VI. *On Rolling-Friction.* *By Professor* OSBORNE REYNOLDS, *of Owens College, Manchester.* *Communicated by Professor* B. STEWART, *F.R.S.*

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Introduction.

ALTHOUGH the motion of wheels and rollers over a smooth plane is attended with much less resistance or friction than the sliding of one flat surface over another, however smooth, yet practically it has been found impossible to get rid of resistance altogether. COULOMB made some experiments on the resistance which wooden rollers meet with when rolling on a wooden plane, from which experiments he deduced certain laws connecting this resistance with the size of the rollers and the force with which they are pressed on to the plane. These laws have been verified and extended to other materials by NAVIER and MORIN, and are now set forth in many mechanical treatises as "*the laws of resistance to rolling.*" It does not appear, however, that any systematic investigation of this resistance has ever been undertaken or any attempts made to explain its nature. When hard surfaces are used it is very small, and it has doubtless been attributed to the inaccuracies of the surfaces and to a certain amount of crushing which takes place under the roller. On closer examination, however, it appears that these causes, although they doubtless explain a great part of the resistance which occurs in ordinary practice, are not sufficient to explain the resistance altogether; and that, if they could be removed, there would still be a definite resistance depending on the size and weight of the roller and on the nature of the material of which it and the plane are composed. If it were not so, a perfectly true roller when rolling on a perfectly true surface ought to experience no resistance, however soft the roller and the plane might be, provided both were made of perfectly elastic material so that the one did not crush the other; and we might expect, although these conditions are not absolutely fulfilled, that a roller of iron would roll as easily on a surface of india-rubber as on one of iron, or that an india-rubber roller would experience no more resistance than one of iron when rolling on a true plane. Such, however, is not the case. The resistance with india-rubber is very considerable; my experiments show it to be ten times as great as with iron. I am not aware that this fact has been previously recognized; and that it has often been overlooked is proved by the numerous attempts which have been made to use india-rubber tires for wheels, the invariable failure of which may, I think, in the absence of any other assigned cause, be fairly attributed to the excessive resistance which attends their use. Another fact which I do not think has been hitherto noticed, but of which I have had ample evidence, and which clearly shows the existence of some hitherto unexplained

cause of resistance to rolling, is the tendency which a roller has to oscillate about any position in which it may be placed on a flat surface.

However true and hard the roller and the surface may be, if the roller is but slightly disturbed it will not move continuously in one direction until it gradually comes to rest, but it will oscillate backwards and forwards through a greater or less angle, depending on the softness of the material. These oscillations are not due to the roller having settled into a hollow. This is strongly implied by the fact that the more care is taken to make the surfaces true and smooth the more regular and apparent do the oscillations become. But even if this is not a sufficient proof—if it is impossible to suppose that an iron roller on an iron plane can be made so true that when the one is resting on the other it will not be able to find some minute irregularities or hollows in which to settle—still we must be convinced when we find the same phenomenon existing when india-rubber is substituted for iron, and in such a marked degree that no irregularities there may be in the surface produce any effect upon it, much less serve to account for it.

These phenomena, with others, have led me to conclude that there is a definite cause for the resistance to rolling besides the mere crushing of the surface or accidental irregularities of shape, a cause which is connected with the softness of the material as well as with the size and weight of the roller.

Such a force, if its existence be admitted, must either be considered as exhibiting some hitherto unrecognized action of matter on matter, or must be supposed to arise in some intelligible manner from the known actions. The latter is the most natural supposition; and it is *my object in this paper to show that this force arises from what is ordinarily known as friction.* It is to imply this connexion that I have gone back to the name *Rolling-Friction* in place of the more general title *resistance to rolling* ("résistance au roulement"), which COULOMB and subsequent writers have chosen avowedly because they did not wish to imply such a connexion.

The assumption that this force is due to friction necessarily implies that there is slipping between the roller and the plane at the point of contact; and on the other hand, if it can be shown that there is slipping, it follows as a natural consequence that there must be friction or resistance to rolling. Therefore the question as to whether the resistance to rolling is due to friction reduces itself into a question as to whether there is any evidence of slipping between the roller and the surface on which it rolls.

My attention was first called* to the possibility of such slipping while considering a phenomenon in the action of endless belts when used to transmit rotary motion from one pulley to another, namely that it is impossible to make the belt tight enough entirely to prevent slipping and cause the surfaces of the two pulleys to move with identically the same velocity. It appears that this slipping is due to the elasticity of the belt, and, since all material is more or less elastic, cannot altogether be prevented.

* The Engineer, November 27, 1874.

This becomes apparent when we consider that of the two parts of the belt which stretch from pulley to pulley the one is tighter and hence more stretched than the other, that is, when the belt is transmitting power. For that side which is most stretched, and consequently thinner, will have to move faster than the slacker side in order to prevent the belt accumulating at one pulley; and the speed of the driving-pulley will be equal to that of the tight side of the belt, while the speed of the following pulley will be equal to that of the slack side. This difference of speed requires that the belt shall slip over the pulleys; and this slipping takes place by the expansion and contraction of the belt on the pulleys as it passes from the tight side to the slack side, and *vice versâ*. With leather belts this slipping is very small; but with soft india-rubber it becomes so great as practically to bar the use of this material for driving-belts.

The recognition of this slipping at once suggested to me that there must be an analogous slipping when a hard roller rolls on a soft surface, or when an india-rubber wheel rolls on a hard surface. A single experiment was sufficient to prove that such was the case—an iron roller rolled through something like three quarters of an inch less in a yard when rolling on india-rubber than when rolling on wood or iron.

Having made this discovery, I proceeded to investigate the subject, and have obtained what I think to be satisfactory evidence that, whatever may be the material of which the plane and the roller are composed, the deformation at the point of contact always causes slipping, although, owing to the hardness of the materials, it may be far too small to be measured.

In the following paper I shall first show that the deformation at the point of contact caused by the weight of the roller must affect the distance rolled through, that it must cause slipping, and that this slipping will be attended with friction. I shall then show that the friction will itself considerably modify the deformation which would otherwise take place, and endeavour to trace the exact nature of the actual deformation. The result of my experiments will then be given, together with the description of certain other causes of rolling-friction which appear under certain circumstances to exist. In conclusion, I shall indicate the direction in which I hope to continue the investigation, consider its bearing on the laws discovered by COULOMB, and discuss certain phenomena connected with the wear of railway-wheels which have been hitherto unexplained, and which serve to illustrate the importance of the subject.

The Distance Rolled through.

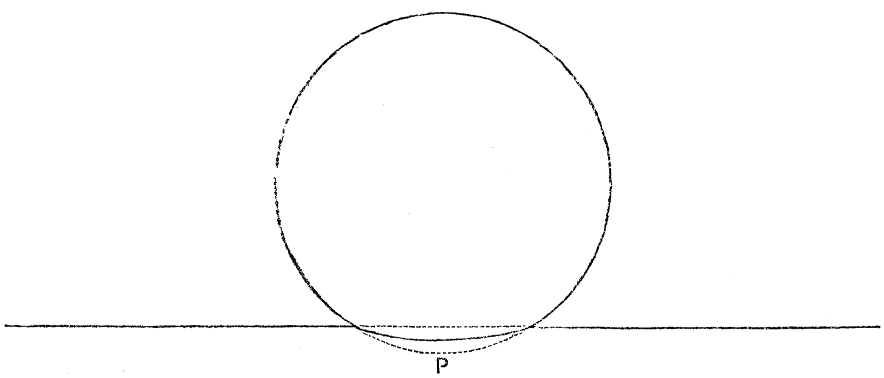
If a perfectly hard cylinder rolled on a perfectly hard plane and there were no slipping, then the distance which the cylinder would pass over in one revolution would be exactly equal to its circumference; but if, from the weight of the cylinder or any cause, the length of the surface either of the cylinder or the plane underwent an alteration near the point of contact, then the distance traversed in one revolution would not be equal to the natural length of the circumference. For example, suppose that an iron cylinder is rolling on a surface of india-rubber across which lines have been drawn at

intervals of ·01 of an inch, and suppose that as the cylinder rolls across these lines the surface of the india-rubber extends so that the intervals become equal to ·011 of an inch, closing again after the cylinder is past, then the cylinder will measure its circumference (so to speak) on the extended plane, and the actual distance rolled through when measured on the contracted surface will be one tenth less than the circumference. In the same way there would be an alteration in the distance rolled through if the surface of the roller extended or if either of the surfaces contracted.

In the subsequent remarks I shall call the distance which the roller would roll through if there were no extension or contraction its geometrical distance.

Since no material is perfectly hard, when a heavy roller rests on a surface the weight of the roller will cause it to indent the surface to a greater or less extent according to the softness of the latter; and in the same way the surface of the cylinder will be flattened at the point of contact in the manner shown in fig. 1.

Fig. 1.



This indentation and flattening will alter the lengths of the surfaces at the point of contact, and will therefore affect the progress of the roller. When a body of any shape is compressed in one direction it extends in the other directions; hence the weight of the roller resting on the plane will, by compressing the material of the plane in a vertical direction, cause it to extend laterally at the point of contact, and thus the length of the surface which the cylinder actually rolls over would be greater than the length measured on the undisturbed plane. From this cause, therefore, the cylinder would roll through less than its geometrical distance.

On the other hand, the surface of the roller would also be extended (squeezed out) in a similar manner by the pressure of the plane at the point of contact; and hence the surface of the roller would be greater than its natural length, and this would cause the roller to roll through more than its geometrical distance.

To a certain extent, therefore, the expansion of the surface of the roller would counteract the expansion of the plane; and if the two were of the same material, then the one of these extensions would, if nothing interfered to prevent it, exactly counteract the other. But if the one was harder than the other, then the effect is that one would be least. Thus an iron cylinder rolling on an india-rubber plane would roll through

less than its geometrical distance; whereas, inversely, an india-rubber roller on an iron plane would roll through *more than* its geometrical distance.

These things actually take place. But there is, besides softness, another circumstance not hitherto mentioned which affects the lateral extension of the surface when compressed by the roller, viz. *the shape of the surface.*

A little consideration will be sufficient to show that a curved indent in a flat surface will have a greater effect to extend the surface than a flat indent on a rounded surface. In the case of the rounded surface it will be seen that the effect of vertical compression to a certain extent counteracts the effect of lateral expansion; whereas in the case of the flat surface these things are reversed, and the effort of the surrounding material to uphold that which is depressed will increase the lateral expansion.

From this cause, therefore, even if the cylinder and the plane were made of the same material, there would still be a difference in the lateral extension of the surfaces at the point of contact depending on the smallness of the diameter of the cylinder, and this difference would still cause the cylinder to roll through less than its geometrical distance.

If, instead of on a plane, the one cylinder rolled on another parallel cylinder under a force tending towards the centre, then, if the two cylinders were of the same material and their diameters were equal, they would roll through their geometrical distance; but if the one was larger than the other, the largest would be most retarded.

It appears, therefore, that there are two independent causes which affect the progress of a roller on a plane—the relative softness of the materials and the diameter of the roller. Of these the curvature of the roller always acts to retard its progress; while the other (the relative softness) to retard or to accelerate according as the plane is softer than the cylinder, or *vice versâ.* These two causes will therefore act in conjunction or in opposition, according to whether the roller is harder or softer than the plane. In the former case the roller will be retarded, whereas in the latter it will depend on the relation between the relative softness and the diameter of the cylinder whether its progress is greater than, less than, or equal to its geometrical progress. Thus an iron roller on an india-rubber plane will make less than its geometrical progress; while an india-rubber cylinder on an iron plane will make more than, less than, or exactly its geometrical progress, according to the relation between its diameter and softness, or, what comes to the same thing, its weight, which conclusions are borne out by experiment.

The Slipping.

The lateral extension of the material, and the effect this has on the progress of the roller, causes slipping between the surface of the roller and that of the plane; for the surface of the roller, owing to the indentation and flattening, really touches the surface of the plane over an area of some extent; and the pressure between these surfaces, which is greatest towards the middle of the area in which they touch, will shade off to

nothing at the edges. Thus deformation is allowed to go on between the surfaces after they have come in contact, and is performed by the slipping of the one over the other.

The Friction.

The slipping is performed against friction, and therefore gives rise to resistance to the motion of the roller.

This resistance will obviously be proportional to the work spent in overcoming the friction between the surfaces during a certain extent of motion; and at first sight it appears as if this would be proportional to the coefficient of friction between these surfaces. When I first commenced this investigation I was under the impression that such would be the case, and that by oiling the surfaces the resistance to rolling might be considerably reduced. Finding by experiment, however, that this was not the case, that although in certain cases the effect of oiling or blackleading the surfaces does reduce the resistance to rolling, yet this reduction is never great, and in some cases the effect appears to be reversed, it occurred to me that the friction would itself modify the deformation which would otherwise take place after contact had commenced, and by preventing slipping might diminish the work that would otherwise have been spent.

The Deformation.

The action of friction to prevent the deformation at any point of the surfaces in contact will obviously depend on two things—the magnitude of the friction, and the force tending to slide the one surface over the other. Now if P (fig. 1) be the point of greatest pressure, the possible friction will gradually diminish with the pressure as the distance from P increases; whereas we may assume that the tendency of the one surface to slip over the other will be nothing at P, and will gradually increase with the distance; so that for a certain distance the friction may be sufficient to prevent slipping altogether, but beyond this distance slipping will go on in an increasing ratio.

The effect of oiling the surface would therefore be to diminish the region of no slipping, and increase the area over which slipping goes on as well as the extent of slipping at each point. These effects would to a certain extent counteract the advantage gained by the reduced coefficient of friction; and it may well be conceived that under certain circumstances they would overbalance it, and that the oil would actually increase the resistance.

The effect which friction has upon the deformation beneath the roller, as well as the general nature of this deformation, will be rendered clearer by examining the effect of friction under circumstances of a less complicated nature than those of rolling.

A Soft Bar between Hard Plates.

Let fig. 2 represent the end or a section of a long rectangular bar of india-rubber, or any elastic material, placed between two flat plates. Suppose these plates to approach

each other, compressing the india-rubber, which will extend laterally. Now if there were no friction between the rubber and the plates, then the surfaces in contact with the plates would extend in the same proportion as the rest of the bar, and the section would preserve its rectilinear form, as shown in fig. 3.

Fig. 2.

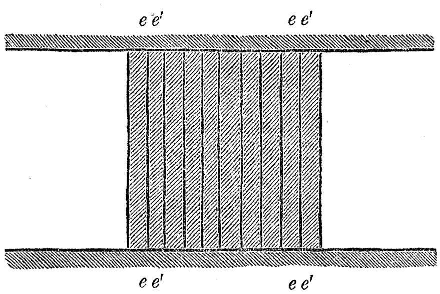
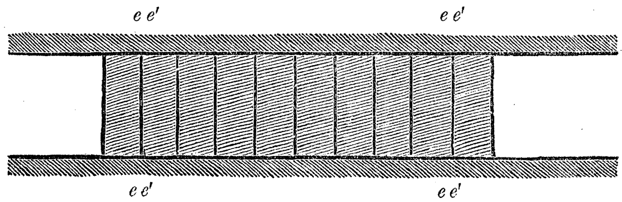


Fig. 3.



With friction, however, the case would be different. The friction would prevent the surface of the india-rubber expanding laterally to the same extent as the rest of the bar, and the section would lose its rectilinear form and bulge out in the middle, as shown in figs. 4 and 5.

Fig. 4.

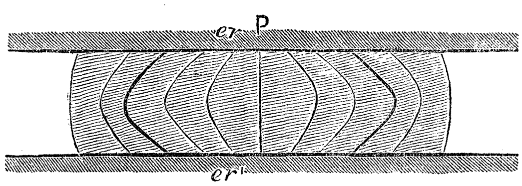
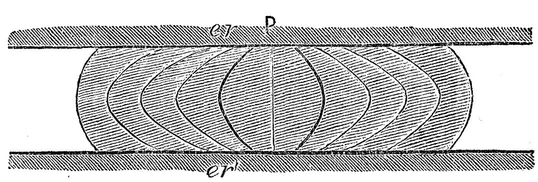


Fig. 5.



If we imagine the section of the bar to have been marked with a series of lines (ee') initially vertical and at equal intervals apart, these lines will when the bar is compressed assume the form shown in the figures.

If there were no friction, then, as shown in fig. 3, the ends of these lines would still be equidistant after compression; but with friction the intervals will not be all equal, but will vary according to their distance from P, the middle of the section. Up to a certain distance (er) the friction will be sufficient to prevent slipping; and hence up to this point the ends of the lines will preserve their original distance. From this point (er), however, slipping will commence and will go on increasing to the edge of the surface. From this point, therefore, the distance between the ends of the lines will continually increase.

With regard to the distribution of the pressure between the india-rubber and the plates:—Without friction this will obviously be uniform over the whole surface. Friction, however, will not only increase the mean intensity of the pressure, but will also alter its distribution, causing it to be greatest at P and gradually diminish towards the edge.

The inclination of the ends of the lines ee' is caused by, and may be taken to repre-

sent, the intensity of the friction at the surface. As long as there is slipping the friction will be proportional to the pressure. Therefore from the edge of the surface to *er* the inclination will continually increase; and it will be greatest at *er*, for from this point inwards the tendency of the india-rubber to slip will obviously diminish until it vanishes at P.

The distance of er *from* P *will not depend on the degree of compression*, at all events so long as this is but small, for the tendency to extend laterally will be proportional to the intensity of the pressure; and since the friction is proportional to the pressure, it will increase at all points in the same ratio as the forces tending to extend the rubber laterally. The distance of *er* from P will, however, obviously depend on the coefficient of friction. The greater this is the greater will be the region over which there is no slipping.

By blackleading the india-rubber, therefore, we should change the shape of the section from that shown in fig. 4 to that shown in fig. 5, in which all the ends of the lines from *er* to the circumference are less inclined than the corresponding lines in fig. 4, and the intervals between them greater, showing that not only is the friction less and the area over which it acts greater, but that each point has also to slip through a greater distance.

It is difficult to say how far these two latter effects will compensate for the former. We may, however, show that there must be some value of the coefficient of friction for which the work spent in overcoming the friction will be a maximum; for when the coefficient was very great *er* would be at the circumference and there would be no slipping, and hence no work spent in friction; whereas if the coefficient were zero, *er* would be at P, and there would be no friction and consequently no work lost in overcoming it. Therefore the work spent in friction, which is a function of the coefficient of friction, is zero for two values of the variable; and since it is positive for all intermediate values, it must pass through a maximum value. Hence for some position of *er* (for some particular coefficient of friction) the work spent in friction would be a maximum. What this value of the coefficient is it is impossible to say; but it seems to be less than that between clean india-rubber and iron, and it may be less than that between blackleaded india-rubber and iron. This was shown by the experiments on rolling-friction.

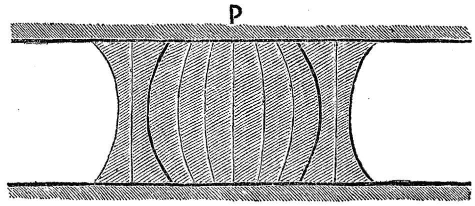
In considering these experiments, however, there is another thing to be taken into account besides the work spent in friction during compression, and that is the effect of friction during restitution; for the action of a roller as it passes over the india-rubber will be first to compress it and then to allow it to expand again in a corresponding manner.

The effect of Friction during Expansion.

If, after the rubber has been compressed as shown in figs. 4 and 5, the surfaces gradually separate again, the shape of the lines will again change. The lines from P up to *er* will assume the same forms which they had at corresponding periods of the

compression; but since that portion of the surface which lies beyond *er* has been extended by the compression, it will have to contract as the surfaces recede, and the friction of the surface will oppose such contraction. Hence the lines which during compression were curved outwards will gradually straighten and curve inwards, as shown in fig. 6. Those at the edges will take the form first, and then those nearer to *er*, until the expansion has become complete.

Fig. 6.



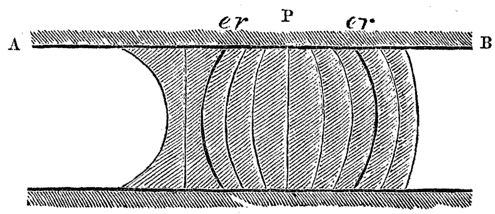
The extent to which friction will deform the india-rubber during this operation will obviously depend on the extent to which friction has allowed the surfaces to expand during compression. The smaller the friction the greater will be this expansion, and consequently the further they will have to contract, and the greater will be the pressure under which contraction must take place. It is obvious, therefore, that the work spent in friction during the recoil will increase up to a certain point as the coefficient of friction diminishes; and it would appear to be this increase which mainly balances the advantage which is gained during compression by reducing the coefficient.

It is evident that the action of friction to prevent contraction during restitution will tend to reduce not only the mean pressure but also the whole pressure, for exactly the same reason as by preventing expansion the friction increases these pressures during compression. Therefore, for every distance between the plates, after the curves become inclined inwards the pressure on the surface would be less than at the same distance with no friction, and in a still greater degree than during compression with friction. We can see at once, therefore, that of the work spent in compressing the material only a part would be returned during restitution. The difference is what is spent in overcoming the friction.

The Direction of the Friction.

In figures 5 and 6 the direction of slipping is opposite on opposite sides of P. If however, we conceive one half of the bar, that towards A, to have been compressed and to be expanding again, while the other half, that towards B, is being compressed and the distance between the plates which hold both parts to be the same, we may imagine the plate A B to have been first inclined towards A and then towards B so as to raise the end A. Then the lines would assume the form shown in fig. 7.

Fig. 7.



In this case we see that the slipping takes place in the same direction on both sides of P, so that the top plate A B would slip backwards in direction A over the india-rubber, while, on the other hand, the india-rubber would slip forwards in the direction D over the lower plate.

The turning of the plate A B, which has been supposed to be going on in figure 7, represents very closely the action of a roller in compressing the material beneath it; and this case affords us an illustration of the way in which the lateral extension of the material under the roller, or of the roller itself, will by causing slipping alter the distance travelled by the roller. If the roller be hard and the surface on which it rolls soft, then the top plate A B may be taken to represent the roller, and, as has just been explained, this slips back; whereas if the roller be soft and the surface hard, then we may take the india-rubber to represent the roller, and this slips forward.

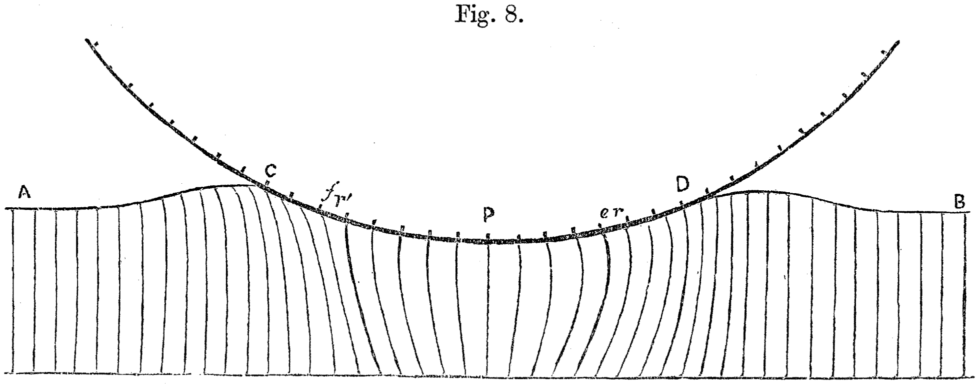
A Continuous Surface.

It is clear that in the case of the bar shown in fig. 7 the slipping will diminish as the coefficient of friction increases. There is, however, an important difference between this case and that of a roller in which it is not the entire breadth of a bar that is compressed, but a portion of a continuous surface; for whatever lateral extension there may be immediately under the roller must be compensated by a lateral compression immediately in front and behind it. The greater the lateral extension under the roller the greater will be the lateral compression; and since the action of the roller is continually to change the one for the other, the one effect will to a certain extent counteract the other; so that in this case we need not expect to find the diminution attended with a corresponding increase in the ostensible slipping. This will be rendered clearer by examining these circumstances as they affect rolling.

The Deformation caused by a Roller.

Fig. 8 may be taken to represent a section of an iron cylinder on an india-rubber plane. The lines on the plane are supposed to represent lines initially vertical and at equal distances apart. The motion of the roller is towards B. P is the point of greatest compression directly below the centre of the roller; *e r* and *f r* limit the surfaces over which there is no slipping; D is the point at which contact commences, and C that at which it ceases.

Fig. 8.



The portions of the india-rubber immediately without C and D are laterally compressed;

this, as has already been pointed out, is to make room for the lateral extension under the roller from C to D. From D towards B, therefore, and from C towards A the parallel lines are somewhat distorted, and at something less than their natural distance apart. From D to *e r* vertical compression and lateral expansion is going on, and the lines are convex outwards. From *e r* to P there is no slipping and the lines straighten. From P to *f r*, which is greater than the corresponding distance from P to *e r*, there is no slipping, and at *f r* the lines are convex outwards. From *f r* to C vertical expansion and lateral contraction take place, so that the lines are all concave outwards. The lateral expansion from D to *e r* and the lateral contraction from *f r* to C can only take place by the slipping of the india-rubber over the iron. Its extent is shown by the distance between the corresponding lines on the india-rubber and those on the iron, which latter have been set out equal to the distance between the lines on the rubber where greatest, namely from *e r* to *f r*.

The Actual and Apparent Slipping.

Since there is no slipping at P, it is clear that the roller will roll through less than its geometrical distance, inasmuch as the geometrical distance between the lines on the plane at P is greater than their natural distance. Therefore the ostensible slipping will be equal to the difference between the intervals marked on the roller, and the initial distance between those on the rubber. The actual slipping, however, is equal to the difference between the intervals on the roller and the intervals on the rubber at D or C, which latter are less than the natural distance; therefore the actual slipping is greater than the ostensible in proportion to the compression at C and D; and since this is increased by diminishing the coefficient of friction, such a diminution will affect the actual slipping in a greater degree than it affects the ostensible. This is in accordance with what has already been stated.

India-rubber Roller.

If the distance between the lines at P were exactly equal to the natural distance, then the roller would roll through its geometrical distance whatever might be the actual slipping. This is very nearly what actually takes place when an india-rubber roller rolls on an iron plane.

In the case of an india-rubber roller on an india-rubber surface the lateral compression in the surface of the roller at D is greater than that in the plane, and the expansion at P is not so large, and hence there is slipping, and the roller will not accomplish its geometrical distance.

In this explanation I have referred to india-rubber because it is much more easy to conceive the effects on it than on a hard substance like iron, the expansion and contraction of which is quite inappreciable to our senses; the reasoning, however, applies equally well to all elastic substances, and is quite independent of their hardness or softness. That friction is sufficient to prevent the expansion of iron at a surface against which it

is squeezed out is amply proved by the fact that when a block of iron, hot or cold, is squeezed on an anvil the iron bulges out in the middle, as shown in fig. 4.

Experimental Verification of the Figures.

The figures which illustrate the foregoing remarks are not altogether ideal, for they have been verified to a certain extent by experiments on india-rubber; for instance, by drawing vertical lines on the edge of a plate of india-rubber, and then observing these lines as the roller passed along as near as possible to this edge; also by observing lines drawn in the same way on the edge of an india-rubber roller. The effect of friction to prevent expansion, shown in figures 4 and 5, was verified by marking the surface of the india-rubber under the plate A B with parallel lines in chalk, which left a mark on the iron and showed how far there had been slipping. The figures are nevertheless intended rather to illustrate the nature of the slipping and various effects than their extent, which latter must be judged of by the experimental results which I now proceed to describe.

The Experiments.

My first object in making these experiments was to ascertain if, and how, oiling the surfaces in contact affected the resistance to rolling.

The apparatus employed consisted of a wooden slab or table-top supported on three set-screws for legs, so that it could be tipped in any required direction. On this table rested one of WHITWORTH's surface-plates. On the surface-plate was placed a surveyor's level, which read divisions to the thousandth of a foot on a staff erected at 50-feet distance; also a bell-glass covered another part of the surface-plate in the manner of the receiver of an air-pump, which could be filled with oil from an aperture in the top. This glass served either to protect the roller from dust or surround it with oil, and thus prevent any surface-tension the oil might exert affecting the results.

The roller was of cast iron, 6 inches in diameter and 2 inches thick, and weighed about 14 lbs. It was not cylindrical, for the edge was somewhat rounded. Originally the roller was turned up so that the edge was curved to a radius of 1 foot; but subsequent grinding somewhat modified this shape.

In the first instance the roller was turned up and polished in the ordinary manner; but some preliminary experiments showed that the surface thus formed was far from perfect, as indeed was apparent when it was examined with a magnifying-glass. The roller was therefore again turned and ground very carefully with Turkey-stone for several days, until the surface appeared through the glass to be as perfect as the iron would allow; there were still some small pits, but these appeared to be in the iron itself.

The roller when thus finished was rolled on various surfaces. First of all it was tried on the cast-iron surface-plate already mentioned; but this surface, which had been formed by scraping, was altogether too rough. Thus when the roller was placed on the plate it immediately rolled into a hollow. Surfaces were then formed by grinding

two plates together with powdered Turkey-stone. In this way the plates were made so true that the roller would remain in any position, and would roll either way with an inclination of 1 in 5000, or about 1 foot in a mile. It appeared impossible, however, to produce surfaces altogether free from inequalities, which may be seen from the results of the experiments.

The effect of Oiling the Surface.

In the first experiments the surface on which the roller was to roll was brought into a level position, so that the roller when placed on it remained at rest. A line of sights, consisting of a mark on the glass and a pin-hole in a plate fixed at some distance, was then brought to bear on a mark on the top of the roller, so that the least motion could be detected, and the position of the roller could be recovered after it had been allowed to roll in one direction. The level was then adjusted to read zero on the staff, and the table tipped until the roller rolled off in one direction. The reading of the level was then noted, and the same operation repeated in the opposite direction, the roller having in the mean time been brought back into its former position. Sundry observations were then taken with different points of the plane and roller in contact. After a considerable number of observations had thus been taken oil was poured into the glass until the roller was covered, and then the observations were repeated. Table I. shows a series of

TABLE I.—Cast-iron Roller on Plate-glass. (The distance of the Staff from the Object-glass of the Level =50 feet. The Divisions on the Scale $= \frac{1}{100}$ foot.)

	Clean.			Oiled.		
	Readings.		Difference.	Readings.		Difference.
	To.	From.		To.	From.	
Starts from rest.	−5·0	1·2	6·2	−5·0	3·2	8·2
	−2·3	3·5	5·8	−3·3	2·5	5·8
	−2·6	2·0	4·6	−4·0	2·0	6·0
	−4·5	1·4	5·9	−4·1	1·0	5·1
	−4·7	2·0	6·7	−1·8	4·0	5·8
	−2·8	3·5	6·3	−3·0	2·0	5·0
	−3·2	4·5	7·7	−5·8	0·5	6·3
	−4·0	3·4	7·4	−5·2	0·3	5·5
	Mean		6·3	Mean.		5·9
Rolls back when set in motion.	−2·6	−0·7	1·9	−3·0	−0·4	2·6
	−3·5	−1·5	2·0	−1·8	+1·0	2·8
	−3·5	−2·0	1·5	−2·5	0·0	2·5
	−4·2	−2·2	2·0	−2·5	+0·2	2·7
	−1·5	+0·6	2·1	−3·9	−1·0	2·9
	−2·5	−0·5	2·0	−3·0	−0·8	2·2
	−1·9	0·0	1·9	−4·4	−2·0	2·4
	−0·7	+1·5	2·1	−1·0	+1·5	2·5
	Mean.		1·9	Mean.		2·6

such observations for a surface of plate-glass both with and without oil. In these particular experiments, however, the surface was simply oiled, it having been found by experience that the effect was the same as when the glass was filled with oil. It will be seen that in these experiments the advantage is slightly in favour of the oiled glass.

The results contained in the second part of this Table were obtained by starting the roller in one direction against the inclination of the plane with just sufficient velocity to carry it up to a certain point, the inclination of the plane being adjusted until it would roll back. In this way the advantage is against the oil. This, however, I think is due to the surface-tension or fluid-friction arising from the motion of the roller.

There is a very marked difference between these inclinations and those required to start the roller from rest, a difference which appears to exist with all the materials tried, and which I think is only in part explained by the roughness of the surface.

In these experiments with a surface of glass the friction was so small that the inequalities of the surface rendered the results very irregular and uncertain. To obviate this a surface of box-wood cut across the grain was next tried. This, being softer, allowed the roller to indent it more than the glass and gave rise to greater friction, and hence the inequalities in the surface are less apparent in the results, which are shown in Table II. These observations were made in the same way as those with the glass, except that *blacklead* was substituted for oil. The effect of the blacklead seems to

TABLE II.—Cast-iron Roller on Box-wood.

	Clean.			Blackleaded.		
	Readings.		Difference.	Readings.		Difference.
	To.	From.		To.	From.	
Starts from rest.	− 3·0	+ 5·0	8·0	−4·8	+ 6·0	10·8
	− 3·0	+ 8·0	11·0	−3·2	+ 7·8	11·0
	− 4·0	+ 5·8	9·8	−7·6	+ 2·4	10·0
	− 4·0	+ 6·0	10·0	−0·5	+ 8·0	8·5
	−10·0	0·0	10·0	+0·8	+10·0	9·2
	+ 3·4	+12·8	9·4	+1·0	+ 8·9	7·9
	− 4·0	+ 7·0	11·0	−9·0	− 1·2	7·8
	− 3·2	+ 8·0	11·2	−9·8	− 1·0	8·8
	Mean ...... 10·05			Mean........ 9·25		
Rolls back when set in motion.	+7·0	+12·2	5·2	−1·0	+1·0	3·5
	−5·0	+ 0·4	5·4	−1·2	+1·2	4·0
	−2·0	+ 4·2	6·2	0·0	+2·0	2·0
	−2·1	+ 3·8	5·9	+2·0	+5·0	3·0
	+3·2	+ 9·0	5·8	+1·2	+4·0	2·8
	+1·3	+ 7·0	5·7	+3·0	+6·2	3·2
	−2·0	+ 4·0	6·0	−6·4	−3·0	3·6
	−2·6	+ 2·9	5·5	−7·6	−3·0	4·6
	Mean........ 5·71			Mean........ 3·34		

have been slightly to diminish friction, not only when starting from rest, but when rolling back, which confirms me in the opinion that the contrary result with oil was due to its obstructive action.

India-rubber was then tried. A plate of this substance three eighths of an inch thick was glued to a piece of wood to prevent it working forward. The results are shown in Table III. The friction was very much greater than in the previous experiments, and the advantage lies with the clean surface.

TABLE III.—Cast-iron Roller on India-rubber.

	Clean.			Blackleaded.		
	Readings.		Difference.	Readings.		Difference.
	To.	From.		To.	From.	
Starts from rest.	−22·0	+14·0	36	−24	+18	42
	−28·0	+15	43	−19	+15	34
	−12·0	+18	30	−18	+19	37
	−19·0	+15	34	−23	+17	40
	−16·0	+16	32	−22	+17	39
	−18·0	+15	33	−23	+14	37
	−25·0	+12	37	−25	+17	42
	−23·0	+15	38	−24	+15	39
	Mean........		35·3	Mean		38·75
Rolls back when set in motion.	−2	+28	30	0	+26	26
	−5	+26	31	− 2	+22	24
	−6	+27	33	− 1	+25	26
	−4	+28	32	− 2	+22	24
	−3	+30	33	−10	+22	32
	−4	+30	34	−14	+19	33
	−6	+24	30	− 6	+24	30
	−7	+25	32	− 6	+23	29
	Mean........		31·8	Mean		28

These results leave no doubt that rolling-friction does not depend greatly on the coefficient of sliding-friction between the roller and the surface. They are, however, completely in accordance with the explanation previously given of the manner in which sliding-friction acts to prevent the deformation of the surfaces at the point of contact.

The Tendency to Oscillate.

Another circumstance which was observed while making these experiments also offers strong evidence of this deformation, namely the tendency which the roller has to oscillate. This was always exhibited whenever the roller was slightly disturbed from rest on the level plane, and it was certainly not due to the fact of its having settled into a hollow; for when on india-rubber it would make several considerable oscillations in *whatever*

position it was placed. By blackleading the surface this tendency was considerably reduced, although not altogether destroyed. These oscillations could not have been caused by the mere resistance which the one surface offered to the sliding of the other over it, unless also this resistance threw the surfaces into constraint from which they are constantly endeavouring to free themselves.

The effect of the Softness of the Materials.

Having found that oil did not reduce the resistance, the experiments were continued with a view to ascertain how far the softness of the material had any thing to do with it. As materials of several degrees of softness had already been tried, the only question was to settle how far the difference in the results observed was due to their softness and how far it might be due to some other difference in their nature. To show this cast iron and brass were tried, which are of much the same hardness as glass, and yet of an altogether different nature in other respects, the surface of the glass being highly polished, while that of the metal was dull as it had been left by the grinding. The results of these experiments are contained in Tables IV. and V.

TABLE IV.—Cast-iron Roller on Brass.

	Clean.			Oiled.		
	Readings.		Difference.	Readings.		Difference.
	To.	From.		To.	From.	
Starts from rest.	−13·2	−5·5	7·7	−2·0	+3·8	5·8
	− 5·5	+2·0	7·5	−4·5	+1·2	5·7
	− 3·2	+5·0	8·2	−2·8	+3·8	6·6
	− 3·5	+4·5	8·0	−2·9	+5·2	8·1
	− 3·5	+3·8	7·3	−1·7	+5·8	7·5
	− 7·0	+1·5	8·5	−5·0	+1·0	6·0
	− 5·0	+2·2	7·2	−3·0	+3·5	6·5
	− 4·6	+3·0	7·6	−3·0	+2·9	5·9
	Mean		7·75	Mean........		6·5
Rolls back when set in motion.	−2·4	−0·8	1·6	−1·5	+1·0	2·5
	−2·4	−0·4	2·0	0·0	+1·8	1·8
	−1·8	+0·7	2·5	−1·2	+1·6	2·8
	−2·0	−0·4	1·6	−2·0	+0·6	2·6
	−2·8	−1·0	1·8	−2·3	+0·5	2·8
	−2·5	+0·2	2·7	−2·0	+1·0	3·0
	−0·2	+2·0	2·2	−1·2	+1·5	2·7
	−2·2	0·0	2·2	−0·9	+1·6	2·5
	Mean		2·07	Mean		2·58

The means of the results for all the materials are contained in Table VI. Comparing these we see at once the effect of softness: the cast iron, brass, and glass are very nearly the same, and the slight difference is not greater than may be accounted for by a slight

TABLE V.—Cast-iron Roller on Cast Iron.

	Clean.			Oiled.		
	Readings.		Difference.	Readings.		Difference.
	To.	From.		To.	From.	
Starts from rest.	−6·5	+0·3	6·8	−1·3	+4·0	5·3
	−2·8	+2·4	5·2	−2·8	+2·5	5·3
	−2·6	+3·5	6·1	−3·5	+2·5	6·0
	−2·5	+2·3	4·8	−2·5	+3·8	6·3
	−0·6	+4·5	5·1	−2·2	+3·2	5·4
	−0·9	+3·9	4·8	−2·3	+3·0	5·3
	−3·0	+2·5	5·5	−5·0	+0·8	5·8
	−2·8	+4·2	7·0	+1·0	+6·5	5·5
	Mean		5·66	Mean		5·61
Rolls back when set in motion.	+4·0	+6·5	2·5	0·0	+2·3	2·3
	−0·7	+1·6	2·3	−1·8	+0·8	2·6
	−3·5	−0·8	2·7	−1·0	+1·3	2·3
	−3·8	−1·0	2·8	+0·2	+2·3	2·1
	−0·5	+1·8	2·3	0·0	+2·2	2·2
	−2·0	+0·1	2·1	−0·6	+2·0	2·6
	−0·8	+2·2	3·0	−0·6	+1·8	2·4
	−1·3	+1·6	2·9	+0·5	+2·9	2·4
	Mean		2·57	Mean		2·36

difference in the smoothness of the surfaces. Of the three, according to hardness cast iron should have given the least results; and so it does, as far as starting from rest is concerned, although when rolling back the result is the other way. Box-wood appears to offer about double the resistance of cast iron; and india-rubber about ten times as much in the case of rolling back, and six times as much in starting from rest.

TABLE VI.—Showing the Mean of the Results for the various conditions of the Surface and manner of Starting.

The nature of the Surface.	Starts from rest.		Started in the opposite direction.		Mean.
	Clean.	Oiled or blackleaded.	Clean.	Oiled or blackleaded.	
Cast iron	5·66	5·61	2·57	2·36	4·05
Glass	6·32	5·96	1·93	2·56	4·19
Brass	7·75	6·53	2·07	2·587	4·73
Box-wood	10·05	9·25	5·71	2·34	7·09
India-rubber	35·37	38·75	31·87	28·00	33·24

Experiments on Actual Slipping.

My object in the second series of experiments was to find by actual measurement how far the roller rolled short of its geometrical distance. Since the exceedingly small slipping on a hard surface precluded all chance of measuring it, these experiments were made on strips of india-rubber glued to wood: these were in general long enough to allow of two complete revolutions of the roller. The strips were of different thicknesses. This difference of thickness has an effect to vary the degree of indentation and the intensity of the pressure, as well as the lateral extension. On the thick india-rubber the indentation was considerable; and, owing to the large bearing-surface thus obtained, the intensity of the pressure beneath the roller must have been comparatively small, as must also the lateral extension; whereas with the thin strips the indentation was small, but the pressure and consequent lateral extension must have been correspondingly great. These considerations serve to explain the differences in the results of the experiments, which are given in Tables VII. and VIII.

TABLE VII.—Showing the Actual Slipping of a Cast-iron Roller.

The nature of the Surface.	The distance travelled.		The amount of the slipping.
	In one revolution.	In two revolutions.	
A steel bar (polished)	17·82	35·64	·00
India-rubber, 0·015 inch thick, glued to wood	....	35·2	·44
Ditto, 0·08 inch thick	....	34·8	·84
Ditto, 0·36 inch thick	....	35·15	·49

TABLE VIII.—Showing the Actual Slipping with an India-rubber Tire 0·75 inch thick glued on to the Roller.

The nature of the Surface.	Distance travelled in one revolution.	Circumference of the ring.	The amount of the slipping.
A steel bar	22·55	22·5	−0·05
India-rubber 0·156 inch thick (clean)	22·55	,,	−0·05
,, ,, (blackleaded)	22·55	,,	−0·05
,, 0·08 inch thick (clean)	22·5	,,	0·0
,, ,, (blackleaded)	22·52	,,	−0·02
,, 0·36 inch thick (clean)	22·39	,,	+0·11
,, ,, (blackleaded)	22·42	,,	+0·08
,, 0·75 inch thick (clean)	22·4	,,	+0·1
,, ,, (blackleaded)	22·4	,,	+0·1

These experiments show that a hard roller on a soft surface rolls short of its geometrical distance, whereas a soft roller on a hard plane rolls more than its geometrical distance, but to a smaller degree, and that when the roller and the plane are of equal hardness the roller rolls through less than its geometrical distance, which results are in exact accordance with what has previously been explained.

The effect of Heat and Viscosity to cause Friction.

While making the experiments which have been described, two other causes of resistance to rolling besides friction suggested themselves to me, and were to a certain extent verified. The first of these is the transference of heat which takes place within both the plate and the roller in the neighbourhood of the point of contact. As the roller moves forward it is continually compressing the material in front of the point of greatest pressure, and this material expands again so soon as the roller is past. During compression there will be a change in the temperature of the material compressed, which change will be readjusted again as the material expands, supposing that in the interval between compression and expansion there has been no heat communicated to or taken from the portion of material affected. But since the change of temperature caused by compression will place the part compressed out of accord with that immediately surrounding it, a transference of heat will necessarily take place. The quantity of heat thus transferred will depend on the length of the interval, *i. e.* the speed of the roller, and on the conducting-power of the material.

This transference will cause resistance to the roller, for the material will not expand to the same temperature, and hence to the same volume, as that from which it was compressed, and hence it will take more work to compress it than it will give out in expanding.

It does not, however, follow that the greater the transference of heat the greater the resistance; for if a sufficient time be allowed the transference of heat will readjust the temperature as fast as expansion takes place. There is some speed, therefore, for which the resistance arising from this cause will be a maximum. If, therefore, the material be a good conductor and the motion slow, the transference of heat will prevent any variation of temperature during either compression or expansion. When such was the case the resistance would increase with the speed, a fact which was very evident when the rolling took place on india-rubber; for it was possible to give the plane such an inclination that the motion of the roller was scarcely perceptible, and any increase in the inclination was followed by a corresponding increase in the speed of the roller.

As already stated, there is another cause of resistance; and this may partly explain the result: this is viscosity.

If we stretch a piece of india-rubber, or any material, when released it does not *immediately* come back to its original length, but at once comes back a certain distance and then recovers the rest more or less slowly. Hence as the roller moves forward the compressed material will require time for its complete expansion, and hence will offer less resistance to the roller when the motion is slow than when it is rapid.

Conclusion.

The foregoing remarks must be regarded as relating only to the *nature* of rolling-friction. I have not attempted to ascertain the laws which connect its magnitude with the various circumstances which affect it. As far as they go I can see no reason to doubt

the two laws propounded by Coulomb, viz. that for the same material the resistance to rolling is proportional to the weight of the roller, and inversely proportional to its diameter. In addition to these laws, however, it appears clear to me that there must be another law connecting rolling-friction in some way with the softness of the tires of the wheels and the road. In addition to the instance of india-rubber tires already mentioned, there are several other phenomena connected with wheels which point to such a law, and can be explained by the recognition of the slipping under the roller.

Steel and Iron Rails.

The very great advantage in point of durability of steel rails over iron has been a matter of much surprise, it not being sufficiently accounted for by the greater hardness of the steel, supposing it to be subjected to the same wearing action as the iron. This is at once explained, however, by the recognition of the fact that hardness tends to reduce the slipping and hence the wearing action, as well as to enable the rail the better to withstand the wear to which it is subjected.

That rails should wear at all in places where they are straight and where brakes are not applied is a matter which calls for an explanation, and this, so far as I am aware, has not hitherto been given; mere crushing, however much it might deform the rail, would not cause such a reduction of weight as actually takes place. The explanation of this phenomenon also at once follows the recognition of the slipping which attends rolling.

A little consideration also serves to show that the scaling of wrought-iron rails is the result of the repeated lateral extension of the surface of the rail under the action of the wheel. The systematic way in which this takes place shows that it is due to something more than the mere imperfection in the iron. There is no doubt that the grain of the iron has a great deal to do with it; but considering the multitudinous ways in which iron is used and that this is the only one in which scaling takes place, it is clear that it must be due to some cause directly connected with the action to which the rail is subjected. Now every time a wheel passes over a point in a rail it tends to slide the upper strata of the rail over those beneath them, and thus causes tangential stress. If the rail were homogeneous this would hardly cause it to scale; but owing to the grain in the iron some strata are stronger than others, and the weaker strata are called upon to do more than their share of the yielding, and so become still weaker and eventually give way.

There are other phenomena which, having been hitherto unnoticed or unexplained, might be shown to arise from the slipping which takes place during rolling; but perhaps those I have mentioned are sufficient to show that the effects of the action are not altogether without practical importance.