VII. *Experiments, by the Method of* LORENTZ, *for the further Determination of the Absolute Value of the British Association Unit of Resistance, with an Appendix on the Determination of the pitch of a Standard Tuning-Fork.*

By Lord RAYLEIGH, *F.R.S., Cavendish Professor of Experimental Physics in the University of Cambridge, and Mrs.* H. SIDGWICK.

Received December 8, 1882,—Read January 11, 1883.

§ 1. IN this method, which was employed by LORENTZ in 1873,* a circular disc of metal is maintained in rotation at a uniform and known rate about an axis passing through its centre, and is placed in the magnetic field due to a battery current which circulates through a coaxal coil of many turns. The revolving disc is touched at its centre and circumference by two wires. If the circuit were simply closed through a galvanometer, the instrument would indicate the current due to the electromotive force of induction acting against the resistance of the circuit. The electromotive force corresponding to each revolution is the same as would be generated in a single turn of wire coincident with the circumference of the disc by the formation or cessation of the battery current. If this be called γ, and M be the coefficient of induction between the coil and the circumference, m the number of revolutions per second, the electromotive force is $m\mathrm{M}\gamma$. In the actual arrangement, however, the circuit is not simply closed, but its terminals are connected with the extremities of a resistance R, traversed by the battery current, and the variable quantities are so adjusted that the electromotive force $\mathrm{R}\gamma$ exactly balances that of induction. When the galvanometer indicates no current, the following relation, independent, it will be observed, of the magnitude of the battery current, must be satisfied—

$$\mathrm{R}=m\mathrm{M}\,;$$

and from this, M being known from the data of construction, the absolute resistance R of the conductor is determined.

One of the principal difficulties to be overcome arises from the smallness of the resistance R, necessary for a balance, even when m and M are both increased as far as possible. LORENTZ employed three resistances, ranging from ·0008 to ·002 of a mercury unit, and he evaded the necessity of comparing these small resistances with ordinary standards by constructing them of actual columns of mercury. His result was accordingly obtained directly in terms of mercury, and was to the effect that

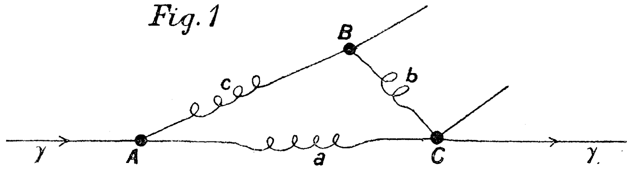
* POGG. Ann., cxlix., p. 251.

$$1 \text{ mercury unit} = {\cdot}9337 \times 10^9 \text{ C.G.S.}$$

differing nearly 1 per cent. from the value (·941) obtained by ourselves.

§ 2. Under the conviction that this method offers in some respects important advantages, and influenced also by the fact that the arrangements for producing and measuring the uniform rotation necessary were ready to our hands, we determined to give it a trial, in the hope of obtaining confirmation of the results already arrived at by ourselves and by GLAZEBROOK with other methods. At first the intention was to follow LORENTZ in using for the resistance a glass tube full of mercury, with two points of which contact would be made by platinum wires passing through the glass. It appeared, however, that there would be difficulty in making the measurements with the degree of accuracy aimed at. If the wires were sealed into the glass, the section would probably be rendered irregular. An attempt was made to avoid this difficulty by using a tube from which the ends had been cut with the aid of heat. After small nicks had been filed sufficiently deep to receive the platinum wires, the ends were replaced in their original positions and secured with shellac. In this way a satisfactory uniformity of section near the points of derivation could be attained, but the measurement of the distance between these points, which is required to be known with full accuracy, was rendered difficult by the presence of the cement. It is possible that these difficulties might have been overcome, but at this point a method of shunting occurred to us, allowing the use of mercury to be dispensed with. Merely for the purpose of connecting the mercury unit with the B.A. unit or other standard of resistance, it would not be desirable to use tubes of such large bore.* This problem may more conveniently be taken by itself, and has already been treated by us in a former communication to the Society.†

Fig. 1



§ 3. In the shunt method the greater part of the main current γ passes on one side through a relatively small resistance a (see fig. 1), and the difference of potentials at the points of derivation B, C, is due to the passage of a small fraction only of the total current, the resistance $(b+c)$ being great compared with a. If at the same time b be small relatively to c, the difference of potentials is doubly attenuated. Its value for a given main current γ is found at once from the consideration that the current divides itself between the two branches in the inverse ratio of the resistances. The current through b is thus $\frac{a}{a+b+c}\gamma$, and the difference of potentials at the points

* If the distance between the points of derivation were 1 metre, R=·002 mercury unit would require a section equal to 500 square millims.

† Phil. Trans., 1883, p. 173.

of derivation is $\frac{ab}{a+b+c}\gamma$. The quantity $\frac{ab}{a+b+c}$ thus takes the place of R in the simple formula, and is called the effective resistance. By taking for instance $a=\frac{1}{2}$, $b=1$, $c=100$, we get an effective resistance of about $\frac{1}{200}$; and the resistances employed may be those of ordinary resistance coils, capable of accurate comparison with the standards.

§ 4. In designing the apparatus we were influenced by the fact that we had at our disposal two very suitable coils of large radius, wound some years ago by Professor CHRYSTAL, the same in fact as were used by Mr. GLAZEBROOK in his investigation by another method. By bringing the two coils close to one another and to the plane of the disc, the inductive effect is rendered a maximum. This arrangement accordingly was the one first experimented with, as being the most likely to prove successful.

The diameter of the disc is limited by two considerations. If it be too small, the whole inductive effect, and with it the sensitiveness of the arrangement, suffers. On the other hand if it be too large, the circumference enters the more intense region of magnetic force which lies near the wire, and the coefficient of induction changes its value rapidly when any alteration occurs in the mean radius of the coils, or in the diameter of the disc, and thus the final result becomes too sensitive to errors in the magnitudes of these elements. In the Phil. Mag. for November, 1882, the reader will find a calculation of the values of M for various cases, and a general comparison of the principal methods for determining absolute resistance especially in respect of errors arising in connexion with the fundamental linear measurements. For the experiments now to be described, the diameter of the disc was chosen so as to be somewhat more than half that of the coils (§§ 22, 23).

§ 5. The disc was of brass and turned upon a solid brass rod as axle. This axle was mounted vertically in the same frame that carried the revolving coil in the experiments described in a former communication to the Society* (see Plate 48), an arrangement both economical and convenient, as it allowed the apparatus then employed for driving the disc and for observing the speed to remain almost undisturbed. The coils were supported horizontally upon wooden pieces screwed on the inner side of the three uprights of the frame.

During the earlier trials, extending over the month of May, 1882, the edge of the disc was bevelled, and contact was made with it by means of a brush of fine copper wires held in a nearly vertical position. No sufficiently regular results could be obtained until the sliding surfaces were amalgamated, and even then there were discrepancies between the work of one day and that of another, whose cause was not discovered until a later period. It soon became manifest, however, that the bevelled edge would not answer the purpose, for it cut its way by degrees into the wires of the brush in such a manner as to render the effective radius uncertain. The substitution of a cylindrical for a bevelled edge promised better results. The width of the edge

* Phil. Trans., Part II., 1882.

(equal to the thickness of the disc) was $4\frac{1}{2}$ millims. and allowed sufficient room for the contact of the brush though placed tangentially. In this way broader bearing surfaces were available, and the small extension of the contact in the direction of the axis is unobjectionable, provided everything is arranged symmetrically with respect to the middle plane of the disc.

As will presently appear, the success of the method is independent of any constant thermo-electric force at the sliding contact, but it is evident that good readings cannot be taken if the thermo-electric force changes its magnitude often and suddenly. It was found advisable to renew the amalgamation of the edge at the commencement of each day's work. The excess of mercury, if any, attaches itself to the brush, and does not appear to render the diameter of the disc uncertain.

The inner contact was made in a similar manner by a brush pressing against the shaft itself at a place a little below that at which the disc was attached. The coefficient of induction to be employed in the calculation is the difference between the coefficients for the coil and the outer and inner circles of sliding contact respectively, but the latter is quite subordinate (§ 25).

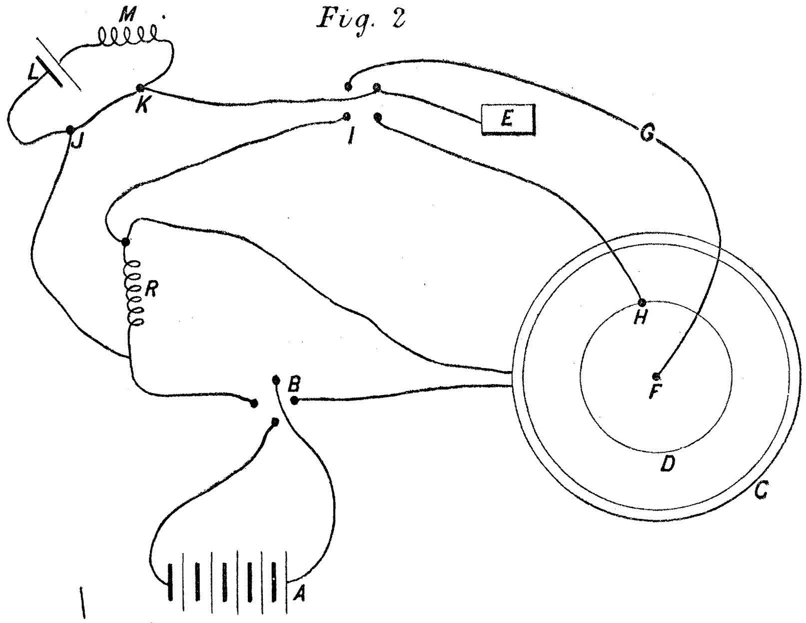
§ 6. The disc was driven by the same water-engine that was employed for the revolving coil of former determinations,* the connexion being made by a long cord passing round a wooden pulley attached to the lower part of the shaft. To the upper face of the disc was cemented a circle of paper on which were marked a series of circles of alternately black and white teeth. One observer looking through the prongs of an electro-magnetically maintained fork regulated the speed of the disc by application of the necessary friction to the driving-cord, which passed through his fingers. When one of the series of circles is seen to be stationary, a simple and easily expressed relation is established between the frequency of the fork and that of revolution. At intervals the number of beats per minute is counted between the notes of a standard fork, and (the octave of) the electric fork. There is no difficulty in thus determining the speed of rotation to within one part in 10,000. With respect to the absolute pitch of the standard fork itself, see the Appendix to this Memoir.

§ 7. When the disc is caused to rotate, and the galvanometer circuit is closed, a deflexion is observed, although the battery which generates the main current is not in action. This deflexion is due to two causes—thermo-electric force at the sliding contact, and induction dependent upon the vertical component of the earth's magnetism. Although not a direct source of error, this deflexion is better avoided, both for convenience in reading the galvanometer and because it implies the actual passage of a not insensible current through the sliding contacts and thus brings into consideration the *resistance* of these contacts. The compensation was effected by the introduction of an opposing electromotive force; for which purpose two terminals of the galvanometer circuit J, K, fig. 2, instead of being connected directly, were attached by binding screws to two points on a stout copper wire forming part of a circuit which

* Proc. Roy. Soc., May 5, 1881, p. 112; Phil. Trans., Part II., 1882.

included a sawdust DANIELL (L) and a resistance coil of 100 ohms (M). By shifting one of the binding screws, the galvanometer reading, in the absence of the main battery current, and after attainment of the proper speed, was made to be nearly the same as when the galvanometer contact was broken.

Fig. 2



§ 8. The general plan of the connexions and the *modus operandi* will now be intelligible from fig. 2. The poles of the battery A, consisting of 20 DANIELL cells, were connected with a mercury reversing key B, the two positions of which were distinguished by the letters E and W (east and west). From thence the current passed through the induction coils C and the equivalent resistance R, of which the details are reserved for the moment. The reflecting galvanometer, G, is placed at a considerable distance in order to avoid the direct influence of the coils, and is connected with the inner sliding contact, F. Its resistance is about $\frac{1}{2}$ ohm; and by the aid of the compensating magnet the vibrations of the needle were made slow enough to be readily observed. The terminals of the galvanometer branch, which includes also a commutator, I, are connected to the extremities of the resistance, R.

If, while the disc is maintained in uniform rotation, the reading of the galvanometer is the same whichever way the battery key may stand (correction being made, if necessary, for a direct effect upon the needle), it is a proof the contemplated balance is actually attained. In this way all disturbance from the earth's magnetism, and from thermo-electric forces whether situated at the sliding contacts, or within the resistance coils of which R is composed, or at any other part of the galvanometer circuit, is eliminated from the result. The adjustment is effected by varying a comparatively large resistance, taken from a box, and placed in multiple arc with one of the components of R.

§ 9. In actual work, however, it is not necessary, or even desirable, to hit off the balance with great accuracy. An unmistakable difference of readings when the battery key is put over, is rather an advantage than otherwise, as giving an indication that the circuits are properly closed. The plan adopted was to take a series of readings of the effect (E—W) of reversing the battery current with an effective resistance R_1, not very different from R. Single readings were liable to considerable irregularity in consequence of change in the friction at the sliding contacts, and of momentary variations in the speed. These errors cannot possibly be systematic, and are in great measure eliminated in the mean of a series. Having thus obtained the difference of galvanometer readings (E—W) corresponding to R_1, we altered the resistance in multiple arc so as to change R_1 into R_2, the difference being some such fraction as $\frac{1}{100}$ of the whole, and in such a direction that the sign of E—W is changed. The two series give by simple interpolation (after correction for the direct effect) the true value of R, that is the effective resistance corresponding to the balance. In order to get the best result relatively to the time occupied, the number of observations of E—W in each set was taken roughly in inverse proportion to the values. To diminish the influence of a progressive change in the strength of the battery current, the observations with R_2 were interspersed between those with R_1 as effective resistance. The readings were usually taken continuously, with no more delay than was necessary to allow the vibrations of the needle to become of moderate extent after each change. When they were completed, the driving cord was reversed, as well as the commutator, I, and a similar set of observations was taken with rotation in the opposite direction.

§ 10. In the earlier experiments the resistance coils composing the effective resistance were arranged as in fig. 1, in which A, B, C may be supposed to represent mercury cups, the bottoms of which were formed of amalgamated copper discs. On these discs rested the amalgamated terminals of the various resistance coils and connecting wires. The shunt a consisted of two unit coils in multiple arc, between which the greater part of the main current was equally divided. The magnitude of the main current was less than $\frac{1}{10}$ ampère. The resistance b between the points of derivation was a unit, while the third resistance c was alternately 105 and 106.

In reckoning the resistance of the galvanometer circuit we have to include b. The remainder scarcely exceeds the $\frac{1}{2}$ ohm due to the galvanometer itself. It appears therefore that the deflections obtained with the arrangement described are only one-third part as great as they would be if a quite small resistance were substituted for the unit in b. As the sensitiveness appeared likely to be inadequate, we afterwards replaced the unit by $\frac{1}{10}$, using for c a coil of ten units. As in this case the addition or subtraction of a whole ohm in c would make too great a difference, the adjustment was obtained by varying a comparatively large resistance placed in multiple arc with a.

In the light of subsequent experience it is doubtful whether this change was an

improvement. The increase of galvanometer deflection was not really of much advantage, since the difficulty of getting sharp results arose from electromotive disturbances, and these were magnified in the same proportion. It would probably have been better to have retained the unit in b, and to have replaced the galvanometer by one of higher resistance.

§ 11. Preliminary trials having given apparently satisfactory results, we proceeded to make regular series of observations in the manner already described. We had not gone far before anomalies revealed themselves of such a character as to prove that we were not yet masters of the method. It usually happened that each day's observations agreed well together, showing that the sensitiveness was sufficient; but when we came to compare the results obtained on different days unaccountable discrepancies became apparent. The first result of the more severe criticism to which the arrangements were then subjected was to show that sufficient thought had not been given to the question of insulation. The wire composing the induction coils, or rather one extremity of it, is necessarily at a high potential, and a very moderate leakage from the coils to the frame, and thence to the disc, might cause great disturbance. Some such leakage was in fact detected on application of appropriate tests. Ebonite insulation was accordingly introduced into the supports of the coils. The battery was carefully insulated from the ground, as was also the frame carrying the revolving disc, and other precautions were taken which it is unnecessary here to detail. For the sake of definiteness one point of the galvanometer commutator was connected to earth. With these improvements tests were satisfied more severe than that of actual use, and these tests were renewed at intervals during the spinnings.

The results however still showed that some defect existed which we had not yet succeeded in detecting. It made no appreciable difference which way the disc rotated, but the means of different days' work failed to exhibit the desired accordance. Two months' work had already been spent upon the experiments, and we had begun to despair of a satisfactory issue, when it occurred to us that the connexion of the coils for compounding the effective resistance was faulty.

§ 12. By reference to fig. 1 it will be seen that the main current traverses part of the cup C, and that part of the same cup is also included in b. Now, although for all ordinary purposes the resistance of the parts of the cup might be neglected, in the present case it is the small effective resistance R with which it comes into comparison. If we aim at an accuracy of $\frac{1}{10000}$, we cannot afford to overlook a resistance entering in this manner, even though it may not exceed $\frac{1}{2000000}$ ohm. The discrepancies were doubtless due to small differences in the position of the wires and coils in cup C, moved as they were from day to day in order to verify the soundness of the contacts.

In order to avoid the difficulty we have only to take care that no part of b can possibly be traversed by the main current, and this is easily done by the introduction of another mercury cup. Fig. 3 shows the arrangement adopted. The main current enters at the cups A and D, and the greater part is taken by the two unit coils in

multiple arc whose terminals rest in these cups. The galvanometer terminals are led into two other cups B and C. The ends of these are beaten flat and the legs of the $\frac{1}{10}$ rest upon them. The connexion between C and D was through a stout copper rod, which may be regarded as part of c. For the first series the connexion between A and B was through a single coil of 10 units' resistance, replaced in subsequent series by other coils giving altogether 16 and 20 units' resistance respectively.

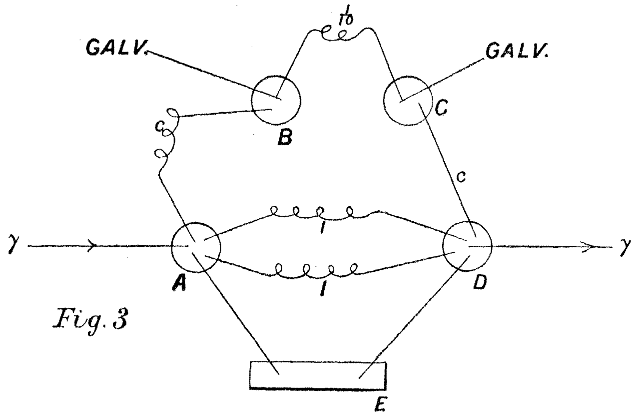


Fig. 3

To make the necessary adjustment and variation of resistance, a box, E, was placed in multiple arc with the two unit coils. The resistances taken from the box were afterwards carefully determined, but they enter into the final results in quite a subordinate manner.

§ 13. Further trials now led to the satisfactory conclusion that the defect was remedied, for the means obtained on different days agreed well together, even although the resistance coils were taken down and remounted in the interval. As we had now every reason to suppose that our experiments would have a successful issue, we proceeded to make the final adjustments preparatory to a complete series of observations.

In the first and second series the two coils were near one another, separated only by three slips of glass, and held firmly together by wooden clamps. The adjustments presented no particular difficulty. By means of an iron finger clamped to the disc and carried gradually round, it could be verified that the coils and disc were concentric and in parallel planes. The coils were gradually wedged into their places, and secured when their mean planes occupied the desired symmetrical positions relatively to the disc. It is evident that errors of maladjustment influence the result only in the second order.

§ 14. Experience in this series having shown that the arrangement was satisfactory, and that the sensitiveness was fully sufficient, we proceeded to make a second series of observations without displacement of the induction coils, but at a speed of rotation lower than before in about the ratio of 16 : 10. This, of course, entailed a corresponding change in R, which was effected by increasing the component c. An agreement between the final results of the two series would give an important con-

firmation, inasmuch as leakage of electricity from the main circuit into the galvanometer branch would exert a different influence in the two cases. The observations were not reduced until some time afterwards, and it then appeared that the agreement was even better than it would have been reasonable to expect.

§ 15. The final number, $\cdot 9867 \times 10^9$, expressing the value of the B.A. unit in absolute measure as determined by these two series of observations, is almost identical with that previously obtained by ourselves, and by GLAZEBROOK, using other methods. With respect to the independence of these determinations, the only thing calling for notice is the fact that the same induction coils were employed both by GLAZEBROOK and in the present investigation. In other respects there has been, we believe, scarcely any point of contact. But it is evident that an error in the measurements of mean radius of these coils must propagate itself into both results. The point to which we now wish to direct attention, is that the error of mean radius will influence the final number in *opposite directions*. In the method employed by GLAZEBROOK, an under-estimate of the mean radius would lead to an under-estimate of the induction coefficient, whereas with us it would lead to an over-estimate of that quantity. So far, therefore, as the error of mean radius is concerned, it would appear that the use of the same coils is far from impairing the value of the results. Even with respect to the number of turns, an error, if that be supposed possible, would affect the results in a different manner, for GLAZEBROOK was concerned with the *product* of the numbers for the two coils, while we evidently are concerned with the *sum*.

§ 16. In researches of this kind it is proper to calculate the influence upon the result of errors in the fundamental measurements. The value of M depends upon three linear quantities: the radius of the disc (a), the mean radius of the two coils (A), and the distance between their mean planes ($2b$). In the present case, however, the latter element enters in a very subordinate degree. From § 25 it appears that

$$\frac{dM}{M} = -1 \cdot 4 \, \frac{dA}{A} + 2 \cdot 4 \, \frac{da}{a}.$$

It has been shown* that these conditions compare favourably with those of most of the other methods that have been employed. From its nature a is much more easily measured than the diameter of a coil.

§ 17. The results deduced from the several days observations, when corrected for slight variations of temperature of the resistance coils, &c., exhibit a remarkable accordance. By reference to the tables (§ 27) the reader will see that the maximum divergence from the mean in Series I. is only about one part in 4000, while in Series II. it is even less. We were thus encouraged to carry out a modification of the method which we had had in view all along, and the results of which would be in great measure independent of those of Series I. and II.

* Phil. Mag., Nov., 1882.

§ 18. The modification referred to relates to the position of the induction coils relatively to the disc. In the arrangement with which we have been dealing hitherto, the mean planes of the coils are nearly coincident with that of the disc, and the accuracy of the final number depends upon an exact knowledge of the mean radius of the coils. It has, on the other hand, the advantages of being practically independent of measurements parallel to the axis, and of giving the maximum coefficient of induction. In the new arrangement the coils are separated to such a distance that the *result is nearly independent of a knowledge of the mean radius.* How this may come about will be readily understood by considering the dependence of the coefficient of induction M upon A, when a and b are given. It is clear that M vanishes, both when A is very small, and also when it is very large; from which it follows that there must be some value of A for which the effect is a maximum, and therefore independent of small variations of A.

In carrying out this idea, it is not necessary to approach the above defined state of things very closely; for of course we have in reality a good approximate knowledge of the value of A. In our apparatus the distance of mean planes was about 30 centims., so that b = about 15 centims. (A=26, a=16). From the calculations in § 25 it appears that with the actual proportions

$$\frac{d\mathrm{M}}{\mathrm{M}}=+\cdot 12\,\frac{d\mathrm{A}}{\mathrm{A}}-\cdot 96\,\frac{db}{b}+1\cdot 8\,\frac{da}{a}\,;$$

so that the error of A enters in quite a subordinate degree. The positive coefficient of dA shows that with the given coils and the given disc the separation was somewhat too great to secure the utmost independence of dA.

§ 19. The success of this arrangement depends principally upon the degree of accuracy with which b can be determined. The two rings upon which the coils are wound were held apart by three equal distance-pieces, against which they were firmly pressed by wooden clamps. The distance-pieces were hollow, of massive brass, and the terminal faces were carefully turned. Central marks upon them facilitated the adjustment of the coils into the symmetrical positions. The distance of mean planes does not however depend solely upon the distance-pieces. Even if we could assume that the mean planes are symmetrically situated relatively to the grooves in which the wire is wound, we should still have to take account of the thicknesses of the flanges. All uncertainty in this matter is eliminated by following the plan adopted by GLAZEBROOK of reversing the rings (without interchange), and then repeating the measurements. Whatever may be the situation of the mean planes and the thicknesses of the flanges, the mean result thus obtained corresponds to a distance equal to the length of the pieces *plus* half the total outside thicknesses of the rings. These quantities can all be measured with great precision, and as easily after the coils are wound as before. Full particulars are given in § 24. There can hardly be a doubt but that the determination is much more accurate than that of the mean radius of a

coil; and, what is also of some importance, it admits of repetition at pleasure with comparatively little trouble.

§ 20. The sensitiveness of this arrangement was about the same as in Series II., and the table shows a good agreement among the results obtained on different days. The final number from this series is ·9868, almost the same as from Series I. and II.

The small difference of effective resistances required for balance in the two positions of the induction coils, amounting to about one part per thousand, is almost exactly accounted for by the small difference of distances of mean planes in the two cases, as deduced from Professor CHRYSTAL'S measurements of the thicknesses of the flanges. In the first position (see § 24) the coils are nearer together by almost exactly one part per thousand, a difference which, according to the formula given above (§ 18), should be reproduced almost without change in M and therefore in R, the greater values of M and R corresponding to the smaller distance.

§ 21. If we combine all the results of the present investigation, giving equal weights to the two arrangements of the induction coils, we have

$$1 \text{ B.A. unit} = {\cdot}98677 \times 10^9 \text{ C.G.S.}$$

With use of the ratio between the mercury unit and the B.A. unit found by us (Proc. Roy. Soc., May, 1882), this gives

$$1 \text{ mercury unit} = {\cdot}94150 \times 10^9 \text{ C.G.S.};$$

or, which is the same thing, the ohm is the resistance of a column of mercury at 0° centigrade whose section is 1 square millim., and whose length is

1062·14 millims.

We now pass on to the details of the measurements.

DETAILS OF MEASUREMENTS.

Diameter of disc.

§ 22. Preliminary measurements of the disc while still mounted were made on August 11, 1882, with callipers by Messrs. ELLIOTT. Read by the vernier of the instrument itself the mean diameter was

$$2a = 310{\cdot}76 \text{ millims.}$$

The opening of the callipers was also determined independently by reference with the aid of microscopes to a verified scale of millimetres. In this way

$$2a = 310{\cdot}77 \text{ millims.}$$

The circumference was also measured by a steel tape, afterwards compared with the millimetre scale. Correction being made for the thickness of the tape, the result was

$$2a = 310{\cdot}84 \text{ millims.}$$

After the disc had been dismounted, the diameter could be determined more advantageously by direct observation through microscopes focussed upon its edge with subsequent reference to the standard scale. It was found (August 19, 1882) that a very appreciable difference existed between the diameter of the upper and lower faces, showing that the edge was somewhat conical. At the upper edge the diameter was 310·80, and at the lower edge 310·58. These were the extremes. At the middle of the thickness the diameter was 310·75. This departure from the truly cylindrical form was undoubtedly a defect in the apparatus, which could easily have been avoided if detected in time. When the apparatus was first set up, the success of the experiment was problematical, and a minute examination of the disc seemed premature. The diameter to be adopted is an average taken with reference to the conductivity of brush contact. The whole width of the brush being decidedly less than the thickness of the disc, and the pressure being greatest at the central parts, we decided (of course without knowing to what precise final result the estimate would lead) to take the mean of 310·75 and $\frac{1}{2}$ (310·58+310·80). Thus

$$2a=310{\cdot}72 \text{ millims.}$$

The error due to the conicality of the edge cannot exceed one part in 5000 at the worst, and thus it appeared scarcely worth while to correct the defect and repeat the spinnings.

The diameter of the shaft at the place where the other brush contact was made, was found to be ·825 inch, or 20·96 millims.

The induction coils.

§ 23. These are the same as were used in Mr. GLAZEBROOK's measurement, and were wound by Professor CHRYSTAL in 1878. The following are the dimensions ; for further particulars reference may be made to Mr. GLAZEBROOK's Memoir.*

	A.	B.	Mean.
Mean radius in centims. (A)	25·753	25·766	25·760
Radial width of section ($2h$)	1·92	1·90	1·91
Axial width of section ($2k$)	1·896	1·899	1·897
Number of windings	797	791	$\frac{1}{2}\times 1588$
Resistance (approximate) in B.A. units	84	83	$\frac{1}{2}\times 167$

Since the coils are so nearly similar and were used symmetrically, it is sufficient to use the numbers in the last column. The section of the ring is shown in fig. 4 full size.

To find the distance of mean planes the following measurements of the thicknesses of the rims are required. They are given in centimetres.

* Phil. Trans., 1883, p. 223.

	A.	B.
Rim (marked side)	·478	·446
Channel	1·896	1·899
Rim (unmarked side) . . .	·488	·465
Total thickness of rim . . .	2·862	2·810

Now that the rings are wound it is difficult to verify these numbers. However, the total thickness of the rings at the places touched by the distance pieces in the arrangement used for Series III. was taken, with the result

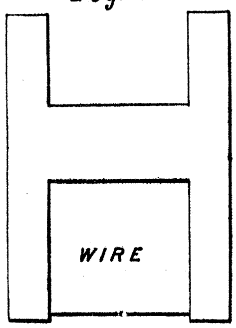
	A.	B.
Mean of three places. . . .	2·8625	2·8067

These latter values of the thicknesses will be used in the calculation of Series III.

In Series I. and II. the rings were not reversed, and we must use the numbers above given for the thicknesses of rims which were contiguous to the slips of glass; but in this case the result is not at all sensitive to changes in the distance of mean planes. The rims contiguous to the glass were for both coils the *marked* rims, of which the aggregate thickness is ·924. If we add to this the thickness of the glass strips ·454, we obtain 1·378 as the distance between the wire sections. Again, adding the mean axial width of section 1·897, we find as the distance of the mean planes

$$2b = 3{\cdot}275 \text{ centims.}$$

Fig. 4



The distance-pieces.

§ 24. The measurement of the distance-pieces used for the third series was made with great care. As only the mean is required, the three pieces were held under the microscopes in one length by a nut and a long bolt running through. Readings were taken in several positions, as the pieces were turned round, and reference was finally

made to the standard scale. Two independent measurements gave 83·580 and 83·579, mean 83·5795 centims., as the aggregate length. This was further verified by measuring each piece separately with callipers, the sum of the lengths thus found being 83·582. For the mean length of these distance-pieces we take

$$27 \cdot 8598 \text{ centims.}$$

As has been already explained, the rings were used in two positions relatively to the distance-pieces, with the view of eliminating any uncertainty as to the situation of the mean planes, and of rendering the final result independent of all measurements of thickness except that of the total thicknesses of the rings. Thus the mean distance of mean planes in the two positions is

$$27 \cdot 8598 + \tfrac{1}{2}(2 \cdot 8625 + 2 \cdot 8067) = 30 \cdot 6944 \text{ centims.}$$

To compare the partial results for the two positions separately, we must use the thicknesses of the rims which were in contact with the distance pieces. In the first position these were the marked rims, and thus the distance of mean planes

$$= 27 \cdot 860 + \cdot 478 + \cdot 446 + 1 \cdot 897 = 30 \cdot 681 \text{ centims.}$$

In like manner for the second position we find

$$27 \cdot 860 + \cdot 488 + \cdot 465 + 1 \cdot 897 = 30 \cdot 710 \text{ centims.}$$

The induction coefficients.

§ 25. Series I. and II. The distance (b) of the mean planes of the coils from the middle plane of the disc is

$$b = 1 \cdot 637 \text{ centim.}$$

The extreme distances, required to be known for the quadrature, are

$$b + k = 2 \cdot 585 \text{ centims.}, \quad b - k = \cdot 689 \text{ centim.}$$

The extreme and mean radii are

$$\text{A} - h = 24 \cdot 805 \text{ centims.}, \quad \text{A} = 25 \cdot 760 \text{ centims.}, \quad \text{A} + h = 26 \cdot 715 \text{ centims.}$$

while $$a = 15 \cdot 536 \text{ centims.}$$

The coefficient of induction between the disc and the middle turn of the coil, denoted by $\text{M}(\text{A}, a, b)$, is equal to $4\pi\sqrt{(\text{A}a)}.f(\gamma)$, where $f(\gamma)$ is a function of γ given by tables.* The angle γ itself is defined by

$$\sin \gamma = \frac{2\sqrt{(\text{A}a)}}{\sqrt{\{(\text{A}+a)^2 + b^2\}}}$$

* MAXWELL's 'Electricity and Magnetism,' 2nd edition, § 706.

It is not necessary to give the details of the calculations, which have been carefully checked. The tabular interval being 6′, it was found desirable in many cases to proceed beyond the simple interpolation by first differences. The results are

$$\begin{aligned} M(A, a, b) &= 215{\cdot}4674 \\ M(A+h, a, b) &= 205{\cdot}1917 \\ M(A-h, a, b) &= 226{\cdot}9835 \\ M(A, a, b+k) &= 211{\cdot}7246 \\ M(A, a, b-k) &= 217{\cdot}5972 \end{aligned}$$

The mean coefficient for the area of the section is found by doubling the first of these values, adding in the others, and then dividing by 6.

Thus

$$M = 215{\cdot}405^{*}$$

The separate values allow us to form an estimate of the effect of errors in the fundamental data. If we write

$$\frac{dM}{M} = \lambda\frac{dA}{A} + \mu\frac{db}{b} + \nu\frac{da}{a},$$

we may take approximately

$$\lambda = \frac{M(A+h, a, b) - M(A-h, a, b)}{2h} \div \frac{M}{A} = -1{\cdot}36$$

In like manner, $\mu = -{\cdot}02$, whence, since $\lambda+\mu+\nu=1$, $\nu = +2{\cdot}38$.

Series III. In this case the data remain precisely as before, except that we now have $b = 15{\cdot}3472$.

We find

$$\begin{aligned} M(A, a, b) &= 110{\cdot}9240 \\ M(A+h, a, b) &= 111{\cdot}2573 \\ M(A-h, a, b) &= 110{\cdot}2442 \\ M(A, a, b+k) &= 104{\cdot}5571 \\ M(A, a, b-k) &= 117{\cdot}6519 \end{aligned}$$

whence

$$M = 110{\cdot}926$$

Determining λ, μ, ν, as in the former case, we find

$$\frac{dM}{M} = +{\cdot}123\frac{dA}{A} - {\cdot}956\frac{db}{b} + 1{\cdot}833\frac{da}{a}.$$

From these values, calculated for the circumference of the disc, we have to subtract

* The factor expressing the number of windings is omitted.

the value (M_0) applicable to the small circuit touched by the inner brush. The area of this is $\frac{1}{4}\pi(2{\cdot}096)^2$. For the first and second series we have

$$M_0=\frac{2\pi A^2}{(A^2+b^2)^{\frac{3}{2}}}\frac{1}{4}\pi(2{\cdot}096)^2={\cdot}836$$

For the third series in like manner

$$M_0={\cdot}534$$

Thus finally for the first and second series

$$M-M_0=214{\cdot}569,$$

and for the third series

$$M-M_0=110{\cdot}392$$

The resistance coils.

§ 26. In all three series the resistance b, fig. 3, was a German-silver coil of about $\frac{1}{10}$, referred to for brevity as the $[\frac{1}{10}]$; and the resistance a was composed of three resistances in multiple arc, the first two being standard singles, and the third a resistance such as 7 B.A. units taken from a box. To make the necessary change, according to the plan already explained in § 9, the 7 would be replaced by 8. The value of a is of course determined principally by the unit resistance coils, and only secondarily by the resistance taken from the box.

The third element of the system of resistances was varied in the different series. In the first series c was a [10], in the second series it was [10]+[5]+[1], and in the third series [10]+[5]+[5′]. Besides the standard singles, whose values at various temperatures was already known in terms of the mean B.A. unit, we had to determine accurately the values of the $[\frac{1}{10}]$, the [10], the [5], and the [5′], as well as the small resistances of the various connecting pieces employed.

The [10] has been determined in various ways, but principally by means of the device referred to in the former paper.* Three German-silver wires of about 3 units each are wound on the same tube, and their terminals are so arranged that by means of a base board containing mercury cups they can be combined either in multiple arc or in series. In the former combination they are compared with a standard single, and the resistance is found to be (say) $1+\alpha$, where α is small. The coils are now without loss of time combined in series, a change which can be effected in a moment. The resistance in series is very approximately $9+9\alpha$; by the addition of the standard single it becomes $10+9\alpha$, and can now be compared with the [10]. If the difference observed be β we have $[10]=10+9\alpha+\beta$. By this method it is easy to obtain an accuracy of at least $\frac{1}{10000}$.

* Phil. Trans., Part II., 1882, p. 697.

The [5]'s were determined in two ways. Five singles were combined in series and compared with one of the [5]'s; afterwards the two [5]'s were compared with one another. In the second method, which is probably preferable, the sum of the two [5]'s was found by comparison with the [10]. From the sum and difference the separate values can of course be deduced.

The measurement of the [$\frac{1}{10}$] demanded some precaution on account of its smallness. Two standard singles, the [10], and the $\frac{1}{10}$, were combined with four insulated mercury cups, and without the use of connecting pieces, so as to form a WHEATSTONE'S balance (fig. 5), care being taken to bring the associated battery and galvanometer terminals

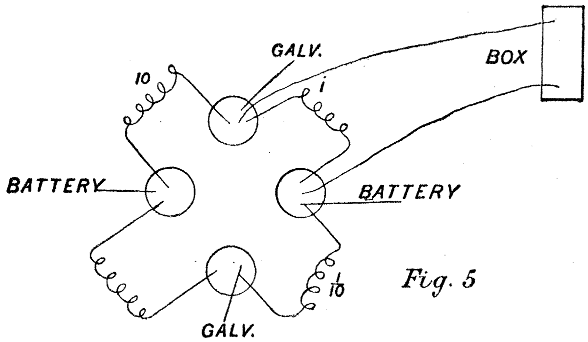


Fig. 5

into immediate contact with the legs of the [$\frac{1}{10}$] (see § 12). To get the means of adjustment, a box, giving resistances up to 10,000, was placed in multiple arc with one of the singles. If, as was the case, the four coils be so nearly in proportion that a resistance of several hundreds from the box is needed for balance, the delicacy of the arrangement is all that can be desired. Readings are taken also with battery reversed, to eliminate thermo-electric disturbances. Especial pains were taken with the measurement of the [$\frac{1}{10}$], and of the [10], errors of which would be propagated into the results of all three series.

§ 27. The various temperatures of the coils at the time of use, and the fluctuations from day to day, complicate the calculation of the effective resistances R_1 and R_2, which in principle is simple enough. The results are given in column II. of the Tables. Thus in the first series on July 14, when the effective resistance was ·0044076 B.A., as calculated from the values of a, b, c, for the observed temperatures of the coils, the effect (E–W) of reversing the battery key (corrected for direct effect) was -30 divisions of the galvanometer scale, the direction of rotation being positive. When the effective resistance was altered to ·0044430, the difference E–W became $+10$ divisions. From these results we infer that E–W would vanish for the effective resistance ·0044341, as given in column V. The corresponding result with negative rotation is given in column VI. These resistances relate to the actual speed of rotation determined by the frequency of vibration of the electric fork (§ 6). To render the results of different days fairly comparable, two small corrections have to be

introduced, the first relating to small alterations in the relative frequencies of the two forks, as shown by the number of beats per minute (column VII.), the second to variations in the frequency of the standard fork itself, dependent upon change of temperature. The temperatures were read by a thermometer which stood between the prongs of the standard, and are given in column IX. The corrections necessary for reduction to a standard number of beats (16 per minute) and to a standard temperature (16°) are tabulated in columns VIII. and X., and the corrected results themselves in XI. and XII. In all cases the electric fork vibrated *more* quickly than the standard.

The degree of accordance in the numbers entered in these columns shows the success of the observations, so far as relates to errors of a casual character. In column XIII. the results of the positive and negative rotations are combined, so as to exhibit the total result of the day's work.

The Table, showing the results of the third series, is divided into two parts, corresponding to the two positions of the induction coils, before and after reversal (§ 19). In each position, it will be seen that two sets of observations were taken upon one of the days. Both sets, however, were complete, and in the interval between them the resistance coils were all dismounted. A similar precaution was taken at least once in each of Series I. and II.

First Series.

Coils near together.

Speed of disc about 12·8 revolutions per second.

Approximate resistances $a=\frac{1}{2}$, $b=\frac{1}{10}$, $c=10$.

Date.	Effective resistance (in B.A. units) used.	Difference of reading of galvanometer on reversal of current.		Effective resistance (in B.A. units) corresponding to zero difference in galvanometer.		Correction for change of speed of fork.				Effective resistance (in B.A. units) as finally corrected.		Means of resistances with both directions of rotation.
July, 1882.		Rotation +	Rotation −	Rotation +	Rotation −	Beats between forks.	Correction to 16 beats.	Temperature of standard fork.	Correction to 16°.	Rotation +	Rotation −	
14th „	·0044076 ·0044430	−30·0 +10·0	+32·0 − 6·3	·0044341	·0044371	17	−·0000006	16·7°	+·0000003	·0044338	·0044368	·00443530
15th „	·0044084 ·0044438	−24·6 +10·5	+25·8 − 9·2	·0044333	·0044346	16	0	17·1	+·0000005	·0044338	·0044351	·00443445
17th „	·0044090 ·0044444	−25·4 +13·4	+27·0 −15·5	·0044322	·0044315	15	+·0000006	17·3	+·0000006	·0044334	·0044327	·00443305
18th „	·0044095 ·0044449	−27·4 +13·7	+25·9 −12·8	·0044331	·0044332	16	0	17·6	+·0000008	·0044339	·0044340	·00443395
19th „	·0044100 ·0044454	−23·9 +13·9	+27·4 −13·5	·0044324	·0044337	16	0	18·0	+·0000010	·0044334	·0044347	·00443405
20th „	·0044100 ·0044454	−24·4 +13·8	+25·4 −14·0	·0044326	·0044328	16	0	17·9	+·0000009	·0044335	·0044337	·00443360
									Means . .	·00443363	·00443450	·00443407

SECOND Series.

Coils near together.
Speed of disc about 8 revolutions per second.
Approximate resistances $a=\frac{1}{2}$, $b=\frac{1}{10}$, $c=16$.

Date.	Effective resistance (in B.A. units) used.	Difference of reading of galvanometer on reversal of current.		Effective resistance (in B.A. units) corresponding to zero difference in galvanometer.		Correction for change of speed of fork.				Effective resistance (in B.A. units) as finally corrected.		Means of effective resistances with both directions of rotation.
August, 1882.		Rotation +	Rotation −	Rotation +	Rotation −	Beats between forks.	Correction to 72 beats.	Temperature of standard fork.	Correction to 16°.	Rotation +	Rotation −	
7th ,,	·0027827 ·0028126	− 8·2 +20·1	+ 9·4 −21·5	·0027914	·0027918	73	−·0000004	17·6°	+·0000005	·0027915	·0027919	·00279170
8th ,,	·0027821 ·0028120	− 8·6 +21·0	+ 9·5 −19·1	·0027908	·0027920	73	−·0000004	17·2	+·0000004	·0027908	·0027920	·00279140
9th ,,	·0027826 ·0028125	− 7·9 +19·5	+ 7·5 −19·1	·0027912	·0027910	72	0	17·5	+·0000005	·0027917	·0027915	·00279160
									Means . .	·00279133	·00279180	·00279157

Third Series.

Coils separated.

Speed of disc about 12·8 revolutions per second.

Approximate resistances $a=\frac{1}{2}$, $b=\frac{1}{10}$, $c=20$.

Date.	Effective resistance in B.A. units used.	Difference of reading of galvanometer on reversal of current.		Effective resistance (in B.A. units) corresponding to zero difference in galvanometer.		Correction for change of speed of fork.				Effective resistance (in B.A. units) as finally corrected.		Means for each day of observations with both directions of rotation.	Means of all the observations with each direction of rotation.		Means of all observations with both directions of rotation.
August, 1882.		Rotation +	Rotation −	Rotation +	Rotation −	Beats.	Correction to 72 beats.	Temperature of standard fork.	Correction to 16°.	Rotation +	Rotation −		Rotation +	Rotation −	
14th „	·0022981 ·0023100	+ ·1 −11·0	− ·3 + 9·1	·0022982	·0022985	73	−·0000003	18°·4	+·0000006	·0022985	·0022988	·00229865			
15th „	·0022976 ·0023095	− ·1 −11·7	− 1·9 +10·0	·0022975	·0022995	72	0	18·4	+·0000006	·0022981	·0023001	·00229910	·00229853	·00229910	·00229881
15th „	·0022973 ·0023092	+ 1·1 − 9·9	− ·6 +11·1	·0022985	·0022979	72	0	18·1	+·0000005	·0022990	·0022984	·00229870			
Second position of induction coils.															
16th „	·0022969 ·0023088	− ·3 −11·9	0 + 7·8	·0022966	·0022969	72	0	17·6	+·0000004	·0022970	·0022973	·00229715			
17th „	·0022965 ·0023084	− ·8 −11·2	+ ·7 +12·1	·0022956	·0022958	72	0	17·4	+·0000004	·0022960	·0022962	·00229610	·00229633	·00229650	·00229642
17th „	·0022966 ·0023085	− ·8 −10·2	+ 1·0 +12·4	·0022956	·0022956	72	0	17·4	+·0000004	·0022960	·0022960	·00229600			
												Means.	·00229743	·00229780	·00229762

§ 28. The results given in these tables are the effective resistances required to obtain a balance, expressed in terms of the B.A. unit. To reduce them to absolute measure we must multiply by 10^9, and by a factor, which we may call x, expressing the absolute value of the B.A. unit in terms of 10^9, and which it is our object to determine.

The actual value of the same quantities in absolute measure is found by multiplying the coefficients of induction $(M-M_0)$ already given (§ 25), by the number of turns in the coils 1588, and by the number of revolutions per second.

In the first series the frequency of vibration (f) of the electric tuning-fork was in the standard case (see Appendix)

$$f=\tfrac{1}{2}(128{\cdot}140+\tfrac{16}{60})=\tfrac{1}{2}\times 128{\cdot}407$$

and the number of revolutions per second is equal to $2f\div 10$. In the second and third series $2f=129{\cdot}340$, a number which in the second series is to be divided by 16, and in the third series by 10, in order to obtain the number of revolutions per second.

The equation to determine x is thus for the first series of observations

$$214{\cdot}569\times 1588\times 12{\cdot}8407=x\times{\cdot}00443407\times 10^9,$$

whence

$$x={\cdot}98674.$$

From the second series

$$x=\frac{214{\cdot}569\times 1588\times 129{\cdot}340}{16\times 10^9\times{\cdot}00279157}={\cdot}98669$$

From the third series

$$x=\frac{110{\cdot}392\times 1588\times 129{\cdot}340}{10\times 10^9\times{\cdot}00229762}={\cdot}98683$$

These are the final results already considered in § 21.

APPENDIX.

Frequency of Vibration of Standard Fork.

All our measurements, both by this method and by that of the revolving coil, being dependent upon the pitch of a standard tuning-fork, we have considered it advisable to determine this element afresh. As in the first determination,* a fork vibrating about 32 times per second rendered intermittent an electric current, which, passing

* Proc. Roy. Soc., May, 1881, p. 137.

through the coils of small electromagnets, maintained in vibration not only the interrupter fork itself, but also a second fork of pitch about 128. After the apparatus has been a short time in operation, the vibrations of the second fork are exactly four times as quick as those of the first, independently of any precise tuning; and they give rise to audible beats when the standard fork is simultaneously excited. In the presence of extraneous noises the observation of the beats is much facilitated by the use of resonators, with one of which the ear may be connected by an indiarubber tube. The object to be aimed at is to make the intensities of the two sounds (as they reach the ear) very nearly equal. The moment of antagonism is then marked by a well defined silence, whose occurrence can be timed to within a second, although the whole duration of the beat may be 20 seconds or more. Without fresh bowing of the standard, the silences can be observed satisfactorily for at least a minute.

In the first determination the comparison between the fork of frequency 32, and the pendulum of the clock was made directly. The observer, looking over a plate carried by the upper prong of the fork, obtained 32 views per second, *i.e.*, 64 views of the pendulum in one complete vibration. The immediate subject of observation is a silvered bead attached to the bottom of the pendulum, upon which as it passes the position of equilibrium the light of a paraffin lamp is concentrated. Close in front of the pendulum is placed a screen perforated by a somewhat narrow vertical slit. If the period of the pendulum were a precise multiple of that of the fork, the flash of light which to ordinary observation would be visible at each passage, would either be visible, or be obscured, in a permanent manner. If, as in practice, the coincidence be not perfect, the flashes appear and disappear in a regular cycle, whose period is the time in which the fork gains (or loses) one complete vibration. This period can be determined with any degree of precision by a sufficient prolongation of the observations.

On account of the large number of views per second, the interval between successive visible positions of the bead, even when it is moving with maximum velocity, is rather small; and thus the adjustment of the apparatus is somewhat delicate.* In order to meet this objection, a modification has been introduced, which must now be explained.†

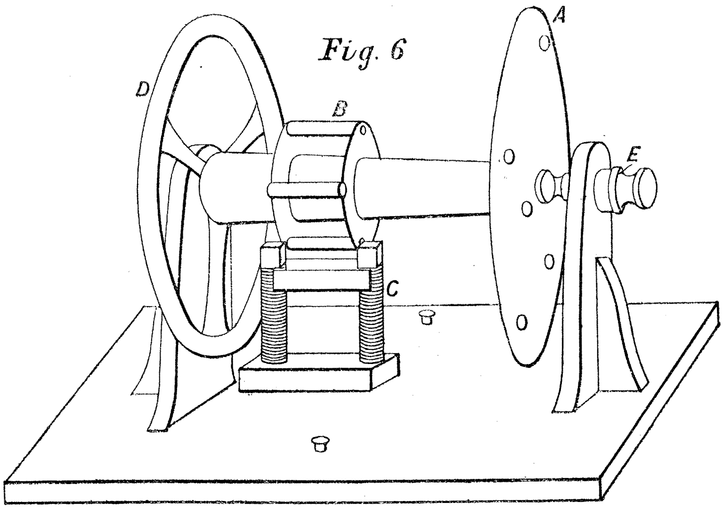
A few years ago it was shown almost simultaneously by La Cour and by Lord Rayleigh, that an electromagnetic engine could be accurately governed by an interrupter-fork. The construction (fig. 6) which has been found most suitable is similar to that of Froment's engine. A horizontal shaft revolving upon steel points carries a

* In the earliest use of this method ("Nature," xvii., p. 12, 1877) the break-fork had a frequency of about 13, and no difficulty of this kind was experienced.

† [July, 1883.—It should be stated, however, that the wheel may easily be dispensed with, if proper care be taken in the illumination of the bead and in the management of the fork. The vibration should be vigorous, and the screens so arranged that the view past the fork at the moment of greatest elongation should be of short duration. Determinations by this method (without the wheel) have often been made successfully by students in the Cavendish Laboratory.]

number of parallel soft iron armatures, disposed symmetrically round the circumference. In the course of the revolution these armatures pass in succession between the poles of a vertical horse-shoe electromagnet, so as almost to complete the magnetic circuit. It is much better that the armatures should pass *between* the poles than *over* them, as in the most usual arrangement, for in the latter case the bearings are subjected to an unnecessary and prejudicial strain. The wheel may be used either with or without an independent driving power. In the former case the power should be very steady, and adjusted so as to give by itself nearly the speed intended. The currents from the interrupter-fork are passed also through the electromagnet of the engine, and give the force required to accelerate or retard the motion so that it may exactly synchronise with the fork, one armature passing for each complete vibration. If the independent power is in excess, the phase of the motion is such that the electromagnet is excited principally after the armatures have passed through the electromagnet; if the independent power is in defect, the electromagnet is excited principally while the armatures are approaching it. Within certain limits any necessary acceleration or retardation is obtained by suitable self-acting adjustment of phase.

Fig. 6



If when the wheel is moving steadily under the influence of the intermittent currents, a slight disturbance is communicated to it, oscillations will set in, the wheel being alternately in advance and in the rear of its proper position. In some cases these oscillations are very persistent, and interfere seriously with the utility of the instrument. To check them, a hollow ring filled with water is attached to the shaft, and revolves with it. When the rotation is perfectly regular, the water behaves as if it were a rigid body and offers no impediment to the motion, but it tends to check variations of speed of moderate period. The oscillations, when they exist, are usually audible; and in any case the behaviour of the wheel in this and other respects may be examined by looking at the interrupter-fork through a paper disc carried by the wheel

and perforated symmetrically along a circle with holes equally numerous with the armatures. When all is regular, the prongs of the fork are seen in one phase only, so long as the eye retains a position fixed in space.

When the wheel runs lightly, independent driving power may be dispensed with, a sufficient amount of work being obtainable from the intermittent governing current. In the present case the whole apparatus, consisting of the two forks and the wheel, was driven by one current supplied from three GROVE cells. The only difficulty experienced is in starting the wheel. By means of string passed once round the shaft, alternately tightened for the advance and slackened for the return, it is easy to cause the wheel to achieve a speed in excess of the necessary eight revolutions per second. But it will not usually happen, every time the speed falls through the proper value, that the wheel will engage with the fork. For this purpose it is necessary that at the moment in question the phase of the wheel should be correct, within limits, which may be narrow when there is no great margin of power; and this can only happen by chance. Several attempts may be necessary before success is reached. With a little practice, however, there is no great loss of time, the ear learning to recognise, by the gradual slowing and subsequent quickening of a sort of beat, when the wheel has passed through the right speed without engagement. A fresh impulse is then given without waiting further. After a start is once effected, the wheel will usually run, keeping perfect time with the fork, until the battery is exhausted.

The wheel employed in the experiments we are now concerned with, has *four* soft iron armatures, and is governed by the interrupter-fork of frequency 32. The speed of the wheel is thus eight revolutions per second; and a single hole in a paper disc carried round with it allows eight views of the pendulum per second, the smallest number of views obtainable by direct use of the fork being 32. Altogether we may regard the frequency of the interrupter-fork as being multiplied four times precisely in the frequency of the auxiliary fork, and as divided four times precisely in the frequency of the wheel. The former is directly comparable with the standard fork, and the latter with the clock. The standard fork was screwed to the table precisely as during the electrical measurements. A thermometer placed between the prongs gave the temperature with fair accuracy.

The calculation of the results is very simple. Supposing in the first instance that the clock is correct, let a be the number of cycles per second (perhaps $\frac{1}{40}$) between the wheel and the clock. Since the period of a cycle is the time required for the wheel to gain, or to lose, one revolution upon the clock, the frequency of revolution is $8\pm a$. The frequency of the auxiliary fork is precisely 16 times as great, *i.e.*, $128\pm 16a$. If b be the number of beats per second between the two forks, the frequency of the standard is

$$128\pm 16a\pm b$$

To give an idea of the magnitudes of the numbers concerned, it will be advisable to

quote in detail the results of one day's observations. On October 19, with a certain loading of the interrupter-fork, the cycle of the pendulum occupied about 78 seconds, and the beats were at the rate of about six per minute. The interrupter was then *sharpened,* after which several observations were taken of the duration of five cycles of the pendulum, and of 16 beats between the forks. For the former the times found were 210, 210, 212 seconds; for the latter by simultaneous observation 58, 58, 59, 59, 59, 60, 60 seconds. The temperature, as given by the thermometer, ranged from 17°·2 to 17°·4. After the sharpening of the interrupter, the frequency both of the wheel and of the auxiliary fork was increased, so that the sign of $16a$ in the expression written above is determined to be $+$ and that of b to be $-$. Using the mean values we find

$$16a = \cdot 3797, \qquad b = \cdot 2712$$

whence

$$128 + 16a - b = 128 \cdot 108$$

To this we must add ·009, making altogether 128·117, to allow for the gaining rate of the clock, which was $6\frac{1}{2}$ seconds per diem. This corresponds to a mean temperature 17°·3.

The procedure adopted was quite good enough for our purpose; but if it were desired to push the power of the method to its limit, the work should be undertaken at an astronomical observatory, and extended over the whole time required to rate the clock by observations of the stars. In this way the comparison of the period of vibration of the standard fork with the mean solar second could be effected with the same degree of accuracy as that to which the former quantity is capable of definition. Without this precaution we cannot be quite sure that the rate of the clock at the time of the observations is identical with the mean rate employed in the calculation. It is scarcely necessary to say that the uncertainty which arises under this head is common to every method by which absolute pitch could be determined.

The results obtained, including those recorded previously,* are given in the accompanying table. They are well represented by the formula

$$128 \cdot 140 \times \{1 - (t - 16)^0 \times \cdot 00011\},$$

in which the temperature coefficient used (·00011) is that found by M'LEOD and CLARKE.† The numbers in the fourth column are calculated from the formula.

* Proc. Roy. Soc., May, 1881, p. 138.

† Phil. Trans., Part I., 1880.

Date.	Temperature.	Frequency by observation.	Frequency by calculation.
1881	13°	128·180	128·182
1881	14°·6	128·161	128·160
October, 1882 . .	15°·98	128·141	128·140
October, 1882 . .	17°·45	128·122	128·120
October, 1882 . .	17°·6	128·119	128·118
October, 1882 . .	17°·3	128·117	128·122

Of the small discrepancies which the table exhibits it is probable that the larger part is due to imperfect knowledge of the actual temperatures of the standard fork. The use of screens to cut off radiation from the observers would probably have effected an improvement. For the highest accuracy some sort of jacket, or chamber, would have to be contrived.

Second Appendix.

(Added July, 1883.)

On the Effect of the Imperfect Insulation of Coils.

In a former paper (Phil. Trans., 1882, Part II.) it was pointed out that the method of the revolving coil, employed by the first B.A. Committee, possesses the important advantage that it is possible to detect the existence of leakage from turn to turn, or from layer to layer, of the coil of wire. The general influence of such leakage, if undetected, upon the final number x expressing the ratio of the resistance of the coil when measured (R) in absolute units to its resistance $r \times 10^9$ as referred to B.A. units, is easily seen by supposing that one turn of the coil is simply short-circuited. The formula in C.G.S. measure is

$$x = \frac{\mathrm{R}}{r \times 10^9} = \frac{\pi^2 n^2 a \, \omega \cot \phi}{r \times 10^9} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

During the revolutions the short circuited turn produces its full effect in deflecting the magnet, and error arises only in the comparison with the standard of resistance. The quantity r will evidently be under-estimated by $1/n$, and this will lead to an *over*-estimate of x, also by $1/n$. This result, however, is modified, if as in practice we take only the *difference* of effects observed when the wire contact is open and closed. The short-circuited turn will produce its effect in *both* cases, and its influence will therefore disappear from the result. For all purposes it will be virtually non-existent, and the error produced is the same as if n had simply been miscounted. The final number x will thus be over-estimated by the fraction $2/n$.

In LORENTZ's method the effect of a short circuit in the induction coil is in the same direction. M, and therefore R and x, will be over-estimated by $1/n$.

If we examine the formulæ applicable to determinations by other methods, we shall see that a similar conclusion holds good, so that in every case leakage leads to an over-valuation of x, at least whenever the result is calculated from the number of turns of wire in a coil.* Even without such an examination, it is pretty evident from consideration of the magnitudes involved that the large factor 10^9 in the denominator of the formula corresponding to (1) can only be compensated by one or more large factors expressive of the number of windings in a coil or coils. An over-valuation of these factors, due to leakage, will therefore lead to an over-valuation of x.

In carefully constructed coils serious leakage is, perhaps, not likely to occur, but its presence in a smaller degree is more probable, and is usually difficult of detection. So far as this argument applies, we may say that the smaller values of the number expressive of the B.A. unit, or of the mercury unit, in absolute measure are to be preferred to the larger.

* The case is different when the constants of a coil of many turns are determined by electrical comparison, as for instance in KOHLRAUSCH's recent correction of the constant of his earth-inductor.