

XXIV. *The principal Properties of the Engine for turning Ovals in Wood or Metal, and of the Instrument for drawing Ovals upon Paper, demonstrated. By the Rev. Mr. Ludlam, Vicar of Norton, near Leicester; communicated by the Astronomer Royal.*

Read May 4, 1780.

THE instrument for drawing ovals upon paper or board is so common, that a particular description of it is needless. It is much in use among the joiners, and called by them *the trammels*. One part of it consists of a cross with two grooves at right angles: the other is a beam carrying two pins which slide in those grooves, and also the describing pencil; we shall distinguish these two parts by the names of the *cross* and the *beam*.

It is very well known, that all the engines for turning ovals are constructed on the same principles with the trammels; the only difference is, that in the trammels the board is at rest, and the pencil moves upon it; in the turning engine, the tool (which supplies the place of the pencil) is at rest, and the board moves against it.

Let

Let Aa and Bb (fig. 1.) be two indefinite lines, intersecting each other at right angles in c . Let LSM be the beam, or a rigid right line, in which assume two fixed points L and s at pleasure. If the fixed point L be kept always sliding upon the line Bcb , and the other point s always sliding upon the line $Ac a$; I say then, that any point M in the line LS , or that line produced, will describe an ellipse.

Bisect LS in E , and through c and E draw the indefinite right line CEH . Upon LS as a diameter with the center E describe a semi-circle, and because Lcs is a right angle, it will pass through c , and $EC=EL$. Through M draw MPH perpendicular to AC meeting CE produced in H ; and because MH is parallel to CL , the triangles MEH and CEL are similar, and $HE=ME$, and $HE+EC=ME+EL$, or $CH=LM$. The point H therefore always falls in the circumference of the circle $HADa$ described with the center c and radius $CH=LM$. Now the similar triangles CHP and SMF give $CH:SM::PH:PM$. But when L arrives at c , then $LM(=CH)$ coincides with CA ; and when s arrives at c , then SM coincides with CB ; therefore $CA:CB::PH:PM$, and $CA^2:CB^2::PH^2:PM^2$, or $CA^2:CB^2::AP \times Pa:PM^2$, which is the property of an ellipse, whose first semi-axe is CA or LM , and second semi-axe is $CB=SM$.

Produce PM till it meets the circle in N, and draw the radius CN; then $PH=PN$ and $CA:CB::PN:PM$. Again, because $PCH=PCN$, therefore $NCD=ECL=ELC$ and CN is parallel to LM, and $CL=NM$. Draw mp perpendicular to Bb , cutting CN in n , and for the like reason $cn=sm=CB$, and $cs=mn$. While the point M describes an oval, the point E describes a circle whose center is c and radius $CE=\frac{1}{2}SL$.

To the ruler MEL (fig. 2. and 3.) fix another ruler or right line mEK passing through E, so that the ruler mEK may be carried about by the ruler MEL, keeping the angle MEM between the two rulers invariable. On mEK take $EV=EK$, and each = ES or EL, I say, the point v will describe a right line $avc\alpha$ passing through c, and making an angle acs with CA, equal to half MEM the angle made by the two rulers; the point K will also describe a right line $bKc\beta$ passing through c, and making an angle bCL , with CL, also equal to half MEM .

On the center E (fig. 2. and 3.) and with the radius EC describe a circle and it will pass through the points s, v, c, L, K; draw the lines vc and Kc, and the angles SEV , and scv , both stand on the same arch sv; the former at the center E, the latter at the circumference c; therefore the former is double the latter. In like manner the angles KEL and KCL both stand on the same arch KL,

the former at the center, the latter at the circumference; therefore the former is double the latter. Now as this holds in every position of the rulers during their joint motion, it is manifest, the points v and k will each describe right lines, namely, $ac\alpha$ and $bc\beta$, passing through c and making the angles aca and bcl ($=bc\beta$) each equal to half MEm .

Hence the lines $ac\alpha$ and $bc\beta$, traced by the points v and k are at right angles, and the ruler $mvek$ moves exactly in the same manner as if it was guided by the the points v and k sliding on the lines $ac\alpha$ and $bc\beta$, at right angles to each other; just in the same manner as the ruler msl is guided by the points s and l , sliding on the lines ac and bc . Therefore if any point m be assumed in the line kvm as a describing point, the figure described will be an ellipse, the position of whose principal axes are the lines $ac\alpha$ and $bc\beta$; the center of the ellipse being still in c as before. If mk is taken equal to ml , the ellipse thus described by the point m will be the same with that described by the point m , only in another position: its greater semi-axis ac making an angle with ac , the greater semi-axis of the former ellipse, equal to half MEm , the angle which the rulers or lines ME and mE make with each other.

Scholium. This proposition is demonstrated in SCHOOTEN'S *Exercitationes*, &c. p. 305.; but he makes twelve cases of it: had he made use of the 20th of the 3d El. they might have been all comprehended under one.

In the turning of ovals, the top of the *rest* which supports the tool is always made to pass through s and L (fig. 1.) the two centers round which the oval engine turns; and in this case the ruler or line MSEL represents the top of the *rest*. If the tool be held on any part of the *rest* between the work-man, and the nearest center as at M, an oval will be turned having its longer axis Aa (in one position of the work) coinciding with the top of the *rest*. As the tool is removed towards s, the oval will grow narrower and at s become a right line. Beyond s towards E it will grow rounder, and at E become a circle; beyond E it will grow narrower, and at L become a right line at right angles to the right line described when the tool was at s. If the tool be removed beyond L, it will describe an oval again, whose longer axis is at right angles to the longer axis of the oval first described when the tool was at M. It may be very convenient to mark the points s and L and also their middle point E on the top or face of the *rest* that supports the tool. If any thing be interposed between the tool and the top of the *rest* so as to raise the tool above the line passing through the centers
s and

s and L, an oval will yet be described, whose center will be the same with that of the oval first described when the tool was at M; but its principal axis will cross the principal axis of that oval (fig. 2. and 3.). Draw right lines both from M the old place of the tool, and from *m* the new place of the tool, to the point E marked on the *rest*. Half the angle which these two lines make with each other will be the angle which the principal axis of the new oval makes with the principal axis of the old one.

It is well known, that when the oval engine is set in order for working, there is a part which slides back, and is then fixed, which separates the two centers of motion and gives the eccentricity; for the difference between the first and second semi-axes will be just as much as the centers are thus separated: call the distance between the two centers E; let now the tool be fixed in any place, upon, above, or below the *rest*; call *mE* the distance of the tool from the middle point between the centers (marked E on the *rest*) D; and the greater semi-axis of the oval so described will be $D + \frac{1}{2}E$, and the lesser semi-axis $D - \frac{1}{2}E$; and thus both the form and position of the oval will be known. All workmen know the tool must never be raised above the place where it was at first held, and we

fee the reason; it would destroy the oval first begun to be turned, and form a new one in a different position.

But there is another difficulty in turning ovals, especially such as have mouldings, as picture-frames, &c. The tool generally has all the mouldings formed upon it: now if it be laid flat upon the *rest*, and the engine set to work; the mouldings will in some places cross the plane of the tool (or the top of the *rest*) at right angles (as in turning circles), in other places obliquely. This will make the several members of the mouldings *leaner* or smaller in one part of the work than another. Nor will the case be altered if the mouldings be turned separately. Analogous to this, when an oval is drawn by the trammels, the line described by the pencil will not, as in a circle, be always at right angles to the beam of the trammels. The oval line so drawn will be at right angles to the describing beam, only at the extremity of the two principal axes where the beam coincides with those axes; in all other places the oval line and beam make an oblique angle. It may be proper therefore to enquire how much this angle deviates from a right angle. This we shall call the angle of *deviation*.

All things as in fig. 1. draw the tangents TM and TN, to the point M in the ellipse and the point N in the circle corresponding to each other; and from the nature of the ellipse

ellipse these tangents will meet each other in the axis CA produced. Draw MG perpendicular to TM, and GMS will be the angle of deviation sought. I say, the angle MTN, between the tangents to corresponding points in the ellipse and circumscribing circle, is equal to the angle of deviation GMS.

For because TNC is a right-angled triangle, and NP perpendicular to TC; therefore TNP=NCP=MSP, that is (in the triangles MTN and GMS) the angles TNM and MSG are equal. In like manner, because TMG is a right-angled triangle and MP perpendicular to TG, therefore TMP=MGP, and (in the triangle MTN and GMS) the angles TMN and MGS are equal; therefore in the same triangles, the remaining angles MTN and GMS are also equal.

To compute the angle MTN, we have by trigonometry $TP^2 + PM \times PN : TP :: MN : \tan. MTN$, radius being unity. Call now $CA=t$, $CB=c$, $CP=x$, $PM=y$, $CA-CB$ (or $t-c$)= d , and we have $PN = \sqrt{tt-xx}$; also $CD : CB :: PN : PM = \sqrt{tt-xx} \times \frac{c}{t}$, whence $PM \times PN = \sqrt{tt-xx} \times \frac{c}{t}$. Again $CP : PN :: PN : PT$, whence $TP = \frac{tt-xx}{x}$. Lastly, $CD : BD :: PN : MN = \sqrt{tt-xx} \times \frac{d}{t}$; whence the tangent of MTN the angle sought is $\frac{dx\sqrt{tt-xx}}{tt-axx}$: and this is a *maximum* when $\frac{ttt}{2t-d}$, or $\frac{ttt}{t+c} = xx$, or when $\frac{ccc}{t+c} = yy$, or when CP and PM have such a proportion that $CP^2 : PM^2 :: CA^3 : CB^3$.

Let

386 *Mr. LUDLAM on the Engine for turning Ovals.*

Let $AMBab$ (fig. 4.) be an ellipse whose center is c ; draw the circumscribed and inscribed circles as before; the former cutting the second axis produced in D , the latter cutting the first axis in d , and the second axis in b . On Db as a diameter describe a circle cutting the first axis Aa in Q , draw DQ and bQ . Set off $CR=DQ$, join DR , and draw DP at right angles cutting the first axis in P , draw PM an ordinate to that axis, and M will be the point in the oval line where the angle of deviation is greatest. Otherwise, upon cb produced set off $cr=bQ$, join dr , and draw dp at right angles cutting the second axis in p : draw pM an ordinate to that axis, and M will be the point where the angle of deviation is greatest.

At the maximum (when $xx=\frac{ttt}{t+c}$) $PN^2=\frac{ttc}{t+c}$, $PM^2=\frac{ccc}{t+c}$, and $TP^2=\frac{tcc}{t+c}$: whence $TP^2=PM \times PN$. Also, PN , PM , TP , are to each other as CA , CB , and $\sqrt{CA \times CB}$, respectively. Therefore, $\frac{CA-CB}{\sqrt{Aa \times Bb}}$ is the tangent of MTN , radius being unity. Also $\sqrt{\frac{CA}{CB}}$ is the tangent of NTP ; and MTP is the complement of NTP : therefore MTN is twice the excess of NTP , above 45° .



A D D E N D A.

At the end of page 386 add,

In fig. 4. draw a circle through the points N, M, T; and at the maximum where $TP^2 = PM \times NP$ this circle will touch CA produced in T. From E the center of this circle draw EF perpendicular to NM, also the radii EN and EM; and PN is the sine of NEF, or half NEM, or of its equal MTN, to the radius EN. But $EN = ET = EF = \frac{PN + PM}{2}$, and $FN = \frac{PN - PM}{2}$. Therefore $PN + PM$ is to $PN - PM$, or $CD + CB$ is to $CD - CB$, or $CA + CB$ is to $CA - CB$, as radius is to the sine of the greatest angle of deviation, which is therefore equal to $\frac{CA - CB}{CA + CB}$, radius being unity.

E R R A T A to Vol. LXX.

Page 6, line antepenult. read be nearly mathematically.

6, l. penult. *dele* yet.

7, l. 13, *dele* section ABC, or

7, l. 18, *at the end of the line add* very nearly.

394, l. 15, *transpose* general equation to the beginning of the line above.

402, l. 6, 7, 8, *for* 9143 r. 9443.

405, l. 7, *for the last*, $1 + \sqrt{-3} r. 1 - \sqrt{-3}$.

405, l. 11, *for the last* $-\frac{\sqrt{-3}}{2} r. + \frac{\sqrt{-3}}{2}$.

443, *end of the 1st line, for* and x r. and X.

548, l. 10, *for* circumstances r. and which are.

*** There are FIFTEEN Plates in this Volume.





