II. *Aſtronomical Obſervations on the Planets Venus and Mars, made with a View to determine the heliocentric Longitude of their Nodes, the annual Motion of the Nodes, and the greateſt Inclination of their Orbits.* *By* Thomas Bugge, *F. R. S. Regius Profeſſor of Aſtronomy at* Copenhagen, *Member of the Academies of* Stockholm, Copenhagen, Manheim, *and* Drontheim, *and Correſpondent of the Academy of Sciences at* Paris.

Read November 26, 1789.

I. *The heliocentric longitude and annual motion of Venus's nodes.*

THE following aſtronomical obſervations were made at the Royal Obſervatory at Copenhagen with a ſix-feet tranſit inſtrument, and with a mural quadrant of ſix-feet radius. It would be too tedious to enumerate all the original obſervations, which either are already printed in the firſt volume of my Aſtronomical Obſervations, or will very ſoon be publiſhed in the ſecond volume. I only ſhall ſet down the obſerved geocentric longitudes and latitudes, corrected for aberration and nutation, and compared with the tables of Dr. HALLEY and of M. DE LA LANDE.

Mean

Mean time at Copenhagen.	h. ′ ″	Observed geocentric longitude of ♀. s. ° ′ ″	Observed geocentric latitude of ♀. ° ′ ″	HALLEY's error, in long. ″	HALLEY's error, in lat. ″	DE LA LANDE's error, in long. ″	DE LA LANDE's error, in lat. ″
1781							
Sept. 13	1 38 6	6 18 39 15	0 28 12 N	+11	+3		
22	1 43 24	6 29 38 48	0 3 9	+17	+1	+39	+3
Oct. 1	1 49 46	7 10 36 42	0 23 48 S	+16	−7	+44	−2
4	1 52 11	7 14 15 17	0 33 3	+12	−4		
1784							
Sept. 20	0 37 17	6 9 40 27	1 6 15 N	+41	+15	+17	+12
25	0 39 57	6 15 52 59	0 57 50	+15	+3		
Oct. 2	0 44 49	6 24 35 34	0 44 21	+36	+6		
14	0 54 7	7 9 30 40	0 16 41	+10	+1		
21	1 0 42	7 18 13 11	0 1 3 S	+17	+1	+18	+3
1786							
Aug. 19	2 25 57	6 4 45 54	0 23 7 N	−5	−3		
20	2 26 14	6 5 56 11	0 19 40	−17	−9		
21	2 26 32	6 7 6 42	0 16 20	−13	−5	+17	−12
28	2 28 30	6 15 7 50	0 8 46 S	−0	−2	+23	+2
29	2 28 47	6 16 27 39	0 12 32	−22	−4		

The angle at the planet or the commutation = P is not directly to be taken out of the table. The difference between the observed geocentric longitude of the planet and the geocentric longitude of the sun, calculated from M. MAYER's solar tables, is the angle at the earth, or the elongation = T. From this elongation, which is to be depended upon to very few seconds, and from the planet's and the earth's distances from the sun, according to the tables, the commutation is calculated, and the geocentric longitude is reduced to the heliocentric longitude. The angles P, T, and S, at the sun, thus found, are likewise used to calculate the heliocentric latitude.

According to the different dimenfions, given to the orbit of the planet in the different tables, the radius vector at a given time will alfo be fomewhat different. Thefe differences in the tables of Dr. HALLEY and M. DE LA LANDE are but fmall: thus, 1784, September 20, at 0 h. 37′ 17″ mean time, the obferved and apparent geocentric longitude of Venus 6 s. 9° 39′ 54″, the aberration and nutation +33″, the corrected and true geocentric longitude 6 s. 9° 40′ 27″, the fun's geocentric longitude 5 s. 28° 5′ 51″; hence the elongation T = 0 s. 11° 34′ 36″. If now the logarithm of the radius vector SP is taken out of Dr. HALLEY's tables = 4.858251, then $\text{fin. } P = \frac{ST \times \text{fin. } T}{SP}$ and P = 16° 11′ 52″; but if the logarithm SP is taken out of the tables of M. DE LA LANDE = 4,858168, the angle P will be found = 16° 12′ 4″, the difference is 12″. This uncertainty in the commutation, and confequently in the heliocentric longitude, would have been ftill greater if the calculations had been made only from the tables, or from the planet's geocentric longitude by the tables; thus this angle P is, according to Dr. HALLEY, = 16° 10′ 52″; and from the tables of M. DE LA LANDE = 16° 10′ 13″.

Mean

Mean time at Copenhagen.		Heliocentric longitude of ♀ in the ecliptic.	Heliocentric latitude of ♀.	HALLEY's error, in long.	HALLEY's error, in lat.	DE LA LANDE's error, in long	DE LA LANDE's error, in lat.
1781	h. ′ ″	s. ° ′ ″	° ′ ″	″	″	″	″
Sept. 13	1 38 6	7 28 40 26	0 56 13 N	+21	+ 5		
22	1 43 24	8 12 59 47	0 6 4	+70	+ 2	+ 63	− 6
Oct. 1	1 49 46	8 27 15 26	0 44 6 S	+41	−13	+ 13	− 6
4	1 52 11	9 1 59 21	1 0 28	−20	− 8		
1784							
Sept. 20	0 37 17	6 25 51 19	2 23 46 N	+28	+34	+106	+27
25	0 39 57	7 3 52 40	2 13 7	+ 1	+ 7		
Oct. 2	0 44 49	7 15 6 32	1 40 46	+74	+13		
14	0 54 7	8 14 13 57	0 36 58	+33	+ 1		
21	1 0 42	8 15 21 40	0 2 17 S	+55	− 2	+ 43	+ 7
1786							
Aug. 19	2 25 57	8 4 12 41	0 37 6 N	+ 25	− 4		
20	2 26 14	8 5 47 32	0 31 22	+ 52	−15		
21	2 26 32	8 7 23 10	0 25 55	+ 68	− 6	+ 69	−16
28	2 28 30	8 18 30 24	0 13 17 S	+108	− 3	+ 89	+ 3
29	2 28 47	8 20 5 16	0 18 52	+ 35	− 6		

Let the difference between two heliocentric longitudes, one before and the other after the passage through the node, be $=a$, the northern heliocentric latitude $=b$, and the southern $=\beta$; let the arc of the ecliptic from the node to the longitude, answering to the southern latitude, be $=x$; then tang. $x = \frac{\text{sin. } a \text{ . tang. } \beta}{\text{tang. } b + \text{cos. } a \text{ tang. } \beta}$. By this formula the distance from every longitude with a southern latitude to the node may be found; and hence the heliocentric longitude of the node.

Observations

Obſervations compared.	Heliocentric longitude of ☋ ♀.	Reduced to 1786.
1781	s. ° ′ ″	
September 13 — October 1	8 14 42 42	
September 13 — October 4	8 14 42 23	
September 22 — October 1	8 14 42 29	
September 22 — October 4	8 14 42 12	
		s. ° ′ ″
Mean	8 14 42 24	8 14 44 53
1784		
October 21 — September 20	8 14 43 38	
October 21 — September 25	8 14 43 40	
October 21 — October 2	8 14 43 42	
October 21 — October 4	8 14 43 30	
Mean	8 14 43 38	8 14 44 34
1786		
Auguſt 19 — Auguſt 29	8 14 45 3	
Auguſt 19 — Auguſt 28	8 14 44 28	
Auguſt 20 — Auguſt 29	8 14 44 16	
Auguſt 20 — Auguſt 28	8 14 44 0	
Auguſt 21 — Auguſt 29	8 14 44 27	
Auguſt 21 — Auguſt 28	8 14 44 36	
Mean	8 14 44 28	8 14 44 28

Hence *the heliocentric longitude of the deſcending node of the planet Venus was,* 1786, *Auguſt* 25, *at* 8 h. 39′ = 8 s. 14° 44′ 38″. I think that this place of the node is to be depended upon to 10 or 15 ſeconds. According to the tables of M. CASSINI, the longitude of the node 8 s. 14° 48′ 31″, the difference, or the error, − 3′ 53″. According to the tables of Dr. HALLEY 8 s. 14° 42′ 39″, the difference + 1′ 59″. From the tables of M. DE LA LANDE 8 s. 14° 45′ 15″, the difference only − 37″.

In order to ascertain the annual motion of the node, this observation is to be compared with the observations of other astronomers. The numbers in the column A are found by setting out from my own observations 1786. In the column B, the observation of M. DE LA LANDE 1769 is taken as the first; the series C is begun with the determination of M. HORROX 1639; and in the series D, M. CASSINI's observation 1698 is taken as the basis.

The Astronomer.	The time of observation.	Heliocentric longitude of ☊ ♀.				Annual motion of ☊ ♀.			
						A.	B.	C.	D.
		s.	°	′	″	″	″	″	″
HORROX	1639, Dec. 4	2	13	27	50	31,3	31,6	— —	
CASSINI	1698, Sept. 4	2	14	1	45	29,2	29,2	36,5	
CASSINI	1705, June 11	2	14	2	52	30,9	31,3	31,9	
CASSINI	1731, April 7	2	14	17	2	30,0	30,2	32,7	24,9
DE LA CAILLE	1746, Dec. 21	2	14	23	10	32,1	34,3	31,0	26,8
DE LA CAILLE	1761, June 5	2	14	31	30	31,2	— —	31,3	27,3
DE LA LANDE	1769, June 3	2	14	36	20	28,1	— —	31,6	29,2
BUGGE	1786, Aug. 25	2	14	44	38	— —	— —	31,3	29,2
		Mean				30,4	31,3	32,3	27,5

If the mean be taken of those four means, *the annual motion of Venus's node will be 30,37″*, or very near 31″, adopted in the tables of Dr. HALLEY and M. DE LA LANDE.

II. *The greatest inclination of the orbit of Venus to the ecliptic.*

In the first place, I shall put down the observed geocentric longitudes and latitudes, corrected for aberration and nutation.

Mean

Mean time at Copenhagen.	h.	′	″	Geocentric longitude of ♀. s.	°	′	″	Geocentric latitude of ♀. °	′	″	Halley's error, in long. ″	Halley's error, in lat. ″	de la Lande's error, in long. ″	de la Lande's error, in lat. ″
1781, July 20	0	1	40	4	11	10	49	1	24	46 N	+ 30	+ 5		
24	1	5	45	4	16	5	52	1	27	21	+ 21	+ 5		
30	1	11	18	4	23	28	41	1	29	21	+ 29	+ 9	+ 69	+ 10
31	1	12	9	4	24	42	29	1	29	36	+ 32	+ 12		
Aug. 1	1	12	58	4	25	55	54	1	29	21	+ 18	+ 3		
4	1	15	21	4	29	37	13	1	29	4	+ 20	+ 7		
1782, July 13	21	9	19	2	10	54	17	2	12	22 S	+ 53	+ 2	+ 47	+ 3
Nov. 5	22	50	43	6	29	46	25	1	21	11 N	+ 40	+ 3	+ 65	+ 4
1783, Sept. 19	2	4	37	7	3	52	29	6	3	1 S	− 115	+ 39	− 58	+ 24
20	2	2	2	7	4	15	51	6	10	26	− 122	+ 49		
26	1	43	58	7	5	55	9	6	50	19	− 190	+ 55		
Oct. 2	1	22	55	7	6	20	38	7	20	29	− 130	+ 58	− 24	+ 44
1784, May 18	22	29	58	1	7	1	56	1	28	35 S	+ 57	+ 6	+ 44	+ 5
Sept. 8	0	30	11	5	24	45	11	1	20	2 N	+ 27	+ 3		
20	0	37	17	6	9	40	27	1	6	15	+ 41	+ 15	+ 77	+ 12
25	0	39	57	6	15	52	59	0	57	50	+ 15	+ 3		
1785, July 29	20	53	21	2	22	5	27	3	53	17 S	− 40	+ 13	− 13	+ 19
Nov. 27	21	29	57	7	9	30	40	1	37	43 N	− 5	+ 4	+ 26	− 2
1786, June 19	1	43	7	3	21	39	32	1	30	40 N	+ 12	+ 12		
24	1	49	20	3	27	44	41	1	35	52	+ 53	+ 19		
29	1	55	7	4	3	47	42	1	38	54	+ 3	+ 5		
July 1	1	57	17	4	6	13	9	1	39	45	+ 9	+ 7	+ 43	+ 8
14	2	9	7	4	21	54	24	1	37	40	− 6	+ 10		
1788, May 6	3	1	24	3	0	23	44	2	43	15 N			+ 20	+ 22
7	3	2	17	3	1	28	48	2	44	22			+ 27	+ 17
9	3	4	4	3	3	38	1[illegible]	2	46	26			+ 39	+ 14

The angle at the planet is found in the manner before mentioned. The following table contains the heliocentric longitudes and latitudes to the moments of mean time in the foregoing table. The heliocentric place of the node is ascertained with a tolerable degree of accuracy; hence the arc of the ecliptic

from

from the node to the circle of latitude, passing through the planet, is given $= d$; the inclination of the orbit to the ecliptic $= y$ is to be calculated by this formula, cot. $y = \frac{\text{fin. } d \times \text{fin. lat.}}{\text{tang. lat. hel.}}$.

	Heliocentric longitude of ♀ in the ecliptic.	Heliocentric latitude of ♀.	HALLEY's error, in long.	HALLEY's error, in lat.	DE LA LANDE's error, in long.	DE LA LANDE's error, in lat.	Inclination of ♀'s orbit.
	s. ° ′ ″	° ′ ″	″	″	″	″	° ′ ″
1781, July 20	5 0 0 53	3 16 55 N	+ 18	+12			3 23 32
24	5 6 30 59	3 21 27	+ 7	+ 7			3 23 31
30	5 16 15 59	3 23 36	+ 12	+21	+115	+21	3 23 41
31	5 17 53 28	3 23 30	+ 16	+29			3 23 49
Aug. 1	5 19 30 31	3 22 43	+ 3	+ 7			3 23 27
4	5 24 22 35	3 20 41	+ 4	+15			3 23 35
1782, July 13	0 4 2 37	3 11 57 S	+116	+ 4			3 23 26
Nov. 5	6 9 46 4	3 4 18 N	+ 40	+ 5			3 23 27
1783, Sept. 19	11 6 41 21	3 21 45 S	− 97	+23	−183	+23	3 23 44
20	11 8 16 11	3 22 31	−105	+27			3 23 48
26	11 17 45 5	3 23 30	−175	+28			3 23 47
Oct. 2	11 27 16 44	3 18 56	−117	+31	·195	+30	3 23 49
1784, May 18	0 5 34 53	3 10 14 S	+ 71	+13	− 1	+11	3 23 34
Sept. 8	6 6 30 11	3 8 56 N	+ 18	+ 7			3 23 27
20	6 25 51 19	2 23 46	+ 28	+30	+116	+27	3 23 58
25	7 3 52 40	2 13 7	+ 1	+ 7			3 23 33
1785, July 29	11 15 44 9	3 23 30 S	− 68	+12	−154	+12	3 23 32
Nov. 27	6 0 50 45	3 15 22 N	+ 1	+ 2	+ 99	− 2	3 23 22
1786, June 19	4 25 53 12	3 12 43 N	+ 1	+26			3 23 41
24	5 4 2 2	3 20 31	+ 44	+43			3 24 0
29	5 12 8 55	3 23 18	− 5	+11			3 23 30
July 1	5 15 23 58	3 23 33	+ 1	+14	+109	+14	3 23 35
14	6 6 28 2	3 9 10	− 8	+18			3 23 38
1788, May 6	5 16 58 34	3 23 37 N			+ 20	+20	3 23 46
7	5 18 36 3	3 23 12			+ 27	+21	3 23 28
9	5 21 50 56	3 22 4			+ 39	+19	3 23 38
						Mean	3 23 37,7

The

The heliocentric latitudes obſerved 1781 July 30, 1783 September 26, 1785 July 29, 1786 July 1, 1788 May 6, are very near the greateſt latitude; the mean of the inclinations found on theſe days is 3° 23′ 40,″2, and very near the mean of all the obſervations 3° 23′ 37″,7. The inclination, or the greateſt heliocentric latitude, may alſo be found by interpolation of the maximum amongſt the obſerved heliocentric latitudes. This maximum is found 1781 = 3° 23′ 39″, 1783 = 3° 23′ 41″, 1786 = 3° 23′ 36″; the mean of thoſe three maximums 3° 23′ 38″,6, which inclination may be depended upon to 1 or 2 ſeconds. The inclination of the orbit of Venus has been ſuppoſed in the tables of M. Cassini, Dr. Halley, and M. de la Lande, = 3° 23′ 20″, and the error of the tables + 18″,6.

III. *The heliocentric longitude and motion of the nodes of Mars.*

In a Paper printed in the Memoirs of the Royal Academy of Sciences at Stockholm, I have determined the *heliocentric longitude of Mars's aſcending node* = 1 s. 17° 54′ 24″,2, *in the year* 1783, *December* 7, 20 h. 23′ 39″, *mean time at Copenhagen:* the error of M. Cassini's tables − 10′ 35″, of Dr. Halley's tables − 23′ 27″, of M. de la Lande's tables − 4′ 37″. I refer the reader to that Paper (Kongliga Svenſka Vetenſkaps Academiens nya Handlingar, Tom. VI. for the year 1785, p. 285—290.). The annual motion of Mars's node may be found by comparing the following obſervations of the longitude of the node. In the column A the numbers are going upwards from the obſervation 1783; in the column B the numbers are going downwards from the obſervation 1595.

The

The name of the astronomer.	Time of observation.	Heliocentric longitude of ☊ ♂.	Annual motion. A.	Annual motion. B.
		s. ° ′ ″	″	″
TYCHO BRAHE	1595, Oct. 28	1 16 24 33	28,7	
CASSINI	1700, May 6	1 17 13 43	28,9	29,4
CASSINI	1721, Nov. 13	1 17 29 49	23,8	31,3
DE LA CAILLE	1747, May 14	1 17 37 11	27,5	28,9
DE LA CAILLE	1753, Nov. 4	1 17 42 5	24,6	29,6
MASKELYNE	1778, April 17	1 17 51 40	28,5	29,3
BUGGE	1783, Dec. 7	1 17 54 24		28,7
		Mean	27,0	29,5

The mean of the two series A and B will give *the most probable annual motion of Mars's node* 28″,2. In the tables of M. CASSINI the annual motion is 34″, in the tables of Dr. HALLEY 38″, and in the tables of M. DE LA LANDE 40″.

IV. *The inclination of the orbit of the planet Mars.*

Mean time at Copenhagen.		Geocentric longitude of ♂.	Geocentric latitude of ♂.	Error of the tables of M. DE LA LANDE, in long.	Error of the tables of M. DE LA LANDE, in lat.
	h. ′ ″	s. ° ′ ″	° ′ ″	″	″
1788, Jan. 9	11 57 7	3 16 27 7	4 5 25N	+17	+ 1
10	11 51 28	3 16 3 35	4 6 10	+19	+ 7
11	11 45 50	3 15 40 15	4 6 44	+23	+10
26	10 24 50	3 10 43 39	4 3 59	+45	+19
Feb. 14	8 58 52	3 8 13 29	3 39 35	− 8	+ 8
Mar. 9	7 37 35	3 11 13 55	3 1 13	− 7	− 4
12	7 29 7	3 11 59 16	2 56 52	+ 2	+ 7
13	7 26 19	3 12 15 21	2 55 24	+ 2	+ 9
14	7 23 35	3 12 31 44	2 53 56	+ 3	+ 8
16	7 18 10	3 13 5 38	2 50 56	−18	+ 7
April 6	6 27 49	3 20 29 42	2 22 46,7	−17	+ 8

The geocentric longitudes of Mars are corrected for aberration and nutation, and compared with M. DE LA LANDE's

newest

neweſt tables, which after the laſt improvements commonly give the true place of Mars within the fourth part of a minute. The error in longitude +17″ ſignifies that the longitude in the tables is 17″ too ſmall; and that thoſe 17″ are to be added to the calculated longitude, in order to make it agree with the obſerved longitude.

Mean time at Copenhagen.		Heliocentric longitude of ♂.	Heliocentric latitude of ♂.	Error of the tables of M. DE LA LANDE,		Inclination of the orbit of ♂.
				in long.	in lat.	
1788	° ′ ″	s. ° ′ ″	° ′ ″	″	″	° ′ ″
Mar. 9	7 37 35	4 15 12 28	1 50 49,1 N	− 7	− 3	1 50 56
12	7 29 7	4 16 31 42	1 50 53,5	+ 2	− 4,5	1 50 56
13	7 26 19	4 16 58 8	1 50 56,0	+ 2	− 3	1 50 56,4
14	7 23 35	4 17 24 28	1 50 57,7	+ 3	− 1,3	1 50 57,7
16	7 18 19	4 18 16 50	1 50 56,7	− 4	− 2,3	1 50 56,7

The inclination of Mars is taken in the tables of M. CASSINI 1° 50′ 54″, and in the tables of M. DE LA LANDE and Dr. HALLEY 1° 51′ 0″.

I ſhall conclude this Paper with the oppoſition of Mars according to the foregoing obſervations. *The oppoſition of Mars to the ſun happened* 1788, *January* 7, *at* 8 h. 19′ 32″ *true time*; *the apparent geocentric longitude of Mars at that moment* = 3 s. 17° 17′ 8″, *and the geocentric latitude* = 4° 4′ 3′ N. Saturn was in oppoſition to the ſun, Auguſt 29, 20 h. 51′ 11″ true time; the apparent longitude ♄ = 11 s. 7° 31′ 34″, and latitude 1° 59′ 33″ S. The new planet was in oppoſition to the ſun January 18, 0 h. 28′ 33″ true time, the longitude = 3 s. 28° 10′ 7″, and latitude 0° 34′ 35″ N.