XXVI. *The Calculus of Chemical Operations; being a Method for the Investigation, by means of Symbols, of the Laws of the Distribution of Weight in Chemical Change.* —Part I. *On the Construction of Chemical Symbols.*

By Sir B. C. BRODIE, *Bart., F.R.S., Professor of Chemistry in the University of Oxford.*

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"*Les formules chimiques, comme nous l'avons dit, ne sont pas destinées à représenter l'arrangement des atomes, mais elles ont pour but de rendre évidentes, de la manière la plus simple et la plus exacte, les relations qui rattachent les corps entre eux sous le rapport des transformations.*"—GERHARDT.

CONTENTS.

EVEN in the earliest times the attention of chemists seems to have been directed to the symbolic expression of the facts of their science, a method which had its origin in the

mystic spirit of alchemy, and the subject has never ceased to occupy a prominent position in chemical philosophy. However, the development of our symbolic system has by no means kept pace with the general progress of the science. Indeed no essential improvement in the method has been effected since its first invention by BERZELIUS; and though this notation has doubtless afforded much aid to memory, and through memory incidentally to reasoning, yet it is difficult to point to even one discovery in the science, for which we are indebted to symbolic operations. In this respect chemical symbols present a marked contrast to other symbolic systems. The application of symbols to geometry and mechanics immediately led to the discovery of important truths, which were followed by the most original and unexpected development of the symbolic method itself. A very slight examination, however, of our present system is sufficient to render evident, that not only are the symbols of the chemist wanting in precision, but that they are of a totally different order from those symbols the employment of which has been attended with such great results.

The question of chemical symbols cannot well be separated from the consideration of the hypothesis which is expressed in them. The actual theory of chemistry is based upon the atomic theory of DALTON, and in the 'New System of Chemical Philosophy' may be found the germ whence our notation has been developed. According to the views of this eminent philosopher, the ponderable matter of any portion of the elemental bodies is assumed to consist of a vast yet finite number of minute, indivisible, and homogeneous particles or atoms, by the varied combinations of which all other substances may be produced. With the object of elucidating his theory, DALTON gave (in the plates at the end of his work) a kind of pictorial representation of the nature of matter from the point of view of his hypothesis. He represented the atoms of the elements by single circles with a characteristic mark, and the molecules of compound substances by systems of circles placed side by side in the figure, as the atoms were supposed to be placed in nature. Such pictures, for instance, are the following*, by which he figured oxygen, hydrogen, nitrogen, water, ammonia, nitric and acetic acids.

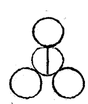
Oxygen ...... 

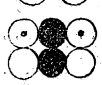
Hydrogen .... 

Nitrogen .... 

Water ...... 

Ammonia .... 

Nitric acid .. 

Acetic acid .. 

* See 'New System of Chemical Philosophy,' part 1, p. 219.

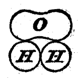
In our present system these signs have been replaced by letters marking the weights of the atoms, and the circles have been removed. But no fundamental change has been made in his conception, and in the arrangement of letters H_2, O_2, H_2O and the like, we still retain the image of DALTON under another form. Indeed where special clearness is required we not unfrequently find the circles restored to the picture; and the most recent representation of the nature of matter consists in a modification of this concrete symbol to meet the necessities of modern ideas*. On this view that arrangement of letters in the symbol which we call a formula is to be regarded as a figure by which the arrangement of atoms in the substance is represented; the symbolic system being a sort of orrery†, in which is imperfectly imitated the structure and movements of that unseen molecular world, on the mechanism of which chemical transformations are assumed to depend. A still more exact comparison would perhaps be to a diagram of Euclid, which bears a certain, though a confessedly inexact, resemblance to the object signified, and serves by this likeness to bring it vividly before the imagination‡.

This molecular interpretation is, it must be admitted, rather a matter of tacit convention than of express statement, and the above remarks, without qualification, would be too general. For it is a striking feature in our science that no system of chemical notation has yet been devised of such a nature as to receive universal and unqualified assent, or even a uniform interpretation. BERZELIUS §, who is generally regarded as the originator of our present method, considered that the letters which he employed simply represented certain weights of matter, and that in the symbol of a chemical substance the sign + was

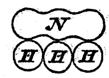
* See KEKULÉ, 'Lehrbuch der organischen Chemie,' 1861, p. 160, where the following diagrams are given:—

Hydrochloric acid ..

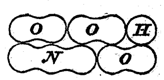
Water ....



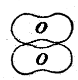
Ammonia ....



Nitric acid ....



Oxygen ....



† "But formulæ may be used in an entirely different and yet perfectly definite manner, and the use of the two distinct points of view will perhaps not be unserviceable. They may be used as an actual image of what we rationally suppose to be the arrangement of the constituent atoms in a compound, as an orrery is an image of what we conclude to be the arrangement of our planetary system."—"On the Constitution of Salts," by A. W. WILLIAMSON, Journal of the Chemical Society, vol. iv. p. 351.

‡ An interesting account of the development of our present system of notation, and its relation to the atomic theory, is given in the article "Notation" in WATTS's Dictionary of Chemistry, by Professor G. C. FOSTER.

§ BERZELIUS, 'Traité de Chimie,' 1845, vol. i. p. 119; and 'Jahresbericht,' vol. xv. p. 201.

to be understood as connecting every letter in the symbol, and was suppressed only from motives of brevity and convenience. So that H_2O was an abbreviated expression for $H+H+O$; H and O being numbers by which the relative weights of the combining proportions of hydrogen and oxygen were expressed. Sir JOHN HERSCHEL, in a paper contained in the Edinburgh Philosophical Journal for 1819*, and more recently in his introductory address to the Chemical Section of the British Association at Leeds, in the year 1858, objected to this system of chemical notation as opposed to algebraic convention, and suggested the replacement of the symbol H_2O, for example, by the expression $\overline{2H+O}$, arguing that the apposition of letters, being the algebraic sign of multiplication, cannot, consistently with the conventional principles of algebra, be employed to express the sum of two weights. GERHARDT, while he admitted the general principles of the atomic mode of representation, ridiculed all attempts to express the grouping and arrangement of atoms. Such, indeed, is the prevailing uncertainty that even in express treatises on chemistry all details on the subject are frequently evaded, and symbols are introduced and employed without any precise meaning being assigned to them. Such latitude is obviously inconsistent with the methods of science, and it has been proposed by more than one chemist, to whose more exact turn of mind it was eminently distasteful, that we should return to the simpler system of BERZELIUS. We are not, however, justified in concluding, as some have done, that because these symbols are wanting in precision, therefore they are utterly without reason or utility, mere idle arrangements of letters to which no serious meaning can be attached†. On the contrary, a more candid appreciation cannot fail to recognize that, notwithstanding many imperfections, they have rendered a most important service, by affording an external and visible image of

* See Edinburgh Philosophical Journal, vol. i. pp. 8, 18, 28.

† See BERTHELOT, 'Chimie organique, fondée sur la synthèse,' 1860, Introduction, p. cxxii, and p. 189. If the principles laid down by M. BERTHELOT are to be regarded as literally correct, the questions raised in the present memoir are unworthy of any serious consideration whatever.

"Ce serait méconnaître étrangement la philosophie des sciences naturelles et expérimentales, que d'attribuer à de semblables mécanismes une portée fondamentale. En effet, dans l'étude des sciences, tout réside dans la découverte des faits généraux, et dans celles des lois qui les rattachent les uns aux autres. Peu importe le langage par lequel on les exprime; c'est une affaire d'exposition, plutôt que d'invention véritable: les signes n'ont de valeur que par les faits dont ils sont l'image. Mais les conséquences logiques d'une idée ne changent point, quelle que soit la langue dans laquelle on la traduit."

It is a fundamental principle of symbolic reasoning, to whatever science it may be applied, that we cannot by the aid of symbols arrive at any conclusion which is not implicitly contained in them. And it might with equal justice be asserted that it was a matter of very little consequence whether we employed for the purposes of calculation Arabic or Roman numerals, the number expressed being in either case precisely the same, and "the language by which it is expressed being of little moment." Or again, that the discovery of the method of denoting the position of points in space by means of the symbols of algebraic geometry was a very unimportant matter. Our conception of a circle is the same as that of the ancient geometricians, and "the logical consequences of an idea do not change into whatever language we translate it." Nevertheless our power of following out and appreciating those consequences may be very materially affected by such a method, as experience has amply proved.

the world of chemistry on which the attention may advantageously be concentrated. Ordinary language is too vague and too diffuse for the purposes of science, and in chemistry especially the facts are so numerous and so complicated that it is only when embodied in a concrete form that they can be stored in the memory, and become the object of reflection. Hence even an imperfect and material picture may, in a certain epoch of the development of the science, be found of indispensable utility.

The actual theory of chemistry may be regarded as an expansion of the hypothesis of DALTON. In science, as in other spheres of thought, hypotheses often pass without question which come to us recommended by early use, and by even a short tradition; and when embodied in symbolic language, and thus intimately blended with our conceptions, they are readily mistaken for facts. No statement, perhaps, would receive more universal or unqualified assent from chemists than that the molecule of ammonia contains three atoms of hydrogen, that ethylamine is derived from ammonia by the substitution of an atom of ethyl for an atom of hydrogen, and that the reason why there are three, and only three, such derivatives of ammonia is, that each derivative is formed by a repetition of the same process, and that, the molecule of ammonia containing only three atoms of hydrogen, this process can only be repeated three times. These conclusions are regarded as so certain as to be almost removed from discussion. But nevertheless they cannot but partake of the hypothetical nature of the theory in which they originated. It is only because our primary hypothesis has led us to take this peculiar view of the atomic constitution of ammonia, and to express it by the symbol NH_3, that chemists have adopted these further hypotheses as to the nature of the process by which ethylamine is formed, and as to the cause of the limitation which exists in regard to the number of these derivatives. Again, the theory of atomicity has a similar origin. Glycol is supposed to be derived from two molecules of water by the substitution of an atom of the diatomic hydrocarbon ethylene for two atoms of hydrogen; the diatomic radical, in the forcible language of the distinguished discoverer of this substance, "welding" and "riveting" together the residues of the two molecules of water*. What is this doctrine? It is simply the expression in language of the relation of the symbols

$$\frac{\begin{matrix}H\\H\end{matrix}O}{\begin{matrix}H\\H\end{matrix}O}\;;\quad \begin{matrix}H\\C_2H_4\\H\end{matrix}\begin{matrix}O\\O\end{matrix}$$

And if the course of the science had been, as might have been the case, such as to have led us to a different view of the atomic constitution of these substances, we should have a different order of chemical ideas, and the theory, in its actual form, would never have existed.

* "Toutes ces molécules sont cimentées, en quelque sorte, par des éléments polyatomiques, qui possèdent la propriété de se souder les uns aux autres."

"Il est bien entendu que dans l'éthyle lui-même les atomes sont rivés ensemble par le carbone tétratomique." —WURTZ, 'Leçons de Philosophie Chimique,' 1864, pp. 138–140.

It is frequently asserted that our present hypothesis affords a clear and simple explanation of chemical phenomena, which is the evidence of its truth. Now it may be considered that such an explanation was perhaps afforded of the incomplete system of facts known to DALTON, but with our present knowledge this account can no longer be regarded as satisfactory. The most important feature in our modern system is the identification of the weight of the chemical molecule with the weight of the unit of gaseous volume, to which we are brought by physical as well as chemical considerations. This great simplification was practically introduced by LAURENT and GERHARDT, and it is generally allowed that this assumption affords the surest basis of chemical theory. Now the atomic theory of DALTON accounts for the fact that the weight of the chemical molecule may be regarded as consisting of an integral number of the atomic weights of those elemental bodies into which it can finally be decomposed. But this is not the only limitation with which we are acquainted. The chemists before mentioned discovered the existence of a peculiar numerical relation between the atomic weights of certain elemental bodies, when combined in the chemical molecule, to which they gave the name of "the law of even numbers." This law may be thus stated:—"The sum of the volumes of the hydrogen, chlorine, bromine, iodine, nitrogen, and generally of that class of elements which goes under the name of the dyad elements, which are formed by the decomposition of two gaseous volumes of any chemical substance, is an even number"*. This statement rests upon evidence quite as satisfactory as that by which the atomic doctrine is supported. A formula containing an uneven number of these elements jointly is rendered as improbable, from our experience, as a formula containing fractions of atoms. But the atomic theory in its present form takes no account of this relation, and so little has this great discovery been appreciated that such formulæ are often met with, even in the works of accomplished chemists; and indeed they are truly admissible, so far as the limitations imposed by our actual theory are concerned. It need not certainly be a matter for surprise or reproach, that the speculations of DALTON should not apply to a class of facts with which he was unacquainted; nor even can this be regarded as conclusive evidence against the truth of his system. But nevertheless this omission indicates some profound defect in chemical theory, and if it should be found that another view of the constitution of matter should cover the whole ground, and account by one and the same hypothesis for both numerical relations, there can be but little room for doubt as to which should be preferred.

Another, although a less important, defect in our method is the singular unit of volume which chemists have been compelled to adopt, for which selection no reason can be assigned except the necessities of the atomic hypothesis. In the so-called "two-volume" and "four-volume" notations the weight of the chemical unit or molecule is assumed as twice or four times the weight of the unit adopted for the purposes of physical measurement, the numbers which express the weight of the chemical molecule being propor-

* See LAURENT, Méthode de Chimie, Ed. 1854, p. 57, "Sur les nombres pairs d'atomes;" 'Chemical Method,' English translation, 1855, p. 46.

tional to, but not identical with, the numbers which express its density. An attempt has been made to remedy this imperfection, retaining the general features of our present method, but the changes proposed have never been adopted, such alterations being always attended with some inconvenience, and the question at issue being one neither of theory nor of fact*.

The following pages contain the outline of a new method for the expression, by means of symbols, of the exact facts of chemistry, and for reasoning upon these facts by their aid. This method is quite independent of any atomic hypothesis as to the nature of the material world, and in it the symbol is placed in immediate relation with the fact, being indeed its symbolic equivalent or expression. It does not, however, preclude or deny such an hypothesis; the question is not raised. This method may be regarded as a special application of the science of algebra, and in its construction I have been guided by the similar applications of that science to geometry, to probabilities, and to logic, to which it presents many curious and interesting analogies. In these branches of science the symbol is not a figure of the object, nor is any resemblance attempted between the symbol and the thing signified by it. The symbols which I shall have occasion to employ are of the same abstract character; they pretend to no resemblance to any object in nature, and are simply to be regarded as arrangements of marks which it is convenient to employ for the purposes of thought. The conditions to be satisfied by such a method are few and simple. It is only necessary that every symbol should be accurately defined; that every arrangement of symbols should be limited by fixed rules of construction, the propriety of which can be demonstrated; and that the symbolic processes employed should lead to results which admit of interpretation.

The object of this method may be considered to be the investigation of the laws of the distribution of weight in chemical changes, and the symbols here employed represent "weights" in the same sense as the symbols of geometry represent lines or surfaces. Now the symbol a in geometry, in its primary sense, may be regarded as the symbol of the operation performed upon the unit of length, by which a line is generated, that is, of which the result is a line. In like manner the symbol a, as a chemical symbol, is to be regarded as the symbol of the operation performed upon a unit of space, by which a weight is generated, that is, of which the result is a weight. Symbols of operation have not hitherto been adopted in chemistry, and their introduction forms a distinctive feature of the present method, which I have hence termed "the Calculus of Chemical Operations."

It is my intention to divide the subject into three parts. The first part, which alone is here given, relates to the construction of chemical symbols. In the second part I

* See LAURENT, 'Méthode de Chimie,' p. 83, English translation, p. 67. Also Mr. J. J. WATERSTON "On Chemical Notation in conformity with the Dynamical Theory of Heat," Phil. Mag. vol. xxvi. pp. 248 and 515, and vol. xxvii. p. 273. Also "Remarks on Chemical Notation," by W. ODLING, *Ib.* vol. xxvi. p. 380, and vol. xxvii. p. 380. The proposition of Mr. WATERSTON is the same as that of LAURENT, and amounts to the obvious expedient of cutting the molecules and atoms in half.

purpose to treat of the theory of chemical equations, which is intimately connected with the general processes of chemical reasoning, and especially with the consideration of the nature of that event which is termed a chemical change, of which I shall give a new analysis founded on its symbolic expression. In the third part it is intended to consider the principles of symbolic classification and the light thrown by this method upon the origin and nature of the numerical laws which limit the distribution of weight in chemical change: I shall then have occasion to contrast the view of the science which is here given with that afforded by our existing system.

It is now some years since the conception of such a work first arose in my mind, and these pages are a very imperfect record of the time and consideration bestowed upon it, which yet can hardly be regarded as excessive in relation to the end I have had in view, which has been no other than to free the science of chemistry from the trammels imposed upon it by accumulated hypotheses, and to endow it with the most necessary of all the instruments of progressive thought, an exact and rational language. "Tout langue est une méthode analytique, et toute méthode analytique est une langue."—CONDILLAC. The views here advocated may appear novel, but nevertheless I strongly feel that one claim, at least, which they have on the consideration of chemists is that they are in truth the rational and simple consequence of opinions at which the most reflecting minds have already arrived, and offer a more complete expression of current ideas than has hitherto been given. Indeed on such subjects novelty is almost inconsistent with truth. For the conceptions through which sciences pass necessarily have their origin in the views which preceded them, of which they are but the natural fruit. The method here developed will be seen, if carefully considered, to be but another step in the direction of the chemical movement of the last twenty years, which some imagine to have found its final consummation in the doctrines of "modern chemistry." Such could never have been the conclusion of the great chemists from whom this impulse emanated, who definitely refused to recognize the atomic doctrine as the adequate exponent of their ideas, and who implanted in the science the germ of a more abstract philosophy, which it has ever since retained.

The object of the following method has been defined as the investigation by means of symbols of the laws of the distribution of weight in chemical change, a problem evidently of the widest range, and embracing many distinct questions. To the consideration of one, and certainly not the least important, of these the first part of this work will be more especially devoted, namely, to the discovery of a system of symbolic expressions by which the composition of the units of weight of chemical substances may be accurately represented, and which may hereafter be employed for the purposes of chemical reasoning. The value of our conclusions must depend upon the degree of precision and certainty with which this point is determined. It will be shown that the problem is of a perfectly real nature, admitting, where the experimental data are adequately supplied, of only one solution. The discussion of this question involves the consideration of the fundamental principles of symbolic expression in chemistry.

SECTION I.—DEFINITIONS.

It is essential for the purposes of exact reasoning that the terms employed should be accurately defined. This can only be effected either by the invention of new words or by the selection from the many shades of meaning which may be attached to existing words of some one definite and appropriate signification, in which it shall be agreed to employ them. I shall adopt the latter and more obvious course.

1. The term "ponderable matter," in its chemical application, is a term by which that class of objects is denoted, the transformations of which form the special study of the chemist. Not that the property of weight can exist apart from those other properties of form, colour, and the like, with which, so far as our experience extends, it is invariably associated, but that in the actual phase of the science this property is chiefly considered, it being the only property of matter in regard to the chemical changes of which we possess any exact knowledge. No further explanation of this term can advantageously be offered, except by exhibiting or enumerating the objects (such as water, silica, oxygen) which are comprehended in the class.

2. A "chemical substance" is a portion of ponderable matter of which every part has the same properties.

It is often difficult to decide whether this condition be satisfied or not, but the propriety of the above definition will be manifest from the line of argument which is applicable to such cases. Take, for example, the case of atmospheric air. To a superficial observation, every part, however minute, of any given portion of the atmosphere has the same properties. But a more exact scrutiny leads us to infer that this is really not the case, but that it consists of parts, which diffuse with different velocities and are unequally soluble in water. On these principles it has been established that the gas procured from the decomposition of acetic acid is truly, in the sense of the above definition, one chemical substance, marsh-gas, and not two chemical substances, hydrogen and methyl.

3. "A weight" is a portion of ponderable matter of any specified kind considered as regards weight.

This application of the term *a weight* is only a slight extension of its ordinary use. A gramme of platinum which serves for the purposes of weighing is termed *a weight*, this being the only property of that portion of matter of which it is necessary to take cognizance*. Now the weight of matter, from a different and far wider point of view, in the laws of its composition and resolution, is the special subject of this investigation;

* The term weight is, in ordinary language, used with two distinct meanings. (1) In what may be termed its abstract signification, as denoting a certain measurable property of matter, as when we inquire, "What is the weight of that loaf?" (2) In its concrete sense, as denoting certain objects, which we discriminate from others by naming them from their most essential property, as when we say "Bring me that box of weights." It is in an extension of this concrete meaning that the word is here employed. It is not, of course, intended to assert that the transformation of *weights* is the only subject of chemical science, but simply to fix the attention upon that subject as the only topic which is here discussed.

and the aspect in which every chemical substance, every portion of ponderable matter, will be here regarded is exclusively as *a weight*. In speaking of such *weights* we habitually employ, by a tacit convention, the terms by which the chemical substances, of which the weight alone is referred to, are usually designated. But this is not a strictly accurate use of language; and it is necessary to observe that in the following pages, where chemical substances, such as chlorine or alcohol or water, are mentioned, or the term "a portion of matter" is employed, the objects referred to are certain weights of the substances under consideration, to the exclusion of all other properties.

4. "A single weight" is a portion of ponderable matter of any specified kind considered as regards weight and as one object, as for example a portion of oxygen, or a portion of ponderable matter consisting of oxygen and hydrochloric acid considered as one object, or two portions of oxygen similarly considered.

5. A "group of weights" is some number of single weights, such as a portion of oxygen and a portion of hydrochloric acid considered as two objects, or two portions of oxygen similarly considered. The single weights of which a group consists are termed the "constituents" of the group, which is said to be "constituted" of them.

6. Two portions of ponderable matter which consist of the same "weights" are said to be identical as regards weight.

It follows from this definition that what may be termed the absolute weight, or weight in grammes, of "identical weights" is equal.

Our knowledge as to the identical relations of ponderable matter is derived exclusively from the science of chemistry. Were we unacquainted with the peculiar phenomena of chemical transmutation, or were we ignorant of the circumstance that in chemical change the total weight of matter is unaltered, the existence of such relations would be unknown to us. It is so important to have a clear perception of that distinction, which is here made I believe for the first time, between equality and identity of weight, that a few words in somewhat fuller explanation of the grounds on which this distinction rests may not be deemed out of place.

If we were to take any portion, say a gramme, of water, and observe its properties on two successive days, the conditions under which the water was placed being assumed as fixed, it would be found that the properties of the water were precisely the same at the second as at the first observation. In this case the identity of properties would be absolute, and as we know that at whatever time the observation had been made, under the same conditions, the same result would have been obtained, we should hence arrive at the conception of the continuous existence of one and the same object, which we should denote with perfect precision by one and the same name, a gramme of water. Now let it be assumed that some condition varies, that the temperature, for example, rises from 60° to 90°, and let another observation be made of the properties of the water. It would be found that these properties were no longer identical with those previously observed; that the bulk of the water had increased, and that some properties had varied while others had remained constant. If the expansion of water were the

point under consideration, those properties which had varied would be of fundamental importance; but as for most purposes these variations may be disregarded the object is still assumed to be the same, and called by the same name, a gramme of water. Let the temperature rise to 100°, and let a new observation be made. The liquid has become a gas, and the change of properties is so great that a new name is assigned to the portion of matter, and it is said that the gramme of water has been converted into a gramme of steam. Nevertheless, many of the properties of the steam are identical with those of the water, and these being the properties with which the chemist is mainly concerned, he asserts the identity of the two objects for the purposes of his science, and says, notwithstanding this transformation, that steam is simply the gaseous form of water. Now let the conditions be again varied. At a further elevation of temperature it will be found that a more profound change has occurred. In the place of one continuous portion of matter, in every part of which the same properties may be recognized, we have two distinct chemical substances, each characterized by a special set of properties. The volume has permanently altered; many chemical as well as physical properties of the water have entirely disappeared. The chemist marks this change by assigning a new name to the portion of matter, and says that the water has been *converted* into oxygen and hydrogen. It has, however, been found that even in this profound change one property has not been affected, namely, weight. This property is constant; and we assert in the most absolute sense that the weight of a gramme of water is identical with the weight of the gramme of oxygen and hydrogen into which it is transformed, for this property, throughout this series of changes, has never varied nor ceased to have a continuous existence. Now a gramme of water will produce the same effect on a balance as a gramme of lead, and it is this relation which is here termed equality of weight. This relation also subsists between the weight of a gramme of water and the weight of the gramme of oxygen and hydrogen into which it is transformed; but these weights are also connected by another relation, which I have termed identity, which does not exist between the gramme of water and the gramme of lead.

It is thus that we are led by experience to the inference comprised in the following statement.

If a portion of matter A be chemically converted into a portion of matter A_1, then the weights (or portions of matter considered as regards weight) of which A consists are identical with the weights (or portions of matter considered as regards weight) of which A_1 consists.

It will be found, on analyzing the process of reasoning by which we conclude the continuous existence of the same weight in a chemical change, that the evidence by which it is supported is precisely of the same order as that by which we are enabled to assert the continuous existence of any external object whatever. This evidence is brought spontaneously and without effort before the mind, and is so perfectly conclusive, however difficult it may be to submit it to analysis, that an eminent writer has actually imagined the doctrine of the permanence of weight in chemical changes to be a truth which

the mind recognizes at once when stated, by some special intuition, as "flowing from the idea of substance"*. As to the nature of this process we are left in the dark. However, such assumptions are happily as unnecessary as they are unmeaning, and it is sufficient for all the objects of science if we admit this axiom to be the undoubted acquisition of those combined processes of reasoning and observation which are the only sources of exact knowledge.

7. A "compound weight" is a single weight of which the whole is identical with two or more weights. Such weights are termed the components of the compound weight, which is said to be composed of them.

It follows from this definition that every part, individually or separately considered, of a compound weight is proportionally identical with the same weights of which the totality is composed; that is to say, if the whole of a compound weight be identical with the weights A and B, $\frac{1}{n}$th part of that compound weight, in whatever way the division into parts be effected, is identical with $\frac{1}{n}$th part of the weight A and $\frac{1}{n}$th part of the weight B, and is similarly composed of those parts; and also it is to be inferred that if every $\frac{1}{n}$th part of a single weight be identical with $\frac{1}{n}$th part of a weight A and $\frac{1}{n}$th part of a weight B, the whole of that single weight is composed of the weights A and B.

A group of two or more weights may, in the sense of this definition, be regarded as a compound weight, if only every constituent of the group have a common component. Thus, for example, a group consisting of two portions of matter, of which the one is composed of A and B, and the other of A and C, may be regarded as a single weight (Sec. I. Def. 4) of which the totality is composed of A and $\overline{\text{B or C}}$; for if, regarding the group as one object, we agree not to effect the separation of its constituents, every $\frac{1}{n}$th part of that weight satisfies proportionally the same condition.

Some difficulty may perhaps be felt in assenting to the above reasoning, from a certain ambiguity in language in the use of the conjunctions "and" and "or." That these conjunctions have the same meaning, so far as the purposes of enumeration are concerned, is apparent on enumerating the constituents of a group with each conjunction. The language of symbols is free from this ambiguity, the two conjunctions being represented by one mark, as will hereafter be explained.

8. A "simple weight" is a weight which is not compound. It may also be defined as a weight which has only one component. Two weights are said to be simple in regard to one another which have no common component.

9. An "integral compound weight" is a weight which is composed of an integral number of simple weights.

* See WHEWELL'S 'Philosophy of the Inductive Sciences,' vol. i. chap. iv. p. 412, ed. 1847, "On the Application of the Idea of Substance in Chemistry."

10. It is necessary to select a "unit of ponderable matter" which may serve as the common measure of those chemical properties which it is our desire to investigate. Such a standard, for example, would be supplied to us were we to select as the common term of comparison that portion of ponderable matter of which the absolute weight is one gramme. This plan would have the great advantage of proceeding upon accurate and certain data, but, on the other hand, the conclusions to which it leads would be of comparatively little interest. The present method aims at effecting a comparison of the chemical properties of those portions of ponderable matter which, in the condition of perfect gases, and compared at the same temperature and pressure, occupy equal volumes.

In the numbers which express the specific gravity of gases a similar comparison is made of the absolute and relative weights of the same portions of matter; and it will be found convenient to refer physical and chemical properties to the same standard. I shall therefore define the "chemical unit of ponderable matter" as that portion of ponderable matter which occupies the volume of 1000 cub. centims. at 0° C. and a pressure of 760 millims. of mercury. The weights of the chemical unit of ponderable matter may be expressed in two ways according to the object in view; as the absolute weight in grammes, and as the relative weight in reference to the weight of some one unit assumed as the standard of comparison. For this purpose the weight of the unit of hydrogen will be selected.

11. The volume of 1000 cub. centims. is here termed "the unit of space."

The system of chemical measurement which has grown up around the atomic theory is of a singular and artificial character. The chemist is accustomed to assume as his standard of comparison, not some real and existing object, but a "molecule," an imaginary arrangement of imaginary atoms, of which no precise definition, by which it can be recognized, has ever been given. It should at least be shown that a system thus constructed offers special facilities for thought. But in truth it has been found so perplexing in practice, that the most skilled teachers* have been forced to admit that the student requires to be initiated into this world of hypothesis by means of more concrete and exact ideas.

12. The term "distribution of weight" may be defined as that operation by which a compound weight is resolved into its component weights, or by which it is made up from those weights, regard being had to some special system of such events which is the subject of consideration.

* Two eminent chemists have recently given independent testimony to the value of a more real standard than is afforded by this imaginary "molecule." Dr. HOFMANN, in his 'Modern Chemistry,' has adopted the term "crith" to denote such a real unit,—a crith being the weight of 1000 cub. centims. of hydrogen at standard temperature and pressure. Professor WILLIAMSON has, from similar motives of utility, adopted an "absolute volume" of 11·2 litres, which is the bulk of a gramme of hydrogen, also at standard temperature and pressure. I now propose to advance another step in the same direction, and to substitute the real for the fictitious unit, not for a special object alone, or to pave the way for more important theories, but for all the purposes of chemistry.

See 'Modern Chemistry,' by A. W. HOFMANN, 1865, pp. 121 and 130; and 'Chemistry for Students,' by A. W. WILLIAMSON, p. 4.

A "distributed weight" is a weight which, in such a system of events, is resolved into two or more weights or made up from such weights.

An "undistributed weight" is a weight which, in the same system of events, is not so resolved or so made up. An undistributed weight may also be defined as a weight which is resolved into one weight, or made up from one weight alone.

This division of weights into distributed and undistributed weights is coextensive with the previous division of the same into compound and simple weights. A distributed weight is necessarily a compound weight, for we can always assert its identity as a whole with the parts into which it is distributed; and a weight which is not distributed can only be regarded as a simple weight, for, by hypothesis, no information is supplied to us from the system of events under consideration, which enables us to assert its identity with any other weights, such an assertion in every case being purely relative to the facts before us, and open to modification by the acquisition of further knowledge.

It is essential to remark that the previous definitions are definitions of abstract conceptions, which have immediate reference to the symbolic method developed in the following section, for the construction of which they afford an adequate basis, and which is necessarily more comprehensive than those special subjects of physical investigation to which it is hereafter to be applied.

Section II.—ON THE SYMBOLS OF CHEMICAL OPERATIONS.

Having explained the nature of those objects and relations which fall under the consideration of the chemist as the investigator of the laws of the distribution of weight, I proceed to consider their symbolic expression.

(1) Let a chemical operation be defined as an operation performed upon the unit of space, of which the result is "*a weight*" (Sec. I. Def. 3), and let x, x_1, x_2 be the symbols of such operations, of which the weights A, A_1, A_2 are the results. Then the symbols of operation, x, x_1, x_2, are termed (for brevity and convenience) the symbols of the weights A, A_1, A_2.

Any symbolic expression into which the symbols of chemical operation enter is termed a chemical function.

(2) Two chemical operations are said to be identical of which the results are identical as regards weight (Sec. I. Def. 6). Now let the symbol $=$ be the symbol of identity. Hence if the weight A be identical with the weight A_1, $x=x_1$.

(3) Further, let the symbol $+$ be the symbol of that operation by which a weight is added to a weight so as to constitute with it one group (Sec. I. Def. 5); and let the symbol $-$ be the symbol of that operation by which a weight is removed from a group of weights. These operations are expressed in language by the words "and" and "without."

From these definitions $x+x_1$ is to be regarded as the symbol of a group constituted of the two weights A and A_1, and $x+x$, that is, $2x$, is the symbol of two weights A, and $x-x_1$ is to be regarded as the symbol of the weight A without the weight A_1. For this

latter operation to be performed it is necessary that the weight A_1 should be part of the weight A.

Now since a group which is constituted of the weights A and A_1 is identical with (Sec. I. Def. 6) a group which is constituted of A_1 and A, $x+x_1=x_1+x$.

Also, since it is immaterial in what order the operation is performed which results in the exclusion of a weight from a group $x-x_1=-x_1+x$.

If, in the expression $x-x_1$, $x_1=x$ the expression becomes the symbol of a group in which no weight appears; for it is the symbol of that weight which is the result of removing the weight A from the weight A, in other words, $x-x$ is the symbol of the weight A without the weight A, but the result of the successive performance of these operations is *no* ponderable matter. By analogy of interpretation to that of the symbol 0, regarded as a numerical symbol, let 0 be the symbol of a group in which no weight appears, and which has had its origin in the several performance of the operations x and $-x$; so that $x-x=0$.

The chemical symbol 0 has the property of the numerical symbol 0 given in the identity $0+x=x$; for the ponderable matter which results from adding the weight A to a group in which no weight appears is identical with the ponderable matter A.

Also, since a group is not affected by removing from it no weight, the symbol of a group is not affected by removing from it the symbol of no weight, and $0+x=+x=x$.

The interpretation which is here assigned to the symbols + and − is strictly analogous to that which has been given to them in the arithmetical and logical systems*. They are the symbols of those operations by which we form a group from its constituents, or remove the constituents from a group. These operations, which may be termed the operations of "aggregation" and "segregation," are found in every department of thought. No uniform meaning has hitherto been attached to the symbols + and − in chemistry, notwithstanding their constant use. The prevalent opinion seems to be in favour of the use of the symbol + as the symbol of "mechanical mixture"†. It is difficult to say what may be the exact signification of this term. In the present method, at any rate, no such interpretation is to be attached to the symbol, it being quite immaterial for the end in view whether the objects referred to be what is termed "mixed" or not. A similar uncertainty prevails in the use of the symbol of identity‡. The symbol = is sometimes employed in chemistry as the symbol of numerical equality, at

* BOOLE, 'Laws of Thought,' p. 32.

† See ODLING'S 'Manual of Chemistry,' vol. i. p. 4. "The sign + signifies addition to, or rather mixture with." Also, WILLIAMSON'S 'Chemistry for Students,' p. 37. "The sign + interposed between symbols denotes addition or mixture of the atoms or molecules which the symbols represent. Thus H+O denotes a mixture of 1 part by weight of hydrogen with 16 parts by weight of oxygen."

‡ ODLING'S 'Chemistry,' vol. i. p. 4. "The sign = signifies equivalency with, or rather conversion into." WILLIAMSON says, p. 37, "The sign = is used in describing chemical changes. It only denotes equality in weight between the sum of the atoms of each kind on one side of it, and the sum of the atoms of the same kind on the other side of it...... $H^2+O=H^2O$ means that 2 parts by weight of hydrogen added to 16 parts by weight of oxygen, can be made to combine to form 18 parts by weight of water."

other times as the symbol of chemical transmutation. So far as I am aware it has never yet been employed with the signification which I have assigned to it, nor has the relation which it here expresses been recognized in the conceptions of the science, among which it occupies so fundamental a position.

(4) Further, let $\overline{x+x_1}$ or $(x+x_1)$ be the symbol of the two weights A and A_1 collectively considered as constituting a single weight. Then $\overline{x+x_1}+\overline{y+y_1}$ will be the symbol of a group of which two such weights are the constituents, and $\overline{(x+x_1)+(y+y_1)}$ will be the symbol of two such weights collectively considered, and as constituting a single weight.

Now, since the result is the same whether we add or remove a group of weights collectively, or add or remove the constituents of the group severally,

$$x+\overline{y+y_1}=x+y+y_1,$$
$$x-\overline{y+y_1}=x-y-y_1,$$
$$x+\overline{y-y_1}=x+y-y_1,$$
$$x-\overline{y-y_1}=x-y+y_1.$$

From this we may infer that the chemical symbols $+$ and $-$ have the properties of the numerical symbols $+$ and $-$, so that

$$++x=+x,$$
$$-+x=-x,$$
$$+-x=-x,$$
$$--x=+x.$$

(5) Now, let W be a compound weight, of which certain portions of matter named A and B are the components (Sec. I. Def. 7). Let φ be the symbol of the weight W, and x and y the symbols of the weights A and B respectively. Further, let xy be selected as the symbol of the weight W, so that

$$\varphi=xy;$$

xy is termed a composite symbol, of which x and y are the factors. The symbol xy is also termed a combination of x and y, which are said to be combined in it.

If $y=x$, then $\varphi=xx$, which may also be written x^2; and generally if φ be the symbol of a compound weight of which the component weights are n weights named A, n_1 weights named A_1, n_2 weights named A_2...., of which x, x_1, x_2,..... are the symbols, then

$$\varphi=x^n x_1^{n_1} x_2^{n_2}\ldots\ldots$$

If in this expression $x=x_1=x_2\ldots\ldots$,

$$\varphi=x^{n+n_1+n_2+\ldots\ldots}.$$

Now, recurring to the symbol xy, since a portion of ponderable matter composed of A and B is identical with a portion of ponderable matter composed of B and A,

$$xy=yx,$$

that is to say, the order in which apposed symbols of chemical operations are written is indifferent. Symbols possessing this property are termed "commutative".

A consequence of this commutative property is that

$$(xy)^n = x^n y^n;$$

for

$$\begin{aligned}(xy)^2 &= xyxy \\ &= xxyy \\ &= x^2y^2.\end{aligned}$$

Further, let there be a compound weight V, of which θ is the symbol, of such a nature that it is identical with the component weight A without the component weight B, and let $\frac{x}{y}$ be selected as the symbol of the weight V. Then

$$\theta = \frac{x}{y},$$

whence, on similar principles, if θ be the symbol of a compound weight V identical with n component weights A, n_1 component weights A_1, n_2 component weights A_2, ... without m component weights B, m_1 component weights B_1, m_2 component weights B_2.....,

$$\theta = \frac{x^n x_1^{n_1} x_2^{n_2} \ldots\ldots}{y^m y_1^{m_1} y_2^{m_2} \ldots},$$

where x, x_1, x_2, ... y, y_1, y_2.... are the symbols of the weights A, A_1, A_2, ..., B, B_1, B_2.... respectively.

We may also reason thus: xy is the symbol of a weight which results from the successive performance upon the unit of space of the operations y and x, yx is the symbol of a weight which results from the performance of these operations in an inverted order, and (xy) is the symbol of a weight which results from their joint performance. Now, since the result is the same in whatever order the operations be performed, and since it is immaterial whether the operations be performed jointly or successively, we infer that

$$xy = yx = (xy).$$

(6) If in the composite symbol φ one of the factors be the symbol of a group, so that

$$\varphi = x(y + y_1),$$

φ is to be interpreted as the symbol of the weight which results from the combination of the weight A with the group of weights B and B_1, the group being collectively considered and as constituting a single weight (Sec. I. Def. 4, 5, and Sec. II. (4)),—not, however, be it observed, a single weight *compounded* of the weights B and B_1, which would be symbolized by yy_1, but a single weight *constituted* of B and B_1 which is symbolized by $(y+y_1)$. (Sec. I. Def. 4, 5, and 7). Now the weight A will be combined with the group of weights *collectively* considered, if it be combined with the constituents of the group *severally*, but the symbol of the weight A combined with the two constituents of the group severally is $xy+xy_1$. We hence arrive at two symbols for the same weight, which express indeed two different aspects of the same object, but which are identical as regards the object signified; whence

$$x(y+y_1) = xy + xy_1.$$

Also by a strictly analogous interpretation to that assigned to the symbol xy as a symbol of operation, the symbol $x(y+y_1)$ is to be interpreted as the symbol of a weight which results from the successive performance upon the unit of space of the single operations $(y+y_1)$ and x. Now the result of performing the *single* operation $(y+y_1)$ is the same as that of performing *severally* the two operations y and y_1; and as regards the result, it is immaterial whether we first perform *severally* the two operations $y+y_1$, and then perform upon these *two* operations the operation indicated by x, as expressed in the symbol $xy+xy_1$, or whether we perform the *single* operation $(y+y_1)$ and then perform x upon that, as expressed in the symbol $x(y+y_1)$. From whichever point of view we regard the symbols, whether as symbols of operation or as symbols of the results of operations, we are brought to the same conclusion, that

$$x(y+y_1)=xy+xy_1.$$

In like manner it may be shown that

$$(x+x_1)(y+y_1)=xy+xy_1+x_1y+x_1y_1,$$

$(x+x_1)(y+y_1)$ being the symbol of a single compound weight, of which the groups A or A_1 and B or B_1 are the components. Symbols which possess this property are termed distributive symbols.

(7) Although the selection of a symbol is in a certain sense arbitrary, it is by no means a matter of indifference; and the symbol xy which is here assigned to a continuous compound weight, so far from being (as might be thought from a superficial consideration) contrary to symbolic analogy, is the only symbol by which the desired end could be attained, consistently with usage. The symbol xy in its abstract interpretation is the symbol of the operations x and y operating *successively* upon the unit or subject of operations; (xy) is the symbol of the same operations but operating *jointly*; $x+y$ is the symbol of the same operations operating upon the same subject but operating *severally*, and $(x+y)$ the symbol of the same operations but operating *collectively*. This fundamental distinction in operations as *successively*, *jointly*, *severally*, or *collectively* performed, appears in various forms in the different sciences, and is found in every branch of knowledge which admits of symbolic treatment. In chemistry it expresses the various ways in which we may conceive of the existence of the same ponderable matter. The language of symbols supplies the means of simply and adequately expressing these conceptions, isolated from every other consideration, which are not only very imperfectly expressed by the usual molecular representation, but are there complicated by many considerations which are totally irrelevant to the real point at issue.

Section III.—ON THE CHEMICAL SYMBOL 1.

(1) The preceding considerations suggest the inquiry as to the symbol of a compound weight of which the weight A and no weight are the components. Now, since any portion of ponderable matter is not altered by the combination with it of no ponderable

matter, a weight of which the weight A and no weight are the components is the same as the weight A. Hence if x be the symbol of the weight A, and y the symbol of no weight,

$$xy=x.$$

Now the symbol 1 regarded as a numerical symbol, possesses the property given in the equation

$$x1=x.$$

From this correspondence of symbolic properties, and guided by the same considerations of analogy as those on which the symbol 0 was selected as the symbol of no weight, regarded as the constituent of a group, I shall select the symbol 1 as the chemical symbol of no weight regarded as a component of a compound weight.

Since any portion of matter whatever may be considered as a compound weight of which that matter itself and no weight are the components, if ϕ be the symbol of any weight,

$$\phi=\phi 1.$$

The symbol 1, therefore, is implicitly contained as a common factor in every chemical symbol, being either expressed or understood as the symbol of the common subject of all chemical operations. Now this subject of chemical operations has been defined as the "unit of space" (Sec. I. Def. 11), a term already appropriated to it in language, for it is in space that we conceive of the existence of ponderable matter. This interpretation of the symbol 1, as the symbol of the unit of space, is identical with the meaning before assigned to it as the symbol of "no weight;" the only property of matter under consideration being weight, by the absence of which the unit of space is defined.

(2) The correctness of the above reasoning is further evident from the identity of the other algebraic forms of the chemical symbol 1 with the algebraic forms of the same numerical symbol, notwithstanding the difference in interpretation.

We have seen that x^n is the symbol of a compound weight, of which n weights A are the components. Hence the symbol of a compound weight, of which 0 (or no) weights A are the components, is x^0. But a weight of this kind is the same as "no weight;" whence

$$x^0=1.$$

The symbols 1 and x^0 correspond to the different ways in which "no weight" may have originated, the result being the same whether the operation performed do not cause weight or whether an operation causing weight be not performed; the former view being expressed by the symbol 1, the latter by x^0.

Again, if in the expression $\frac{x}{y}$ (Sec. II. (5)), $y=x$, this expression becomes the symbol of a compound weight composed of the weight A without the weight A, that is to say, which is composed of no weight; whence also

$$\frac{x}{x}=1.$$

This third form of the symbol 1 corresponds to a third origin of the absence of weight, which we may also regard as effected by the simultaneous performance of inverse opera-

tions upon the unit of space, the result of one of which is to cause "a weight," and of the other to remove the same. A correct system will take cognizance, not of one only, but of every way in which a given result can be attained.

There is considerable difficulty in the use of language for the expression of such abstract ideas, and these points would hardly become clearer by fuller explanation. Let it be sufficient, in conclusion, to state that the chemical symbol 1, while it is a necessary constituent of the system of chemical symbols, and may be and indeed must be employed to give effect to the purposes of the chemical calculus, is not to be interpreted in weight.

(3) An inquiry not without theoretical interest is immediately suggested by the previous considerations. We have arrived at the symbol of no ponderable matter, regarded as a component of a compound weight; what is the symbol of all ponderable matter, similarly regarded? Now all ponderable matter is characterized by the property that the addition to it of any finite weight does not alter our conception of it. Hence a compound weight, of which all ponderable matter and a finite weight are the components, is the same as a compound weight of which all ponderable matter is the single component. Hence, if y be the symbol of all ponderable matter thus regarded,

$$yx=y.$$

Now the numerical symbols 0 and ∞ satisfy this condition, since $0x=0$, and $\infty\, x=\infty$; and either symbol, so far as this equation is concerned, may be with equal propriety selected as the symbol of all ponderable matter. This is by no means contrary to analogy. As the numerical symbols 0 and ∞ are symbols of which all numbers are factors, so the chemical symbols 0 and ∞ are symbols of which all other chemical symbols are components,

$$\infty = x\,x_1\,x_2 \ldots\ldots \infty,$$
$$0 = x\,x_1\,x_2 \ldots\ldots 0.$$

In the same sense as the symbol 1 is to be interpreted as the symbol of space, so it will appear, on consideration, that the symbol ∞ is to be interpreted as the symbol of the ponderable universe regarded as a whole. Neither object can be presented to the imagination, but, nevertheless, they are to be treated as realities in the order of ideas, and appear in the chemical system as the necessary limits of our conceptions.

(4) Similar ideas occur in every symbolic method. In symbolic chemistry 1, as the symbol of the unit of space, the subject of chemical operations, occupies the place held in the geometric calculus by 1, the symbol of the unit of length considered as the subject of the operations of geometry. Again, the chemical symbol ∞ holds a position analogous to that occupied by the symbol 1 in the calculus of probabilities, as denoting the total subject matter of the science, and the chemical symbols 1 and ∞, the symbols of space and of the ponderable universe, represent in the calculus of chemistry the limits between which the values of all other symbols are comprised, precisely as in arithmetical algebra the corresponding limits are represented by the symbols 0 and ∞, and in the calculus of logic by the symbols 0 and 1*.

* BOOLE, 'Laws of Thought,' p. 47.

SECTION IV.—ON THE FUNDAMENTAL CHEMICAL EQUATIONS.

(1) xy is the symbol of a single weight which is composed of the same weights as those of which that group of weights is constituted of which $x+y$ is the symbol. Now according to the definition which I have given of chemical identity, two weights are said to be identical which consist of the same weights (Sec. I. Def. 6). Hence the weight of which xy is the symbol is identical with the weight of which $x+y$ is the symbol; and

$$xy=x+y.$$

In like manner, since $\frac{x}{y}$ is the symbol of a single weight composed of the same weights as that of which the group of weights $x-y$ is constituted,

$$\frac{x}{y}=x-y.$$

These equations may justly be termed the fundamental equations of the Chemical Calculus, for from them chemical symbols derive their distinctive character, and, through the limitations thus imposed upon them, are discriminated from numerical symbols, which in many respects they resemble*.

(2) If, in the equation $xy=x+y$, $y=1$, $x1=x+1$; whence since $x=x1$ and $x-x=0$, we infer that

$$0=1.$$

This equation informs us of the identity of the ponderable matter of which 0 and 1 are the symbols, which has already been shown.

The same point may be proved in a similar manner as regards the other forms of the symbol 1. For since

$$x-x=x-1x,$$

$$x-x=xx^{-1},$$

and

$$0=x^0.$$

Or, since

$$mx=x^m,$$

$$0=x^0.$$

And again, since

$$\frac{x}{y}=x-y,$$

* This equation occupies a somewhat similar place in the chemical calculus to that held in the logical system by the equation $x^2=x$ (BOOLE, 'Laws of Thought,' p. 31), as being expressive of a characteristic property by which the symbols are distinguished. The possibility of the existence of a class of symbols, other than the symbols of the logarithms of numbers, which should satisfy the condition

$$f(x)+f(y)=f(xy),$$

was indicated by D. F. GREGORY in his paper "On the Real Nature of Symbolical Algebra" (Edin. Phil. Trans. vol. xiv. p. 208). This anticipation is here realized.

if $y=x$,

$$\frac{x}{x}=x-x,$$

and

$$\frac{x}{x}=0.$$

We thus arrive from the general properties of chemical symbols at the same result as regards the forms of the symbol 1, and the interpretation of that symbol, as was inferred from the special interpretations of each form of that symbol.

(3) If, in the equation $xy=x+y$, $x=1$ and $y=1$,

$$1^2=1+1$$

and

$$1=2.$$

From this and the previous equation, $0=1$, it is to be inferred that

$$0=1=2=3=\ldots\ldots n.$$

It hence follows that any number of numerical symbols of this class may be added to a chemical function without affecting its interpretation; a property which will hereafter be shown to admit of important applications. The reason of this is that in every independent symbol of number which enters into a chemical function the chemical symbol 1 is understood as the subject of operation, so that $2=2\times1$, and that this symbol has no interpretation in weight. We have a parallel to this property of chemical symbols in the property conferred upon numerical symbols by the factor 0, where

$$0=1\times0=2\times0=3\times0=\ldots\ldots n\times0.$$

The chemical equation $0=1$ may at the first glance appear paradoxical. But this apparent paradox arises merely from the associations connected with the interpretation of these symbols in those symbolic systems with which we are most familiar. In these systems there is a profound antithesis between the symbols, which reaches its climax in the logical system, where 0 is the symbol of nothing and 1 the symbol of the universe of thought*. It need not, however, be a matter of surprise that in the chemical system we should have two symbols for "no weight," since in that system the same ponderable matter may be denoted by xy and $x+y$. Indeed it might even be expected from analogy that as a real weight may have several symbols, so the absence of weight should be expressed in more than one way. Nor is it, in truth, more singular or paradoxical that in chemistry 0 and 1 should be symbols denoting the same object, than that in geometry x^0 and 1 should have the same interpretation.

Now it would appear that the symbols 0 and 1 may occur in a chemical function with two distinct interpretations, as chemical symbols and as arithmetical symbols, and that to prevent ambiguity, it might be desirable to make evident by some special notation the meaning to be assigned to them. But this is not necessary. The chemical symbol 1, although implicitly contained as the subject of operations in every chemical

* Boole, 'Laws of Thought,' p. 48.

function, yet conformably to the principles of algebraic notation may be invariably suppressed, and every numerical symbol which appears in this calculus may be interpreted with its usual arithmetical signification, regard being had to those special properties which are derived from the subject on which it operates. If, for a moment, we discriminate between the chemical and arithmetical symbols 0 and 1, marking the former as $0'$ and $1'$, and the latter as (0) and (1), it is at once evident that $(1)=(1)1'=1'$, and that $(0)=(0)x=0'$. Hence we may in every case replace the chemical symbols $0'$ and $1'$ by the arithmetical symbols (0) and (1), which are, so far as the purposes of this calculus are concerned, identical with them both in interpretation and in properties. These symbols 0 and 1 may be termed the zero-symbols of the chemical system, being marks by which we denote the absence of ponderable matter. That such symbols may serve most important ends is evident from the use which has been made in arithmetic of the zero-symbol 0, which is the very key-stone of the arithmetical system; and yet it is not too much to assert that the system of chemical symbols without the zero-symbol 1 is as incomplete and as little adapted to the purpose which it is destined to fulfil as the arithmetical system would be deprived of the symbol 0.

(4) No other known system of symbols is characterized by the same property as that by which chemical symbols are defined, but the equation $x+y=xy$ is similar in form to the equation connecting the logarithms of numbers; and the relation which subsists between the absolute weight (or weight in grammes) of the ponderable matter of which x and y are the symbols is the same as the logarithmic relation. For, writing $w(x)$ and $w(y)$ as the absolute weights of the ponderable matter symbolized by x and y,

$$w(x)+w(y)=w(xy),$$
$$w(1)+w(y)=w(y),$$
$$w(1)=0,$$

similar in form with the logarithmic equations

$$l(x)+l(y)=l(xy),$$
$$l(1)+l(y)=l(y),$$
$$l(1)=0.$$

The property of chemical symbols given in the equation $x+y=xy$ may from these analogies appropriately be termed the "logarithmic" property of these symbols.

(5) It is sufficiently obvious that we may operate between chemical equations by means of addition and subtraction as with numerical equations. This is a consequence of the axiom that if identical weights be added to or removed from identical groups the resulting groups are identical. So that, if $x=y$, and $x_1=y_1$, $x\pm x_1=y\pm y_1$. The operations, however, which correspond to the algebraic operations of multiplication and division can only be performed under certain conditions, which will be considered in a subsequent part of this memoir.

Section V.—ON THE SYMBOLS OF SIMPLE WEIGHTS.

(1) A simple weight has been defined as a weight which is not compound, and two weights as simple in regard to one another which have no common component.

It follows from this definition that the symbol of a simple weight cannot be expressed by more than one factor, and also that the symbols of weights simple in regard to one another cannot have a common factor.

The symbol of a simple weight is termed a prime factor, and the symbols of weights simple in regard to one another are said to be prime to one another.

The symbols of simple weights have the following properties:—

(2) The operations of algebraic subtraction and division cannot be performed between such symbols. For let a and b be two symbols of simple weights, and, if possible, let $a-b=a_1$. Then $a=a_1+b=a_1b$, that is, a is the symbol of a compound weight, which is contrary to the hypothesis.

Or again, if possible, let $\frac{a}{b}=c$. Then $a-b=c$, and $a=b+c=bc$, which is, as before, contrary to the hypothesis.

(3) If a and b be two symbols of weights simple in regard to one another, and if $aa_1=bb_1$, then

$$a_1=bk, \text{ and } b_1=ak;$$

for since $aa_1=bb_1$, $a+a_1=b+b_1$, and $a_1=b+b_1-a$; and since by hypothesis b is the symbol of a weight simple in regard to a, no part of a is a constituent of b, therefore a must be a constituent of b_1, so that $b_1=a+k$ and $a_1=b+k$. Whence also $b_1=ak$, and $a_1=ak$.

(4) Hence also if a_1 be the symbol of a weight simple in regard to the weights a and b, a_1 is the symbol of a weight simple in regard to the weight ab.

For otherwise, if possible, let a_1 and ab have a common component k, so that $a_1=ck$, and $ab=c_1k$; then, since by hypothesis k is a factor of a_1, k is by hypothesis prime to a. Therefore k is a factor of b, which is also contrary to the hypothesis.

We may also argue thus. If possible, let k be the factor common to a_1 and ab. Then $ab=kd$, and $d=a+b-k$. But by hypothesis no part of k is a constituent of a; therefore k is a constituent of b, and $b=k+c=kc$, which is contrary to the hypothesis.

(5) Hence if a is prime to b, a^p is prime to b^q, and no part of qb is a constituent of pa.

(6) Also, if a, b, c, d, ... be prime to a_1, b_1, c_1, d_1, ..., then $a^p b^{p_1} c^{p_2} d^{p_3}$ ..... is prime to $a^q b^{q_1} c^{q_2} d^{q_3}$ .....; and also the operation of subtraction cannot be performed between the weights $pa+p_1b+p_2c+p_3d+$...... and $qa_1+q_1b_1+q_2c_1+q_3d_1+$.....

(7) Whence, if a, b, c, d, ... be symbols prime to one another, and if

$$a^p b^{p_1} c^{p_2} d^{p_3} \ldots\ldots = 1,$$

$$p=0, p_1=0, p_2=0, p_3=0 \ldots\ldots;$$

nd if

$$pa+p_1b+p_2c+p_3d+\ldots\ldots=0,$$
$$p=0, p_1=0, p_2=0, p_3=0.\ldots.$$

(8) Lastly, a composite symbol can only be expressed in one manner by means of prime factors. That is to say, a compound weight can only be assumed to be composed of one set of simple weights. This proposition may be proved in the same manner as the corresponding numerical proposition.

This assertion does not imply that we cannot make more than one hypothesis as to the expression of any given composite symbol by means of prime factors, that is, as to the simple weights of which a given compound weight is composed, but only that two or more such hypotheses cannot simultaneously be true.

There is a close analogy between the symbols of simple weights in chemistry and the symbols of prime numbers in arithmetic, but owing to the condition imposed on chemical symbols, given in the equation $x+y=xy$, a chemical symbol which has only one factor is also incapable of partition.

The prime symbols of chemistry may be indifferently defined by either property, the one being a consequence of the other, and constitute a new and peculiar order of symbols. There is, however, one numerical symbol of the class, namely, the symbol 1, which has only one factor and one part, and like the primes of chemistry is incapable of division or partition.

(9) An integral compound weight has been defined (Sec. I. Def. 9) as a weight which is composed of an integral number of simple weights. If φ be the symbol of such a weight, $a, b, c, \ldots\ldots$, as before, the symbols of simple weights, and $n, n_1, n_2, \ldots.$ integral numbers,

$$\varphi=a^n b^{n_1} c^{n_2}\ldots\ldots$$

This symbol is termed an integral composite symbol. It is identical in form with the symbol of an integral number expressed by means of its prime factors.

(10) It remains to consider the method by which we may arrive at the expression of chemical symbols by means of an integral number of prime factors in a given system of equations, if such an expression be possible, and further may select from the various forms of symbols which satisfy this condition that form in which the symbols are expressed by the smallest possible number of such factors. In this form the symbol is said to be expressed in the simplest possible manner by means of prime factors, it being the only symbolic expression which is at once both necessary and sufficient to satisfy the conditions of the problem.

To these conditions it is to be added that the prime factors thus chosen are to be the symbols of real weights, it being possible to find symbolic expressions which satisfy the requirements of the equation, but which do not admit of interpretation, the weights of which they are the symbols being affected with the negative sign. Such expressions will here be rejected.

Now, first let the system of equations in which it is required to express the chemical

symbols by means of prime factors consist of one equation, and, to render the problem determinate, let the equation contain only two undetermined symbols, and be of the form

$$m\phi+m'\phi_1+m''\phi_2=0,$$

where m, m', m'' are known, being positive or negative numerical symbols; and let $\phi_2=a^nb^{n_1}$, a and b being the symbols of simple weights, and n, n_1 given positive and integral numbers.

Then putting $\phi=a^pb^{p_1}$, $\phi_1=a^qc^{q_1}$,

$$ma^pb^{p_1}+m'a^qb^{q_1}+m''a^nb^{n_1}=0,$$

whence, from the fundamental equation $x+y=xy$,

$$(a^pb^{p_1})^m(a^qb^{q_1})^{m'}(a^nb^{n_1})^{m''}=1,$$

and from the property of simple weights before given (Sec. V. (7))

$$mp\ +m'q\ +m''n\ =0,$$

$$mp_1+m'q_1+m''n_1=0.$$

The integral and positive solutions of these equations as regards p, q, p_1, q_1, if such can be found, will give all the possible ways by which the symbols ϕ and ϕ_1 can be expressed by means of prime factors in the above equation, the symbol ϕ_2 being of the form given, and the minimum solution in whole numbers of these equations, as regards the same indeterminate quantities, will give the simplest expression of the symbols by means of prime factors, subject to the same condition.

The number of admissible forms of these symbols is, however, further limited by the requirement that the factors a and b are to be the symbols of real weights.

Putting W, W_1, W_2 as the known absolute weights of the portions of matter of which ϕ, ϕ_1, ϕ_2 are the symbols, and $w(a)$, $w(b)$ as the unknown absolute weights of the simple weights of which a and b are the symbols, we have for the determination of $w(a)$ and $w(b)$ the equations

$$pw(a)+p_1w(b)=W,$$

$$qw(a)+q_1w(b)=W_1,$$

$$nw(a)+n_1w(b)=W_2,$$

which, subject to the equation of condition

$$mW+m'W_1+m''W_2=0,$$

are equivalent to two independent equations. All values, therefore, of p, p_1, q, q_1 are to be rejected which would give a negative value for $w(a)$ or $w(b)$ in the above equations.

If in the original equation the given symbol ϕ_2 be expressed by more than two factors, so that $\phi_2=a^nb^{n_1}c^{n_2}$, the problem is indeterminate unless the absolute weight of one of the simple weights be given; for in this case we should have only two equations to determine the three values $w(a)$, $w(b)$, $w(c)$.

Or again, if no symbol were given, so that $\phi_2=a^rb^{r_1}$, r and r_1 being indeterminate

quantities, the indeterminate equations, whence the value in whole numbers of p, p_1, q, q_1, r, r_1 are to be ascertained, become

$$mp+m'q+m''r=0,$$
$$mp_1+m'q_1+m''r_1=0,$$

the two equations containing three indeterminate quantities.

If in the original equation two symbols were given as determined from other considerations, as that

$$\varphi_2=a^s b^{s_1},$$
$$\varphi_1=a^t b^{t_1},$$

s, s_1, t, t_1 being given positive and integral, the indeterminate equations would contain only one unknown quantity, and the problem would be possible in that case alone where the values of p and p_1 derived from them were positive and integral, and where the conditions before referred to and given in the equations connecting $w(a)$ and $w(b)$ were satisfied.

The course to be pursued in other cases is sufficiently obvious from the above instance. It remains only to state the nature of the problem in its most general form.

If there be a system of N equations connecting the chemical symbols $\varphi, \varphi_1, \varphi_2, \varphi_3, \ldots$ of the form

$$m\varphi+m'\varphi_1+m''\varphi_2+m'''\varphi_3+\ldots=0,$$

where $m, m', m'', \ldots$ are numerical symbols, negative, positive, or 0, putting as before

$$\varphi=a^p b^{p_1} c^{p_2}\ldots,$$
$$\varphi_1=a^q b^{q_1} c^{q_2}\ldots,$$
$$\varphi_2=a^r b^{r_1} c^{r_2}\ldots,$$

we shall have N sets of indeterminate equations connecting $p, q, r, \ldots$ and $p_1, q_1, r_1, \ldots$ and $p_2, q_2, r_2, \ldots$ of the form

$$mp+m'q+m''r+\ldots=0,$$
$$mp_1+m'q_1+m''r_1+\ldots=0,$$
$$mp_2+m'q_2+m''r_2+\ldots=0,$$
$$\cdots\cdots\cdots$$
$$\cdots\cdots\cdots$$

If a common positive solution in whole numbers of these N sets of equations for

$$p, q, r, \ldots, \quad p_1, q_1, r_1, \ldots, \quad p_2, q_2, r_2, \ldots$$

can be found, then the symbols $\varphi, \varphi_1, \varphi_2, \ldots$ can be expressed in the given system of equations by means of the prime factors $a, b, c, \ldots$; if such a solution does not exist, then the symbols cannot be so expressed; and the simplest expression of the symbols $\varphi, \varphi_1, \varphi_2$ in that system of equations by means of the prime factors $a, b, c, \ldots$ is that expression in which the indices $p, p_1, p_2, \ldots, q, q_1, q_2, \ldots, r, r_1, r_2, \ldots$ have the minimum integral values which satisfy the above N sets of indeterminate equations.

The admissible values are limited by the conditions

$$pw(a)+p_1w(b)+p_2w(c)\ldots\ldots=\mathrm{W},$$
$$qw(a)+q_1w(b)+q_2w(c)\ldots\ldots=\mathrm{W}_1,$$
$$rw(a)+r_1w(b)+r_2w(c)\ldots\ldots=\mathrm{W}_2,$$
$$\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot$$
$$\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot$$

where $w(a)$, $w(b)$, $w(c)$ are positive, and W, W_1, W_2, ... are connected by N equations of the form

$$m\mathrm{W}+m'\mathrm{W}_1+m''\mathrm{W}_2+m'''\mathrm{W}_3+\ldots\ldots=0.$$

SECTION VI.—ON THE CONSTRUCTION OF CHEMICAL EQUATIONS FROM THE DATA AFFORDED BY EXPERIMENT.

Our knowledge as to the identical relations of ponderable matter is derived, as has already been observed, exclusively from the science of chemistry. The next step in this inquiry is to embody in a system of chemical equations the information on this subject which experiment affords to us. The process is very simple by which this may be effected. To take, for example, a single instance. Let it be supposed that we have ascertained by experiment that 3000 cub. centims. of chlorine and 2000 cub. centims. of ammonia have been converted into 6000 cub. centims. of hydrochloric acid and 1000 cub. centims. of nitrogen. We hence infer the identity of the ponderable matter of which the two groups respectively consist, and putting

ϕ as the symbol of a unit of chlorine,
ϕ_1 as the symbol of a unit of ammonia,
ϕ_2 as the symbol of a unit of hydrochloric acid,
ϕ_3 as the symbol of a unit of nitrogen,

we assert this identity in the chemical equation

$$3\phi+2\phi_1=6\phi_2+\phi_3.$$

Proceeding in other cases in a similar manner, we should arrive at a system of equations corresponding in number to the experiments of which the results were thus recorded. It would soon, however, be perceived that we could not in this manner indefinitely add to our knowledge, but that the information thus supplied to us was soon exhausted, the equations not being independent, but capable of being derived from one another by the processes of addition and subtraction; and that, in fact, they could be replaced by a single system of equations connecting every chemical symbol equal in number to the total number of chemical substances, exclusive of the elemental bodies. Such a system is afforded to us by those equations which express the relations of identity which subsist between the ponderable matter of compound substances and the ponderable matter of the elemental bodies of which they are composed, which we may consider as a solution of the entire system of chemical equations in regard to the symbols of the elements. From this primary system every other chemical equation may be derived, and our total

knowledge as to the identical relations of ponderable matter is implicitly comprised in it. Indeed it may readily be shown that, however numerous may be our experiments, we can never arrive at any greater number of independent equations without effecting the decomposition of the elements. For if such an independent equation were discovered, it either would be an equation connecting the symbols of the elements themselves, or if it contained the symbols of other substances we might eliminate between it and the other equations of the system, and thus derive such an equation. It is possible that the limited range of physical conditions under which we necessarily operate, or other obstacles equally insuperable, may for ever preclude such an addition to our knowledge, but nevertheless we can form a conception of another and a wider chemistry, of which our actual system should be but an imperfect fragment, and in which we should have n independent equations containing $n+1$ symbols admitting of a solution of the form

$$\phi=m\theta,$$

when we should recognize but one primary elemental form of ponderable matter, and the great problem of analysis would be completely and finally resolved.

Every chemical equation is necessarily the expression of a hypothesis; for even the most accurate experiments are attended with error, and can only be regarded as affording a certain approximation to that true result at which it is our object to arrive. Even the assertion that two gaseous volumes of water consist of the same ponderable matter as two volumes of hydrogen and one volume of oxygen involves hypotheses as to the gaseous densities of those substances, and the relations of absolute weight before and after chemical change, which go beyond our actual experience. Experiment proves this proposition to be true within certain limits of error, but in the equation

$$2\phi=2\phi_1+\phi_2,$$

an assertion is made in which the errors of observation are not included. Regard being had to the total evidence on which it rests, no statement of the kind is perhaps more credible than this; and the above equation may serve to mark the extreme limit to which chemical certainty has attained. Such equations form the true basis of the science.

It is, however, only in comparatively few instances that we are able to ascertain by direct observation the gaseous densities of all the chemical substances which enter into a reaction; and where this cannot be effected we are compelled to have recourse to indirect methods of a less satisfactory character, to attain the desired end.

There are many admirable examples of such chemical reasoning*, which, divested of the theoretical considerations with which they are unnecessarily complicated, may be regarded as arguments based upon actual observation of the laws of chemical change, by which certain forms of these equations are established with superior probability, to

* For example, ODLING "On the Atomic Weight of Oxygen and Water," Journal of the Chemical Society, vol. xi. p. 107. Also the article in WATTS's Dictionary of Chemistry by the same author on the atomic weights of the metals; and WURTZ, "On the Oxide of Ethylene considered as a link between Organic and Inorganic Chemistry," Journal of the Chemical Society, vol. xv. p. 387.

the exclusion of other forms. The conclusions thus arrived at must obviously have very different values, and while some are in the highest degree probable, others can only be regarded as tentative and conjectural. But, nevertheless, uncertain as such results may often appear, a profound distinction is to be drawn between this order of hypothesis and those molecular speculations which can neither be confirmed nor disproved by facts. In the former case experiment is constantly controlling our conclusions, and we have the most positive evidence that the methods pursued by the chemist are in the main correct; since in numerous cases he has been able to anticipate the results of direct observation, and in others even to correct by theory the erroneous results which observation apparently afforded.

It is essential to have clear ideas upon this point, that we may not over-estimate the value of our results, since any uncertainty attached to the data must undoubtedly attend the conclusions which are derived from them; but nevertheless the question does not fall within the scope of a deductive and symbolic method, the province of which commences only where the task of experiment terminates; and in the consideration of chemical equations I have not, in uncertain cases, attempted any full discussion of the evidence on which they rest, but have limited myself to arguments, in regard to which the application of symbolic reasoning afforded some peculiarity or advantage.

Section VII.—ON THE SYMBOLS OF THE UNITS OF CHEMICAL SUBSTANCES.

Group 1.—*Symbols of Hydrogen, Oxygen, Sulphur, Selenium, Chlorine, Iodine, Bromine, Nitrogen, Phosphorus, Arsenic, and Mercury.*

(1) *Symbol of Hydrogen.*—I am about to show by the aid of the principles which have been established in the previous pages that the units of chemical substances are composed of an integral number of simple weights; that is to say, according to the definition previously given (Sec. I. Def. 9 & 10), that these units are "integral compound weights." The point will be demonstrated if it be found possible to express the symbols of these units in the actual system of chemical equations by means of an integral number of prime factors, these factors being the symbols of real weights (Sec. V. (9 & 10)). Now it will be found that such an expression is not only possible, but possible in a great variety of ways; in other words, many assumptions may be made as to the composition of ponderable matter which are consistent with the above fundamental hypothesis. From these possible expressions, that one will in each case be selected, as the correct inference from the facts, in which the symbol is expressed by the smallest possible number of such factors; since any other expression, as has before been indicated, must involve hypotheses which are unnecessary.

The problem is not dissimilar to that of the determination of the density or relative weight of the same units. We are about to estimate the number and the absolute weight of the simple weights of which the units (Sec. I. Def. 8 & 10) of ponderable matter are composed; and, as in the former case the problem is unmeaning unless the

standard of absolute or relative weight be previously determined, so in the latter case also it is necessary that some one simple weight shall be selected from external considerations as the standard of comparison, before any statement can be made upon the subject. This case, however, differs from the preceding in the circumstance that the selection of a simple weight is not the choice of an arbitrary unit, to be determined by considerations of convenience alone, but involves the assertion of a hypothesis as to the actual composition of the chemical units of ponderable matter, which may be verified and tested by experience.

Now the hypothesis on which the present method is based, and which is the only assumption of the kind which I shall have occasion to make, is that the unit of hydrogen is a simple weight, that is to say, that in chemical transformations this weight is never distributed (Sec. I. Def. 12). The symbol of this "weight" (Sec. I. Def. 3) will be expressed by the letter α, which may be termed the "modulus" of the symbolic system, it being that symbol by which the form of every other symbol is regulated. The absolute weight of the portion of ponderable matter thus symbolized, that is to say, of 1000 cub. centims. of hydrogen at 0° C. and 760 millims. pressure, is 0·089 grm.

In considering this question I shall select certain examples which may serve to illustrate the way in which the subject may be treated, and the difference in the result arrived at, according to the degree of information supplied to us by experiment.

The first, and for the present object the most important, group of symbols to be considered are the symbols of those elements of which the density in the gaseous condition can be experimentally determined, and which also form with one another gaseous combinations. I shall then consider the symbols of carbon and its combinations with the previous group, and subsequently the symbols of certain other elements and their combinations as to which we possess less adequate information.

(2) *Symbol of Oxygen.*—It is known from experiment that 2 units of water can be decomposed into 2 units of hydrogen and 1 unit of oxygen. We hence infer the identity of the weights of which these portions of ponderable matter consist, and putting

ϕ as the symbol of the unit of water,

ϕ_1 as the symbol of the unit of hydrogen,

ϕ_2 as the symbol of the unit of oxygen,

$$2\phi=2\phi_1+\phi_2.$$

Now, if possible, let

$$\phi=\alpha^m\xi^{m_1},$$
$$\phi_1=\alpha,$$
$$\phi_2=\alpha^n\xi^{n_1},$$

where α and ξ are prime factors, that is to say, the symbols of simple weights, and m, m_1, n, n_1 positive integers. Then

$$2\alpha^m\xi^{m_1}=2\alpha+\alpha^n\xi^{n_1},$$

and from the fundamental equation which connects chemical symbols, $x+y=xy$ (Sec. IV. (1)),

$$(\alpha^m\xi^{m_1})^2=\alpha^2\alpha^n\xi^{n_1},$$

whence

$$2m=2+n \text{ and } 2m_1=n_1,$$

to which is attached the condition

$$w(\alpha)=1,$$

$$m+m_1w(\xi)=9;$$

1 and 9 being the densities of hydrogen and of water, and $w(\alpha)$ and $w(\xi)$ being positive.

The integral and positive solutions of these equations as regards m, m_1, n, n_1, give all the possible hypotheses which can be made as to the components of oxygen and of water, which are consistent with the hypothesis that the unit of each of these substances is composed of an integral number of simple weights, and that the unit of hydrogen is a simple weight, and the minimum solution selects from these that one hypothesis which is both necessary and sufficient to satisfy the condition given in the equation

$$2\phi=2\phi_1+\phi_2.$$

This solution is

$$n=0, \quad m=1,$$

$$n_1=2, \quad m_1=1,$$

whence the symbols of water and oxygen as determined from considering the above equation are

Symbol of water $\alpha\xi$,

Symbol of oxygen ξ^2,

and the relative weights corresponding to the prime factors α and ξ are

$$w(\alpha)=1,$$

$$w(\xi)=8,$$

the equation being thus expressed,

$$2\alpha\xi=2\alpha+\xi^2.$$

It is not to be assumed without proof that these symbols will satisfy the conditions afforded by other equations. This is a matter for inquiry. But we have arrived at the knowledge that no symbol can be found for these substances composed of a smaller number of prime factors, and also that if these symbols can be so expressed the indices of these factors will be found among the integral solutions of the above equations, which are given in the forms

$$m=1+t, \quad n=2t,$$

$$m_1=1+t_1, \quad n_1=2(1+t_1).$$

Hence we arrive at the following general forms for the symbols of oxygen and water,

Oxygen $\alpha^{2t}\xi^{2(1+t_1)}$,

Water $\alpha^{1+t}\xi^{1+t_1}$,

which include all the possible forms of symbols which satisfy the above conditions.

From the equation

$$m+m_1w(\xi)=9$$

we have, substituting for m and m_1 the above values,

$$w(\xi)(1+t_1)=8-t,$$

whence t is not greater than 8. If $t=8$, either $w(\xi)=0$ and $\xi=1$ (Sec. III. (1)), or $t_1=-1$, when $n_1=0$, and $m_1=0$; in which case $w(\xi)=\frac{0}{0}$ and may have any value. In either case we have

Symbol of oxygen α^{16},
Symbol of water α^9.

In this case, therefore, owing to the peculiar numerical relation which subsists between the densities, it is possible to express all the symbols by means of the one factor α. The different forms of symbol are given by assigning to t and t_1 all possible values; thus, for example,

		Symbol. Oxygen.	Symbol. Water.
$t=0$	$t_1=0$	ξ^2	$\alpha\xi$
	$t_1=1$	ξ^4	$\alpha\xi^2$
	$t_1=2$	ξ^6	$\alpha\xi^3$
	. . . . .	. . . . .	. . . . .
$t=1$	$t_1=0$	$\alpha^2\xi^2$	$\alpha^2\xi$
	$t_1=1$	$\alpha^2\xi^4$	$\alpha^2\xi^2$
	$t_1=2$	$\alpha^2\xi^6$	$\alpha^2\xi^3$
	. . . . .	. . . . .	. . . . .
$t=2$	$t_1=0$	$\alpha^4\xi^2$	$\alpha^3\xi$
	$t_1=1$	$\alpha^4\xi^4$	$\alpha^3\xi^2$
	$t_1=2$	$\alpha^4\xi^6$	$\alpha^3\xi^3$
	. . . . .	. . . . .	. . . . .
$t=3$	$t_1=0$	$\alpha^6\xi^2$	$\alpha^4\xi$
	$t_1=1$	$\alpha^6\xi^4$	$\alpha^4\xi^2$
	$t_1=2$	$\alpha^6\xi^6$	$\alpha^4\xi^3$
. . .	. . . . .	. . . . .	. . . . .
. . .	. . . . .	. . . . .	. . . . .
. . .	. . . . .	. . . . .	. . . . .
$t=7$	$t_1=0$	$\alpha^{14}\xi^2$	$\alpha^8\xi$
	$t_1=1$	$\alpha^{14}\xi^4$	$\alpha^8\xi^2$
	$t_1=2$	$\alpha^{14}\xi^6$	$\alpha^8\xi^3$
	. . . . .	. . . . .	. . . . .
$t=8$	$t_1=-1$	α^{16}	α^9

The problem before us is the selection from this system of symbols of that symbol for oxygen (if such can be found) which shall satisfy the conditions afforded by the other equations of the system, and in which it shall be expressed by the smallest possible number of prime factors. Our hypothesis must be necessary as well as sufficient. Owing to the peculiar and simple laws which prevail in the actual system of chemical

changes this selection is attended with no difficulty. It will be found by trial that the symbol of oxygen, ξ^2, that symbol in which it is expressed by the smallest number of prime factors in the above equation, will satisfy all other known conditions.

There is only one other known compound of oxygen and hydrogen, the peroxide of hydrogen. Our knowledge as to this substance is very imperfect, but let us assume that it can be decomposed with the formation of water and oxygen, in the ratio of two units of water to one of oxygen. Then putting z as the unknown number of the volumes of the peroxide decomposed, and y as the number of volumes of oxygen formed, and φ_3 as the symbol of peroxide of hydrogen,

$$z\varphi_3=2y\varphi+y\varphi_2, \quad \ldots \quad (1)$$

whence, substituting for φ its value $\varphi_1+\frac{\varphi_2}{2}$,

$$z\varphi_3=2y\varphi_1+2y\varphi_2;$$

and putting $\alpha^p\xi^{p_1}$ as the symbol of the peroxide of hydrogen, α as the symbol of hydrogen, and ξ^2 of oxygen,

$$z\alpha^p\xi^{p_1}=2y\alpha+2y\xi^2 \quad \ldots \quad (2)$$

and

$$(\alpha^p\xi^{p_1})^z=\alpha^{2y}(\xi^2)^{2y},$$

whence

$$zp=2y,$$

$$zp_1=4y.$$

Now, if p and p_1 be positive integers, since z is prime to y, z must be either 1 or 2. If

$$z=1,\ p=2y,\ p_1=4y;$$

if

$$z=2,\ p=y,\ p_1=2y.$$

In either case the above equation (2) is of the form

$$(\alpha\xi^2)^{y_1}=y_1(\alpha+\xi^2).$$

Now, although it undoubtedly cannot be considered that this equation is established with the same certainty as the equation

$$2\alpha\xi=2\alpha+\xi^2,$$

yet regarded as a question of probability the evidence in favour of this view is very conclusive. For the only hypotheses involved in it are (1) that in this equation the symbol of the peroxide of hydrogen can be expressed by an integral number of prime factors, and (2) that the symbol of hydrogen can be expressed as α, and the symbol of oxygen as ξ^2, in favour of which hypotheses we have (as will hereafter be seen) the evidence of an extensive and uniform experience.

We may proceed by the aid of further hypotheses, and still with great probability, to determine the value of y_1 in this symbol. The most probable hypothesis is that $y_1=1$, and that the symbol of peroxide of hydrogen is $\alpha\xi^2$, in which case the above equation is thus expressed,

$$\alpha\xi^2=\alpha+\xi^2.$$

The chief arguments by which this view is supported are derived from the observations (1) that it is very rarely that the indices of the prime factors of chemical symbols have a common measure; (2) that on this assumption the symbols of hydrogen, water, and peroxide of hydrogen constitute a series of the form α, $\alpha\xi$, $\alpha\xi^2$; and that the densities of these substances form an arithmetical progression, being 1, 9, 17; and that such series so frequently occur in the actual system of chemical symbols as to render probable their existence in the future system.

The conclusions at which we thus arrive are not to be regarded as necessarily final. Not only is it possible that further information as to the chemical properties of the peroxide of hydrogen might lead us to the adoption of a more complex symbol, but we can even specify the very facts, the discovery of which would induce us to modify our opinion. But, nevertheless, the choice of the expression $\alpha\xi^2$ as the symbol of the unit of this substance is not an arbitrary and conventional selection. It expresses the most probable opinion which, with our actual knowledge, we can form as to the nature of the equation from which it is derived, and which we provisionally embody in the symbol for the purpose of tracing the consequences of our hypothesis.

The weight of that portion of any chemical substance which I have termed the chemical unit of ponderable matter, is (Sec. I. Def. (10)) the weight of that portion of each substance which in the gaseous condition, at 0° C. and 760 millims. pressure, occupies the space of 1000 cub. centims. This weight may be measured in two ways; either by comparison with the weight of a cubic centimetre of distilled water at 4° C., or by comparing it with the weight of the chemical unit of hydrogen. We shall hence have two series of numbers by which the weights of the portions of ponderable matter resulting from any chemical operation are expressed, viz.

1. The absolute weight in grammes.
2. The relative weight or density as compared with the weight of the unit of hydrogen.

This second series of numbers may also be regarded as expressing the absolute weight of the units of ponderable matter as estimated in "criths"*.

Combinations of the Prime Factors α and ξ.

Name of substance.	Prime factors.	Absolute weight, in grammes.	Relative weight.
	α	0·089	1
	ξ	0·715	8
	Symbol.		
Hydrogen	α	0·089	1
Oxygen†	ξ^2	1·430	16
Water	$\alpha\xi$	0·805	9
Peroxide of hydrogen	$\alpha\xi^2$	1·520	17

* HOFMANN, *loc. cit.* (Sec. I. (11)).

† In calculating the absolute weight, it is necessary to assume the absolute weight of the gaseous litre at 0° and 760 millims. pressure, of some one substance as accurately determined. The weight of a litre of oxygen is here assumed as the standard, and the other numbers are calculated from it.

(3) *Symbol of Sulphur.*—It has been shown by the recent experiments of DEVILLE and TROOST that the density of the vapour of sulphur above a temperature of 860° C. becomes constant, and approximates to 32, the density of hydrogen being 1. This being the case, the ponderable matter of 2 units of sulphide of hydrogen is identical with the ponderable matter of 2 units of hydrogen and 1 unit of sulphur. Hence, putting $\alpha^m\theta^{m_1}$ as the symbol of sulphide of hydrogen, and $\alpha^n\theta^{n_1}$ as the symbol of sulphur,

$$2\alpha^m\theta^{m_1}=2\alpha+\alpha^n\theta^{n_1},$$

and

$$(\alpha^m\theta^{m_1})^2=\alpha^2\alpha^n\theta^{n_1},$$

and

$$2m=2+n, \quad 2m_1=n_1,$$

whence

$$n=0, \quad m=1,$$
$$n_1=2, \quad m_1=1,$$

a minimum.

And putting the density of sulphide of hydrogen as 17,

$$m+m_1w(\theta)=17,$$

and

$$w(\theta)=16.$$

Hence we have, as satisfying the conditions afforded by the above equation,

Symbol of sulphur . . . . . θ^2,
Symbol of sulphide of hydrogen $\alpha\theta$.

If we assume the density of sulphur to be correct as determined before the recent experiments referred to, the simplest statement which can be made as to the decomposition of sulphide of hydrogen is, that 6 volumes of sulphide of hydrogen are decomposed into 6 volumes of hydrogen and 1 volume of sulphur-vapour; in which case

$$(\alpha^m\theta_1^{m_1})^6=\alpha^6\alpha^n\theta_1^{n_1},$$

and

$$6m=6+n, \quad 6m_1=n_1,$$

which gives as the minimum solution

$$n=0, \quad m=1,$$
$$n_1=6, \quad m_1=1,$$

and

$$w(\theta_1)=16.$$

Hence

$$\theta_1=\theta.$$

The symbols, therefore, may be in either case expressed by the same prime factors.

In the latter case, we have

Symbol of sulphur . . . . θ^6,
Symbol of sulphide of hydrogen as before $\alpha\theta$.

By similar reasoning to that employed in the determination of the symbol of the peroxide of hydrogen, we arrive at the following symbols.

Combinations of the Prime Factors α, ξ, and θ.

Name of substance.	Prime factor.	Absolute weight, in grammes.	Relative weight.
	θ	1·43	16
	Symbol.		
Sulphur	θ^2	2·861	32
Protosulphide of hydrogen	$\alpha\theta$	1·520	17
Bisulphide of hydrogen	$\alpha\theta^2$	2·950	33
Sulphurous anhydride	$\theta\xi^2$	2·861	32
Sulphuric anhydride	$\theta\xi^3$	3·576	40
Sulphurous acid	$\alpha\theta\xi^3$	3·665	41
Sulphuric acid	$\alpha\theta\xi^4$	4·380	49
Nordhausen sulphuric acid	$\alpha\theta^2\xi^7$	7·956	89
Hyposulphurous acid	$\alpha\theta^2\xi^3$	5·095	57
Dithionic acid	$\alpha\theta^2\xi^6$	7·241	81
Trithionic acid	$\alpha\theta^3\xi^6$	8·671	97
Tetrathionic acid	$\alpha\theta^4\xi^6$	10·101	113
Pentathionic acid	$\alpha\theta^5\xi^6$	11·532	129

(4) *Symbol of Selenium.*—The vapour-density of selenium exhibits similar anomalies to the vapour-density of sulphur. But from the experiments of DEVILLE there can be little doubt that at a sufficiently high temperature it would accord with theory. DEVILLE found for the vapour-density of selenium at 860°, 8·2, and at 1040°, 6·37. On the hypothesis that 2 volumes of selenide of hydrogen are decomposed into 2 volumes of hydrogen and 1 volume of selenium, the vapour-density of selenium would be expressed by the number 5·44. I shall assume this as the correct number.

We have, then, putting $\alpha^m\lambda^{m_1}$ as the symbol of selenide of hydrogen, and $\alpha^n\lambda^{n_1}$ as the symbol of selenium,

$$2\alpha^m\lambda^{m_1}=2\alpha+\alpha^n\lambda^{n_1},$$

whence, as in the two last examples,

$$2m=2+n, \quad 2m_1=n_1,$$

and

$$n=0, \quad n_1=2,$$
$$m=1, \quad m_1=1,$$

a minimum.

Assuming the density of selenide of hydrogen as 41,

$$w(\lambda)=40.$$

Combinations of the Prime Factors α, ξ, θ, and λ.

Name of substance.	Prime factor.	Absolute weight, in grammes.	Relative weight.
	λ	3·576	40
	Symbol.		
Selenium	λ^2	7·151	80
Selenide of hydrogen	$\alpha\lambda$	3·665	41
Selenious anhydride	$\lambda\xi^2$	5·006	56
Selenic anhydride	$\lambda\xi^3$	5·721	64
Selenious acid	$\alpha\lambda\xi^3$	5·810	65
Selenic acid	$\alpha\lambda\xi^4$	6·526	73

(5) *Symbol of Chlorine.*—It is ascertained by experiment that 2 volumes of hydro chloric acid can be decomposed into 1 volume of hydrogen and 1 volume of chlorine.

Hence, putting $\alpha^m\chi^{m_1}$ as the symbol of the unit of hydrochloric acid, and $\alpha^n\chi^{n_1}$ as the symbol of the unit of chlorine,

$$2\alpha^m\chi^{m_1}=\alpha+\alpha^n\chi^{n_1},$$

and

$$(\alpha^m\chi^{m_1})^2=\alpha\alpha^n\chi^{n_1};$$

whence

$$2m=1+n,$$
$$2m_1=n_1,$$

and

$$m=1, \quad m_1=1,$$
$$n=1, \quad n_1=2,$$

a minimum.

Since the density of hydrochloric acid is 18·25, we have to determine the absolute weight of the simple weight χ,

$$m+m_1w(\chi)=18\cdot25,$$

whence

$$w(\chi)=17\cdot25,$$

which gives for the

Symbol of hydrochloric acid $\alpha\chi$,
Symbol of chlorine . . . $\alpha\chi^2$,

in which case the above equation becomes

$$2\alpha\chi=\alpha+\alpha\chi^2.$$

The general solutions of the above equations, which contain all possible values of the indices of the symbols, give

$$m=1+t, \qquad m_1=1+t_1,$$
$$n=1+2t, \qquad n_1=2(1+t_1),$$

whence we arrive at the following general forms,

Hydrochloric acid $\alpha^{1+t}\chi^{1+t_1}$,
Chlorine . . . $\alpha^{1+2t}\chi^{2(1+t_1)}$,

and

$$w(\chi)=\frac{17\cdot25-t}{1+t_1};$$

whence

$$t \text{ is not } >17,$$

and

$$t_1 \text{ is not } <0.$$

Combinations of the Prime Factors α, ξ, θ, λ, and χ.

Name of substance.	Prime factor.	Absolute weight, in grammes.	Relative weight.
	χ	1·542	17·25
	Symbol.		
Chlorine	$\alpha\chi^2$	3·173	35·5
Hydrochloric acid	$\alpha\chi$	1·631	18·25
Protoxide of chlorine	$\alpha\chi^2\xi$	3·888	43·5
Teroxide of chlorine	$\alpha\chi^2\xi^3$	5·319	59·5
Tetroxide of chlorine	$\alpha\chi^2\xi^4$	6·034	67·5
Hypochlorous acid	$\alpha\chi\xi$	2·346	26·25
Chlorous acid	$\alpha\chi\xi^2$	3·062	34·25
Chloric acid	$\alpha\chi\xi^3$	3·777	42·25
Perchloric acid	$\alpha\chi\xi^4$	4·492	50·25
Hydrate of chlorine	$\alpha^{11}\chi^2\xi^{10}$	11·219	125·5
Protosulphide of chlorine	$\alpha\chi^2\theta$	4·604	52·5
Bisulphide of chlorine	$\alpha\chi^2\theta^2$	6·034	67·5
Biselenide of chlorine	$\alpha\chi^2\lambda^2$	10·325	115·5
Tetrachloride of selenium	$\alpha^2\chi^4\lambda$	9·922	111
Chlorosulphurous acid	$\alpha\chi^2\theta\xi$	5·319	59·5
Hydrochlorosulphurous acid	$\alpha\chi\theta\xi^3$	5·207	58·25
Chlorosulphuric acid	$\alpha\chi^2\theta\xi^2$	6·034	67·5

(6) *Symbol of Iodine.*—Two volumes of hydriodic acid are decomposed into 1 volume of hydrogen and 1 volume of iodine.

Hence, putting $\alpha^m\omega^{m_1}$ as the symbol of the unit of hydriodic acid, and $\alpha^n\omega^{n_1}$ as the symbol of the unit of iodine,

$$2\alpha^m\omega^{m_1}=\alpha+\alpha^n\omega^{n_1},$$

and,

$$(\alpha^m\omega^{m_1})^2=\alpha\alpha^n\omega^{n_1},$$

and

$$2m=1+n,\ 2m_1=n_1,$$

$$m=1,\quad m_1=1,$$

$$n=1,\quad n_1=2,$$

a minimum.

Assuming the density of iodine vapour to be 127,

$$1+2w(\omega)=127,$$

$$w(\omega)=63,$$

and the symbol of hydriodic acid is $\alpha\omega$, and of iodine $\alpha\omega^2$, in which case the above equation is thus expressed,

$$2\alpha\omega=\alpha+\alpha\omega^2.$$

Combinations of the Prime Factors α, ξ, θ, λ, χ, and ω.

Name of substance.	Prime factor.	Absolute weight, in grammes.	Relative weight.
	ω	5·632	63
	Symbol.		
Iodine	$\alpha\omega^2$	11·353	127
Hydriodic acid	$\alpha\omega$	5·721	64
Iodic anhydride	$\alpha\omega^2\xi^5$	14·928	167
Per-iodic anhydride	$\alpha\omega^2\xi^7$	16·359	183
Iodic acid	$\alpha\omega\xi^3$	7·866	88
Protochloride of iodine	$\alpha\omega\chi$	7·263	81·25
Terchloride of iodine	α^2	10·436	116·75

(7) *Symbol of Bromine.*—Two volumes of hydrobromic acid are decomposed into 1 volume of hydrogen and 1 volume of bromine.

Hence, putting $\alpha^m\beta^{m_1}$ as the symbol of the unit of hydrobromic acid, and $\alpha^n\beta^{n_1}$ as the symbol of the unit of bromine,

$$2\alpha^m\beta^{m_1}=\alpha+\alpha^n\beta^{n_1},$$

and by similar reasoning to that by which the symbols of chlorine and iodine have been ascertained, we have

Symbol of hydrobromic acid $\alpha\beta$,
Symbol of bromine . . . $\alpha\beta^2$,

and assuming 80 as the density of bromine,

$$w(\beta)=39{\cdot}5.$$

Combinations of the Prime Factors α, ξ, θ, λ, χ, ω, and β.

Name of substance.	Prime factor.	Absolute weight, in grammes.	Relative weight.
	β	3·531	39·5
	Symbol.		
Bromine	$\alpha\beta^2$	7·151	80
Hydrobromic acid	$\alpha\beta$	3·620	40·5
Protoxide of bromine	$\alpha\beta^2\xi$	7·866	88
Hypobromous acid	$\alpha\beta\xi$	4·335	48·5
Bromic acid	$\alpha\beta\xi^3$	5·766	54·5

(8) *Symbol of Nitrogen.*—Two volumes of ammonia can be decomposed into 3 volumes of hydrogen and 1 volume of nitrogen.

Hence, putting $\alpha^m\nu^{m_1}$ as the symbol of the unit of ammonia, and $\alpha^n\nu^{n_1}$ as the symbol of the unit of nitrogen,

$$2\alpha^m\nu^{m_1}=3\alpha+\alpha^n\nu^{n_1},$$

and

$$(\alpha^m\nu^{m_1})^2=\alpha^3\alpha^n\nu^{n_1};$$

whence

$$2m=3+n,\quad 2m_1=n_1,$$

and

$$m=2, \quad m_1=1,$$
$$n=1, \quad n_1=2,$$

a minimum.

Hence the symbol of nitrogen as determined from this equation is $\alpha\nu^2$, and the symbol of ammonia is $\alpha^2\nu$.

Since the density of ammonia is 8·5,

$$2+w(\nu)=8{\cdot}5,$$

and

$$w(\nu)=6{\cdot}5.$$

The general solutions of the above equations give

$$m=2+t, \quad m_1=1+t_1,$$
$$n=1+2t, \quad n_1=2(1+t_1),$$

whence we have as the

Symbol of ammonia . . . $\alpha^{2+t}\nu^{1+t_1}$,
Symbol of nitrogen . . . $\alpha^{1+2t}\nu^{2(1+t_1)}$,

and

$$w(\nu)=\frac{6{\cdot}5-t}{1+t_1},$$

where t is not >6, and t_1 not <0.

Combinations of the Prime Factors α, ξ, θ, χ, λ, ω, β and ν.

Name of substance.	Prime factor.	Absolute weight, in grammes.	Relative weight.
	ν	0·581	6·5
	Symbol.		
Nitrogen	$\alpha\nu^2$	1·251	14
Ammonia	$\alpha^2\nu$	0·760	8·5
Protoxide of nitrogen	$\alpha\nu^2\xi$	1·967	22
Binoxide of nitrogen*	$\alpha\nu^2\xi^2$	2·682	30
Teroxide of nitrogen	$\alpha\nu^2\xi^3$	3·397	38
Tetroxide of nitrogen	$\alpha\nu^2\xi^4$	4·112	46
Pentoxide of nitrogen	$\alpha\nu^2\xi^5$	4·827	54
Nitrous acid	$\alpha\nu\xi^2$	2·101	23·5
Nitric acid	$\alpha\nu\xi^3$	2·816	31·5
Bisulphide of nitrogen	$\alpha\nu^2\theta^2$	4·112	46
Chloride of nitrogen	$\alpha^2\nu\chi^3$	5·386	60·25
Nitrite of ammonium	$\alpha^3\nu^2\xi^2$	2·861	32
Nitrate of ammonium	$\alpha^3\nu^2\xi^3$	3·576	40
Sulph-hydrate of ammonium	$\alpha^3\nu\theta$	2·279	25·5
Protosulphide of ammonium	$\alpha^5\nu^2\theta$	3·039	34
Bisulphide of ammonium	$\alpha^5\nu^2\theta^2$	4·470	50
Acid sulphite of ammonium	$\alpha^3\nu\theta\xi^3$	4·424	49·5
Acid sulphate of ammonium	$\alpha^3\nu\theta\xi^4$	5·139	57·5
Sulphite of ammonium	$\alpha^5\nu^2\theta\xi^3$	5·285	58
Sulphate of ammonium	$\alpha^5\nu^2\theta\xi^4$	6·000	66
Thionamic acid†	$\alpha^2\nu\theta\xi^2$	3·620	40·5
Sulphamic acid	$\alpha^2\nu\theta\xi^3$	4·336	48·5
Thionamide	$\alpha^3\nu^2\theta\xi$	3·576	40
Sulphamide	$\alpha^3\nu^2\theta\xi^2$	4·281	48
Acid sulphate of azotyl	$\alpha\nu\theta\xi^5$	5·676	63·5
Neutral sulphate of azotyl	$\alpha\nu^2\theta\xi^6$	6·972	78
Anhydro-sulphate of azotyl	$\alpha\nu^2\theta\xi^9$	9·118	102
Chloride of ammonium*	$\alpha^3\nu\chi$	2·391	26·75
Chloride of azotyl†	$\alpha\nu\chi\xi$	2·928	32·75
Chloride of nitryl†	$\alpha\nu\chi\xi^2$	3·642	40·75
Bichloride of azotyl	$\alpha^3\nu^2\chi^2\xi^2$	5·944	66·5
Chlorate of ammonium	$\alpha^3\nu\chi\xi^3$	4·537	50·75
Iodide of ammonium	$\alpha^3\nu\omega$	6·481	72·5
Diniodamide	$\alpha^2\nu\omega^2$	12·023	134·5
Bromide of ammonium	$\alpha^3\nu\beta$	4·380	49

(9) *Symbol of Phosphorus.*—Let the symbol of the unit of phosphorus be $\alpha^n\varphi^{n_1}$, and the symbol of the unit of gaseous phosphide of hydrogen $\alpha^m\varphi^{m_1}$. Then, since 4 volumes of the gaseous phosphide of hydrogen are decomposed into 6 volumes of hydrogen and

* The symbols of nitric oxide, chloride of ammonium, and certain other substances, the densities of which are apparently anomalous, will be subsequently considered (see Section VIII.).

† See ODLING'S 'Chemistry,' pages 269, 254, and 261.

1 volume of phosphorus-vapour,

$$4\alpha^m\phi^{m_1}=6\alpha+\alpha^n\phi^{n_1},$$

and

$$(\alpha^m\phi^{m_1})^4=\alpha^6\alpha^n\phi^{n_1},$$

whence

$$4m=6+n, \qquad 4m_1=n_1,$$

and

$$m=2, \qquad m_1=1,$$
$$n=2, \qquad n_1=4,$$

a minimum; and we have for the

Symbol of phosphide of hydrogen $\alpha^2\phi$,
Symbol of phosphorus . . . $\alpha^2\phi^4$

Assuming the density of phosphorus-vapour as 62,

$$2+4w(\phi)=62,$$
$$w(\phi)=15.$$

Combinations of the Prime Factors α, ξ, θ, λ, χ, ω, β, ν, and ϕ.

Name of substance.	Prime factor.	Absolute weight, in grammes.	Relative weight.
	ϕ	1·341	15
	Symbol.		
Phosphorus	$\alpha^2\phi^4$	5·541	62
Phosphide of hydrogen	$\alpha^2\phi$	1·519	17
Teroxide of phosphorus	$\alpha\phi^2\xi^3$	4·916	55
Pentoxide of phosphorus	$\alpha\phi^2\xi^5$	6·346	71
Hypophosphorous acid	$\alpha^2\phi\xi^2$	2·949	33
Phosphorous acid	$\alpha^2\phi\xi^3$	3·664	41
Orthophosphoric acid	$\alpha^2\phi\xi^4$	4·379	49
Pyrophosphoric acid	$\alpha^3\phi^2\xi^7$	7·956	89
Metaphosphoric acid	$\alpha\phi\xi^3$	3·574	40
Protosulphide of phosphorus	$\alpha\phi^2\theta$	4·201	48
Tersulphide of phosphorus	$\alpha\phi^2\theta^3$	7·061	80
Pentasulphide of phosphorus	$\alpha\phi^2\theta^5$	9·921	112
Terchloride of phosphorus	$\alpha^2\phi\chi^3$	6·145	68·75
Pentachloride of phosphorus	$\alpha^3\phi\chi^5$	9·319	104·25
Oxychloride of phosphorus	$\alpha^2\phi\chi^3\xi$	6·869	76·75
Terbromide of phosphorus	$\alpha^2\phi\beta^3$	12·112	135·5
Pentabromide of phosphorus	$\alpha^3\phi\beta^5$	19·264	215·5
Oxybromide of phosphorus	$\alpha^2\phi\beta^3\xi$	12·827	143·5
Bromide of phosphonium	$\alpha^3\phi\beta$	5·140	57·5
Biniodide of phosphorus	$\alpha^3\phi^2\omega^4$	25·476	285
Teriodide of phosphorus	$\alpha^2\phi\omega^3$	18·413	206
Iodide of phosphonium	$\alpha^3\phi\omega$	7·240	81
Sulphochloride of phosphorus	$\alpha^2\phi\chi^3\theta$	7·575	84·75

(10) *Symbol of Arsenic.*—The density of the chloride of arsenic has been determined by DUMAS, the density of arsenic-vapour by MITSCHERLICH.

Four volumes of the chloride of arsenic are decomposed into 1 volume of arsenic and 6 volumes of chlorine.

Hence, putting $\alpha^m\chi^{m_1}\varrho^{m_2}$ as the symbol of the unit of chloride of arsenic, and $\alpha^n\chi^{n_1}\varrho^{n_2}$ as the symbol of the unit of arsenic, and $\alpha\chi^2$ as the symbol of the unit of chlorine,

$$4\alpha^m\chi^{m_1}\varrho^{m_2}=6\alpha\chi^2+\alpha^n\chi^{n_1}\varrho^{n_2},$$

and

$$(\alpha^m\chi^{m_1}\varrho^{m_2})^4=(\alpha\chi^2)^6\alpha^n\chi^{n_1}\varrho^{n_2};$$

whence

$$4m=6+n,\ 4m_1=12+n_1,\ 4m_2=n_2.$$

The minimum solution of these equations gives

$$\begin{array}{ll} m=2, & n=2, \\ m_1=3, & n_1=0, \\ m_2=1, & n_2=4; \end{array}$$

and we have, as thus determined,

Symbol of arsenic . . . . $\alpha^2\varrho^4$,
Symbol of terchloride of arsenic . . $\alpha^2\chi^3\varrho$.

Assuming 150 as the density of the vapour of arsenic,

$$2+4w(\varrho)=150,$$
$$w(\varrho)=37.$$

Combinations of the Prime Factors α, ξ, θ, χ, ... and ϱ.

Name of substance.	Prime factor.	Absolute weight, in grammes.	Relative weight.
	ϱ	3·308	37
	Symbol.		
Arsenic	$\alpha^2\varrho^4$	13·408	150
Arsenide of hydrogen......	$\alpha^2\varrho$	3·485	39
Teroxide of arsenic	$\alpha^2\varrho^4\xi^6$	17·699	198
Pentoxide of arsenic	$\alpha\varrho^2\xi^5$	10·279	115
Arsenious acid	$\alpha^2\varrho\xi^3$	5·630	63
Arsenic acid	$\alpha^2\varrho\xi^5$	6·345	71
Bisulphide of arsenic	$\alpha\varrho^2\theta^2$	9·563	107
Tersulphide of arsenic	$\alpha\varrho^2\theta^3$	10·995	123
Pentasulphide of arsenic ..	$\alpha\varrho^2\theta^5$	13·856	155
Terchloride of arsenic	$\alpha^2\varrho\chi^3$	8·111	90·75
Oxychloride of arsenic	$\alpha\varrho\chi\xi$	5·653	63·25
Teriodide of arsenic	$\alpha^2\varrho\omega^3$	20·379	228
Terbromide of arsenic	$\alpha^2\varrho\beta^3$	14·078	157·5

(11) *Symbol of Mercury.*—Assuming 100 as the density of the vapour of mercury, and

135·5 as the density of the vapour of mercuric chloride, 1 volume of mercuric chloride is decomposed into 1 volume of chlorine and 1 volume of mercury.

Hence, putting $\alpha^{m}\chi^{m_1}\delta^{m_2}$ as the symbol of the unit of mercuric chloride, and $\alpha^{n}\chi^{n_1}\delta^{n_2}$ as the symbol of the unit of mercury,

$$\alpha^{m}\chi^{m_1}\delta^{m_2}=\alpha\chi^2+\alpha^{n}\chi^{n_1}\delta^{n_2},$$

and

$$\alpha^{m}\chi^{m_1}\delta^{m_2}=\alpha\chi^2\alpha^{n}\chi^{n_1}\delta^{n_2};$$

whence

$$m=1+n,\quad m_1=2+n_1,\quad m_2=n_2.$$

The minimum solution of these equations gives

$$\begin{aligned} m&=1, & n&=0,\\ m_1&=2, & n_1&=0,\\ m_2&=1, & n_2&=1; \end{aligned}$$

whence we have

Symbol of mercuric chloride . . $\alpha\chi^2\delta$,
Symbol of mercury . . . . . . δ,

and

$$w(\delta)=100.$$

Combinations of the Prime Factors α, ξ, θ, λ, χ, ... and δ.

Name of substance.	Prime factor.	Absolute weight, in grammes.	Relative weight.
	δ	8·939	100
	Symbol.		
Mercury	δ	8·939	100
Mercurous oxide	$\delta^2\xi$	18·593	208
Mercuric oxide	$\delta\xi$	9·654	108
Mercurous sulphide	$\delta^2\theta$	19·309	216
Mercuric sulphide	$\delta\theta$	10·369	116
Mercuric sulphite	$\delta\theta\xi^3$	12·514	140
Basic mercuric sulphite	$\delta^2\theta\xi^4$	22·169	248
Mercurous sulphate	$\delta^2\theta\xi^4$	22·169	248
Mercuric sulphate	$\delta\theta\xi^4$	13·230	148
Basic mercuric sulphate (Turpeth's mineral)	$\delta^3\theta\xi^6$	32·539	364
Sulphate and sulphide of mercury	$\delta^3\theta^2\xi^4$	32·539	364
Selenide of mercury	$\delta\lambda$	12·515	140
Mercurous selenite	$\delta^2\lambda\xi^3$	23·600	264
Mercuric selenite	$\delta\lambda\xi^3$	14·661	164
Mercurous chloride	$\alpha\chi^2\delta^2$	21·051	235·5
Mercuric chloride	$\alpha\chi^2\delta$	12·113	135·5
Oxychloride of mercury, 1	$\alpha\chi^2\delta^3\xi^2$	31·421	351·5
Oxychloride of mercury, 2	$\alpha\chi^2\delta^4\xi^3$	41·075	459·5
Oxychloride of mercury, 3	$\alpha\chi^2\delta^5\xi^4$	50·730	567·5
Mercurous chlorate	$\alpha\chi^2\delta^2\xi^6$	25·342	283·5
Mercuric chlorate	$\alpha\chi^2\delta\xi^6$	16·404	183·5
Mercurous perchlorate	$\alpha\chi^2\delta^2\xi^8$	26·772	299·5
Mercuric perchlorate	$\alpha\chi^2\delta\xi^8$	17·834	199·5
Chloride of mercury and sulphur	$\alpha^2\chi^4\delta^2\theta$	25·655	287
Mercurous iodide	$\alpha\omega^2\delta^2$	29·231	327
Intermediate iodide	$\alpha^3\omega^6\delta^4$	69·814	781
Mercuric iodide	$\alpha\omega^2\delta$	20·292	227
Mercurous iodate	$\alpha\omega^2\delta^2\xi^6$	33·521	375
Mercuric iodate	$\alpha\omega^2\delta\xi^6$	24·583	275
Mercurous bromide	$\alpha\beta^2\delta^2$	25·029	280
Mercuric bromide	$\alpha\beta^2\delta$	16·090	180
Trimercuramine	$\alpha\nu^2\delta^3$	28·069	314
Mercurous nitrate	$\alpha\nu^2\delta^2\xi^6$	23·420	262
Mercuric nitrate	$\alpha\nu^2\delta\xi^6$	14·481	162
Nitrate and sulphide of mercury	$\alpha\nu^2\delta^3\theta^2\xi^6$	35·220	394
Nitrate and iodide of mercury	$\alpha\nu\omega\delta\xi^3$	17·386	194·5
Mercuramine	$\alpha^3\nu^2\delta^4\xi^3$	39·332	440
*A	$\alpha^2\nu\chi\delta^2$	20·179	225·75
B	$\alpha^2\nu\chi\delta$	11·240	125·75
C	$\alpha^5\nu^2\chi^2\delta$	13·632	152·5
Mercurous phosphate	$\alpha\phi^2\delta^6\xi^8$	62·128	695
Mercuric phosphate	$\alpha\phi^2\delta^3\xi^8$	35·310	395

* A. Formed by the action of ammonia on calomel. B. Formed by the action of ammonia on corrosive sublimate. C. Formed by the action of B on sal-ammoniac.

The preceding Tables comprise a large number of the ascertained combinations of the prime factors to which they relate, and afford sufficient illustration of the method. Hereafter I shall limit myself, in the case of each factor, to a few examples.

With regard to the selection of letters by which the symbols of simple weights may be expressed, it is a mistake to confuse the objects of a symbolic system with those of a "memoria technica," and I am inclined to believe that a purely accidental distribution of letters among the weights to be expressed would be the best. In the selection here made, however, I have not proceeded rigidly upon this principle, a certain reminiscence of the name being retained in the symbol, as for example, the ξ of ὀξύς, the θ of θεῖον, the χ of χλωρός, and the δ of ὑδράργυρος. Facility of writing and reading the symbols is, however, far more important than any aid to memory which can be thus afforded, and these points are to be mainly considered. The unit of hydrogen, which occupies a peculiar position as the "modulus" of the system, is indicated by a special symbol, α. With regard to names, I cannot pretend to be more successful than others. In the confusion which at present prevails on this point it is almost impossible to use a language which shall be universally understood, new names having been frequently and inconsiderately assigned to chemical substances as the expression of some transitory theory, or even individual speculation, rather than to fulfil the main purpose of words, as the common medium for the exchange of ideas. Hence, as in the language of barbarous tribes* and from causes similar to those which there prevail, the same object comes to be denoted by a variety of appellations, and chemists of different schools can hardly understand one another. It is to be hoped that, as the science gradually assumes a more exact form, the use of symbols will enable the chemist to dispense, to a great extent, with any other nomenclature, and afford a satisfactory solution to this difficult problem.

Group 2.—*Symbols of Carbon, Silicon, and Boron.*

(1) *Symbol of Carbon.*—Although the density of the vapour of carbon has never been determined by experiment, we are yet able to construct numerous chemical equations which connect the vapour-density of carbon with known vapour-densities, and in which the number of volumes of carbon-vapour which enters into the chemical equation appears as an indeterminate quantity. From these equations we are able to determine with a high degree of probability, though doubtless only by the aid of hypothesis, the symbols of many gaseous compounds of carbon, the prime factor of carbon, and, within certain limits, the symbol of the element itself.

It is known from experiment that marsh-gas can be decomposed into hydrogen and carbon, and that the number of volumes of hydrogen formed in this decomposition is twice the number of volumes of marsh-gas decomposed.

* "Any feature that struck the observing mind as peculiarly characteristic could be made to furnish a new name. In common Sanskrit dictionaries we find five words for land, 11 for light, 15 for cloud, 20 for moon, &c."—MAX MÜLLER, 'Lectures on the Science of Language,' Ed. v. p. 426. It would be easy to find parallel examples in the nomenclature of chemistry.

Hence, putting y as the number of units of marsh-gas thus decomposed, and x as the number of units of the vapour of carbon formed, and putting $\alpha^m\chi^{m_1}$ as the symbol of marsh-gas, and $\alpha^n\chi^{n_1}$ as the symbol of carbon,

$$y\alpha^m\chi^{m_1}=2y\alpha+x\alpha^n\chi^{n_1}, \quad \ldots \ldots \ldots \ldots \quad (1)$$

whence

$$(\alpha^m\chi^{m_1})^y=\alpha^{2y}(\alpha^n\chi^{n_1})^x,$$

and

$$m\,y=2\,y+nx,$$
$$m_1y=\ n_1x.$$

Now, whatever be the values of y and x, these equations will admit of a minimum integral solution, provided only that y and x be integral numbers.

This solution is

$$m=2, \qquad m_1=x,$$
$$n=0, \qquad n_1=y.$$

As determined, therefore, from this equation, we have

Symbol of marsh-gas . . . $\alpha^2\chi^x$,
Symbol of carbon . . . . χ^y.

Assuming 8 as the density of marsh-gas,

$$2+xw(\chi)=8,$$

whence

$$xw(\chi)=6\,;$$

and putting W as the density of carbon-vapour,

$$W=\frac{6y}{x},$$

the above equation (1) being thus expressed,

$$y(\alpha^2\chi^x)=2y\alpha+x\chi^y.$$

We now should proceed to ascertain whether the same symbol, χ^y, for carbon will satisfy the conditions afforded by other equations. I will give one or two examples of the process by which this is effected.

It is known from experiment that y_1 volumes of olefiant gas are decomposed into $2y_1$ volumes of hydrogen and x_1 volumes of carbon-vapour. Hence the absolute weight of carbon formed by the decomposition of a unit of olefiant gas is $\frac{x_1}{y_1}$W.

But this weight is determined by experiment, and is equal to twice the weight of carbon formed by the decomposition of a unit of marsh-gas, which weight is equal to $\frac{x}{y}$W. Hence $\frac{x_1}{y_1}=\frac{2x}{y}$; and putting $\alpha^p\chi^{p_1}$ as the symbol of olefiant gas, and χ^y, as before, as the symbol of carbon,

$$y\alpha^p\chi^{p_1}=2y\alpha+2x\chi^y,$$

and

$$p=2, \quad p_1=2x,$$

whence we have the symbol of olefiant gas $\alpha^2\chi^{2x}$; and the symbol of carbon can be expressed in this equation also as χ^y, where y has the value given in equation (1).*

Again, by similar reasoning, putting $\alpha^{q}\varkappa^{q_1}$, for example, as the symbol of styrol,

$$y\alpha^{q}\varkappa^{q_1}=4y\alpha+(\varkappa^{y})^{8x},$$

whence

$$q=4, \qquad q_1=8x,$$

and we have as the symbol of styrol $\alpha^4\varkappa^{8x}$.

The following are examples of symbols thus determined:—

Marsh-gas	$\alpha^2\varkappa^{x}$.
Acetylene	$\alpha\varkappa^{2x}$.
Olefiant gas	$\alpha^2\varkappa^{2x}$.
Methyl	$\alpha^3\varkappa^{2x}$.
Propylene	$\alpha^3\varkappa^{3x}$.
Ethyl	$\alpha^5\varkappa^{4x}$.
Allyl	$\alpha^5\varkappa^{6x}$.
Formic acid	$\alpha\varkappa^{x}\xi^2$.
Methylic alcohol	$\alpha^2\varkappa^{x}\xi$.
Oxide of ethylene	$\alpha^2\varkappa^{2x}\xi$.
Acetic acid	$\alpha^2\varkappa^{2x}\xi^2$.
Butylic alcohol	$\alpha^5\varkappa^{4x}\xi$.
Benzoic acid	$\alpha^3\varkappa^{7x}\xi^2$.
Chloride of methyl	$\alpha^2\chi\varkappa^{x}$.
Chloride of ethyl	$\alpha^3\chi\varkappa^{2x}$.
Chlorine derivatives of chloride of ethyl, 1	$\alpha^3\chi^2\varkappa^{2x}$.
,, ,, ,, ,, 2	$\alpha^3\chi^3\varkappa^{2x}$.
,, ,, ,, ,, 3	$\alpha^3\chi^4\varkappa^{2x}$.
Chloride of butylene	$\alpha^5\chi^2\varkappa^{4x}$.
Chloride of amyl	$\alpha^6\chi\varkappa^{5x}$.
Chloride of benzoyl	$\alpha^3\chi\varkappa^{7x}\xi$.
Cyanogen	$\alpha\nu^2\varkappa^{2x}$.
Cyanide of methyl	$\alpha^2\nu\varkappa^{2x}$.
Ethylamine	$\alpha^4\nu\varkappa^{2x}$.
Cyanide of butyl	$\alpha^5\nu\varkappa^{5x}$.
Cyanide of amyl	$\alpha^6\nu\varkappa^{6x}$.

Were we thus to proceed to construct the symbols of the gaseous compounds of carbon with the elements of which the symbols have already been determined, it would be found that in all cases these symbols could be expressed by an integral number of the prime factors α, ξ, θ, λ, χ, ω, ν, ..., and that the index of the factor $\varkappa$ was always of the form mx, m being a positive integer.

The only hypothesis which can be made as to the value of x, which shall be at once necessary and sufficient, is that $x=1$, in which case $w(\varkappa)=6$, and W, the density of carbon, $=y\times 6$. This hypothesis is based upon a very large number of observations of

the most varied character, and consequently a very high degree of probability is attached to it. For we cannot but believe that if any chemical substance could exist, in the symbol of which the index of $\varkappa$ should not be of the form mx, among the great variety of known substances some one such substance would have been discovered, and that the reason why the weight $\varkappa^x$ is never distributed in the chemical changes with which we are acquainted is that this weight is a simple weight, and that $x=1$.

It is to be observed that the weight to be given to an argument of this kind may become very small, if the observations on which it is founded are few, and made exclusively on one class of substances. Thus, for example, if the course of chemical inquiry had been such as to make us acquainted only with the following substances: olefiant gas, methyl, ethyl, butylene, oxide of ethylene, glycol, alcohol, ether, acetic acid, and other substances of which the symbols can be expressed by the factor $\varkappa_1^z$, and of which the symbols are $\alpha^2\varkappa_1^z$, $\alpha^3\varkappa_1^z$, $\alpha^5\varkappa_1^{2z}$, $\alpha^4\varkappa_1^{2z}$, $\alpha^2\varkappa_1^z\xi$, $\alpha^3\varkappa_1^z\xi^2$, $\alpha^3\varkappa_1^z\xi$, $\alpha^5\varkappa_1^{2z}\xi$, $\alpha_1^2\varkappa^z\xi^2$, $\alpha^3\varkappa_1^{2z}\xi^3$, and the like, where $z=2x$, we should, by similar reasoning, have concluded that the symbols of the compounds of carbon could be expressed by the prime factor $\varkappa_1$, of which the absolute weight $w(\varkappa_1)=12$, and that $\varkappa_1$ was the symbol of a simple weight, a result which would not have been justified by a more extended experience.

We are able to bring to bear upon the symbol of carbon certain arguments of a very general application, and which are derived from direct experiment. If we compare the chemical equations into which enter the symbols of the units of volume of those elements of which the density can be experimentally determined, it will be perceived that, putting A as the smallest weight of the element which is in any case formed in the decomposition of the unit of any chemical substance, and V as the density of the element, either $A=V$, as in the case of mercury, or $A=\frac{V}{2}$, as in the case of hydrogen, chlorine, iodine, bromine, nitrogen, oxygen, sulphur, selenium, or $A=\frac{V}{4}$, as in the case of phosphorus and arsenic.

The truth of the above observation will be seen on inspecting the following equations, which have already been interpreted.

I. $A=V$. Mercury:—

$$\alpha\chi^2\delta=\alpha\chi^2+\delta.$$

II. $A=\frac{V}{2}$. Hydrogen, chlorine, iodine, bromine, nitrogen, oxygen, sulphur, selenium:—

$$2\alpha\chi=\alpha+\alpha\chi^2,$$
$$2\alpha\omega=\alpha+\alpha\omega^2,$$
$$2\alpha\beta=\alpha+\alpha\beta^2,$$
$$2\alpha\xi=2\alpha+\xi^2,$$
$$2\alpha\theta=2\alpha+\theta^2,$$
$$2\alpha\lambda=2\alpha+\lambda^2,$$
$$2\alpha^2\nu=3\alpha+\alpha\nu^2.$$

III. $A=\frac{V}{4}$. Phosphorus and arsenic:—

$$4\alpha^2\phi=6\alpha+\alpha^2\phi^4,$$
$$4\alpha^2\chi^3\varrho=6\alpha\chi^2+\alpha^2\varrho^4.$$

This weight A, which may be regarded as the limit beyond which the chemical division of the weight of unit of the elemental body cannot be effected, is, it will be observed, half the atomic weight of the element on the most recent and approved system. And it is further true, as a matter of observation, that if X be the weight of an elemental body which is formed in the decomposition of the unit of volume of any chemical substance, X is a multiple of A, so that X=NA, where N is a positive integer. The relation which this weight holds to the thermal properties of the element will hereafter be pointed out.

From these considerations a certain probability is raised in regard to the value of V, where the value cannot be experimentally determined, in favour of the values V=A, V=2A, V=4A.

The smallest weight of carbon, A, which is formed by the decomposition of the unit of volume of any chemical substance, in regard to which the point can be experimentally ascertained, is that which is formed by the decomposition of the unit of volume of marsh-gas, formic acid, methylic alcohol, and a few other substances.

Now, the equation which asserts the identity of the unit of weight of marsh-gas with the units of hydrogen and carbon into which it is resolved, is

$$y\alpha^2\varkappa^x=2y\alpha+x\varkappa^y,$$

whence

$$A=\frac{x}{y}V.$$

There are, therefore, from these considerations, three hypotheses more probable than others which may be made as to the value $\frac{x}{y}$.

1. $A=V,\ \frac{x}{y}=1,\ x=1,\ y=1$:—

Symbol of carbon . . $\varkappa$,
Symbol of marsh-gas . $\alpha^2\varkappa$,

in which case the equation is of the form

$$\alpha^2\varkappa=2\alpha+\varkappa.$$

2. $A=\frac{V}{2},\ \frac{x}{y}=\frac{1}{2},\ x=1,\ y=2$:—

Symbol of carbon . . $\varkappa^2$,
Symbol of marsh-gas . $\alpha^2\varkappa$,

and

$$2\alpha^2\varkappa=4\alpha+\varkappa^2.$$

3. $A=\frac{V}{4}, \frac{x}{y}=\frac{1}{4}, x=1, y=4$:—

Symbol of carbon . . $\varkappa^4$,

Symbol of marsh-gas . $\alpha^2\varkappa$,

and

$$4\alpha^2\varkappa=8\alpha+\varkappa^4.$$

The symbol of marsh-gas (and therefore the symbols of every other compound of carbon) is the same, whichever hypothesis be preferred; being, so far, independent of the form of the primary equation.

There is no class of symbols, in regard to which the direct evidence of experiment either entirely or partially fails us, as to which we have more positive knowledge than the symbols of those substances of which the vapour-density can be experimentally determined, and which can be finally decomposed into carbon and the gaseous elements. The formulæ of these substances are usually given with unhesitating confidence, and even the vapour-density of carbon is treated as a reality. The evidence, however, on the former point is far more satisfactory than that on the latter; and with our present information, all that can be asserted with any high degree of probability is that the weight of the unit of carbon is a multiple of 6.

As the extreme limit of chemical certainty is marked by the equation

$$2\alpha\xi=2\alpha+\xi^2,$$

so the equation

$$y\alpha^2\varkappa=2y\alpha+\varkappa^y$$

may serve to indicate another degree in the same scale of chemical probability; the assumption here made being all that is truly required to determine the symbol of marsh-gas, by which the symbols of the other compounds of carbon are implicitly determined.

Between the forms of equation

$$\alpha^2\varkappa=2\alpha+\varkappa$$

and

$$2\alpha^2\varkappa=4\alpha+\varkappa^2,$$

we have no adequate means of selection.

Combinations of the Prime Factors α, ξ, θ, λ, χ, ω, ... and $\varkappa$.

Name of substance.	Prime factor.	Absolute weight, in grammes.	Relative weight.
	$\varkappa$	0·536	6
	Symbol.		
Carbon*	$\varkappa^y$	$y\times 0·536$	$y\times 6$
Acetylene	$\alpha\varkappa^2$	1·161	13
Marsh-gas	$\alpha^2\varkappa$	0·704	8
Olefiant gas	$\alpha^2\varkappa^2$	1·251	14
Benzole	$\alpha^3\varkappa^6$	3·486	39
Carbonic oxide	$\varkappa\xi$	1·251	14
Carbonic acid	$\varkappa\xi^2$	1·967	22
Alcohol	$\alpha^3\varkappa^2\xi$	2·056	23
Ether	$\alpha^5\varkappa^4\xi$	3·308	37
Allylic alcohol	$\alpha^3\varkappa^3\xi$	2·592	29
Benzylic alcohol	$\alpha^4\varkappa^7\xi$	4·827	54
Glycol	$\alpha^3\varkappa^2\xi^2$	2·771	31
Glycerine	$\alpha^4\varkappa^3\xi^3$	4·112	46
Anhydrous acetic acid	$\alpha^3\varkappa^4\xi^3$	4·559	51
Acetic peroxide	$\alpha^3\varkappa^4\xi^4$	5·274	59
Lactic acid	$\alpha^3\varkappa^3\xi^3$	4·023	45
Tetrachloride of carbon	$\alpha^2\chi^4\varkappa^2$	7·411	83
Chloride of ethylene	$\alpha^3\chi^2\varkappa^2$	4·425	49·5
Chloroform	$\alpha^2\chi^3\varkappa$	5·341	59·75
Chloride of acetyl	$\alpha^2\chi\varkappa^2\xi$	3·509	39·25
Chloracetic acid	$\alpha^2\chi\varkappa^2\xi^2$	4·224	47·25
Trichloracetic acid	$\alpha^2\chi^3\varkappa^2\xi^2$	7·308	81·75
Chlorocarbonic acid	$\alpha\chi^2\varkappa\xi$	4·425	49·5
Iodide of ethyl	$\alpha^3\omega\varkappa^2$	6·973	78
Chloriodide of ethylene	$\alpha^3\chi\omega\varkappa^2$	8·515	95·25
Cyanogen	$\alpha\nu^2\varkappa^2$	2·324	26
Hydrocyanic acid	$\alpha\nu\varkappa$	1·207	13·5
Methylamine	$\alpha^3\nu\varkappa$	1·386	15·5
Kakodyl	$\alpha^7\varrho^2\varkappa^4$	9·386	105
Cyanide of kakodyl	$\alpha^4\varrho\nu\varkappa^3$	5·855	65·5
Iodide of phosphotetrethylium	$\alpha^{11}\omega\varphi\varkappa^8$	12·247	137
Mercuric ethide	$\alpha^5\varkappa^4\delta$	11·532	129

(2) *Symbol of Silicon.*—In the decomposition of chloride of silicon it has been ascertained that the volume of chlorine formed is double the volume of the chloride of silicon decomposed.

Hence, putting $\alpha^m\chi^{m_1}\sigma^{m_2}$ as the symbol of the unit of chloride of silicon, and $\alpha^n\chi^{n_1}\sigma^{n_2}$ as the symbol of the unit of silicon, and z as the number of units of chloride of silicon decomposed, and z_1 as the number of units of silicon formed, we have

$$z\alpha^m\chi^{m_1}\sigma^{m_2}=2z\alpha\chi^2+z_1\alpha^n\chi^{n_1}\sigma^{n_2},$$

* y being the number of units of marsh-gas in the equation (1), Sec. VII. Group 2 (1).

and

$$(\alpha^m\chi^{m_1}\sigma^{m_2})^z=(\alpha\chi^2)^{2z}(\alpha^n\chi^{n_1}\sigma^{n_2})^{z_1};$$

whence

$$zm=2z+z_1n,$$
$$zm_1=4z+z_1n_1,$$
$$zm_2=z_1n_2,$$

which give

$$m=2,\quad n=0,$$
$$m_1=4,\quad n_1=0,$$
$$m_2=z_1,\quad n_2=z,$$

a minimum, whatever be the values of z and z_1, if only z and z_1 be positive and integral.

This gives as the symbol of chloride of silicon $\alpha^2\chi^4\sigma^{z_1}$, and the symbol of silicon σ^z; and assuming 85 as the density of the chloride of silicon,

$$2+4\times 17{\cdot}25+z_1w(\sigma)=85,$$

and

$$w(\sigma)=\frac{14}{z_1}.$$

Again, in the decomposition of silicon-ethyl it has been experimentally ascertained that putting z_2 as the number of volumes of silicon-ethyl decomposed, and z_3, z_4, z_5 as the number of volumes of hydrogen, carbon, and silicon respectively formed by its decomposition,

$$\frac{z_3}{z}=10,\quad \frac{z_4}{z_2}=\frac{8x}{y},\quad \text{and}\quad \frac{z_5}{z_2}=\frac{z_1}{z},$$

where $\frac{x}{y}$ is the ratio of the number of volumes of carbon formed to the number of volumes of marsh-gas decomposed, in the equation expressing the result of the decomposition of that substance (Sec. VII. Group 2 (1)), and z and z_1 have the values assigned to them in the previous equation. Hence, putting $\alpha^m\chi^{m_1}\sigma^{m_2}$ as the symbol of silicon-ethyl, κ^y as the symbol of carbon, and σ^z as the symbol of silicon, and substituting the above values for z_2, z_3, z_4, z_5 in the equation

$$z_2\alpha^m\kappa^{m_1}\sigma^{m_2}=z_3\alpha+z_4\kappa^y+z_5\sigma^z,$$

we have

$$yz\times\alpha^m\kappa^{m_1}\sigma^{m_2}=10yz\times\alpha+8xz\times\kappa^y+yz_1\times\sigma^z,$$

whence

$$(\alpha^m\kappa^{m_1}\sigma^{m_2})^{yz}=\alpha^{10yz}(\kappa^4)^{8xz}(\sigma^z)^{yz_1}$$

and

$$m=10,\quad m_1=x,\quad m_2=z_1;$$

and we have for the symbol of silicon-ethyl, as determined from experiment, $\alpha^{10}\kappa^{8x}\sigma^{z_1}$, or, putting $x=1$ (Sec. VII. Group 2 (1)), as in the symbols of the other compounds of carbon, $\alpha^{10}\kappa^8\sigma^{z_1}$.

Proceeding in a similar manner with the other gaseous compounds of silicon, we have

	Symbol.
Chloride of silicon	$\alpha^2\chi^4\sigma^{z_1}$.
Silicon-ethyl	$\alpha^{10}\varkappa^8\sigma^{z_1}$.
Silicate of ethyl	$\alpha^{10}\varkappa^8\xi^4\sigma^{z_1}$.
Silicate of amyl	$\alpha^{22}\varkappa^{20}\xi^4\sigma^{z_1}$.
Monochlorhydrine of silicate of ethyl . .	$\alpha^8\chi\varkappa^6\xi^3\sigma^{z_1}$.

Now there is but one hypothesis which can be made as to the value of z_1, which is at once necessary and sufficient, namely that $z_1=1$. The reasoning here employed is of the same kind as that by which the symbols of the combinations of carbon were determined; but the observations being few, the conclusion is of a less certain character. By a similar argument also to that before used in regard to the value of A, it may be shown that there is great reason to believe that $z=1$, or $=2$, or $=4$. It is not necessary, however, to make any other assumption than that $z=1$, in which case the density of silicon-vapour is a multiple of 14, which is all that can be asserted with probability.

Putting $z_1=1$, we have the following symbols:—

Combinations of the Prime Factors α, ξ, θ, χ, ... and σ.

Name of substance.	Prime factor.	Absolute weight, in grammes.	Relative weight.
	σ	1·251	14
	Symbol.		
Silicon	σ^z	$z\times 1{\cdot}251$	$z\times 14$
Silica	$\sigma\xi^2$	2·682	30
Monohydrated silicic acid	$\alpha\sigma\xi^3$	3·486	39
Sulphide of silicon	$\sigma\theta^2$	4·112	46
Chloride of silicon	$\alpha^2\chi^4\sigma$	7·598	85
Bromide of silicon	$\alpha^2\beta^4\sigma$	15·554	174
Silicon-ethyl	$\alpha^{10}\varkappa^8\sigma$	6·436	72
Silicate of ethyl	$\alpha^{10}\varkappa^8\xi^4\sigma$	9·297	104
Silicate of amyl	$\alpha^{22}\varkappa^{10}\xi^4\sigma$	11·442	128
Monochlorhydrin of silicate of ethyl	$\alpha^8\chi\varkappa^6\xi^3\sigma$	8·872	99·25

(3) *Symbol of Boron.*—In the decomposition of the chloride of boron into its elements, one volume of that substance is decomposed with the formation of $1\frac{1}{2}$ volume of chlorine. Hence, if y_1 be the smallest integral number of units of the chloride of boron decomposed, and y_2 the number of units of boron formed in the decomposition, $\frac{y_1}{y_2}=\frac{2}{3}$; and putting $\alpha^m\chi^{m_1}\beta_1^{m_2}$ as the symbol of the terchloride of boron, and $\alpha^n\chi^{n_1}\beta_1^{n_2}$ as the symbol of boron,

$$2y_1\alpha^m\chi^{m_1}\beta_1^{m_2}=3y_1\alpha\chi^2+y_2\alpha^n\chi^{n_1}\beta_1^{n_2};$$

whence

$$(\alpha^m\chi^{m_1}\beta_1^{m_2})^{2y_1}=(\alpha\chi^2)^{3y_1}(\alpha^n\chi^{n_1}\beta_1^{n_2})^{y_2},$$

and

$$2y_1m = 3y_1 + y_2n,$$
$$2y_1m_1 = 6y_1 + y_2n_1,$$
$$2y_1m_2 = y_2n_2.$$

From the first of these equations it appears (since y_2 and y_1 have no common measure) that y_2 cannot be an even number, if the equation is to admit of an integral solution. Making this assumption, we have, putting $y_2=1+2z$,

$$m = 2+z, \quad n = y_1,$$
$$m_1 = 3, \quad n_1 = 0,$$
$$m_2 = 1+2z, \quad n_2 = 2y_1,$$

a minimum, which give

Symbol of chloride of boron . $\alpha^{2+z}\chi^3\beta_1^{1+2z}$,
Symbol of boron . . . . . $\alpha^{y_1}\beta_1^{2y_1}$.

Also, since 5·5 parts of boron are formed by the decomposition of 1 volume of chloride of boron of the density 58·75, putting W as the density of boron-vapour,

$$\frac{y_2\mathrm{W}}{2y_1} = \frac{y_2(1+2w(\beta_1))}{2} = 5{\cdot}5,$$

and putting $y_2=1+2z$,

$$w(\beta_1) = \frac{5-z}{1+2z}.$$

Again, putting $\alpha^m\varkappa^{m_1}\beta_1^{m_2}$ as the symbol of boric methide, and (as before) $\frac{x}{y}$ as the number of volumes of carbon formed by the decomposition of one volume of marsh-gas, and giving to y_1 and y_2 the same values as in the last equation, it is known from experiment that

$$2yy_1\alpha^m\varkappa^{m_1}\beta_1^{m_2} = 9yy_1\alpha + 6xy_1\varkappa^y + yy_2\alpha^{y_1}\beta_1^{2y_1};$$

whence

$$2m = 9 + y_2,$$
$$2m_1 = 6x,$$
$$m_2 = y_2;$$

and putting $y_2=1+2z$, and $x=1$,

$$m = 5+z,$$
$$m_1 = 3,$$
$$m_2 = 1+2z,$$

and we have for the symbol of boric methide $\alpha^{5+z}\varkappa^3\beta_1^{1+2z}$.

Proceeding in a similar manner with the other gaseous combinations of boron, we arrive at the following symbols:—

	Symbol.
Boron	$\alpha^{y_1}\beta_1^{2y_1}$.
Chloride of boron	$\alpha^{2+z}\beta_1^{1+2z}\chi^3$.
Boric methide	$\alpha^{5+z}\beta_1^{1+2z}\kappa^3$.
Boric ethide	$\alpha^{8+z}\beta_1^{1+2z}\kappa^6$.
Trimethylic borate	$\alpha^{5+z}\beta_1^{1+2z}\kappa^3\xi^3$.
Triethylic borate	$\alpha^{8+z}\beta_1^{1+2z}\kappa^6\xi^3$.
Triamylic borate	$\alpha^{17+z}\beta_1^{1+2z}\kappa^{15}\xi^3$.

By similar reasoning to that employed in the case of silicon, we are led to assume in this system of symbols $z=0$, which results in the system given in the following Table.

Combinations of the Prime Factors α, ξ, θ, χ, ... and β_1.

Name of substance.	Prime factor.	Absolute weight, in grammes.	Relative weight.
	β_1	0·447	5
	Symbol.		
Boron	$\alpha^y\beta_1^{2y}$	$y\times 0{\cdot}983$	$y\times 11$
Teroxide of boron	$\alpha\beta_1^2\xi^3$	3·129	35
Boracic acid	$\alpha^2\beta_1\xi^3$	2·771	31
Terchloride of boron	$\alpha^2\beta_1\chi^3$	5·252	58·75
Nitride of boron	$\alpha\beta_1\nu$	1·117	12·5
Boric methide	$\alpha^5\beta_1\kappa^3$	2·503	28
Trimethylic borate	$\alpha^5\beta_1\kappa^3\xi^3$	4·648	52
Triethylic borate	$\alpha^8\beta_1\kappa^6\xi^3$	6·534	73
Triamylic borate	$\alpha^{17}\beta_1\kappa^{15}\xi^3$	12·157	136
Boric ethide	$\alpha^8\beta_1\kappa^6$	4·380	49

If we proceed to determine the most probable symbol of boron by aid of the hypothesis $A=V$, or $A=\frac{V}{2}$, or $A=\frac{V}{4}$ (Sec. VII. Group 2 (1)), we have in case (i) $A=V$, $\frac{y_2}{2y_1}=1$, which admits of no integral solution, y_2 being odd and y_1 prime to y_2; in case (ii) $A=\frac{V}{2}$, $\frac{y_2}{2y_1}=\frac{1}{2}$, and $y_2=1$, $y_1=1$; in case (iii) $A=\frac{V}{4}$, $\frac{y_2}{2y_1}=\frac{1}{4}$, and $y_2=1$, $y_1=2$. Hence the more probable symbols for boron (from these considerations) are $\alpha\beta_1^2$ and $\alpha^2\beta_1^4$, between which we cannot decide.

The symbols which have been assigned to the gaseous compounds of the preceding elements, silicon and boron, are to be regarded as the symbolic expression of the most probable hypothesis as to their chemical constitution, which is consistent with the known facts of gaseous combination. What weight, we may ask, is to be attached to such conclusions? Now, it has already been remarked that the weight to be given to such hypotheses primarily depends upon the number of cases to which they are applicable. But in the case of these elements we are acquainted only with a very limited number of gaseous compounds; and it must be admitted that, regarded exclusively from this point

of view, but little value could be attached to any inference at which we thus arrive; for the conclusions drawn from six or seven instances, accidentally selected, would not improbably be negatived by a more extended experience. But our judgment is in truth based upon considerations of a far more complex character; and it would be unreasonable not to extend our view to the probabilities derived from other sources of the inferences to which the various hypotheses lead, and which often enable us to select among them.

For example, in the symbol of the chloride of boron, $\alpha^{2+z}\chi^3\beta_1^{1+2z}$, the value of z is necessarily limited only by the condition $w(\beta_1)=\frac{5-z}{1+2z}$, whence z is less than 5. But if we proceed to assign to z the different values 0, 1, 2, 3, 4, it will be found that on the first hypothesis, $z=0$, the combinations of the prime factor β_1 form a system strictly similar in the laws of their construction to the system of the combinations of the prime factors ν, ϕ, and ϱ; whereas on the other hypotheses they are analogous to no existing system whatever.

Thus, putting $z=0$, we have the following parallel systems, limiting our view for the moment to the gaseous compounds of boron.

	$z=0.$		
Boron	$(\alpha\beta_1^2)^{y_1}$	Nitrogen	$\alpha\nu^2$
Chloride of boron	$\alpha^2\beta_1\chi^3$	Chloride of nitrogen	$\alpha^2\nu\chi^3$
Boric methide	$\alpha^5\beta_1\varkappa^3$	Methylamine	$\alpha^5\nu\varkappa^3$
Boric ethide	$\alpha^8\beta_1\varkappa^6$	Ethylamine	$\alpha^8\nu\varkappa^6$
Triethylic borate	$\alpha^8\beta_1\varkappa^6\xi^3$		
Phosphorus	$(\alpha\phi^2)^2$	Arsenic	$(\alpha\varrho^2)^2$
Chloride of phosphorus	$\alpha^2\phi\chi^3$	Chloride of arsenic	$\alpha^2\varrho\chi^3$
Trimethyl-phosphine	$\alpha^5\phi\varkappa^3$	Trimethyl-arsine	$\alpha^5\varrho\varkappa^3$
Triethyl-phosphine	$\alpha^8\phi\varkappa^6$	Triethyl-arsine	$\alpha^8\varrho\varkappa^6$
Phosphite of ethyl	$\alpha^8\phi\varkappa^6\xi^3$		

If we put $z=1, 2, 3, 4$, the system of the gaseous compounds of boron appears with the following symbols:—

	$z=1.$	$z=2.$	$z=3.$	$z=4.$
Chloride of boron	$\alpha^3\beta^3\chi^3$	$\alpha^4\beta^5\chi^3$	$\alpha^5\beta^7\chi^3$	$\alpha^6\beta^9\chi^3$
Boric methide	$\alpha^6\beta^3\varkappa^3$	$\alpha^7\beta^5\varkappa^3$	$\alpha^8\beta^7\varkappa^3$	$\alpha^9\beta^9\varkappa^3$
Boric ethide	$\alpha^9\beta^3\varkappa^6$	$\alpha^{10}\beta^5\varkappa^6$	$\alpha^{11}\beta^7\varkappa^6$	$\alpha^{12}\beta^9\varkappa^6$
Triethylic borate	$\alpha^9\beta^3\varkappa^6\xi^3$	$\alpha^{10}\beta^5\varkappa^6\xi^3$	$\alpha^{11}\beta^7\varkappa^6\xi^3$	$\alpha^{12}\beta^9\varkappa^6\xi^3$

to which no parallel can be found among known and existing systems.

This coincidence cannot be regarded as of an accidental character. It is doubtless the result of the profound analogy of chemical properties by which this group of elements is connected, and which is revealed to us in the similarity of the symbolic forms of their combinations.

Group 3.—*Symbols of Antimony, Bismuth, Tin, Zinc, Cadmium, and Silver.*

It is evident that if some property of matter were discovered which admitted of accurate estimation, and which should vary with the gaseous density according to a known law, we should be able to infer the density from this property. Now in the numbers which represent the relative specific heats of chemical substances certain remarkable relations have been observed, which render it probable that general laws of this kind will hereafter be discovered, connecting the gaseous density of chemical substances with their specific heat, and which will afford a more solid foundation than we at present possess for the construction of a complete system of theoretical chemistry.

The law of DULONG and PETIT is the most important of these numerical relations which has been as yet ascertained. This law may be regarded as an experimental truth, and thus stated:

If A, A_1, A_2, ... A_n be the smallest weights of the elemental bodies formed by the decomposition of the unit of any chemical substance, and if h, h_1, h_2, ... h_n be the specific heats of these elements, either in the liquid or solid condition, then

$$hA=h_1A_1=h_2A_2= \ldots . h_nA_n.$$

In the following Table the value of the ratio $\frac{hA}{h_1A_1}$ is given in the case of those elements of which the symbols have been considered, and of which the specific heats have been experimentally determined, with the exception of the elements carbon, boron, and silicon, which do not appear to satisfy the condition. But these substances affect several allotropic forms, and have more than one specific heat; and it is not improbable that some variety of these elements may yet be discovered which shall conform to the law.

In the last column h_mA_m is assumed as the mean of the values given in the preceding column, namely, 3·289.

	A.	h.	hA.	$\frac{hA}{h_mA_m}$.
Sulphur	16	0·2026	3·241	0·985
Selenium	40	0·0837	3·348	1·018
Iodine	63·5	0·0541	3·436	1·045
Bromine	40	0·0843	3·372	1·025
Phosphorus	15·5	0·2120	3·285	0·999
Arsenic	37·5	0·0814	3·052	0·928

Hence if the specific heat, h, of an element be known, we are able, from the equation $A=\frac{h_mA_m}{h}=\frac{3\cdot289}{h}$, to calculate the value of A. The reasons have already been given which lead us to assume, with a certain probability, that W being the density of the element, $A=W$, or $=\frac{W}{2}$, or $=\frac{W}{4}$. Whence, if A be known, W is determined within certain limits.

Symbol of Antimony.—In the decomposition of the terchloride of antimony into

its elements, if y be the number of units of the terchloride of antimony decomposed, and y_1 the number of units of chlorine formed in the decomposition,

$$\frac{y}{y_1}=\frac{2}{3};$$

whence, if y_2 be the greatest common measure of y and y_1, $y=2y_2$, and $y_1=3y_2$; and putting y_3 as the number of units of antimony formed, W as the density of antimony, V as the density of chlorine, and V_1 as the density of terchloride of antimony,

$$y_3W=y_2(2V_1-3V),$$

and since $V=35{\cdot}5$, and $V_1=114{\cdot}25$, as experimentally determined,

$$y_3W=y_2 122.$$

Now the specific heat of antimony, as determined by REGNAULT, is 0·05077; whence, putting $A=\frac{h_m A_m}{h}$,

$$A=\frac{3{\cdot}289}{0{\cdot}05077}=64{\cdot}8;$$

and since $2\times 64{\cdot}4=128{\cdot}8$ may be regarded as approximately equal to 122, we may assume, within the limits of error, that in the above decomposition

$$y_3W=2y_2A.$$

Now three hypotheses may, as has been shown, be made as to the probable value of W.

1. $W=\ A$, in which case $y_3=2y_2$.
2. $W=2A$, in which case $y_3=\ y_2$.
3. $W=4A$, in which case $2y_3=\ y_2$.

Further, putting $\alpha^m\chi^{m_1}\sigma_1^{m_2}$ as the symbol of the terchloride of antimony, $\alpha\chi^2$ as the symbol of chlorine, and $\alpha^n\chi^{n_1}\sigma_1^{n_2}$ as the symbol of antimony, we have

$$2y_2(\alpha^m\chi^{m_1}\sigma_1^{m_2})=3y_2\alpha\chi^2+y_3\alpha^n\chi^{n_1}\sigma_1^{n_2},$$

$$(\alpha^m\chi^{m_1}\sigma_1^{m_2})^{2y_2}=(\alpha\chi^2)^{3y_2}(\alpha^n\chi^{n_1}\sigma_1^{n_2})^{y_3};$$

whence

$$2m\ y_2=3y_2+ny_3,$$
$$2m_1y_2=6y_2+n_1y_3,$$
$$2m_2y_2=n_2y_3.$$

1. Now on the first hypothesis $y_3=2y_2$, which is incompatible with any solution in whole numbers of the first of these equations, which then becomes

$$2m=3+2n:$$

this hypothesis is therefore to be rejected.

2. In the second case $y_3=y_2$, and we have

$$2m\ =3+n,$$
$$2m_1=6+n_1,$$
$$2m_2=n_2,$$

the minimum integral solutions of which equations give

$$m=2, \quad n=1,$$
$$m_1=3, \quad n_1=0,$$
$$m_2=1, \quad n_2=2,$$

and we have as the

Symbol of terchloride of antimony . $\alpha^2\chi^3\sigma_1$,
Symbol of antimony . . . . $\alpha\sigma_1^2$.

3. On the third hypothesis $2y_3=y_2$, and

$$4m=6+n,$$
$$4m_1=12+n_1,$$
$$4m_2=n_2,$$

and

$$m=2, \quad n=2,$$
$$m_1=3, \quad n_1=0,$$
$$m_2=1, \quad n_2=4,$$

a minimum, and the symbol of the terchloride is, as before, $\alpha^2\chi^3\sigma_1$, and the symbol of antimony $\alpha^2\sigma_1^4$, the equations on the two hypotheses being thus expressed:

Hypothesis II. . . . . $2\alpha^2\chi^3\sigma_1=3\alpha\chi^2+\alpha\sigma_1^2$.
Hypothesis III. . . . . $4\alpha^2\chi^3\sigma_1=6\alpha\chi^2+\alpha^2\sigma_1^4$.

The symbol of the terchloride, and of all other compounds of antimony, is the same on either view.

Combinations of the Prime Factors α, ξ, θ, χ, ... and σ_1.

Name.	Prime factor.	Absolute weight, in grammes.	Relative weight.
	σ_1	5·408	60·5
	Symbol.		
Antimony, Hypothesis 1 ..	$\alpha\sigma_1{}^2$	10·905	122
Antimony, Hypothesis 2 ..	$\alpha^2\sigma_1{}^4$	21·810	244
Teroxide of antimony	$\alpha\sigma_1{}^2\xi^3$	13·051	146
Tetroxide of antimony	$\alpha\sigma_1{}^2\xi^4$	13·767	154
Pentoxide of antimony	$\alpha\sigma_1{}^2\xi^5$	14·482	162
Tersulphide of antimony ..	$\alpha\sigma_1{}^2\theta^3$	15·198	170
Pentasulphide of antimony	$\alpha\sigma_1{}^2\theta^5$	18·057	202
Terchloride of antimony....	$\alpha^2\sigma_1\chi^3$	10·213	114·25
Pentachloride of antimony..	$\alpha^3\sigma_1\chi^5$	133·86	149·75
Oxychloride of antimony ..	$\alpha\sigma_1\chi\xi$	7·755	86·75

Symbol of Bismuth.—We are able to make precisely similar statements as to the relation which subsists between the densities of the terchloride of bismuth and its elements, chlorine and bismuth, to those which have been made respecting terchloride

of antimony; and putting W as the density of bismuth, V the density of chlorine, and V_1 the density of terchloride of bismuth, we have, as before,

$$y_3W=y_2(2V_1-3V);$$

and putting $V=35{\cdot}5$ and $V_1=157{\cdot}25$,

$$y_3W=y_2\times 208.$$

Now, the specific heat of bismuth, h, $=0{\cdot}03084$, and putting $A=\frac{3{\cdot}289}{0{\cdot}03084}$,

$$A=106{\cdot}6.$$

We may hence assume that $y_3W=2y_2A$, and, as before, putting

$$\begin{array}{lll} 1. & W=\ A, & y_3=2y_2; \\ 2. & W=2A, & y_3=\ y_2; \\ 3. & W=4A, & 2y_3=\ y_2; \end{array}$$

and by precisely similar reasoning to that in the last example, putting $\alpha^m\chi^{m_1}\beta_2^{m_2}$ as the symbol of terchloride of bismuth, and $\alpha^n\chi^{n_1}\beta_2^{n_2}$ as the symbol of bismuth, from the equation

$$(\alpha^m\chi^{m_1}\beta_2^{m_2})^{2y_2}=(\alpha\chi^2)^{3y_2}(\alpha^n\chi^{n_1}\beta_2^{n_2})^{y_3}$$

we arrive at the following symbols, rejecting, for the same reason as in the case of antimony, the first hypothesis.

On the second hypothesis, $W=2A$,

Symbol of bismuth $\alpha\beta_2^2$,

Symbol of terchloride of bismuth $\alpha^2\chi^3\beta_2$.

On the third hypothesis, $W=4A$,

Symbol of bismuth $\alpha\beta_2^4$,

Symbol of terchloride of bismuth $\alpha^2\chi^3\beta_2$.

We have hence the following system:—

Combinations of the Prime Factors α, ξ, θ, χ, ... and β_2.

Name.	Prime factor.	Absolute weight, in grammes.	Relative weight.
	β_2	9·252	103·5
	Symbol.		
Bismuth, Hypothesis 1	$\alpha\beta_2{}^2$	18·594	208
Bismuth, Hypothesis 2	$\alpha^2\beta_2{}^4$	37·188	416
Teroxide of bismuth	$\alpha\beta_2{}^2\xi^3$	20·739	232
Pentoxide of bismuth	$\alpha\beta_2{}^2\xi^5$	22·169	248
Hydrate of bismuth	$\alpha\beta_2\xi^2$	10·772	120·5
Bismuthic acid	$\alpha\beta_2{}^2\xi^3$	20·739	232
Bisulphide of bismuth	$\alpha\beta_2{}^2\theta^2$	21·454	240
Tersulphide of bismuth	$\alpha\beta_2{}^2\theta^3$	22·884	256
Terchloride of bismuth	$\alpha^2\beta_2\chi^3$	13·967	156·25
Oxychloride of bismuth	$\alpha\beta_2\chi\xi$	11·598	129·75
Triethyl bismuth	$\alpha^8{}_2\kappa^6$	13·185	147·5
Chloride of protethyl bismuth	$\alpha^4\beta_2\chi^2\kappa^2$	13·767	154

Symbol of Tin.—In the decomposition of the gaseous chloride of tin into tin and chlorine, the volume of chlorine formed is double the volume of the chloride decomposed. Hence, putting y_1 as the number of units of stannic chloride decomposed, and $\alpha^m\chi^{m_1}\varkappa_1^{m_2}$. $\alpha^n\chi^{n_1}\varkappa_1^{n_2}$ as the symbols, respectively, of stannic chloride and of tin,

$$y_1\alpha^m\chi^{m_1}\varkappa_1^{m_2}=2y_1\alpha\chi^2+y_2\alpha^n\chi^{n_1}\varkappa_1^{n_2}$$

and

$$(\alpha^m\chi^{m_1}\varkappa_1^{m_2})^{y_1}=(\alpha\chi^2)^{2y_1}(\alpha^n\chi^{n_1}\varkappa_1^{n_2})^{y_2},$$

whence

$$y_1m=2y_1+y_2n,$$
$$y_1m_1=4y_1+y_2n_1,$$
$$y_1m_2=y_2n_2.$$

In all cases

$$m=2,\quad n=0,$$
$$m_1=4,\quad n_1=0,$$
$$m_2=y_2,\quad n_2=y_1,$$

a minimum, and we hence have, as determined from the above equation,

Symbol of tin $\varkappa_1^{y_1}$,

Symbol of bichloride of tin $\alpha^2\chi^4\varkappa_1^{y_2}$;

and, assuming 130 as the density of bichloride of tin,

$$2w(\alpha)+4w(\chi)+y_2\varkappa_1=130;$$

whence

$$y_2w(\varkappa_1)=59,$$

and

$$W=y_1w(\varkappa_1)=\frac{y_1}{y_2}59.$$

Now, proceeding to construct the symbols of the other known gaseous compounds of tin by processes precisely similar to those of which sufficient examples have already been given in the case of the elements silicon and boron, and which it is unnecessary here to repeat, we arrive at the following symbols:—

	Symbol.
Bichloride of tin	$\alpha^2\chi^4\varkappa_1^{y_2}$.
Chloride of stannic dimethyl . .	$\alpha^4\chi^2\varkappa^2\varkappa_1^{y_2}$.
Chloride of stannic diethyl. . .	$\alpha^6\chi^2\varkappa^4\varkappa_1^{y_2}$.
Bromide of stannic diethyl . . .	$\alpha^6\beta^2\varkappa^4\varkappa_1^{y_2}$.
Iodide of stannic trimethyl. . .	$\alpha^5\omega\varkappa^3\varkappa_1^{y_2}$.
Chloride of stannic triethyl . .	$\alpha^8\chi\varkappa^6\varkappa_1^{y_2}$.
Bromide of stannic triethyl . .	$\alpha^8\beta\varkappa^6\varkappa_1^{y_2}$.
Stannic dimethyl-diethyl . . .	$\alpha^8\varkappa^6\varkappa_1^{y_2}$.
Stannic tetrethyl	$\alpha^{10}\varkappa^8\varkappa_1^{y_2}$.

As in the corresponding cases which have already been discussed, there is but one hypothesis which can be made as to the value of y_2, which is at once necessary and sufficient, namely that $y_2=1$, which gives $\alpha^2\chi^4\varkappa_1$ as the symbol of bichloride of tin.

Now the specific heat of tin is 0·05623. Hence $A=\frac{3·289}{0·05623}=58·5$; and it is known from experiment that $\frac{y_2}{y_1}W=59$. It may therefore be assumed that $\frac{y_2}{y_1}W=A$.

Hence (1) If $A=W$, $\frac{y_2}{y_1}=1$, and $y_2=1$, $y_1=1$.

(2) If $A=\frac{W}{2}$, $\frac{y_2}{y_1}=\frac{1}{2}$, and $y_2=1$, $y_1=2$.

(3) If $A=\frac{W}{4}$, $\frac{y_2}{y_1}=\frac{1}{4}$, and $y_2=1$, $y_1=4$.

Hence it appears that from these considerations also (whichever hypothesis be preferred) $y_2=1$, and the symbol of bichloride of tin is $\alpha^2\chi^4\varkappa_1$. The symbol of tin is, from these data, either $\varkappa_1$, $\varkappa_1^2$, or $\varkappa_1^4$, between which values we have no means of selecting.

Combinations of the Prime Factors α, ξ, θ, χ, ... and $\varkappa_1$.

Name of substance.	Prime factor.	Absolute weight, in grammes.	Relative weight.
	$\varkappa_1$	5·274	59
	Symbol.		
Tin. Hypothesis 1	$\varkappa_1$	5·274	59
Tin. Hypothesis 2	$\varkappa_1^2$	10·548	118
Tin. Hypothesis 3	$\varkappa_1^4$	21·096	236
Protoxide of tin	$\varkappa_1\xi$	5·989	67
Binoxide of tin	$\varkappa_1\xi^2$	6·704	75
Hydrated stannic acid	$\alpha\varkappa_1\xi^3$	7·509	84
Protosulphide of tin	$\varkappa_1\theta$	6·704	75
Bisulphide of tin	$\varkappa_1\theta^2$	8·135	91
Bichloride of tin	$\alpha\chi^2\varkappa_1$	8·448	94·5
Tetrachloride of tin	$\alpha^2\chi^4\varkappa_1$	11·621	130
Chloride of stannic dimethyl	$\alpha^4\chi^2\kappa^2\varkappa_1$	9·788	109·5
Chloride of stannic diethyl	$\alpha^6\chi^2\kappa^4\varkappa_1$	11·040	123·5
Bromide of stannic diethyl	$\alpha^6\beta^2\kappa^4\varkappa_1$	15·018	168
Iodide of stannic trimethyl	$\alpha^5\omega\kappa^3\varkappa_1$	12·962	145
Chloride of stannic triethyl	$\alpha^8\chi\kappa^6\varkappa_1$	10·749	120·25
Bromide of stannic triethyl	$\alpha^8\beta\kappa^6\varkappa_1$	12·738	142·5
Stannic tetrethyl	$\alpha^{10}\kappa^8\varkappa_1$	10·459	117
Stannic dimethyl-diethyl	$\alpha^8\kappa^6\varkappa_1$	9·207	103

Symbol of Zinc.—It is known from experiment that one volume of zinc-ethyl, of which the density is 61·75, can be decomposed into 5 volumes of hydrogen, 24 parts by weight of carbon, and 32·5 parts by weight of zinc.

Now, since 6 parts by weight of carbon are formed by the decomposition of one volume of marsh-gas, putting y_1 as the number of volumes of zinc-ethyl decomposed, and y_2 as the number of volumes of vapour of carbon formed in this decomposition,

$$\frac{y_2}{y_1}=4\frac{x}{y},$$

where x and y have the values previously assigned to them (Sec. VII. Group 2 (1)), and assuming $x=1$,

$$\frac{y_2}{y_1}=\frac{4}{y};$$

whence, putting y_3 as the number of volumes of zinc formed, $\alpha^m\kappa^{m_1}\zeta^{m_2}$ as the symbol of zinc-ethyl, $\alpha^n\kappa^{n_1}\zeta^{n_2}$ as the symbol of zinc, α as the symbol of hydrogen, and κ^y as the symbol of carbon,

$$yy_1(\alpha^m\kappa^{m_1}\zeta^{m_2})=5yy_1\alpha+4y_1\kappa^y+yy_3\alpha^n\kappa^{n_1}\zeta^{n_2},$$

and

$$(\alpha^m\kappa^{m_1}\zeta^{m_2})^{yy_1}=\alpha^{5yy_1}(\kappa^y)^{4y_1}(\alpha^n\kappa^{n_1}\zeta^{n_2})^{yy_3};$$

whence

$$y_1m=5y_1+y_3n,$$
$$y_1m_1=4y_1+y_3n_1,$$
$$y_1m_2=y_3n_2,$$

which equations have for all integral values of y_1 and y_2 the minimum solutions,

$$m=5,\qquad n=0,$$
$$m_1=4,\qquad n_1=0,$$
$$m_2=y_3,\qquad n_2=y_1,$$

which give the following expressions for the symbols,

Zinc-ethyl . . . $\alpha^5\kappa^4\zeta^{y_3}$.
Zinc . . . . . . ζ^{y_1}.

In a similar manner we arrive at $\alpha^3\kappa^2\zeta^{y_3}$ as the symbol of zinc-methyl.

These are the only gaseous compounds of zinc known.

The specific heat of zinc is 0·0955. Hence it may be assumed that

$$A=\frac{3\cdot289}{0\cdot0955}=34\cdot4.$$

Now from the above equtions

$$\frac{y_3}{y_1}W=32\cdot75,$$

W being the density of zinc in the gaseous condition, and approximately

$$\frac{y_3}{y_1}W=A.$$

(1) If $A=W$, $y_3=1$, and $y_1=1$.

(2) If $A=\frac{W}{2}$, $y_3=1$, and $y_1=2$,

(3) If $A=\frac{W}{4}$, $y_3=1$, and $y_1=4$.

On each hypothesis the value of y_3 is the same. Hence we have—

Name of substance.	Prime factor.	Absolute weight, in grammes.	Relative weight.
	ζ	2·905	32·5
	Symbol.		
Zinc. Hypothesis 1	ζ	2·905	32·5
Zinc. Hypothesis 2	ζ^2	5·811	65
Zinc. Hypothesis 3	ζ^4	11·621	130
Oxide of zinc	$\zeta\xi$	3·620	40·5
Hydrated oxide of zinc	$\alpha\zeta\xi^2$	4·425	49·5
Sulphide " "	$\zeta\theta$	4·335	48·5
Sulphate " "	$\zeta\theta\xi^4$	7·196	80·5
Hyposulphite " "	$\zeta\theta^2\xi^3$	7·911	88·5
Chloride " "	$\alpha\chi^2\zeta$	6·079	68
Carbonate " "	$\zeta\varkappa\xi^3$	5·587	62·5
Zinc-methyl	$\alpha^3\varkappa^2\zeta$	4·246	47·5
Zinc-ethyl	$\alpha^5\varkappa^4\zeta$	5·498	61·5
Zinc-amide	$\alpha^3\nu^2\zeta$	4·157	46·5

Symbol of Cadmium.—The density of the vapour of cadmium, as ascertained by DEVILLE and TROOST, is 56·7 on the hydrogen scale, and in the decomposition of the chloride of cadmium equal volumes of chlorine and cadmium are formed.

Hence putting

$\alpha^m\chi^{m_1}\varkappa_2^{m_2}$ as the symbol of chloride of cadmium,

$\alpha^n\chi^{n_1}\varkappa_2^{n_2}$ as the symbol of cadmium,

$\alpha\chi^2$ as the symbol of chlorine,

$$y_1\alpha^m\chi^{m_1}\varkappa_2^{m_2}=y_2\alpha\chi^2+y_2\alpha^n\chi^{n_1}\varkappa_2^{n_2},$$

and

$$(\alpha^m\chi^{m_1}\varkappa_2^{m_2})^{y_1}=(\alpha\chi^2)^{y_2}(\alpha^n\chi^{n_1}\varkappa_2^{n_2})^{y_2},$$

whence

$$y_1m = y_2+y_2n,$$
$$y_1m_1=2y_2+y_2n_1,$$
$$y_1m_2= y_2n_2.$$

Now the specific heat of cadmium is 0·05669, whence the calculated value of A $\left(\text{putting } A=\frac{3·289}{0·05669}\right)$ is 58, and W=A.

Hence, assuming in conformity with the principle previously laid down, that $\frac{y_2W}{y_1}=XA$, where X is a positive integer, since W=A, and y_1 and y_2 have no common measure, $y_1=1$, and the above equations become

$$m = y_2+y_2n,$$
$$m_1=2y_2+y_2n_1,$$
$$m_2= y_2n_2,$$

which in all cases admit of the minimum solutions,

$$m = y_2, \qquad n = 0,$$
$$m_1 = 2y_2, \qquad n_1 = 0,$$
$$m_2 = y_2, \qquad n_2 = 1,$$

and we have

Symbol of chloride of cadmium . . $(\alpha\chi^2\varkappa_2)^{y_2}$,
Symbol of cadmium . . . . $\varkappa_2$.

Or, assuming as the most probable hypothesis (in default of further information) that $y_2=1$, we have

Symbol of chloride of cadmium . . $\alpha\chi^2\varkappa_2$.

We hence arrive at the following symbols:—

Combinations of the Prime Factors α, ξ, θ, χ, ... and $\varkappa_2$.

Name.	Prime factor.	Absolute weight, in grammes.	Relative weight.
	$\varkappa_2$	5·006	56
	Symbol.		
Cadmium	$\varkappa_2$	5·006	56
Oxide of cadmium	$\varkappa_2\xi$	5·721	64
Sulphide of cadmium	$\varkappa_2\theta$	6·436	72
Sulphate of cadmium	$\varkappa_2\theta\xi^4$	9·297	104
Chloride of cadmium	$\alpha\chi^2\varkappa_2$	8·179	91·5
Carbonate of cadmium	$\varkappa_2\varkappa\xi^3$	7·688	86

Symbol of Silver.—Lastly, I will give one example of a class of symbols in regard to which we have even less positive knowledge, and are thrown almost exclusively upon hypothesis.

The specific heat of silver is 0·05701, whence the calculated value of A $\left(\text{putting } A=\frac{3\cdot289}{0\cdot05701}\right)$ is 57·6.

Now the percentage composition of chloride of silver is

Silver	75·26
Chlorine	24·74
	100·00

whence, putting

$\alpha^m\chi^{m_1}\varrho_1^{m_2}$ as the symbol of chloride of silver,
$\alpha^n\chi^{n_1}\varrho_1^{n_2}$ as the symbol of silver,
$\alpha\chi^2$ as the symbol of chlorine,

$$y_1\alpha^m\chi^{m_1}\varrho_1^{m_2} = y_2\alpha\chi^2 + y_3\alpha^n\chi^{n_1}\varrho_1^{n_2};$$

and

$$y_1 m = y_2 + y_3 n,$$
$$y_1 m_1 = 2y_2 + y_3 n_1,$$
$$y_1 m_2 = y_3 n_2,$$

and putting W as the density of the vapour of silver,

$$\frac{y_3 W}{y_2}=\frac{35{\cdot}5\times 75{\cdot}24}{24{\cdot}66}=108,$$

whence we may infer that

$$\frac{y_3 W}{y_2}=2A.$$

Also from the general considerations previously given,

$$\frac{y_3 W}{y_1}=XA,$$

where X is a positive integer.

Hypothesis I.—Let us assume that $W=A$. Then

$$\frac{y_3}{y_2}=2,\ \frac{y_3}{y_1}=X, \text{ and } \frac{y_2}{y_1}=\frac{X}{2}.$$

There are two cases, according as X is assumed to be odd or even.

(1) Let $X=2x+1$, an odd number. Then $y_1=2$, $y_2=X$, $y_3=2X$; and substituting these values in the equation

$$y_1 m=y_2+y_3 n,$$

we have

$$2m=(2x+1)(1+2n),$$

to which equation there is no integral solution. This hypothesis is therefore untenable.

(2) Let $X=2x_1$, an even number. Then $y_1=1$, $y_2=x_1$, $y_3=2x_1$, and

$$\begin{aligned} m &= x_1(1+2n),\\ m_1 &= 2x_1(1+n_1),\\ m_2 &= 2x_1 n_2. \end{aligned}$$

These equations in all cases admit of the minimum solution,

$$\begin{aligned} m &= x_1, & n &= 0,\\ m_1 &= 2x_1, & n_1 &= 0,\\ m_2 &= 2x_1, & n_2 &= 1. \end{aligned}$$

In which case the above equation becomes

$$(\alpha\chi^2\varrho_1^2)^{x_1}=x_1(\alpha\chi^2+2\varrho_1),$$

the symbols being thus expressed:

	Symbol.	Weight in grm.	Relative weight.
Silver	ϱ_1	$4{\cdot}827$	54
Chloride of silver .	$(\alpha\chi^2\varrho_1^2)^{x_1}$	$x_1\times 12{\cdot}828$	$x_1\times 143{\cdot}5$

Hypothesis II.—Now, let $W=2A$. Then

$$\frac{y_3}{y_2}=1,\ \frac{2y_3}{y_1}=X, \text{ and } \frac{2y_2}{y_1}=X.$$

(1) Let X be odd, $=2x+1$; then $y_1=2$, $y_2=X$, $y_3=X$, and

$$\begin{aligned} 2m &= (2x+1)(1+n),\\ 2m_1 &= (2x+1)(2+n_1),\\ 2m_2 &= (2x+1)n_2. \end{aligned}$$

These equations in all cases admit of the minimum solution,

$$\begin{aligned} m &= 2x+1, & n &= 1, \\ m_1 &= 2x+1, & n_1 &= 0, \\ m_2 &= 2x+1, & n_2 &= 2, \end{aligned}$$

and the above equation becomes

$$2(\alpha\chi\varrho_1)^{2x+1}=(2x+1)(\alpha\chi^2+\alpha\varrho_1^2),$$

the symbols being thus expressed:

	Prime factor. ϱ_1. Symbol.	Weight in grm.	Relative weight.
		4·783	53·5
Silver	$\alpha\varrho_1^2$	9·654	108
Chloride of silver . .	$(\alpha\chi\varrho_1)^{2x+1}$	$(2x+1)6{\cdot}423$	$(2x+1)71{\cdot}75$

(2) If $X=2x_1$, an even number, $y_1=1$, $y_2=x_1$, $y_3=x_1$, and

$$\begin{aligned} m &= x_1(1+n), \\ m_1 &= x_1(2+n_1), \\ m_2 &= x_1 n_2, \end{aligned}$$

which admit of the minimum solution,

$$\begin{aligned} m &= x_1, & n &= 0, \\ m_1 &= 2x_1, & n_1 &= 0, \\ m_2 &= x_1, & n_2 &= 1. \end{aligned}$$

This hypothesis is, however, untenable for the following reason.

According to the definition given of the weight A (Sec. VII. Group 2 (1)), the weight A is the smallest weight of silver formed by the decomposition of the unit of any chemical substance. Hence, if y be the number of units of the substance decomposed, and x the number of units of silver in that equation in which the weight A appears,

$$\frac{xW}{y}=A.$$

And putting ϱ_1 as the symbol of silver,

$$y\varrho_1^t=x\varrho_1,$$

and

$$ty=x:$$

but since $W=2A$, $y=2x$, whence $2t=1$, to which equation there is no integral solution. This hypothesis is therefore to be rejected.

If, however, in the above equation we select 2 as the value of n_2,

$$\begin{aligned} m &= x_1, \\ m_1 &= 2x_1, \\ m_2 &= 2x_1, \end{aligned}$$

in which case

$$(\alpha\chi^2\varrho_1^2)^{x_1}=x_1(\alpha\chi^2+\varrho_1^2);$$

and we have

$$y\varrho_1^t=x\varrho_1^2,$$

and $ty=2x$ and $t=1$.

It appears, therefore, that so far as any information extends which is afforded to us by the specific heat of silver, the symbol of this metal may be regarded as identical in form with that of zinc or mercury, the equation under consideration being expressed thus,

$$\alpha\chi^2\varrho_1^2=\alpha\chi^2+2\varrho_1,$$

or as being similar to the equation which expresses the relation existing between mercurous chloride and its elements,

$$\alpha\chi^2\delta^2=\alpha\chi^2+2\delta.$$

Or again, it may also be regarded as identical in form with the symbols of chlorine and of nitrogen; in which case the above equation is thus expressed,

$$2\alpha\chi\varrho_1=\alpha\chi^2+\alpha\varrho_1^2,$$

and is similar to the equation which connects the symbols of the chloride of iodine with those of chlorine and iodine,

$$2\alpha\chi\omega=\alpha\chi^2+\alpha\omega^2.$$

And, lastly, the facts are not even inconsistent with the assumption that the symbol of silver is identical in form with the symbols of oxygen and sulphur, so that

$$\alpha\chi^2\varrho_1^2=\alpha\chi^2+\varrho_1^2,$$

which is similar to the equation which connects the symbol of the bisulphide of chlorine with the symbols of its elements

$$\alpha\chi^2\theta^2=\alpha\chi^2+\theta^2.$$

The symbols of silver and its compounds appear, on the two more probable hypotheses, as follows:

Hypothesis I. $W=A$, $X=2x_1$.

Name.	Prime factor.	Absolute weight, in grammes.	Relative weight.
	ϱ	4·827	54
	Symbol.		
Silver	ϱ_1	4·827	54
Oxide of silver	$\varrho_1^2\xi$	10·369	116
Sulphide of silver	$\varrho_1^2\theta$	11·085	124
Sulphate of silver	$\varrho_1^2\theta\xi^4$	13·945	156
Chloride of silver	$\alpha\chi^2\varrho_1^2$	12·828	143·5
Nitrate of silver	$\alpha\nu^2\varrho_1^2\xi^6$	15·197	170
Metaphosphate of silver	$\alpha\varphi^2\rho_1^2\xi^6$	16·716	187
Pyrophosphate of silver	$\alpha\varphi^2\varrho_1^4\xi^7$	27·086	303
Orthophosphate of silver . .	$\alpha\varphi^2\varrho_1^6\xi^8$	37·455	419
Cyanide of silver.	$\alpha\nu^2\rho_1^2\kappa^2$	11·979	134

Hypothesis II. W=2A, X=2x+1.

Name.	Prime factor.	Absolute weight, in grammes.	Relative weight.
	ϱ_2	4·782	53·5
	Symbol.		
Silver	$\alpha\varrho_2{}^2$	9·654	108
Oxide of silver	$\alpha\varrho_2{}^2\xi$	10·369	116
Sulphide of silver	$\alpha\varrho_2{}^2\theta$	11·085	124
Sulphate of silver	$\alpha\varrho_2{}^2\theta\xi^4$	13·945	156
Chloride of silver	$\alpha\chi\varrho_2$	6·414	71·75
Nitrate of silver	$\alpha\nu\varrho_2\xi^3$	7·598	85
Metaphosphate of silver	$\alpha\phi\varrho_2\xi^3$	8·358	93·5
Pyrophosphate of silver	$\alpha^3\phi^2\varrho_2{}^4\xi^7$	27·086	303
Orthophosphate of silver ..	$\alpha^2\phi\varrho_2{}^3\xi^4$	18·728	209·5
Cyanide of silver	$\alpha\nu\varrho_2\varkappa$	5·989	67

This example may serve to show how inadequate are those considerations which are frequently regarded as affording a satisfactory solution of these important problems. If we are to pronounce an opinion on such slender data, the more probable hypothesis seems to be that which associates silver and the allied metals with the other electro-positive elements. In the two cases (namely mercury and cadmium) in which the fact can be experimentally determined, W=A. In other instances also, such as that of zinc, there is every reason to believe that this is the case.

As hypotheses thus accumulate the probability diminishes of the conclusions which are based upon them. It is, however, to be remembered that such uncertainty is not peculiar to chemistry, but that in every inductive science our exact knowledge lies within a very narrow sphere, as compared with the total field of observation; and after every deduction has been made on this account, there remains, as the solid nucleus of the science, the combinations of carbon and the gaseous elements, which hold in the theory of chemistry a position somewhat analogous to that occupied in astronomy by our solar system, as the area of exact observation.

Section VIII.—ON THE APPARENT EXCEPTIONS TO THE LAW OF PRIME FACTORS.

(1) On proceeding with the construction of the symbols of chemical substances, it will be found that in a certain limited number of cases the primary equations are apparently of such a nature as to render impossible the expression in them of the symbols by means of the same system of prime factors α, χ, ξ, ... by which the symbols in other cases can be expressed. It is probable that this anomaly admits of a very simple explanation, but it is not without interest to consider the modifications which such a fact, if it were truly established, would render necessary in our chemical ideas.

The density of sal-ammoniac in the gaseous condition, as experimentally determined, is 14·44, so that 4 volumes of sal-ammoniac are apparently decomposed into 4 volumes

of hydrogen, 1 volume of chlorine, and 1 volume of nitrogen. Whence, putting $\alpha^m\chi^{m_1}\nu^{n_2}$ as the symbol of the unit of sal-ammoniac,

$$(\alpha^m\chi^{m_1}\nu^{m_2})^4=\alpha^4\alpha\chi^2\alpha\nu^2,$$

and

$$2m=3,$$
$$2m_1=1,$$
$$2m_2=1,$$

and the symbol of sal-ammoniac is $(\alpha^3\chi\nu)^{\frac{1}{2}}$, and cannot be expressed by an integral number of the prime factors α, χ, ν.

We may now inquire whether, seeing this expression of the symbol of chloride of ammonium to be impossible on the assumption that the symbols of the elements hydrogen, chlorine, and nitrogen are of the forms α, $\alpha\chi^2$, $\alpha\nu^2$, it be possible on any other hypothesis as to these symbols, consistent with known facts. Now every possible hypothesis as to these symbols consistent with their expression by an integral number of prime factors in the equations from which the symbols have been derived, and with the expression of the symbol of hydrogen by one prime factor, α, is, as has been shown (Sec. VII. (5) and (8)), implicitly contained in the general forms of the symbols of chlorine and nitrogen, which are respectively $\alpha^{1+2p}\chi^{2(1+p_1)}$ and $\alpha^{1+2q}\nu^{2(1+q_1)}$; whence, putting

$$(\alpha^m\chi^{m_1}\nu^{m_2})^4=\alpha^4\alpha^{1+2p}\chi^{2(1+p_1)}\alpha^{1+2q}\nu^{2(1+q_1)},$$
$$2m=3+\overline{p+q},$$
$$2m_1=1+p_1,$$
$$2m_2=1+q_1,$$

and

$$m=2, \qquad p+q=1,$$
$$m_1=1, \qquad p_1=1,$$
$$m_2=1, \qquad q_1=1,$$

a minimum.

On the hypothesis, $p=0$, $q=1$, we have

Symbol of chlorine $\alpha\chi^4$,
Symbol of nitrogen $\alpha^3\nu^4$.

On the hypothesis, $p=1$, $q=0$, we have

Symbol of chlorine $\alpha^3\chi^4$,
Symbol of nitrogen $\alpha\nu^4$.

Neither hypothesis is absolutely inconsistent with any known fact, for it is possible thus to express the symbols of chlorine and nitrogen in every equation into which those symbols enter; and if it were placed beyond doubt that the true density of chloride of ammonium in the gaseous condition were 12·88, we might thus accept the fact and assert that the factors χ and ν were composite, so that either $\chi=\chi_1^2$ and $\nu=\alpha^2\nu_1^2$, or $\chi=\alpha^2\chi_2^2$ and $\nu=\nu_2^2$, and our view of the possible system of chemical substances, and of the laws of combination to which they were subject, would be profoundly modified. I

am far from believing that such is the true solution of this apparent anomaly. Such a solution, although not absolutely precluded to us, is in the highest degree improbable; and the facts admit of an obvious and simple explanation on the hypothesis that chloride of ammonium is decomposed, at the temperature at which its vapour-density is supposed to have been taken, into equal volumes of hydrochloric acid and ammonia, of which very satisfactory evidence has been given *.

(2) Again, the density of the binoxide of nitrogen is 14·989, as determined by experiment. If this be correct, 2 volumes of binoxide of nitrogen are decomposed into 1 volume of nitrogen and 1 volume of oxygen; whence, putting $\alpha^{m}\nu^{m_1}\xi^{m_2}$ as the symbol of the binoxide, $\alpha\nu^2$ as the symbol of nitrogen, and ξ^2 as the symbol of oxygen,

$$(\alpha^{m}\nu^{m_1}\xi^{m_2})^2=\alpha\nu^2\xi^2,$$

and

$$2m=1,$$
$$m_1=1,$$
$$m_2=1,$$

and the symbol of the binoxide of nitrogen, as expressed by the factors α, ν, ξ, is $\alpha^{\frac{1}{2}}\nu\xi$.

If we now inquire, as before, whether any hypothesis as to the symbols of nitrogen and oxygen can be made which shall be consistent with the fundamental assumption that the symbol of hydrogen is expressed by one factor, we have, putting $\alpha^{2p}\xi^{2(1+p_1)}$ and $\alpha^{1+2q}\,\nu^{2(1+q_1)}$ as the general symbols of oxygen and nitrogen (Sec. VII. (2) and (8),

$$(\alpha^{m}\nu^{m_1}\xi^{m_2})^2=\alpha^{1+2q}\nu^{2(1+q_1)}\alpha^{2p}\xi^{2(1+p_1)},$$

and

$$2m=1+2(q+p),$$
$$m_1=1+q_1,$$
$$m_2=1+p_1.$$

Now no positive and integral solution as regards m, q, and p, can be found which shall satisfy the first of these equations. The above equation, therefore, which expresses the relation which exists between the ponderable matter of the binoxide of nitrogen and its elements, is incompatible with the expression of the symbols by an integral number of prime factors, on the assumption that the symbol of hydrogen is α.

The anomaly in the density of the binoxide of nitrogen was long since observed by LAURENT and GERHARDT, who discovered the empirical law of even numbers. But such has been the influence upon the mind of chemists of an arbitrary hypothesis as to the constitution of matter, and of an uncertain system of notation, that this anomaly, the most singular exception known to the general laws of chemistry, is even now imperfectly recognized, and has never yet been submitted to any serious or adequate investigation.

(3) The following are the chief exceptions, real or apparent, to the law of prime factors †.

* See PEBAL, Ann. Chem. Pharm., vol. cxxiii. p. 199.

† See LAURENT's Chemical Method, p. 81; WATTS's Dictionary of Chemistry, vol. i. p. 469; GERHARDT, Traité de Chimie, vol. i. p. 581, vol. iv. p. 897.

The second column contains the symbol, as expressed by the factors α, χ, ξ, ... ; the third column the temperatures at which the observation is made; column A, the density of the substance, the density of air being assumed as 1; column B, the same density, the density of hydrogen being 1; column C contains the density as calculated from the symbols given in the first column.

Substance.	Symbol.	Temp. C.	A.	B.	C.
Binoxide of nitrogen	$\alpha^{\frac{1}{2}}\nu\xi$	. . . .	1·038	14·989	15
Peroxide of nitrogen	$\alpha^{\frac{1}{2}}\nu\xi^2$	. . . .	1·527	22·050	23
" "	"	97·5	1·783	25·746	"
" "	"	24·5	2·520	36·389	"
" "	"	11·3	2·645	38·194	"
Mercurous chloride	$\alpha^{\frac{1}{2}}\chi\delta_1$	. . . .	8·35	120·574	117·75
Chloride of ammonium	$\alpha^{\frac{3}{2}}\chi^{\frac{1}{2}}\nu^{\frac{1}{2}}$	1040	1·00	14·44	13·375
Bromide of ammonium	$\alpha^{\frac{3}{2}}\beta^{\frac{1}{2}}\nu^{\frac{1}{2}}$	. . . .	1·69	24·40	24·5
Iodide of ammonium	$\alpha^{\frac{3}{2}}\omega^{\frac{1}{2}}\nu^{\frac{1}{2}}$	. . . .	2·68	38·77	36·25
Cyanide of ammonium	$\alpha^{\frac{3}{2}}\varkappa^{\frac{1}{2}}\nu^{\frac{1}{2}}$	. . . .	0·79	11·41	11
Hydrosulphate of ammonia	$(\alpha^3\nu\theta)^{\frac{1}{2}}$	. . . .	0·89	12·85	12·75
Iodide of phosphonium	$(\alpha^3\omega\varphi)^{\frac{1}{2}}$	. . . .	2·77	39·999	40·5
Pentachloride of phosphorus	$(\alpha^3\chi^5\varphi)^{\frac{1}{2}}$	182	5·078	73·326	52·125
" "	"	336	3·656	52·793	"
Oxychloride of phosphorus	$(\alpha^2\chi^3\varphi\xi)^{\frac{1}{2}}$	. . . .	5·40	77·976	76·75
Sulphide of mercury	$(\delta\theta)^{\frac{2}{3}}$	. . . .	5·51	79·564	77·33
Perchlorinated methylic ether	$\alpha^{\frac{3}{2}}\chi^3\varkappa\xi^{\frac{1}{2}}$	. . . .	4·67	67·435	63·25
Perchlorinated sulphide of methyl. .	$\alpha^{\frac{3}{2}}\chi^3\varkappa\theta^{\frac{1}{2}}$	. . . .	5·68	82·019	67·25

It will be seen on inspection of the preceding Table that, in many instances, the vapour-density, as determined by experiment, does not sufficiently agree with any hypothesis even remotely probable.

Many of these apparent exceptions obviously admit of a similar simple explanation to that which has been suggested in the case of chloride of ammonium. Indeed, it would be truly surprising if in the varied transformations of matter no example of such decomposition should occur. In more than one case actual evidence of it has been adduced *; and while undoubtedly it must be allowed that the question is not to be answered by theoretical considerations alone, but that every case of such apparent anomaly should be submitted to the most rigid tests of experiment, there is every reason to believe that the simple weights α, ξ, θ, χ, ... are the ultimate known components of the units of ponderable matter, and represent a limit to chemical decomposition which has not as yet been passed.

* See PLAYFAIR and WANKLYN, Journ. Chemical Society, vol. xv. p. 142; WANKLYN and ROBINSON, "On Diffusion of Vapours," Proceedings, Royal Society, vol. xii. p. 507.

The atomic theory may be compared to a sort of "abacus" or simple mechanical instrument which chemists have invented to facilitate their calculations. It is useless to pretend that any demonstration can be given of this theory, which, at best, can only be regarded as a possible hypothesis suggested by the facts; but nevertheless it has a very real claim upon our consideration from the practical advantages which it has afforded in the study of the science. The atoms of the chemist fulfil a similar purpose in his calculations to that fulfilled by balls in the estimation of probabilities. They afford a simple and not inaccurate image of the subject with which he is concerned, by which he is enabled to reduce his problems to a concrete form, and thus at once to realize and to isolate them. To forbid the use of such an image would be to impose a very unnecessary restriction upon scientific methods. A ball as the concrete symbol of an indivisible whole, may advantageously represent, as occasion requires, a unit of weight, a simple weight, an event. We are perfectly free, when it suits our purpose, to make use of such conceptions. It is, however, a fatal illusion to mistake the suggestions of fancy for the realities of nature, and such a symbol becomes open to serious objection unless we carefully discriminate between conjecture and fact. Under the baneful influence of such hypotheses the methods of positive science lose their hold upon the mind, until at length we are actually informed by the consistent advocates of these ideas that the science of chemistry has no other field for its activity than the obscure region of atomic speculations.

Now a symbolic calculus affords the same indispensable aid which is given by the atomic theory, but in a more truthful and effectual way. In the place of molecules and atoms it offers, as the subject of scientific contemplation, a system of marks and combinations of letters, which, however, we are not free to arrange and to interpret according to the dictates of caprice, but of which each has a specific meaning assigned to it in the calculus, from which the laws are deduced according to which it is permitted to operate upon it. We are thus enabled to construct an accurate symbolic representation of the phenomena before us, on the fidelity of which we can rely. Such a system is indeed based, in the most absolute sense, upon fact, for it presents only two objects to our consideration, the symbol and the thing signified by the symbol, the object of thought and the object of sense; and it is not the least among the advantages which such a method affords, that through it we are enabled to dispense altogether with less truthful modes of representation, as no longer calculated to serve even a useful purpose.

Every mark or sign which we employ for the purposes of thought is in a certain sense a symbol, and in their actual system of chemical notation, chemists are already in possession of an imperfect symbolic method. It appeared to me inexpedient to attempt any interference with this method, which has already been subject to so many modifications, and which, moreover, satisfies certain real demands. I must confess also that it seemed to me incapable of development, as being destitute of those essential conceptions, in the growth of which the development of such a method consists.

Now it is the introduction of the conception of chemical operations which, as has before been said, especially distinguishes this calculus. The symbols here employed

are symbols not of quantities (which may be replaced by numbers), but of operations (which cannot be thus replaced), which are defined by their results; and the units of ponderable matter are primarily conceived of in this calculus as made up from their component weights by the successive performance upon the unit of space of the operations indicated by the symbols of those weights.

It is through this order of conceptions that we are enabled to introduce into the chemical calculus the zero-symbol 1, regarded as the symbol of the unit of space, the subject of chemical operations, without which symbol, as will hereafter be still more clearly evident, the construction of a chemical calculus would appear to be impossible, and the absence of which symbol, perhaps more than any other defect, marks the radical imperfection of the present notation.

It is moreover from this point of view that it has been found possible to assign to the composite symbol xy, as the symbol of a compound weight, an exact interpretation in harmony with symbolic analogies, and it is as symbols of operation that chemical symbols have been proved to possess the properties given in the equations

$$xy=yx,$$
$$x(y+y_1)=xy+xy_1,$$
$$xy=x+y,$$

which afford an adequate basis for a symbolic method, and enable us to apply to these symbols those algebraic processes through which symbols become an instrument of reasoning.

But further, symbols of this class afford the most real and the most obvious expression of the facts with which the chemist deals. That such operations as are here indicated are the primary and immediate object of his study, and therefore the most essential particular to be embodied in the symbol, has been already, to a certain extent, recognized by more than one master of the science, adverse to the atomic mode of representation. Thus GERHARDT, in the remarkable words which I have placed as a fitting motto to this paper, thus defines the object of a chemical formula. "Les formules chimiques, comme nous l'avons dit, ne sont pas destinées à représenter l'arrangement des atomes, mais elles ont pour but de rendre évidentes, de la manière la plus simple et la plus exacte, les relations qui rattachent les corps entre eux sous le rapport des transformations"*. Now if this be the object of a formula, how unreasonable is it to attempt the expression of that formula by symbols which not only permit, but even compel us to regard it from the atomic point of view. We cannot adopt the atomic symbol and at the same time declare ourselves free from the atomic doctrines. The symbols which are here employed impose no such limitation upon our view. They are simply the symbols of the operations, from whatever point of view these operations may be regarded, by which chemical transformations are effected. In the symbol of the unit of water $\alpha\xi$, we assert an indisputable fact as to the operations by which that unit is com-

* Chimie Organique, Paris, 1856, vol. iv. p. 566.

posed and decomposed in the actual system of chemical transformations. The symbol asserts that the unit of water is composed by two indivisible operations—indivisible, that is, so far as our experience extends—operating successively upon the unit of space, which are known to us through their results and are defined by their results. Again, we assert that the units of certain other substances are similarly composed, the units of hydrochloric acid and of hydrosulphuric acid, for example, of which $\alpha\chi$ and $\alpha\theta$ are the symbols, and that in this respect these substances are similar to water. Or, again, we say that the units of hydrogen, of water, and of peroxide of hydrogen are connected by a certain serial relation between the operations by which they are composed which is given by the interpretation of the symbols α, $\alpha\xi$, $\alpha\xi^2$, and that this relation is similar to that which exists between the units of hydrogen, hydrochloric acid, and chlorine, α, $\alpha\chi$, $\alpha\chi^2$. Now these are those very relations "qui rattachent les corps entre eux sous le rapport des transformations," which GERHARDT discerned to be the true object of symbolic expression, but which are not indicated, except accidentally, by our present system, which is based upon a different order of ideas.

But there is another aspect of the science equally real with that in which GERHARDT regarded it, and which he declined to consider. Surely we may be permitted to ask with DALTON, not only by what operations water is composed, but what water *is*? What is the nature of ponderable matter as revealed to us by the science of Chemistry? To this inquiry also, in the only form in which such an inquiry is real and intelligible, the symbol supplies an answer. This answer is given by interpreting the symbol with reference to the results of the operations. The unit of water (Sec. I. Def. 10), we may reply to such a question, is an integral compound weight (Sec. I. Def. 7) of which the whole is identical with the two simple weights (Sec. I. Def. 8) α and ξ of Section VII., which we recognize as simple weights from the fact that in the total system of chemical transformations these weights are not distributed (Sec. I. Def. 12). The science of Chemistry, we may add, affords no further information whatever as to the composition of water than that which is comprised in this assertion, which is not only the true but the only real form of answer which it is possible to give to inquiries as to the chemical composition of ponderable matter. There is no difference whatever, as regards facts, between this and the preceding statement; the difference lies in the way in which the facts are regarded. From the former point of view we consider the operations, from the latter the result of the operations. The symbols of geometry have a similar double interpretation. They may be regarded, with equal truth, as the symbols of lines and surfaces, or of the operations by which lines and surfaces are generated.

A symbol, however, should be something more than a convenient and compendious expression of facts. It is, in the strictest sense, an instrument for the discovery of facts, and is of value mainly with reference to this end, by its adaptation to which it is to be judged. Now in the present paper I have considered not only the principles of the chemical calculus, as regards the formal construction of symbols, but also the primary application of these principles to the construction of a special symbolic system. In

the symbol of each chemical substance a distinct assertion is made as to the chemical properties of the substance, which any one is at liberty to test by an appeal to facts. Now as no symbolic system similar to the present has yet been devised, and as this system cannot be deduced from any existing system, every symbol not only makes an assertion but expresses a discovery as to the chemical properties of the substance symbolized. It is obvious, from the way in which the symbol is constructed, that the properties symbolized are the properties of that system of chemical equations into which the symbols enter, and from which the laws of the science are to be deduced. The further development of these ideas must be reserved for another communication, in which the nature of the numerical laws which are thus expressed will be more fully considered.

We may also regard the symbolic system as the expression, in the language of reason, of those conceptions as to the composition of ponderable matter to which we are inevitably brought by the contemplation of chemical phenomena. Our conclusions on this point are so remarkable, and so contrary to anticipation, that doubtless we could never trust them but for the simple and exact process by which they are deduced. Now the conceptions which we form of the nature of the elemental bodies constitute the fundamental theory of the science, for these conceptions comprise and determine every similar conception. The unit of the element hydrogen is here conceived of as a simple weight, and symbolized by the letter α. That, to say the least, this view may be permitted is proved by constructing the symbols of chemical substances upon this hypothesis. There are, however, certain exceptions, be they real or apparent, in which this mode of expression is impossible, and it will be seen on reference to the table of exceptions (Sec. VIII. (4)) that this hypothesis rejects as inadmissible, not only the cases which are rejected by the atomic theory, but those also which are rejected by the empirical law of even numbers (p. 786). The symbolic system which is here given is the expression of these laws, by the truth of which it must stand or fall. The unit of the element mercury, and the units of several other metals, such as zinc, cadmium, and tin, so far as our imperfect experience extends, appear to be analogous in this respect to hydrogen. But these are the only elements of this simple composition. The units of a second group, of which the element oxygen, symbolized as ξ^2, may be taken as a type, and to which belong sulphur, θ^2, and selenium, λ^2, are composed of two identical simple weights, and the facts of the science do not permit us to assume these units as otherwise composed. Lastly, another group of elements appears in this system of a different and more complex composition, to which group belong the elements chlorine $\alpha\chi^2$, bromine $\alpha\beta^2$, iodine $\alpha\omega^2$, nitrogen $\alpha\nu^2$, phosphorus $(\alpha\phi^2)^2$, arsenic $(\alpha\varrho^2)^2$, and in all probability numerous other elements. The simplest view which, consistently with the fundamental hypothesis (Sec. VII. Group 1 (1)), can be taken of the composition of these elements, regard being had to the total system of chemical combinations, is that they are severally composed of a unit of hydrogen and of two identical simple weights, as, for example, in the case of chlorine, of the simple weight α and two of the simple weights symbolized by χ, so

that the elements of this group are to be considered as combinations of elements of the two previous forms respectively. It is further found, as a matter of experience, that the unit of every chemical substance may be regarded as a combination of the same simple weights, α, ξ, θ, χ, ω, ν, ..., which are the component weights of the units of the elements. Now from the fundamental equation $xy=x+y$,

$$\xi^2=2\xi,$$
$$\alpha\chi^2=\alpha+2\chi,$$
$$\alpha^2\phi^4=2\alpha+4\phi,$$
$$\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot$$
$$\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot$$

whence we unavoidably have suggested to us as the ultimate origin of our actual system of combinations, and as affording an adequate and probable (doubtless we cannot say the only possible or conceivable) explanation of the peculiar phenomena there presented to us, a group of elements ξ, θ, χ, β, ω, ν, ϕ, ... of the densities indicated by these symbols, and which, though now revealed to us through the numerical properties of chemical equations only as "implicit and dependent existences," we cannot but surmise may some day become, or may in the past have been, "isolated and independent existences." Examples of these simple monad forms of material being are preserved to us in such elements as hydrogen and mercury, which appear in the chemical system, as records suggestive of a state of things different from that which actually prevails, but which has passed away, and which we are unable to restore.

Such a hypothesis is not precluded to us, but nevertheless we are not to imagine that it is a necessary inference from the facts. So far as the principles or conclusions of this method are concerned, the "simple weights" ξ, θ, χ, β, ω, ν, ϕ, ... may be treated purely as "ideal" existences created and called into being to satisfy the demands of the intellect, to enable us to reason and to think in reference to chemical phenomena, but destined to vanish from the scene when their purpose has been served; and the existence of which as external realities we neither assume nor deny.