XI. *On the Tides of the Arctic Seas.*

By the Rev. Samuel Haughton, *M.D. Dubl., D.C.L. Oxon., F.R.S., Fellow of Trinity College, Dublin.*

Part IV. *On the Tides of Northumberland Sound, at the Northern Outlet of Wellington Channel.*

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These Tidal Observations were made on board H.M.S. 'Assistance,' Captain Sir Edward Belcher, R.N., K.C.B., from 24th May to 6th July 1853, the exact position of the ship being 76° 52′ N. lat. and 97° 00′ W. long. Sir Leopold M'Clintock kindly procured for me, from Sir Edward Belcher, a copy of the Observations; and in forwarding them to me writes thus:—"Sir Edward Belcher wishes me to tell you how his Tidal Observations in 1853 were made. He says they did not depend upon the guess of any one, but resulted from machinery connected with the bottom, which moved a ratchet-wheel, each cog or inch of gauge ringing a bell; and the rise and fall was not that of the ship, but of the whole floe in which she was fixed. This machinery is described in his narrative, 'The last of the Arctic Voyages,' vol. i. p. 141. He further states that this rise was repeatedly verified by Theodolite Observations."

The following Table contains the Time and Height of High Water and Low Water, extracted from the original observations (which are forwarded with this paper); also the Diurnal Tide at High Water and Low Water, calculated from the heights by means of the formula

$$\text{Diurnal Tide}=\frac{h_1-4h_2+6h_3-4h_4+h_5}{16}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)^*$$

which gives the fourth difference of the successive heights.

* This expression for the Diurnal Tide is used and explained by Mr. Airy in his paper "On the Tides of the Coasts of Ireland" (Phil. Trans. 1845), and by the author in his paper "On the Diurnal Tides of the Coasts of Ireland" (Trans. Royal Irish Academy, 1855).

A. *Diurnal Tide* (Heights).

Northumberland Sound.

Time.		High Water. Height.	Low Water. Height.	Diurnal Tide at High Water.	Diurnal Tide at Low Water.
1853.	h m	ft. in.	ft. in.	ft.	ft.
May 27.	4 0 P.M.	16 1			
27.	10 0 "	......	14 4½		
28.	4 40 A.M.	16 4½	......	0·187	
28.	12 5 P.M.	......	15 4	......	0·481
28.	4 0 "	15 11	......	0·193	
28.	11 0 "	......	14 4½	......	0·443
29.	4 30 A.M.	16 3	......	0·201	
29.	1 0 P.M.	......	15 3	......	0·381
29.	6 0 "	15 10	......	0·219	
29.	Midnight	......	14 8	.....	0·339
30.	6 40 A.M.	16 4	......	0·224	
30.	1 40 P.M.	......	15 5	......	0·292
30.	7 0 "	15 11	......	0·208	
31.	12 50 A.M.	......	14 11	......	0·230
31.	8 0 "	16 4	......	0·198	
31.	3 30 P.M.	......	15 3	......	0·187
31.	9 0 "	16 0	......	0·214	
June 1.	2 40 A.M.	......	14 10	......	0·146
1.	8 32 "	16 6	......	0·234	
1.	3 30 P.M.	......	15 1	......	0·062
1.	9 40 "	16 0	......	0·229	
2.	3 30 A.M.	......	15 0¾	......	0·088
2.	9 40 "	16 5	......	0·209	
2.	4 20 P.M.	......	14 10	......	0·083
2.	10 30 "	16 0	......	0·172	
3.	4 35 A.M.	......	14 11	......	0·121
3.	10 30 "	16 3¾	......	0·182	
3.	5 0 P.M.	......	14 7	......	0·177
3.	11 0 "	16 2	......	0·062	
4.	5 0 A.M.	......	15 0	......	0·224
4.	10 48 "	16 3	......	0·010	
4.	5 30 P.M.	......	14 6	......	0·255
5.	12 15 A.M.	16 3	......	0·015	
5.	5 30 "	......	15 0	......	0·313
5.	11 45 "	16 2	......	0·031	
5.	6 0 P.M.	......	14 3½	......	0·370
6.	12 50 A.M.	16 3½	......	0·010	
6.	6 30 "	......	15 2	......	0·380
6.	12 15 P.M.	16 5½	......	0·021	
6.	6 20 "	......	14 6½	......	0·391
7.	1 10 A.M.	16 8	......	0·088	
7.	7 22 "	......	15 5	......	0·438
7.	12 25 P.M.	16 5	......	0·117	
7.	6 21 "	......	14 6	......	0·474
8.	1 0 A.M.	16 8	......	0·117	
8.	7 20 "	......	15 6½	......	0·475
8.	12 40 P.M.	16 6	......	0·128	
8.	8 0 "	......	14 8½	......	0·474
9.	2 0 A.M.	16 10½	......	0·110	
9.	8 22 "	......	15 9	......	0·477
9.	1 15 P.M.	16 8½	......	0·107	
9.	7 43 "	......	14 10	......	0·479
10.	2 38 A.M.	16 10½	......	0·112	
10.	8 50 "	......	15 9	......	0·484
10.	2 0 P.M.	16 6½	......	0·125	

Northumberland Sound (continued).

Time.		High Water. Height.	Low Water. Height.	Diurnal Tide at High Water.	Diurnal Tide at Low Water.
1853.	h m	ft. in.	ft. in.	ft.	ft.
June 10.	8 30 P.M.	......	14 8	......	0·428
11.	3 40 A.M.	16 8	......	0·151	
11.	9 35 „	......	15 6¼	......	0·256
11.	2 30 P.M.	16 3	......	0·214	
11.	10 7 „	......	15 5	......	0·234
12.	4 0 A.M.	16 9	......	0·240	
12.	10 30 „	......	15 8	......	0·251
12.	3 15 P.M.	16 3	......	0·203	
12.	10 0 „	......	14 9¾	......	0·339
13.	4 53 A.M.	16 8	......	0·068	
13.	11 15 „	......	15 6	......	0·339
13.	10 30 P.M.	......	14 10	......	0·328
14.	5 45 A.M.	16 8	......	0·151	
14.	12 15 P.M.	......	15 5¼	......	0·286
14.	6 0 „	15 11¾	......	0·307	
14.	Midnight	......	14 11	......	0·219
15.	6 45 A.M.	16 8	......	0·354	
15.	1 10 P.M.	......	15 3½	......	0·143
15.	8 0 „	15 11½	......	0·360	
16.	12 50 A.M.	......	15 2	......	0·005
16.	8 0 „	16 9	......	0·339	
16.	2 15 P.M.	......	15 2¾	......	0·005
16.	8 10 „	16 2	......	0·307	
17.	2 10 A.M.	......	15 3	......	0·078
17.	8 30 „	16 9½	......	0·271	
17.	3 0 P.M.	......	14 11½	......	0·151
17.	9 30 „	16 4	......	0·224	
18.	3 12 A.M.	......	15 4	......	0·187
18.	9 15 „	16 10	......	0·088	
18.	4 35 P.M.	......	14 11	......	0·214
18.	10 45 „	16 9	......	0·099	
19.	4 50 A.M.	......	15 3	......	0·250
19.	10 12 „	17 0	......	0·078	
19.	4 55 P.M.	......	14 8	......	0·344
19.	Midnight	16 10¾	......	0·041	
20.	5 30 A.M.	......	15 6	......	0·427
20.	11 20 „	17 0½			
20.	6 0 P.M.	......	14 7½		
21.	Noon	——			
21.	7 0 P.M.	......	——		
22.	1 0 „	——			
22.	7 20 „	......	13 2		
23.	2 0 A.M.	15 7			
23.	8 0 P.M.	......	14 1		
24.	2 0 „	16 8			
24.	8 30 „	......	——		
25.	3 0 P.M.	16 5			
25.	9 55 „	......	14 8		
26.	4 30 A.M.	17 0	......	0·333	
26.	11 30 „	......	15 6	......	0·407
26.	4 0 P.M.	16 3¼	......	0·307	
26.	10 40 „	......	14 8½	......	0·401
27.	5 45 A.M.	16 10	......	0·266	
27.	12 16 P.M.	......	15 7½	......	0·339
27.	6 0 „	16 5	......	0·266	
27.	11 0 „	......	15 2	......	0·250

Northumberland Sound (continued).

Time.		High Water. Height.	Low Water. Height.	Diurnal Tide at High Water.	Diurnal Tide at Low Water.
1853.	h m	ft. in.	ft. in.	ft.	ft.
June 28.	6 40 A.M.	17 0½	......	0·302	
28.	1 38 „	......	15 8	......	0·052
28.	6 20 P.M.	16 5½	......	0·453	
28.	Midnight...........	......	15 6	......	0·240
29.	7 20 A.M.	17 3	......	0·635	
29.	2 10 P.M.	......	13 11	......	0·297
29.	7 40 „	14 9	......	0·656	
30.	1 30 A.M.	......	13 11½	......	0·125
30.	8 30 „	15 4	......	0·364	
30.	3 45 P.M.	......	13 11	......	0·057
30.	9 0 „	14 10¾	......	0·193	
July 1.	12 40 A.M.	......	14 1	......	0·107
1.	9 5 „	15 2½	......	0·135	
1.	4 35 P.M.	......	13 10	......	0·172
1.	9 40 „	15 0	......	0·088	
2.	4 20 A.M.	......	14 3	......	0·229
2.	9 38 „	15 2	......	0·115	
2.	4 10 P.M.	......	13 8	......	0·281
2.	10 45 „	15 1	......	0·000	
3.	4 45 A.M.	......	14 2	......	0·339
3.	10 30 „	14 11½	......	0·026	
3.	5 4 P.M.	......	13 4	......	0·151
3.	11 13 „	15 0	......	0·026	
4.	5 0 A.M.	......	14 2	......	0·448
4.	10 45 „	14 11¾	......	0·021	
4.	5 30 P.M.	......	13 2½	......	0·490
5.	12 15 A.M.	15 1	......	0·015	
5.	6 0 „	......	14 2	......	0·500
5.	12 45 P.M.	15 1	......	0·031	
5.	6 15 „	......	13 1	......	0·511
6.	1 0 A.M.	15 1½	......	0·073	
6.	8 0 „	......	14 0	......	0·521
6.	1 0 P.M.	14 10	......	0·028	
6.	6 40 „	......	12 9½	......	0·521
7.	2 0 A.M.	15 0			
7.	8 0 „	......	13 8		

The general expression for the Diurnal Tide is the following:—

$$D=S \sin 2\bar{\sigma} \cos (s-i_s)+M \sin 2\bar{\mu} \cos (m-i_m), \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

where

D=height of tide,

$\bar{\sigma}$, $\bar{\mu}$=Solar and Lunar Declinations, corrected for Age of Tide,

s, m=Solar and Lunar Hour-angles,

i_s, i_m=Diurnal Solitidal and Lunitidal Intervals,

S, M=Solar and Lunar Coefficients, uncorrected for Parallax, &c.

It would be impossible to obtain any result as to the Diurnal Tides from so short a series of observations, only for a lucky chance which simplifies the calculation at this station, and enables us to obtain the Solar Diurnal Tide, although it is not easy to

determine the Lunar Diurnal Tide*. It so happened, during the observations, that the time of vanishing of the whole Diurnal Tide at Low Water corresponded very closely with the time of vanishing of the Moon's declination.

So that we have, at the same times,

$$\mu=0, \quad D=0,$$

which reduces the general expression at these times to

$$S \sin 2\sigma \cos(s-i_s)=0. \quad \quad (3)$$

The times corresponding to

$$\mu=0, \quad D=0$$

were

	h m
1st June	3 0 A.M.
15th „	4 30 P.M.
28th „	9 30 A.M.

If we now take the hours of Low Water of the Tides occurring nearest to the time of the Moon's declination vanishing, we find:—

		h m
1st June	$s=$	2 40 A.M.
15th „		1 10 P.M.
28th „		1 38 A.M.
Mean value of s		1 49

Now from equation (3) we have

$$s-i_s=6^h \text{ or } 18^h,$$

hence

$$1^h\ 49^m-i_s=6^h \text{ or } 18^h;$$

and, finally,

$$i_s=-4^h\ 11^m$$

$$\text{or}\dagger \quad +7^h\ 49^m.$$

The Diurnal Tide at High Water, when $\mu=0$, is represented by

$$D=S \sin 2\sigma \cos(s-i_s),$$

and had the following values:—

1st June	D=+0·234 ft.
16th „	−0·360 „
28th „	+0·302 „
Mean value of D	0·299 „

* See Note A, p. 327.

† An examination of the signs of the numerical values of the tide shows that the negative value of i_s must be chosen.

The hours of High Water corresponding to these values, and nearest to the time of the Moon's declination vanishing, were :—

	h m
1st June	8 32
15th ,,	8 00
28th ,,	6 40
Mean	7 44

Hence, using the mean value of 2σ during the observations ($45° 50'$), we obtain

$$\pm 0{\cdot}299 = \mathrm{S} \sin 45° 50' \cos(s - i_s);$$

but

$$\begin{array}{rr} & \text{h} \quad \text{m} \\ s = & 7 \quad 44 \\ i_s = - & 4 \quad 11 \\ \hline s - i_s = & 11 \quad 55 \end{array}$$

which corresponds to 180°, or the cosine equal to unity. Hence we have

$$0{\cdot}299 = \mathrm{S} \sin 45° 50',$$

or, finally,

$$\mathrm{S} = \frac{0{\cdot}299 \text{ ft.}}{\sin 45° 50'}$$

$$= 0{\cdot}417 \text{ ft.} = 5{\cdot}00 \text{ inches.}$$

B. *Semidiurnal Tide* (Heights).

If the preceding Table be plotted to scale, it is easy to separate the Semidiurnal Tide from the Diurnal Tide just discussed; but it is not possible, from observations made at the Solstice only, to separate the Solar and Lunar Tide and determine their coefficients.

The general expression for the Semidiurnal Tide is

$$\mathrm{T} = \mathrm{S}' \cos 2(s - i_s) + \mathrm{M}' \cos 2(m - i_m), \quad \ldots \quad (3^*)$$

where

S', M' = Solar and Lunar Coefficients, not corrected for declination or parallax,

s, m = Hour-angles of Sun and Moon,

i_s, i_m = true Solitidal and Lunitidal Intervals.

This expression may be thrown into the form

$$\mathrm{T} = \mathrm{A} \cos 2(m - \mathrm{B}), \quad \ldots \quad (4)$$

where

$$\left.\begin{aligned} \mathrm{A} &= \sqrt{\mathrm{M}'^2 + \mathrm{S}'^2 + 2\mathrm{M}'\mathrm{S}' \cos 2(\overline{m - s} - \overline{i_m - i_s})}, \\ \tan 2\mathrm{B} &= \frac{\mathrm{M}' \sin 2i_m + \mathrm{S}' \sin 2(\overline{m - s} + i_s)}{\mathrm{M}' \cos 2i_m + \mathrm{S}' \cos 2(\overline{m - s} + i_s)}. \end{aligned}\right\} \quad \ldots \quad (5)$$

A is the apparent height of the tide, and

B is the apparent Lunitidal Interval at High Water and Low Water.

At Spring-Tides we have

$$\left.\begin{array}{r} m-s=i_m-i_s, \\ A=M'+S'; \end{array}\right\} \quad (6)$$

at Neap-Tides we have

$$\left.\begin{array}{r} \overline{m-s}-\overline{i_m-i_s}=90°, \\ A=M'-S'. \end{array}\right\} \quad (7)$$

The maximum Spring-Tides occurred:—

	Sun's Hour-Angle. h m	Moon's Hour-Angle. h m
6th June . . .	12 50 A.M.	+1 18
	12 15 P.M.	+0 21
19th June . . .	Midnight.	+1 6
20th ,, . . .	11 20 A.M.	+0 2
Mean . .	0 6	+0 42

The minimum Neap-Tides occurred:—

	Sun's Hour-Angle. h m	Moon's Hour-Angle. h m
30th May	6 40 A.M.	−0 15
	7 0 P.M.	−0 19
12th June . . .	4 0 A.M.	−0 30
	3 15 P.M.	−1 40
Mean . .	+5 14	−0 26

From the Spring-Tides we find, by equation (6),

$$i_m-i_s=36^m;$$

and from the Neap-Tides, by equation (7), we obtain

$$i_m-i_s=40^m.$$

The mean of these values gives us

$$i_m-i_s=38^m. \quad (8)$$

We have no means of determining i_m and i_s separately.

The maximum and minimum ranges of the Tide, corrected for Diurnal Inequality only, were:—

Springs.	Range.
6th June	19·3 inches.
20th June	23·7 ,,
Mean . .	21·5 ,,

Neaps.		Range.
30th May	. . . . .	12·0 inches.
12th June	. . . . . .	11·3 „
	Mean . .	11·65 „

Substituting these values in (6) and (7), we find

$$2(M'+S') = 21{\cdot}5,$$
$$2(M'-S') = 11{\cdot}65,$$

and, finally,

$$\frac{S'}{M'}=0{\cdot}297. \qquad (9)$$

It will be observed that the Diurnal Solar Tide Range, already determined (9·40 in.), bears a very large proportion to the Semidiurnal Tide Ranges.

C. *Diurnal Tide* (Times).

The following Table contains the hour in local time of High Water and Low Water, and also the Lunitidal Intervals at High Water and Low Water elapsed from the Moon's passage of the meridian of the place. The Diurnal Tide in time might be calculated from the Lunitidal Intervals by first or second differences, as in the case of heights; but it is not worth the trouble to make the calculations, as the results can be more readily obtained by plotting the Lunitidal Intervals carefully to scale.

When this is done the diagram shows a fairly regular Diurnal Tide, with vanishing epochs and range well marked.

The maximum accelerations and retardations of the time of High or Low Water occasioned by the Diurnal Inequality amounted, generally, to from 35 minutes to 40 minutes, and on 1st July, at Low Water, reached 65 minutes.

D. *Semidiurnal Tide* (Times).

Northumberland Sound.—Lunitidal Intervals.

	High Water.	Low Water.		High Water.	Low Water.
1853. h m	h m	h m	1853. h m	h m	h m
May 27. 4 0 P.M.	−0 46		June 1. 2 40 A.M.	......	6 38
27. 10 0 „	......	5 14	1. 8 32 „	+0 12	
28. 4 40 A.M.	−0 37		1. 3 30 P.M.	......	7 10
28. 12 5 P.M.	......	6 48	1. 9 40 „	+0 56	
28. 4 0 „	−1 41		2. 3 30 A.M.	......	6 46
28. 11 0 „	......	5 19	2. 9 40 „	+0 39	
29. 4 30 A.M.	−1 38		2. 4 20 P.M.	......	7 31
29. 1 0 P.M.	......	6 52	2. 10 30 „	+1 5	
29. 6 0 „	−0 32		3. 4 35 A.M.	......	7 10
29. Midnight.	......	5 28	3. 10 30 „	+0 48	
30. 6 40 A.M.	−0 15		3. 5 0 P.M.	......	7 18
30. 1 40 P.M.	......	6 45	3. 11 0 „	+0 54	
30. 7 0 „	−0 19		4. 5 0 A.M.	......	6 54
31. 12 50 A.M.	......	5 31	4. 10 48 „	+0 24	
31. 8 0 „	+0 22		4. 5 30 P.M.	......	7 6
31. 3 30 P.M.	......	7 52	5. 12 15 A.M.	+1 27	
31. 9 0 „	+0 58		5. 5 30 „	......	6 42

Northumberland Sound.—Lunitidal Intervals (continued).

	High Water.	Low Water.
1853. h m	h m	h m
June 5. 11 45 P.M.	+0 37	
5. 6 0 „	......	6 52
6. 12 50 A.M.	+1 18	
6. 6 30 „	......	6 58
6. 12 15 P.M.	+0 21	
6. 6 20 „	......	6 26
7. 1 10 A.M.	+0 52	
7. 7 22 „	......	7 4
7. 12 25 P.M.	−0 18	
7. 6 21 „	......	5 38
8. 1 0 A.M.	−0 7	
8. 7 20 „	......	6 13
8. 12 40 P.M.	−0 53	
8. 8 0 „	......	6 27
9. 2 0 A.M.	+0 3	
9. 8 22 „	......	6 25
9. 1 15 P.M.	−1 9	
9. 7 43 „	......	5 19
10. 2 38 A.M.	−0 10	
10. 8 50 „	......	6 2
10. 2 0 P.M.	−1 15	
10. 8 30 „	......	5 15
11. 3 40 A.M.	+0 1	
11. 9 35 „	......	5 56
11. 2 30 P.M.	−1 36	
11. 10 7 „	......	6 1
12. 4 0 A.M.	−0 30	
12. 10 30 „	......	6 0
12. 3 15 P.M.	−1 40	
12. 10 0 „	......	5 5
13. 4 53 A.M.	−0 26	
13. 11 15 „	......	5 56
13. ——	——	
13. 10 30 P.M.	......	4 47
14. 5 45 A.M.	−0 22	
14. 12 15 P.M.	......	6 8
14. 6 0 „	−0 30	
14. Midnight	......	5 30
15. 6 45 A.M.	−0 9	
15. 1 10 P.M.	......	6 16
15. 8 0 „	+0 42	
16. 12 50 A.M.	......	5 32
16. 8 0 „	+0 18	
16. 2 15 P.M.	......	6 33
16. 8 10 „	+0 3	
17. 2 10 A.M.	......	6 3
17. 8 30 „	−0 1	
17. 3 0 P.M.	......	6 29
17. 9 30 „	+0 31	
18. 3 12 A.M.	......	6 13
18. 9 15 „	−0 8	
18. 4 35 P.M.	......	7 12
18. 10 45 „	+0 51	
19. 4 50 A.M.	......	6 56
19. 10 12 „	−0 6	
19. 4 55 P.M.	......	6 36
19. Midnight	+1 6	
20. 5 30 A.M.	......	6 36
20. 11 20 „	+0 2	
20. 6 0 P.M.	......	6 42
20. ——	——	
21. ——	......	——
21. ——	——	
1853. h m	h m	h m
June 21. ——	......	——
21. ——	——	
22. ——	......	——
22. ——	——	
22. 7 20 P.M.	......	5 55
23. 2 0 A.M.	−0 5	
23. ——	......	——
23. ——	——	
23. 8 0 P.M.	......	5 31
24. ——	——	
24. ——	......	——
24. 2 0 P.M.	−1 28	
24. 8 30 „	......	5 2
25. ——	——	
25. ——	......	——
25. 3 0 P.M.	−1 23	
25. 9 55 „	......	5 32
26. 4 30 A.M.	−0 18	
26. 11 30 „	......	6 42
26. 4 0 P.M.	−1 12	
26. 10 40 „	......	5 28
27. 5 45 A.M.	+0 11	
27. 12 16 P.M.	......	6 42
27. 6 0 „	+0 2	
27. 11 0 „	......	5 2
28. 6 40 A.M.	+0 23	
28. 1 38 P.M.	......	7 21
28. 6 20 „	−0 21	
28. Midnight	......	5 19
29. 7 20 A.M.	+0 21	
29. 2 10 P.M.	......	7 11
29. 7 40 „	+0 17	
30. 1 30 A.M.	......	6 7
30. 8 30 „	+0 50	
30. 3 45 P.M.	......	8 5
30. 9 0 „	+0 56	
July 1. 12 40 A.M.	......	4 36
1. 9 0 „	+0 38	
1. 4 35 P.M.	......	8 13
1. 9 40 „	+0 54	
2. 4 20 A.M.	......	7 34
2. 9 38 „	+0 33	
2. 4 10 P.M.	......	7 5
2. 10 45 „	+1 16	
3. 4 45 A.M.	......	6 0
3. 10 30 „	+0 39	
3. 5 4 P.M.	......	7 13
3. 11 13 „	+0 58	
4. 5 0 A.M.	......	6 45
4. 10 45 „	+0 6	
4. 5 30 P.M.	......	6 51
5. 12 15 A.M.	+1 12	
5. 6 0 „	......	6 57
5. 12 45 P.M.	+1 17	
5. 6 15 „	......	6 47
6. 1 0 A.M.	+1 8	
6. 8 0 „	......	8 8
6. 1 0 P.M.	+0 40	
6. 6 40 „	......	6 20
7. 2 0 A.M.	+1 16	
7. 8 0 „	...	7 16
Mean	+0 7·05	6 35·35

Having corrected the curve of Lunitidal Intervals for the Diurnal Inequality, the remainder is the acceleration or retardation on the time of the Semidiurnal Tide.

We have, by equation (5),

$$\tan 2B = \frac{M' \sin 2i_m + S' \sin 2(m-s+i_s)}{M' \cos 2i_m + S' \cos 2(m-s+i_s)}.$$

Differentiating this expression so as to obtain for B a maximum value, we find, as the equation of condition,

$$0 = M' \cos 2(\overline{m-s} - \overline{i_m - i_s}) + S'. \quad \ldots \ldots \ldots \quad (10)$$

Substituting in (5) we obtain

$$\tan 2B = \frac{\sqrt{M'^2 - S'^2} \sin 2i_m + S' \cos 2i_m}{\sqrt{M'^2 - S'^2} \cos 2i_m - S' \sin 2i_m}; \quad \ldots \ldots \quad (11)$$

and assuming

$$\frac{S'}{M'} = \sin 2\theta,$$

we find, after reduction,

$$\tan 2B = \tan 2(i_m + \theta);$$

and, finally,

$$2(B - i_m) = 2\theta$$

and

$$\frac{S'}{M'} = \sin 2(B - i_m). \quad \ldots \ldots \ldots \quad (12)$$

On examining the Lunitidal Curve, corrected for Diurnal Inequality, we find the following ranges from Springs to Neaps:—

High Water.	Low Water.
h m	h m
+1 0	7 18
−1 6	5 30
2 6	1 48

or mean maximum range

$$2B = 1^h\ 57^m.$$

Although we have not found the value of i_m, we may take as an approximation to it the Moon's mean Hour-Angle at High Water, already given in the Table,

$$i_m = +0^h\ 7^m.$$

Hence we have

$$2B - 2i_m = 1^h\ 57^m - 0^h\ 14^m,$$

or

$$\frac{S'}{M'} = \sin(1^h\ 43^m) = \sin(24°\ 55') = 0{\cdot}421. \quad \ldots \ldots \quad (13)$$

Collecting together the partial results obtained at this most interesting Tidal Station, we obtain:—

Diurnal Tide.

1. Solitidal Interval,

$$i_s = -4^h\ 11^m.$$

2. Solar Coefficient, corrected for Declination,

$$S = 5\cdot0 \text{ inches.}$$

3. Lunitidal Interval,

$$i_m = -8^h\ 8^m.$$

4. Solar Coefficient, corrected for Declination,

$$M = 4\cdot0 \text{ inches.}$$

Semidiurnal Tide.

1. Mean Lunitidal Interval,

High Water.	Low Water.
h m	h m
+0 7·05	6 35·35

2. Difference between Lunitidal and Solitidal Intervals,

$$i_m - i_s = 38^m.$$

3. Approximate ratios of uncorrected Solar and Lunar Coefficients,

$$\frac{S'}{M'} = 0\cdot297 \text{ (Heights)}$$
$$= 0\cdot421 \text{ (Intervals).}$$

NOTE A.—Added July 1, 1875.

At the time of writing this paper I abandoned the attempt to determine the Lunar Diurnal Tide, in consequence of the breakdown of the observations which occurred in the neighbourhood of the 23rd June, which corresponds with one of the maxima of the Lunar Declination. This Tide may, however, be found from the tides of 8th and 9th June and 5th and 6th July, which also correspond to maxima of the Declination. The Lunar Diurnal Tide is the difference between the Total Diurnal Tide and the Solar Diurnal Tide, which is determined in the paper.

In the following Tables, the Solar Diurnal Tide is calculated from the formula

$$\text{Solar Tide} = 3\cdot58 \cos(s + 4^h\ 11^m),$$

founded on the constants

$$S = 5\cdot00 \text{ inches,}$$
$$i_s = -4^h\ 11^m.$$

North Declination of Moon a Maximum.

High Water.

	Diurnal Tide.	Solar.	Lunar.
h m	inches.	inches.	inches.
June 7. 12 45 P.M.	−1·40	+1·20	−2·60
8. 1 0 A.M.	+1·40	−0·76	+2·16
8. 12 40 P.M.	−1·53	+1·06	−2·59
9. 2 0 A.M.	+1·32	+0·17	+1·15
9. 1 15 P.M.	−1·28	+0·53	−1·81
10. 2 38 A.M.	+1·34	+0·78	+0·56
		Mean ...	±1·812

Low Water.

June 7. 6 21 P.M.	−5·69	−3·28	−2·41
8. 7 20 A.M.	+5·70	+3·51	+2·19
8. 8 0 P.M.	−5·69	−3·58	−2·11
9. 8 22 A.M.	+5·72	+3·52	+2·20
9. 7 43 P.M.	−5·75	−3·55	−2·20
10. 8 05 A.M.	+5·81	+3·58	+2·23
		Mean ...	±2·223

High Water.

July 4. 10 45 A.M.	−0·25	+2·25	−2·50
5. 12 15 A.M.	+0·18	−1·42	+1·60
5. 12 45 P.M.	−0·37	+0·54	−0·91
6. 1 0 A.M.	+0·87	−0·76	+1·63
6. 1 0 P.M.	−0·33	+0·76	−1·09
		Mean ...	±1·546

Low Water.

July 4. 5 30 P.M.	−5·88	−2·91	−2·97
5. 6 0 A.M.	+6·00	+3·18	+2·82
5. 6 15 P.M.	−6·13	−3·25	−2·88
6. 8 00 A.M.	+6·25	+3·58	+2·67
6. 6 40 P.M.	−6·25	−3·38	−2·87
		Mean ...	±2·842

If m denote the Moon's Hour-Angle at High Water, we have at High Water,

$$\text{Lunar Tide}=\text{M}\sin\overline{2\mu}\cos\overline{m-i_m}, \quad \ldots\ldots\ldots \quad (a)$$

and at Low Water,

$$\text{Lunar Tide}=-\text{M}\sin\overline{2\mu}\sin\overline{m-i_m}. \quad \ldots\ldots\ldots \quad (b)$$

Hence we find (observing that the Lunar Tide has the same sign at High Water and Low Water)

$$\text{June 7, 8, 9} \quad \ldots\ldots \quad \overline{2\mu}=49^\circ\ 30'\ \text{N}.$$

$$\tan\overline{m-i_m}=-\frac{2223}{1812},$$

$$m-i_m=-50^\circ\ 49' \text{ or } +129^\circ\ 11'$$

$$=-3^h\ 23^m \text{ or } +8^h\ 37^m.$$

The value of m, as found from all the observations, is given in the paper,

$$m = 0^{\text{h}}\ 7^{\text{m}}.$$

Hence we find

$$i_m = +3^{\text{h}}\ 30^{\text{m}} \text{ or } -8^{\text{h}}\ 30^{\text{m}}.$$

The signs of the Lunar Tide show that the negative value of i_m is the proper one; hence

$$i_m = -8^{\text{h}}\ 30^{\text{m}}. \quad \text{(c)}$$

We have also, from (a) and (b),

$$\text{M} \sin \overline{2\mu} = \sqrt{(2 \cdot 223)^2 + (1 \cdot 812)^2} = 2 \cdot 861,$$

and, finally,

$$\text{M} = 3 \cdot 77 \text{ inches}. \quad \text{(d)}$$

July 4, 5, 6 . . . . $\overline{2\mu} = 49° \ 30' \text{ N}.$

$$\tan \overline{m - i_m} = -\frac{2842}{1546},$$

$$m - i_m = -61° \ 27' \text{ or } +118° \ 33'$$

$$= -4^{\text{h}}\ 6^{\text{m}} \text{ or } +7^{\text{h}}\ 54^{\text{m}},$$

$$i_m = +4^{\text{h}}\ 13^{\text{m}} \text{ or } -7^{\text{h}}\ 47^{\text{m}}, \quad \text{(e)}$$

of which the latter value must be used.

We have also

$$\text{M} \sin \overline{2\mu} = \sqrt{(1 \cdot 546)^2 + (2 \cdot 842)^2} = 3 \cdot 235,$$

and, finally,

$$\text{M} = 4 \cdot 26 \text{ inches}. \quad \text{(f)}$$

The mean values of i_m and M, deduced from the preceding equations, are

$$i_m = -8^{\text{h}}\ 8^{\text{m}}, \quad \text{(g)}$$

$$\text{M} = 4 \cdot 00 \text{ inches}. \quad \text{(h)}$$

The Lunar Diurnal Tide is therefore expressed by the equation

$$\text{Lunar Tide} = 4 \sin \overline{2\mu} \cos(m + 8^{\text{h}}\ 8^{\text{m}}). \quad \text{(i)}$$