

XXXIII. *Account of an Observation of the Right Ascension and Declination of Mercury out of the Meridian, near his greatest Elongation, Sept. 1786, made by Mr. John Smeaton, F. R. S. with an Equatorial Micrometer, of his own Invention and Workmanship; accompanied with an Investigation of a Method of allowing for Refraction in such Kind of Observations; communicated to the Rev. Nevil Maskelyne, D. D. F. R. S. and Astronomer Royal, and by him to the Royal Society.*

Read June 7, 1787.

**M** DE LA LANDE having announced to some of my astronomical friends the utility of accurate observations of Mercury, at his two elongations the last year, in August and September; I tried to get observations of that planet in crossing the meridian, for some days before and after the greatest elongation in August; and though the state of the atmosphere about that time was not very favourable to the purpose, yet there was one day that I thought unexceptionable, but could not perceive the least appearance of Mercury; at which I was the rather surprised, as I had formerly seen that planet in the like situation, with the same instrument, with perfect perspicuity\*: and as I did not hear of any one else having succeeded in

\* The instrument mentioned is a transit, made by myself in the year 1768; at which time achromatic object-glasses not having been, so far as I knew, applied to astronomical instruments by others, but having found the good effects thereof for

in this observation, I thought it might be very possible for the same disappointment again to happen, with respect to the approaching elongation in September. I judged, therefore, that it might be of some utility to astronomy, if, by *any means*, a good observation of Mercury could be got; and also, that it would be a proper subject whereon to make trial of an instrument for such purposes, the idea of which I had conceived, and begun to construct, above forty years before; but which, from various avocations, I did not perfect to my satisfaction till the year 1770\*: since which time it has lain by, in hopes that something might happen, by which a full and effectual trial might be made thereof.

This instrument was originally intended as an improvement of the common wire micrometer, for the purposes of taking differences of right ascension and declination, in a more commodious and effectual manner than could be done in the method then practised of using that instrument †; and at the same

for other purposes, I resolved to use a double achromatic object-glass, made by Mr. DOLLOND; which being of equal aperture with the simple object-glass, then in the transit of the Royal Observatory, this telescope I therefore esteemed to be of nearly equal validity, as to quantity of light, with that at Greenwich, but reduced to the more commodious length of three feet six inches.

\* Some observations made therewith, after it was completed, I transmitted to my friend Mr. AUBERT; the consistency of which induced him to procure a similar instrument to be made for *cometary* and such kind of observations as cannot be commodiously made in the meridian. Of this instrument the Rev. Mr. WOLLASTON has made honourable mention in *Phil. Trans.* Vol. LXXV. for 1785, p. 348.

† The common wire micrometer, as used by Dr. BRADLEY, and described from a paper of the Doctor's hand-writing, is given by Dr. MASKELYNE in the *Philosophical Transactions*, Vol. LXII. for the year 1772; and in addition to which I must beg leave to observe, that the telescopes then in use for the

same time more effectually to answer the purpose of GRAHAM's *astronomical sector*, which was contrived by him (as Dr. SMITH informs us) to supply the deficiencies of the micrometer then in use\*.

The most necessary and fundamental improvements seemed to be; first, that of rendering the micrometer telescope manageable upon an equatorial motion; and, secondly, the contrivance of a stand of such solidity and steadfastness that the telescope might preserve the position in which it was placed, for a length of time: for it occurred to me, that if the telescope could be maintained at rest; or in a degree of stability superior to that of the astronomical sector; then the necessity of taking in a greater compass in declination than could be commodiously given to the field of a telescope would be the less necessary: for, instead of confining the object to a comparison with a star not differing more than a few minutes of time, or at most a quarter of an hour in right ascension, those comparisons could be extended to an hour or two, or even on occasion to three or four hours; there being scarcely any part of the heavens so devoid of stars, of a suitable magnitude for these observations, but that a proper one may be found within that compass in right ascension, provided there is allowed

micrometer were from ten to fifteen feet long, made with wooden tubes, supported at each end upon two wooden supports, by which the telescope could be managed in altitude and azimuth; but not to follow a celestial object in its proper motion on one center: which apparatus, I believe, is still remaining in the Royal Observatory.

\* This instrument is described in SMITH's *Optics*, Vol. II. p. 350.; and the original one, made by Mr. GRAHAM, was, at his death, placed in the Royal Observatory, and is mounted upon a three-legged stand of wood.

the difference of a degree, either north or south of the object, in declination \*.

Considering, however, that the approaching elongation would be in the morning; and that the best chance of seeing Mercury with this instrument would be some time in the twilight, between Mercury's rising and the rising of the Sun; yet, on supposition of catching the planet in his passage over the wires, there would be no chance of seeing any star pass over the field, wherewith to compare him, till the following evening, which being at least fourteen hours, the certain position of the telescope for so great a length of time was *almost* more than I could reasonably hope for. To judge how far I might form an expectation, by way of a previous trial, I compared Saturn with  $\gamma$  Capricorni †, and found the return

\* The field of the telescope of Mr. AUBERT's instrument is two degrees; but that of the original, wherewith this observation was made, is  $1^{\circ} 17'$ : to gain which, the eye-glass being immovable (and *achromatic* to prevent the indistinctness that would otherwise have taken place near the border) the magnifying power was obliged to be considerably reduced, in respect of what has been usual for micrometers, that is, so as not to exceed 20 times: in consequence, there is, therefore, no need for so long a telescope, this being but  $34\frac{2}{3}$  inches focal length of object-glass; but being a double achromatic, made by the late Mr. JOHN DOLLOND, it is capable of as great an aperture as could be given to the simple object-glasses of twelve or fifteen feet telescopes, that were then generally given to micrometers; but the pencil of light being greatest in this, is attended with this advantage, that the small stars can be seen very distinct and in great abundance, like the modern night-glasses: and there is in reality no need of great magnifying powers for the present purpose; for the place of the wire being viewed by an eye-glass, of about  $1\frac{3}{4}$  inches focus, its place may be distinguished to less than a 2300th part of an inch, which, on the radius of  $34\frac{2}{3}$  inches, is scarce  $2''\frac{1}{2}$  of a degree; and which, as I apprehend, is nearer the truth than can reasonably be expected from instruments out of the meridian.

† According to this observation  $\delta$  preceded  $\gamma$   $\gamma$  the 2d of Sept. at 9 h. 15' P.M. mean time by  $30' 9''.7$  MT, and with greater declination south than  $\gamma$  by  $41' 23''$ .

of the star to the same place two evenings afterwards, both in right ascension and declination, was so near, that I concluded I might very well expect a good observation of Mercury, in case I could get a sight of him, though the stars wherewith he was to be compared lay at the distance of the following evening at the soonest.

The micrometer is furnished with five horary wires, denominated in their order *a*, A, B, C, D (B being the middle horary wire), and the two declination wires are denominated *A* and *B*, each moveable by a separate and independent micrometer-screw, from the outside of the field to the center, and a little beyond it; so that each wire can be moved into the place of the other when at or near the center\*.

The morning of the 23d of September, about a quarter past five o'clock, the air being clear and perfectly serene, it being then about an hour after Mercury's rising, and near three-quarters of an hour before the rising of the sun, I very readily found Mercury with the telescope, and when found could easily see him with an opera glass; and Mercury being then in a state of very little alteration of declination, I adjusted one of the declination wires to his apparent run, by making him traverse the *whole* field. The observations were then taken as

\* In Dr. BRADLEY's Paper it is said, that before the *late* alterations, both the declination wires were made moveable; and that it was an improvement to make one of them fixed, and one only moveable. But however they might be immediately preceding the Doctor's time, I believe, the original micrometers by Mr. TOWNLEY were with one fixed and one moveable declination wire, as I have seen one in this form among the remaining apparatus of Mr. ABRAHAM SHARPE. In an instrument, however, fitted up for the purposes of the *equatorial micrometer*, I believe, it will be found most convenient to have both those wires moveable; as by this means they not only are enabled to slide into *each other's* place, but every part of the frame of the instrument remains fixed during the whole of the observation, the two slides carrying these two wires excepted.

in the first table; and in the evening I was lucky enough to get those of  $\lambda$  Ceti and  $\sigma$  Tauri, intending to repeat the whole the next morning and evening. The next morning proved cloudy, and so continued, that I saw the planet no more; but in the evening of the 26th, I found the stars come again so near the same declination, that I was encouraged to continue the observation to see what change would happen. It then came on bad rainy weather till the 30th, when I again repeated the observation, and found the stars to come so near the same declination that I was fully satisfied of the stability of the instrument, so far at least as could regard twenty-four hours; but as I was then appointed to go a journey, and could have no other use for it, I locked the door of the Observatory, leaving the instrument in its position, that I might see what change would happen by the time of my return and was quite astonished to find, on the 13th of October, that it had remained in a manner unmoved; for it had suffered no more apparent alteration than what might occur by the error of observing, and alterations of the clocks and transit.

It must, however, be remarked, that, besides that in the construction of the instrument every thing was contrived that appeared likely to give it firmness, it was rested upon the *frustum* of an hexagonal pyramid of stone, in the founding whereof great care was taken as to its solidity, and was detached from the floor for supporting the observer.

This Observatory at *Austhorpe* I esteem in the latitude of  $53^{\circ} 47' 54''$  N. and  $5^{\circ} 50''$  of time W. from *Greenwich*.

Table I. Observations of Mercury at his elongation Sept. 1786, with an Equatorial Micrometer.

Day, object, and wires.	Hour.	Time as taken by the clock.	Time reduced to min. & sec.	Reduced to the middle wire.	Mean of the wires.	Parts of the microm.	Micrometer reduced.
Sept. 23.	AM.	M. q. bea.				Rev.Pts.	Rev.Pts.
Merc. to wire <i>a</i>	5	24 3 5	24 47.5	26 34.8			
A —	—	25 3 14½	25 52.3	26 34.8			
Middle wire B	—	26 2 9½	26 34.7	26 34.7	26 34.7	B 28.85	S 0.74
C —	—	27 1 6	27 18	26 34.7	26 34.7 { N. B. The telescope's center was pointed to horary circle VI. 34½ Decl. N. 7° 43'.		
D —	—	28 1 15	28 22.5	26 34.6			
	PM.						
$\lambda$ Ceti to B	9	15 1 27	15 28.5	15 28.5	15 28.4		
C —	—	16 0 23	16 11.5	15 28.3			
$\phi$ Tauri to B	—	40 1 25	40 27.5	40 27.5	40 27.4	B 8.39	N 19.72
C —	—	41 0 21	41 10.5	40 27.3			
Sept. 26.							
$\lambda$ Ceti to <i>a</i>	9	2 2 13	2 36.5	4 23.5			
A —	—	3 2 23	3 41.5	4 23.9	4 23.8	B 16.97	N 11.14
C —	—	5 0 14½	5 7.3	4 24.1			
$\phi$ Tauri to A	—	28 2 21	28 40.5	29 22.9			
B —	—	29 1 16	29 23	29 23	29 23	B 8.47	N 19.64
Sept. 30.							
$\lambda$ Ceti to <i>a</i>	8	47 0 3½	47 1.8	48 48.8			
A —	—	48 0 13	48 6.5	48 48.9			
B —	—	48 3 8	48 49	48 49	48 49	B 16.97	N 11.14
C —	—	49 2 4½	49 32.3	48 49.1			
$\phi$ Tauri to A	9	13 0 11	13 5.5	13 47.9			
B —	—	13 3 6	13 48	13 48	13 48.1	B 8.48	N 19.63
C —	—	14 2 3	14 31.5	13 48.3			
$\alpha$ Orionis to A	11	41 3 12	41 51	43 38			
A —	—	42 3 20½	42 55.3	43 37.7			
B —	—	43 2 17	43 38.5	43 38.5	43 37.9	A 15.07	S 15.77
C —	—	44 1 12	44 21	43 37.8			
D —	—	45 1 20½	45 25.3	43 37.7			
Oct. 13.							
$\lambda$ Ceti to A	7	58 3 0	58 45	59 27.4			
B —	—	59 1 25½	59 27.7	59 27.7	59 27.5	B 16.97	N 11.14
C —	8	0 0 21½	0 10.7	59 27.5			
$\phi$ Tauri to C	—	25 0 20	25 10	24 26.8	24 26.8	B 8.50	N 19.61
$\alpha$ Orionis to <i>a</i>	10	52 2 0½	52 30.2	54 17.2			
A —	—	53 2 9	53 34.5	54 16.9			
B —	—	54 1 4½	54 17.2	54 17.2	54 17.1	A 15.07	S 15.77
C —	—	55 0 0½	55 0.2	54 17			
D —	—	56 0 10	56 5	54 17.4			
1	2	3	4	5	6	7	8

Table II. For reducing the horary wires of the Equatorial Micrometer to that of the middle, when taken in mean solar time.

	Wires.	Equatorial object.		Declination 7° 48'	
		☉'s run	*'s run	☉'s run	*'s run
The 1st wire precedes the middle, add	a	1' 46.2	1' 46	1' 47.3	1' 47
2d _____	A	0 42.1	0 42	0 42.5	0 42.4
3d, or middle wire — —	B	— —	— —	— —	— —
4th is subsequent to the middle, subtract	C	0 42.9	0 42.8	0 43.3	0 43.2
5th _____	D	1 46.8	1 46.6	1 47.9	1 47.6
<b>I</b>	2	3	4	5	6



Table III. Containing the observations of Tab. I. reduced so as to shew the correct differences of right ascension and declination between Mercury and the stars wherewith he was compared.

1786. Date and object.	Hour.	Passage over mid. hor. wire by journ. clock.	Correction to reduce the clock to mean time.	Correct mean time of the observation.	Intervals of mean time of different observations.	Parts of microme- ter from the tele- scope's center.	Pts. of micr. reduced= declin. from the tele- scope's center.
Sept. 23. ♄ to the mid. wire	AM 5	26 34.7	+ 3 59.8	h. 5 22 34.9		Rev.Pts. S 0.74	S 1 8
λ Ceti to mid. wire	PM 9	15 28.4	-4 0.1	9 11 28.3	h. 15 48 53.4		
♉ Tauri to the same	9	40 27.4	-4 0.1	9 36 27.3	0 24 59	N 19.72	N 30 26
Sept. 26.							
λ Ceti to mid. wire	9	4 23.8	-4 43.2	8 59 40.6		N 11.14	N 17 11
♉ Tauri to the same	9	29 23	-4 43.2	9 24 39.8	0 24 59.2	N 19.64	N 30 18
Sept. 30.							
λ Ceti to mid. wire	8	48 49	-4 50.9	8 43 58.1		N 11.14	N 17 11
♉ Tauri to the same	9	13 48.1	-4 50.9	9 8 57.2	0 24 59.1	N 19.63	N 30 17
♈ Orion. to the same	11	43 37.9	-4 50.8	11 38 47.1	2 29 49.9	S 15.77	S 24 20
Oct. 13.							
λ Ceti to mid. wire	7	59 27.5	-6 36.4	7 52 51.1		N 11.14	N 17 11
♉ Tauri to the same	8	24 26.8	-6 36.8	8 17 50	0 24 58.9	N 19.61	N 30 15
♈ Orion. to the same	10	54 17.1	-6 37	10 47 40.1	2 29 50.1	S 15.77	S 24 20
I	2	3	4	5	6	7	8

Table IV. Deviations in the direction of the axis of the telescope of the Equatorial Micrometer in right ascension and declination in 20 days, from the 23d of September to the 13th of October, both inclusive.

Objects observed.	3 days from the 23d to 26th.	4 days from the 26th to 30th.	7 days from the 23d to 30th	13 days from the 30th to 13th.	17 days from the 26th to 13th	20 days from the 23d to 13th.
$\alpha$ Ceti { R. Ascension Declination too late South 8. 0.2	exact " O not taken the 23d	too late 1.1 exact O. too late 1.0 South 1.	too late 1.1 not taken the 23d too late 1.2 South 9.	" too soon 0.3 exact O. too soon 0.5 South 2.	" too late 0.8 exact O. too late 0.5 South 3.	" too late 0.8 not taken the 23d too late 0.7 South 11.
$\alpha$ Orionis { R. Ascension Declination	- - -	- - -	- - -	too soon 0.3 exact O.	- - -	- - -

N. B. The right ascension is expressed in the integer and decimal parts of a second of time. The declination is expressed in seconds of a degree.

*Explanation of the less obvious parts of the Tables of the Observation of Mercury near his Elongation, Sept. 1786.*

The third column of Tab. I. contains the times of observation as they were taken down from the half-second *journeyman* clock, in minutes, quarters, and *beats*, according to the following method; which was, by taking up the beat when the second hand came to 15, 30, 45, or 60, and then counting 30 beats repeatedly till the arrival of the object at the middle of the wire it was approaching; after its arrival, the beats (or interval between two beats) being retained in memory, and the eye cast upon the dial-plate, it was easily seen whether it was so many beats more than the quarter, the half, three-quarters, or the whole minute, and was set down accordingly. Those reduced to minutes, seconds, and tenths of seconds, by allowing .2 or .3 for the quarter second, .5 the half, and .7 or .8 for the three-quarters of a second, are contained in the fourth column. The reduction of the fourth column to the fifth was by means of the auxiliary Tab. II.; and Mercury being then nearly stationary respecting the sun, the sun's run was used for the planet instead of that of a star. The mean of each set of observations of the fifth column is carried into the sixth.

The seventh column contains the parts of the micrometer as they were read off; to render which intelligible, it is to be noted, that the declination wire *A* travels from the upper side of the field of view of the telescope towards the center, and somewhat beyond it: and upon it are taken all the objects that pass the field of view on the upper side, answerable (by inversion of the object) to the southern half of the field: and in like manner those that pass the field of view on the lower

half are taken upon the wire *B*, and for the same reason denote a declination north. The scale of the micrometers of each wire begins from a point assumed somewhat without the field, and the number increases from thence towards the center of the field, and continues beyond it; the integral parts are the turns of the screw, and the centesimal the divisions of the index plate, being divided into 100 parts. The point of the scale, answerable to the center of the field of view, having been found by observations on each scale respectively; when the wire *A* (*Australis*) stands at 30.84, it is in the center of the field; and when the wire *B* (*Borealis*) is at 28.11, it also cuts the same center. Hence the parts of the micrometer being respectively taken from those two numbers (which may therefore be called *constant* numbers) the remainder will be the distance of each respective wire from the center in parts of the micrometer. Thus, in the observation of  $\alpha$  Tauri upon the 23d, the parts are *B* 8.39; this taken from 28.11, leaves *N* 19.72, which are placed in col. 8. as the distance, in parts of the micrometer, that  $\alpha$  Tauri passed north of the center of the field of view, or axis of the telescope.

In like manner, in the observation of Mercury on the 23d, the parts are *B* 28.85; but this being greater than the constant number 28.11, the excess will be .74 parts; which being the parts reaching beyond the center, they will be so much *south* of it, and are set down therefore in col. 8. *S* 0.74: and in this manner the declinations of the rest are made out, from their respective numbers of parts of the micrometer, and set down in col. 8.

The numbers of the sixth column of Tab. I. are transferred to the third column of Tab. III.; and the declinations set down in parts of the micrometer, Tab. I. col. 8. are transferred to col. 7. of Tab. III.

Col. 4. of this table contains the corrections of the times deduced from the journeyman clock (as *per* col. 3.) to reduce it to mean time; which corrections are made out from the general account of the goings of the transit clock, corrected by transits of the sun, taken the 22d, 23d, 27th, and 30th of September, and the 12th, 13th, and 14th of October\*. The journeyman clock was regularly compared at nights and mornings with the transit clock; and generally immediately after the observation. The *meridian* and *rotative* observatories in which the clocks respectively were, are at the distance of 53 yards E. and W.; the comparisons were made by a seconds stop watch †.

The numbers of the fourth column being properly applied to those of the third produce the fifth; and which, with the sixth column, will be sufficiently explained by their titles. The parts of the micrometer in the seventh column, being reduced into minutes and seconds, are contained in col. 8. and respectively shew the minutes and seconds at which each object passed to the north or south of the center of the telescope. The value of the parts of the micrometer were obtained by previous observations, from whence the following rule was deduced: the numbers of turns and centesimal parts being considered as integral, and divided by 1.08, the quotient will be the number of seconds. Thus, in the observation of *♂* Tauri

\* The transit clock was made by HINDLEY, and has a pendulum rod of cedar wood.

† The journeyman clock was generally set to the transit clock on Sunday mornings; and when from home the former was suffered to go down. The journeyman will generally agree with the transit clock to 2" in 24 hours; but during the period of these observations, went remarkably well.

upon the 23d, the parts 1972, divided by 1.08, gives  $1826'' = 30' 26''$ ; and the parts of mercury .74, divided by 1.08 =  $68'' = 1' 8''$ . Now the telescope being fixed to one point of the heavens during the whole period of these observations, without any motion of any of the parts, the screws commanding the declination wires *A* and *B* excepted, we are enabled to judge of its steadiness to this point by the following remarks. If it varied in declination, this would be shewn by the passage of the same star at a different distance from the center of the telescope at different revolutions; and if it varied in right ascension, it would be shewn by its not passing the horary wires at the due time, according to the acceleration of the stars upon the mean time of the sun. Both the right ascension and declination may be varied by differences of refraction of the air at the same altitude; and the right ascension is further liable to be *apparently* varied, by the errors of the transit instrument, the transit clock, the transferring of its time to the journeyman clock, the intermediate errors of the same, and of the observation itself; and as there passed an interval of almost 16 hours betwixt the passage of Mercury over the field of view of the telescope and that of  $\lambda$  Ceti, which was the nearest star wherewith a comparison could be made, it will be a satisfaction to see, as before intimated, what variations arose in still greater intervals of time.

In right ascension.

Thus $\lambda$ Ceti upon Sept. 23. passed the horary wires at	h. m. s.
and ————— 26. —————	9 11 28.3
	8 59 40.6
$\lambda$ Ceti therefore came sooner in three days by	11 47.7
but ——— ought to accelerate on mean time	11 47.7
————— therefore came after three days exactly to the time.	

Again,

Again, $\alpha$ Tauri upon Sept. 23. passed the horary wires at	-	h. 9 36 27.3
and _____ 26. _____	-	9 24 39.8
<hr/>		<hr/>
$\alpha$ Tauri therefore came sooner after three days by	-	11 47.5
_____ ought to accelerate on mean time	-	11 47.7
<hr/>		<hr/>
_____ therefore came too late in three days by	-	.2
<hr/>		<hr/>

## In declination.

$\alpha$ Tauri upon Sept. 23. passed north of telescope's center	-	30 26
_____ 26. _____	-	30 18
<hr/>		<hr/>
_____ therefore passed less north, or more south, than before by	-	8
<hr/>		<hr/>

In like manner every comparison that Tab. III. affords is particularly set down in Tab. IV. which containing thirteen comparisons in right ascension and ten in declination, the greatest deviation in right ascension is  $1''.2$ , and  $11''$  of a degree in declination. This supposes every error before mentioned to reside in the instrument, and every other instrument and observation, which were concerned in the result, to be perfect; which, from the smallness of the total errors, seems to indicate a degree of steadiness in the instrument unexperienced or unnoticed before.

*Deduction of the position of Mercury from the preceding observations as set down in Tab. III.*

## In right ascension from col. 6.

$\alpha$ Tauri followed $\lambda$ Ceti Sept. 23.	-	-	-	24 59
_____ 26.	-	-	-	24 59.2
_____ 30.	-	-	-	24 59.1
_____ Oct. 13.	-	-	-	24 58.7
<hr/>				<hr/>
_____ at a mean of the four	-	-	-	24 59
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$\alpha$  Orionis

$\alpha$ Orionis followed $\sigma$ Tauri Sept. 30.	—	—	—	h. ' "
————— Oct. 13.	—	—	—	2 29 49.9
————— at a mean	—	—	—	2 29 50.1
				—————
Now Mercury preceded $\lambda$ Ceti Sept. 23.	—	—	—	15 48 53.4
$\lambda$ Ceti preceded $\sigma$ Tauri by mean of four	—	—	—	24 59
$\sigma$ Tauri preceded $\alpha$ Orionis by mean of two.	—	—	—	2 29 50
				—————
Mercury therefore preceded $\alpha$ Orionis by	—	—	—	18 43 42.4
				—————

In declination from col. 8.

Sept. 23. A.M. Mercury passed the middle horary wire, south of its center	1' 8"
Same evening $\sigma$ Tauri passed the middle horary wire, north of it	— 30 26
	<hr/>
Therefore Mercury passed the middle horary wire more S. than $\sigma$ Tauri by	31 34
	<hr/>

But Sept. 26. $\lambda$ Ceti passed N. of center	17 11	} Diff. 13 7
————— $\sigma$ Tauri —————	30 18	
————— 30. $\lambda$ Ceti —————	17 11	} — 13 6
————— $\sigma$ Tauri —————	30 17	
————— Oct. 13. $\lambda$ Ceti —————	17 11	} — 13 4
————— $\sigma$ Tauri —————	30 15	

From the smallness of the above differences we may infer, that very little uncertainty in declination had attended the passage of  $\sigma$  Tauri upon Sept 23.

■ Upon Sept. 30. $\sigma$ Tauri passed N	30 17	} Sum	54 37
————— $\alpha$ Orionis — S	24 20		
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Upon Oct. 13. $\sigma$ Tauri passed N	30 15	} —	54 35
————— $\alpha$ Orionis — S	24 20		
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$\alpha$ Orionis then at a mean passed more south than $\sigma$ Tauri	—	54 36
		—————
Merc. therefore on the 23d passed with more N. declination than $\alpha$ Orionis	23	2
		—————



*Investigation of the effects of refraction.*

The preceding deductions and remarks shew the consistency of the observations with themselves; yet, from the position of the telescope, it being only elevated  $11^{\circ}\frac{1}{2}$  above the horizon\*, it is necessary to examine how far the deductions above specified were capable of being affected by refraction. And in this respect it will appear, that if it be supposed, there is no difference in the quantity of refraction of such objects as appear within the limits of the field of view of this instrument (which is  $1^{\circ} 17'$ ), then their relative positions to each other will not be affected thereby: for if in fig. 1. (Tab. XIII.) we suppose the circle VHRO to represent the boundary of the field of view, HO being an horizontal and VR a vertical line, each passing through the center of the field at L; and if PLP denotes a part of a parallel of declination, then BLX perpendicular thereto, will be a part of an horary circle, both passing through the same center. Now let  $d*$  be the apparent path of a star, supposing it unaffected by refraction till it comes to the vertical line at \*, and there to be lifted up by refraction in the said vertical to L. Let  $e+$  denote another star, also unaffected by refraction, to pass along the different parallel of declination  $e+$  till it comes to +; then, if it be supposed that the two stars are both situated in the same horary circle, if at the point + refraction takes place, and by hypothesis this is lifted up equally with the other, in the perpendicular  $+L$ , then the line  $+*$  being drawn through the places of the two stars, will be cotemporary and parallel to LX; and the figure  $+*L$  being evidently a rhomboides, the two stars, so altered by

\* This will readily be deduced by inspection of the celestial globe.

refraction, will arrive together at the horary circle LX at the same time, and with the same difference of declination, as if no refraction had taken place. It is therefore only the *difference* of refraction which takes place in objects at different heights in the *same field*, that can alter their relative situations: however, it appears necessary to examine what this may amount to.

Let the letters in fig. 2. denote the same things as before; to which we will add, that  $a, A, B, C, D$ , denote the parallel horary wires of the micrometer, and  $AA, BB$ , the declination wires, denoted  $A$  and  $B$  in the tables: now from the celestial globe we shall also readily obtain the horary angle  $VLP = 54^{\circ}\frac{1}{2} = Lbc$ . Let now an object pass along the wire  $AA$  from the horizontal line at  $d$  to the vertical line at  $b$ ; in this it will pass through a difference of refraction, according as it gets more and more elevated above the horizontal line HO; and let the elevation  $Lb$  be half a degree or 30 minutes: then, according to Dr. BRADLEY's Table of Refraction\*, the difference of refraction betwixt the 78th and 79th degrees of zenith distance is  $23''.6$ , half of which  $11''.8$ , may be esteemed the difference of refraction for a difference of half a degree of altitude at  $78^{\circ}\frac{1}{2}$  zenith distance, or of  $11^{\circ}\frac{1}{2}$  altitude: the object, therefore, in passing from the horizontal line at  $d$  to the vertical line at  $b$  passes through every difference of refraction from  $0''$  to  $11''.8$ ; and the question is, how much it is at a *medium*, that is, when it arrives at the middle wire at the point  $c$ ? From this point let fall the perpendicular  $ce$ . Now, the proportion of the sides of the triangle  $dbL$  being given from construction, they may be taken off by a scale, *viz.*

\* Inserted in Dr. MASKELYNE's Observations, Vol. I. p. 15.

Suppose  $Lb = 174$

$db = 299$

$dL = 242$

and assuming the side  $Lb = 30$

the other sides by proportion  $\left\{ \begin{array}{l} db = 51.6 \\ dL = 41.7 \end{array} \right.$

as above will be

The triangles  $Lbc$  and  $dce$  are similar to  $dbL$ ; therefore say, as  $db = 51.6 : dL = 41.7 :: Lb = 30 : Lc = 24$ ; and as  $Lb = 30 : Lc = 24 :: dL = 41.7 : dc = 33.5$ ; and again, as  $db = 51.6 : dc = 33.5 :: Lb = 30 : ce = 19.5$ : but this will affect the declination, only in proportion of the line  $ef$  drawn parallel to  $LX$ ; and it will affect the right ascension according to the line  $fc$ : but the triangle  $ecf$  being similar to the original one  $dbL$ , we shall have  $db = 51.6 : Lb = 30 :: ce = 19.5 : fc = 11.3$  for the line affecting the right ascension; and also, as  $db = 51.6 : dL = 41.7 :: ce = 19.5 : ef = 15.8$  for the line affecting the declination. But the effect of difference of refraction upon the line  $Lb = 30'$  being only  $11''.8$ , the respective effects of the lines  $fc$  and  $ef$  will be in proportion; that is,

as  $30' : 11.3 :: 11''.8 : 4.4$  for the effect in right ascension,  
and as  $30 : 15.8 :: 11.8 : 6.2$  ————— declination;  
but as it has been determined, that when the line  $Lb$  is 30 minutes, the line  $LC$ , or the corresponding declination, will be only 24 minutes; the effects of refraction above stated will be therefore due to  $24'$ .

#### *Correction for the position of the wires.*

The above corrections take place on supposition that the several wires of the micrometer were strictly parallel to the respective parts of the circles of declination, and horary circles in the heavens; but in the practical use of this instrument it

is found more convenient, on account of a ready and certain adjustment, to place one of the wires *AA* or *BB* parallel to the apparent track of the star wherewith the *planetary* body is to be compared: in consequence, when the star  $\star$ , fig. 1. is lifted up to *L*, it will not strictly pursue the line *LP*; but being less and less lifted up as it mounts higher, it will apparently fall more and more below the line *LP* as it ascends above the line *HO*, and will therefore take a course, suppose *Lp*. The wire *PLP* being therefore adjusted to agree with *pLp*; by construction of the instrument, the wire *BLX* will assume the position *qLx* perpendicular to *pLp*. The star, therefore, that ran along the parallel *e+* before it suffered refraction, and at *+* was supposed to be lifted up to *l*, there not meeting *LX* will take the course *ly*, nearly parallel to *Lp*, and have some distance, as *lz*, to travel before it arrives at the new-placed wire *Lx*; and it is now proper to examine what this quantity may be.

Through the point *z* draw the line *rxos* parallel to *HO*, and cutting the vertical *RLV* in *a*, and let *Lo* be assumed =  $30'$ ; then, since the angle *XLx* is supposed to be *minute*, the *gross* proportions of the sides of the triangles *Lyx* and *Lyl* may be, for this purpose, supposed the same\*, and the same as *Lbc*, *dbL*, fig. 2. to which the triangle *Lzo*, fig. 1. will also be similar; as likewise the triangle *yzo*, and also the little triangle *zlv*: but making the side *lv* of the triangle *zlv* equal to the effect of refraction in perpendicular =  $11''.8$ ; then, to find the side *lz*,

\* I am aware, that the supposition of the sides of the triangles *Lyx* and *Lyl* being the same cannot be strictly so; nor can they have the same proportions; nor are any of the lines concerned right lines, that are supposed such; but assumptions near the truth are allowable for the correction of an error in the *greatest part*, that if uncorrected would scarcely amount to a *gross error*.

the distance run from the first to the last supposed place of the wire, we need only say, as  $Lb = 30 : db = 51.6 :: lv = 11''.8 : lz = 20''.3$ ; and this will be its value when the declination  $Lc$ , fig. 2. is  $24'$ ; but then the declination  $Ll$  or  $Lz$ , fig. 1. being greater than the perpendicular side  $Lo$  (assumed  $30'$ ) in the proportion of  $Lz : Lo$ , say, by similarity of triangles conversely, as  $dL = 41.7 : db = 51.6 :: Lo = 30' : Lz = 38'.2$ ; but as the correction before stated of  $20''.3$  is an angular error, taking place in proportion to the distance from the center, or the declination; for the declination given of  $24'$  say, as  $38.2 : 24' :: 20''.3 : 13$ ; to which adding  $4''.4$ , we shall have  $17''.4$  for the whole error in right ascension, supposing it in the equator, but must be again increased in the proportion in which a star having declination is slower than a star in the equator; that is, it must be increased in the proportion of any of the numbers in the fourth column of Tab. II. to the similar ones in col. 6. of the same table; that is, as  $1' 46'' : 1' 47''$  or as  $106 : 107$ ,  $:: 17''.4 : 17''.6$  \*.

As all these errors, arising from difference of refraction, are in proportion of the distance of the object from the center of the telescope, they will take place in proportion to the difference of declination of the two objects to be compared, whether they have passed the field on the same, or on different sides of the center. Now the difference of declination of Mercury and  $\alpha$  Orionis being only  $23' 2''$ , and the quantities being made out for  $24'$  say (rejecting the 2 seconds), as  $24' : 23' :: 17''.4 : 16''.7$ , which turned into time in the run of the star will be  $1''.1$  in right ascension.

\* My friend Dr. MASKELYNE observes, that in *strictness* each star ought to have its own proper reduction, on account of *difference* of declination, which in *extreme cases* will amount to a sensible quantity.

Say again, as  $24' : 23' :: 6''.2 : 6''$ , the correction in declination. From the near equality of the lines  $Ll$  and  $Lz$ , it is evident, that no correction of *declination* is necessary on account of the inclination of the wires, the whole difference falling in right ascension. As therefore Mercury passed with  $23' 2''$  more north declination than  $\alpha$  Orionis, and passed through a part of the *medium* that lifted him up less; it therefore gave him less north declination than it did to  $\alpha$ , and therefore apparently diminished the real difference; hence  $6''$  must be added to the apparent difference  $23' 2''$ , making it  $23' 8''$  difference of declination: and as Mercury was lifted up less than  $\alpha$ , he would not so soon come to the middle wire by  $1''.1$  as he should have done, he therefore came too late by  $1''.1$ , which must be subtracted from the time of Mercury's passage the 2d of Sept. which will increase the time in which he preceded  $\alpha$  Orionis; that is,  $18\text{ h. } 43' 42''.4$  increased by  $1.1$  will become  $18\text{ h. } 43' 43''.5$  difference of right ascension.

I have been the more particular in the investigation of this observation, first of all to ascertain the degree of dependance that may be formed on an instrument of the kind; and, secondly, to infer such easy and simple rules, that other similar observations may be the more easily reduced. Being therefore satisfied of the stability of the instrument; if we had concluded the observation with that of Mercury in the morning, and of  $\alpha$  Tauri in the evening of the 23d, then the result from Tab. III. should have been

				h.	'	''
Mercury passed the wires at	-	-	-	17	22	34.9
And $\alpha$ Tauri passed at	-	-	-	9	36	27.3
Difference of right ascension	=	=	=	15	13	52.4

which

which is the very same as was before deduced from the mean of the whole :

And if to Mercury's declination south of telescope's center	-	1' 8"
We add $\sigma$ Tauri's ————— north —————	-	30 26
We shall have for the difference of declination		<hr/> 31 34 <hr/>

the same as before determined. Our observation would, therefore, in this case simply have been, that Mercury preceded  $\sigma$  Tauri in right ascension 15 h. 13' 52''.4 mean time, and passed the wire with more south declination than  $\sigma$  Tauri by 31' 34".

After this,  $\sigma$  Tauri would have required to be compared with some well rectified star by meridian instruments; but in the present case  $\alpha$  Orionis, one of Dr. MASKELYNE's Catalogue of 34 principal stars happened to lie sufficiently near the same parallel of declination, to admit of  $\sigma$  Tauri to be compared therewith by the same instrument, while pointed to the same place of the heavens. The operations which were subsequent, therefore, must be considered as intended to save those of a meridian instrument.

Now had our observation concluded with the above, then the correction would have taken place upon the difference of declination of  $\sigma$  Tauri with Mercury, instead of the ultimate one with  $\alpha$  Orionis; but it must be observed, that whatever quantity of correction the difference of declination would occasion, it would be compensated in the difference of refraction of  $\sigma$  Tauri and  $\alpha$  Orionis, when they came to be observed on the meridian; however, in the present case it happens to be more commodious, as both can be done under one.

Preparatory then to the laying down the simple rule for the correction of refraction, it is proper to premise, that it is evident,

dent, the lines, fig. 2.  $Lb$ ,  $Lc$ ,  $ce$ ,  $ef$ , being in continued proportion  $Lb$  will be to  $ef$  in triplicate proportion of  $Lb$  to  $Lc$ ; and that  $Lc$  will be to  $ef$  in duplicate proportion of  $Lb : Lc$ . The difference of declination, therefore, due to  $30'$  difference of elevation will be as  $Lb$  to  $Lc$  simply; but the effect of difference of refraction in declination will be less than the difference of declination in the proportion of  $Lb^2 : Lc^2$ ; and that the effect of difference of refraction in right ascension will be less than the difference of refraction in declination in the proportion of  $Lb : cb$  simply.

Now it has been remarked, that the elevation of the telescope's center above the horizon, and the horary angle  $VLP$ , will always be readily given near enough for the purpose by the globe. A triangle given  $Lbd$  can therefore be constructed, and the side  $Lb$  being made  $30'$  (or any convenient aliquot part of a degree) the other sides will be found by proportion: say then, as in the present case,  $db = 51.6 : dL = 41.7 :: Lb = 30 : Lc = 24$ , for the difference of declination corresponding to half a degree of altitude: say then, as  $51.6^2 : 41.7^2$ , that is, as  $2663 : 1739 :: 24 : 15.7 = ef$ . But without troubling ourselves with high numbers, if we take the proportion  $51.6$  to  $41.7$  by the slide-rule *twice*, we shall arrive at  $15.7$ , near enough for the value of the line  $ef$ : say then, as  $Lb = 30 : ef = 15.7 :: 11''.8 : 6''.2$  for the refraction in declination: and as  $dL = 41.7 : Lb = 30 :: 6''.2 : 4''.4$  for the refraction in right ascension, according to the *true* position of the wires: and, for the correction of right ascension in the position of the wires, say,

Fig. 2.	Fig. 1.
As $Lb = 30$ : $db = 51.6 ::$	$lv = 11''.8 : lz = 20''.3 :$
and again, $dL = 41.7 : db = 51.6 ::$	$Lo = 30' : Lx = 38'.2.$

Take



Take now  $Lb$ , fig. 1. =  $Lc$ , fig. 2. =  $24'$ , and draw the line  $bik$ , fig. 1. parallel to the line  $lzy$ , and then say, as  $Lz = 38'.2 : Lb$  (or  $Li$ ) =  $24' :: lz = 20''.3 : bi = 13''$ , which  $+ 4''.4 = 17''.4$  for the whole error in right ascension, with a declination or distance from the center of  $24'$ ; but as the errors both of right ascension and declination are in proportion to distance from the center, as the difference of the planet and star is only  $23'$ , say, as  $24' : 23' :: 17''.4 : 16''.7 = 1''.1$  time; and for the declination, say again, as  $24' : 23' :: 6''.2 : 6''$  declination\*.

*Reduction of Mercury's comparison with  $\alpha$  Orionis, to right ascension and declination.*

We have laid it down, that the 23d Sept. 1786, A.M. at 5 h.  $22' 34''.9$  mean time, Mercury preceded  $\alpha$  Orionis 18 h.  $43' 43''.5$ , and had then a more northern declination by  $23' 8''$ .

According to Dr. MASKELYNE's Catalogue of 34 stars, the right ascension of  $\alpha$  Orionis reduced to the time when he was observed is  $85^\circ 54' 12''$ .

Now as the whole circle of the sphere makes a revolution in the time that  $\alpha$  Orionis makes one turn, which is

$$\begin{array}{r} \text{h.} \\ 23 \ 56 \ 4.1 \text{ then from this deduct} \\ 18 \ 43 \ 43.5 \\ \hline \end{array}$$

5 12 20.6 remains for the time that  $\alpha$  Orionis preceded

\* If the comparison had been with  $\sigma$  Tauri, then we must have said,

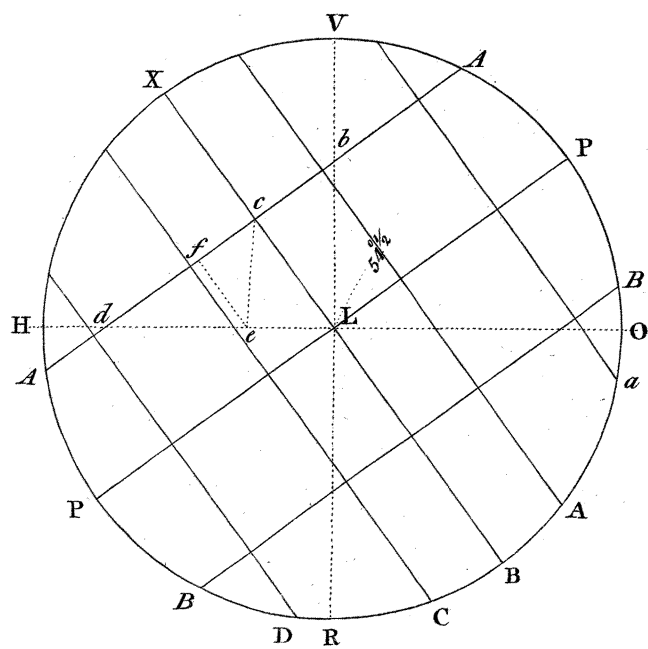
$$\text{As } 24' : 31\frac{1}{2}'' :: 17.5 : 23 = 1.5 \text{ of time,}$$

$$\text{and } 24' : 31\frac{1}{2}'' :: 6.2 : 8.1 \text{ correction for declination.}$$

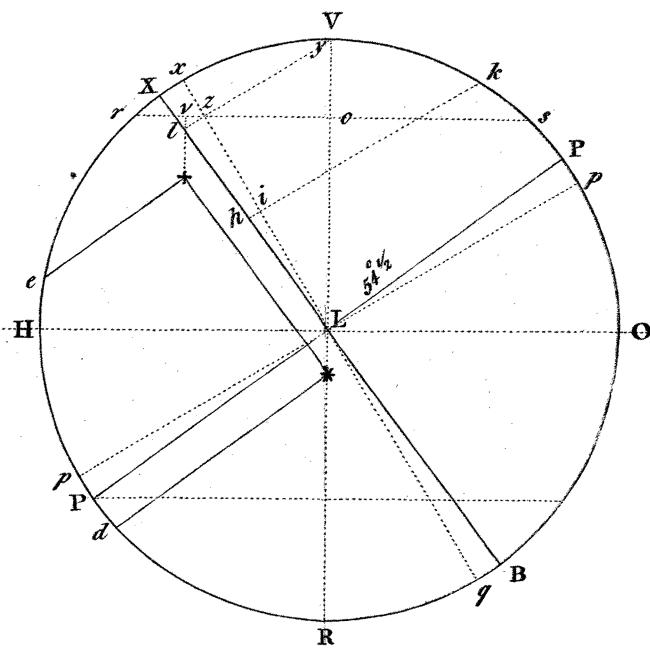
N. B. All these and the above proportions will be commodiously wrought with the slide-rule.

*Mercury,*

*Fig. 2.*



*Fig. 1.*



Mercury in right ascension; but if  $\alpha$  ran the whole rotation  $\approx 360^\circ$  in 23 h. 56' 4''.1, what portion of it will be run in 5 h. 12' 20''.6  $\approx 18740''.6$ ?

But 24 h.  $\approx 86,400$  seconds, and  $360 \approx 1,296,000$  seconds:

	Time.	Time.	Of degrees.	Of degrees			
Say then, as	86400"	18740''.6	::	1296000"	:	281109"	$\approx$ 78 5 9

But, according to Dr. MASKELYNE's select Catalogue, the right ascen-

fion of $\alpha$ Orionis for Sept. 30, 1786, was, (which add)	-	85 54 12
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The right ascension of Mercury at the time of observation was therefore	163 59 21
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According to Dr. MASKELYNE's select Catalogue $\alpha$ Orionis had decli-		0 ' "
nation north, corrected for precession	-	7 21 8.8

The sum of aberration and nutation from <i>Connoissance des Temps</i>	-	+ 8.4
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The correct declination north of $\alpha$ Orionis	-	7 21 17.2
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To which add that Mercury passed more north	-	23 8
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Mercury's declination therefore was	-	7 44 25.2
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### The result.

1786, Sept 23. A.M.	} Mercury's {	right ascension	-	163 59 21
at 5 h. 22' 35" M.T.		declination north	-	7 44 25

