



PHILOSOPHICAL
TRANSACTIONS.

- I. *Observations on the Graduation of Astronomical Instruments; with an Explanation of the Method invented by the late Mr. Henry Hindley, of York, Clock-maker, to divide Circles into any given Number of Parts. By Mr. John Smeaton, F. R. S.; communicated by Henry Cavendish, Esq. F. R. S. and S. A.*

Read November 17, 1785.

PERHAPS no part of the science of Mechanics has been cultivated by the ingenious with more assiduity, or more deservedly so, than the art of dividing Circles for the purpose of Astronomy and Navigation. It is said, that TYCHO BRAHE

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and HEVELIUS laboured this part of their instruments with their own hands; and though public rewards have at length brought forth different methods of dividing from our best artists, which have been communicated to the Public; yet I trust it will be thought, that if any thing relative to this business remains yet behind, that may tend to furnish the ingenious artists, who are cultivating this field, with any new or curious idea upon the subject, it will be well worth communicating to this learned Society: since, if an hint, which is essentially different from any thing that (so far as I know) the Public is in possession of, be once started, and is pursued and worked upon by ingenious men, it is not possible to say, to what valuable purposes it may be converted.

This, perhaps, will better appear by taking a short review of the labours of others, from the time of TYCHO BRAHE and HEVELIUS (who did not use telescopic sights) to the present time.

The very learned, ingenious, and inventive Dr. HOOK, in his Animadversions on the *Machina Caelestis* of HEVELIUS, published in the year 1674, has given us an elaborate description of a quadrant, whose divisions were formed, and afterwards read off, by means of an endless screw, working upon the outermost border of the limb of a quadrant; which, he says, *does not at all depend upon the care and diligence of the instrument-maker in dividing, graving, or numbering the divisions, for the same screw makes it from end to end*; yet he has given us no account of any particular care or caution that he used, in preventing the same screw from making larger or smaller paces, in consequence of unequal resistance, from a different hardness of the metal in different parts of the limb; nor any method of correcting or checking the same; nor of making a screw, the

angle of whose threads with the axis shall be equal in every part of the circumference; therefore the whole of this business (in which accurate *mechanists* well know consists the whole of the difficulty) he refers to the *ingenious workman*; and, in particular, to the then celebrated Mr. TOMPION, whom, he says, he employed to make his instrument, and who had thereby *seen and experienced the difficulties that do occur therein*: but was any ingenious workman now to pursue the directions of Dr. Hook, so far as his communication thereof extends, we may conclude, that he would make a very inaccurate piece of work, far inferior in performance to what the Doctor seems to expect from it*. But yet, I believe, it was the *first* attempt to apply the endless screw and wheel, or arch, to the purpose of forming divisions for astronomical instruments; for, the Doctor says himself, the perfection of this instrument is the *way* of making the divisions; that it *excels all the common ways of division*: and in the table of contents it is intitled, *An Explication of the new Way of dividing*.

This method, however, of Dr. Hook's was not laid aside without a very full and sufficient trial: for Mr. FLAMSTEED, in the *Prolegomena* of the third volume of *Historia Cœlestis*, informs us, that *he* contrived the sextant, wherewith his observations were chiefly made, from his entrance into the Royal Observatory in the year 1676 to the year 1689. This sextant was first made of wood, and afterwards of iron, with a brass limb of two inches broad, by Mr. TOMPION, at the expence of Sir JONAS MOORE; the radius thereof was 6 feet $9\frac{1}{4}$ inches; it was furnished with an endless screw upon its limb of 17

* This was indeed verified in an attempt upon the same plan by the Duc DE CHAULNES, published in a Memoir of the Royal Academy of Sciences at Paris, for the year 1765.

threads in an inch, and with telescopic sights*. Of this instrument Mr. FLAMSTEED gives the figure at the latter end of his *Prolegomena* before-mentioned, sufficiently large to see the general design; the whole being mounted upon a strong *polar axis* of iron, of three inches diameter.

Though, in the full description of this instrument, Mr. FLAMSTEED mentioned the Limb's being furnished with *diagonal divisions*, distinguishing the arch to 10 seconds†; yet it is pretty clear, that it had not these originally upon it; but that the dependance was wholly upon the screw divisions, when it came out of Mr. TOMPION's hands. This one may reasonably infer from the observations themselves; for the first observation, set down as taken with this instrument, being upon the 29th of October, 1676, it was not till the 11th of September, 1677, that the column which contained the *check* angle by diagonal lines was filled up; and there was also a space of time, antecedent to that last mentioned, wherein no observations are recorded as taken with this instrument, in which time the diagonal divisions might be put on; and this will be put beyond a doubt, as he says expressly, that finding, in the year 1677, that the threads of the screw had worn the border of the limb, he divided the limb into degrees himself, and drew a set of diagonal divisions‡; and then comparing the two sets of divisions together, he sometimes found them to differ a whole minute; wherefore, for correction thereof, he constructed a new table for conversion of the revolutions and parts of the screw into degrees, minutes, and seconds; and

* “— Qualem nemo, cœlo adhibens;—” Preface to *Historia Cœlest.* printed in one vol. 1712.

† *Prolegomena Histor. Cœlest.* vol. III. p. 104.

‡ *Ibid.* p. 106. “Gradus in limbo distribui ac diagonales duxi.”

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which he applied in the observations taken in 1678.—However, notwithstanding this correction, in looking over the observations noted down as deduced each way, I frequently find a difference of half a minute; not unfrequently $40''$; but in an observation of the moon, of the 9th June, 1687, I find a difference of $55''^*$, which upon a radius of 6 feet 9 inches amounts to more than $\frac{1}{30}$ th part of an inch.

In the year 1689, Mr. FLAMSTEED compleated his *mural arc* at GREENWICH; and, in the *Prolegomena* before-mentioned, he makes an ample acknowledgement of the particular assistance, care, and industry of Mr. ABRAHAM SHARP; whom, in the month of August, 1688, he brought into the observatory, as his *amanuensis*; and being, as Mr. FLAMSTEED tells us, not only a very skilful mathematician, but exceedingly expert in mechanical operations †, he was principally employed in the construction of the mural arc; which in the compass of fourteen months he finished, so greatly to the satisfaction of Mr. FLAMSTEED, that he speaks of him in the highest terms of praise ‡.

This celebrated Instrument, of which he also gives the figure at the end of the *Prolegomena*, was of the radius of 6 feet $7\frac{1}{2}$ inches; and, in like manner as the sextant, was furnished both

* Vol. I. of Hist. Cœlest. p. 343.

† “Qui mechanices perquam expertus, pariter ac matheos peritus.” *Prolegomena*, vol. III. p. 108.

‡ “SHARPEIUS servus meus fidelissimus, ac omnibus quidem dotibus & facultatibus erat imbutus, quæ ipsum tam subtili & difficili operi obeundo idoneum redderent.” *Prolegom. ibid.*

And on finishing the instrument, he says, “Gradus describuntur five narrantur et exculpantur, artificiosa manuali opera dicti domini SHARP, qui limbum partitus est, diagonales duxit, totumque organum absolvit et perfecit: adeo ut præstantissimi quivis artifices postquam illud conspexerunt et considerarunt, se exactius id peragere non potuisse, agnoverint.” *Prolegom. p. 111.*

with screw and diagonal divisions, all performed by the accurate hand of Mr. SHARP. But yet, whoever compares the different parts of the table for conversion of the revolutions and parts of the screw belonging to the mural arc into degrees minutes and seconds *, with each other, at the same distance from the zenith on different sides; and with their halves, quarters, &c. will find as notable a disagreement of the screw-work from the hand-divisions, as had appeared before in the work of Mr. TOMPION: and hence we may conclude, that the method of Dr. HOOK, being executed by two such masterly hands as TOMPION and SHARP, and found defective, is in reality not to be depended upon in nice matters.

From the account of Mr. FLAMSTEED it appears also, that Mr. SHARP obtained the zenith point of the instrument, or *line of collimation*, by observation of the zenith stars, with the face of the instrument on the east and on the west side of the wall †: and that having made the index stronger (to prevent flexure) than that of the sextant, and thereby heavier, he contrived, by means of pullies and balancing weights, to relieve the hand that was to move it from a great part of its gravity ‡.

I have been the more particular relating to Mr. SHARP, in the business of constructing this mural arc; not only because we may suppose it the first good and valid instrument of the kind, but because I look upon Mr. SHARP to have been the first person that cut accurate and delicate divisions upon astronomical instruments; of which, independent of Mr. FLAMSTEED's testimony, there still remain considerable proofs: for, after leaving Mr. FLAMSTEED, and quitting the department above-mentioned †,

* Hist. Cœlest. vol. II. Appendix.

† Prolegom. p. 109.

‡ Mr. SHARP continued in strict correspondence with Mr. FLAMSTEED so long as he lived, as appeared by letters of Mr. FLAMSTEED's found after Mr. SHARP's death; many of which I have seen.

he retired into *Yorkshire*, to the village of *Little Horton*, near *Bradford*, where he ended his days about the year 1743; and where I have seen not only a large and very fine collection of mechanical tools (the principal ones being made with his own hands), but also a great variety of scales and instruments made therewith, both in wood and brass, the divisions whereof were so exquisite, as would not discredit the first artists of the present times: and I believe there is now remaining a quadrant, of four or five feet radius, framed of wood, but the limb covered with a brass plate; the subdivisions being done by diagonals, the lines of which are as finely cut as those upon the quadrants at *Greenwich*. The delicacy of Mr. SHARP's hand will indeed permanently appear from the copper plates in a quarto book, published in the year 1718, intituled, *Geometry improved by A. Sharp, Philomath.* whereof not only the geometrical lines upon the plates, but the whole of the engraving of letters and figures, were done by himself, as I was told by a person in the mathematical line, who very frequently attended Mr. SHARP in the latter part of his life. I therefore look upon Mr. SHARP as the first person that brought the affair of hand division to any degree of perfection.

Some time about the establishment of the mural arc at *Greenwich*, the celebrated Danish Astronomer OLAUS ROEMER began his domestic Observatory, which he finished in the year 1715, as we are informed by his historian PETER HORREBOW, in the third volume of his works, in the tract, intituled, *Basis Astronomiæ*, published in the year 1741. In this tract is the description of an instrument, Tab. III. which not only answered the purpose of the meridian arc; but, its telescope being mounted upon a long axis, became also in reality what we now call a *Transit Instrument*; and which furnished, so far

as I have been able to learn, the first idea thereof. One end of the axis of this instrument being the center of the meridian arc, and carrying its index, M. ROEMER thereby avoided the errors arising from the plane of the mural arc not being accurately a vertical plane; and which Mr. FLAMSTEED endeavoured to check, by observing the passage of known stars nearly in the same parallel of declination; that is, passing nearly over the same part of the plane of the arc; by which he was enabled to correct or check the errors of the arc in right ascension. But it is the peculiar method in which ROEMER *divided* his instruments, that occasions him here to be introduced.

Though it is a very simple problem by which geometers teach how to divide a given right line into any number of parts required; yet it is still a much more simple thing to set off upon a given right line, from a point given, any number of equal parts required, where the total length is not exactly limited; for this amounts to nothing more than assuming a convenient opening of the compasses, and beginning at the point given, to set off the opening of the compasses as many times in succession, as there are equal parts required; which process is as applicable to the arch of a circle as it is to a right line. Of this simple principle ROEMER endeavoured to avail himself.

For this purpose M. ROEMER took two stiff, but very fine-pointed, pieces of steel, and fixed them together, so as to avoid, as much as possible, every degree of spring that would necessarily attend long-legged compasses, or even those of the shortest and stiffest kind when the points are brought near together. The distance of the points that he chose was about the $\frac{1}{16}$ or $\frac{1}{12}$ of an inch. This, upon a radius of $2\frac{1}{2}$ or three feet, would be about 10 minutes. With this opening, beginning
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at the point given, he set off equal spaces in succession to the end of his arch, which was about 75° . Those were distinguished upon the limb of the instrument by very fine points, which were referred to by a grosser division, the whole being properly numbered. The subdivision of those arches of 10 minutes each was performed by a double microscope, carried upon the arm or radius of the instrument, the common focus being furnished with parallel threads of single silk, whereof eleven being disposed at ten equal intervals, comprehending together one ten-minute division, the distance of the nearest threads became a very visible space, answerable to one minute each, and therefore capable of a much further subdivision by estimation.

The divisions of this instrument were therefore, properly speaking, not degrees and minutes; but yet, if exactly equal, would serve the purpose as well, when their true value was found, which was done by comparison with larger instruments.

Now, if it be considered, that in going step by step of ten minutes each, through a space of 75 degrees, there will be a succession of 450 divisions, dependant upon each other; if it be also considered, that the least degree of extuberance in the surface of the metal, where each new point is set down, or the least hard particle (wherewith all the base metals seem to abound) will cause a deviation in the first impression of a taper point, and thereby produce an inequality in the division; it is evident, that though this inequality may be very small, and even imperceptible between neighbouring divisions, yet among distant ones, it may and will arise to something considerable; which, in the mensuration of angles, will have the same ill tendency as in near ones. Now, as M. ROEMER has given us no

means of checking the distant divisions, in respect of each other, it is very probable that no one has followed his steps, in cases where great accuracy was required, in a considerable number of divisions. For in reality this method is likely to fall far short of Dr. HOOK'S; as Dr. HOOK'S divisions being cut in a similar successive manner, by the rotation of the sharp edge of the threads of a screw against the exterior edge of the limb of the instrument, a very slight degree of pressure will bring a fine screw of thirty threads in an inch (which he prescribes) to touch against an arch whose radius is four or five feet in more than one, two, three, or four threads at once; so that the threads supporting one another, a small extuberance, or even a small hard particle in the metal, will be cut through or removed by the grinding or rather sawing motion of the screw; and which, in regard to its contact, being in reality an edge, will be much more effectual (that is, more firm) in its retention than a mere simple point: and a repetition of the operation, from the support of the threads to each other, will tend to mend the first traces; whereas, in ROEMER'S way, a repetition will make them worse; for, whatever drove forward or backward the point on first entering, will, from the sloping of the point, be confirmed and increased in driving it deeper.

When Dr. HALLEY was chosen Astronomer Royal (Mr. FLAMSTEED'S instruments being taken away by his executors), Mr. GRAHAM undertook to make a new mural quadrant, about the year 1725; who, uniting all that appeared valuable in the different methods of his predecessors, executed it with a degree of contrivance, accuracy, and precision, before unknown: and the division thereof he performed with his own hand. The model of this quadrant, for strength, easy management, and convenience, has been ever since pursued as the most perfect.

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What I apprehend to be peculiar in it, was the application of the arch of 96° ; not only as a check upon the arc of degrees and minutes, but as superior thereto, being derived from the more simple principle of *continual bisection*.

To make room for this, he has entirely rejected the subdivision by diagonals, and has adopted the method of the *Vernier*; but the subdivision of the vernier divisions he, as I apprehend for the first time, measured by the turns of the detached adjusting screw, making it in fact a micrometer, by which the distance of the *set* of the instrument was to be measured from the perfect coincidence of one of the actual divisions of the limb with the next stroke of the vernier; by which means the observation could not only be read off with all the precision that the division of the instrument was capable of, but the two sets of divisions could be checked and compared with each other. Another thing that I apprehend to be peculiar in this instrument, was the more certain method of transferring and cutting the divisions, from the original divided points, by means of the *beam-compass*, than could possibly be done from a *fiducial edge*, as had doubtless been constantly the practice in cutting diagonals; for, placing the steady point of the beam-compass in the tangent line to that part of the arc where each division was to be cut, the opening of the compass being nearly the length of the tangent, the other point would cut the division in the direction of the radius nearly; and though in reality an arch of a circle, yet the small part of it in use would be so nearly a right line, as perfectly to answer the same end; all which advantages put together, it is probable, induced Mr. GRAHAM to reject the diagonals.

Soon after the completion of this quadrant, Mr. GRAHAM undertook to execute a *zenith sector* for the Rev. Dr. BRADLEY,

which was fixed up at *Wanstead*, in *Essex*, in the year 1727. The very simple construction that he adopted for this instrument (the plumb line itself being the index) did not admit of the use of a vernier: he therefore contented himself with dividing the arch of the limb of this instrument by primary points, as close as he thought necessary, that is, by divisions of five minutes each, and measuring the distance from the *set* of the instrument to the next point of division by a *micrometer* screw, in the construction of which screw he used uncommon care and delicacy. I have mentioned this instrument to introduce this observation; that I think it highly probable, had Mr. GRAHAM constructed the great quadrant *after* the zenith sector had been fully tried, he would have rejected not only the diagonals but the verniers also, as containing a source of error within themselves which may be avoided by a well-made screw*.

It seems also, that Mr. GRAHAM, at the time he constructed both these instruments, was not aware how much error could arise from the unequal expansions of different metals by heat or cold: for in both the radius, or frame of the instrument, was iron, while the limbs were of brass. They, however, remain in the Royal Observatory, perfect models, in all other respects, of every thing that is likely to be attained in their respective destinations, and monuments of the superlative abilities of that great mechanician Mr. GRAHAM†.

* This has been found consentaneous to the experience of my friend Mr. AUBERT, who, on my suggestion, has long since laid aside the use of his vernier, measuring always by the micrometer screws the distance between the set of the instrument, and the coincidence of the first stroke of the vernier with the next primary division of the limb.

† I have been informed, that Dr. MASKELYNE has caused this objection to the sector to be rectified, since its removal to the Royal Observatory, by substituting an iron limb instead of that of brass, the points being made upon studs of gold.

Mr. GRAHAM lived till the year 1751; and during his time there were few instruments of consequence constructed without his advice and opinion. They were for many years done by Mr. SISSON, to whom doubtless Mr. GRAHAM would fully communicate his method of division; and from this school arose that very eminent and accurate artist Mr. BIRD, whose delicate hand, joined with great care and assiduity, enabled him still further to promote this branch of division; and which being carried by him to a great pitch of perfection, the Commissioners of Longitude did themselves the credit, by an handsome reward, to induce him to publish to the world his particular method of dividing astronomical instruments; which being drawn up by himself, in the year 1767, this matter is fully set forth to the public: I shall therefore only take this opportunity of observing, that there seems to be one article in which Mr. BIRD's method may be still improved.

I must here observe, that I apprehend no quadrant, that has ever undergone a severe examination, has been found to form a perfect arch of 90° ; nor is it at all necessary it should: the perfect equality of the divisions throughout the whole is the first and primary consideration; as the proportion of error, when ascertained by proper observations, can be as easily and readily applied, when the whole error of the rectangle is fifteen seconds, as when it is but five.

In this view, from the radius taken, I would compute the chord of sixteen degrees only. If I had an excellent plain scale, I would use it; because I should expect the deviation from the right angle to be less than if taken from a scale of more moderate accuracy; but if not, the equality of the divisions would not be affected, though taken from any common diagonal scale. This chord, so prepared, I would lay off five times in succession, from

the primary point of 0 given, which would compleat eighty degrees; I would then bisection each of those arches of 16° , as prescribed by Mr. BIRD, and laying off one of them beyond the 80th, would give the 88th degree: proceeding then by bisection, till I came to an arch of two degrees, laying that off from the 88th degree, would give the point of ninety degrees. Proceeding still by bisection, till I had reduced the degrees into quarters = fifteen minutes each, I would there stop; as from experience I know, that when divisions are over close, the accuracy of them, even by bisection, cannot be so well attained as where they are moderately large. If a space of $\frac{1}{4}^\circ$ of an inch, which is a quarter of a degree, upon an eight-feet radius, is thought too large an interval to draw the index over by the micrometer screw, this may be shortened by placing another line at the distance of one-third of a division on each side of the index line, in which case the screw will never have to move the index plate more than one-third of a division, or five minutes; and the perfect equality of those side lines from the index line may be obtained, and adjusted to five minutes precisely, by putting each of the side lines upon a little plate, capable of adjustment to its true distance from the middle one, by an adjusting screw.

The above hint is not confined to the chord of sixteen degrees, which prohibits the subdivisions going lower than fifteen minutes: for if it be required to have divisions equivalent to five minutes upon the limb itself; then I would compute the chord of $21^\circ 20'$ only; and laying it off four times from the primary point, the last would mark out the division $85^\circ 20'$, pointed out by Mr. BIRD; *supplying the remainder* to a quadrant, from the bisected divisions as they arise, and not by the application of other computed chords.

In my Introduction to M. RÖEMER'S Method of Division, I have shewn, that divisions laid off in succession, by the same opening of the compasses, either in a right line, or in the arch of a circle, being in its idea geometrically true, and in itself the most simple of all processes, it has the fairest chance of being mechanically and practically exact, when cleared of the disturbing causes. The objection therefore to his method is, the great number of repetitions, which depending upon each other in succession (requiring no less than 540 to a quadrant, when subdivided to ten minutes each), the smallest error in each, repeated 540 times, without any thing to check it by the way, may arise to a very sensible and large amount: but in the method I have hinted, this objection will not lie; for, in the first case, the assumed opening is laid off but five times; and in the latter case but four times; nor does this *repetition* arise out of the nature of the thing; for, if you like it better, you may, in the former case, at once compute the chord of 64° ; and in the latter that of $85^{\circ} 20'$, and then proceed wholly by bisection; supplying what is wanted to make up the quadrant, from the bisected divisions, as they arise. Mr. BIRD prescribes this method himself, for the division of HADLEY'S sextants and octants.

He, I suppose, was the first, who conceived the idea of laying off chords of arches, whose subdivisions should be come at by continual bisection; but why he mixed therewith divisions that were derived from a different origin (as prescribed in his method of dividing) I do not easily conceive. He says, that after he had proceeded by the bisections, from the arc of $85^{\circ} 20'$, the several points of 30° , 60° , 75° , and 90° , (all of which were laid down from the principle of the chord of 60° being equal to radius), *fell in without sensible inequality*; and so indeed they might; but yet it does not follow that they were equally true in their places

as if they had been (like the rest) laid down from the bisection from $85^{\circ} 20'$, and therefore being the first made, whatever error was in them, would be communicated to all connected with them, or taking their departure from them. Every heterogeneous mixture should be avoided where equal divisions are required. It is not the same thing (as every good artist will see) whether you *twice* take a measure from a scale as *nearly* the same as you can, and lay them off separately; or lay off *two* openings of the compasses, in succession, *unaltered*; for though the same opening, carefully taken off from the same scale a second time, will doubtless fall into the points made by the first, without sensible error; yet as the sloping sides of the conical cavities made by the first point will conduct the points themselves to the center, there may be an error which, though insensible to the sight, would have been avoided by the *more simple process* of laying off the opening twice, without ever altering the compasses.

The 96 arc was, I have no doubt, invented by Mr. GRAHAM, from having perceived, in common with all preceding artists, how very much more easy a given line was to bisect, than to trisect, or quinquesect; and therefore the 96 arc which proceeded by bisections only (or by laying off the same identical openings, which, as already shewn, is still more simple and unexceptionable) was wholly intended by him by way of checking the division of the arc of 90, which required trisections and quinquesections. But experience soon shewed the superior advantage of it so strongly, that the use of the 90 arc is now wholly set aside, where accuracy is required; whereas the ingenuity of Mr. BIRD having shewn a way to produce the 90 arc by bisection, when this is really pursued quite through the piece, by rejecting all divisions derived from any other origin, the 90 arc will have nothing in it to prevent its being
equally

equally unexceptionable with the 96 arc; and consequently if, instead of the 96 arc, another arc of 90 was laid down (which being upon a different radius, its divisions will stand totally unconnected with the former), then these two arcs would in *reality* be a check upon each other; for being of equal validity, the mean might be taken: and if (in lieu of vernier divisions) strokes at the distance of any odd number, as 7, 9, 11, or 13, are marked upon, and carried along with, the index plate; these will produce a check upon neighbouring divisions; and the angle may then be deduced from the medium of no less than four readings.

The last works that have been made known to the public in the line of graduation (so far as has come to my knowledge) are those of the very ingenious Mr. RAMSDEN, which were published, by order of the Board of Longitude, in the year 1777.

From his own information, I learn, that in the year 1760 he turned his thoughts towards making an engine for dividing mathematical instruments; and this he did in consequence of a reward offered by the Board of Longitude to Mr. BIRD, for publishing his method of graduating quadrants; for as, several years previous to that period, he had taken great pains to accomplish himself in the art of hand-dividing, in which line Mr. BIRD had acquired his eminence, he conceived by this publication of Mr. BIRD's he should be reduced to the same standard of performance with the rest of the trade. He, therefore, partly to save time, and that kind of weariness to an ingenious mind that ever must attend the endless repetition of the same thing from morning to night; partly still to preserve the pre-eminence he had then gained; and partly to procure dispatch in the great increase of demand for HADLEY's sextants and octants, in consequence of the successful application of the moon's motion to the

purpose of ascertaining the longitude at sea (which instruments for this purpose required a degree of accuracy and certainty in the division, by no means necessary thereto when applied to the simple purpose of observing latitudes); I say, for these considerations, Mr. RAMSDEN determined to set about something in the instrumental way, that should be sufficient effectually to answer these purposes.

Accordingly, considering the nature of the endless screw, he set himself to work upon an engine whose divided wheel or plate was of thirty inches diameter; and though the performance of this first essay was inferior to his expectations and wishes, yet with it he was able to divide theodolites with a degree of precision far superior to any thing of the kind that had been exhibited hitherto to public.

This engine I myself saw in the spring of the year 1768; and it appeared to me not only a very laudable attempt towards instrumental divisions, but a very good model for the construction of an engine of the most accurate kind for that purpose. And he furthermore, at the same time, shewed me the model or pattern for casting a wheel of a much larger size, which he proposed to make upon the same plan, and with considerable improvements. This being effected some time in or about the year 1774, its accuracy was proved by making a sextant, afterwards subjected to the examination of Mr. BIRD; who in consequence approved the method, not only as fully sufficient for the division of HADLEY's sextants and octants for any purpose whatsoever, but in fact for dividing any instrument whose radius did not exceed that of the dividing wheel, which was forty-five inches in diameter: whereupon the Board of Longitude, ever ready to encourage all endeavours that tend to the certainty and perfection of any thing subservient to the purpose of finding the longitude at sea, very properly and usefully resolved to confer

an handsome reward on Mr. RAMSDEN, for delivering a full explanation of his method of making the said engine; which, in consequence, was published by order of the Board of Longitude in the year already mentioned, viz. 1777: the designs whereof are so full and explicit, that whoever could not understand that description, so as to enable him to make it, would be unfit to undertake it on other accounts.

From what I have said upon the works of the different artists that I have mentioned, it would seem that the art of graduation was brought to that degree of perfection, that nothing material can now be added thereto: and I should have been apt to have thought so myself, if I had not happened, in the course of my life, to have had a communication made to me (under the seal of secrecy) which seems to promise yet further light and assistance in perfecting that important art; and every impediment to the discovery thereof being now removed, I shall in the remainder of this essay give the clearest description thereof that I am able, with such elucidations and improvements as seem to be naturally pointed out by the method itself.

In the autumn of the year 1741, I was first introduced to the acquaintance of that then eminent artist, Mr. HENRY HINDLEY, of York, clockmaker;—he immediately entered with me into the greatest freedom of communication, which founded a friendship that lasted to his death, which did not happen till the year 1771, at the age of 70. On the first interview, he shewed me not only his general set of tools, but his *engine*, at that time furnished with a dividing plate, with a great variety of numbers for cutting the teeth of clock wheels, &c. and also, for more nice and curious purposes, furnished with a wheel of about thirteen inches diameter, very stout and strong, and cut into 360 teeth; to which was applied an endless screw, adapted thereto. The

threads of this screw were not formed upon a cylindric surface, but upon a solid whose sides were terminated by arches of circles. The whole length contained fifteen threads; and as every thread (on the side next the wheel) pointed towards the center thereof, the whole fifteen were in contact together; and had been so ground with the wheel, that, to my great astonishment, I found the screw would turn round with the utmost freedom, interlocked with the teeth of the wheel, and would draw the wheel round without any shake or sticking, or the least sensation of inequality.

How long this engine might have been made before this first interview, I cannot now exactly ascertain: I believe not more than about a couple of years; but this I well remember, that he then shewed me an instrument intended for astronomical purposes, which must have been produced from the engine, and which of itself must have taken some time in making*.

Fin

* This instrument was of the equatorial kind; the wheel parallel to the equator, the quadrant of latitude, and semi-circle of declination, being all furnished with screws containing fifteen threads each, framed and moved in the same manner as that of the engine; the whole of which instrument was already framed, and the telescope tube in its place, which was intended to be of the inverting refracting kind, and to be furnished with a micrometer. This, however, was not completed till some years after; but, in the year 1748, I received it in London for sale. It staid with me two years, in which time I shewed it to all my mechanical and philosophical friends, amongst whom was Mr. SHORT, who afterwards published in the *Philosophical Transactions*, vol. XLVI. N^o 493. p. 241. an Account of a portable Observatory, but without claiming any particular merit from the contrivance. However; the model of it differs from HINDLEY's equatorial only in the following articles. He added an azimuth circle and compass at the bottom. He omitted the endless screws, placing verniers in their stead; and at the top, a reflecting telescope instead of a refractor. This instrument of HINDLEY's being afterwards returned to him unfold, I pointed out the principal deficiencies that I found therein; viz. that, in putting the instrument into different positions,

I in reality thought myself much indebted to Mr. HINDLEY for this communication ; but though he shewed me his engine, and told me, that the screw was cut by the rotation of the point of a tool, carried round upon a strong arm, at the distance of the radius of the wheel from the center of motion, which arm was carried forward by the wheel itself, and the wheel was put forward by an endless screw, formed upon a cylinder to a proper size of thread, cut by his chock lathe ; though he shewed me also this chock lathe, and the method employed to make the threads of the screw *equiangular* with the axis, that is, to free the screw from what workmen term *drunkenness* ; and also shewed me how, by the single screw of his lathe, he could cut, by means of wheel-work, screws of every necessary degree of fineness * (and, by taking out a wheel, could cut a left-handed screw of the very same degree of fineness) ; by which means he was enabled not only to adapt his plain screw to the size of the teeth of his wheel, but also to prevent any drunkenness that otherwise the curved screw would be subject to in consequence of being produced from the plain

tions, the springing of the materials was such as in some positions to amount to considerable errors. This remained with him in the same state till the year of the first *Transit of Venus*, viz. 1761 ; when it was sold to ——— CONSTABLE, Esq. of Burton Constable, in *Holderness*. Mr. HINDLEY, to remedy the evil above-mentioned, applied balances to the different movements. He soon afterwards completed one, *de novo*, upon this improved plan, for his Grace the late Duke of NORFOLK. A method of balancing in much the same way, without the knowledge that it had been done before, has been fully explained, and laid before the Society, by our ingenious and worthy brother Mr. NAIRNE. *Phil. Transf.* vol. LXI. p. 108.

* A machine for cutting the endless screw of Mr. RAMSDEN's engine, upon principles exactly similar, is fully and accurately set forth in his Description of his dividing Engine above-mentioned.

one; furthermore, that the screw and wheel, being ground together as an optic glass to its tool, produced that degree of smoothness in its motion that I observed; and, lastly, that the wheel was cut from the dividing plate: yet, how the dividing plate was produced, he for particular reasons reserved to himself.

Nor can he be blamed for the reservation of this one secret; as he had, even at the time of my early acquaintance with him, a kind of foresight that from the superior merit of HADLEY's quadrant, a demand for that, and other instruments for the purpose of navigation, was likely to increase; and that he might live to see a public reward offered for a method of dividing them with greater accuracy and dispatch than had at that time appeared. Indeed, he had himself an idea, from the satisfactory success that had attended his operations in dividing, that a screw and wheel, produced from his engines of one foot diameter, would have as much truth as the eight-feet quadrant at Greenwich: and though he doubtless greatly over-rated the accuracy of these miniature performances, yet it does not follow, as his methods were not confined to so narrow a compass, but that, his scale of operation being proportionably enlarged, a degree of accuracy in the graduation of astronomical instruments may be attained in proportion.

I must here beg leave to observe, that there appears to me to be a natural limitation to the accuracy of instruments, consisting of considerable portions of a circle, such as quadrants, &c. *. I do not find that the finest stroke upon the limb of a quadrant, made by BIRD's own hand, if removed from its

* The zenith sector consists but of *few* degrees, with little variation of its position in using it.

coincidence with its index, can be replaced with any degree of certainty nearer than the 4000th part of an inch, though aided by a magnifying glass *.

A 4000th part of an inch being then determined to be the *minimum visibile* by the strokes of an instrument, this will be less than one second of a degree upon a radius of four feet; and therefore, if the whole set of divisions upon the limb could be preserved true to this aliquot part of an inch, the eight-feet quadrants of Greenwich might be expected to be true to half a second. How far they are from this, I do not exactly know; but I have reason to think they vary from it some seconds: nay, I believe it is generally allowed, that our largest quadrants, even when executed by the accurate hand of Mr. BIRD, do not exceed those of a less size, by the same hand, in proportion to their increase of radius: nor can it well be expected that they should; since, as the weight necessarily increases in a triplicate ratio of the radius, the great weight of the Greenwich quadrants in moving and fixing them (as they could not be divided in their place) may easily derange the framing; or even the *internal elasticity* of the materials may give way, by a change of position, to so minute a quantity as a 4000th part of an inch. It therefore appears to me, that since the divisions of a quadrant of four-feet radius are more than sufficient, and even those of three feet admit of all the distinctness that in other respects is wanted, a three-feet quadrant, in point of

* It will be to little purpose to attempt it with a greater power. Double microscopes can doubtless be formed to magnify objects, far less than a 4000th part of an inch, to distinct surfaces; but then the advantage of such degrees of magnifying power is chiefly upon the organized bodies of nature. Let a dot, or the finest point that can be made by human art, be so viewed, and it will appear not round, but a very ragged irregular figure.

size, is capable of all attainable exactness; and would be as much to be depended on as any of those now in being of eight feet. By adopting quadrants of this smaller size, we shall of course get rid of $\frac{1}{100}$ ths of the present weight; and consequently of much cumber, unhandiness, and derangement, that must arise from that weight, as well as the fear of totally discomposing them, if ever moved out of their place.

It now comes to be time to open a principle upon which there is a prospect of effecting such an improvement. I have shewn that a 4000th part of an inch is the *ultimatum* that we are to expect from *sight*, though aided by glasses, when observing the divisions of an instrument. But in the XLVIIIth volume of the Philosophical Transactions for the year 1754, I have shewn the mechanism of a new *pyrometer*, and experiments made therewith; whereby it appears, that, upon the principle of *contact*, a 24,000th part of an inch is a very definite quantity. I remembered very well that I did not then go to the extent of what I might have asserted, being willing to keep within the bounds of *credibility*: but on occasion of the present subject, I have re-examined this instrument, and find myself very well authorised to say, that a 60,000th part of an inch, with such an instrument, is a more definite and certain quantity than a 4000th part of an inch is to the *sight*, conditioned as above specified. The certainty of contact is, therefore, fifteen times greater than that of vision, when applied to the divisions of an instrument; and if this principle of certainty in contact did not take place even much beyond the limit I have now assigned, we never should have seen those exquisite mirrors for reflecting telescopes, that have already been produced.

These reflections apply immediately to my present subject, as HINDLEY's method of division proceeds *wholly by contact*, and
that

that of the firmest kind ; there being scarcely need of magnifying glasses in any part of the operation.

In the year 1748 I came to settle in London ; and the first employment I met with was that of making philosophical instruments and apparatus. In this situation, my friend HINDLEY, from a principle the reverse of jealousy, fully communicated to me, by letter, his method of division ; and though I was enjoined secrecy respecting others (for the reasons already mentioned), yet the communication was expressly made with an intention that I might apply it to my own purposes.

The following are extracts from two letters, which contain the whole of what related to this subject ; and since I have many things to observe thereon, so that the paraphrase would be much greater than the text, I think it best not to interrupt the description with any commentary, as perhaps his own mode of expression will more briefly and happily convey the general idea of the work than any I can use instead of it.

MY DEAR FRIEND,

York, 14 Nov. 1748.

AS to what you was mentioning about my brother's knowing how I divided my engine plate, I will describe it as well as I can myself ; but you will want a good many things to go through with it.

The manner is this : first chuse the largest number you want, and then chuse a long plate of thin brass ; mine was about one inch in breadth, and eight feet in length, which I bent like an hoop for an hoghead, and soldered the ends together ; and turned it of equal thickness, upon a block of smooth-grained wood, upon my great lathe in the air (that is, upon the end of the mandrel) : one side of the hoop must be rather wider than

the other, that it may fit the better to the block, which will be a short piece of a cone of a large diameter : when the hoop was turned, I took it off, cut, and opened it straight again.



The next step was to have a piece of steel bended into the form as *per margin* * ; which had two small holes bored in it, of equal bigness, one to receive a small pin, and the other a drill of equal size. I ground the holes after they were hardened, to make them round and smooth. The chaps formed by this steel plate were as near together as just to let the long plate through. Being open at one end, the chaps so formed would spring a little, and would press the long plate close, by setting in the *vise*. Then I put the long plate to a right angle to the length of the steel chaps, and bored one hole through the long plate, into which I put the small pin ; then bored through the other hole ; and by moving the steel chaps a hole forward, and putting in the pin in the last hole, I proceeded till I had divided the whole length of the plate.

The next thing was to make this into a circle again. After the plate was cut off at the end of the intended number, I then proceeded to join the ends, which I did thus : I bored two narrow short brass plates † as I did the long one, and put one on the inside, and the other on the outside of the hoop, whose ends were brought together ; and put two or three turned screw pins, with flat heads and nuts to them, into each end, which held them together till I rivetted two little plates, one on each side of the narrow plate, on the outside of the hoop. Then I took out the screws, and turned my block down, till the hoop

* The figure is considerably less than the real tool should be.

† These I shall hereafter distinguish by the name of *saddle-plates*.

would fit close on ; and by that means my right line was made into an equal divided circle of what number I pleased.

The engine plate was fixed on the face of the block, with a steel hole fixed before it, to bore through ; and I had a point that would fall into the holes of the divided hoop ; so by cutting shorter, and turning the block less, I got all the numbers on my plate.

I need not tell you, that you get as many prime numbers as you please ; nor that the distance of the holes in the steel chaps must be proportioned to the length of the hoop.

You may ask my brother what he knows about my method of dividing ; but need not tell him what I have said about it ; for I think neither *he* nor *John Smith* know so much as I have *told you*, though I believe they got some knowledge of it in general terms *.——I desire you to keep the method of dividing to *yourself*, and conclude with my best wishes,

and am, dear Sir, yours, &c.

HEN. HINDLEY.

Though the above letter was in itself very clear and explicit, as to the general traces of the method, yet some doubts occurring to me, a further explanation became necessary. A copy of my letter not being preserved, the purport of it may be inferred from the answer, which was as follows :

* The persons here referred to were both bred with him. His brother, Mr. ROGER HINDLEY, who has many years followed the ingenious profession of a watch-cap-maker in London, was so much younger as to be an apprentice to him. Mr. JOHN SMITH, now dead, had some years past the honour to work in the instrument way, under the direction of the late Dr. DEMAINBRAY, for his present MAJESTY.

DEAR FRIEND,

York, 13 March, 1748-9.

I THINK in your last you seem to be apprehensive of some difficulties in drilling the hoop for dividing: First, that the center of the hole in the hoop might not be precisely in the center of the hole of the steel chaps, it was drilled in; but if I described fully to you the method I used, I can see no danger of error there: for my chaps were very thick, and the two corresponding holes were a little conical, and ground with a steel pin; first one pair, and then the other, alternately, till the pin would go the same depth into each. Then for drilling the hoop, I took any common drill that would pass through, and bore the hole. After that I took a five-sided broach, which opened the hole in the brass betwixt the steel chaps, but would not touch the steel; so consequently the center of the holes in the brass must be concentric with the holes in the chaps: and for alterations by air, heat, cold, &c. I was not above two or three hours in drilling a row of holes, as far as I remember.

2dly, For drilling, in a right line, I had a thin brass plate, fastened between the steel chaps, for the edge of the hoop to bear against, whilst I thrust it forward from hole to hole. What you propose of an iron frame with a lead outside, will be better than my wooden block; but considering the little time that past, betwixt transferring the divisions of the hoop to the divisions of my dividing plate, I did not suffer much that way. It was when I drilled the holes in my dividing plate that I used a frame for drilling, which had one part of it that had a steel hole, that in lying upon the plane of the dividing plate was fixed fast in its place for the point of the drill to pass through: then, at the length of the drill, there was another piece of

steel, with a hole in it, to receive the other end of the drill to keep it at right-angles to the plane of the plate. This piece was a spring, which bended at the end, where it was fastened to the frame of the lathe, at about eighteen inches from the end of the drill; so it pushed the drill through with any given force the drill would bear: and though that end of the drill moved in the arch of a circle, it was a very small part of it, being no more than equal to the thickness of the dividing plate.

My good wishes conclude me yours,

HEN. HINDLEY.

Whoever attentively considers the communication contained in the above letters will see, that more happy expedients could not have been devised to procure a set of divisions, where there should be the most exact equality among *neighbours*; and which, for the purposes of clock-making, is the principal thing to be wished for. But herein, as in M. ROEMER's method, there were no means of checking the distant divisions, which run on to 360: now such a check, when the expansion of metals is considered, and particularly the difference of expansion between brass and steel, seems absolutely necessary for the purpose of divisions upon instruments, where the accurate mensuration of large angles is required, as much as the equality of neighbouring divisions*.

With this view the invention of this ingenious person suggested to him the thought of making his curved screw to lay

* The ingenious Mr. STANCLIFFE (some years a workman of HINDLEY's) has suggested; that the difference of expansion between the steel chaps and the brass hoop may be avoided by making the chaps of brass also, with hard steel holes set separately therein, somewhat similar to the jewelled holes of watches.

hold of fifteen teeth or degrees together : this, in effect, becomes a pair of compasses, 24 removes of which complete the whole circle, and produce 24 checks in the circumference : and whoever considers the very exquisite degree of truth that results from the grinding of surfaces in contact, as already noticed, must expect a very great degree of rectification of whatever errors might subsist in the wheel after its first cutting.

What degree of truth it might in reality be capable of upon its first production and adjustment, is not now to be ascertained, he never having used it for the graduation of any capital instrument. Those that he made with a view to an accurate measure of angles, he always made with a screw and wheel, or parts of circles cut by his engine into teeth, and ground together as before-mentioned ; but I have reason to think, that its performance, if put to a strict test, was never capable of that accuracy that he himself supposed it to have.

The method itself, however, from its simplicity, and ease of execution, seems to me to be a foundation for every thing that can be expected in truth of graduation ; and in consequence for reducing instruments to the least size that is capable of bringing out all that can be expected from the largest ; when it shall, like manual division, have received those advantages that the joint labours of the most ingenious men can bestow upon it. That I may not appear to be without grounds for my expectations, I will beg leave to propose, what near forty years occasional contemplation has suggested to me on the subject ; and as I can describe the process I would pursue, where different from HINDLEY's, in fewer words than I could make out a regular criticism upon his letters, I will immediately proceed to the description of it.

Proposed Improvements of HINDLEY's Method.

I would recommend the number of parts into which the circle is to be reduced to be 1440; that is 4 times 360; which divisions will therefore be quarters of a degree; the distances of the holes in the chaps will therefore, to a three-feet radius, be $\frac{1.57}{1000}$ of an inch nearly; that is, between the one-sixth and one-seventh of an inch distance center and center.

Having provided myself with a stout mandrel, or arbor, for a *chock Lathe*, properly framed, that would turn a circle of six feet diameter, I would prepare a chock, or platform, for the end of it, of that diameter, or a little more, composed of clean-grained mahogany plank, all cut out of the same log; which, when finished, to be about $1\frac{1}{2}$ inch thick, and formed in sectors of circles, suppose 16 to make the circle; the middle line of each sector lying in the direction of the grain of the wood, this will consequently every where point outward: the method of framing this kind of work is well known.

The way of getting a slip of brass to answer the circumference of this platform is suggested in Mr. BIRD's Account of constructing Mural Quadrants. Let a parallelogram of brass of about three feet long, and of a competent substance (suppose half an inch) to make it when finished about one-twentieth of an inch in thickness, be cast of the finest brass; and this to be rolled down till it becomes of sufficient length for the hoop, and about one-fifth part more. I would then cut off, from the whole length, somewhat better than one-sixth part, the whole being sufficiently reduced to a thickness by the rollers. Perhaps no way will be more ready and convenient to

get such a long strip of brass reduced to an equal breadth, than the method prescribed by HINDLEY; viz. by turning it upon the chock prepared; but I would not make it wider on one side than the other, like the hoop of a cask, as he describes, but exactly to fit the chock, when truly cylindric; for the internal elasticity of the brass, in so great a length, will be very sufficient for fitting it on tight enough, without any tapering. This I will now suppose done; and a pair of steel *chaps*, as described by HINDLEY, to be also prepared, and ready for grinding; which, by such a careful admeasurement as can easily be made, will give the length of the hoop sufficiently near, on its first preparation.

Method of forming a Pair of Straps as a check to the Divisions.

The part first cut off must be again cut into two equal parts in length; which, for distinction sake, I will call the *straps*; and which are to serve as checks to every 60th and every 120th division of the circle.

A steel plate, of about half an inch in breadth, the same thickness as the straps, and in length equal to the breadth of the hoop plate, must be folded with silver folder to one end of each of the straps, by which means their length will be increased half an inch by the steel. An hole must then be made through each steel plate, of the same size as those through the chaps, and answerable to the middle of the straps; but so near the border of the steel, that when the chaps are put on, and adapted to the steel hole, the next hole will fall through the brass. The steel plates must then be hardened; and a pin being put through the two holes and the two plates, these must
be

be wrought to a right line in contiguity to each other; by this means the straight edge of each of the straps will be reduced to the same distance from the steel hole: the hard steel edges may be rectified by the grindstone, if necessary.

This being done, not only the holes in the chaps, but the holes in the two steel plates, applied to each other, like the two sides of the chaps, must be respectively ground together; not with a taper pin, as prescribed by HINDLEY; but so as not only to be cylindrical, but that the same cylindrical pin shall equally fit them all, and leave them smooth and polished; which is a process no ways difficult to a curious artist, and of which therefore a minute description is unnecessary.

The chaps being then put upon one of the straps, with its straight edge uppermost, and a pin put through the holes on the left-hand, and through the steel hole in the strap under operation, the chaps must be set upright, so that the line joining the centers of the holes shall be parallel to the upper edge of the strap; the brass plate, mentioned by HINDLEY, between the chaps, as a guide for directing them always to that upright position, may be then adjusted and fixed to the inside of the chap next the operator*.

The performance of the ensuing part of this work should be at a season when the temper of the air is not very variable; rather above the mean temper (suppose at 60°) than below it;

* It would be well, previous to the drilling of the steel chaps, that another hole was drilled in the chaps, that should be somewhat above the upper edge of the straps, and in the middle betwixt side and side, to receive a *steady pin* therein, antecedent to drilling the main holes; for then a tempered steel pin, a little taper, will, by driving it in as far as necessary, constantly answer this purpose from first to last, so as to regulate the holes in grinding, to be truly opposite: proper holes should also be drilled for fixing the brass guide plate to one of the chaps.

but above all things the artist should be himself cool ; that is, not in a state of sensible perspiration ; and there should be a free circulation of air in the room. Things being thus conditioned in respect to temperature, he may begin to drill the holes in one of the straps ; the pin being first put through the chaps and through the steel hole of the strap ; and the next hole, being drilled through the brads with a common drill, that and every hole as it goes is to be finished with a taper broach, as prescribed by HINDLEY ; and he may then prove or finish every hole by the application of a thorough broach, made so full as to require a degree of pressure to force it through ; and this broach being a little tempered, and the holes quite hard, there will be no fear of injuring the steel holes*.

Calling the hole in the steel plates o, and observing the time of beginning, you may proceed to drill 60 holes as prescribed by HINDLEY ; and noting how long you have been about it, you may lay the work aside a length of time, equal to the time you took in drilling ; that any addition of warmth it may have acquired in handling or working may be again lost in a great degree†. After this pause you may begin again, and go on to finish 60 holes more ; that is, to the length of

* The steel holes in the chaps need not to be above one-twentieth of an inch in diameter ; and though it may be proper to make the steel plate, of which they are formed, one-tenth of an inch thick, in order to give the spring formed between them a convenient degree of stiffness, yet they may be reduced (by chamfering the outsides) to half that thickness.

† As there is not much occasion for the artist to touch his work, the effects of that may also be very much avoided by wearing thick gloves ; and the friction being but slight, and the work almost continually in the vise, the variation of temperature in the metals concerned cannot be sensible or considerable.

120 holes from the beginning; you then proceed in the same manner with the other strap.

Method of drilling the Hoop.

You are now prepared to commence the work upon the long or *hoop-plate*; and you proceed therewith, in forming the first hole with the chaps, as before directed by HINDLEY, and this first hole you call 0. You then place the straps one on each side the hoop, with their gaged edges upward, and put the pin through the holes denominated 60 upon the straps, and through the first hole already made, and denominated 0 upon the hoop; then, bringing the gaged edges of the steel plates to be even with the upper or working side of the hoop, you pinch them together in the vise, and drill and broach the hole through the steel plates, which will make the hole, number 60, upon the hoop. This done, you put the pin through the left-hand hole of the chaps, and the hole marked 0 upon the hoop-plate first made, and proceed to drill with the chaps to 59 holes inclusive, which will fill up the whole space from 0 to the 60th division before obtained.

You now again have recourse to the straps, and placing them one on each side the hoop-plate, you put the pin through the 120th hole of the straps, and through the hole marked 0 upon the hoop-plate; and regulating the steel plates to the hoop-plate as before, you drill and form a hole with the steel plates, which will correspond with the 120th hole upon the hoop-plate; and afterwards filling up the 59 holes wanting, by means of the chaps, you then have all completed to the 120th division, which is one-twelfth of the whole circle.

You then proceed, in like manner, with another set of 120 holes; that is, placing the 60th hole of the straps to the 120th hole of the hoop-plate, and from it producing the 180th hole; you, in like manner as before, fill up this 60 with the chaps; and afterwards placing the 120th hole on the straps in the 120th hole on the hoop-plate, you will obtain the 240th hole; so that filling up this last set of 60 divisions, you have obtained 241 holes, including 240 spaces or divisions of the hoop; and repeating this process ten times more, you will, in like manner, obtain 1441 holes, comprehending 1440 spaces*. And this process being carried on in temperate weather, the manner of working produces twelve similar operations, wherein the materials and tools concerned will not only be subject to very little change of temperature, but that change, whatever it is, will be nearly similar in each set of 120 holes: we may therefore infer, that the greatest inequality, or indeed any that can be sensible, must be, at every 60 divisions, that is, between the 59th and 60th, and between the 119th and 120th, both which will be equally repeated 12 times, in the whole length which is to compose the *circumference of a Circle*, and which will thus be checked thereby 12 times in the circumference, and 12 times more at the intermediate distances; that is, with 12 master checks, and 12 subordinate ones, in the whole round.

It is proper here to observe, that in M. ROEMER's method even sixty divisions could scarcely be trusted in an affair of great accuracy, on account of the objections already made, arising from the points having such slight hold in the surface of the brass; but here the parts are held so exceedingly firm, and the

* It will be proper, for reasons hereafter to be mentioned, to continue the divisions to 20 holes more, making in the whole 1461 holes.

operation carried on with so much power, that any small inequality in the hardness of the brass, or irregularity of surface, cannot be supposed to affect the place of the center of the hole; nor will any small inequality that may be suspected from the wear of the steel holes sensibly affect the *center* of the hole, to which every thing is ultimately referred.

Method of joining the Hoop.

A more happy thought than that of HINDLEY's, for joining the two ends of the hoop, could scarcely have been wished for, in regard to preserving the same equality of the space between the holes contiguous to the joint, as in the other parts: for though, geometrically speaking, the two *saddle* plates, in which the little cylindrical bolts are fixed, for bringing the terminating holes of the hoop plate to their due distance, being one applied within the hoop, and the other without, will belong to circles of different *radii*; yet this difference being exceedingly small in such thin metal, and so great a radius, and one being as much too big for the hoop as the other is too little, when the bolts are put in, and the hoop in that part set nearly to a circle by a mould; the mean between them assumed by the hoop, from the elastic compressibility of the materials, will be the truth.

It must, however, be remarked, that in the use of the straps, the joining of the hoop should not be made at any part betwixt an 119th and an 120th division, as some inequality must be supposed there, unless the saddle plates were adapted thereto. The method the most easily practised, will be to continue the division upon the hoop, about twenty more than the completion of the number intended to form the circle, and to cut off all the overplus ones at the beginning.

The faddle plates I would recommend to contain ten holes each ; so that if the divisions are carried on to twenty more than what will be contained in the circle, there will be a piece containing twenty to cut off ; and this again being cut in the middle will afford ten holes to make each faddle plate ; so that there will be a place for a bolt on each side the joint, and then putting a bolt through every other hole, there will be three bolts at an end.

The pieces destined for the faddle plates, thus obtained, being broader than can be admitted when put to this use, I would advise to divide the breadth of the plate into three equal parts ; and with a cutting hook (which perhaps will be attended with the least violence in the separation) to separate the two outside pieces from the middle piece : by this means the two faddle plates (though double) will occupy one third only of the breadth of the hoop in the middle ; and two of the pieces cut off being applied, one on each side of the faddle plate on the outside, will answer in like manner for the *rivet* plates.

The last operation to compleat the joining of the hoop is the putting on the rivet plates : to compleat this, I would advise a piece of brass, of three or four inches in length, to be filed so as to answer to the inside of the hoop, when reduced to a true circular form ; and being three-eighths, or one-half an inch in thickness, to file the opposite side somewhat nearly concentric thereto ; apply the middle of its convex arch to the inside of the hoop at the joint, and then bringing on the middle of one of the rivet plates to the joint of the hoop, confine the three together by a couple of narrow-chapped hand vices, leaving a space between them capable of receiving a couple of pins as rivets on each side the joint ; the holes for the rivets are then to be drilled through all, and a little smoothed with a broach at their

their entry, into which smooth taper pins are to be driven; not with violence, but moderately, that no sensible stretching of the solid parts may take place thereby; then cutting off and smoothing the heads, shift the vises so as to receive another couple of holes, and a third couple in the same end of the hoop; and proceed progressively in the same manner, from the middle to the other end of the rivet plate; then gently separate the internal brass mould with a thin knife, or such like instrument; and cutting off, and very lightly rivetting the inner ends, proceed to fix the other rivet plate, in the same manner, on the other side: by this means the hoop will be firmly joined in the very position given it by the saddle plates and mould. These plates may then be removed, the inside of the hoop cleared and smoothed, if necessary; and the outside will have the middle part clear where the divisions lie, and that without sensible loss or gain in the juncture.

Method of transferring the Divisions of the Hoop to a dividing Plate.

The hoop being thus refitted for the chock, that should be turned down to leave a shoulder on one side, that the hoop, now reduced to an equal breadth, may be forced against it; and the divisions, being equally distant from one of its edges, will be all found in a circle, as if turned upon it. It should be very carefully fitted to the chock, that it may go on with a sufficient degree of tightness, and without the necessity of much forcing; and it will be no inconvenience now, if it goes on upon a very slight degree of taper of the chock, as the internal spring of the materials will easily accommodate it to this shape without any injury to its general truth: a slight degree of a groove should

be turned in the place where the divisions will come, that any conical pin, that is to serve as an index, let drop into the divisions or holes, may not, by reaching through this thin plate, abut upon the wood, rather than upon the sides of the holes: and thus this hoop is made into a wheel of 1440 equal divisions, moveable round upon its own axis, whereon it was formed.

Against the time that this is compleated, there must be prepared a flat circular plate or wheel of brass, the rim of which should be of about $3\frac{1}{2}$ inches breadth, and about two-tenths of an inch in thickness when finished, to make a dividing plate; the external diameter of this is to be such, that when laid flat upon the surface of the mahogany platform, its extreme edge will exceed the diameter of the hoop by about half an inch all round. There must also be prepared brass arms (suppose eight in number) of an equal substance with the outer rim, and all connected with a circular plate in the middle; and, the whole of this work being framed beforehand, is to be let on flat upon the mahogany platform; whose face is supposed to be turned truly flat, and sufficiently affixed with screws: in this situation, the outward edge is to be turned, and the outward face of the rim turned flat. The *center plate*, which may be about twelve inches diameter, is also to be turned as flat as possible, and a center hole, of about half an inch diameter, to be very carefully turned therein.

A piece of clean, straight-grained, well seasoned mahogany, of about two feet long, three inches thick, and five or six inches broad, is then to be well affixed to some part of the general frame of the lathe, which must now have its position altered, so that the platform will become horizontal; and therefore the frame should
be

be originally made with this view *. The piece of mahogany is to be affixed so that one of its larger faces shall be in a parallel plane to the face of the platform, and so low as to clear the under side of the platform in its rotation; and so far distant from the center, that an index may be fixed upon this upper face of the piece of wood, so as conveniently to drop into the holes of the hoop; while the common *cutter frame* of a clock-maker's engine shall be firmly attached upon the same face of the wood, and so fixed as to cut the edge of the dividing plate into teeth, answerable to the several divisions of the hoop. The teeth need only to be cut with a common cutter, making a parallel notch: and here it will be proper to observe, that not only both the index and cutter are to be founded on the same piece or base of wood; but that the nearer they are together, the more free they will be from the effects of all variations of expansions by variations of temperature †.

The equalising the Teeth of the dividing Plate by grinding.

The object of transferring the divisions of the hoop to the teeth of the dividing plate, is still farther to equalise the teeth by grinding; especially those that, falling within the compass of

* After changing the position of the lathe, the collar of its mandrel should be removed, and the neck made to move within three planes, so as to preserve an exact center, in the manner of an *equal altitude* instrument.

† It is proper to observe, that as it may be impracticable to get the rim of the dividing plate cast of the proper size, in one entire piece, it will be very practicable, if cast of a less size (suppose half), but of a sufficient thickness, to roll it down; and by having the outward edge originally thicker than the inner, in the proportion of the *radii*, it may be so managed by the rollers as to be of an equal thickness when brought to its proper size. But the arms and center plate should be of the same metal, rolled in the same degree.

each set of 120 divisions, may be supposed, if any, to be mended thereby; but as it may be incommodious to construct a curved screw, of such a length and size, in HINDLEY's method, as would be sufficient for the purpose, I would propose to use two screws of brass, cut from a cylinder in the way set forth by Mr. RAMSDEN, each of which, with a very little grinding upon this large circumference, would lay hold of ten or twelve teeth together. I would place the two screws, that is, their middles, to be ninety divisions asunder; of consequence, when one of the screws is between the 59th and the 60th, or between the 119th and 120th division of each set, the other will be in the middle of the space divided by the chaps only*.

The threads of these screws I would advise to be cut a little taper, so that as they grind in, they may fill the notches of the teeth; which also, by this means, will acquire a little tapering towards their extremities; and by cutting the notches parallel, as I have mentioned, the true ground part will always be certain of being at the extremity.

When the screws have been used in grinding till they are found to have the effect of a perfectly equal and easy rotation all round, and all the teeth reduced to a sensible taper, and regular bearing, I would then totally remove the screws from the square block of wood, upon whose upper face I suppose them to have been mounted; in like manner as I suppose the index and cutting frame to have been removed, to make room for the mounting of

* The best way of giving an equal motion to those two screws, seems to be by a detached axis, carrying two common flat wheels; one acting upon a like flat wheel, upon the axis of one screw, and the other, in the same manner, upon the other; and applying the pulley for communicating the power to the middle of the detached axis between the two wheels, the spring or twist will be equal both ways; so that in turning the contrary way round, they will still be in equal advance.

the screws. I now consider the teeth of the dividing plate, so formed, as having all the equality that the present known state of human art has pointed out; and the whole convertible upon the axis or mandrel upon which it has been originally formed, and the central hole of the plate concentric therewith: I therefore consider the ground faces of the teeth of the plate as the actual divisions. It now remains to shew how they are to be transferred, to form the divisions of an instrument.

Preparation of the dividing Plate for graduating Instruments.

If a small cylinder of hard steel is duly polished, and made of a size so as just to chock in betwixt the extremities of the teeth, then the center of that cylinder will be a fixed point, in respect to the circumference of the wheel: if another cylinder is applied in like manner, at the distance of a number of divisions (suppose it a prime number, so as to cross all former divisions, viz. 17 or 19), then the middle of the line joining the centers of the two cylinders will remain in the direction of the *same radius*, though one of them should force in a minute quantity further than the other; and if a point is assumed in the direction of a tangent to a circle at this middle-point, then though both the cylinders should drop in a minute quantity further at one time than another, yet the middle-point would remain at the same distance from the point in the tangent; provided that point was removed to a competent distance, that is, to five or six inches. On this principle I would construct an index, the two cylinders being fixed in a frame, convertible about the middle-point, and to be centered in the end of the lever, representing the tangent; then this lever being again convertible about the point in the tangent line, the middle-point would always have a fixed distance

from the point in the tangent, and there hold it steadily fast; the tangent point being placed upon the fixed block before-mentioned.

Use of the dividing Plate in the Graduation of Instruments.

Our dividing plate is now ready for the reception of an instrument; suppose it a quadrant, whose radius, however, must not exceed the radius of the dividing plate: It is to be laid upon the face of the dividing plate, and a weight, or weights, equivalent to that of the quadrant, is placed on the opposite side, to balance it. It must also be supposed, that the quadrant is made with a view to be divided by this engine; and consequently, that the central cylinder is so well adapted, and nicely fitted to the center hole of the quadrant, that the center cylinder can be removed, in order for the limb to be divided, and again replaced, without sensibly altering its center. This being the case, let a piece of metal be turned, to apply to the quadrant, perfectly like its center cylinder at the upper end, and turned nicely to fit the central hole in the dividing plate, at the lower end; then, the quadrant being fixed with proper fastening screws, I would cut the divisions with a beam compass; and, if a fixed point is assumed, *viz.* the center of the tangent point for the index; then the beam compass being always opened to the computed length of the tangent of the circle of divisions, it will be sufficiently near for cutting the divisions, square to the circular arches between which they are placed.

It will also be proper (to prevent unequal expansions) that the beam of the compass should be formed of a piece of clean-grained *white fir*; and that the length between the points be inclosed in a tube of tin or brass; without touching the beam,
except

except at the terminations, which will in a great measure protect it from both alteration of moisture, and of heat from the body of the artist, during the operation.

It will be likewise proper to have a lever, or some equivalent contrivance, to bring the dividing plate forward; that after lifting the little cylinders out of the divisions, and resting them upon the tops of the teeth, they may be brought gently forward with an equal drag, and ultimately snap in between the teeth, by the strength of the spring commanding the index; by this means the drag of the friction of the whole will be constantly the same way.

Conclusion.

Now, if, as it has been shewn, a quadrant of any radius may be read off to the 40000th part of an inch, then this quantity upon a radius of three feet will not be so much as $1\frac{1}{2}$ second; and as the whole of the process is carried on by contact, in which a greater error than that of a 60,000th part of an inch cannot be admitted in any single operation, I should assuredly expect a three-foot quadrant, so divided, to be true in its divisions, and read off to at most two seconds.

But, after all, in an instrument like this, I should expect the greatest source of error to be in the want of perfect coincidence of the center of the divisions with the actual center upon which the index revolves; and therefore, that if, instead of a quadrant of three-feet radius, a complete circle of five feet diameter was divided, and its divisions read off from the two opposite points (taking the mean), then the errors of the center will be wholly avoided. For this reason, I am very clearly of opinion, that the sagacious proposition of Mr. RAMS-

DEN, to use circles instead of quadrants, or other portions of circles, will bid much the fairest for perfection in actual practice; and that his ingenious method of making them both stiff and light, by the use of hollow conical tubes by way of spokes, in the manner of a common wheel, will enable him to mount them of five feet diameter, upon hollow axes, in the nature of a *transit*. By this means we shall have all the good properties of both the quadrant and transit united in one instrument; and observations both of right ascension and declination, through the very same telescope, as long since attempted by M. ROEMER; and to a degree of perfection and certainty, in point of declination, hitherto unattainable by the largest instruments that have yet been made.

N. B. In matters of very nice determination, small circumstances often come to be of consequence; and it is in this view that I mention what follows. It was a practice of HINDLEY's of many years standing, and since followed by myself and others, wherever he made any use of the *vernier*, to lay the vernier plate in the same plane, or cylindrical surface continued, whereon the principal divisions are cut. It is of equal utility, though the vernier be rejected, to lay the index stroke in the plane of the divisions. In this way the divisions being by convenience upon the external border of the limb*, two sets of divisions are thereby rendered inconvenient; but those

* It has been objected, that laying the divisions upon the extreme edge of the limb of the instrument subjects it to injury: but, to obviate this, in an HADLEY's quadrant made for me, by my direction, by the late Mr. MORGAN, in the year 1756, wherein the vernier is laid even with the divisions, those are protected by a projection of the solid part of the limb, beyond the divisions; a *Rabbit* being sunk in the edge of the limb, to clear the vernier.

that with two sets, as a check, will in a great measure aid themselves, by reading from two different parts of the same set of divisions; which is very easily provided for, by putting an additional stroke upon the index plate, at the distance of 9, 11, or any prime number of divisions to 19, 23, or more; and reading off from that stroke also; as before recommended for great quadrants, where the vernier is proposed to be rejected *: so that they will thereby be mutually checked by divisions that had no correspondence in their original formation.

* I would not have it thought, from my proposal of rejecting the vernier, that I have any quarrel with it; I think it a very simple and ingenious contrivance, where it is properly applicable; that is, where the strokes of the vernier, or their estimated halves, are sufficient for all the precision required or expected from the instrument, as in HADLEY's quadrants, theodolites, &c.: but where still more minute divisions are required than can easily be had by estimation from the vernier; to do this by a screw, as a *supplement* to the vernier, appears to me in the light of bringing a more accurate tool to supply the deficiencies of one less accurate; when the former might, with more propriety, supply the place of the latter altogether.

