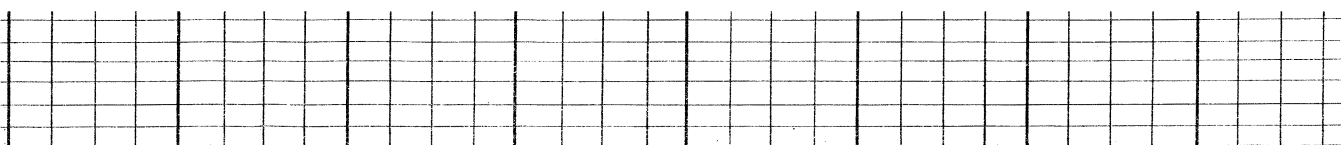
<i>Hausmannite</i>	<i>Idocrase</i>	<i>Zircon</i>	<i>Somervillite</i>	<i>Sarcobite</i>	<i>Scapolite</i>	<i>Apophyllite</i>	<i>Faujasite</i>
0 / 1	0 / 1	0 / 1	0 / 1	0 / 1	0 / 1	0 / 1	0 / 1
	14 10 104					14 3 104	
	72 44 308			22 41 50 12	33 46 409		19 30 1 14
28 58 2 25	28 9 338						
		32 38 203	32 51 203		31 50 449	32 2 508	30 32 55 12
	37 7 304						
		42 10 223		41 35 558			
44 24 101	43 51 101		44 5 101	45 7 101	44 44 101	45 1 01	44 37 101
	45 16 101						
	46 37 334						
49 36 605				51 27 504		51 21 504	52 45 403
	56 33 302						
58 57 665							
		61 6 443			61 50 443		60 32 554
	66 14 304						
		69 48 221					
	71 43 301						

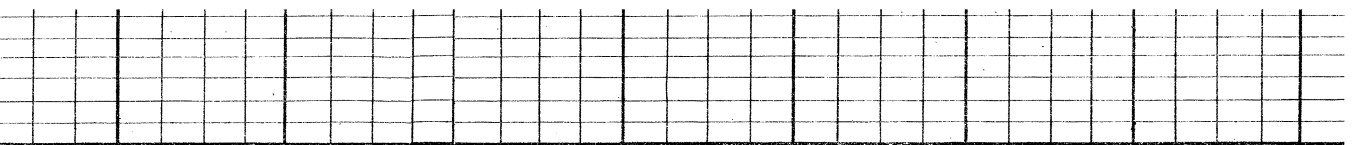
<i>Auvasite</i>	<i>Edingtonite</i>	<i>Scheelite</i>	<i>Stolzite</i>	<i>Wulfenite</i>	<i>Xenotime</i>	<i>Uranite</i>	<i>Chiolite</i>	
0 / 0 /	0 / 0 /	0 / 0 /	0 / 0 /	0 / 0 /	0 / 0 /	0 / 0 /	0 / 0 /	0 /
				7 55 906				
		16 32 30 10						
	25 26 113			26 18 102				
				27 41 3 38				
						32 4 508		
		34 58 112						
		36 34 3 04		36 34 3 04				
				38 12 99 16		39 53 506		
					41 6 07			
1 01	43 40 223	44 41 1 01	44 31 1 01	44 41 1 01				44 31
	45 21 1 01				45 24 1 01	45 5 1 01	45 21 1 01	
		46 22 3 34	47 56 908	46 22 3 34			47 8 334	
4 03						51 25 504		
	53 28 4 03							
		56 1 3 02		57 27 998	57 34 998		56 43 3 02	
		61 31 3 32	65 43 9 04	65 48 9 04	67 2 27 16			67 55
				73 19 27 08				
			77 17 9 02					



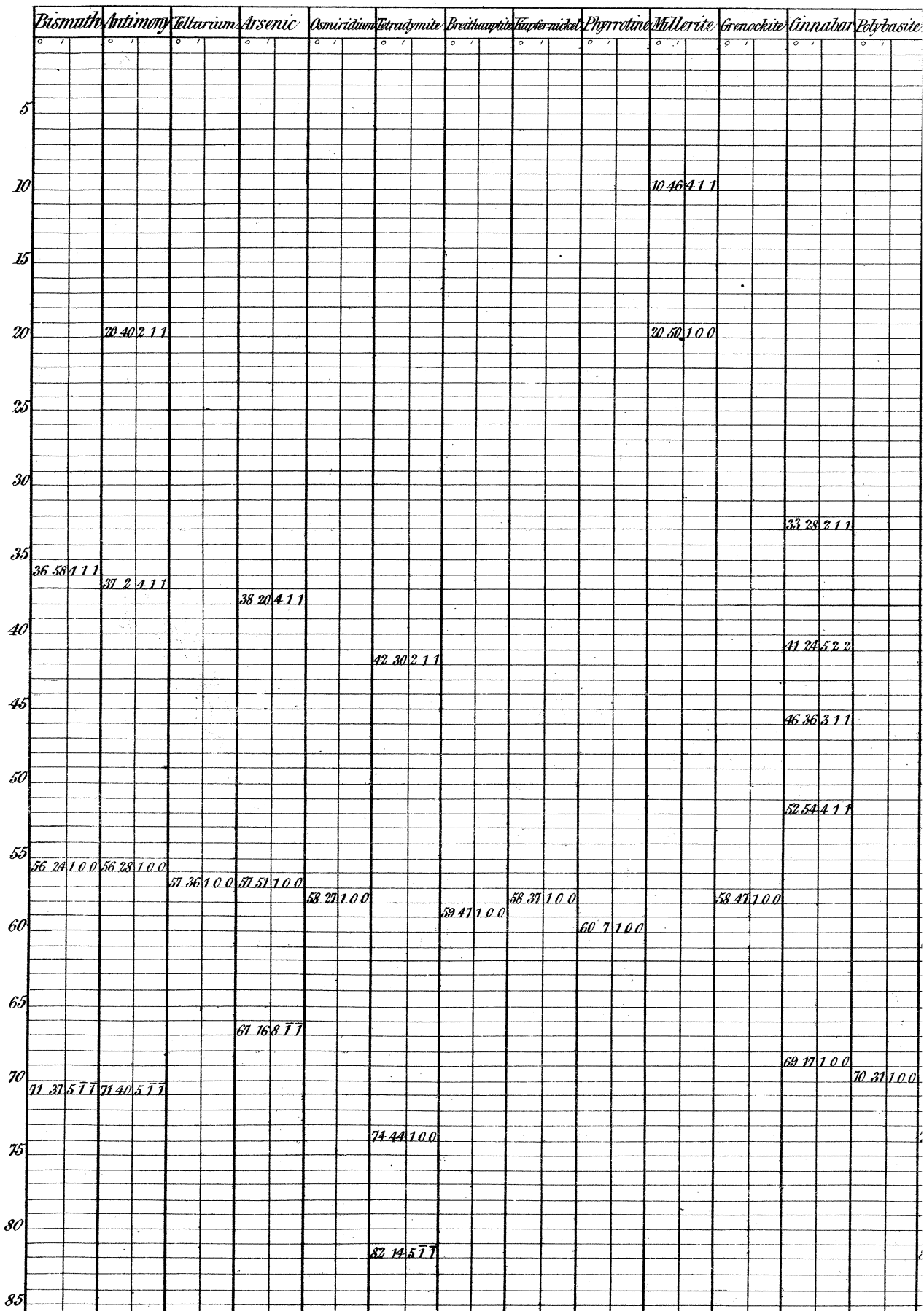








[illegible]

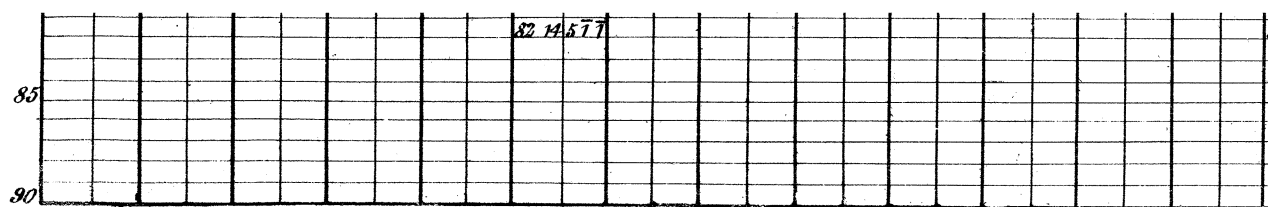


Polybasite	Pyrrhotite	Proustite	Xanthoconite	Spartulite	Hematite	Corundum	Quartz	Willemite	Emerald	Phenakite	Tourmaline	Endialyte	Nepheline	Bio.
					5 36 6 5 5									
					11 6 40 7 7									
12 49 2 1 1		13 3 2 1 1									14 79 4 1 1			
					17 26 7 4 4									
					21 25 2 1 1					20 52 4 1 1				
24 23 4 1 1	24 53 4 1 1				24 29 15 5									
					27 41 5 2 2						27 20 1 0 0			
							30 7 1 0 0						31 22 2 1 1	
							32 25 4 1 1							
				37 30 1 0 0							37 19 1 0 0			
				38 7 4 1 1	38 12 4 1 1									
42 18 1 0 0	42 54 1 0 0				44 27 6 1 1				44 56 1 0 0		45 57 7 1 1			
							49 14 5 7 7							
							51 47 1 0 0						50 38 4 1 1	
		54 18 8 7 7												
					57 30 1 0 0	57 34 1 0 0				56 44 5 7 7				
													59 10 1 0 0	
61 13 5 7 7	61 41 5 7 7										61 4 16 5 5			
							64 43 13 2 2				64 11 3 7 7			
					66 59 8 7 7									
							68 31 5 7 7				68 51 11 4 4	67 42 1 0 0		
70 31 1 0 0		69 30 1 0 0												70 -
					72 20 5 7 7	72 22 5 7 7								
74 38 3 7 7					75 42 4 7 7		75 19 7 2 2							
							77 19 16 5 5							
							78 52 3 7 7						78 25 5 7 7	
		79 25 5 7 7			80 57 3 7 7									79 41
82 11 11 4 4					82 44 11 4 4		81 57 8 3 3							
							82 31 13 5 5							
							83 35 5 2 2							
							85 54 23 10 10							

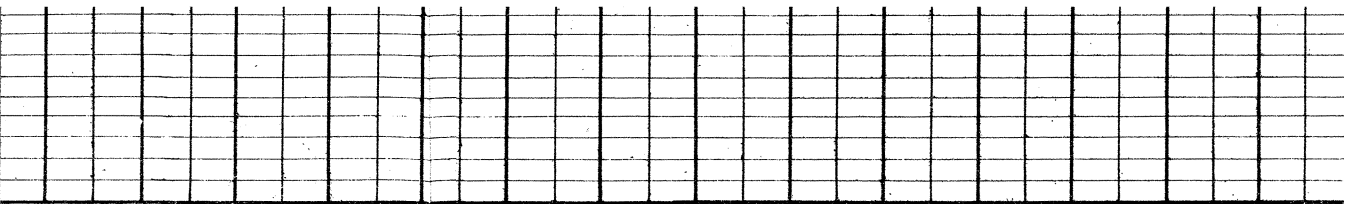


[illegible]





82	11	11	4	4	82	44	11	4	4	82	31	13	5	5
										83	35	5	2	2
										85	54	23	10	10



[illegible]

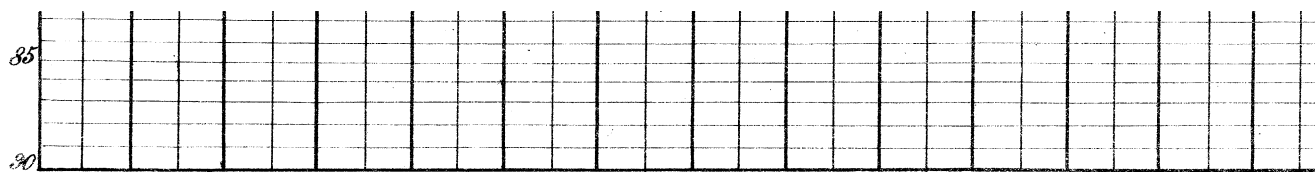
[illegible]

site	Pyargyrite	Proustite	Xanthorone	Spartalite	Hematite	Corundum	Quartz	Willemite	Emerald	Phenakite	Tourmaline	Eudialyte	Nepheline	Biotite
					5 36 6.5.5									
					11 6 10 1 1									
12 43 14 9.9		13 3 14 9.9									14 29 8.5.5			
					17 26 1 4 4									
					21 25 2 1 1					20 52 2 1 1				
24 28 26 11 11	24 53 25 11 11			24 9 11 5.5										
					27 41 5 2 2						27 20 5 2 2			
							30 7 3 1 1					31 22 3 1 1		
							32 25 22 7 7							
			37 30 4 1 1		38 7 4 1 1	38 12 4 1 1				37 19 4 1 1				
42 18 6 1 1	42 51 6 1 1													
					44 27 6 1 1				44 56 7 1 1		45 57 13 7 7			
								49 14 13 1 1						
							51 47 16 1 1					50 38 13 1 1		
55 31 1 0 0	54 18 46 1 1							55 25 1 0 0					55 42 1 0 0	
56 2 1 0 0	56 48 1 0 0	56 55 1 0 0		57 30 1 0 0	57 34 1 0 0		56 44 1 0 0	56 1 1 0 0	56 44 1 0 0	57 11 1 0 0		56 44 1 0 0		57 30 1 0 0
61 13 14 1 1	61 41 14 1 1										67 4 20 1 1			
							64 43 63 7 7							
					66 59 8 1 1									
							68 31 13 2 2			68 51 13 2 2		67 42 7 1 1		
		69 30 6 1 1												
72 20 5 1 1	72 22 5 1 1													70 - 6 1 1
					75 42 4 1 1		15 18 4 1 1							
74 38 4 1 1							77 19 62 23 23							
							78 52 23 7 7							
		79 25 16 5.5										78 25 35 11 7 7		79 41 16 5.5
					80 57 3 1 1									
82 11 58 1 1					82 44 11 1 1		81 51 122 4 43							
							82 31 11 5 4							
							83 59 76 22 29							
							85 58 16 4 2 43							



[illegible]





53 59 76 22 22)

55 54 16 72 43)

