

XX. *Tables of the Numerical Values of the Sine-integral, Cosine-integral, and Exponential-integral.* By J. W. L. GLAISHER, *Trinity College, Cambridge.* Communicated by Professor CAYLEY, *F.R.S.*

Received February 10,—Read March 10, 1870.

It has for a long time been evident that the extension of the Integral Calculus would require the introduction of new functions; or, rather, that certain functions should be regarded as primary, so that forms reduced to dependence on them might be considered known.

Thus, in the evaluation of Definite Integrals, the three transcendents

$$\int_0^x \frac{\sin u}{u} du, \quad \int_{-\infty}^x \frac{\cos u}{u} du, \quad \int_{-\infty}^{-x} \frac{e^{-u}}{u} du,$$

called the sine-integral, the cosine-integral, and the exponential-integral, have become recognized elementary functions, and great use has been made of them to express the values of more complicated forms. They were introduced by SCHLÖMILCH to evaluate the integral $\int_0^{\infty} \frac{a \sin x\theta}{a^2 - \theta^2} d\theta$, and several allied forms*, and denoted by him $\text{Si } x$, $\text{Ci } x$, $\text{Ei } x$.

ARNDT also employed them in a similar manner about the same time.

The first two functions had, however, previously received some attention from BRETSCHNEIDER, who appears to have been led to their consideration by their analogy with the logarithm-integral, as in a paper in the 17th volume of CRELLE'S Journal he announces his intention of not only tabulating this integral, but also of forming "tabulas aliarum quarundam functionum, cum logarithmo integrali arcte junctarum." The Tables are published in the third volume of GRUNERT'S 'Archiv der Mathematik und Physik' for 1843, and contain ten positive and negative values of the logarithm-integral, and ten values of the sine-integral and cosine-integral, besides tables of several other functions.

The exponential-integral was introduced in its present form by SCHLÖMILCH, though for all real values it is the same as the logarithm-integral $\text{li } x = \int_0^x \frac{du}{\log u}$, the relation between the two forms being

$$\text{li } e^x = \text{Ei } x.$$

The logarithm-integral appears to have been first discussed by MASCHERONI†, and in 1809 a work was published by SOLDNER at Munich concerning its theory, which also contained a Table of its values. This Table is reprinted in DE MORGAN'S 'Differential and Integral Calculus,' p. 662.

* CRELLE'S Journal, vol. xxxiii. p. 316.

† Referred to by BRETSCHNEIDER, CRELLE'S Journal, vol. xvii. p. 257.

The sine-integral and cosine-integral occur in a Memoir by BIDONE in the Turin Transactions for 1812, where they are expanded in series, the same as those marked (1) on the next page.

From the moment of its introduction the logarithm-integral excited considerable interest, but it is only in the last twenty-five years that the other functions have become of importance. A complete list of all the memoirs in which these functions are considered is given by Professor BIERENS DE HAAN, on page 83 of his 'Supplément aux tables d'intégrales définies,' published in the tenth volume of the Transactions of the Royal Academy of Amsterdam; and in the second volume reference is made to several other works in a memoir by the same author.

Since 1845 the three integrals have been practically regarded as primary functions in the integral calculus; and how well suited they are to this purpose is evident from the success which has attended the labours of those analysts who have sought to reduce more complicated integrals to dependence on them.

Professor DE HAAN, in the fifth volume of the Amsterdam Transactions, has evaluated a very large number of integrals by means of them; and in the great Tables* of the same author there are given nearly 450 functions dependent for their evaluation on that of these integrals. Considering therefore their extreme importance as a means of extending the Integral Calculus, and the probable value of many of the integrals evaluated in physical inquiries, it seemed very desirable that they should be systematically tabulated, so as to be known, not only by convention, but in reality; and on this subject Professor DE HAAN has strongly expressed his opinion of the value of such Tables.

BRETSCHEIDER, in the memoir previously cited, has computed $\text{Si } x$, $\text{Ci } x$, $\text{Ei } x$, $\text{Ei}(-x)$ for the values 1, 2, 3 . . . 10 of the argument to 20 places of decimals (except the values for $x=1$, which are extended to 35 places). This Table is reprinted by SCHLÖMILCH at the end of his 'Analytische Studien,' and a portion of it is quoted by the same author at the end of a paper in the thirty-third volume of CRELLE's Journal.

The Tables given in the present paper are the following:—

Tables I., II., III., IV.— $\text{Si } x$, $\text{Ci } x$, $\text{Ei } x$, $\text{Ei}(-x)$ from $x=0$ to $x=1$ at intervals of $\cdot 01$ to 18 places of decimals, with differences to the third order.

Tables V., VI., VII., VIII.— $\text{Si } x$, $\text{Ci } x$, $\text{Ei } x$, $\text{Ei}(-x)$ from $x=1$ to $x=5$ at intervals of $0\cdot 1$ to 11 places of decimals, with differences to the third order.

Table IX.— $\text{Si } x$, $\text{Ci } x$, $\text{Ei } x$, and $\text{Ei}(-x)$ from $x=5$ to $x=15$ at intervals of unity, to 11 places of decimals.

Table X.— $\text{Si } x$ and $\text{Ci } x$ from $x=20$ to $x=100$ at intervals of 5, from $x=100$ to $x=200$ at intervals of 10, from $x=200$ to $x=1000$ at intervals of 100, and for several higher values of x to 7 places of decimals.

Table XI.—Maxima and minima values of $\text{Si } x$ to 7 places of decimals.

Table XII.—Maxima and minima values of $\text{Ci } x$ to 7 places of decimals.

In the course of the work BRETSCHEIDER's values have been verified as far as they

* Nouvelles tables d'intégrales définies, Leyden, 1867.

coincided with those in the present paper, though from the great care he used, and the mode of verification he adopted, there was little doubt of their accuracy. One error was detected in $Ei(-5)$, which should be $\cdot 00114\dots$ instead of $\cdot 00144\dots$. This has doubtless arisen in the final copying or printing.

Expressed in series the functions are:

$$\left. \begin{aligned} Si\ x &= x - \frac{1}{3} \cdot \frac{x^3}{1 \cdot 2 \cdot 3} + \frac{1}{5} \cdot \frac{x^5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} - \frac{1}{7} \cdot \frac{x^7}{1 \cdot 2 \dots 7} + \dots \\ Ci\ x &= \gamma + \frac{1}{4} \log_e(x^4) - \frac{1}{2} \cdot \frac{x^2}{1 \cdot 2} + \frac{1}{4} \cdot \frac{x^4}{1 \cdot 2 \cdot 3 \cdot 4} - \frac{1}{6} \cdot \frac{x^6}{1 \cdot 2 \dots 6} + \dots \\ Ei\ x &= \gamma + \frac{1}{4} \log_e(x^4) + x + \frac{1}{2} \cdot \frac{x^2}{1 \cdot 2} + \frac{1}{3} \cdot \frac{x^3}{1 \cdot 2 \cdot 3} + \frac{1}{4} \cdot \frac{x^4}{1 \cdot 2 \cdot 3 \cdot 4} + \dots \end{aligned} \right\} \dots \quad (1)$$

γ being EULER's constant $0\cdot 5772156\dots$

From these expressions it is evident that $Si\ x$, $Ci\ x$, and $Ei\ x$ are connected by the relation

$$Ei(x\sqrt{-1}) = Ci\ x + \sqrt{-1}\ Si\ x.$$

The logarithms are written as above to indicate that they are real when x is negative, or of the form $a\sqrt{-1}$. The logarithm-integral differs in this respect only from the exponential-integral, for

$$li\ e^x = \gamma + \frac{1}{2} \log_e(x^2) + x + \frac{1}{2} \cdot \frac{x^2}{1 \cdot 2} + \frac{1}{3} \cdot \frac{x^3}{1 \cdot 2 \cdot 3} + \frac{1}{4} \cdot \frac{x^4}{1 \cdot 2 \cdot 3 \cdot 4} + \dots$$

The series in (1) are clearly always convergent, however large x may be.

The following series are easily obtained by integration by parts:

$$\left. \begin{aligned} Si\ x &= \frac{\pi}{2} - \cos x \left\{ \frac{1}{x} - \frac{1 \cdot 2}{x^3} + \frac{1 \cdot 2 \cdot 3 \cdot 4}{x^5} - \frac{1 \cdot 2 \dots 6}{x^7} + \dots \right\} \\ &\quad - \sin x \left\{ \frac{1}{x^2} - \frac{1 \cdot 2 \cdot 3}{x^4} + \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{x^6} - \frac{1 \cdot 2 \dots 7}{x^8} + \dots \right\} \\ Ci\ x &= \sin x \left\{ \frac{1}{x} - \frac{1 \cdot 2}{x^3} + \frac{1 \cdot 2 \cdot 3 \cdot 4}{x^5} - \frac{1 \cdot 2 \dots 6}{x^7} + \dots \right\} \\ &\quad - \cos x \left\{ \frac{1}{x^2} - \frac{1 \cdot 2 \cdot 3}{x^4} + \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{x^6} - \frac{1 \cdot 2 \dots 7}{x^8} + \dots \right\} \\ Ei\ x &= e^x \left\{ \frac{1}{x} + \frac{1}{x^2} + \frac{1 \cdot 2}{x^3} + \frac{1 \cdot 2 \cdot 3}{x^4} + \frac{1 \cdot 2 \cdot 3 \cdot 4}{x^5} + \dots \right\} \end{aligned} \right\} \dots \quad (2)$$

These series are ultimately divergent, though for values of x greater than unity they begin by converging, and when $x=17$, seven places of decimals are obtainable from them for $Si\ x$ and $Ci\ x$.

Formulae (1) were used for values where the argument was less than 16, formulae (2) where it was greater.

Tables I. to IX. were calculated in the following manner. The denominators of the terms in the series (1) were first computed, the first 20 figures of which, as far as the 71st power, and the logarithms of their reciprocals, are given in the following Table.

Table of the Constants.

x .	First twenty figures of $x\Gamma(x+1)$.	$-\log \{x\Gamma(x+1)\}$.	x .	First twenty figures of $x\Gamma(x+1)$.	$-\log \{x\Gamma(x+1)\}$.
2	4	$\bar{1} \cdot 397\ 940\ 008\ 7$	37	509 258 864 375 374 766 71	$\bar{45} \cdot 293\ 061\ 402\ 7$
3	18	$\bar{2} \cdot 744\ 727\ 494\ 9$	38	198 748 594 637 308 422 46	$\bar{47} \cdot 701\ 695\ 933\ 6$
4	96	$\bar{2} \cdot 017\ 728\ 767\ 0$	39	795 517 401 166 700 290 98	$\bar{48} \cdot 099\ 350\ 316\ 1$
5	600	$\bar{3} \cdot 221\ 848\ 749\ 6$	40	326 366 113 299 159 093 73	$\bar{50} \cdot 486\ 294\ 940\ 5$
6	432 0	$\bar{4} \cdot 364\ 516\ 253\ 2$	41	137 155 359 113 971 609 14	$\bar{52} \cdot 862\ 787\ 218\ 4$
7	352 80	$\bar{5} \cdot 452\ 471\ 423\ 5$	42	590 102 569 456 209 557 38	$\bar{53} \cdot 229\ 072\ 494\ 3$
8	322 560	$\bar{6} \cdot 491\ 389\ 489\ 6$	43	259 785 631 172 507 493 24	$\bar{55} \cdot 585\ 384\ 873\ 6$
9	326 592 0	$\bar{7} \cdot 485\ 994\ 457\ 7$	44	116 963 949 290 691 745 79	$\bar{57} \cdot 931\ 947\ 976\ 2$
10	362 880 00	$\bar{8} \cdot 440\ 236\ 967\ 1$	45	538 299 993 894 660 875 52	$\bar{58} \cdot 268\ 975\ 625\ 1$
11	439 084 800	$\bar{9} \cdot 357\ 451\ 596\ 8$	46	253 120 619 351 356 091 69	$\bar{60} \cdot 596\ 672\ 475\ 5$
12	574 801 920 0	$\bar{10} \cdot 240\ 481\ 789\ 9$	47	121 552 923 510 249 044 90	$\bar{62} \cdot 915\ 234\ 591\ 3$
13	809 512 704 00	$\bar{11} \cdot 091\ 776\ 331\ 3$	48	595 867 948 441 731 488 20	$\bar{63} \cdot 224\ 849\ 974\ 5$
14	122 049 607 680 0	$\bar{13} \cdot 913\ 463\ 612\ 3$	49	298 058 113 376 791 104 82	$\bar{65} \cdot 525\ 699\ 051\ 8$
15	196 151 155 200 00	$\bar{14} \cdot 707\ 409\ 129\ 8$	50	152 070 466 008 566 890 21	$\bar{67} \cdot 817\ 955\ 123\ 2$
16	334 764 638 208 000	$\bar{15} \cdot 475\ 260\ 423\ 6$	51	791 070 564 176 564 962 91	$\bar{68} \cdot 101\ 784\ 775\ 3$
17	604 668 627 763 200 0	$\bar{16} \cdot 218\ 482\ 563\ 5$	52	419 422 510 888 908 168 57	$\bar{70} \cdot 377\ 348\ 264\ 2$
18	115 242 726 703 104 000	$\bar{18} \cdot 938\ 386\ 474\ 6$	53	226 568 814 055 181 354 90	$\bar{72} \cdot 644\ 799\ 868\ 6$
19	231 125 690 776 780 800 0	$\bar{19} \cdot 636\ 151\ 777\ 8$	54	124 655 596 563 190 345 45	$\bar{74} \cdot 904\ 288\ 218\ 5$
20	486 580 401 635 328 000 00	$\bar{20} \cdot 312\ 845\ 387\ 5$	55	698 302 184 451 205 175 92	$\bar{75} \cdot 155\ 956\ 599\ 4$
21	107 290 978 560 589 824 00	$\bar{22} \cdot 969\ 436\ 793\ 7$	56	398 159 209 170 723 533 03	$\bar{77} \cdot 399\ 943\ 234\ 9$
22	247 280 160 111 073 689 60	$\bar{23} \cdot 606\ 810\ 726\ 7$	57	231 003 441 177 800 135 50	$\bar{79} \cdot 636\ 381\ 550\ 5$
23	594 596 384 994 354 462 72	$\bar{24} \cdot 225\ 777\ 735\ 5$	58	136 332 557 214 406 957 16	$\bar{81} \cdot 865\ 400\ 419\ 1$
24	148 907 616 415 977 465 44	$\bar{26} \cdot 827\ 083\ 088\ 1$	59	818 230 309 419 570 030 85	$\bar{82} \cdot 087\ 124\ 389\ 4$
25	387 780 251 083 274 649 60	$\bar{27} \cdot 411\ 414\ 312\ 5$	60	499 259 226 764 483 408 65	$\bar{84} \cdot 301\ 673\ 900\ 2$
26	104 855 779 892 917 465 25	$\bar{29} \cdot 979\ 407\ 625\ 2$	61	309 623 930 465 107 127 26	$\bar{86} \cdot 509\ 165\ 480\ 6$
27	293 999 475 161 295 508 34	$\bar{30} \cdot 531\ 653\ 444\ 9$	62	195 113 834 214 405 212 65	$\bar{88} \cdot 709\ 711\ 936\ 6$
28	853 687 364 912 798 809 40	$\bar{31} \cdot 068\ 701\ 146\ 4$	63	124 904 323 870 479 724 03	$\bar{90} \cdot 903\ 422\ 527\ 2$
29	256 411 097 818 451 356 68	$\bar{33} \cdot 591\ 063\ 181\ 9$	64	812 076 365 989 658 650 26	$\bar{91} \cdot 090\ 403\ 128\ 7$
30	795 758 579 436 573 175 90	$\bar{34} \cdot 099\ 218\ 670\ 4$	65	536 097 288 485 360 593 33	$\bar{93} \cdot 270\ 756\ 389\ 4$
31	254 907 998 279 515 607 34	$\bar{36} \cdot 593\ 616\ 537\ 4$	66	359 267 659 791 112 422 24	$\bar{95} \cdot 444\ 581\ 874\ 9$
32	842 018 678 187 819 296 53	$\bar{37} \cdot 074\ 678\ 274\ 6$	67	244 356 443 151 864 191 43	$\bar{97} \cdot 611\ 976\ 205\ 1$
33	286 549 481 420 792 254 35	$\bar{39} \cdot 542\ 800\ 373\ 2$	68	168 642 416 885 704 480 77	$\bar{99} \cdot 773\ 033\ 182\ 4$
34	100 379 151 673 465 407 88	$\bar{41} \cdot 998\ 356\ 479\ 0$	69	118 074 492 175 417 504 84	$\bar{101} \cdot 927\ 843\ 913\ 6$
35	361 660 178 823 515 072 53	$\bar{42} \cdot 441\ 699\ 307\ 3$	70	838 500 016 897 892 425 72	$\bar{102} \cdot 076\ 496\ 924\ 3$
36	133 917 597 644 364 438 28	$\bar{44} \cdot 873\ 162\ 350\ 1$	71	603 839 797 883 182 245 44	$\bar{104} \cdot 219\ 078\ 266\ 9$

The powers of all numbers from 1 to 100 were then formed up to the 20th, and in some extreme cases up to the 40th, the numbers under each power being entered in Tables*. The divisions were then made, and the values of the following four series,

$$x + \frac{1}{5} \cdot \frac{x^5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} + \frac{1}{9} \cdot \frac{x^9}{1 \cdot 2 \dots 9} + \dots$$

$$\frac{1}{2} \cdot \frac{x^2}{1 \cdot 2} + \frac{1}{6} \cdot \frac{x^6}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \frac{1}{10} \cdot \frac{x^{10}}{1 \cdot 2 \dots 10} + \dots$$

* The powers of numbers from 1 to 100 as far as the tenth are given in HUTTON'S 'Powers of Numbers,' published by the Commissioners of Longitude in 1781. In this Table, which was only used as a check, the fifth power of 81 and the seventh power of 98 were found to be inaccurate.

$$\frac{1}{3} \cdot \frac{x^3}{1 \cdot 2 \cdot 3} + \frac{1}{7} \cdot \frac{x^7}{1 \cdot 2 \dots 7} + \frac{1}{11} \cdot \frac{x^{11}}{1 \cdot 2 \dots 11} + \dots$$

$$\gamma + \log x + \frac{1}{4} \cdot \frac{x^4}{1 \cdot 2 \cdot 3 \cdot 4} + \frac{1}{8} \cdot \frac{x^8}{1 \cdot 2 \dots 8} + \dots$$

were formed, which, when suitably combined by additions and subtractions, gave the values of the functions.

In Tables I. to IV. the highest power of x included was x^{20} .

In Table IX. x^{63} was required for the value 15. In the intermediate Tables x^{29} was the highest power included.

The values of the functions when the argument was above 16 were calculated from formulæ (2), the log sines and log cosines being taken from TAYLOR's logarithms.

As the formulæ (2) are divergent, to remove every shade of doubt that might attach to their use, the functions for $x=20$ were computed from both formulæ, and the agreement was perfect to the eighth place, which was as far as the second formulæ could give correct results for this value of x .

The calculation of the values from $x=10$ to $x=20$ was extremely difficult and laborious, owing to the great number of terms and high powers necessary to be included. The term involving x^{76} was the first one rejected in the calculation for $x=20$; and to show how extremely unmanageable the formulæ (1) had become, it may be stated that this value, calculated as before described, required the formation of about 22,000 figures exclusive of verifications. Great confidence may, however, be placed in the truth of these results (from $x=10$ to $x=20$), as they were also calculated entirely independently by deducing each term from its predecessor, and in addition the value of $Ei(-x)$, which, on account of its extreme smallness, admitted of being obtained from formulæ (2), served as a rigorous verification of the whole process, excepting the final additions and subtractions.

The functions for $x=20$ were obtained correct to the 12th place, the values being

Si20	= +	1.548 241 701 043
Ci20	= +	0.044 419 820 845
Ei20	= +	256 156 52.664 056 588 820
Ei(-20)	= -	0.000 000 000 098.

Having obtained the values for $x=20$, it was a matter of comparative ease to give the values of the functions for $x=2$ to a great many places; they are to 43 places as follows:

Si2	= +	1.605 412 976 802 694 848 576 720 148 198 588 940 848 5834
Ci2	= +	0.422 980 828 774 864 995 698 565 153 198 255 894 135 7378
Ei2	= +	4.954 234 356 001 890 163 379 505 130 227 035 275 518 0536
Ei(-2)	= -	0.048 900 510 708 061 119 567 239 835 228 049 522 314 4922

agreeing with BRETSCHNEIDER's values to the first 20 places, which is as far as he has computed them. The value of γ was taken from a paper by Mr. SHANKS in No. 114 of the 'Proceedings of the Royal Society.' BRETSCHNEIDER has calculated the functions for $x=1$ to 35 places.

The maxima and minima values of the sine-integral correspond to multiples of π . Those above 4π were calculated by formulæ (2), the others were deduced by TAYLOR'S theorem from other values, Si π from Si 3 and Si 3.1; Si 2π from Si 6; Si 3π from Si 9; and Si 4π from Si 13.

The cosine-integral has its maxima and minima values for odd multiples of $\frac{1}{2}\pi$. Those above $\frac{9}{2}\pi$ were calculated from formulæ (2), the others were deduced from previously calculated values, Ci $\frac{1}{2}\pi$ from Ci 1.6; Ci $\frac{3}{2}\pi$ from Ci 4.7; Ci $\frac{5}{2}\pi$ from Ci 8; Ci $\frac{7}{2}\pi$ from Ci 11; and Ci $\frac{9}{2}\pi$ from Ci 14. The difference formulæ in the form best adapted for logarithmic computation are:

Si $(x+h)$ —Si x

$$\begin{aligned}
 &= \frac{h \sin x}{x} \left\{ \begin{aligned} &1 - \frac{h}{2x} \\ &+ h^2 \left(\frac{1}{3} - \frac{h}{4x} \right) \left(\frac{1}{x^2} - \frac{1}{1.2} \right) \\ &+ h^4 \left(\frac{1}{5} - \frac{h}{6x} \right) \left(\frac{1}{x^4} - \frac{1}{1.2x^2} + \frac{1}{1.2.3.4} \right) \\ &+ h^6 \left(\frac{1}{7} - \frac{h}{8x} \right) \left(\frac{1}{x^6} - \frac{1}{1.2x^4} + \frac{1}{1.2.3.4x^2} - \frac{1}{1.2.3.4.5.6} \right) \\ &+ \dots \end{aligned} \right\} \\
 &+ \frac{h^2 \cos x}{x} \left\{ \begin{aligned} &\frac{1}{2} - \frac{h}{3x} \\ &+ h^2 \left(\frac{1}{4} - \frac{h}{5x} \right) \left(\frac{1}{x^2} - \frac{1}{1.2.3} \right) \\ &+ h^4 \left(\frac{1}{6} - \frac{h}{7x} \right) \left(\frac{1}{x^4} - \frac{1}{1.2.3x^2} + \frac{1}{1.2.3.4.5} \right) \\ &+ \dots \end{aligned} \right\}
 \end{aligned}$$

Ci $(x+h)$ —Ci x

$$\begin{aligned}
 &= \frac{h \cos x}{x} \left\{ \begin{aligned} &1 - \frac{h}{2x} \\ &+ h^2 \left(\frac{1}{3} - \frac{h}{4x} \right) \left(\frac{1}{x^2} - \frac{1}{1.2} \right) \\ &+ h^4 \left(\frac{1}{5} - \frac{h}{6x} \right) \left(\frac{1}{x^4} - \frac{1}{1.2x^2} + \frac{1}{1.2.3.4} \right) \\ &+ \dots \end{aligned} \right\} \\
 &- \frac{h^2 \sin x}{x} \left\{ \begin{aligned} &\frac{1}{2} - \frac{h}{3x} \\ &+ h^2 \left(\frac{1}{4} - \frac{h}{5x} \right) \left(\frac{1}{x^2} - \frac{1}{1.2.3} \right) \\ &+ \dots \end{aligned} \right\}
 \end{aligned}$$

$\text{Si } \pi$ was calculated both from $\text{Si } 3$ and $\text{Si } 3.1$, thus verifying the formula as well as the value of $\text{Si } \pi$.

In all cases the values calculated from the difference formulæ on the preceding page were verified to as many places as could be obtained from formulæ (2).

In Table XI. $\text{Si } (x\pi) - \frac{1}{2}\pi$ is tabulated in preference to $\text{Si } x\pi$, as the changes in the function are thus rendered more apparent.

The curves $y = \text{Si } x$, $y = \text{Ci } x$, $y = \text{Ei } x$ were easily drawn from the values in the Tables. It is thus seen how rapidly the two former curves become flattened, and it is worth notice that the radius of curvature at any maximum or minimum point is equal to the abscissa of that point.

The point where the exponential-integral curve cuts the axis of x has the abscissa $0.37249680\dots$, in other words, this is the only real root of the equation $\text{Ei } x = 0$.

It is my intention to determine the points where the sine-integral and cosine-integral curves cut the lines with which they ultimately coincide, and with this view I have already determined the ordinates corresponding to points midway between the maxima and minima values as data for a first approximation. This will amount to finding the roots of the equations $\text{Si } x = \frac{1}{2}\pi$; $\text{Ci } x = 0$.

In Tables I. to IV., the work has been performed *correct* to the 20th place, and the last two figures have been finally rejected; in Tables V. to IX., the twelfth place has been corrected throughout the work, and the last figure only rejected. In the other Tables two figures have been generally rejected.

In Tables I. to VIII., all the results were verified as far as 7-figure logarithms were available, and in some cases 10-figure logarithms have been used. A very large portion of the work was done in duplicate, and every figure has been carefully examined either by my father or myself; in all cases great pains were taken to ensure accuracy, by independent methods as by logarithms, or in the way by which the values from 10 to 20 were verified.

It may be mentioned also that differences as far as the ninth order have been taken of the numbers in Tables I. to IV., thus affording a rigid test of the accuracy of more than the first ten, and in the sine-integral of the whole eighteen figures.

The Exponential-integral Curve,
 $y = \text{Ei } x$.

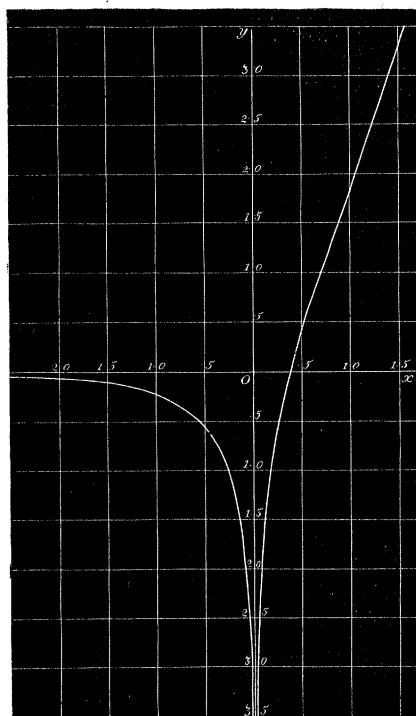


TABLE I.

Values of the Sine-integral from 0 to 1 at intervals of .01.

x .	$\text{Si } x$.	Δ	Δ^2	Δ^3
.00	+0.000 000 000 000 000 000	+0.009 999 944 444 611 111	-0.000 000 333 328 333 369	-0.000 000 333 308 333 845
.01	.009 999 944 444 611 111	9 999 611 116 277 742	00 666 636 667 214	333 268 336 226
.02	.019 999 955 560 888 853	9 998 944 479 610 527	00 999 905 003 440	333 208 343 369
.03	.029 998 500 040 499 380	9 997 944 574 607 087	01 333 113 346 809	333 128 359 558
.04	.039 996 444 615 106 467	9 996 611 461 260 278	01 666 241 706 367	333 028 390 507
.05	.049 993 056 076 366 745	9 994 945 219 553 911	01 999 270 096 874	332 908 443 856
.06	.059 988 001 295 920 656	9 992 945 949 457 037	02 332 178 540 231	332 768 526 673
.07	.069 980 947 245 377 693	9 990 613 770 916 806	02 664 947 066 904	332 608 650 451
.08	.079 971 561 016 294 499	9 987 948 823 849 903	02 997 555 717 354	332 428 826 108
.09	.089 959 509 840 144 402	9 984 951 268 132 548	03 329 984 543 462	332 229 066 489
.10	.099 944 461 108 276 950	9 981 621 283 589 086	03 662 213 609 951	332 009 385 859
.11	.109 926 082 391 866 036	9 977 959 069 979 135	03 994 222 995 810	331 769 799 909
.12	.119 904 041 461 845 172	9 973 964 846 983 325	04 325 992 795 719	331 510 325 749
.13	.129 878 006 308 828 497	9 969 638 854 187 606	04 657 503 121 468	331 230 981 910
.14	.139 847 645 163 016 103	9 964 981 351 066 138	04 988 734 103 378	330 931 788 339
.15	.149 812 626 514 082 241	9 959 992 616 962 760	05 319 665 891 717	330 612 766 404
.16	.159 772 619 131 045 001	9 954 672 951 071 043	05 650 278 658 121	330 273 938 884
.17	.169 727 292 082 116 044	9 949 022 672 412 922	05 980 552 597 005	329 915 329 975
.18	.179 676 314 754 528 966	9 943 042 119 815 917	06 310 467 926 980	329 536 965 282
.19	.189 619 356 874 344 884	9 936 731 651 888 938	06 640 004 892 261	329 138 871 820
.20	.199 556 088 526 233 821	9 930 091 646 996 676	06 969 143 764 081	328 721 078 013
.21	.209 486 180 173 230 498	9 923 122 503 232 595	07 297 864 842 095	328 283 613 689
.22	.219 409 302 676 463 093	9 915 824 638 390 501	07 626 148 455 784	327 826 510 079
.23	.229 325 127 314 853 593	9 908 198 489 934 717	07 953 974 965 863	327 349 799 815
.24	.239 233 325 804 788 310	9 900 244 514 968 854	08 281 324 765 678	326 853 516 926
.25	.249 133 570 319 757 164	9 891 963 190 203 176	08 608 178 282 604	326 337 696 837
.26	.259 025 533 509 960 340	9 883 355 011 920 572	08 934 515 979 441	325 802 376 367
.27	.268 908 888 521 880 913	9 874 420 495 941 131	09 260 318 355 808	325 247 593 721
.28	.278 783 309 017 822 044	9 865 160 177 585 323	09 585 565 949 529	324 673 388 495
.29	.288 648 469 195 407 367	9 855 574 611 635 794	09 910 239 338 024	324 079 801 666
.30	.298 504 043 807 043 161	9 845 664 372 297 770	10 234 319 139 690	323 466 875 591
.31	.308 349 708 179 340 931	9 835 430 053 158 079	10 557 786 015 282	322 834 654 007
.32	.318 185 138 232 499 010	9 824 872 267 142 798	10 880 620 669 289	322 183 182 023
.33	.328 010 010 499 641 808	9 813 991 646 473 509	11 202 803 851 312	321 512 506 117
.34	.337 824 002 146 115 317	9 802 788 842 622 197	11 524 316 357 429	320 822 674 137
.35	.347 626 790 988 737 514	9 791 264 526 264 768	11 845 139 031 566	320 113 735 250
.36	.357 418 055 515 002 282	9 779 419 387 233 201	12 165 252 766 857	319 385 740 146
.37	.367 197 474 902 235 484	9 767 254 134 466 345	12 484 638 507 003	318 638 740 627
.38	.376 964 729 036 701 829	9 754 769 495 959 342	12 803 277 247 630	317 872 790 009
.39	.386 719 498 532 661 171	9 741 966 218 711 712	13 121 150 037 639	317 087 942 912
.40	.396 461 464 751 372 883	9 728 845 068 674 073	13 438 237 980 550	316 284 255 300
.41	.406 190 309 820 046 956	9 715 406 830 693 523	13 754 522 235 851	315 461 784 478
.42	.415 905 716 650 740 480	9 701 652 308 457 673	14 069 984 020 328	314 620 589 081
.43	.425 607 368 959 198 152	9 687 582 324 437 344	14 384 604 609 409	313 760 729 075
.44	.435 294 951 283 635 496	9 673 197 719 827 935	14 698 365 338 484	312 882 265 752
.45	.444 968 149 003 463 431	9 658 499 354 489 450	15 011 247 604 237	311 985 261 722
.46	.454 626 648 357 952 882	9 643 488 106 885 214	15 323 232 865 959	311 069 780 912
.47	.464 270 136 464 838 095	9 628 164 874 019 255	15 634 302 646 871	310 135 888 557
.48	.473 898 301 338 857 350	9 612 530 571 372 384	15 944 438 535 429	309 183 651 200
.49	.483 510 831 910 229 734	9 596 586 132 836 955	16 253 622 186 629	308 213 136 681
.50	+0.493 107 418 043 066 689	+0.009 580 332 510 650 327	-0.000 016 561 835 323 309	-0.000 000 307 224 414 136

TABLE I. (continued).

Values of the Sine-integral from 0 to 1 at intervals of .01.

x .	$\text{Si } x$.	Δ	Δ^2	Δ^3
.50	+0.493 107 418 043 066 689	+0.009 580 332 510 650 327	-.000 016 561 835 323 309	-.000 000 307 224 414 136
.51	502 687 750 553 717 016	9 563 770 675 327 017	16 869 059 737 445	306 217 553 991
.52	512 251 521 229 044 033	9 546 901 615 589 572	17 175 277 291 436	305 192 627 954
.53	521 798 422 844 633 606	9 529 726 338 298 137	17 480 469 919 389	304 149 709 013
.54	531 328 149 182 931 742	9 512 245 868 378 747	17 784 619 628 403	303 088 871 429
.55	540 840 395 051 310 490	9 494 461 248 750 345	18 087 708 499 831	302 010 190 727
.56	550 334 856 300 060 834	9 476 373 540 250 513	18 389 718 690 559	300 913 743 696
.57	559 811 229 840 311 348	9 457 983 821 559 955	18 690 632 434 255	299 799 608 379
.58	569 269 213 661 871 302	9 439 293 189 125 700	18 990 432 042 634	298 667 864 067
.59	578 708 506 850 997 002	9 420 302 757 083 065	19 289 099 906 702	297 518 591 296
.60	588 128 809 608 080 067	9 401 013 657 176 364	19 586 618 497 997	296 351 871 835
.61	597 529 823 265 256 430	9 381 427 038 678 366	19 882 970 369 832	295 167 788 687
.62	606 911 250 303 934 797	9 361 544 068 308 534	20 178 138 158 519	293 966 426 077
.63	616 272 794 372 243 331	9 341 365 930 150 015	20 472 104 584 596	292 747 869 447
.64	625 614 160 302 393 346	9 320 893 825 565 419	20 764 852 454 043	291 512 205 450
.65	634 935 054 127 958 765	9 300 128 973 111 377	21 056 364 659 493	290 259 521 943
.66	644 235 183 101 070 142	9 279 072 608 451 884	21 346 624 181 436	288 989 907 981
.67	653 514 255 709 522 026	9 257 725 984 270 448	21 635 614 089 417	287 703 453 807
.68	662 771 981 693 792 474	9 236 090 370 181 031	21 923 317 543 225	286 400 250 849
.69	672 008 072 063 973 505	9 214 167 052 637 806	22 209 717 794 074	285 080 391 711
.70	681 222 239 116 611 311	9 191 957 334 843 732	22 494 798 185 785	283 743 970 164
.71	690 414 196 451 455 043	9 169 462 536 657 948	22 778 542 155 948	282 391 081 143
.72	699 583 658 988 112 991	9 146 683 994 501 999	23 060 933 237 091	281 021 820 736
.73	708 730 342 982 614 990	9 123 623 061 264 908	23 341 955 057 827	279 636 286 178
.74	717 853 966 043 879 898	9 100 281 106 207 081	23 621 591 344 005	278 234 575 845
.75	726 954 247 150 086 979	9 076 659 514 863 075	23 899 825 919 850	276 816 789 241
.76	736 030 906 664 950 054	9 052 759 688 943 226	24 176 642 709 091	275 383 026 997
.77	745 083 666 353 893 280	9 028 583 046 234 135	24 452 025 736 088	273 933 390 860
.78	754 112 249 400 127 415	9 004 131 020 498 047	24 725 959 126 948	272 467 983 684
.79	763 116 380 420 625 462	8 979 405 061 371 099	24 998 427 110 633	270 986 909 425
.80	772 095 785 481 996 560	8 954 406 634 260 466	25 269 414 020 057	269 490 273 128
.81	781 050 192 116 257 026	8 929 137 220 240 409	25 538 904 293 186	267 978 180 927
.82	789 979 329 336 497 435	8 903 598 315 947 223	25 806 882 474 112	266 450 740 027
.83	798 882 927 652 444 658	8 877 791 433 473 111	26 073 333 214 140	264 908 058 705
.84	807 760 719 085 917 769	8 851 718 100 258 971	26 338 241 272 845	263 350 246 295
.85	816 612 437 186 176 740	8 825 379 858 986 126	26 601 591 519 140	261 777 413 180
.86	825 437 817 045 162 866	8 798 778 267 466 986	26 863 368 932 320	260 189 670 790
.87	834 236 595 312 629 852	8 771 914 898 534 665	27 123 558 603 110	258 587 131 584
.88	843 008 510 211 164 517	8 744 791 339 931 555	27 382 145 734 694	256 969 909 049
.89	851 753 301 551 096 072	8 717 409 194 196 861	27 639 115 643 743	255 338 117 686
.90	860 470 710 745 292 933	8 689 770 078 553 117	27 894 453 761 429	253 691 873 005
.91	869 160 480 823 846 050	8 661 875 624 791 688	28 148 145 634 434	252 031 291 514
.92	877 822 356 448 637 739	8 633 727 479 157 254	28 400 176 925 948	250 356 490 709
.93	886 456 083 927 794 993	8 605 327 302 231 306	28 650 533 416 657	248 667 589 068
.94	895 061 411 230 026 299	8 576 676 768 814 649	28 899 201 005 725	246 964 706 039
.95	903 638 087 998 840 948	8 547 777 567 808 925	29 146 165 711 764	245 247 962 033
.96	912 185 865 566 649 873	8 518 631 402 097 161	29 391 413 673 797	243 517 478 412
.97	920 704 496 968 747 033	8 489 239 988 423 364	29 634 931 152 209	-.000 000 241 773 377 483
.98	929 193 736 957 170 397	8 459 605 057 271 155	-.000 029 876 704 529 692	
.99	937 653 342 014 441 552	+0.008 429 728 352 741 463		
1.00	+0.946 083 070 367 183 015			

TABLE II.

Values of the Cosine-integral from 0 to 1 at intervals of .01.

x .	$\text{Ci } x$.	Δ	Δ^2	Δ^3
.00	$-\infty$			
.01	-4.027 979 520 982 392 072	+693 072 182 122 430 726	-287 732 067 243 586 944	+169 899 043 044 911 376
.02	3.334 907 338 859 961 346	.405 340 114 878 843 782	.117 833 024 198 675 568	.053 244 523 267 548 460
.03	2.929 567 223 981 117 564	.287 507 090 680 168 214	.064 588 500 931 127 108	.023 716 537 864 691 258
.04	2.642 060 133 300 949 350	.222 918 589 749 041 107	.040 871 963 066 435 849	.012 651 131 298 823 265
.05	2.419 141 543 551 908 243	.182 046 626 682 605 257	.028 220 831 767 612 585	.007 551 606 006 198 032
.06	2.237 094 916 869 302 986	.153 825 794 914 992 673	.020 669 225 761 414 552	.004 870 948 972 724 095
.07	2.083 269 121 954 310 313	.133 156 569 153 578 120	.015 798 276 788 690 458	.003 325 858 202 350 686
.08	1.950 112 552 800 732 193	.117 358 292 364 887 663	.012 472 418 586 339 772	.002 372 207 871 050 106
.09	1.832 754 260 435 844 530	.104 885 873 778 547 891	.010 100 210 715 289 665	.001 751 559 256 444 941
.10	1.727 868 386 657 296 639	.094 785 663 063 258 226	.008 348 651 458 844 724	.001 330 162 206 136 986
.11	1.633 082 723 594 038 413	.086 437 011 604 413 502	.007 018 489 252 707 738	.001 033 964 991 797 193
.12	1.546 645 711 989 624 911	.079 418 522 351 705 763	.005 984 524 260 910 545	.000 819 668 534 466 454
.13	1.467 227 189 637 919 148	.073 433 998 090 795 218	.005 164 855 726 444 091	.000 660 786 482 471 572
.14	1.393 793 191 547 123 930	.068 269 142 364 351 127	.004 504 069 243 972 519	.000 540 489 674 574 183
.15	1.325 524 049 182 772 803	.063 765 073 120 378 608	.003 963 579 569 398 336	.000 447 732 469 653 596
.16	1.261 758 976 062 394 196	.059 801 493 550 980 272	.003 515 847 099 744 740	.000 375 059 007 749 541
.17	1.201 957 482 511 413 924	.056 285 646 451 235 532	.003 140 788 091 995 199	.000 317 311 760 910 951
.18	1.145 671 836 060 178 393	.053 144 858 359 240 333	.002 823 476 331 084 248	.000 270 845 208 527 120
.19	1.092 526 977 700 938 060	.050 321 382 028 156 085	.002 552 631 122 557 128	.000 233 032 694 222 349
.20	1.042 205 595 672 781 975	.047 768 750 905 598 956	.002 319 598 428 334 780	.000 201 948 944 446 945
.21	0.994 436 844 767 183 019	.045 449 152 477 264 177	.002 117 649 483 887 835	.000 176 160 845 749 172
.22	0.948 987 692 289 918 843	.043 331 502 993 376 342	.001 941 488 638 138 663	.000 154 586 643 260 153
.23	0.905 656 189 296 542 501	.041 390 014 355 237 679	.001 786 901 994 878 510	.000 136 399 373 475 365
.24	0.864 266 174 941 304 822	.039 603 112 360 359 169	.001 650 502 621 403 145	.000 120 959 486 540 614
.25	0.824 663 062 580 945 652	.037 952 609 738 956 024	.001 529 543 134 862 531	.000 107 767 092 340 306
.26	0.786 710 452 841 989 629	.036 423 066 604 093 493	.001 421 776 042 522 225	.000 096 427 625 744 883
.27	0.750 287 386 237 896 135	.035 001 290 561 571 269	.001 325 348 416 777 342	.000 086 626 832 352 608
.28	0.715 286 095 676 324 866	.033 675 942 144 793 927	.001 238 721 584 424 733	.000 078 112 321 492 205
.29	0.681 610 153 531 530 939	.032 437 220 560 369 194	.001 160 609 262 932 529	.000 070 679 808 267 854
.30	0.649 172 932 971 161 745	.031 276 611 297 436 665	.001 089 929 454 664 675	.000 064 162 744 941 555
.31	0.617 896 321 673 725 080	.030 186 681 842 771 991	.001 025 766 709 723 119	.000 058 424 430 296 270
.32	0.587 709 639 830 953 089	.029 160 915 133 048 871	.000 967 342 279 426 849	.000 053 351 950 025 087
.33	0.558 548 724 697 904 217	.028 193 572 853 622 022	.000 913 990 329 401 763	.000 048 851 483 588 263
.34	0.530 355 151 844 282 195	.027 279 582 524 220 260	.000 865 138 845 813 500	.000 044 844 640 358 638
.35	0.503 075 569 320 061 936	.026 414 443 678 406 760	.000 820 294 205 454 862	.000 041 265 577 858 653
.36	0.476 661 125 641 655 175	.025 594 149 472 951 838	.000 779 028 627 596 209	.000 038 058 719 145 214
.37	0.451 066 976 168 703 278	.024 815 120 845 355 689	.000 740 969 908 450 995	.000 035 176 932 744 862
.38	0.426 251 855 323 347 588	.024 074 150 936 904 695	.000 705 792 975 706 133	.000 032 580 072 291 218
.39	0.402 177 704 336 442 894	.023 368 357 961 198 562	.000 673 212 903 414 915	.000 030 233 797 814 381
.40	0.378 809 346 425 244 332	.022 695 145 057 783 646	.000 642 979 105 609 535	.000 028 108 619 007 410
.41	0.356 114 201 367 460 686	.022 052 165 952 183 112	.000 614 870 486 593 125	.000 026 179 114 520 912
.42	0.334 062 035 415 277 574	.021 437 295 465 589 987	.000 588 691 372 072 213	.000 024 423 291 667 800
.43	0.312 624 739 949 687 587	.020 848 604 093 517 774	.000 564 268 080 404 413	.000 022 822 058 752 324
.44	0.291 776 135 856 169 812	.020 284 336 013 113 361	.000 541 446 021 652 089	.000 021 358 788 215 773
.45	0.271 491 799 843 056 452	.019 742 889 991 461 271	.000 520 087 233 436 316	.000 020 018 953 384 443
.46	0.251 748 909 851 595 180	.019 222 802 758 024 955	.000 500 068 280 051 873	.000 018 789 825 156 349
.47	0.232 526 107 093 570 225	.018 722 734 477 973 082	.000 481 278 454 895 524	.000 017 660 217 724 651
.48	0.213 803 372 615 597 143	.018 241 456 023 077 558	.000 463 618 237 170 873	.000 016 620 274 595 416
.49	0.195 561 916 592 519 586	.017 777 837 785 906 685	.000 446 997 962 575 458	.000 015 661 287 855 561
.50	-0.177 784 078 806 612 901	+0.017 330 839 823 331 227	-0.000 431 336 674 719 896	+0.000 014 775 544 989 064

TABLE II. (continued).

Values of the Cosine-integral from 0 to 1 at intervals of .01.

x .	$\text{Ci } x$.	Δ	Δ^2	Δ^3
.50	-0.177 784 078 806 612 901	+0.17 330 839 823 331 227	-0.000 431 336 674 719 896	+0.000 014 775 544 989 064
.51	.160 453 238 983 281 674	16 899 503 148 611 331	416 561 129 730 832	13 956 198 605 652
.52	.143 553 735 834 670 344	16 482 942 018 880 499	402 604 931 125 180	13 197 155 297 124
.53	.127 070 793 815 789 845	16 080 337 087 755 318	389 407 775 828 056	12 492 980 518 590
.54	.110 990 456 728 034 527	15 690 929 311 927 262	376 914 795 309 466	11 838 816 941 275
.55	.095 299 527 416 107 265	15 314 014 516 617 796	365 075 978 368 191	11 230 314 167 701
.56	.079 985 512 899 489 469	14 948 938 538 249 605	353 845 664 200 490	10 663 568 060 746
.57	.065 036 574 361 239 864	14 595 092 874 049 115	343 182 096 139 744	10 135 068 231 990
.58	.050 441 481 487 190 749	14 251 910 777 909 371	333 047 027 907 755	09 641 652 475 284
.59	.036 189 570 709 281 378	13 918 863 750 001 616	323 405 375 432 471	09 180 467 128 916
.60	.022 270 706 959 279 763	13 595 458 374 569 145	314 224 908 303 555	08 748 932 512 471
.61	-.008 675 248 584 710 618	13 281 233 466 265 590	305 475 975 791 084	08 344 712 718 958
.62	+0.004 605 984 881 554 972	12 975 757 490 474 506	297 131 263 072 126	07 965 689 154 386
.63	.017 581 742 372 029 478	12 678 626 227 402 380	289 165 573 917 741	07 609 937 309 740
.64	.030 260 368 599 431 858	12 389 460 653 484 639	281 555 636 608 000	07 275 706 327 822
.65	.042 649 829 252 916 497	12 107 905 016 876 639	274 279 930 280 178	06 961 400 992 214
.66	.054 757 734 269 793 136	11 833 625 086 596 461	267 318 529 287 964	06 665 565 820 091
.67	.066 591 359 356 389 596	11 566 306 557 308 496	260 652 963 467 874	06 386 870 986 369
.68	.078 157 665 913 698 093	11 305 653 593 840 623	254 266 092 481 505	06 124 099 845 379
.69	.089 463 319 507 538 715	11 051 387 501 359 118	248 141 992 636 126	05 876 137 848 932
.70	.100 514 707 008 897 833	10 803 245 508 722 992	242 265 854 787 194	05 641 962 687 398
.71	.111 317 952 517 620 825	10 560 979 653 935 798	236 623 892 099 796	05 420 635 504 037
.72	.121 878 932 171 556 622	10 324 355 761 836 002	231 203 256 595 758	05 211 293 052 883
.73	.132 203 287 933 392 625	10 093 152 505 240 244	225 991 963 542 875	05 013 140 687 697
.74	.142 296 440 438 632 868	09 867 160 541 697 368	220 978 822 855 178	04 825 446 084 212
.75	.152 163 600 980 330 237	09 646 181 718 842 190	216 153 376 770 966	04 647 533 610 477
.76	.161 809 782 699 172 427	09 430 028 342 071 224	211 505 843 160 490	04 478 779 271 008
.77	.171 239 811 041 243 651	09 218 522 498 910 734	207 027 063 889 481	04 318 606 159 767
.78	.180 458 333 540 154 385	09 011 495 435 021 253	202 708 457 729 714	04 166 480 365 071
.79	.189 469 828 975 175 638	08 808 786 977 291 539	198 541 977 364 643	04 021 907 276 523
.80	.198 278 615 952 467 177	08 610 244 999 926 896	194 520 070 088 120	03 884 428 250 105
.81	.206 888 860 952 394 073	08 415 724 929 838 776	190 635 641 838 016	03 753 617 592 816
.82	.215 304 585 882 232 849	08 225 089 288 000 760	186 882 024 245 200	03 629 079 832 820
.83	.223 529 675 170 233 609	08 038 207 263 755 560	183 252 944 412 379	03 510 447 245 054
.84	.231 567 882 433 989 169	07 854 954 319 343 181	179 742 497 167 326	03 397 377 605 691
.85	.239 422 836 753 332 350	07 675 211 822 175 855	176 345 119 561 634	03 289 552 151 954
.86	.247 098 048 575 508 205	07 498 866 702 614 221	173 055 567 409 680	03 186 673 726 348
.87	.254 596 915 278 122 425	07 325 811 135 204 540	169 868 893 683 332	03 088 465 086 809
.88	.261 922 726 413 326 966	07 155 942 241 521 209	166 780 428 596 523	02 994 667 366 229
.89	.269 078 668 654 848 175	06 989 161 812 924 686	163 785 761 230 294	02 905 038 666 700
.90	.276 067 830 467 772 860	06 825 376 051 694 392	160 880 722 563 594	02 819 352 775 327
.91	.282 893 206 519 467 252	06 664 495 329 130 798	158 061 369 788 267	02 737 397 989 950
.92	.289 557 701 848 598 049	06 506 433 959 342 531	155 323 971 798 317	02 658 976 044 266
.93	.296 064 135 807 940 580	06 351 109 987 544 214	152 664 995 754 051	02 583 901 123 017
.94	.302 415 245 795 484 794	06 198 444 991 790 162	150 081 094 631 034	02 511 998 958 817
.95	.308 613 690 787 274 956	06 048 363 897 159 128	147 569 095 672 217	02 443 106 003 102
.96	.314 662 054 684 434 084	05 900 794 801 486 911	145 125 989 669 116	02 377 068 664 402
.97	.320 562 849 485 920 995	05 755 668 811 817 795	142 748 921 004 714	+0.000 002 313 742 607 896
.98	.326 318 518 297 738 790	05 612 919 890 813 081	-0.000 140 435 178 396 818	
.99	.331 931 438 188 551 871	+0.005 472 484 712 416 263		
1.00	+0.337 403 922 900 968 135			

TABLE III.

Values of the Exponential-integral from 0 to 1 at intervals of .01.

x .	$\text{Ei}x$.	Δ	Δ^2	Δ^3
-00	$-\infty$			
-01	-4.017 929 465 426 669 387	+703 222 571 016 515 484	-287 631 400 546 641 026	+169 899 376 444 219 825
-02	3.314 706 894 410 153 902	415 591 179 469 874 458	117 732 024 102 421 201	053 244 856 728 419 607
-03	2.899 115 723 940 279 444	297 859 146 367 453 257	064 487 167 374 001 594	023 716 871 408 300 820
-04	2.601 256 577 572 826 187	233 371 978 993 451 663	040 770 295 965 700 774	012 651 464 946 686 004
-05	2.367 884 598 579 374 524	192 601 683 027 750 888	028 118 831 019 014 771	007 551 939 780 169 192
-06	2.175 282 915 551 623 636	164 482 852 008 736 117	020 566 891 238 845 579	004 871 282 895 000 827
-07	2.010 800 063 542 887 518	143 915 960 769 890 539	015 695 608 343 844 752	003 326 192 295 473 483
-08	1.866 884 102 772 996 980	128 220 352 426 045 787	012 369 416 048 371 269	002 372 512 157 804 231
-09	1.738 663 750 346 951 193	115 850 936 377 674 518	009 996 873 890 567 038	001 751 893 760 561 864
-10	1.622 812 813 969 276 675	105 854 062 487 107 480	008 244 980 130 005 174	001 330 496 950 195 812
-11	1.516 958 751 482 169 195	097 609 082 357 102 306	006 914 483 179 809 362	001 034 300 000 726 105
-12	1.419 349 669 125 066 889	090 694 599 177 292 944	005 880 183 179 083 257	000 820 003 832 044 144
-13	1.328 655 069 947 773 945	084 814 415 998 209 687	005 060 179 347 039 113	000 661 122 093 428 678
-14	1.243 840 653 949 564 258	079 754 236 651 170 574	004 399 057 253 610 435	000 540 825 623 894 734
-15	1.164 086 417 298 393 684	075 355 179 397 560 139	003 858 231 629 715 700	000 448 068 782 676 449
-16	1.088 731 237 900 833 545	071 496 947 767 844 439	003 410 162 847 039 252	000 375 395 710 169 824
-17	1.017 234 290 132 989 106	068 086 784 920 805 187	003 034 767 136 869 428	000 317 648 878 781 511
-18	0.949 147 505 212 183 919	065 052 017 783 935 760	002 717 118 258 087 916	000 271 182 768 259 970
-19	0.884 095 487 428 248 159	062 334 899 525 847 843	002 445 935 489 827 947	000 233 370 722 590 115
-20	0.821 760 587 902 400 316	059 888 964 036 019 897	002 212 564 767 237 832	000 202 287 468 584 321
-21	0.761 871 623 866 380 419	057 676 399 268 782 065	002 010 277 298 653 511	000 176 499 893 154 373
-22	0.704 195 224 597 598 354	055 666 121 970 128 553	001 833 777 405 499 139	000 154 926 241 796 379
-23	0.648 529 102 627 469 801	053 832 344 564 629 415	001 678 851 163 702 760	000 136 739 551 372 245
-24	0.594 696 758 062 840 385	052 153 493 400 926 655	001 542 111 612 330 515	000 121 300 272 395 693
-25	0.542 543 264 661 913 730	050 611 381 788 596 140	001 420 811 339 934 822	000 108 108 515 120 495
-26	0.491 931 882 873 317 589	049 190 570 448 661 319	001 312 702 824 814 327	000 096 769 714 787 934
-27	0.442 741 312 424 656 270	047 877 867 623 846 992	001 215 933 110 026 393	000 086 969 617 368 590
-28	0.394 863 444 800 809 278	046 661 934 513 820 599	001 128 963 492 657 804	000 078 455 832 564 978
-29	0.348 201 510 286 988 679	045 532 971 021 162 795	001 050 507 660 092 825	000 071 024 075 856 537
-30	0.302 668 539 265 825 884	044 482 463 361 069 970	000 979 483 584 236 269	000 064 507 799 882 085
-31	0.258 186 075 904 755 915	043 502 979 776 833 701	000 914 975 784 354 183	000 058 770 303 802 776
-32	0.214 683 096 127 922 214	042 588 003 992 479 517	000 856 205 480 551 408	000 053 698 673 691 462
-33	0.172 095 092 135 442 697	041 731 798 511 928 110	000 802 506 806 859 945	000 049 199 089 389 648
-34	0.130 363 293 623 514 587	040 929 291 705 068 165	000 753 307 717 470 297	000 045 193 160 652 919
-35	0.089 434 001 918 446 422	040 175 983 987 597 867	000 708 114 556 817 278	000 041 615 045 387 977
-36	0.049 258 017 930 848 555	039 467 869 430 780 489	000 666 499 511 429 402	000 038 409 167 037 497
-37	-0.009 790 148 500 068 066	038 801 369 919 351 087	000 628 090 344 391 905	000 035 528 394 515 312
-38	+0.029 011 221 419 283 021	038 173 279 574 959 182	000 592 561 949 876 593	000 032 932 581 843 860
-39	0.067 184 500 994 242 204	037 580 717 625 082 589	000 559 629 368 032 733	000 030 587 389 443 588
-40	0.104 765 218 619 324 793	037 021 088 257 049 856	000 529 041 978 589 145	000 028 463 327 399 437
-41	0.141 786 306 876 374 650	036 492 046 278 460 711	000 500 578 651 189 708	000 026 534 974 755 443
-42	0.178 278 353 154 835 360	035 991 467 627 271 003	000 474 043 676 434 266	000 024 780 839 219 494
-43	0.214 269 820 782 106 363	035 517 423 950 836 737	000 449 263 337 214 772	000 023 180 329 492 372
-44	0.249 787 244 732 943 109	035 068 160 613 621 966	000 426 083 007 722 400	000 021 718 318 413 462
-45	0.284 855 405 346 565 066	034 642 077 605 899 566	000 404 364 689 308 938	000 020 379 779 708 722
-46	0.319 497 482 952 464 632	034 237 712 916 590 628	000 383 984 909 600 216	000 019 151 984 677 409
-47	0.353 735 195 869 055 260	033 853 728 606 990 412	000 364 832 924 922 807	000 018 023 747 915 507
-48	0.387 558 923 876 045 672	033 488 895 082 067 604	000 346 809 177 007 300	000 016 985 213 333 501
-49	0.421 077 818 958 113 276	033 142 085 905 060 304	000 329 823 963 673 799	000 016 027 673 424 323
-50	+0.454 219 904 863 173 580	+032 812 261 941 386 505	-000 313 796 290 249 476	+000 015 143 416 079 574

TABLE III. (continued).

Values of the Exponential-integral from 0 to 1 at intervals of .01.

x .	$Ei\ x$.	Δ	Δ^2	Δ^3
.50	+0.454 219 904 863 173 580	+0.032 812 261 941 386 505	—0.000 313 796 290 249 476	+0.000 015 143 416 079 574
.51	0.487 032 166 804 560 085	32 498 465 651 137 028	298 652 874 169 903	14 325 594 318 221
.52	0.519 530 632 455 697 113	32 199 812 776 967 126	284 327 279 851 682	13 568 115 142 926
.53	0.551 730 445 232 664 239	31 915 485 497 115 444	270 759 164 708 756	12 865 544 421 295
.54	0.583 645 930 729 779 683	31 644 726 332 406 688	257 893 620 287 461	12 213 025 238 691
.55	0.615 290 657 062 186 371	31 386 832 712 119 227	245 680 595 048 770	11 606 207 613 426
.56	0.646 677 489 774 305 598	31 141 152 117 070 458	234 074 387 435 344	11 041 187 825 826
.57	0.677 818 641 891 376 055	30 907 077 729 635 114	223 033 199 609 517	10 514 455 906 593
.58	0.708 725 719 621 011 169	30 684 044 530 025 596	212 518 743 702 925	10 022 850 070 374
.59	0.739 409 764 151 036 766	30 471 525 786 322 671	202 495 893 632 551	09 563 517 077 949
.60	0.769 881 289 937 359 437	30 269 029 892 690 120	192 932 376 554 602	09 133 877 673 091
.61	0.800 150 319 830 049 557	30 076 097 516 135 518	183 798 498 881 512	08 731 596 374 710
.62	0.830 226 417 346 185 075	29 892 299 017 254 006	175 066 902 506 802	08 354 555 016 438
.63	0.860 118 716 363 439 081	29 717 232 114 747 205	166 712 347 490 363	08 000 829 518 616
.64	0.889 835 948 478 186 286	29 550 519 767 256 841	158 711 517 971 748	07 668 669 455 143
.65	0.919 386 468 245 443 127	29 391 808 249 285 094	151 042 848 516 605	07 356 480 042 458
.66	0.948 778 276 494 728 220	29 240 765 400 768 489	143 686 368 474 146	07 062 806 232 359
.67	0.978 019 041 895 496 709	29 097 079 032 294 342	136 623 562 241 787	06 786 318 636 166
.68	1.007 116 120 927 791 051	28 960 455 470 052 555	129 837 243 605 621	06 525 801 046 404
.69	1.036 076 576 397 843 607	28 830 618 226 446 934	123 311 442 559 217	06 280 139 354 884
.70	1.064 907 194 624 290 541	28 707 306 783 887 717	117 031 303 204 332	06 048 311 693 796
.71	1.093 614 501 408 178 258	28 590 275 480 683 385	110 982 991 510 537	05 829 379 650 047
.72	1.122 204 776 888 861 642	28 479 292 489 172 848	105 153 611 860 489	05 622 480 423 165
.73	1.150 684 069 378 034 490	28 374 138 877 312 358	099 531 131 437 324	05 426 819 814 266
.74	1.179 058 208 255 346 848	28 274 607 745 875 034	094 104 311 623 059	05 241 665 948 303
.75	1.207 332 816 091 221 883	28 180 503 434 251 976	088 862 645 674 756	05 066 343 644 439
.76	1.235 513 319 435 473 858	28 091 640 788 577 220	083 796 302 030 316	04 900 229 360 204
.77	1.263 604 960 224 051 079	28 007 844 486 546 904	078 896 072 670 113	04 742 746 644 486
.78	1.291 612 804 710 597 983	27 928 948 413 876 791	074 153 326 025 627	04 593 362 042 462
.79	1.319 541 753 124 474 774	27 854 795 087 851 164	069 559 963 983 165	04 451 581 402 545
.80	1.347 396 548 212 325 938	27 785 235 123 867 999	065 108 382 580 620	04 316 946 541 485
.81	1.375 181 783 336 193 938	27 720 126 741 287 379	060 791 436 039 136	04 189 032 229 030
.82	1.402 901 910 077 481 317	27 659 335 305 248 244	056 602 403 810 106	04 067 443 458 089
.83	1.430 561 245 382 729 560	27 602 732 901 438 138	052 534 960 352 017	03 951 812 970 353
.84	1.458 163 978 284 167 699	27 550 197 941 086 122	048 583 147 381 664	03 841 799 010 783
.85	1.485 714 176 225 253 820	27 501 614 793 704 458	044 741 348 370 881	03 737 083 287 429
.86	1.513 215 791 018 958 278	27 456 873 445 333 577	041 094 265 083 452	03 637 369 115 694
.87	1.540 672 664 464 291 855	27 415 869 180 250 125	037 366 895 967 759	03 542 379 728 489
.88	1.568 088 533 644 541 980	27 378 502 284 282 366	033 824 516 239 270	03 451 856 735 785
.89	1.595 467 035 928 824 346	27 344 677 768 043 096	030 372 659 503 485	03 365 558 718 867
.90	1.622 811 713 696 867 441	27 314 305 108 539 610	027 007 100 784 619	03 283 259 946 176
.91	1.650 126 018 805 407 052	27 287 298 007 754 992	023 723 840 838 442	03 204 749 199 040
.92	1.677 413 316 813 162 043	27 263 574 166 916 550	020 519 091 639 402	03 129 828 696 825
.93	1.704 676 890 980 078 593	27 243 055 075 277 148	017 389 262 942 577	03 058 313 112 134
.94	1.731 919 946 055 355 741	27 225 665 812 334 571	014 330 949 830 443	02 990 028 667 662
.95	1.759 145 611 867 690 311	27 211 334 862 504 128	011 340 921 162 780	02 924 812 307 161
.96	1.786 356 946 730 194 439	27 199 993 941 341 348	008 416 108 855 619	02 862 510 933 738
.97	1.813 556 940 671 535 787	27 191 577 832 485 729	005 553 597 921 881	+0.000 002 802 980 709 425
.98	1.840 748 518 504 021 516	27 186 024 234 563 848	—0.000 002 750 617 212 456	
.99	1.867 934 542 738 585 363	+0.027 183 273 617 351 392		
1.00	+1.895 117 816 355 936 755			

TABLE IV.

Values of the Exponential-integral from 0 to 1 at intervals of .01.

x .	$Ei(-x)$.	Δ	Δ^2	Δ^3
.00	$-\infty$			
.01	-4.037 929 576 538 113 832	+683 221 793 228 404 301	-287 632 733 939 975 455	+169 898 709 647 547 372
.02	3.354 707 783 309 709 531	395 589 059 288 428 847	117 734 024 292 428 082	053 244 189 811 732 869
.03	2.959 118 724 021 280 684	277 855 034 996 000 764	064 489 834 480 695 214	023 716 204 331 581 697
.04	2.681 263 689 025 279 920	213 365 200 515 305 550	040 773 630 149 113 517	012 650 797 669 904 970
.05	2.467 898 488 509 974 370	172 591 570 366 192 034	028 122 832 479 208 546	007 551 272 263 282 429
.06	2.295 306 918 143 782 336	144 468 737 886 983 487	020 571 560 215 926 118	004 870 615 097 947 363
.07	2.150 838 180 256 798 849	123 897 177 671 057 370	015 700 945 117 978 754	003 325 524 178 172 337
.08	2.026 941 002 585 741 479	108 196 232 553 078 615	012 375 420 939 806 418	002 371 873 680 151 542
.09	1.918 744 770 032 662 864	095 820 811 613 272 198	010 003 547 259 654 875	001 751 224 882 428 031
.10	1.822 923 958 419 390 666	085 817 264 353 617 322	008 252 322 377 226 844	001 329 827 631 422 625
.11	1.737 106 694 065 773 344	077 564 941 976 390 478	006 922 494 745 804 219	001 033 630 201 123 876
.12	1.659 541 752 089 382 866	070 642 447 230 586 259	005 888 864 544 680 343	000 819 333 511 388 831
.13	1.588 899 304 858 796 606	064 753 582 685 905 916	005 069 531 033 291 512	000 660 451 211 459 020
.14	1.524 145 722 172 890 691	059 684 051 652 614 404	004 409 079 821 832 492	000 540 154 140 309 362
.15	1.464 461 670 520 276 287	055 274 971 830 781 912	003 868 925 681 523 131	000 447 396 657 131 013
.16	1.409 186 698 639 494 375	051 405 046 149 258 781	003 421 529 024 392 117	000 374 722 902 274 108
.17	1.357 780 652 540 235 594	047 984 517 124 866 664	003 046 806 122 118 010	000 316 975 348 096 544
.18	1.309 796 135 415 368 931	044 937 711 002 748 654	002 729 830 774 021 466	000 270 508 474 295 132
.19	1.264 858 424 412 620 276	042 207 880 228 727 188	002 459 322 299 726 333	000 232 695 624 800 250
.20	1.222 650 544 183 893 088	039 748 557 929 000 855	002 226 626 674 926 083	000 201 611 526 366 829
.21	1.182 901 986 254 892 234	037 521 931 254 074 772	002 025 015 148 559 254	000 175 823 065 846 310
.22	1.145 380 055 000 817 462	035 496 916 105 515 517	001 849 192 082 712 944	000 154 248 488 671 547
.23	1.109 883 138 895 301 945	033 647 724 022 802 573	001 694 943 594 041 397	000 136 060 831 638 280
.24	1.076 235 414 872 499 371	031 952 780 428 761 176	001 558 882 762 403 116	000 120 620 545 191 144
.25	1.044 282 634 443 738 195	030 393 897 666 358 060	001 438 262 217 211 973	000 107 427 739 511 901
.26	1.013 888 736 777 380 135	028 955 635 449 146 087	001 330 834 477 700 072	000 096 087 849 766 893
.27	0.984 933 101 328 234 048	027 624 800 971 446 015	001 234 746 627 933 179	000 086 286 621 848 827
.28	0.957 308 300 356 788 033	026 390 054 343 512 836	001 148 460 006 084 352	000 077 771 665 379 399
.29	0.930 918 246 013 275 197	025 241 594 337 428 484	001 070 688 340 704 953	000 070 338 695 754 303
.30	0.905 676 651 675 846 712	024 170 905 996 723 531	001 000 349 644 950 650	000 063 821 165 525 578
.31	0.881 505 745 679 123 181	023 170 556 351 772 882	000 936 528 479 425 072	000 058 082 373 764 756
.32	0.858 335 189 327 350 299	022 234 027 872 347 810	000 878 446 105 660 316	000 053 009 406 452 021
.33	0.836 101 161 455 002 489	021 355 581 766 687 494	000 825 436 699 208 295	000 048 508 443 333 252
.34	0.814 745 579 688 314 995	020 530 145 067 479 200	000 776 928 255 875 042	000 044 501 094 065 429
.35	0.794 215 434 620 835 795	019 753 216 811 604 158	000 732 427 161 809 613	000 040 921 516 453 654
.36	0.774 462 217 809 231 637	019 020 789 649 794 545	000 691 505 645 355 959	000 037 714 133 836 006
.37	0.755 441 428 159 437 092	018 329 284 004 438 586	000 653 791 511 519 953	000 034 831 815 018 716
.38	0.737 112 144 154 998 507	017 675 492 492 918 632	000 618 959 696 501 237	000 032 234 413 913 604
.39	0.719 436 651 662 079 874	017 056 532 796 417 396	000 586 725 282 587 633	000 029 887 590 827 474
.40	0.702 380 118 865 662 479	016 469 807 513 829 763	000 556 837 691 760 159	000 027 761 855 728 595
.41	0.685 910 311 351 832 716	015 912 969 822 069 604	000 529 075 836 031 564	000 025 831 787 541 285
.42	0.669 997 341 529 763 112	015 383 893 986 038 040	000 503 244 048 490 279	000 024 075 393 850 669
.43	0.654 613 447 543 725 072	014 880 649 937 547 761	000 479 168 654 639 610	000 022 473 583 231 700
.44	0.639 732 797 606 177 312	014 401 481 282 908 150	000 456 695 071 407 910	000 021 009 728 394 867
.45	0.625 331 316 323 269 161	013 944 786 211 500 240	000 435 685 343 013 043	000 019 669 302 934 150
.46	0.611 386 530 111 768 921	013 509 100 868 487 197	000 416 016 040 078 893	000 018 439 578 013 741
.47	0.597 877 429 243 281 723	013 093 084 828 408 305	000 397 576 462 065 152	000 017 309 368 091 453
.48	0.584 784 344 414 873 419	012 695 508 366 343 153	000 380 267 093 973 699	000 016 268 816 936 490
.49	0.572 088 836 048 530 266	012 315 241 272 369 454	000 363 998 277 037 209	000 015 309 216 897 384
.50	-0.559 773 594 776 160 812	+011 951 242 995 332 245	-000 348 689 060 139 825	+000 014 422 855 718 195

TABLE IV. (continued).

Values of the Exponential-integral from 0 to 1 at intervals of .01.

x .	$Ei(-x)$.	Δ	Δ^2	Δ^3
.50	-0.559 773 594 776 160 812	+0.11 951 242 995 332 245	-0.000 348 689 060 139 825	+0.000 014 422 855 718 195
.51	547 822 351 780 828 566	11 602 553 935 192 420	334 266 204 421 630	13 602 886 267 210
.52	536 219 797 845 636 146	11 268 287 730 770 790	320 663 318 154 421	12 843 215 393 249
.53	524 951 510 114 865 356	10 947 624 412 616 369	307 820 102 761 171	12 138 408 806 911
.54	514 003 885 702 248 987	10 639 804 309 855 198	295 681 693 954 260	11 483 609 433 366
.55	503 364 081 392 393 789	10 344 122 615 900 938	284 198 084 520 894	10 874 467 127 542
.56	493 019 958 776 492 851	10 059 924 531 380 044	273 323 617 393 351	10 307 078 003 175
.57	482 960 034 245 112 807	09 786 600 913 986 693	263 016 539 390 177	09 777 931 921 152
.58	473 173 432 331 126 115	09 523 584 374 596 516	253 238 607 469 025	09 283 866 923 079
.59	463 649 848 956 529 599	09 270 345 767 127 491	243 954 740 545 946	08 822 029 593 446
.60	454 379 503 189 402 108	09 026 391 026 581 545	235 132 710 952 500	08 389 840 496 472
.61	445 353 112 162 820 563	08 791 258 315 629 045	226 742 870 456 028	07 984 963 968 245
.62	436 561 853 847 191 518	08 564 515 445 173 017	218 757 906 487 783	07 605 281 656 282
.63	427 997 338 402 018 501	08 345 757 538 685 234	211 152 624 831 501	07 248 869 291 509
.64	419 651 580 863 333 267	08 134 604 913 853 733	203 903 755 539 992	06 913 976 255 089
.65	411 516 975 949 479 534	07 930 701 158 313 741	196 989 779 284 903	06 599 007 567 397
.66	403 586 274 791 165 793	07 733 711 379 028 838	190 390 771 717 505	06 302 507 980 815
.67	395 852 563 412 136 955	07 543 320 607 311 333	184 088 263 736 691	06 023 147 903 883
.68	388 309 242 804 825 622	07 359 232 343 574 642	178 065 115 832 807	05 759 710 922 972
.69	380 950 010 461 250 979	07 181 167 227 741 835	172 305 404 909 835	05 511 082 720 336
.70	373 768 843 233 509 144	07 008 861 822 832 000	166 794 322 189 499	05 276 241 215 202
.71	366 759 981 410 677 144	06 842 067 500 642 501	161 518 080 974 298	05 054 247 778 083
.72	359 917 913 910 034 644	06 680 549 419 668 203	156 463 833 196 214	04 844 239 388 669
.73	353 237 364 490 366 441	06 524 085 586 471 989	151 619 593 807 545	04 645 421 624 773
.74	346 713 278 903 894 452	06 372 465 992 664 444	146 974 172 182 773	04 457 062 384 569
.75	340 340 812 911 230 008	06 225 491 820 481 671	142 517 109 798 204	04 278 486 256 939
.76	334 115 321 090 748 336	06 082 974 710 683 467	138 238 623 541 265	04 109 069 465 619
.77	328 032 346 380 064 869	05 944 736 087 142 202	134 129 554 075 647	03 948 235 322 169
.78	322 087 610 292 922 667	05 810 606 533 066 555	130 181 318 753 478	03 795 450 130 887
.79	316 277 003 759 856 112	05 680 425 214 313 077	126 385 868 622 591	03 650 219 495 731
.80	310 596 578 545 543 035	05 554 039 345 630 486	122 735 649 126 860	03 512 084 985 411
.81	305 042 539 199 852 548	05 431 303 696 563 626	119 223 564 141 449	03 380 621 118 022
.82	299 611 235 503 288 922	05 312 080 132 422 176	115 842 943 023 428	03 255 432 631 193
.83	294 299 155 370 866 746	05 196 237 189 398 748	112 587 510 392 235	03 136 152 007 684
.84	289 102 918 181 467 998	05 083 649 679 006 514	109 451 358 381 551	03 022 437 229 855
.85	284 019 268 502 461 484	04 974 198 320 621 963	106 428 921 154 696	02 913 969 739 468
.86	279 045 070 181 839 521	04 867 769 399 467 267	103 514 951 415 228	02 810 452 581 925
.87	274 177 300 782 372 254	04 764 254 448 052 039	100 704 498 833 302	02 711 608 716 403
.88	269 413 046 334 320 215	04 663 549 949 218 737	097 992 890 116 899	02 617 179 475 390
.89	264 749 496 385 101 478	04 565 557 059 101 838	095 375 710 641 509	02 526 923 158 907
.90	260 183 939 325 999 641	04 470 181 348 460 328	092 848 787 482 603	02 440 613 750 339
.91	255 713 757 977 539 312	04 377 332 560 977 725	090 408 173 732 264	02 358 039 742 136
.92	251 336 425 416 561 587	04 286 924 387 245 461	088 050 133 990 129	02 279 003 060 944
.93	247 049 501 029 316 126	04 198 874 253 255 333	085 771 130 929 185	02 203 318 082 785
.94	242 850 626 776 060 793	04 113 103 122 326 148	083 567 812 846 400	02 130 810 729 881
.95	238 737 523 653 734 645	04 029 535 309 479 748	081 437 002 116 519	02 061 317 641 600
.96	234 707 988 344 254 897	03 948 098 307 363 229	079 375 684 474 919	01 994 685 412 731
.97	230 759 890 036 891 668	03 868 722 622 888 310	077 380 999 062 188	+0.000 001 930 769 893 030
.98	226 891 167 414 003 358	03 791 341 623 826 121	-0.000 075 450 229 169 158	
.99	223 099 825 790 177 237	+0.003 715 891 394 656 963		
1.00	-0.219 383 934 395 520 274			

TABLE V.

Values of the Sine-integral from 1 to 5 at intervals of 0.1.

x .	$\text{Si } x$.	Δ	Δ^2	Δ^3
1.0	+0.946 083 070 37	+0.082 602 148 31	−.003 240 167 97	−.000 211 002 31
1.1	1.028 685 218 67	79 361 980 34	3 451 170 28	190 916 06
1.2	1.108 047 199 01	75 910 810 06	3 642 086 34	169 839 01
1.3	1.183 958 009 08	72 268 723 72	3 811 925 35	147 918 33
1.4	1.256 226 732 80	68 456 798 37	3 959 843 68	125 307 21
1.5	1.324 683 531 17	64 496 954 70	4 085 150 88	102 162 36
1.6	1.389 180 485 87	60 411 803 81	4 187 313 24	078 643 74
1.7	1.449 592 289 68	56 224 490 57	4 265 957 08	054 913 36
1.8	1.505 816 780 26	51 958 533 49	4 320 870 44	031 133 17
1.9	1.557 775 313 75	47 637 663 05	4 352 003 61	−.000 007 464 83
2.0	1.605 412 976 80	43 285 659 44	4 359 468 45	+0.000 015 932 01
2.1	1.648 698 636 24	38 926 191 00	4 343 536 43	038 900 83
2.2	1.687 624 827 24	34 582 654 56	4 304 635 61	061 289 33
2.3	1.722 207 481 81	30 278 018 96	4 243 346 28	082 950 68
2.4	1.752 485 500 76	26 034 672 68	4 160 395 60	103 744 46
2.5	1.778 520 173 44	21 874 277 09	4 056 651 14	123 537 81
2.6	1.800 394 450 53	17 817 625 95	3 933 113 33	142 206 29
2.7	1.818 212 076 47	13 884 512 61	3 790 907 05	159 634 84
2.8	1.832 096 589 08	10 093 605 56	3 631 272 21	175 718 53
2.9	1.842 190 194 65	06 462 333 35	3 455 553 68	190 363 35
3.0	1.848 652 528 00	+0.003 006 779 67	3 265 190 33	203 486 76
3.1	1.851 659 307 67	−.000 258 410 65	3 061 703 57	215 018 33
3.2	1.851 400 897 02	03 320 114 22	2 846 685 25	224 900 05
3.3	1.848 080 782 80	06 166 799 47	2 621 785 19	233 086 83
3.4	1.841 913 983 33	08 788 584 66	2 388 698 36	239 546 60
3.5	1.833 125 398 67	11 177 283 02	2 149 151 75	244 260 50
3.6	1.821 948 115 65	13 326 434 77	1 904 891 26	247 222 88
3.7	1.808 621 680 88	15 231 326 03	1 657 668 38	248 441 28
3.8	1.792 390 354 85	16 888 994 40	1 409 227 10	247 936 14
3.9	1.776 501 360 45	18 298 221 50	1 161 290 96	245 740 62
4.0	1.758 203 138 95	19 459 512 46	0 915 550 34	241 900 13
4.1	1.738 743 626 49	20 375 062 80	0 673 650 21	236 471 94
4.2	1.718 368 563 69	21 048 713 01	0 437 178 27	229 524 51
4.3	1.697 319 850 68	21 485 891 28	−.000 207 653 75	221 136 90
4.4	1.675 833 959 41	21 693 545 03	+0.000 013 483 15	211 398 04
4.5	1.654 140 414 38	21 680 061 88	224 881 19	200 405 87
4.6	1.632 460 352 50	21 455 180 69	425 287 06	188 266 51
4.7	1.611 005 171 81	21 029 893 64	613 553 56	+0.000 175 093 36
4.8	1.589 975 278 17	20 416 340 07	+0.000 788 646 92	
4.9	1.569 558 938 10	−.019 627 693 15		
5.0	+1.549 931 244 94			

TABLE VI.

Values of the Cosine-integral from 1 to 5 at intervals of 0.1.

x .	$\text{Ci } x$.	Δ	Δ^2	Δ^3
1.0	+0.337 403 922 90	+0.047 469 454 52	—0.011 883 649 05	+0.001 577 228 22
1.1	384 873 377 42	35 585 805 47	10 306 420 83	1 295 053 76
1.2	420 459 182 89	25 279 384 64	09 011 367 07	1 093 081 60
1.3	445 738 567 53	16 268 017 57	07 918 285 47	0 944 753 11
1.4	462 006 585 10	08 349 732 10	06 973 532 36	0 833 179 87
1.5	470 356 317 19	+0.001 376 199 74	06 140 352 50	0 747 270 49
1.6	471 732 516 93	—0.004 764 152 76	05 393 082 00	0 679 537 03
1.7	466 968 364 18	10 157 234 76	04 713 544 98	0 624 803 80
1.8	456 811 129 42	14 870 779 74	04 088 741 17	0 579 421 15
1.9	441 940 349 68	18 959 520 91	03 509 320 02	0 540 772 16
2.0	422 980 828 77	22 468 840 93	02 968 547 86	0 506 955 15
2.1	400 511 987 84	25 437 388 79	02 461 592 72	0 476 574 11
2.2	375 074 599 05	27 898 981 51	01 985 018 60	0 448 597 64
2.3	347 175 617 54	29 884 000 11	01 536 420 96	0 422 261 73
2.4	317 291 617 43	31 420 421 07	01 114 159 23	0 397 002 10
2.5	285 871 196 36	32 534 580 30	00 717 157 13	0 372 405 58
2.6	253 336 616 06	33 251 737 43	—0.000 344 751 55	0 348 175 05
2.7	220 084 878 63	33 596 488 99	+0.000 003 423 50	0 324 103 84
2.8	186 488 389 64	33 593 065 49	00 327 527 34	0 300 056 00
2.9	152 895 324 16	33 265 538 15	00 627 583 34	0 275 952 00
3.0	119 629 786 01	32 637 954 81	00 903 535 34	0 251 757 02
3.1	086 991 831 20	31 734 419 48	01 155 292 36	0 227 472 21
3.2	055 257 411 72	30 579 127 11	01 382 764 58	0 203 127 38
3.3	+0.024 678 284 61	29 196 362 54	01 585 891 95	0 178 775 28
3.4	—0.004 518 077 93	27 610 470 58	01 764 667 24	0 154 486 69
3.5	032 128 548 51	25 845 803 35	01 919 153 92	0 130 346 43
3.6	057 974 351 86	23 926 649 42	02 049 500 35	0 106 449 88
3.7	081 901 001 28	21 877 149 07	02 155 950 22	0 082 899 95
3.8	103 778 150 36	19 721 198 85	02 238 850 17	0 059 804 56
3.9	123 499 349 21	17 482 348 68	02 298 654 73	0 037 274 27
4.0	140 981 697 89	15 183 693 94	02 335 929 01	+0.000 015 420 19
4.1	156 165 391 83	12 847 764 94	02 351 349 19	—0.000 005 647 73
4.2	169 013 156 77	10 496 415 75	02 345 701 46	025 822 48
4.3	179 509 572 51	08 150 714 29	02 319 878 98	045 001 30
4.4	187 660 286 80	05 830 835 30	02 274 877 68	062 087 18
4.5	193 491 122 10	03 555 957 62	02 211 790 51	079 989 86
4.6	197 047 079 73	—0.001 344 167 12	02 131 800 65	095 626 89
4.7	198 391 246 84	+0.000 787 633 53	02 036 173 75	—0.000 109 924 67
4.8	197 603 613 31	02 823 807 29	+0.001 926 249 08	
4.9	194 779 806 02	+0.004 750 056 37		
5.0	—0.190 029 749 66			

TABLE VII.

Values of the Exponential-integral from 1 to 5 at intervals of 0.1.

x .	Ei x .	Δ	Δ^2	Δ^3
1.0	+ 1.89 511 781 636	+ .272 260 463 21	+ .002 453 542 42	+ .002 139 046 99
1.1	2.16 737 827 956	.274 714 005 63	.004 592 589 41	01 909 399 47
1.2	2.44 209 228 519	.279 306 595 04	.006 501 988 88	01 767 412 19
1.3	2.72 139 888 023	.285 808 583 92	.008 269 401 06	01 687 113 85
1.4	3.00 720 746 415	.294 077 984 98	.009 956 514 91	01 652 237 53
1.5	3.30 128 544 913	.304 034 499 89	.011 608 752 44	01 652 351 36
1.6	3.60 531 994 902	.315 643 252 33	.013 261 103 80	01 680 669 53
1.7	3.92 096 320 135	.328 904 356 13	.014 941 773 33	01 732 766 26
1.8	4.24 986 755 749	.343 846 129 46	.016 674 539 59	01 805 795 01
1.9	4.59 371 368 695	.360 520 669 05	.018 480 334 60	01 898 001 91
2.0	4.95 423 435 600	.379 001 003 65	.020 378 336 51	02 008 414 85
2.1	5.33 323 535 965	.399 379 340 16	.022 386 751 37	02 136 642 15
2.2	5.73 261 469 981	.421 766 091 52	.024 523 393 51	02 282 739 71
2.3	6.15 438 079 133	.446 289 485 02	.026 806 133 21	02 447 123 76
2.4	6.60 067 027 635	.473 095 618 23	.029 253 256 97	02 630 513 26
2.5	7.07 376 589 458	.502 348 875 21	.031 883 770 23	02 833 892 91
2.6	7.57 611 476 979	.534 232 645 44	.034 717 663 15	03 058 491 02
2.7	8.11 034 741 522	.568 950 308 59	.037 776 154 17	03 305 767 14
2.8	8.67 929 772 381	.606 726 462 75	.041 081 921 31	03 577 408 12
2.9	9.28 602 418 656	.647 808 384 06	.044 659 329 43	03 875 329 94
3.0	9.93 383 257 062	.692 467 713 49	.048 534 659 37	04 201 684 43
3.1	10.62 630 028 411	.741 002 372 86	.052 736 343 80	04 558 870 07
3.2	11.36 730 265 697	.793 738 716 65	.057 295 213 87	04 949 546 62
3.3	12.16 104 137 362	.851 033 930 52	.062 244 760 49	05 376 652 89
3.4	13.01 207 530 414	.913 278 691 01	.067 621 413 38	05 843 427 61
3.5	13.92 535 399 515	.980 900 104 39	.073 464 841 00	06 353 433 87
3.6	14.90 625 409 954	1.054 364 945 38	.079 818 274 86	06 910 586 18
3.7	15.96 061 904 492	1.134 183 220 24	.086 728 861 04	07 519 181 29
3.8	17.09 480 226 516	1.220 912 081 28	.094 248 042 33	08 183 932 32
3.9	18.31 571 434 644	1.315 160 123 61	.102 431 974 65	08 910 006 54
4.0	19.63 087 447 006	1.417 592 098 27	.111 341 981 19	09 703 067 21
4.1	21.04 846 656 832	1.528 934 079 46	.121 045 048 40	10 569 319 70
4.2	22.57 740 064 778	1.649 979 127 86	.131 614 368 10	11 515 562 27
4.3	24.22 737 977 564	1.781 593 495 96	.143 129 930 37	12 549 241 83
4.4	26.00 897 327 160	1.924 723 426 34	.155 679 172 20	13 678 515 39
4.5	27.93 369 669 794	2.080 402 598 54	.169 357 687 60	14 912 317 43
4.6	30.01 409 929 648	2.249 760 286 13	.184 270 005 03	16 260 433 83
4.7	32.26 385 958 261	2.434 030 291 16	.200 530 438 85	+ .017 733 583 15
4.8	34.69 788 987 377	2.634 560 730 01	+ .218 264 022 01	
4.9	37.33 245 060 378	+ 2.852 824 752 02		
5.0	+ 40.18 527 535 580			

TABLE VIII.

Values of the Exponential-integral from 1 to 5 at intervals of 0.1.

x .	$Ei(-x)$.	Δ	Δ^2	Δ^3
1.0	-0.219 383 934 40	+0.33 393 029 86	-0.005 810 562 18	+0.001 185 573 49
1.1	185 990 904 54	27 582 467 68	4 624 988 69	0 899 154 96
1.2	158 408 436 85	22 957 479 00	3 725 833 72	0 693 918 60
1.3	135 450 957 85	19 231 645 28	3 031 915 12	0 543 433 67
1.4	116 219 312 57	16 199 730 16	2 488 481 45	0 430 922 04
1.5	100 019 582 41	13 711 248 71	2 057 559 41	0 345 385 15
1.6	086 308 333 70	11 653 689 30	1 712 174 26	0 279 410 41
1.7	074 654 644 40	09 941 515 04	1 432 763 85	0 227 880 13
1.8	064 713 129 36	08 508 751 19	1 204 883 72	0 187 185 44
1.9	056 204 378 18	07 303 867 47	1 017 698 27	0 154 733 53
2.0	048 900 510 71	06 286 169 20	0 862 964 75	0 128 630 17
2.1	042 614 341 51	05 423 204 45	0 734 334 57	0 107 470 93
2.2	037 191 137 06	04 688 869 88	0 626 863 64	0 090 200 47
2.3	032 502 267 18	04 062 006 24	0 536 663 18	0 076 016 18
2.4	028 440 260 94	03 525 343 06	0 460 647 00	0 064 300 91
2.5	024 914 917 88	03 064 696 07	0 396 346 09	0 054 575 48
2.6	021 850 221 81	02 668 349 98	0 341 770 61	0 046 464 45
2.7	019 181 871 82	02 326 579 38	0 295 306 15	0 039 671 06
2.8	016 855 292 45	02 031 273 22	0 255 635 09	0 033 959 29
2.9	014 824 019 23	01 775 638 13	0 221 675 80	0 029 139 74
3.0	013 048 381 09	01 553 962 33	0 192 536 06	0 025 059 75
3.1	011 494 418 76	01 361 426 27	0 167 476 31	0 021 595 39
3.2	010 132 992 50	01 193 949 96	0 145 880 92	0 018 645 54
3.3	008 939 042 54	01 048 069 04	0 127 235 39	0 016 127 26
3.4	007 890 973 51	00 920 833 65	0 111 108 12	0 013 972 27
3.5	006 970 139 86	00 809 725 53	0 097 135 85	0 012 124 02
3.6	006 160 414 33	00 712 589 67	0 085 011 83	0 010 535 52
3.7	005 447 824 66	00 627 577 84	0 074 476 31	0 009 167 65
3.8	004 820 246 82	00 553 101 53	0 065 308 66	0 007 987 56
3.9	004 267 145 28	00 487 792 87	0 057 321 10	0 006 967 78
4.0	003 779 352 41	00 430 471 77	0 050 353 32	0 006 085 09
4.1	003 348 880 64	00 380 118 45	0 044 268 23	0 005 319 92
4.2	002 968 762 18	00 335 850 23	0 038 948 31	0 004 655 68
4.3	002 632 911 96	00 296 901 92	0 034 292 63	0 004 078 28
4.4	002 336 010 04	00 262 609 29	0 030 214 35	0 003 575 74
4.5	002 073 400 76	00 232 394 94	0 026 638 61	0 003 137 83
4.6	001 841 005 82	00 205 756 33	0 023 500 78	0 002 755 80
4.7	001 635 249 49	00 182 255 55	0 020 744 98	+0.000 002 422 18
4.8	001 452 993 94	00 161 510 57	-0.000 018 322 80	
4.9	001 291 483 36	+0.000 143 187 77		
5.0	-0.001 148 295 59			

TABLE IX.

Values of $\text{Si } x$, $\text{Ci } x$, $\text{Ei } x$, and $\text{Ei}(-x)$ from 6 to 15 at intervals of unity.

x .	$\text{Si } x$.	$\text{Ci } x$.	$\text{Ei } x$.	$\text{Ei}(-x)$.
6	+1.424 687 551 28	-0.068 057 243 89	+ 85.989 762 142 44	-0.000 360 082 45
7	1.454 596 614 25	+0.076 695 278 48	191.504 743 335 50	115 481 73
8	1.574 186 821 71	+0.122 433 882 53	440.379 899 534 84	037 665 62
9	1.665 040 075 83	+0.055 347 531 33	1037.878 290 717 09	012 447 35
10	1.658 347 594 22	-0.045 456 433 00	2492.228 976 241 88	004 156 97
11	1.578 306 806 95	-0.089 563 135 49	6071.406 374 098 61	001 400 30
12	1.504 971 241 53	-0.049 780 006 88	14959.532 666 397 53	000 475 11
13	1.499 361 722 87	+0.026 764 125 57	37197.688 490 689 03	000 162 19
14	1.556 211 050 08	+0.069 396 355 93	93192.513 633 965 37	000 055 66
15	+1.618 194 443 70	+0.046 278 677 67	+234955.852 490 768 32	-0.000 000 019 18

TABLE X.

Table of the Sine-integral and Cosine-integral for values of the argument above 20.

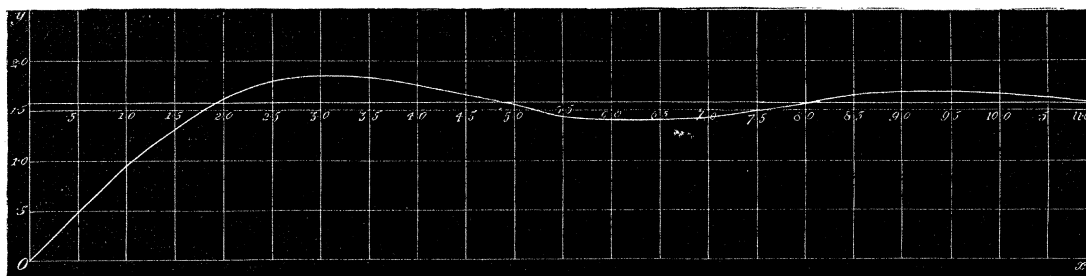
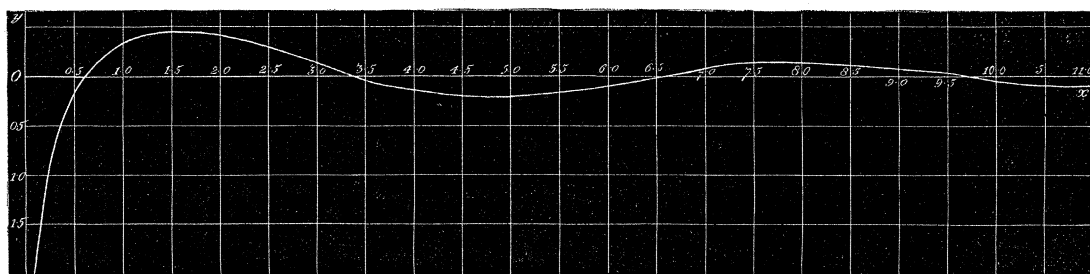
x .	$\text{Si } x$.	$\text{Ci } x$.	x .	$\text{Si } x$.	$\text{Ci } x$.
20	+1.54 824 17	+0.04 441 98	190	+1.57 041 97	+0.00 524 95
25	1.53 148 26	- 0 684 86	200	1.56 838 23	- 437 84
30	1.56 675 65	- 3 303 24	300	1.57 088 11	- 333 22
35	1.59 692 22	- 1 147 99	400	1.57 211 49	- 212 40
40	1.58 698 51	+ 1 902 00	500	1.57 256 59	- 093 20
45	1.55 871 50	+ 1 863 17	600	1.57 246 12	+ 007 64
50	1.55 161 71	- 0 562 84	700	1.57 199 39	+ 077 88
55	1.57 072 41	- 1 817 26	800	1.57 135 51	+ 111 82
60	1.58 674 56	- 0 481 32	900	1.57 072 15	+ 110 86
65	1.57 747 11	+ 1 284 74	1000	1.57 023 31	+ 082 63
70	1.56 159 49	+ 1 092 20	2000	1.57 097 98	+ 046 51
75	1.55 857 96	- 0 533 23	3000	1.57 112 15	+ 007 32
80	1.57 233 09	- 1 240 25	4000	1.57 097 89	- 017 08
85	1.58 239 84	- 0 193 48	5000	1.57 076 54	- 019 76
90	1.57 566 34	+ 0 998 61	6000	1.57 064 57	- 007 13
95	1.56 303 63	+ 0 710 98	7000	1.57 067 32	+ 007 24
100	1.56 222 55	- 0 514 88	8000	1.57 078 81	+ 012 47
110	1.57 988 05	- 0 031 96	9000	1.57 088 39	+ 006 84
120	1.56 397 22	+ 0 478 12	10,000	1.57 089 15	- 003 06
130	1.57 367 63	- 0 713 21	11,000	1.57 082 20	- 008 72
140	1.57 215 91	+ 0 701 11	100,000	1.57 080 63	+ 000 04
150	1.56 616 68	- 0 479 65	1,000,000	1.57 079 54	- 000 04
160	1.57 688 50	+ 0 140 94	10,000,000	1.57 079 64	+ 000 00
170	1.56 526 71	+ 0 200 65	100,000,000	+1.57 079 63	+0.00 000 00
180	+1.57 414 56	-0.00 443 21	∞	$\frac{1}{2}\pi$	0.0

TABLE XI.—Maxima and Minima Values of the Sine-integral.

x .	$\text{Si}(x\pi) - \frac{\pi}{2}$.	x .	$\text{Si}(x\pi) - \frac{\pi}{2}$.	x .	$\text{Si}(x\pi) - \frac{\pi}{2}$.	x .	$\text{Si}(x\pi) - \frac{\pi}{2}$.
1	+ 0.28 114 07	21	+ 0.01 515 07	50	− 0.00 636 57	350	− 0.00 090 95
2	− 15 264 47	22	− 01 446 26	51	+ 624 09	351	+ 90 69
3	+ 10 396 56	23	+ 01 383 43	60	− 530 49	400	− 79 58
4	− 07 863 54	24	− 01 325 83	61	+ 521 79	401	+ 79 38
5	+ 06 316 85	25	+ 01 272 83	70	− 454 71	450	− 70 74
6	− 05 276 24	26	− 01 223 90	71	+ 448 31	451	+ 70 58
7	+ 04 528 92	27	+ 01 178 60	80	− 397 87	500	− 63 66
8	− 03 966 50	28	− 01 136 53	81	+ 392 96	501	+ 63 53
9	+ 03 528 06	29	+ 01 097 36	90	− 353 67	600	− 53 05
10	− 03 176 72	30	− 01 060 79	91	+ 349 78	601	+ 52 96
11	+ 02 888 93	31	+ 01 026 59	100	− 318 30	700	− 45 47
12	− 02 648 88	32	− 00 994 52	101	+ 315 15	701	+ 45 41
13	+ 02 445 62	33	+ 00 964 40	150	− 212 20	800	− 39 79
14	− 02 271 31	34	− 00 936 04	151	+ 210 80	801	+ 39 74
15	+ 02 120 16	35	+ 00 909 31	200	− 159 15	900	− 35 37
16	− 01 987 87	36	− 00 884 06	201	+ 158 36	901	+ 35 33
17	+ 01 871 10	37	+ 00 860 17	250	− 127 32	1000	− 31 83
18	− 01 767 29	38	− 00 837 54	251	+ 126 82	1001	+ 31 80
19	+ 01 674 38	39	+ 00 816 07	300	− 106 10	2000	− 15 92
20	− 0.01 590 75	40	− 0.00 795 67	301	+ 0.00 105 75	2001	+ 0.00 15 91

TABLE XII.—Maxima and Minima Values of the Cosine-integral.

x .	$\text{Ci} \frac{x\pi}{2}$.	x .	$\text{Ci} \frac{x\pi}{2}$.	x .	$\text{Ci} \frac{x\pi}{2}$.	x .	$\text{Ci} \frac{x\pi}{2}$.
1	+ 0.4 720 007	39	− 0.0 163 149	77	+ 0.0 082 667	501	+ 0.0 012 707
3	− .1 984 076	41	+ 155 198	79	− 80 574	599	− 10 628
5	+ .1 237 723	43	− 147 986	99	− 64 300	601	+ 10 593
7	− .0 895 640	45	+ 141 415	101	+ 63 027	699	− 09 108
9	+ .0 700 653	47	− 135 401	119	− 53 494	701	+ 09 082
11	− .0 575 011	49	+ 129 879	121	+ 52 610	799	− 07 968
13	+ .0 487 422	51	− 124 789	139	− 45 798	801	+ 07 948
15	− .0 422 916	53	+ 120 082	141	+ 45 148	899	− 07 081
17	+ .0 373 449	55	− 115 718	159	− 40 038	901	+ 07 066
19	− .0 334 321	57	+ 111 660	161	+ 39 540	999	− 06 373
21	+ .0 302 601	59	− 107 877	179	− 35 564	1001	+ 06 360
23	− .0 276 371	61	+ 104 341	181	+ 35 171	1099	− 05 793
25	+ .0 254 320	63	− 101 030	199	− 31 990	1101	+ 05 782
27	− .0 235 525	65	+ 097 923	201	+ 31 672	1199	− 05 310
29	+ .0 219 314	67	− 095 001	299	− 21 291	1201	+ 05 301
31	− .0 205 189	69	+ 092 248	301	+ 21 150	1299	− 04 901
33	+ .0 192 772	71	− 089 650	399	− 15 955	1301	+ 04 893
35	− .0 181 771	73	+ 087 195	401	+ 15 876	1399	− 04 551
37	+ 0.0 171 958	75	− 0.0 084 870	499	− 0.0 012 758	1401	+ 0.0 004 544

The Sine-integral Curve, $y=Si\ x$, for positive abscissæ.The Cosine-integral Curve, $y=Ci\ x$.

Note added July 30, 1870.

Professor OPPERMAN, of Copenhagen, who was present at the reading of this paper, shortly afterwards presented to the Royal Society two pamphlets, "Tabulæ logarithmi integralis, auctore L. STENBERG, Malmogiæ, Pars I. 1861, Pars II. 1867," containing values of $\text{li } 10^x$ from $x=-15$ to $x=3.5$ at intervals of $.01$ to 18 places of decimals; the arguments differ therefore from those in this paper by the modulus of the common logarithms as a factor. From a reference in the second of these tracts the author found that Tables of $Si\ x$, $Ci\ x$, $Ei\ x$, and $Ei(-x)$ from 0 to 1 at intervals of $.01$ and from 1 to 7.5 at intervals of 0.1 , had been computed by BRETSCHNEIDER, and published in the 6th volume of SCHLÖMILCH's 'Zeitschrift für Mathematik und Physik.' The referees recommended the comparison of the parts common to these Tables and those given in this paper; this has been made, and the following errors have been found in BRETSCHNEIDER's values:—

$\text{li } e^a$ for $a=0.34$ should be $-0.13036\ 32936$ instead of $-0.13030\ 32936$

$\text{li } e^{-a}$ for $a=1.9$ should be $-0.05620\ 43781$: instead of $-0.05620\ 43780$:

a	$\text{li } e^a$	$\text{li } e^{-a}$	$Si\ a$	$si\ a$
4.9	37.33237 06037:	-0.00121 14833.	18.66679 10435:	1.56963 89381

should be

4.9	37.33245 06037:	-0.00129 14833.	18.66687 10435:	1.56955 89381
-----	-----------------	-----------------	-----------------	---------------

the error previously alluded to in $Ei(-5)$ is corrected in this paper.

BRETSCHNEIDER has indicated by dots certain limits between which the eleventh figure must lie, and the agreement between these and the eleventh figure in the Tables V. to VIII. was so close, that it seemed worth while to retain this figure, on the understanding that it may be in error to the extent of a unit.