

III. *The Force of fired Gun-powder, and the initial Velocities of Cannon Balls, determined by Experiments; from which is also deduced the Relation of the initial Velocity to the Weight of the Shot and the Quantity of Powder.* By Mr. Charles Hutton, of the Military Academy at Woolwich. Communicated by Samuel Horsley, LL.D. Sec. R. S.

Read Jan. 8, 1778. **T**HESE experiments I made at Woolwich in the summer of the year 1775, assisted by several able officers of the royal artillery at that place, and other ingenious gentlemen. The object of them was the determination of the actual velocities with which balls are impelled from given pieces of cannon, when fired with given charges of powder. These experiments were made according to the method invented by Mr. ROBINS, and described in his treatise, intitled, *New Principles of Gunnery*, of which an account was printed in the *Philosophical Transactions* for the year 1743. Before the discoveries of that ingenious gentleman very little progress had been made in the true theory of military projectiles. His book, however, contained such important discoveries, that it was soon

soon translated into several of the languages on the continent, and the famous Mr. L. EULER honoured it with a very extensive commentary in his translation of it into the German language. That part of it hath always been particularly admired which relates to the experimental method of ascertaining the actual velocities of shot, and in imitation of which were made the experiments related in this paper. Experiments in the manner of Mr. ROBINS were generally repeated by his commentators and others, with universal satisfaction, the method being so just in theory, so simple in practice, and altogether so ingenious, that it immediately gave the fullest conviction of its excellence, and of the abilities of its author. The use which that gentleman made of this invention was, to obtain the actual velocities of bullets experimentally, in order to compare them with those which he computed *a priori* from his new theory, and thereby to verify the principles on which it is founded. The success was fully answerable to his expectations, and left no doubt of the truth of his theory, when applied to such pieces and bullets as he had used: but these were very small, being only musket balls of about one ounce weight; for, on account of the great size of the machinery necessary for such experiments, Mr. ROBINS and other ingenious gentlemen had not ventured to extend

their practice beyond bullets of that kind, and satisfied themselves with earnestly wishing for experiments to be made in a similar manner with balls of a larger sort. By the experiments in this paper I have endeavoured, in some degree, to supply this defect, having made them with small cannon balls of above twenty times the size, or from one pound to near three pounds weight. These are the only experiments that I know of which have been made with cannon balls for this purpose, although the conclusions to be deduced from such are of the greatest importance to those parts of natural philosophy which are dependent on the effects of fired gunpowder; nor do I know of any other practical method of ascertaining the initial velocities of military projectiles within any tolerable degree of the truth. The knowledge of this velocity is of the utmost consequence in gunnery: by means of it, together with the law of the resistance of the medium, every thing is determinable relative to that business; for, besides its being an excellent method of trying the strength of different sorts of powder, it gives us the law relative to the different quantities of powder, to the different weights of shot, and to the different lengths and sizes of guns. Besides these, there does not seem to be any thing wanting to determine any inquiry that can be made concerning the flight and ranges of shot, except the effects arising from the resistance of the medium.

Of

*Of the nature of the experiment, and of the machinery
used in it.*

The intention of the experiment is to discover the actual velocity with which a ball issues from a piece, in the usual practice of artillery. This velocity is very great; from one thousand to two thousand feet in a second of time. For conveniently estimating so great a velocity, the first thing necessary is to reduce it, in some known proportion, to a small one. This we may conceive to be effected thus: suppose the ball, with a great velocity, to strike some very heavy body, as a large block of wood, from which it will not rebound, so that they may proceed forward together after the stroke. By this means it is obvious, that the original velocity of the ball may be reduced in any proportion, or to any slow velocity which may conveniently be measured, by making the body struck to be sufficiently large; for it is well known, that the common velocity, with which the ball and block of wood would move forward after the stroke, bears to the original velocity of the ball only, the same ratio which the weight of the ball hath to that of the ball and block together. Thus then velocities of one thousand feet in a second are easily reduced to
those

those of two or three feet only; which small velocity being measured by any convenient means, let the number denoting it be increased in the proportion of the weight of the ball to the weight of the ball and block together, and the original velocity of the ball itself will thereby be obtained. In these experiments, this reduced velocity is rendered very easy to be measured by a very simple and curious contrivance, which is this: the block of wood, which is struck by the ball, is not left at liberty to move straight forward in the direction of the motion of the ball, but it is suspended, as the weight or bob of a pendulum, by a strong iron stem, having a horizontal axis at top, on the ends of which it vibrates freely when struck by the ball. The consequence of this simple contrivance is evident: This large ballistic pendulum, after being struck by the ball, will be penetrated by it to a small depth, and it will then swing round its axis and describe an arch, which will be greater or less according to the force of the blow struck; and from the size of the arch described by the vibrating pendulum, the velocity of any point of the pendulum itself can be easily computed; for a body acquires the same velocity by falling from the same height, whether it descend perpendicularly down, or otherwise; therefore, the length of the arch described, and of its radius, being

given, its verfed sine becomes known, which is the height perpendicularly defcended by the correſponding point of the pendulum. The height defcended being thus known, the velocity acquired in falling through that height becomes known from the common rules for the deſcent of bodies by the force of gravity; and this is the velocity of that point of the pendulum: this velocity of any known point whatever is then to be reduced to the velocity at the center of oſcillation, by the proportion of their radii or diſtances from the axis of motion; and the velocity of this center, thus obtained, is to be eſteemed the velocity of the whole pendulum itſelf; which being now given, that of the ball before the ſtroke becomes known from the given weights of the ball and pendulum. Thus then the menſuration of the very great velocity of the ball is reduced to the obſervation of the magnitude of the arch deſcribed by the pendulum, in conſequence of the blow ſtruck. This arch may be meaſured after various ways: in the following experiments it was aſcertained by meaſuring the length of its chord by means of a piece of tape or ſmall ribband, the one end of which was faſtened to the bottom of the pendulum, and the reſt of it made to ſlide through a ſmall machine contrived for the purpoſe, which will be hereafter deſcribed; for thus the length of the tape drawn out,

out, was equal to the length of the chord of the arch described by the bottom of the pendulum.

This description may convey a general idea of the nature and principle of the experiment; but besides the center of oscillation and the weights of the ball and pendulum, the effect of the blow depends also on the place of the center of gravity and the point of impact: it will, therefore, be now necessary to give a more particular description of the machine, and of the methods of finding the abovementioned requisites, and then investigate our general rule for determining the velocity of the ball, in all cases, from them and the chord of the arch of vibration.

Of the particular description of the machine, and of the determination of the centers of gravity and oscillation.

Tab. I. Fig. 1. is a representation of the machine used in the first three courses of experiments; and fig. 2. of that which was used in the other two. I shall here describe the former of these, and afterwards take notice of the few particulars in which the other differs from it when I come to treat of the use of the latter.

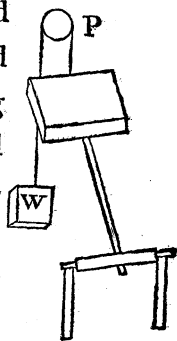
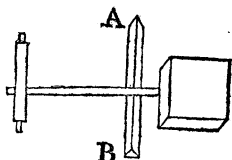
The first pendulum consisted of a block of sound and dry elm, being nearly a cube of twenty inches long,
which

which was fastened to a strong iron stem on the back part of it by screw-bolts, having a thick iron axis at the top, whose ends were turned truly cylindrical, to roll pretty freely in sockets made to receive them; the whole being supported by a four-legged stand of very strong timber, which was firmly fixed in the ground. A is the face of the cube into which the balls were fired; by means of the blow it is made to swing round the axis BC, and the chord of the arch thereby described is measured by the tape DEF fastened to the bottom of the wood at D, and sliding with some slight friction through a little machine of brass, fixed at E for that purpose, the tape being marked with inches and tenths, for the more easily measuring of the chord or part of it drawn through by the pendulum. The whole length of this pendulum, from the middle of the axis to the ribband at D, was $102\frac{1}{2}$ inches. The weight and the other dimensions were taken each day when the experiments were made, and then registered; and the manner of discovering the places of the centers of gravity and oscillation was as follows:

To find the center of oscillation, the pendulum was hung up, and made to vibrate in small arcs, and the time of making two or three hundred vibrations was observed by a half-second pendulum. Having thus obtained the

time answering to a certain number of vibrations, the finding of the center of oscillation is easy: for if v denote the number of vibrations made in s seconds, then it is well known, that as $vv:ss::39.2:\frac{39.2ss}{vv}$ = the distance in inches from the axis of motion to the center of oscillation; and by this rule the place of that center was found for each day.

The center of gravity was ascertained by one or both of the two following methods. First, a triangular prism of iron AB, being placed upon the ground with an edge upwards, the pendulum was laid across it, and moved forward or backward on the stem or block as the case required, till the two ends exactly balanced each other; then, as it lay, the distance was measured from the middle of the axis to the part which rested on the edge of the prism, or the center of gravity of the pendulum. The other method was as in the latter of the two annexed figures, where the ends of the axis being supported on fixed uprights, and a chord fastened to the lower end of the pendulum, was passed over a pulley at P, different weights w were fastened to the other end of it, till the pendulum was



brought

brought to a horizontal position. Then, taking also the whole weight of the pendulum, and its length from the axis to the bottom where the chord was fixed, the place of the center of gravity is found by this proportion, as p the weight of the pendulum : w the appended weight :: d the whole length from the axis to the bottom : $\frac{dw}{p}$ = the distance from the axis to the center of gravity. Either of these two methods gave the place of the center of gravity sufficiently exact; but the coincidence of the results of both of them was still more satisfactory.

Of the rule for computing the Velocity of the ball.

Having described the methods of obtaining the necessary dimensions, I proceed now to the investigation of the theorem by which the velocity of the ball is to be computed. The several weights and measures being found, let then

- b denote the weight of the ball,
- p the whole weight of the pendulum,
- g the distance of the center of gravity below the axis,
- h the distance to the center of oscillation,
- k the distance to the point struck by the ball,
- x the velocity of this point struck after the blow,

v the original velocity of the ball,

c the chord of the arch measured by the tape, and

r its radius, or the distance from the axis to the bottom of the pendulum.

Then the effect of the blow struck by the ball is as $\frac{gb}{kk}$; or, as $kk : gb :: p : \frac{gbp}{kk}$ = the weight of a body, which, being placed at the point struck, would acquire the same velocity from the blow as the pendulum does at the same point. Here then are two bodies, b and $\frac{gbp}{kk}$, the former of which, with the velocity v , strikes the latter at rest, so that after the blow they both proceed uniformly forward together with the velocity z ; in which case it is well known, that $b : b + \frac{gbp}{kk} :: z : v$; and therefore the velocity z is $= \frac{bkkv}{bkk + gbp}$. But because of the accession of the ball to the pendulum, the place of the center of oscillation will be changed; and from the known property of that point we find $\frac{bkk + gbp}{bk + gp}$ = to its distance from the axis. Call this distance of the center of oscillation, of the mass compounded of the ball and pendulum, H . Then, since z is the velocity of the point whose distance is k , we have this proportion, as $k : H :: z : \frac{zH}{k} = \frac{bkkv}{bk + gp}$ = the velocity of this compound center of oscillation.

Again,

Again, since $\frac{cc}{2r}$ is the versed sine of the described arc c , its radius being r ; therefore as $r : H :: \frac{cc}{2r} : \frac{cc}{2rr} \times \frac{bkk + gbp}{bk + gp} =$ the versed sine to the radius H , or the versed sine of the arc described by the center of oscillation, which call v ; then is v the perpendicular height descended by this center, and the velocity it acquires by the descent through this space is thus easily found, *viz.* as $\sqrt{16\frac{1}{12}} : \sqrt{v} :: 32\frac{1}{6} : \frac{8.02c}{r\sqrt{2}} \times \sqrt{\frac{bkk + gbp}{bk + gp}} =$ the velocity of the center of oscillation deduced from the chord of the arc which is actually described.

Having thus obtained two different expressions for the velocity of this center, independent of each other, let an equation be made of them, and it will express the relation of the several quantities in the question; thus then we have $\frac{bkv}{bk + gp} = \frac{8.02c}{r\sqrt{2}} \sqrt{\frac{bkk + gbp}{bk + gp}}$ from which we obtain $v = \frac{8.02c}{bkr\sqrt{2}} \sqrt{bk + gp \times bkk + gbp}$ the true expression for the original velocity of the ball the moment before it struck the pendulum.

COROLLARY. But this theorem may be reduced to a form much more simple and fit for use, and yet be sufficiently near the truth. Thus, let the root of the compound factor $\sqrt{bk + gp \times bkk + gbp}$ be extracted, and it will be equal to $\sqrt{b \times pg + bk \times \frac{b - k}{2h}}$ within the 100000th part

of

of the truth in such cases as generally happen. But since $b\dot{k} \times \frac{b+\dot{k}}{2b}$ is usually but about the 300th or 400th part of pg , and that $b\dot{k}$ differs from $b\dot{k} \times \frac{b+\dot{k}}{2b}$ but by about the 80th or 100th part of itself, therefore $pg+b\dot{k}$ is within about the 20000th or 30000th part of $pg+b\dot{k} \times \frac{b+\dot{k}}{2b}$. Consequently v is $= 8.02 c \sqrt{\frac{1}{2}b} \times \frac{pg+b\dot{k}}{bkr}$ very nearly. Or, farther, if g be written for \dot{k} in the last term $b\dot{k}$, then finally v is $= 8.02 cg \sqrt{\frac{1}{2}b} \times \frac{p+b}{bkr}$, or $v = 5.672 cg \sqrt{b} \times \frac{p+b}{bkr}$, which is an easy theorem to be used on all occasions; and being within about the 3000th part of the truth, it is sufficiently exact for all practical purposes whatever. Where it must be observed, that c, g, k, r , may be taken in any measures, either feet or inches, &c. provided they be but all of the same kind; but b must be in feet, because the theorem is adapted to feet.

SCHOLIUM. As the balls remain in the pendulum during the time of making one whole set of experiments, by the addition of their weight to it, both its weight and the centers of gravity and oscillation will be changed by the addition of each ball which is lodged in the wood, and therefore p, g , and b , must be corrected after every shot in the theorem for determining the velocity v . Now the succeeding value of p is always $p+b$;
or

or p must be corrected by the continual addition of b : and g is corrected by taking always $g + \frac{k-g}{p+b} b$, or $g + \frac{k-g}{p} b$ nearly for each successive value of g ; or g is corrected by adding always $\frac{k-g}{p} b$ to the next preceding value of g : and lastly, b is corrected by taking for its new values successively $\frac{pgh+bkk}{pg+bk}$, or by adding always $\frac{k-b}{pg+bk} bk$, or $\frac{k-b}{p} b$ nearly, to the preceding value of b ; so that the three corrections are made by adding always,

b to the value of p ,

$\frac{k-g}{p} \times b$ to the value of g ,

$\frac{k-b}{p} \times b$ to the value of b .

Before we proceed to the experiments it may not be improper to take notice of three seeming causes of error, which have not been brought to account in our theorem for determining the velocity of the shot; and to examine here whether their effects can sensibly affect the conclusion. These are the penetration of the ball into the wood of the pendulum, the resistance of the air to the back of it, and the friction on the axis: by each of these three causes the motion of the pendulum seems to be retarded. The principle on which our rule is founded supposes the momentum of the ball to be communicated to the pendulum in an instant; but this is not accurately

the case, because that this force is communicated during the small time in which the ball makes the penetration; but as this is generally effected before the pendulum has moved one-tenth of an inch out of its vertical position, and usually amounts to scarcely more than the 200th part of a second, its effect will be quite imperceptible, and therefore it may safely be neglected in these experiments. As to the second retarding force, or the resistance of the air to the back of the pendulum, it is manifest that it will be quite insensible, when it is considered that its velocity is not more than three feet in a second, that its surface is but about twenty inches square, and that its weight is four or five hundred pounds. Neither can the effect of the last cause, or the friction on the axis, ever amount to a quantity considerable enough to be brought into account in these experiments: for, besides that care was taken to render this friction as small as possible, the effect of the little part which does remain is nearly balanced by the effect it has on the distance of the center of oscillation; for as this center was determined from the actual vibrations of the pendulum, the friction on the axis would a little retard its motion, and cause its vibrations to be slower, and the consequent distance of this center to be greater; so that the other parts of our theorem being multiplied by \sqrt{h} , or the root of this distance, which is

as the time of a vibration, it is evident that the friction in the one case operates against that in the other; and that the difference of the two is the real efficacious cause of resistance, and which therefore is either equal to nothing, or very nearly so.

These general causes of error in the principles of the experiments are therefore safely omitted in the theorem: and our only care must be to guard against accidental errors in the actual execution of the business.

Of the experiments.

The gun, with which the experiments were made, was of brass; the diameter of the bore or cylinder at the muzzle was $2\frac{4}{5}$, or 2.16 inches; but its diameter next the breech was a small matter less, being there only $2\frac{2}{5}$, or 2.08 inches; so that the greatest cast-iron ball it would admit was just $19\frac{1}{2}$ ounces avoirdupois, or $1\frac{1}{4}$ pound wanting half an ounce; but sometimes leaden balls were used, which weighed above $1\frac{3}{4}$ pound, and sometimes long or cylindrical shot which weighed near three pounds; the length of the bore was $42\frac{3}{5}$, or 42.6 inches, so that it was nearly $20\frac{1}{2}$ calibers long.

The powder used was of the sort which is commonly made for government; the quantity was two, four, or

eight ounces to a charge, which was always put into a light flannel bag, and rammed more or less, as expressed in each day's experiments, but without ever using any wad before it.

The distance of the gun from the pendulum was 29 or 30 feet; which distance was found by firing the piece, with eight ounces of powder without a ball, at different distances, till the force of the elastic fluid was found not to move the pendulum.

The penetrations of the balls into the wood were attempted to be taken, but were soon neglected on account of their uncertainty, because of so many balls striking in or near the same part of the wood. The depth of the penetration seemed to be near about three inches in solid wood when two ounces of powder was used.

The first course of experiments was on the 13th of May, 1775, it being a clear, dry day. The weights and measures then taken were thus, *viz.*

$p = 328$ pounds, the weight of the pendulum,

$g = 72$ inches, the distance of the center of gravity,

$b = 88$ inches $= 7\frac{1}{3}$ feet, the distance of the center of oscillation,

$r = 102\frac{1}{2}$ inches, the distance to the bottom or tape.

The value of $b = 88$ was determined from the number of forty vibrations being made in a minute; for as

$40^2 : 60^2 :: 4 : 9 :: 39.2 : 88$. The number of shot was eight, and the circumstances and results as exhibited in the following table.

Number.	Weight of powder.	Diam. of the ball.	Height of the charge.	Struck below the axis, <i>k</i> .	Weight of the ball.	Weight of the ball, <i>b</i> .	Values of <i>p</i> .	Values of <i>g</i> .	Chord of the arc, <i>c</i> .	Veloc. per second.
	Oz.	In.	Inches.	Inches.	Oz.	Pounds	Pounds	Inches.	Inches.	Feet.
1	2	1.98		92.5	17½	1.094	328.0	72.0	13.0	458
2	2	1.98		92.5	17½	1.094	329.1	72.1	17.8	631
3	2	1.98	3.15	91.6	17½	1.094	330.2	72.2	18.1	650
4	2	1.97	3.15	91.	17½	1.078	331.3	72.3	17.6	646
5	2	1.97	3.15	90.5	17½	1.078	332.3	72.3	16.3	604
6	2	1.96	3.15	92.4	17	1.063	333.4	72.4	16.2	598
7	4	1.97	4.5	92.	17½	1.078	334.4	72.5	24.0	881
8	4	1.96	4.5	90.5	17	1.063	335.5	72.5	25.0	950

By computing the velocities from our theorem investigated in the corollary, they come out as they are here registered in the last column of the table, and they are all pretty regular excepting the first one, which is about one-fourth part less than the rest with the same weight of powder, and which irregularity must have been caused by some unperceived accident. The values of *p* and *g* were each corrected by their respective theorems; but the value of *b* was kept the same ($7\frac{1}{2}$ feet) through-


out, because that its correction was so small as not to make a difference of above a foot or two at most in the velocity: and for the same reason this correction is neglected, as quite unnecessary, in the rest of the experiments of the other days following.


The mean velocity of the second, third, fourth, fifth, and sixth numbers is 626, and of the seventh and eighth it is 915; that is, the velocity with two ounces of powder was 626 feet *per* second, and that with four ounces was 915 feet; and these two velocities are in the ratio of 1 to 1.46. But the mean weight of the balls in the former case was $17\frac{1}{3}$ ounces, and in the latter it was $17\frac{1}{8}$ ounces; and the ratio of the quantities of powder was that of 1 to 2. But the direct sub-duplicate ratio of the powder, compounded with the inverse sub-duplicate ratio of the weights of the shot, forms the ratio of 1 to 1.42, which is nearly equal to the ratio (1 to 1.46) of the velocities; that is, in this instance the velocities are very nearly as the square roots of the quantities of powder directly, and the square roots of the weights of the balls inversely. The powder was forced up with only one stroke of the rammer.

The second course was performed on the 3d of June, 1775, which was a clear, dry day, but windy. Some of the

the experiments of this day are doubtful, as indeed is evident from their irregularity, on account of the wind blowing the tape, which was not very properly secured by the little brazen machine through which it was made to slide.

The powder was taken from the bottom of a barrel, and the charges rammed a little closer than those of the former day; and so tight did the shots fit towards the breech, that many strokes of the rammer were necessary to drive them home.

The fourth and fifth shots were of a long form, which may be called spherico-cylindrical, as they were cylinders terminated by hemispherical ends, so that their section through the axis was of this form , and the length of the axis was near double the diameter of the shot.

The fourth shot, or first of the long sort, struck sideways, making a hole of the shape of the above section, only its length or axis was not horizontal but vertical, thus .

The last shot lay obliquely in the wood; it appeared to have struck with its end foremost, or nearly so, as the oblique position in which it lay seemed to be caused by its striking against a former shot lodged in the wood, with the hance of its end, so as to flatten it in that part.

Of

Of the pendulum, the weight, length, and centers of gravity and of oscillation were the same as when taken the former day before the experiments were made; the former balls having been extracted, and the holes filled up with wood.

Number.	Weight of powder.	Diam. of the ball.	Height of the charge.	Struck below the axis, <i>k</i> .	Weight of the ball.	Weight of the ball, <i>b</i> .	Values of <i>p</i> .	Values of <i>g</i> .	Chord of the arc, <i>c</i> .	Veloc. per second.
	Oz.	In.	Inches.	Inches.	Oz.	Pounds	Pounds	Inches.	Inches.	Feet.
1	2	2.08	2.85	88½	19½	1.219	328.0	72.0	24.3	800
2	2	2.08	2.85	89	19½	1.219	329.2	72.1	30.5	1003
3	2	2.08	2.85	93½	19½	1.219	330.4	72.1	30.0	943
4	2	2.08	3.35	92½	46½	2.906	331.6	72.2	57.0	767
5	2	2.08	3.35	93	46½	2.906	334.5	72.4	54.0	731

Here the first shot is again so much smaller than the two following ones, that some irregularity must have attended it, on which account we cannot make any use of it. The mean between the second and third is 973; and between the fourth and fifth the mean is 749; that is, the velocity of the 19½ ounce ball is 973, and that of the 46½ ounce shot 749 feet *per* second, which two numbers are in the ratio of 1.3 to 1. But the reciprocal subduplicate ratio of the weights (19½ and 46½) is the ratio of 1.54 to 1: therefore, in this instance, the velocity of the heavier shot is a little less than would arise from the
inverfe

inverse ratio of the square roots of the weights of the shot. But the accurate ratio cannot certainly be drawn from these numbers, on account of the doubtfulness of some of them, as was before observed.

It is very remarkable, that in the experiments of this day, the mean velocity with two ounces of powder is 973, whereas it was no more than 626 in the former day with the same quantity of powder, notwithstanding the balls were heavier with the greater velocity in the proportion of 19 to 17 nearly. This remarkable difference must be chiefly owing to the windage in the first course: and from hence we may perceive the great advantage to be gained by the use of balls approaching in proportion nearer to the diameter of the bore of the gun than what is prescribed in the present establishment. Possibly, however, some part of this difference might be owing to some small inequality in the powder, as that which was used this day was taken from the bottom of a barrel. Perhaps also some part of the effect may be owing to the greater degree of ramming which the powder had in this course.

The third course was made on the 12th of June, 1775, it being a clear, dry, and calm day. The powder in the experiments of this day was rammed in the same
degree

degree as in the last one. It was also nearly the same in the succeeding days, as may be perceived by inspecting the fourth column of each course, which, denoting the height of the charge, shews the degree of compactness with which the powder was lodged in the piece. The dimensions, as taken this day, were thus:

$p = 324$ pounds, the weight of the pendulum.

$g = 71.4$ inches, the distance of the center of gravity.

$b = 88$ inches $= 7\frac{1}{3}$ feet, the distance of the center of oscillation.

$r = 102\frac{1}{2}$ inches, the whole length to the tape.

Number.	Weight of powder.	Diam. of the ball.	Height of the charge.	Struck below the axis, k .	Weight of the ball.	Weight of the ball, b .	Values of p .	Values of g .	Chord of the arc, c .	Veloc. per second.
	Oz.	Inches.	Inches.	Inches.	Oz.	Pounds	Pounds	Inches.	Inches.	Feet
1	2	2.080	2.85	94	$19\frac{1}{2}$	1.219	324.0	71.4	23.0	700
2	2	2.036	2.85	94	$18\frac{1}{4}$	1.141	325.2	71.5	24.5	799
3	2	2.045	2.85	$93\frac{1}{2}$	$18\frac{1}{2}$	1.156	326.4	71.6	22.0	715
4	4	2.062	4.	$92\frac{1}{4}$	19	1.188	327.5	71.7	27.3	880
5	4	2.036	4.	$93\frac{1}{2}$	$18\frac{1}{4}$	1.141	328.7	71.7	35.0	1163
6	4	2.045	4.	$93\frac{1}{2}$	$18\frac{1}{2}$	1.156	329.9	71.8	33.0	1087

Here the common mean weight of the ball is $18\frac{2}{3}$ ounces, the mean velocity with two ounces of powder is 738, and that with four ounces of powder is 1043 feet *per* second. The ratio of these two velocities is that of

1 to

1 to 1.414; that is, accurately the ratio of the square roots of the quantities of powder.

Of the experiments made with the other pendulum, which is represented in fig. 2.

The first pendulum was gradually more and more rent and shattered by the firing of so many balls into it, till at the end of the last course of experiments it had become quite useless. Another was then fitted up, and with it were performed the two following courses.


This second pendulum consisted of a cubical block of sound elm, of near two feet long, fixed to the iron stem, but not exactly in the manner of the former; for in this the stem was placed vertically over the center of the top-end, to which point it continued whole, but there divided in two, each passing to right and left over the top down the sides, and returning along the bottom, and being at proper intervals fastened to the wood with iron pins. A thick sheet of lead was fastened over each of the two upright faces into which the shot were to be fired, both to guard them from splintering very much, and to add to the weight of the pendulum. The whole was then firmly secured by two very thick iron bands or hoops, passed horizontally quite around the wood, and

firmly fixed to it, the one next the upper end, and the other near the lower, so as strongly to resist the endeavours of the shot to split the wood.

The whole weight of the pendulum, thus fitted up, was 552 pounds; its whole length, from the middle of the axis to the tape at the bottom, was 101 inches; the distance to the center of gravity was 78 inches; and the distance to the center of oscillation was 88 inches equal to $7\frac{1}{3}$ feet, which was exactly the same as that of the former pendulum, their numbers of vibrations being alike in the same time.

Instead of suspending this pendulum, after the manner of the former, by the ends of its axis in grooves turned to fit them, they were only placed on flat, level pieces of wood, on which this pendulum vibrated much freer than the other did; but a small nail was driven into the supporting wood, just behind each end of the axis, to prevent the stroke of the shot from throwing it off the stand.

To this pendulum was adapted a better machine for the tape to slide through than the former one was, the inconvenience of which had often been experienced by its catching and entangling the tape, so as to interrupt its free motion, and once indeed to break it. This new one, however, is at once very simple, and perfectly free from
every

every inconvenience, giving just the necessary degree of friction to the tape, without ever stopping its motion; so that of the real quantity drawn out by the vibration of the pendulum there could not possibly be the least doubt. This simple contrivance consisted barely of about six or eight inches of the list of woollen cloth fastened upon the arch of a small piece of wood, which was shaped into the form of the segment of a circle thus , the tape being made to pass through between the curved side and the list, which was moderately stretched and fastened by its two ends to those of the little arch.

Upon the whole, the machinery was all so perfect, and every circumstance attending the experiments of the two ensuing days so carefully observed, that I can with great safety rely on the conclusions resulting from them. And as those of the one day were made with leaden balls, and those of the other with iron ones, which differ greatly in weight, every other circumstance being the same, they afford very good means for discovering the law of the different weights of shot, while the variations in the powder from two to four and eight ounces furnish us with the rule for the different quantities of it.

The fourth course was on the 20th of July, a fine clear day. The powder was a mixture of several of the forts made for government, and the balls were of lead. The quantities of powder were two, four, and eight ounces alternately; and the dimensions at first were thus:

$p = 552$ pounds, the whole weight of the pendulum.

$r = 101$ inches, its whole length.

$g = 78$ inches, the distance of the center of gravity.

$b = 88$ inches $= 7\frac{1}{3}$ feet, that of the center of oscillation.

Number.	Weight of powder.	Diam. of the ball.	Height of the charge.	Struck below the axis, k .	Weight of the ball.	Weight of the ball, b .	Values of p .	Values of g .	Chord of the arc c .	Veloc. per second.
	Oz.	Inches.	Inches.	Inches.	Oz.	Pounds	Pounds	Inches.	Inches.	Feet
1	2	2.021	2.85	90.	$28\frac{1}{4}$	1.766	552.0	78.0	14.8	622
2	4	2.021	4.4	87.	$28\frac{1}{4}$	1.766	553.8	78.0	20.5	879
3	8	2.032	7.1	87.	$28\frac{3}{4}$	1.797	555.5	78.1	27.5	1164
4	2	2.026	2.85	90.	$28\frac{1}{2}$	1.781	557.3	78.1	15.0	622
5	4	2.026	4.4	88.	$28\frac{1}{2}$	1.781	559.1	78.1	20.5	871
6	8	2.032	7.1	92.	$28\frac{3}{4}$	1.797	560.9	78.2	28.5	1154
7	2	2.021	2.85	89.8	$28\frac{1}{4}$	1.766	562.7	78.2	14.3	605
8	4	2.026	4.4	91.3	$28\frac{1}{2}$	1.781	564.5	78.2	21.0	870
9	8	2.026	7.1	87.	$28\frac{1}{2}$	1.781	566.2	78.3	26.8	1169

Let us now collect together the several velocities belonging to the same quantity of powder, in order to take their means, thus:

Veloc.

Veloc. with 2 ounces.	Veloc. with 4 ounces.	Veloc. with 8 ounces.
612	879	1164
622	871	1154
605	870	1169
<hr/>	<hr/>	<hr/>
3)1839	3)2620	3)3487
<hr/>	<hr/>	<hr/>
The means, 613	873	1162
<hr/>	<hr/>	<hr/>

The uniformity of these velocities is very striking, and the means with two, four, and eight ounces of powder are 613, 873, and 1162, which are in the ratio of 1, 1.424, and 1.9; these numbers are nearly in the ratio of the square roots of the quantities (2, 4, and 8) of powder, the numbers in this latter ratio being 1, 1.414, and 2, where the small difference lies chiefly in the last number. A small part of this defect in the greatest velocity is to be attributed to the mean weight of the balls used with it being greater than in the others; for the mean weight of the balls used with eight ounces of powder is $28\frac{2}{3}$ ounces, while that with the two and four ounces is only $28\frac{1}{3}$; the reciprocal sub-duplicate ratio of these is that of 1 to 1.006, in which proportion, increasing 1.9 the number for the greater velocity, it becomes 1.91, which still falls short of 2 by .09, which is about the $\frac{1}{22}$ part too small for the sub-duplicate ratio of the

the powder. This defect of a 2^d part is owing to three evident causes, *viz.* 1. The less length of cylinder through which the ball was impelled; for by inspecting the fourth column, denoting the height of the charge, it appears, that the balls lay three or four inches nearer to the muzzle of the piece with the eight ounce charge than with the others. 2. The greater quantity of elastic fluid which escaped in this case than in the others by the windage; this happens from its moving with a greater velocity, in consequence of which a greater quantity escapes by the vent and windage than with the smaller velocities. 3. The third cause is the greater quantity of powder blown out unfired in this case than in that of the less velocities; for the ball which was impelled with the greater velocity would be sooner out of the piece than the others, and the more so as it had a less length of the bore to move through; and if powder fire in time, which cannot be denied, although indeed that time is manifestly very short, a greater quantity of it must remain unfired when the ball with the greater velocity issues from the piece, than when that which has the less velocity goes out, and still the more so as the bulk of powder which was at first to be inflamed in the one case so much exceeded that in the others. The effect, however, will arise chiefly from the first and last
of

of these three causes, as that of the second will amount to very little; because that the effect arising from the greater velocity with which the fluid escapes at the vent and windage, is partly balanced by the shorter time in which it acts.

From the above reflections we may also perceive, how small the quantity of powder is which is blown out unfired in any of these cases, and the amazing quickness with which it fires in all cases: for although the time in which the ball passed through the barrel, when impelled by the eight ounces of powder, was not greatly different from the half only of the time in which it was impelled by the two ounces, it is evident that in half the time there was nearly four times the quantity of powder fired.

The fifth or last course was on the 21st of September, 1775, fine clear weather, but a little windy.

The machinery and the balls were of iron, but powder the same as in the last course, and the dimensions as follows:

$p = 553$ pounds, the weight of the pendulum.

$r = 101$ inches, its length.

$g = 78\frac{1}{8}$ inches, the distance of the center of gravity.

$b = 84.775$ inches = 7.065 feet, that of the center of oscillation, the pendulum making 68 vibrations in 100 seconds.

Number

Number.	Weight of powder.	Diam. of the ball.	Heigh. of the charge.	Struck below the axis, f.	Weight of the ball.	Weight of the ball, lb.	Values of p.	Values of g.	Chord of the arc, c.	Velo. per second.
	Oz.	Inches	Inches	Inches.	Oz. Dr.	Pounds	Pounds	Inches.	Inch.	Feet.
1	2	2.062	3.	88.3	19 0	1.188	553.0	78.1	11.4	702
2	4	2.062	4.3	88.3	19 0	1.188	554.2	78.1	17.3	1068
3	8	2.062	6.7	91.0	19 0	1.188	555.5	78.2	23.6	1419
4	2	2.070	3.	90.7	19 3 $\frac{1}{2}$	1.201	556.8	78.2	11.4	682
5	4	2.080	4.3	90.7	19 8 $\frac{1}{2}$	1.221	558.1	78.2	17.3	1020
6	8	2.064	6.7	90.7	19 0 $\frac{3}{4}$	1.190	559.4	78.2	22.3	1352
7	2	2.060	3.	91.	18 15	1.184	560.6	78.3	11.4	695
8	4	2.058	4.3	90.	18 14	1.180	561.9	78.3	15.3	948
9	8	2.049	6.7	90.	18 9 $\frac{3}{4}$	1.163	563.1	78.3	22.9	1443
10	2	2.047	3.	88.3	18 9	1.160	564.3	78.3	10.9	703
11	4	2.037	4.3	88.3	18 4 $\frac{1}{2}$	1.142	565.5	78.4	14.8	973
12	8	2.036	6.7	88.3	18 3 $\frac{1}{4}$	1.140	566.6	78.4	20.6	1360
13	2	2.034	3.	92.	18 3	1.137	567.8	78.4	11.4	725
14	4	2.034	4.3	92.	18 3	1.137	569.0	78.4	15.0	957
15	8	2.031	6.7	94.3	18 1 $\frac{1}{2}$	1.131	570.1	78.5	22.5	1412

Let us now take the means among those of the same quantity of powder, thus:

Veloc.

Veloc. with 2 ounces.	Veloc. with 4 ounces.	Veloc. with 8 ounces.
702	1068	1419
682	1020	1352
695	948	1443
703	973	1360
725	957	1412
<hr/> 5)3507	<hr/> 5)4966	<hr/> 5)6986
<hr/> The means, 701	<hr/> 993	<hr/> 1397

And these mean velocities with two, four, and eight ounces of powder, are as the numbers 1, 1.416, and 1.993; but the sub-duplicate ratio of the weights (two, four, and eight) of powder gives the numbers 1, 1.414, and 2, to which the others are sufficiently near. It is obvious, however, that the greatest difference lies in the last number which answers to the greatest velocity, and which is again in defect. It will still be a little more in defect if we make the allowance for the weights of the balls; for the mean weight of the balls with the two and four ounces is $18\frac{3}{4}$ ounces, but of the eight ounces it is $18\frac{3}{5}$; diminishing therefore the number 1.993 in the reciprocal sub-duplicate ratio of $18\frac{3}{5}$ to $18\frac{3}{4}$, it becomes 1.985, which falls short of the number 2 by .015 or the 133d part of itself; which defect is to be

attributed to the same causes as it was in the last course of experiments before explained.

Let us now compare the corresponding velocities in this course and the last.

In this course they are 701, 993, 1397;

In the last they were 613, 873, 1162.

Now the ratio of the first two numbers, or the velocities with two ounces of powder, is that of 1 to 1.1436; the ratio of the next two, is that of 1 to 1.1375; and the ratio of the last is that of 1 to 1.2022. But the mean weight of the shot was, for two and four ounces of powder $28\frac{1}{3}$ ounces in the last course, and $18\frac{3}{4}$ ounces in this; and for eight ounces of powder, it was $28\frac{2}{3}$ in the last, and $18\frac{3}{5}$ in this: taking now the reciprocal subduplicate ratios of these weights of shot, we obtain the ratio of 1 to 1.224 for that of the balls which were fired with two ounces and four ounces of powder, and the ratio of 1 to 1.241 for the balls which were fired with eight ounces. But the real ratios above found are not greatly different from these. And the variation of the actual velocities from this law of the weights of shot incline the same way in this course, as they appeared to do in the second course of these experiments.

We may now collect into one view the principal inferences that have resulted from these experiments.

1. And first, it is made evident by them, that powder fires almost instantaneously, seeing that almost the whole of the charge fires though the time be much diminished.

2. The velocities communicated to balls, or shot of the same weight, with different quantities of powder, are nearly in the sub-duplicate ratio of those quantities. A very small variation, in defect, taking place when the quantities of powder become great.

3. And when shot of different weights are fired with the same quantity of powder, the velocities communicated to them are nearly in the reciprocal sub-duplicate ratio of their weights.

4. So that, universally, shot which are of different weights, and impelled by the firing of different quantities of powder, acquire velocities which are directly as the square roots of the quantities of powder, and inversely as the square roots of the weights of the shot, nearly.

5. It would therefore be a great improvement in artillery to make use of shot of a long form, or of heavier matter; for thus the momentum of a shot, when fired

with the same weight of powder, would be increased in the ratio of the square root of the weight of the shot.

6. It would also be an improvement to diminish the windage; for by so doing, one-third or more of the quantity of powder might be saved.

7. When the improvements mentioned in the last two articles are considered as both taking place, it is evident that about half the quantity of powder might be saved, which is a very considerable object. But important as this saving may be, it seems to be still exceeded by that of the article of the guns; for thus a small gun may be made to have the effect and execution of one of two or three times its size in the present mode, by discharging a shot of two or three times the weight of its natural ball or round shot. And thus a small ship might discharge shot as heavy as those of the greatest now made use of.

Finally, as the above experiments exhibit the regulations with regard to the weights of powder and balls, when fired from the same piece of ordnance, &c.; so by making similar experiments with a gun, varied in its length, by cutting off from it a certain part before each course of experiments, the effects and general rules for the different lengths of guns may be certainly determined by them. In short, the principles on which these
expe-

Fig: 1.

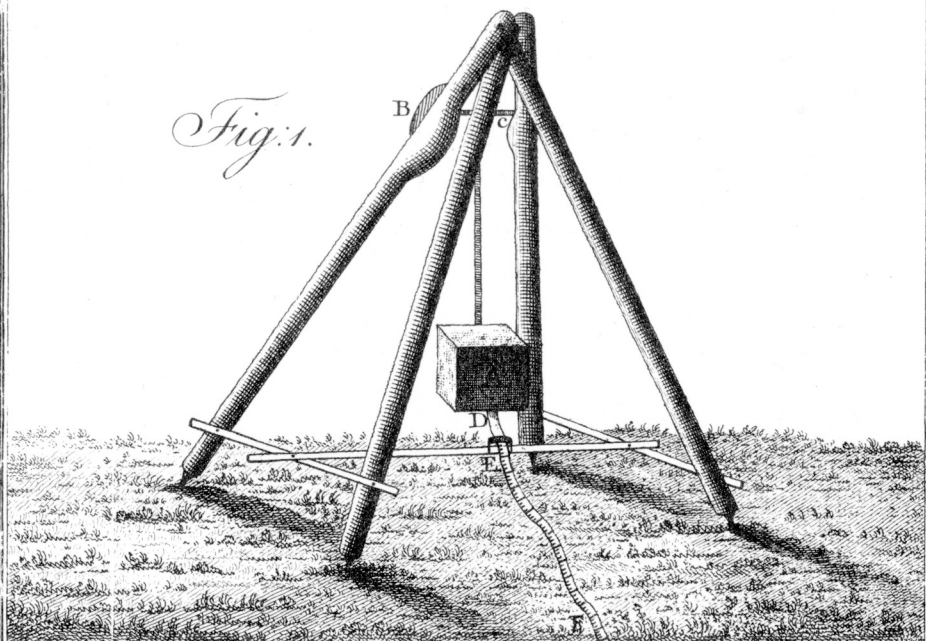
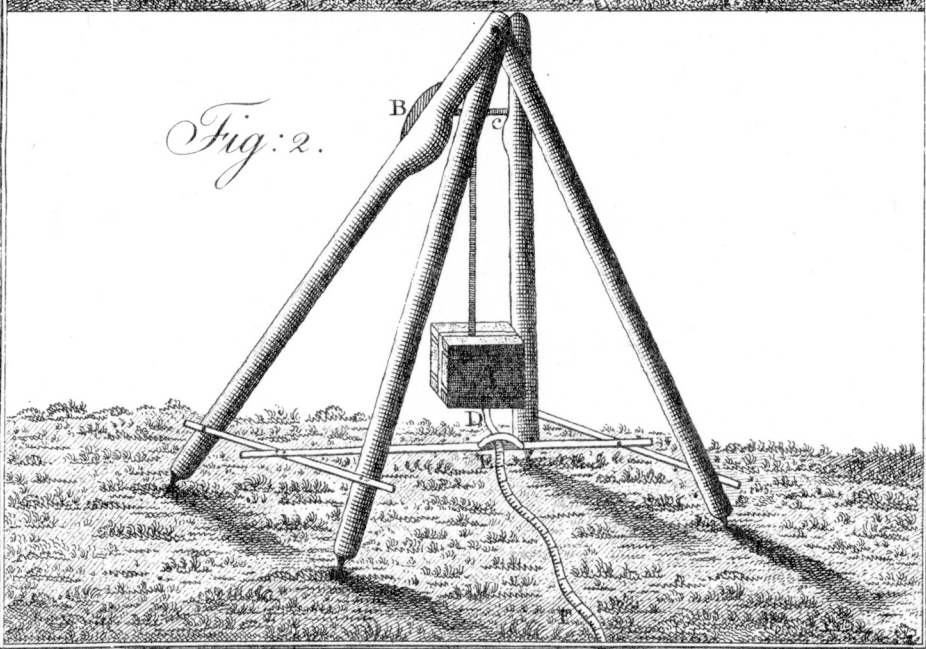


Fig: 2.



experiments were made, are so fruitful in consequences, that, in conjunction with the effects resulting from the resistance of the medium, they seem to be sufficient for answering all the enquiries of the speculative philosopher, as well as those of the practical artillerist.

