

VI. *Addition to Memoir on the Resultant of a System of two Equations.*

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THE elimination tables in the Memoir on the Resultant of a System of two Equations (Phil. Trans. 1857, pp. 703–715), relate to equations of the form $(a, b \dots \mathfrak{X}x, y)^m = 0$, *without* numerical coefficients; but it is, I think, desirable to give the corresponding tables for equations in the form $(a, b, \dots \mathfrak{X}x, y)^m = 0$ *with* numerical coefficients, which is the standard form in quantics. The transformation can of course be effected without difficulty, and the results are as here given. It is easy to see *à priori* that the sum of the numerical coefficients in each table ought to vanish; these sums do in fact vanish, and we have thus a verification as well of the tables of the present Addition as of the tables of the original memoir, by means whereof the present tables were calculated.

Table (2, 2).

Resultant of
 $(a, b, c \mathfrak{X}x, y)^2$,
 $(p, q, r \mathfrak{X}x, y)^2$.

e^2	p^2
$+1$	
ab	qr
-4	
ac	pr
-12	p^2
$+4$	
bc	pq
-4	
e^2	p^2
$+1$	

Table (3, 2).

Resultant of
 $(a, b, c, d \mathfrak{X}x, y)^3$,
 $(p, q, r \mathfrak{X}x, y)^2$.

e^3	p^3
$+1$	
cd	qr^2
-6	
ac	pr^2
-6	q^2r
$+9$	$+12$
ad	pqr
$+6$	q^3
bc	-18
-18	-8
bd	p^2r
-6	pq^2
$+9$	$+12$
cd	p^2q
-6	
e^3	p^3
$+1$	

Resultant of
 $(a, b, c, d, e \wr (x, y))^4,$
 $(p, q, r \wr (x, y))^2.$

The diagram illustrates the multiplication table for the 27 elements of the Heisenberg group. The elements are arranged in a diamond shape, with the identity element '1' at the top and 'z^3' at the bottom. The elements are grouped into three sets of nine, each forming a 3x3 sub-diagram. The multiplication is defined by the relations: $x^3 = y^3 = z^3 = 1$, $[x, y] = z$, $[x, z] = 1$, $[y, z] = 1$. The diagram shows that the product of two elements is another element in the group, and that the group is nilpotent of class 2.

Resultant of
 $(a, b, c, d \wr x, y)^3$
 $(p, q, r, s \wr x, y)^3$.

Figure 1 consists of three diamond-shaped grids of numbers, each representing a different type of 3D coordinate system. The top grid is labeled a^2b^2 and b^2a^2 at its top vertices. The middle grid is labeled abc and pqr at its top vertices. The bottom grid is labeled acd and pqs at its top vertices. Each grid contains numbers in its vertices and midpoints, with some numbers being positive and others negative. The grids are arranged in a vertical sequence, with the top grid being the largest and the bottom grid being the smallest.

Grid	Top-Left	Top-Right	Left-Mid	Right-Mid	Bottom-Left	Bottom-Right
a^2b^2 / b^2a^2	+1	-9	+27	+27	-27	-27
abc / pqr	-3	+27	+27	-81	-27	-27
acd / pqs	-9	-54	+81	+81	+9	+54

* N.B. In the corresponding table of the memoir, there is an error in the signs of the last two terms; they should be

The image shows two diamond-shaped grids, each representing a 2D Ising model configuration. The top grid is labeled with 'a22', 'b21', 'p23', 'p22', 'p21', 'p20' and contains values +27, -27, +81, -27, +27. The bottom grid is labeled with 'a21', 'b20', 'p22', 'p21', 'p20' and contains values -1, +5, -27, +18, -27.

Table (4, 3).

Resultant of
 $(a, b, c, d, e)(x, y)^4$,
 $(p, q, r, s)(x, y)^3$.

e^4	d^4	c^4	b^4	a^4
$+1$				
e^3d	d^3c	c^3b	b^3a	
-12				
e^2d^2	d^2c^2	c^2b^2	b^2a^2	
-36				
ed^3	d^2c^3	c^3b^3	b^3a^3	
$+48$				
e^2d	d^3c	c^4	b^4	a^4
-12	$+108$	-108		
ed^2	d^2c^3	c^3b^3	b^3a^3	
$+72$	-216			
e^3d	d^3c^2	c^2b^2	b^2a^2	
-64				
e^2d^2	d^2c^3	c^3b^3	b^3a^3	
$+12$	$+18$	-108	$+81$	
ed^3	d^3c^4	c^4b^4	b^4a^4	
-48	-288	$+432$		
e^4d	d^4c	c^4b	b^4a	
-216	$+324$			
e^3d^2	d^3c^2	c^2b^2	b^2a^2	
$+288$				
e^2d^3	d^2c^3	c^3b^3	b^3a^3	
-60	$+36$	$+324$	-324	
ed^4	d^4c^4	c^4b^4	b^4a^4	
$+72$	$+432$	-648		
e^4d^2	d^4c^2	c^2b^2	b^2a^2	
$+384$	-576			
e^3d^3	d^3c^3	c^3b^3	b^3a^3	
-432				

ace	$2a^2c$	pqr^3	c^3	$2p^2q$	$2p^2r$	$2p^2r^2$
-18	$+216$	-324	-324	$+432$	$+486$	
b^2e	$+48$	-432	0			
ad^2	$+48$	-432	$+432$			
bcd	-288	$+864$				
c^3	$+216$					

bce	$2b^2c$	pqr^3	c^3	$2p^2q$	$2p^2r$	$2p^2r^2$
-60	$+36$	$+324$	-324			
bde	$+72$	$+432$	-648			
bcd^2	$+384$	-576				
c^3d	-432					

ade	$2a^2d$	pqr^3	c^3	$2p^2q$	$2p^2r$	$2p^2r^2$
$+12$	$+18$	-108	$+81$			
bde	-48	-288	$+432$			
acd^2	-216	$+324$				
b^2e	$+288$					

ade	$2a^2d$	pqr^3	c^3	$2p^2q$	$2p^2r$	$2p^2r^2$
-60	$+36$	$+324$	-324			
acd	$+72$	$+432$	-648			
b^2d	$+384$	-576				
bce	-432					

bce	$2b^2c$	pqr^3	c^3	$2p^2q$	$2p^2r$	$2p^2r^2$
-12	$+108$	-108				
bde	$+72$	-216				
bcd^2	-64					

ace	$2a^2c$	pqr^3	c^3	$2p^2q$	$2p^2r$	$2p^2r^2$
-36	$+54$					
bce	$+48$					
c^3	$+1$					

Table (4, 4).

Resultant of

$$(a, b, c, d, e)(x, y)^4,$$

$$(p, q, r, s, t)(x, y)^4.$$

a^4	t^4	+1	
a^3b	st^3	-16	
a^2c	r^2s^2	-72	
a^2c^2	s^2t^2	+96	+96
a^2d	s^2t^2	-48	
a^2bc	r^2st	+288	+288
ab^3	s^3t	-256	-256
a^3e	st^3	-4	
a^2bd	st^3	+64	+64
a^2c^2	r^2st	+72	-256
ab^2c	r^2st	+72	-1152
ab^3	s^3t	+256	+1536
	s^3t	+1536	+1536
	s^3t	+1296	+1296
	s^3t	+1536	+1536
	s^3t	+256	+256

a^2de	pqr^2	$prst$	q^2s	q^2t	p^2s	qr^2	s^2
$abce$	+80	+192	-1280	+576	-768	+4608	-3456
b^2e	+192	-4608	+1536	+6912	+4608	-9216	
$cdde^2$	-768	+4608	0	0	-4096		
ae^2d	-1280	+1536	+8192	-9216			
bd^2d	+576	+6912	-9216				
bc^2d	+4608	-9216					
bc^2s	-3456						

a^2e^2	p^2s	$prst$	p^2t	q^2s	qr^2	s^2
$cdde$	+6	-128	-144	+384	+512	-2304
ae^2e	-128	+2560	0	-1536	-8192	+9216
b^2ce	-144	0	+5184	-6192	+9216	
acd^2	+384	0	-1536	-6912	+9216	
bd^2e	+384	-1536	-6912	+9216		
bc^2d	+512	-8192	+9216			
bc^2s	-2304	+9216				
e^4	+1296					

ace^2	p^2st	pqt	q^2t	pqs^2	pr^2s	q^2rs	s^3
$acde$	+80	+192	-768	-1280	+576	+4608	-3456
bde	+192	-4608	+4608	+1536	+6912	-9216	
bc^2e	-1280	+1536	0	+8192	-9216		
acd^2	+576	+6912	0	-9216			
bcd^2	-768	+4608	-4096				
c^2d	+4608	-9216					
c^2d^2	-3456						

ace^2	p^2st	pqt	q^2t	pqs^2	pr^2s	q^2rs	s^3
$acde$	+72	-192	-288	+2304	-3072	-2592	+3456
bde	-288	+768	+768	-4008	0	+3456	
bc^2e	+192	+768	-256	-4008	+9216	+4096	
acd^2	+2304	-4008	-4008	0	+3456		
bcd^2	-2592	0	+3456				
c^2d	-3072	+4096					
c^2d^2	+3456						

	p^2q^2	p^2q^3	p^3q^3	p^4q^3	q^4
ade^2	-16	-480	+256	+1728	-1536
bce^2	-480	+576	+3072	-3456	
bd^2c	+256	+3072	-3456		
c^2de	+1728	-3456			
cd^2e	-1536				

	p^2q	p^2q^2	p^3q^2	p^4q^2	q^3
ae^3	-4	+64	+72	-384	+256
be^3	+64	-256	-1152	+1536	
ce^3	+72	-1152	+1296		
cd^2e	-384	+1536			
d^4	+256				

	p^2q^3	p^3q^3	p^4q^3
be^3	-48	+288	-256
cd^2e	+288	-576	
d^4e	-256		

	p^2q	p^2q^2
ce^3	-72	+96
d^2e^2	+96	

	p^2q
de^3	-16
e^4	+1