# XII. *An Account of the Trigonometrical Operation, whereby the Diſtance between the Meridians of the Royal Obſervatories of* Greenwich *and* Paris *has been determined. By Major-general* William Roy, *F. R. S. and A. S.*

Read February 25, 1790.

## INTRODUCTION.

THE trigonometrical operation which becomes the ſubject of the preſent Paper, had its commencement, as will be remembered, in the meaſurement of a baſe on Hounſlow-Heath in 1784, an account of which was given to the Royal Society in the following year.

On the completion of that firſt part of the buſineſs, it was little expected, that nearly three full years would have elapſed before, even in this country, an inſtrument could be obtained for taking the angles!

In the ſpring of 1787, there were indeed appearances, that Mr. RAMSDEN would have enabled us to embrace the early part of the ſeaſon, by proceeding with the execution of the main deſign; and therefore Sir JOSEPH BANKS had opened (through the official intercourſe of his Majeſty's Secretary of State, the Marquis of CARMARTHEN, with the Ambaſſador at the Court of France) a correſpondence with the Academy of Sciences, regarding the co-operation expected on their part, for connecting the triangles which we were now preparing to extend

extend along the Englifh coaft, with thofe formerly executed on the coaft of France, oppofite to Dover. And Dr. BLAGDEN (who for this purpofe confented to lay afide his intention of making a tour through Germany that fummer) had engaged to affift in the bufinefs, on the appointment of the Royal Society, whenever we fhould be enabled to affign any probable time, for the different parties to repair to their refpective coafts, for the aforefaid co-operation.

About the fame time likewife, a Paper was laid before the Royal Society, intended as a fketch of the mode propofed to be followed in carrying the fcheme into execution; for which purpofe it was accompanied with a general map, fhewing nearly the difpofition of the triangles, and containing alfo various inveftigations concerning the figure of the earth, whereon, it is hoped, the refult of the prefent operation will throw fome additional light.

For feveral months of the fpring and fummer of 1787, Mr. RAMSDEN had been ferioufly at work in endeavouring to finifh the inftrument. Not having employed a fufficient number of workmen upon it at the outfet, it was now evident, that he had even deceived himfelf, by leaving too much to be done at the latter end. At length, however, the inftrument was produced, and placed on the 31ft of July at the ftation near Hampton Poor-houfe, on the very fpot where, about thirty-five months before, the meafurement of the bafe had been compleated.

By commencing an operation of this nature, at fo advanced a feafon of the year, it was fufficiently obvious, that only very faint hopes could be entertained of bringing it to a conclufion before the bad weather would fet in. But it being of much importance to get the triangles, which extend acrofs the

Channel, at all events executed, it was therefore propofed to Comte DE CASSINI, who by this time had been appointed by the Academy of Sciences to fuperintend their part of the bufinefs, that he fhould fix the time that might fuit him beft for our meeting on the coaft; that we would then difcontinue the operation to the weftward, and, having in concert executed the coaft triangles, we would refume the inland parts of our own feries at fome more convenient opportunity.

This propofition being readily acceded to by Comte DE CASSINI, the 20th of September was appointed for our repairing to the coafts of Dover and Calais refpectively.

In the mean time our operation was continued, with all imaginable care and affiduity, through the firft ten ftations of the feries of triangles from Hampton Poor-houfe to that at Wrotham Hill inclufively.

The inftrument, and the various parts of the apparatus, were then removed to Dover, at which place Meff. DE CASSINI, MECHAIN, and LE GENDRE, three diftinguifhed Members of the Academy of Sciences, arrived on the 23d of September.

In the courfe of two days that thefe Gentlemen honoured us with their company at Dover (and we regretted exceedingly that the latenefs of the feafon did not admit of our enjoying that pleafure for a much longer period) every thing was fettled in the moft amicable manner poffible, with regard to the times of reciprocal obfervation.

A great number of white lights, fitted for long diftances, and feveral reverberatory lamps had been previoufly provided. Having been fupplied with fuch a proportion of the lights as feemed neceffary for their fide of the channel, and one of the lamps, the French Gentlemen departed for Calais on the 25th, accompanied by Dr. BLAGDEN, who attended them during the

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time of the co-operation, until it was finally closed on the 17th of October.

For the greater part of the time, the weather was extremely bad; nevertheless, on the particular nights when the most important observations on our side were made, namely, those at Dover and Fairlight Down, the nights happened very fortunately to be favourable, so as to enable us to intersect, with great accuracy, the two distant points on the French coast of *Blancnez* and *Montlambert* *, and thereby to establish for ever, the triangular connection between the two countries.

The Duke of RICHMOND, Master General of his Majesty's Ordnance, had, in the most liberal manner possible, given every assistance to the operation (from that great department over which he presides with so much honour to himself and advantage to the publick) by furnishing an officer and a detachment of artillery-men for the work; ordering the laboratory at Woolwich † to supply whatever fire-works might be wanted for signals; and temporary scaffolds to be erected at Greenwich Observatory, Shooter's Hill, and Dover Castle, for the reception of the instrument. But what was still of more importance than any of these, his Grace had permitted Lieut. FIDDES (one of the engineers on the survey then under my direction) to be employed, in the summers of 1786 and 1787, in making a very accurate plan of that part of Romney Marsh where the base of verification was to be measured. In a country so much intersected by ditches, and where there were so many ponds of water to be avoided, without such a plan raised before-hand,

* The name of this hill is vulgarly pronounced *Boulembcrg*, and it is even written in the same manner in the book, *La Méridienne vérifiée*.

† Major Congreve, of the Royal Artillery, had the management of the lights at Shooter's Hill; and his assistance was found to be most essentially useful.

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an operation of ſo delicate and difficult a nature could not have been effected.

The apparatus for the meaſurement of the baſe with the ſteel chain, notwithſtanding the urgency of the caſe, was not ſent to its deſtination until the end of the firſt week of October. To Lieut. FIDDES the engineer, was then joined Lieut. BRYCE of the Royal Artillery; and it was not before the beginning of December, that theſe two gentlemen, with the moſt unremitting labour and perſeverance, were able to accompliſh the meaſurement, as will be ſeen in the detailed account of that operation given in the firſt ſection of this Paper.

In finiſhing the co-operation with the French Commiſſioners, at Lydd on the 17th of October, our inſtrument had now paſſed through ſixteen ſtations out of twenty-three. There of courſe remained yet ſeven ſtations where it was to be placed, and obſervations to be made. Eagerly wiſhing to bring the buſineſs to a concluſion, we ſtruggled on through five of the ſeven. But the weather at length became ſo tempeſtuous, that it was utterly impoſſible to continue it, with any hopes of being able to make ſatisfactory obſervations. Perched on the tops of high ſteeples, ſuch as Lydd and Tenterden, or on heights, ſuch as Hollingborn Hill, we ſufficiently experienced, that operations of this ſort, where the moſt important obſervations could only be made at night, by means of the white lights, ſhould never be undertaken in the latter ſeaſon.

On the ſecond of November, the inſtrument was accordingly removed from the top of Hollingborn Hill, and ſent to town, leaving the ſtations on Goudhurſt and Frant Churches, both likewiſe ſituated on eminences, unoccupied until the enſuing ſeaſon.

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The winter months were employed in calculating the obſervations that had been made; and from theſe we were very well enabled to judge to what a degree of accuracy we had arrived in determining the ſides and angles: for Frant and Goudhurſt, having been interſected from Botley Hill, Wrotham Hill, and Hollingborn Hill; Goudhurſt having been obſerved from Tenterden, and Frant having (contrary to our expectation) been ſeen and obſerved from Fairlight Down, we had thereby the certain means of determining very nearly what difference there would be between the meaſured and computed length of either baſe as given by the other, although obſervations had not been made at the two intermediate ſtations of Goudhurſt and Frant. This difference, it was ſeen, would ſcarcely amount to one foot, or about $\frac{1}{28000}$th part of the whole diſtance. In as far, therefore, as the reſults of the plane triangles were concerned, we might have proceeded with the computations, and drawn the conſequent concluſions, without heſitation, or any riſk of ſenſible error.

But, beſides that it might ſtill have been ſaid that the inſtrument had not been placed at theſe two ſtations, there were reaſons of a different kind, which rendered it in ſome degree neceſſary to place the inſtrument not only at Goudhurſt and Frant, but alſo at Botley Hill and Folkſtone Turnpike, where it had formerly ſtood.

In 1787, when at the ſtation of St. Ann's Hill, in a very high wind, the box containing the axis level was blown from the ſcaffold, and unluckily broken. Mr. RAMSDEN replaced it with one not ſo good as the firſt; and it was with this ſecond level that the obſervations of the pole ſtar had been made at Dover Caſtle. This caſtle, although lofty, and ſituated on a high chalk cliff, that raiſes its northern turret about 466 feet

above low water at ſpring tides, is nevertheleſs ſurrounded on the land ſide with eminences, at the diſtance of ſix or ſeven miles, ſtill higher than itſelf. From this circumſtance we found it impoſſible to connect it with the great triangles to the weſtward, otherwiſe than by a ſhort ſide. It was therefore ſufficiently obvious, that it would be eligible to make obſervations of the pole ſtar for determining the difference of longitude, and the convergence of the meridians, at ſome other intermediate ſtations between Greenwich and Dover, from whence our longeſt ſides could be diſtinctly ſeen. For this purpoſe none ſeemed ſo proper as Botley Hill, Goudhurſt, and Folkſtone Turnpike. The firſt of theſe three is only 171½ feet eaſtward from the meridian of Greenwich, Goudhurſt is about 23 miles ſouth-eaſtward from the former, and Folkſtone Turnpike, the ſtation neareſt to Dover, is ſo ſituated, that from it can be ſeen the end of the baſe of verification at High Nook, Fairlight Down, and other diſtant ſtations.

With this object in view, whereon ſo much depended, we had again the mortification to be thrown into the latter ſeaſon of 1788.

Beſides a better level for the axis of the teleſcope, the microſcope B wanted to be better ſupported. Another ſort of clamp, alſo an eye-piece, with a diagonal priſm for obſervations near the zenith, or for thoſe of the pole ſtar in high latitudes, were neceſſary improvements, which might have been executed in a ſhort ſpace of time. With theſe alterations the inſtrument was at laſt returned, but ſo late, that it could not be placed on Goudhurſt Steeple till the 9th of Auguſt, 1788.

The obſervations at Goudhurſt, Frant, Botley Hill, and Folkſtone Turnpike, having been finiſhed early in September,

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the inſtrument was brought back to town, in the neighbourhood of which it was employed for three days for the following purpoſe.

In 1787, when at the ſtations of Hundred Acres, Norwood, Greenwich, and Shooter's Hill, we had only been able to determine, in a ſatisfactory manner, two points within the limits of the Capital, namely, St. Paul's Church and Argyll Street, the laſt by means of the white lights. Bearings of ſome others, it is true, were obtained; but, in order that theſe might be interſected in the beſt manner, it became neceſſary to place the inſtrument at one or more ſtations to the northward of the town.

With the view, therefore, of laying the foundation hereafter for a much more accurate plan of London than could poſſibly be obtained in any other way, the inſtrument was placed, firſt, at Hornſey Hill, to the eaſtward of Highgate; and, ſecondly, on Primroſe Hill, between London and Hampſtead.

Although the weather was rather unfavourable at the time of making the obſervations from theſe two new ſtations; and that the ſmoke conſtantly hanging over the town in the latter ſeaſon impeded us greatly; nevertheleſs, the former bearings were interſected, and the ſituations of a conſiderable number of remarkable ſteeples within London and its environs, were accurately determined, as will more fully appear in treating of the ſecondary triangles.

Having thus briefly ſhewn the order with regard to time in which the recent operation, through its various ſteps, was progreſſively carried on and compleated, it is proper that I ſhould mention, that Mr. Dalby, who had been recommended as an aſſiſtant, has acquitted himſelf throughout the whole perfectly to my ſatisfaction, as a diligent and accurate

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obſerver, as well as an able and indefatigable calculator. This teſtimony, which is juſtly due to his merit, joined to the ſpecimen which he gives of his mathematical abilities in the fifth ſection of this Paper, can ſcarcely fail of making him better known hereafter; and it is hoped, that he will have opportunities of exerting his talents, by aſſiſting in the continuation of the future operations that are projected and recommended to be carried on in the concluſion of this Memoir; the various parts of which are arranged as follows:

*Section First.*

Deſcription of the apparatus made uſe of in the meaſurement of the baſe of verification in Romney Marſh, with the hundred-feet ſteel chain, in the autumn of 1787, with the reſult of that operation. Reference to be had to Plate I. and II., and alſo to the table containing the general detail of the meaſurement.

*Section Second.*

General deſcription of the great inſtrument with which the angles in the recent trigonometrical operation were obſerved; ſhewing alſo its various adjuſtments for practice. Reference to be had to Plate III. a general view of the entire machine; Plate IV. a plan and two ſections of it; Plate V. various parts repreſented on large ſcales; and Plate VI. the microſcopes and eye pieces.

*Section Third.*

Deſcription of various articles of machinery made uſe of in the courſe of the trigonometrical operation, referred to in Plate VII. Alſo the diſtinction of the ſtations into two ſets; thoſe of the ſecond ſet being referred to in Plate VIII.

*Section*

*Section Fourth.*

Calculation of the ſeries of triangles extending from Windſor to Dunkirk, whereby the geodetical diſtance between the meridians of the Royal Obſervatories of Greenwich and Paris is determined. Reference to be had to Plate IX.

*Section Fifth.*

On the difference between horizontal angles on a ſphere and ſpheroid. Plate X.

*Section Sixth.*

Manner of determining the latitudes of the ſtations. Application of the pole ſtar obſervations to computations on different ſpheres, and alſo on M. BOUGUER's ſpheroid, for the determination of the difference of longitude. Ultimate reſult of the trigonometrical operation, whereby the difference of the meridians of the Royal Obſervatories of Greenwich and Paris is determined. Plate X.

*Section Seventh.*

An account of the obſervations made during the courſe of the trigonometrical operation for the determination of terreſtrial refraction. Plate X.

*Section Eighth.*

Secondary triangles, ſubdivided into two ſets, for the improvement of the maps of the country, and the plan of the City of London and its environs. Plate XI.

*Concluſion,*

Containing Propoſitions for extending trigonometrical operations over Great Britain.

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## SECTION FIRST.

*Description of the apparatus made use of in the measurement of the base of verification in Romney Marsh, with the hundred-feet steel chain, in the autumn of 1787, with the result of that operation. Reference to be had to Plates I. and II.; and also to the table containing the general detail of the measurement.*

### ARTICLE I. *Preamble.*

IN the account of the measurement of the base on Hounslow Heath in 1784, which appeared in the Philosophical Transactions of the subsequent year, we had occasion to shew, how very accurately distances might be determined by the steel chain, when applied in the ordinary way on the natural surface of the ground, if that surface happened to be tolerably smooth, which was the case in the instance alluded to. By the comparison of the measurement of a length of one thousand feet with the glass rods, and with the chain when used with an apparatus adapted to the purpose, it further appeared, that the difference between the results was so very small as scarcely to be discernible, since it would not have exceeded half an inch on the whole length of that base of 27404.7 feet.

Having always considered the experiment on Hounslow Heath, just now mentioned, as positive proof of the excellency of the chain, it had been resolved on to apply it to the mensuration of the base of verification in Romney Marsh, even if no other reasons had existed to make that choice eligible.

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But befides the danger of having the glafs rods broken, in tranfporting to fo great a diftance from London, and, on fuch an event happening, the impoffibility of getting them replaced with others at the advanced feafon of the year in which we were unfortunately thrown with the operation, it was obvious, that in a plain of the breadth of fix miles, fo much interfected with ditches full of water as Romney Marfh in reality is, the laying of bridges for the tripod ftands, which muft have been ufed with the glafs rods, would alone have been a very troublefome and tedious operation.

ART. II. *Beech Pofts.*

In the firft place, about thirty pofts made of beech wood, three inches in diameter, and of different lengths, from two feet three inches to three feet fix, and a few of them ftill longer, were provided. They were fhod with iron, and each of them carried on its top a caft-iron ferrule, with two dovetails projecting from it; care being taken in driving them into the ground, that the dovetails fhould ftand in or nearly in the direction of the bafe, as reprefented by the plan and fection of a fingle poft, in the middle part of Plate I. The arrangement of twenty-four of thefe pofts may be feen at the top of the faid plate, for the meafurement of a portion of the bafe equal to one hundred yards, or the length of three chains. Sixteen of the pofts reckoning from that which ftands in the center of the firft group, to that which ftands in the center of the fecond, and fo on from right to left, were placed at the diftance of twenty feet from each other. The firft is fuppofed to co-incide with the mouth of the pipe funk into the earth, at the eaftern extremity of the bafe, at a place called High Nook near Dymchurch; and every fifth poft from that towards the left

left marks the end of a chain. The other eight poſts in the arrangement, that is to ſay, the right and left poſts of each of the four groups, are ſuppoſed to ſtand twelve or fifteen inches from thoſe in the center. By referring to the elevation near the top, and the plans and ſection in the middle part of Plate I. it will be perceived, that theſe poſts, together with certain other iron parts of the apparatus fixed to them, hereafter to be deſcribed, ſupport the ends of the coffering for each chain, free and independently of the central poſts, to which laſt the braſs ſcales alone are attached.

## ART. III. *Deal Coffers.*

Fifteen deal coffers, numbered from one to fifteen, were neceſſary for the length of three chains, being five to each. Six of them, that is to ſay, the firſt and fifth, the ſixth and tenth, the eleventh and fifteenth, being the firſt and laſt of each chain, were only nineteen feet four or five inches in length. The other nine, being the three in the middle of each chain, were of the complete length of twenty feet. Theſe coffers perfectly reſembled in ſhape, and nearly in dimenſions, the caſes of the glaſs rods, being ten inches broad in the middle, and uniformly of that depth throughout their whole length. But from the middle they became gradually narrower, in a curvilinear manner, towards each end, where they were only two inches wide. The two cheeks or ſides were about half an inch thick, and the bottom, which entered into a ſhallow groove in the middle of the cheeks, was an inch in thickneſs. Thus the cheeks being thin, bent and applied eaſily to the bottom, to which they were firmly nailed, and the whole was fortified by ſmall blocks of wood faſtened at intervals in the inſide, ſometimes above and ſometimes below the bottom.

From the elevation it will be perceived, that nine or ten inches of the under extremities of the cheeks were cut off, ſo as to permit the bottom itſelf to reſt on the irons. This conſtruction of the coffers was found to anſwer very well, that is to ſay, they were, conſidering their length, not ſo heavy as to be unmanageable, at the ſame time that by their general figure, and particularly the depth of the cheeks, they were entirely prevented from warping.

In addition to the fifteen coffers, juſt now deſcribed, a ſixteenth, not repreſented in the plate, was afterwards prepared at Hythe by Lieut. FIDDES, to be uſed occaſionally, when the end of one chain, and commencement of another, co-incided with a deep ditch or one of the ſewers full of water, and where of courſe it would have been extremely difficult, if not impoſſible, to have fixed ſteadily the group of three poſts in the uſual manner. In this coffer there was a double or falſe bottom, with grooves adapted for the purpoſe; and the braſs ſcale, pulley, &c. were removed from the irons, and placed on this bottom.

ART. IV. *Apparatus of caſt iron, &c. for the ends of the Chain.*

By referring to the plate, where the ſeveral parts of the apparatus for the extremities of the chain are repreſented in plan and ſection, by a ſcale equal to one-fourth of their real dimenſions, it will appear, that the caſt-iron pieces were of two different forms, one long, and the other ſhort; but both applied in the ſame manner, on the ferrules binding the tops of the poſts, as has been already mentioned. Of the long kind there were in all fifteen or ſixteen, that is to ſay, one for each poſt in a length of three chains. Each iron had two clamps on its

under ſide, which being ſlackened, it was placed on its ferrule at right angles to the line of meaſurement; and being turned round 90°, the dovetails of the ferrule, ſtanding originally in the direction of the baſe, came within the clamps, which were then tightened by four ſcrews, turned with ſquare keys adapted to the purpoſe.

It is ſufficiently obvious, that ſo many irons, with ſuch a number of ſcrews to each, could not fail of rendering this operation tedious! The buſineſs would have been greatly expedited if there had been only two ſuch ſcrews, one on each end, in a middle ſituation; and, inſtead of the four ſcrews, there ſhould have been four ſteady pins, entering eaſily into holes prepared for them in the under ſide. A ſhort groove, of two or three inches in length, in each extremity of the bottom, would, on this ſuppoſition, have been neceſſary to ſuffer the ſquare heads of the ſcrews to paſs; and it will be readily conceived, that the thickneſs of the bottom would have effectually ſecured the chain from touching them, prevented the mutilation of its handles, and ſaved much loſs of time. Indeed the ſame purpoſe might have been effected, but not ſo advantageouſly, by laying the original four ſcrews lower in the iron, which its thickneſs eaſily admitted of. Finally, in order to avoid ſuch like inconveniencies in future, there is ſtill one imperfection more, which it is incumbent on me to remark, namely, that caſt-iron ferrules will not anſwer; for the force that was found to be neceſſary to drive the poſts into the ground, burſt almoſt the whole of them, ſo that before the operation was compleated, they were obliged to be replaced with others made of hammered iron, forged for the purpoſe.

Of the ſhort irons only three were neceſſary, one for each end of the chain, and a ſpare one in caſe of accident. They

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were placed, turned, and clamped on the ferrules, in all refpects fimilarly with thofe of the long kind. By infpection of the plate it will be feen, that each of them carried on its furface a brafs fcale of fix inches in length, divided into inches and quarters, and moveable in a flide, either backwards or forwards, by a finger-fcrew adapted to the right-hand end.

The right-hand poft of each group is called the *drawing-poft*, becaufe the iron fixed on its top carries a fmall apparatus of brafs, which being connected with the flat iron rod and hooks formerly ufed at Hounflow Heath, for a like purpofe, lays hold of the rear handle of the chain, and draws it back until zero co-incides with the point of commencement. The left-hand poft in each group is called the *weight-poft*, becaufe it carried a brafs pulley, over which a weight of 28 lbs. was hung by a fmall rope attached to the hooks that laid hold of the front handle of the chain. This weight acting againft the force of the fcrew at the other end, the chain was thereby kept perfectly ftraight in the coffers, and conftantly in the fame degree of tenfion, until fome certain divifion (the neareft for inftance) of the fcale could be brought, by means of the fcrew, accurately to coincide with 100 feet at the front end. That divifion, whatever it might be, was of courfe regiftered in the field book of the operation, together with the true temperature of the chain, as fhewn by five thermometers, one being laid for that purpofe in each coffer, and fecured with white cloth from the fun's rays, as occafion might require.

Fifteen coffers were always arranged on the ground at the fame time, comprehending a fpace of the bafe equal to the length of three chains or 100 yards. The extremities of the firft chain having been accurately transferred, in the manner above mentioned, to the brafs fcales on the tops of the central

2 pofts,

posts, and these remaining firm and motionless, as being wholly unconnected with any other parts of the apparatus, the chain was then moved forward into the second set of coffers, where the thermometers were also placed. In the mean time, the first set of coffers now vacated, with their posts, &c. were carried on and arranged in the front, for the measurement of the second 100 feet; and so on continually with the others in succession.

### Art. V. *Of the survey of Romney Marsh previously to the measurement of the Base.*

In the introduction to this Paper it has been mentioned, that the Duke of Richmond had permitted Lieut. Fiddes, of the Royal Engineers, to be employed in 1786 and 1787 in raising a plan of that part of the Marsh where, on examination, it should be found, that the base of verification might be the best executed. In justice to that officer, I consider it as incumbent on me to say, that it was impossible for any person to fulfil the duties entrusted to him better than he did, either in the course of the survey, or subsequent measurement of the base, whereof he also had the direction. The general instructions given to him were, that after having by a base of his own determined certain triangles in the neighbourhood of Dymchurch, Ruckinge, and Romney, by way of foundation for his work, he should preserve Ruckinge as the point whereon the *allignement* of the great base was to be directed, and vary the position of that end next the sea-wall in such a manner as to meet with the fewest local obstructions to the measurement between the two extremities. By inspection of the plan Plate II. which comprehends a tract of country of two miles in breadth, one on each side of the base line, it will be perceived, that

that besides the numberless ditches with which this singular plain is intersected, and which it was impossible to avoid crossing, there is almost in every field a watering pond for the cattle, many of them of considerable depth. Nevertheless, so very attentive had Mr. FIDDES been to the accuracy of his survey, that he was enabled, after several trials of other directions, to run a line from High Nook on Dymchurch Sea-wall, upon the small spire of Ruckinge Church, of the length of nearly six miles, without interfering with any one of the watering ponds, or meeting with any other local obstruction of consequence. So very minute was he in his remarks, and so accurate in the situation of particular trees, that in tracing his line with the telescope, he managed so as to avoid them all, a few insignificant bushes excepted; which I believe to be an instance of exactness scarcely to be equalled.

### ART. VI. *Pipes sunk in the ground.*

Permission having previously been obtained from the proprietors of the soil, pipes were sunk into the ground at the two extremities of the base, and also one on Allington Knoll, which last point with Lydd Church * form that side of one of the great triangles depending on the base on Hounslow Heath,

* It will be perceived, that several of the names of places differ in their orthography, from that whereby they were expressed in the plan of the intended triangles given in the Paper of 1787. This has been done, on procuring better information in that respect than had formerly been obtained. Mr. COBB, of Lydd, an ingenious gentleman, well acquainted with Romney Marsh, was so obliging as to present me with a manuscript map of that singular plain, compiled by himself from actual surveys, where the names and boundaries of the *waterings*, and many other curious particulars, are very distinctly expressed.—Our plan of the base has therefore derived advantage by adhering to such respectable authority.

Heath, to be firſt verified by the meaſurement of this new baſe. Every field is ſurrounded with a ditch, in cleaning of which the earth and mud are continually thrown out on each ſide, whereby flat dykes are gradually formed on either ſide. That the meaſurement might be carried on as nearly as poſſible in the ſame plane, that is to ſay, about fifteen or eighteen inches above the common ſurface, therefore, narrow grooves were cut in theſe flat dykes, which the different farmers readily conſented to without murmuring. Here it is to be obſerved, that there was no occaſion for levelling the line, Romney Marſh having been formerly covered by the ſea, and a conſiderable part of it, particularly towards the bottom of the range of hills that ſeparate it from the Wealds of Kent, being ſtill lower than the ſea at high water, would again be overflowed by it, if much care and expence were not annually beſtowed in ſecuring and repairing the dykes, whereby it is protected. Thus the line of the baſe may be conſidered as an inclined plane, deſcending gradually about five feet from the mouth of High Nook pipe to within 246 yards of the Ruckinge end, where the ground in that direction ſeems to be the loweſt. Thence it riſes comparatively ſuddenly, about fifteen feet, to the mouth of the pipe ſituated in a ſmall field immediately adjoining to Ruckinge Church-yard.

### ART. VII. *Reſult of the meaſurement.*

Lieut. FIDDES, in the courſe of his trigonometrical ſurvey, and of the different meaſurements he had actually made of the line with a common iron chain, which from time to time was compared with ſtandard rods of deal, had determined the total length of the baſe within a few feet of the truth, before the ultimate operation began. He had likewiſe driven into the ground, at the end of every

 thouſand

thousand feet, a strong picket, which were numbered 1, 2, 3, &c. from the pipe at High Nook to the 28th near Ruckinge. In all this preparatory part of the business he had no other assistants than the artillery-men of his surveying party. But for the ultimate determination, it being absolutely necessary that he should have the aid of some person in whom he could confide for the management of the operation in general, and particularly for the adjustment of the scale at one end of the chain, while he himself was adjusting that at the other; therefore Lieut. Bryce, of the Royal Artillery (now of the Corps of Royal Engineers), an attentive officer and excellent mathematician, was left with him for those essential purposes. These two gentlemen began the operation on the 15th of October, and, after experiencing many difficulties arising from the badness of the weather in that late season of the year, and the defectiveness of the apparatus, it was only by dint of great labour, and the utmost perseverance, that they were enabled to accomplish the measurement on the 4th of December following.

The annexed general table of the base, which contains five columns, shews the progress that was made in the work from day to day. The first column contains the date; the second, the spaces measured each day, reckoned by hundreds of yards, and denoted in the general plan by strong dots; the third shews the temperature of the chain deduced from the mean of fifteen thermometers, five for each chain; the fourth expresses the difference of temperature above or below 62° of Fahrenheit; and the fifth shews the correction answering to that difference, additive to the apparent length with the sign +, and subtractive from it with the sign −.

Feet. In. Pts.

From inſpection of the table it will appear, that the total apparent length of the baſe, as given directly by the ſteel chain, was 9512$\frac{2454}{10000}$ yards; which are equal to 28536 . 8.835

But when the new points, at the diſtance of twenty-five feet from each other, were laid off on the chain in Mr. RAMSDEN's ſhop from the original points on the great plank of New-England deal, the temperature was 55°, that is, 7° below 62°; wherefore the contraction of the chain by 1° of FAHRENHEIT being = 0.00763 in. this × 7° × 285.37 chains = 15.242 in. is the reduction for the total contraction below 62°, to be taken from the apparent length; which are equal to - - - - 1 . 3.242

The apparent length is likewiſe to be leſſened by the exceſs of the corrections with the ſign − above thoſe with the ſign + in the annexed table; becauſe the temperature of the chain, when actually applied to the meaſurement, being ſo much below 62°, the apparent length became thereby too great by 30.65 inches, which are equal to 2 . 6.65

To be deducted alſo from the apparent length, the reduction on two hypothenuſal diſtances, meaſured at the Ruckinge extremity of the baſe, which is ſuddenly elevated above the loweſt part fifteen feet, amounting to - - 0 . 3.023

The ſum of theſe three reductions, to be taken from the apparent length, amounts to - 4 . 0.915

| | Feet. In. Pts. |
|---|---|
| And confequently there remains for the length - - - - | 28532 . 7.92 |
| But when the chain was adjufted in Mr. RAMSDEN's fhop, as above-mentioned, the temperature was 55°. Being then carried into St. James's Church-yard, its length was laid off on brafs pins inferted into the ftone coping of the church-yard wall, for the purpofe of comparifon on any future occafion, at which time the temperature had changed to $55°\frac{1}{2}$. After the meafurement in Romney Marfh had been finifhed, the chain in the temperature of 39°, being ftretched out on the wall, its length was found to fall fhort of the original points on the brafs pins $\frac{103}{1000}$ of an inch. Now, $55°.5 - 39° = 16°.5$, and $16°.5 \times 0.00763 = 0.126$ in.; hence $0.126 - \frac{103}{1000} = 0.023$ in. is the fpace which the chain had only lengthened during an operation which continued above fix weeks; and one-half of this fpace, *viz.* 0.0115 multiplied by 285.37 chains is = 3.282 in., the correction to be added to the apparent length for the wear of the chain during the operation - - - | +0 . 3.282 |
| Whence the length becomes - | 28532 . 11.202 |
| Laftly, the bafe is to be fhortened for its height of $15\frac{1}{2}$ feet above the mean level of the fea, fuppofed to be 6 feet 8 inches above low-water fpring tides at High Nook, which gives for the reduction - - - | −0 . 0.166 |

And

Feet. In. Pts.

And hence there remains for the ultimate or true length of the base of verification, in the temperature of 62° of FAHRENHEIT's thermometer, being the heat to which that on Hounslow Heath was reduced, - - 28532. 11.036

Which make 28532.92 feet.

ART. VIII. *Remarks on the comparative accuracy of the two bases.*

With regard to the accuracy of the measurement of this base, compared with that executed on Hounslow Heath in 1784, from the infinite pains and care bestowed in both operations, it is very difficult to say, to which the preference should be given. The expansion of glass being so much less than that of steel, if manageable glass rods of equal length with the chain could have been obtained; then, as far as that single circumstance might have affected the result, a measurement made with such glass rods would undoubtedly have deserved the preference to one with the steel chain. But when it is considered, that the expansion of steel was determined by the pyrometer with the same care as that of glass; that the wear of the chain is so very small, as we have shewn it to be, in six weeks use; that coffers were laid for it, and its length transferred by means of the brass scales to the tops of immovable posts; that, in the present case, there was but one-fifth part of the error arising from faulty co-incidences as with the twenty-feet glass rods; on this view of the matter, the preference seems to be due to the measurement by the steel chain, supposing always the error in excess, caused by the deviation from the *allignement* horizontally or vertically, to have affected both equally.

As

As a proof that the expanſion of the chain was accurately determined, I ſhall cloſe this ſection with a remark repeatedly made by the two gentlemen entruſted with the execution of this laſt meaſurement. At the cloſe of each day's work, the two ſcales marking the extremities of the laſt chain (after regiſtering the diviſions of co-incidence) were left upon their reſpective poſts until the next morning. They were ſecured during the night, from being diſturbed by cattle, with a certain number of the ſpare poſts driven into the ground around them. A tent was alſo pitched between the two, where ſome men of the party conſtantly lay, by way of a guard for the whole apparatus. On the recommencement of the operation the ſubſequent morning, the chain being applied anew to the braſs ſcales; if the temperature continued the ſame, the co-incidences were found to be equally accurate as on the preceding night; but if it had changed one or two degrees, the chain never failed unequivocally to ſhew it, by falling ſhort of the diviſions on the ſcales, if the cold had increaſed, or by over-reaching them if it had diminiſhed.

Finally, with reſpect to the ſubject of theſe baſes, it is here to be remarked, that the baſe of verification in Romney Marſh makes with the meridian of the pipe at High Nook an angle of $54^\circ\ 28'\ 56''\frac{1}{2}$ north-weſtward; and *that* on Hounſlow Heath makes with the meridian of the pipe at Hampton Poor-houſe an angle of $44^\circ\ 41'\ 49''$, alſo north-weſtward.

GENERAL

GENERAL TABLE of the Meafurement of the BASE of VERIFICATION in ROMNEY MARSH,
to be $9512\frac{2454}{10000}$ Yards, and the true, or corrected Length i

| Days. | Spaces meafured. Yards. | Temperature. Mean by 15Therm. | diff. from 62°. | Correction for the difference. | Days. | Spaces meafured. Yards. | Temperature. Mean by 15Therm. | diff. from 62°. | Correction for the difference. | Days. | Spaces meafured. Yards. | Temperature. Mean by 15Therm. | diff. from 62°. | Corre for t differ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Oct | | ° | ° | In. Parts. | Oct. | | ° | ° | In. Parts. | Nov. | | ° | ° | n. P |
| 15 | 100 | 54.7 | − 7.3 | 0.16710 | | 2100 | 65.0 | + 3.0 | +0.06867 | | 4100 | 55.2 | − 6.8 | 0.1 |
| 16 | 200 | 62.7 | + 0.7 | +0.01602 | | 2200 | 64.1 | + 2.1 | +0.04807 | 10 | 4200 | 55.3 | − 6.7 | 0.1 |
| | 300 | 61.3 | − 0.7 | 0.01602 | | 2300 | 56.7 | − 5.3 | 0.12132 | | 4300 | 53.6 | − 8.4 | 0.1 |
| 17 | 400 | 57.0 | − 5.0 | 0.11445 | 30 | 2400 | 58.7 | − 3.3 | 0.07554 | | 4400 | 49.0 | − 13.0 | 0.2 |
| | 500 | 52.2 | − 9.8 | 0.22432 | | 2500 | 59.5 | − 2.5 | 0.05722 | 12 | 4500 | 50.1 | − 11.9 | 0.2 |
| | 600 | 53.6 | − 8.4 | 0.19228 | 31 | 2600 | 57.3 | − 4.7 | 0.10758 | | 4600 | 47.9 | − 14.1 | 0.3 |
| 20 | 700 | 46.8 | − 15.2 | 0.34793 | | 2700 | 54.6 | − 7.4 | 0.16939 | 13 | 4700 | 44.7 | − 17.3 | 0.3 |
| | 800 | 58.9 | − 3.1 | 0.07096 | Nov. | 2800 | 53.9 | − 8.1 | 0.18541 | | 4800 | 44.8 | − 17.2 | 0.3 |
| 23 | 900 | 53.9 | − 8.1 | 0.18541 | 1 | 2900 | 49.0 | − 13.0 | 0.29757 | | 4900 | 41,3 | − 20.7 | 0.4 |
| | 1000 | 55.3 | − 6.7 | 0.15336 | | 3000 | 54.0 | − 8.0 | 0.18312 | 14 | 5000 | 41.8 | − 20.2 | 0.4 |
| 24 | 1100 | 55.7 | − 6.3 | 0.14421 | | 3100 | 50.9 | − 11.1 | 0.25408 | | 5100 | 42.9 | − 19.1 | 0.4 |
| | 1200 | 50.0 | − 12.0 | 0.27468 | 2 | 3200 | 49.1 | − 12.9 | 0.29528 | 15 | 5200 | 45.3 | − 16.7 | 0.3 |
| | 1300 | 55.2 | − 6.8 | 0.15565 | | 3300 | 50.4 | − 11.6 | 0.26552 | | 5300 | 44.1 | − 17.9 | 0.4 |
| 26 | 1400 | 59.1 | − 2.9 | 0.06638 | | 3400 | 48.5 | − 13.5 | 0.30901 | | 5400 | 40.4 | − 21.6 | 0.4 |
| | 1500 | 60.0 | − 2.0 | 0.04578 | | 3500 | 42.6 | − 19.4 | 0.44407 | 16 | 5500 | 41.5 | − 20.5 | 0.4 |
| 27 | 1600 | 59.1 | − 2.9 | 0.06638 | 5 | 3600 | 52.3 | − 9.7 | 0.22203 | | 5600 | 44.8 | − 17.2 | 0.3 |
| | 1700 | 63.1 | + 1.1 | +0.02518 | | 3700 | 53.0 | − 9.0 | 0.20601 | | 5700 | 44.6 | − 17.4 | 0.3 |
| | 1800 | 68.1 | + 6.1 | +0.13963 | | 3800 | 52.4 | − 9.6 | 0.21974 | 17 | 5800 | 40.6 | − 21.4 | 0.4 |
| | 1900 | 57.9 | − 4.1 | 0.09385 | | 3900 | 47.3 | − 14.7 | 0.33648 | | 5900 | 39.4 | − 22.6 | 0.5 |
| 29 | 2000 | 60.8 | − 1.2 | 0.02747 | 7 | 4000 | 55.6 | − 6.4 | 0.14650 | | 6000 | 41.3 | 20.7 | 0.4 |
| | | | | −2.16540 | | | | | −3.77913 | | | | | −7.5 |

From the above Table it appears, that the total apparent length of the Bafe, as given imm
The corrections in the above Table, and others fpecified in the Text, being fubtracted fr

There remains, for the true length of the bafe in the temperature of 62° of Fahrenheit,

ARSH, executed in the Autumn of 1787, whereby the apparent Length is found
ngth in the Temperature of 62°, 28532 $\frac{92}{100}$ Feet.

| — 2°. | Correction for the difference. | Days. | Spaces measured. Yards. | Temperature. Mean by 15 Therm. | Temperature. diff. from 62°. | Correction for the difference. | Days. | Spaces measured. Yards. | Temperature. Mean by 15 Therm. | Temperature. diff. from 62°. | Correction for the difference. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | n. Parts. | | | ° | ° | In. Parts. | | | ° | ° | In. Parts. |
| .8 | 0.15565 | | 6100 | 42.1 | −19.9 | 0.45551 | | 8100 | 36.3 | −25.7 | 0.58827 |
| .7 | 0.15336 | Nov. | 6200 | 39.3 | −22.7 | 0.51960 | | 8200 | 39.2 | −22.8 | 0.52189 |
| .4 | 0.19228 | 21 | 6300 | 43.3 | −18.7 | 0.42804 | Nov. | 8300 | 39.5 | −22.5 | 0.51502 |
| .0 | 0.29757 | | 6400 | 46.5 | −15.5 | 0.35479 | 29 | 8400 | 34.8 | −27.2 | 0.62261 |
| .9 | 0.27239 | | 6500 | 45.6 | −16.4 | 0.37540 | | 8500 | 40.5 | −21.5 | 0.49213 |
| .1 | 0.32275 | 22 | 6600 | 42.5 | −19.5 | 0.44635 | | 8600 | 42.3 | −19.7 | 0.45093 |
| .3 | 0.39600 | | 6700 | 42.2 | −19.8 | 0.45322 | 30 | 8700 | 30.5 | −31.5 | 0.72103 |
| .2 | 0.39371 | | 6800 | 41.2 | −20.8 | 0.47611 | | 8800 | 44.3 | −17.7 | 0.40515 |
| .7 | 0.47382 | 23 | 6900 | 39.8 | −22.2 | 0.50816 | Dec. | 8900 | 46.0 | −16.0 | 0.36624 |
| .2 | 0.46238 | | 7000 | 39.0 | −23.0 | 0.52647 | 1 | 9000 | 43.0 | −19.0 | 0.43491 |
| .1 | 0.43720 | | 7100 | 37.7 | −24.3 | 0.55623 | | 9100 | 45.6 | −16.4 | 0.37540 |
| .7 | 0.38226 | 24 | 7200 | 36.2 | −25.8 | 0.59056 | | 9200 | 48.7 | −13.3 | 0.30444 |
| .9 | 0.40973 | | 7300 | 42.1 | −19.9 | 0.45551 | 3 | 9300 | 41.1 | −20.9 | 0.47840 |
| .6 | 0.49442 | | 7400 | 40.5 | −21.5 | 0.49213 | | 9400 | 46.9 | −15.1 | 0.34564 |
| .5 | 0.46924 | 26 | 7500 | 35.2 | −26.8 | 0.61345 | 4 | 9512.2454 | 48.4 | −13.6 | 0.34942 |
| .2 | 0.39371 | | 7600 | 39.8 | −22.2 | 0.50816 | | | | | |
| .4 | 0.39829 | | 7700 | 38.5 | −23.5 | 0.53791 | | | | | − 6.97148 |
| .4 | 0.48985 | 27 | 7800 | 33.6 | −28.4 | 0.65098 | | | | | −10.14802 |
| .6 | 0.51731 | | 7900 | 38.9 | −23.1 | 0.52876 | | | | | − 7.58574 |
| .7 | 0.47382 | 28 | 8000 | 32.7 | −29.3 | 0.67068 | | | | | − 3.77913 |
| | | | | | | | | | | | − 2.16540 |
| | −7.58574 | | | | | −10.14802 | | | | | |
| | | | | | | | | Total Correction | | | −30.64977 |

| | Feet. | In. |
|---|---|---|
| ven immediately by the Steel Chain, was 9512 $\frac{2454}{10000}$ yards, which are equal to | 28536 | 8.835 |
| racted from the apparent length, — — — | 3 | 9.799 |
| enheit, — — — — — — | 28532 | 11.036 |
| equal to — | 28532 $\frac{92}{100}$ Feet. | |

SECTION

## SECTION SECOND.

*General Deſcription of the great inſtrument with which the angles, in the recent trigonometrical operation, were obſerved; ſhewing alſo its various adjuſtments for practice. Reference to be had to Plate III. a general view of the entire machine; Plate IV. a plan and two ſections; Plate V. various parts repreſented to large ſcales; and Plate VI. the microſcopes and eye-pieces.*

### ARTICLE I. *Preamble.*

IN endeavouring to deſcribe the curious inſtrument made uſe of for obſerving the angles in the recent trigonometrical operation, it has been judged beſt to confine ourſelves to the principal parts, without entering into any detail of the *minutiæ*: for even to have mentioned theſe, with the almoſt infinite number of little ſcrews that ſerve to unite them into one entire machine, which could only have been done by references to a multitude of great and ſmall Roman and Greek characters, would have been a diſguſting labour. By the help of the four plates which this deſcription refers to, and which have been executed with great care, that fewer words might ſuffice, it is hoped, that the inſtrument may be underſtood by two claſſes of people for whom it is chiefly intended; firſt, by thoſe who being poſſeſſed of ſuch a machine would wiſh to make themſelves maſters of its uſe; and, ſecondly, by ſuch ingenious artiſts as would attempt to conſtruct ſuch another; for theſe laſt, in particular, the parts that are of braſs, of bell-metal,

or

or of ſteel, are diſtinguiſhed from each other. And here it is neceſſary to obſerve, that the plates muſt not only be frequently conſulted, but alſo attentively conſidered, and repeatedly compared with each other, in the courſe of this deſcription.

Art. II. *General view of the inſtrument.*

It is a braſs circle, three feet in diameter, and may be called a great theodolet, rendered extremely perfect; having this advantage in particular, which common theodelets have not, that its tranſit teleſcope can be nicely adjuſted by inverſion on its ſupports; that is to ſay, it can be turned upſide down, in the ſame manner that tranſit-inſtruments are, in fixed obſervatories.

The circle is attached by ten conical tubes, as ſo many *radii*, to a large vertical, conical hollow axis of twenty-four inches in height, which may be called the exterior axis. Within the baſe of this hollow axis, a collar of caſt ſteel is ſtrongly driven; and on its top there is inſerted a thick bell-metal plate, with ſloping cheeks, which, by means of five ſcrews, can be raiſed or depreſſed a little.

The inſtrument reſts on three feet, which are firmly united to each other at the place where they branch off, by a ſtrong circular plate of bell-metal, upon which riſes another vertical hollow cone, of leſs ſize than the former, being included within it, and is therefore called the interior axis. On its top is inſerted a caſt-ſteel pivot, with ſloping cheeks, paſſing through the bell-metal plate on the top of the exterior axis, the cheeks of the one being nicely ground to fit the cheeks of the other. The bell-metal baſe of this interior axis is in like manner ground to fit the caſt-ſteel collar in the baſe of that which is without it. Thus the circle being lifted up by two

men

men laying hold of its *radii*, and the exterior being placed upon the interior axis, the cheeks at the top being at the ſame time adjuſted to their proper bearing, it turns round very ſmoothly, and is perfectly, or at leaſt as to ſenſe, free from any central ſhake. This mode of centering is one of the chief excellencies of the inſtrument. From the uſe that has been made of it both years, it ſeems not to have ſuffered in the leaſt; and it is perhaps the only conſtruction that could have anſwered for a machine of ſuch magnitude, undergoing ſo many quick tranſitions from place to place, and ſo often raiſed to high ſituations without any riſk of being thereby hurt.

ART. III. *Mahogany Planes under the inſtrument.*

By inſpection of the plates, but more particularly the IIId, and the ſection towards the right hand in the Vth, it will be ſeen, that there are three planes of mahogany under the metal parts of the inſtrument; namely, that which forms the top of the ſtand, which, although a ſquare of about three feet four inches at bottom, becomes, by the ſeparation of the legs, an octagon at top. In the center there is a circular opening of nine inches diameter, the uſe of which will appear hereafter. Over the top of the ſtand lies another plane of mahogany, likewiſe an octagon, of ſomewhat greater dimenſions than the former, with a circular curb running around it, about half an inch within the planes of its ſides. This octagon hath in its center an open conical ſocket of braſs, three inches in diameter; and on four of its oppoſite ſides there are fixed four ſtrong braſs ſcrews, one on each ſide, which acting againſt pieces of braſs inlaid into the oppoſite ſides of the top of the ſtand, the octagon plane, with every thing that reſts on it, may thereby be moved in four oppoſite directions, until the plummet ſuſ-

pended from the center of the inftrument above, is accurately brought to co-incide with the point marking the ftation underneath. The third or uppermoft plane of mahogany is in fact a part of the inftrument itfelf, being at all times by fcrews or otherwife united to it, and carrying the handles whereby it is lifted out for ufe, or in again into its cafe, to be tranfported from place to place. In the middle of this plane or bottom to the inftrument, there is another conical brafs focket, of three inches and a quarter in diameter, fitted to flip over and turn eafily on that in the center of the octagon underneath. In the brafs cover of this focket, there is a very fmall hole concentric with the inftrument, to fuffer the thread or wire to pafs, which fufpends the plummet; and in the view, Plate III. may be feen another fmall box that contains the thread, with a winch-handle for raifing or lowering the plummet, according as the height of the inftrument above the ftation on the ground, or edifice where it ftands, may require.

## Art. IV. *Feet Screws for levelling the inftrument.*

By attending to the group reprefenting the front elevation of the feet fcrews, with its fide nuts, in the right hand upward angle of Plate V. it will appear, that they are flackened, which is always the cafe before the inftrument is levelled, to give room for that operation by the action of the fcrews. This being done, the fide nuts are brought to prefs gently on the horizontal plate that embraces the whole group, and thereby keeps the inftrument as it were united to the mahogany until fome frefh adjuftment becomes neceffary. When the inftrument is to be put into its cafe, then the feet are let down, and by the fide nuts the horizontal plate is brought to prefs ftrongly

on

on the whole group, whereby it is kept perfectly fast and secure from motion in carrying from one situation to another.

ART. V. *Blocks of box wood and conical rollers under the feet screws.*

By referring also to Plate V. it will appear, that directly below each foot there is fixed to the lower surface of the mahogany a small block of box wood, curvilinear in the direction of its motion. On these three blocks rests the whole weight of the instrument, which nevertheless can be moved circularly on them alone. But to render the motion perfectly easy, three conical brass rollers, placed somewhat nearer to the center, are, by means of their respective springs and regulating screws, brought to act and receive such a proportion of the weight as it may be necessary to lay upon them. The head of one of these screws, which give more or less action to the rollers, may be seen at D in the principal view of the instrument Plate III. as well as in the plan and section Plate V.

ART. VI. *Screws giving motion to the whole instrument.*

By examining attentively the general view of the instrument may be seen, in two positions, the great screw with the flat ivory head, whereby the entire machine received a circular motion. In one, it is attached to the curbs, as when in use in 1787; in the other, it is laid upon the mahogany bottom, as was the case the same year every time it was carrried to a new situation. But this ivory-headed screw having been found to act by jerks in moving so great a weight, and consequently to be troublesome in adjusting the instrument to the fixed point, or that of commencement in measuring angles; it was therefore laid aside in 1788, and another apparatus or clamp was

 adapted

adapted for the ſame purpoſe. This laſt may be ſeen attached to the curbs, as repreſented towards the right hand of Plate V. It conſiſts of a braſs cock, fixed to, and projecting outwards from, the curb of the inſtrument; which cock is acted upon by two ſcrews working on the oppoſite ſides againſt it, and which are clamped to the curb of the octagon.

ART. VII. *Mahogany Baluſtrade and Cover.*

The curb, whereon the three feet of the inſtrument reſt, carries a baluſtrade of mahogany fitted to receive, on the top thereof, a mahogany cover, no where repreſented except in the two ſections in Plate IV. In this cover there are only four ſmall openings (beſides that which allows the great vertical axis to paſs), *viz.* one for each vertical microſcope, one for the clamp of the circle, and one for the ſocket of the Hook's-joint. The two laſt are leſs than the former. At the ſame time that this cover effectually ſecures the circle with its cones from dirt and from accidents, it ſerves conveniently for laying the Hook's-joint upon, or any thing that may be conſtantly wanted near at hand; but more particularly for placing the lanterns uſed at night for reading off the diviſions on the limb of the inſtrument that come immediately under the vertical microſcopes.

ART. VIII. *Achromatic Teleſcopes.*

Two achromatic teleſcopes, each of thirty-ſix inches focal length, with double object-glaſſes of two inches and a half aperture, belong to the inſtrument. They are excellent of their kind, and are furniſhed with eye-pieces of different magnifying powers, for erect as well as inverted viſion. The lower teleſcope lies exactly under the center of the inſtrument, and is directed through one of the openings of the baluſtrade. Being only

only uſed for terreſtrial objects, it requires but a ſmall elevation or depreſſion, and therefore is only ſupplied with a ſhort axis of ſeventeen inches in length, ſupported by braces attached to the feet. The eye end of this teleſcope is purpoſely made heavier than the object end; and reſting on an horizontal arm, that is raiſed or depreſſed by rack-work, it is thereby readily brought to bear, and remain very ſteadily, upon its object. The rack-work may be ſeen in the view of the inſtrument, and alſo on the left ſide of the right hand ſection in Plate IV. But there is a ſmall horizontal motion that can be given to the right hand end of the axis of this teleſcope, which is effected by means of a handle inſerted through the vacancy of the baluſtrades, and placed on a dovetail at E, which could not be ſhewn in the plate. Thus the inſtrument being nicely levelled, the upper teleſcope at zero, and likewiſe on its object, the lower teleſcope, by the help of this adjuſtment, is brought accurately to the ſame object, ſuppoſed to be the point of commencement, or that from which angles are meaſured.

By referring to Plates III. and IV. and likewiſe to the ſection on the left ſide of Plate V. it will be ſeen, that a horizontal bar extends acroſs the top of the vertical axis, ſupported by two ſide braces that ſpring from the cone,. about one-third of its height above the plane of the inſtrument. The horizontal bar carries the Y's or ſupports, in which the pivots of the upper teleſcope move. They are of ſuch height as to permit a ſemicircle of ſix inches radius, attached to the axis of the tranſit, to paſs freely, and conſequently the teleſcope to be directed to the ſun or ſtars in high elevations, but not to be brought to the zenith. The arc of exceſs of the ſemicircle likewiſe admits of ſeveral degrees of depreſſion being meaſured thereon.

ART.

ART. IX. *Spirit Levels.*

The inſtrument has two very good ſpirit levels, that are fitted with the ſeveral means of adjuſtment, as is uſual in ſuch caſes, the detail of which it is unneceſſary here to enter into. The firſt or axis level, becauſe it is only applied on the axis of the teleſcope, is that whereby it is ſet horizontal, as in the ordinary tranſit inſtrument; and it is likewiſe uſed for placing the conical axis truly vertical, ſo that the inſtrument may turn round without ſenſible alteration of the level, previouſly to obſervations of the pole ſtar, or of other heavenly bodies.

The ſecond, or elevation level, is that whereby the teleſcope is brought to be truly horizontal, when angles of elevation or depreſſion are to be taken. At ſuch times it is ſuſpended on a rod attached to the outſide of the teleſcope, to whoſe axis of viſion the rod, by adjuſtment, can be made parallel, as will readily be conceived, by obſerving the repreſentation of theſe parts in the right hand ſection of Plate IV.

When the angles of elevation or depreſſion to be determined are very ſmall, they are meaſured by the motion of an horizontal wire in the focus of the eye-glaſs of the teleſcope; but when great, their quantity is meaſured by the arc of motion of the ſemicircle, as ſhewn by its proper horizontal microſcope.

The elevation level is likewiſe made uſe of for levelling the inſtrument when horizontal angles only are to be taken, for which purpoſe it is ſuſpended on two pins, which are ſeen projecting from the horizontal bar in the plan, and one of them in each of the ſections in Plate IV. This was the ordinary poſition of the elevation level when the angles of the

 triangles

triangles were obſerved, and thereby it was eaſily ſeen in the courſe of the operation, whether the inſtrument had ſuffered any change to render a re-adjuſtment neceſſary.

## Art. X. *Lanterns for the Illumination of the Wires.*

The axis of the tranſit teleſcope is hollow, and in the middle there is placed, at an angle of 45° with the axis of viſion, a perforated elliptical illuminator for throwing light on the wires in night obſervations. The light is communicated from a ſmall lantern attached to the horizontal bar at its junction with the brace, directly oppoſite to the end of the axis, which has a bit of thin glaſs placed before it to prevent duſt from entering. There is another ſuch lantern for the lower teleſcope, not however repreſented in the plate. As the light given by theſe lanterns was found to be rather too weak, eſpecially that for the upper teleſcope, therefore it was cuſtomary in practice to illuminate the wires, by holding up frontwiſe one of thoſe ſeen in the ſection in Plate IV. againſt the end of the axis of the upper teleſcope, when directed to the pole ſtar. The ſame method was uſed by preſenting it obliquely to the object-glaſs of the lower teleſcope, when it became neceſſary to examine whether the interſection of the wires continued without ſenſible variation on a reverberatory lamp, commonly placed twelve or fifteen miles off, and ſometimes even at the great diſtance of twenty or twenty-four miles.

## Art. XI. *Lanterns for throwing light on the Diviſions of the inſtrument.*

Beſides the two ſmall lanterns for illuminating the wires of the teleſcopes in night obſervations, two larger ones may be ſeen, as already mentioned, ſtanding on the mahogany cover in the ſection

ſection in Plate IV. uſed for reading off the diviſions of the inſtrument, under the vertical microſcopes. The front of one of theſe is ſhewn, and the back, or that to which the handle is fixed, of the other. Their narrow ſides are preſented towards the microſcopes, there being in each a ſilvered reflector of copper at FF; and oppoſite to it, at GG, a ſcreen of talc or tranſparent oiled paper. The light from a wax candle being thrown on the reflectors, and thence back again through the ſcreens, on the diviſions of the inſtrument under the microſcopes, theſe could be very diſtinctly read off and regiſtered: for the light communicated in this way was very ſtrong, at the ſame time that the glare of it, which otherwiſe would have been diſagreeable to the ſight, was removed by paſſing through the ſcreen.

Art. XII. *Arms projecting from the bell-metal plate under the plane of the inſtrument.*

By referring to Plates III. and IV. but more particularly the latter, it will be perceived, that there are three flat arms, ſtrongly fixed by ſcrews to the edge of the circular bell-metal plate, forming, as has been already mentioned, the baſis of the interior vertical axis. Theſe arms, which are alſo firmly braced to the feet of the inſtrument, riſe gradually as they project outwards towards the circumference of the circle, whoſe radius they exceed about an inch and a quarter, and their extremities are about an inch lower than its upper ſurface. One arm, lying directly over one of the feet, is that to which are attached the wheels and ſcrew moved by the Hook's-joint, and alſo the clamp of the circle, as repreſented in Plate V. The other two arms, whereof one lies alſo over a foot, and the other directly oppoſite to it, become thereby a diameter to the

circle,

circle, having their extremities terminated in a kind of blunted triangular figure, forming the bases of pedestals whereon stand the vertical microscopes hereafter to be described. The arms, together with the horizontal bar and braces carrying the transit telescope, are every where pierced, in order to lessen the weight without diminishing the strength of the parts.

### Art. 13. *Vertical Microscopes.*

Two vertical microscopes, distinguished A and B, are used for reading off the divisions on the opposite sides of the circle immediately under them. They are exactly of the same construction, and the chief parts of that marked A are represented in their real dimensions towards the left hand of Plate VI.; where, beside the general, may be seen particular plans of the slides, and also that of the pedestal, containing within it the gold tongue, with its axis and screws for adjustment. Next to these plans stand the elevation and optical lines, shewing the position of the glasses with the magnified scale at the bottom.

Each microscope contains two slides, one lying immediately over the other, their contiguous surfaces being in the focus of the eye-glasses. The uppermost, or that nearest the eye, is a very thin plate of brass, to the lower surface of which is attached the fixed wire, having no other motion than what is necessary for adjustment, by the left hand screw to its proper dot, as hereafter to be explained.

The steel slide immediately under the former is made of one entire piece, of sufficient thickness to permit the micrometer screw, of about 72 threads in an inch, to be formed of it. To its upper surface is fixed the moveable wire, which changes its place by the motion of the micrometer head, seen in the plan and elevation towards the right hand. The head is divided

into 60 equal parts, each of which repreſents one ſecond of angular motion of the teleſcope. By examining the particular plan of this ſteel ſlide, it will be ſeen, that it is attached by a chain to the ſpring of a watch, coiled up in the uſual manner, within a ſmall barrel adjacent to it in the frame. By this proviſion no time whatever is loſt; the ſmalleſt motion of the head being inſtantly ſhewn, by a proportionable motion of the wire, to one hand or the other, in the field of the microſcope.

It is neceſſary to remark, that the whole microſcope between its pillars can be raiſed or depreſſed a little more or leſs, with regard to the plane of the circle, by the help of two ſteel levers, ſeen one on each ſide of the elevation, which for that purpoſe are applied in the holes repreſented above and below the projecting plate that unites the tops of the pillars. By means of this motion, diſtinctneſs is obtained at the wires; and by the motion of the proper ſcrew of the object lens, which neceſſarily follows that given to the whole microſcope, the ſcale is ſo adjuſted as that fifteen revolutions of the head ſhall move the wire over fifteen minutes, or one grand diviſion, on the limb, equal to nine hundred ſeconds, each degree on the circle being only divided into four parts. This operation being delicate, requires great patience and many repetitions, before the purpoſe can be exactly, or even nearly, effected: for at the ſame time that the fixed wire muſt biſect the dot on the gold tongue, the moveable wire muſt alſo biſect the dot at $180^{\circ}$ on the limb, as well as the firſt notch in the magnified ſcale at the bottom of the plate, where the minutes in the field of the microſcope are repreſented in the proportion of between fifteen and ſixteen to one as painted on the eye of the obſerver. In this adjuſtment there is yet another circumſtance to be

attended to, which is, that ſixty on the micrometer head ſhould ſtand nearly vertical, ſo as to be conveniently ſeen. A few ſeconds of inclination to one ſide or the other are of no moment, becauſe the dart or index being brought to that poſition, whatever it may be, muſt at all times remain there without alteration, unleſs ſome derangement that may have happened to the inſtrument, in tranſporting from one place to another, ſhould have rendered a freſh adjuſtment neceſſary. But if, when the wires co-incide with their reſpective dots and the firſt notch, ſixty on the micrometer head ſhould happen to be underneath, or ſo far over from the vertex on either ſide as to be ſeen with difficulty, then the gold tongue muſt be moved a little by means of the capſtan-headed ſcrews, which act againſt each other on the oppoſite extremities of its axis. Thus, by repeated trials, the wiſhed-for object will at length be effected, that is to ſay, ſixty, to which the dart is to be ſet, will ſtand in a place eaſily ſeen. But it is not to be expected, that each microſcope will give juſt nine hundred ſeconds for the run of fifteen minutes. Without great loſs of time this cannot be done; beſides that two obſervers, of different ſights, will adjuſt the microſcopes differently. Accordingly, in 1787, after many trials of the runs in meaſuring fifteen minutes on the different parts of the limb, microſcope A was found to give only 896″, while B gave at a medium 901″. But in 1788, microſcope A gave 900″, while B gave no more than 894″. Theſe differences were of courſe regiſtered and allowed for in the eſtimation of the angles for computation, whereby any difference between them almoſt wholly diſappeared.

The gold tongue, which is extremely thin, applies very cloſely to the ſurface of the circle. In the plan it is ſuppoſed to be ſeen through a thin plate of braſs covering the whole

pedeſtal, and alſo through a ſmall ſquare plate lying over the former, and faſtened to it by three ſcrews. In the under ſide of this laſt, there is a cavity for the projecting part of the tongue. This contrivance of the tongue with its dot was to guard againſt any error that might ariſe from accidental motion given to the inſtrument between one obſervation and another, which from this precaution could never happen, without being immediately diſcovered: for the wires being adjuſted to their dots under the microſcopes reſpectively, if the inſtrument be then turned round 180°, the wires will reciprocally biſect the dots that were originally oppoſite to them, and thereby ſhew, that they are accurately in the diameter of the circle; and ſo on with regard to any other dots whatever. Hence this becomes the moſt ſevere mode of trying the juſtneſs of the diviſions of the inſtrument.

Art. XIV. *Manner of reading off angles with the microſcopes.*

By attending to the magnified ſcale at the bottom of the plate, it will appear, that the dot on the gold tongue, which is here inverted, is about one minute to the left of zero, and alſo of the firſt notch, with which the moveable wire alone co-incides. Now it will eaſily be conceived, from what has been ſaid in this deſcription, how readily, as well as accurately, any obſervation of an angle can be read off with ſuch an inſtrument; for the degrees and quarters, that is to ſay, the 15′, 30′, or 45′, being ſeen with the naked eye, and regiſtered, the value of the fractional ſpace between zero and the laſt paſt grand diviſion, ſeen in the field of the microſcope, is obtained by turning the micrometer head until the moveable wire biſects the dot at that grand diviſion. The number of notches towards the right hand paſſed over on the ſcale, equal to ſo many revolu-

tions

tions of the head, are the number of minutes, always lefs than 15′, to be added. If there be no odd feconds, the dart will then ftand at 60″ on the head; but, if any number of feconds are to be added, the dart will fhew, by its pofition with regard to 60″, what that number is. Thus, by adding the parts together, the meafure of the total angle is obtained.

The conftruction, adjuftment, and application of thefe vertical microfcopes have been given more fully, becaufe they form a moft effential part of the inftrument: for the fixed wire conftantly remaining on its dot, the fractional fpace may be repeatedly meafured many times over, if neceffary, and a mean refult may then be taken. But it rarely happens that two obfervers, reading off with the oppofite microfcopes, differ more than half a fecond from each other at the very firft reading. If time therefore permits, and the circumftances of the weather fhould alfo be favourable for repeating the obfervation with the telefcope, it is fufficiently obvious to what a wonderful degree of accuracy the meafure of angles may in this way be obtained.

ART. XV. *Horizontal Microfcopes.*

Befides the two vertical microfcopes, applied in the manner that has been defcribed to the meafurement of the fractional fpace in horizontal angles, there is yet another to be mentioned, which is placed horizontally on the bar that carries the tranfit telefcope, and is directed to the divifions on the femicircle attached to its axis, for the meafurement of angles of elevation or depreffion, as has already been taken notice of. This microfcope, which is of the fame conftruction with the others, but larger, being upwards of nine inches in length, is reprefented in its full dimenfion in Plate VI. It has, like the others, a flide made of fteel, of fuch thicknefs as to permit the

micrometer ſcrew to be formed of it; and it carries a vertical wire placed in the focus of the eye-glaſſes, in which poſition it is moved parallel to itſelf from left to right, by turning the micrometer head. This ſlide is alſo attached to a watch ſpring which acts in a contrary direction to the head, as in other microſcopes of this ſort.

Each degree of the ſemicircle being divided into two parts or 30′, and one revolution of the micrometer head moving the wire in the field of the microſcope 3′; therefore in 10 revolutions it changes its place half a degree or 30′, which are ſhewn by a ſcale of 10 notches in the upper part of the field of the microſcope, and alſo repreſented towards the top of the plate. Each notch correſponds to 3 minutes or 180 ſeconds, and the head being divided into 3 minutes, and each minute into 12 equal parts, therefore each part is of the value of five ſeconds.

## ART. XVI. *Concerning the Semicircle.*

With regard to the ſemicircle, which has been repeatedly mentioned in the courſe of this deſcription, it is yet neceſſary to make ſome remarks; and particularly to ſhew how, by its means, the axis of viſion of the teleſcope, when adjuſted, is brought and kept truly horizontal, which is effected in the following manner.

On the oppoſite ſides of the horizontal bar that carries the teleſcope there are fixed four ſmall, but finely poliſhed bell-metal planes, two on each ſide, on the right and left of the top of the vertical axis, in ſuch a manner as that the ſurfaces of the two on either ſide are directed to or in the ſame plane with the center of the axis of the teleſcope. Theſe planes will be beſt conceived

conceived by obſerving attentively the top of the vertical axis in the ſection towards the right hand of Plate IV. On the edge of the ſemircircle may likewiſe be ſeen a moveable clamp, eaſily made to ſlip, with the hand only, around its circumference, and it carries with it a very fine ſteel ſcrew. When the ſemicircle is towards the left hand of the teleſcope, which is its ordinary poſition, the point of the ſteel ſcrew reſts, or may be made to reſt, perpendicularly on the ſurface of the plane that is on the left of the vertical axis. But when the teleſcope is inverted in its Y's, or turned upſide down, as is the caſe in adjuſting the line of collimation, the ſemicircle being then on the right of the teleſcope, and the clamp neceſſarily brought down, the point of the ſteel ſcrew accordingly reſts perpendicularly on the ſurface of the plane to the right of the vertical axis. Thus it will be readily conceived, that in adjuſting the teleſcope by the level for elevations, which is then conſtantly ſuſpended on its proper rod, parallel to the axis of viſion, the action of the ſteel ſcrew on the bell-metal plane ſerves not only for the adjuſtment of the teleſcope in a truly horizontal poſition, for angles of elevation or depreſſion, by the motion of a wire in the focus of its eye-glaſs, in the manner hereafter to be deſcribed, but alſo to keep it in that poſition, by the ſuperior weight of the eye end, rendered ſo on purpoſe. By the ſame means the teleſcope remains ſteadily on any object that it may be directed to for interſection, whether above or below the plane of the horizon.

One thing more with regard to the ſemicircle muſt be mentioned, namely, that it gives angles of elevation 12″ too great, and thoſe of depreſſion 12″ too little. It is very eaſy to conceive, that this aroſe from the impoſſibility of dividing it on the axis of the teleſcope to which it is fixed, and through the

centers

centers of whofe pivots an imaginary line paffes that fhould at the fame time have paffed through the center of the femicircle. Mr. Ramsden took the beft method that could be devifed to render the excentricity as little as poffible. Having framed the femicircle, and fcrewed it in its place on the axis, he made a fteel point firmly fixed to the horizontal bar defcribe the concentric arcs whereon the divifions were afterwards to come, and then marked the point for zero, when the telefcope by adjuftment had been brought as nearly horizontal as poffible. Thefe previous fteps being taken, the femicircle was removed, divided on the engine, and replaced in its orginal fituation. Neverthelefs, when the inftrument was carried into the field, and fcrupuloufly adjufted, the error was found, as has above been faid, 12″, which of courfe became the conftant quantity to be applied with its proper fign, when angles of elevation or depreffion were taken.

Art. XVII. *Eye-glaffes of the telefcopes, and mechanifm of the wires in their foci.*

It has been already mentioned, that the telefcopes of the inftrument are furnifhed with eye-glaffes of different magnifying powers for erect and inverted vifion, fix for each telefcope, as follows, *viz.*

| | Erect vifion. | | Inverted vifion. | |
|---|---|---|---|---|
| | N° | Power. | N° | Power. |
| For the lower telefcope, | N° 1. | 58. | N° 1. | 43. |
| | 2. | 88. | 2. | 59. |
| | 3. | 117. | 3. | 87. |
| For the upper telefcope, | N° 1. | 54. | N° 1. | 40. |
| | 2. | 81. | 2. | 55. |
| | 3. | 108. | 3. | 80. |

With

With regard to these eye-glasses, it is only necessary here to mention, that those of the least magnifying powers were found both in day and night observations to answer the best.

In the focus of the eye-glass of the lower telescope there are only two wires crossing each other in acute angles, which are vertical, instead of being placed at right angles, horizontally and vertically, as was the ancient method. Since the lower telescope never moves through more than a few degrees of a vertical arc, the wires require little or no adjustment. Nevertheless this was provided for, by allowing room for a small circular motion of the end-piece, which, when adjusted, is then fastened by its proper screws, and never afterwards needs any alteration.

By referring to the middle part of Plate VI. two representations of the eye end of the upper telescope will be seen, with the eye-piece removed. Five wires are shewn in this end, namely, two that intersect each other in acute angles, similarly to those in the lower telescope; and three that lie horizontally or parallel to each other. Four of these, *viz.* the two that form the acute angles, and the two extreme horizontal wires, are fixed in the focus of the eye-glasses to the farther surface of a thin brass slide, supposed to be seen through the outward brass, and therefore shaded more dark than the rest. This slide, as will be conceived, lies nearest the eye, and is moveable from right to left, and, *vice versâ*, horizontally, for the adjustment of the line of collimation, by the insertion of a small mill-head key, on a square pin fitted to receive it, and secured by a socket on the right hand side. The fifth or middlemost horizontal wire is attached to the nearest surface of a steel slide, that lies contiguously to, but beyond the former. It is made of one entire thick piece, like those of the micro-

ſcopes, to permit the micrometer ſcrew to be formed of it; and it is repreſented in the uppermoſt figure attached to a watch ſpring coiled up in the uſual manner.

By the motion of the micrometer head, the ſlide, and with it the wire, moves upwards or downwards in the field of the teleſcope, a ſpace equal to half the diſtance of the extreme wires from each other. This motion above or below the central point, which was made to correſpond with the acute interſection of the wires placed in the axis of viſion of the teleſcope, is performed in ten revolutions of the head, as denoted by the motion of the dart, ten diviſions upwards or downwards, in the narrow groove ſeen at the top of the figure.

Now, by the means of this piece of mechaniſm in the eye-end of the teleſcope, it will appear ſufficiently obvious, that ſmall angles of elevation or depreſſion may be determined with great accuracy, when the value of a certain number of revolutions and parts (the circumference of the head being divided into 100) have been once aſcertained by repeated obſervations of the altitude of any well-defined object taken by the ſemicircle. Thus it was found, by experiment, that $7\frac{77}{100}$ revolutions of the micrometer head were equal to an angle of elevation or depreſſion of 10′ 59″, or 659″, on the ſemicircle. Whence it follows, that one revolution raiſes or depreſſes the wire above or below the central point 1′ 24″.8134, or a little more than 84″81. And hence a motion of one diviſion on the head raiſes or depreſſes the wire nearly $\frac{85}{100}$ths of a ſecond.

In this manner were determined the reciprocal elevations or depreſſions of the ſeveral ſtations of the ſeries of triangles with regard to each other.

By obſerving attentively the four ſcrews repreſented in the outward end of the teleſcope, a dotted groove will be ſeen

under

under the head of each. And in the uppermoſt figure there appears a flat braſs ring, ſoldered to the inſide of the tube about half an inch from the outward end, which carries on its ſurface four ſtuds to receive the lower extremities of the four ſcrews. Thus the grooves allow room for a ſmall circular motion to be given to the end-piece for the vertical adjuſtment of the fork of the wires, thoſe that are horizontal being by conſtruction at right angles with it. This being done, the ſcrews are made very faſt in the ſtuds below, and thereby the whole machinery of the end-piece is rendered perfectly firm and ſecure.

There remains yet one piece more to be barely mentioned. It is the priſm eye-tube, repreſented by dotted lines towards the right-hand ſide of Plate VI. as attached to the eye end of the tranſit teleſcope, inſtead of the common eye-piece with two convex glaſſes. In leaning over our inſtrument to obſerve the pole ſtar, highly elevated in theſe latitudes, the body is neceſſarily thrown into an inconvenient fatiguing poſture, whereby ſome riſk is run of deranging the inſtrument, and conſequently of making the obſervations leſs accurately than when the obſerver can look directly forward, without bending the body ſo much. For this purpoſe, Mr. RAMSDEN promiſed to ſupply the priſm tube in 1787; but it was only, and with great difficulty, obtained in 1788, by which time Mr. DALBY had accuſtomed himſelf to obſerve very well without it, ſo that it was never uſed.

By employing this piece, light is no doubt loſt; becauſe the image paſſes through more glaſſes before it reaches the eye, than when the common eye-piece is uſed. But for obſervations of ſtars nearer the zenith than the pole ſtar is in our latitudes, it would be indiſpenſably neceſſary. It would likewiſe be advantageouſly

ufed in looking at the meridian fun in fummer, for which purpofe it is furnifhed with dark glaffes, placed in a flide moved by rack-work, as may be feen from infpection of the plate. They confift of three prifms, laid clofe to each other, fo as to form, when thus affembled, a parallelopipid. Here the green prifm ftands neareft to the eye, a dark one fartheft from it, and between the two, one of white flint glafs, for correction of the refraction which would otherwife take place. It will eafily be conceived, from the difpofition of the prifms, that the darkeft medium is here towards the left; and that it becomes gradually lighter towards the right hand, where a void part in the frame is brought into the field when the ftars are obferved; or when, from the circumftances of the weather, it may be unneceffary to fcreen the eye from the fun's rays.

### Art. XVIII. *General management of the inftrument for obfervation.*

When the inftrument is ufed on the ground, it is covered from the weather, under a circular tent, eight feet in diameter. Four fhort piles, hooped and fhod with iron, are driven into the earth, and their heads levelled, by laying acrofs from one to the other a mahogany ftraight ruler, having a fpirit level attached to one fide of it. The feet of the ftand being then placed on piles, are firmly faftened to them by means of long fquare-headed fcrews, only one of which may be feen in the view of the inftrument, belonging to that foot which ftands neareft the eye. By working with the four fcrews fixed in the octagonal mahogany plane, the plummet fufpended from the center of the inftrument is brought accurately over the point on the ground that marks the ftation. The fcrews of the feet, with the fide nuts appertaining to them, are then flackened,

to

to give ſufficient room for the adjuſtment of the inſtrument, which by them is brought to be level.

ART. XIX. *Adjuſtment of the axis Level.*

The axis of the upper or tranſit teleſcope being brought over any one of the feet, and the circle being clamped, hang the axis level on the pivots or *anſæ* of the teleſcope, and bring the bubble to the two indexes; then reverſe the level, that is, turn it end for end, and note the difference. Biſect this difference, one half by the level's proper adjuſting ſcrew, and the other half by that foot-ſcrew only which is in a line with the axis. This operation being repeated until the difference wholly vaniſhes, the level will be truly adjuſted, that is to ſay, the bubble will reſt between the ſame points in both poſitions.

ART. XX. *Adjuſtment of the elevation Level.*

This level being ſuſpended on the rod attached to the outſide of the tranſit teleſcope, ſcrew the erect eye-tube on, to make that end preponderate. Adjuſt the bubble to the indexes by the ſteel finger-ſcrew at the tail of the ſemicircle's clamp. Reverſe the level, and note the difference. Then biſect that difference, and correct one half by the finger ſcrew, and the other half by the proper adjuſting ſcrew under the level, and ſo on repeatedly until the difference wholly vaniſhes. The level may then be hung on the two pins that project from the horizontal bar which carries the teleſcope, where, being parallel to the axis level, it will ſhew when that is removed (as is commonly the caſe when terreſtrial objects only are obſerved) whether the plane of the inſtrument ſuffers any alteration. If this ſhould have happened, the level on the horizontal bar is at all times ſufficient to correct it.

## Art. XXI. *To set the vertical Axis perpendicular.*

This may be done by either level, but best with the axis level, which being suspended on its pivots, must be brought parallel with two of the feet of the instrument; and by the screws of these two feet, the bubble is to be brought between its indexes. The circle being then turned round 180°, if the bubble changes its place, half the difference is to be corrected by one of the feet screws, and the other half by two capstan-headed screws, that act against each other, under and belonging to one of the Y's, or supports, in which the pivots rest. When the bubble is found to be just in these two positions, turn the circle 90°, which will necessarily bring the axis over the third foot of the instrument. Then correct any error there may be by that foot screw. In this manner the circle will be made to revolve again and again, without any alteration whatever of the bubble, which shews that the vertical axis is then truly perpendicular to the horizon.

## Art. XXII. *To make the line of Collimation in the telescope at right angles with the transverse Axis.*

The pivots resting in their Y's, direct the telescope to some distant well-defined object, and let the circle be clamped. Then reverse the axis, that is, turn the telescope upside down. If the intersection of the wires does not co-incide with the object in both positions, half the difference must be corrected by the motion of the circle with the Hook's-joint, and the other half by the motion of the brass slide in the eye end of the telescope, by applying the milled-head key in the small socket seen on the right hand side in Plate VI. and so repeatedly until the difference wholly disappears.

ART. XXIII. *To ſet the Rod on which the elevation level hangs parallel to the line of Collimation.*

The vertical axis being ſuppoſed to be nearly vertical, hang the level on its rod, and rectify the bubble by the finger ſcrew of the clamp. Set the horizontal wire on the ſteel ſlide, to interſect the center of the oblique wires, and place the dart or index at zero on the micrometer head. Then obſerve ſome diſtant diſtinct object covered by the horizontal wire. Invert the ſemicircle, that is, turn the azimuth circle 180°, and the teleſcope upſide down, ſo as to bring the wire upon or nearly upon the ſame object. Now, if the level be not right, rectify it by the finger ſcrew at the tail of the clamp. If the teleſcope does not now accurately cover the ſame object as in the former poſition, biſect the difference by the finger ſcrew of the clamp, and then rectify the bubble by the capſtan-nuts under one end of the rod. Repeat this operation until the level is right, when the teleſcope ſees the ſame objects in both poſitions, and thereby the rod will be brought parallel in altitude to the line of collimation or axis of viſion.

The adjuſtments of the microſcopes having been already ſufficiently explained, in giving the deſcription of the eſſential parts of the inſtrument, it is unneceſſary here to repeat them.

ART. XXIV. *Of the weight of the inſtrument, and mode of tranſporting it from place to place.*

The inſtrument, whoſe deſcription and uſes we have here attempted to give in a general way, without reference to its minute parts, by a multitude of different characters, weighed in the whole about 200 lbs. It is contained in two deal boxes; one

one of a circular form for the body of the inſtrument; and the other of an oblong ſquare figure, for the tranſit teleſcope. Within this laſt box there is one of mahogany, that holds all the ſmaller parts of the apparatus. The ſtand, ſteps, ſtools, pullies, ropes, tent, and canopy for the ſcaffold, &c. &c. weighed at leaſt as much more. The whole attirail was tranſported from place to place, in a four-wheeled ſpring carriage, drawn by two, and ſometimes by four horſes. The carriage part, originally that of a crane-necked phaeton, was preſented, with his uſual liberality, by Sir Joseph Banks; and upon it was built a kind of caravan, covered with painted oil-cloth, whereby every thing within was kept dry and ſecure.

## SECTION THIRD.

*Deſcription of various articles of machinery made uſe of in the trigonometrical operation referred to in Plate VII. Alſo the diſtinction of the ſtations into two ſets, thoſe of the ſecond ſet being referred to in Plate VIII.*

### Article I. *Portable Scaffold.*

IN the account of the meaſurement of the baſe on Hounſlow Heath we have ſhewn, that the ſurface of that remarkable plain is not elevated more than fifty or ſixty feet above the mean level of the ſea. From this ſmall elevation, and the circumſtance of its being ſurrounded, almoſt on every ſide, with lofty trees, it was from the beginning ſufficiently obvious, that, in order to be enabled to make the obſervations of the collateral

ſtations

ſtations from the extremities of the baſe, it would be abſolutely neceſſary to raiſe the inſtrument, by ſome means or other, to a conſiderable height above the ground. For this purpoſe the portable ſcaffold, whoſe plan and elevation are repreſented on the left hand ſide of Plate VII. was conſtructed. It conſiſted, as may be ſeen, of an inward ſcaffold for ſupporting the inſtrument, and an outward one for the obſervers, wholly free and independent of each other, the platforms of both being framed about thirty-two feet above the lower ends of the ſcantlings, which reſt on the ground. Theſe being made of ſquared deal, and the ſeveral parts being bolted and ſcrewed together with many iron ſcrews ſecured by nuts, the whole could be readily taken to pieces, carried in a waggon (for which it made a complete load), and replaced again in any new ſituation. This ſcaffold anſwered very well the purpoſe for which it was intended; for the ſtep-ladders, or ſtairs leading to the platform, being attached to the outward frame, the inward one that carried the inſtrument remained undiſturbed by the motion of thoſe who went up and down, or walked around the top. The ſilk thread, that ſuſpended the plummet, was ſecured from the effects of the wind by a ſort of funnel or trunk, compoſed of three deals (one ſide being left open), and ſo contrived as to be eaſily turned round to any quarter of the heavens, whereby the open ſide was always preſented to leeward. The inſtrument was covered from the weather by a canvas canopy, about ſeven feet ſquare, to which ſide walls could be hooked for ſcreening it from the wind, as occaſion might require. By referring to the elevation it will be ſeen, that the ſcaffolds, both outward and inward, might be divided horizontally into two parts, ſo as to permit the uppermoſt half alone to be uſed when it became unneceſſary to raiſe the inſtru-

ment to a greater height than fifteen or ſixteen feet above the ground. The whole together was never made uſe of, except at the two extremities of the Hounſlow Heath baſe. The uppermoſt half was applied at three of the ſtations only, namely, St. Ann's Hill, Botley Hill, and Padleſworth near Dover.

### Art. II. *Tripod Ladder.*

Next to the ſcaffold the plate repreſents, in plan and ſection, a tripod ladder, about thirty-five feet in height. It carries on its top a globe lamp, of about one foot in diameter, in which was uſed a ſimple Argand's burner, of a large ſize, made for that purpoſe. The lamp being removed, a ſocket for a white light might occaſionally be ſubſtituted in its place; or (as was the caſe when we obſerved the ſtation at King's Arbour from St. Ann's Hill) a flag-ſtaff might be added at the top, which was ſecured in a truly vertical poſition, by braces fixed to the legs of the ladder underneath. It will be readily conceived, that by a contrivance of this ſort a white light could be raiſed to a conſiderable height above the ground, if the circumſtances at any time had rendered ſuch elevation neceſſary; and that it could, by the help of a heavy plummet, be always placed in a truly vertical poſition over the point on the ground marking the ſtation. The globe lamp was found to anſwer very well for ſhort diſtances of ſix or eight miles, when the weather was favourable; but it could not be depended upon in obſervations of diſtances that were conſiderably greater.

### Art. III. *Common Flag-ſtaff.*

After the tripod ladder, comes in the plate the plan and elevation of a common flag-ſtaff with its braces, carrying

likewiſe two reverberatory lamps. Theſe two were attached to the ſame iron bar, at the diſtance of three feet from each other. They had concave copper reflectors, nine inches in diameter, extremely well poliſhed and ſilvered. They were intended at firſt for experiments near London, and were very well ſeen at the diſtance of fifteen or ſixteen miles. To ſecure us from any uncertainty that might have ariſen, by miſtaking other lights for our own, one lamp was placed over the other. But when we came afterwards to be better acquainted with the appearance of theſe lamps, that precaution was found to be entirely unneceſſary; wherefore ſingle reverberatories were provided, with *ſpecula* of ten inches diameter, and they were ſupplied with ſtill larger burners, which could be ſeen at the diſtance of twenty or twenty-four miles. But here it is proper to remark, that theſe lamps muſt be carefully watched, eſpecially in expoſed windy ſituations; for if the cotton be drawn out a little too far, they are apt to ſmoke, whereby the front glaſs becomes obſcure, and therefore muſt be wiped frequently. They are eaſily turned on the poſts that ſupport them; and were, by the help of a teleſcope laid on one ſide, parallel to the axis of the rays (for which a contrivance was provided in the tin work) accurately preſented towards the ſtation occupied by the inſtrument at the time from whence they were to be obſerved. There was conſtantly one of theſe lamps, and ſometimes two, at two different ſtations, burning each night, when we were making obſervations of the pole ſtar, or white lights of ſhort duration, placed at other diſtant ſtations.

## Art. IV. *Tripod for White Lights.*

Next after the flag-ſtaff (whereon a ſocket for white lights could likewiſe be placed, when the flag itſelf was removed) is repreſented a ſmall tripod intended for white lights only. The ſame ſocket that fitted the top of the flag-ſtaff, or lamp-poſt, could be applied to the tripod, by the help of three ſmall ſockets ſoldered for that particular purpoſe to the ſides of the principal one. Deal rods, of five or ſix feet in height, or hazels cut from the neareſt hedge, ſerved as the legs of this ſtand. The ſockets themſelves were made of copper, becauſe thoſe of iron would have been diſſolved by the ſulphur; and the upper part, which was only an inch, or an inch and an half, in height, was ſquare or round, according to the figure of the boxes containing the compoſition, ſometimes of one kind, and ſometimes of the other. Theſe white-light tripods, being readily placed by the help of a plummet over the point marking the ſtation, were found to be very convenient on the top of an open hill, or on the leads of a church ſteeple, as the perſon attending them could eaſily light the box with the portfire, without the aid of a ladder.

## Art. V. *Portable Crane.*

On the right hand ſide of the plate is repreſented, in plan and ſection, and by a larger ſcale than the others, a portable crane for weighing up the inſtrument to the tops of ſuch towers, church ſteeples, or other buildings, as became ſtations in the ſeries of triangles. It was conſtructed in the Tower of London, and anſwered very well the purpoſe for which it was intended, although it might ſtill be improved. Before we

were

were ſupplied with this crane, we made ſhift, by the help of a long beam, and a moveable treſtle by way of fulcrum for it to reſt upon, to get the inſtrument up to the top of its own proper ſcaffold, and one that was ſtill higher, erected over the tranſit room of the Royal Obſervatory at Greenwich.

ART. VI. *Reaſons for changing certain Stations.*

In the courſe of the trigonometrical operation, the center of the inſtrument has conſtantly been brought, even almoſt to mathematical exactneſs, over the preciſe point marking the ſtation, whereby reductions to the center on account of excentricity have been avoided; and the ſtations have been diſtinguiſhed, as far as poſſible, by permanent marks in ſuch a manner, that, while theſe remain, the center of this or any other inſtrument may be again brought into the ſame vertical line. By theſe means our recent obſervations may be repeated on any future occaſion, and connected with others, which it is to be hoped will be made hereafter: for this operation, the firſt of its kind in Britain, ſhould only be conſidered as the foundation or commencement of a ſeries of others, which by degrees will be carried to the remoteſt parts of the iſland.

By compariſon of the annexed plan of the triangles with that communicated to the Royal Society in 1787, as only a ſketch of the ſcheme then propoſed to be carried into execution, it will be perceived, that ſome few ſtations are omitted entirely, and others ſubſtituted in lieu of ſome that were then intended to be occupied. Of this laſt number Hanger-hill Tower has been made uſe of inſtead of Kew Pagoda. This laſt had been propoſed on a ſuppoſition, that without a ſcaffold of an enormous height, it would have been impoſſible to ſee Hanger-hill Tower from King's Arbour. Nevertheleſs, after

I a good

a good deal of trouble, by cutting off the tops of certain trees, lopping the branches of others, and raiſing a flag-ſtaff on the center of the ſcaffold, theſe two ſtations were rendered reciprocally viſible. By theſe means we not only avoided making uſe of Kew Pagoda, which, from the nature of the building, would have been a very incommodious ſtation; but we thereby got rid of Clermont Tower altogether; and thus, inſtead of two ſmall triangles, one was conſtituted, larger and better, being nearly equilateral.

In the introduction there has been occaſion to take notice of the advantage that was gained by being able to ſee Frant and Fairlight Down reciprocally. From this circumſtance the ſeries from Frant eaſtward to the baſe of verification becomes in reality a double one, and conſequently affords better means of aſcertaining the correctneſs of the work.

The ſingularity of the ſituation of Dover Caſtle has likewiſe been mentioned. Inſtead of two ſtations near Tatterlees Barn and Barefriſtan, whereby it was hoped, that Dover Caſtle might have been connected with the ſeries to the weſtward, it was found neceſſary to make uſe of three ſtations; one at Padleſworth, one at Folkſtone Turnpike, and a third at Swingfield. Thus the ſide which connects that ancient fort with the other triangles is ſhorter than was intended. But with ſuch an inſtrument as ours, and where all the angles of the triangles were obſerved, no uncertainty ariſes on that account.

## ART. VII. *Diſtinction of the Stations.*

Having aſſigned the reaſons that rendered it eligible or neceſſary to change ſome few of the ſtations propoſed in the original ſcheme, it only now remains to enumerate the whole as diſtinguiſhed

distinguished into two sets. First, those which are permanently marked by pipes sunk in the earth; and, secondly, those where the instrument was elevated to the top of some tower, church steeple, or other building. The plans of the platforms of this last set are given in Plate VIII. along with such dimensions as are necessary to shew, with regard to the side walls, the precise spot over which the center of the instrument was placed. As often as was possible, these situations were further defined, by means of concentric circles described on the leads.

The stations of the first set, marked with pipes, are fourteen in number, *viz.*

| | |
|---|---|
| Hampton Poor-house, King's Arbour, | the extremities of Hounslow Heath base. |
| St. Ann's Hill, . | about the middle on the east edge. |
| Hundred Acres, . | near the west end of the garden. |
| Norwood, . . . | towards the Croydon end of the heights. |
| Botley Hill, . . . . | in a field belonging to Limpsfield Lodge Farm. |
| Wrotham Hill . . | in a field belonging to Mr. JOHNSTON. |
| Hollingborn Hill, . | in a field belonging to Mr. DUPPER. |
| Fairlight Down, . | 347 feet southward from the Windmill, which makes with Fairlight Church, an angle of 105° 53′ 20″. |
| Ruckinge, . . . High Nook, . . | the extremities of the base of verification. |
| Allington Knoll, . | an artificial mount belonging to Sir JOHN HONEYWOOD. |
| Padlesworth, . . | eastward from the Church, in the Broom-field belonging to Mr. BROCKMAN. |

Folkstone

Folkstone Turnpike . . westward from the Public-house.

The stations of the second set, where the instrument was elevated on buildings, are nine in number, *viz.*

Hanger-hill Tower.
Transit-room of Greenwich Royal Observatory.
North-west turret of Severndroog Castle, on Shooter's Hill.
Swingfield Church Steeple.
North turret of the Keep of Dover Castle.
Lydd Steeple.
Tenterden Steeple.
Goudhurst Steeple.
Frant Steeple.

## SECTION FOURTH.

*Calculation of the series of triangles extending from Windsor to Dunkirk, whereby the geodetical distance between the meridians of the Royal Observatories of Greenwich and Paris is determined. Reference to be had to Plate IX.*

### Article I. *Excess of the angles of spherical above those of plane Triangles.*

IF the earth, or any considerable portion of its surface, was a perfect plane, an instrument, such as has been formerly described, when applied on that surface, to determine by trigonometrical measurement the extent of the plane part, would every where have its axis parallel to itself; and the sum of the three

three angles of each of the triangles, into whatever number, great or ſmall, it might be divided, would conſtantly amount to 180°. But the earth being a ſphere or ſpheroid, it follows, that the ſame inſtrument, ſucceſſively adjuſted at each of the ſtations, will have its axis perpendicular, on a ſphere, to an equally curved ſurface; on a ſpheroid, to one unequally curved, in either caſe forming the horizon of the ſtation; and the ſum of the three angles of ſuch a ſpherical or ſpheroidical triangle muſt, as is known, always exceed 180°, leſs or more, in proportion to the lengths of the ſides. When the triangles are very ſmall, the exceſs being of courſe ſmall cannot poſſibly be diſcernible by common inſtruments. Even the fineſt, ſuppoſing them free from error of diviſion, will ſcarcely render it perceptible, without the utmoſt care in making the obſervations. This will be ſufficiently exemplified in the following calculations, where a column is inſerted containing the ſpherical exceſs; and another for the difference or error between that and the exceſs of the ſum of the obſerved angles above 180°. From theſe it will appear, that, notwithſtanding the goodneſs of our inſtrument, and the pains taken in uſing it, we have frequently failed in bringing out an exceſs; and indeed the reſults have even ſometimes been in a ſmall degree defective.

It had been at firſt propoſed to multiply the obſervations as much as poſſible, and particularly by ſucceſſively changing the zero of the inſtrument to new points (Phil. Tranſ. 1787, p. 219.), to meaſure the ſame angles on different parts of the circle, ſo as to ſubdivide any errors that might ariſe from inaccuracy of diviſion, or ſhake at the center. This principle, perfectly good in theory, and which was adhered to as far as the circumſtances would permit, was nevertheleſs found, on many occaſions, to be impoſſible in practice, without ſacri-

ficing much more time than we could afford, consistently with the engagements entered into with the French Gentlemen, for the co-operation on the Coast. At particular times, especially in hot weather, there was such a tremulous motion or boiling in the air, that it was only during a very short space, chiefly in the mornings and evenings, that the objects were sufficiently distinct to be observed with accuracy. So difficult it is to do any thing perfectly good in this way, that a whole day has frequently been spent, after watching with anxious care, in obtaining a single one that was perfectly satisfactory! At such times as these it would have been absurd to have attempted to change the zero, which always rendered it necessary to re-adjust the instrument by its levels.

In very favourable circumstances of the weather a good observation by day is preferable to one by the white lights at night; because, in the first case, the observer has time at his leisure nicely to bisect a fine flag-staff, and repeatedly to read off the angle; whereas, in the short duration of the burning of the light, he is somewhat hurried, from the fear of losing some of the lights at other distant stations, if two of them happened to come together, which now and then they did, from the irregularity of the rates of the watches of the artillery-men attending at the different stations. It was, however, by the assistance of the white lights only, that the most distant stations could be rendered visible; and there cannot be a doubt that, in great trigonometrical operations of this sort, they will be universally adopted hereafter.

Sometimes an observation has been entirely lost, or at least that which had been obtained was not thought a very good one. In such cases a blank has been left in the column of observed angles, and also in that expressing the error. But no

bad consequence has arisen on that account, there being always such other checks from the collateral stations, as to leave nothing doubtful.

On the whole, although, for the reasons already assigned, we have repeated the observations seldomer than was at first proposed; yet it will obviously appear from the results, and particularly from the near agreement between the measured and computed length of the base of verification, that a few very good observations are greatly preferable to a mean that might perhaps have been obtained of many made in a hurry, which at best would have been but indifferent.

The quantity by which the sum of the three observed angles of spherical triangles should have exceeded 180° was found as follows.

Because the excess of the three angles of a spherical triangle above 180° × earth's radius = its area, therefore $\frac{\text{Area}}{\text{Earth's rad.}}$ = excess above 180° in seconds, if the area and radius are taken in seconds. Now, 60859.1 fathoms being = 1° on a mean sphere, we get the log. of the feet in a second = 2.0061743, and twice this, or 4.0123486 is the log. of the square feet in a square second. Therefore log. area in feet − 4.0123486 = log. area in seconds; and the log. of the earth's radius in seconds being 5.3144251, we have area in feet − 4.0123486 − 5.3144251 = log. area in feet − 9.3267737 = log. excess in seconds; that is to say, *from the logarithm of the area of the triangle taken as a plane one, in feet, subtract the constant logarithm* 9.3267737, *and the remainder is the logarithm of the excess above* 180° *in seconds nearly.*

 ART.

## ART. II. *Calculation of the Triangles.*

| N° of triangles. | Names of the stations. | Observed angles. | Spherical excess. | Diff. or error. | Angles corrected for calculation. | Distances. |
|---|---|---|---|---|---|---|
| | | ° ′ ″ | ″ | ″ | ° ′ ″ | Feet. |
| I. | Hanger-hill Tower | 42 2 32 | | | 42 2 34 | |
| | Hampton Poor-house | 67 55 39 | | | 67 55 39 | |
| | King's Arbour . | 70 1 48 | | | 70 1 47 | |
| | | 179 59 59 | 0.29 | − 1.29 | | |
| | The BASE between Hampton Poor-house and King's Arbour . . . | | | | . . . . | 27404.7 |
| | Hanger-hill Tower from Hampton Poor-house | | | | . . . . | 38461.12 |
| | Hanger-hill Tower from King's Arbour . | | | | . . . . | 37922.57 |
| II. | St. Ann's Hill . | 44 18 51.5 | | | 44 18 51.5 | |
| | Hampton Poor-house | 61 26 33.1 | | | 61 26 33.5 | |
| | King's Arbour . | 74 14 35 | | | 74 14 35 | |
| | | 179 59 59.6 | 0.21 | − 0.61 | | |
| | St. Ann's Hill from Hampton Poor-house | | | | . . . . | 37754.25 |
| | St. Ann's Hill from King's Arbour . . | | | | . . . . | 34455.8 |

Hence, in the quadrilateral formed by *Hampton Poor-house*, *King's Arbour*, *Hanger-hill Tower*, and *St. Ann's Hill*, making use of the two obtuse angles, as contained within their respective known sides, we have for the mean distance of the points of the acute angles at Hanger-hill Tower and St. Ann's Hill, expressed by a dotted line in the plan of the triangles, 68897.165 feet.

| | | | | | | |
|---|---|---|---|---|---|---|
| III. | Wardrobe Tower of Windsor Castle | . . . . | . . | . . | 58 9 58.5 | |
| | King's Arbour . | 62 40 27.5 | | . | 62 40 27.5 | |
| | St. Ann's Hill . | 59 9 14 | | | 59 9 14 | |
| | | | 0.25 | | | |
| | Windsor Castle from King's Arbour . | | | | . . . . | 34819.4 |
| | Windsor Castle from St. Ann's Hill . | | | | . . . . | 36032.37 |

N°

| N° of triangles. | Names of the ſtations. | Obſerved angles. | Spherical exceſs. | Diff. or error. | Angles corrected for calculation. | Diſtances. |
|---|---|---|---|---|---|---|
| | | ° ′ ″ | ″ | ″ | ° ′ ″ | Feet. |
| IV. | Hundred Acres . | 53 58 35.75 | | | 53 58 36.5 | |
| | Hanger-hill Tower | 68 24 44 | | | 68 24 44 | |
| | St. Ann's Hill . | 57 36 39 5 | | | 57 36 39.5 | |
| | | 179 59 59.25 | 1.08 | − 1.83 | | |
| | Hundred Acres from { Hanger-hill Tower | | | | . . . | 71934.2 |
| | St. Ann's Hill . | | | | . . . . | 79211.22 |
| V. | Severndroog Caſtle, Shooter's Hill | 53 31 10 | | | 53 31 9.75 | |
| | Hanger-hill Tower | 55 53 44.3 | | | 55 53 44 | |
| | Hundred Acres . | 70 35 6.75 | | | 70 35 6.25 | |
| | | 180 0 1.05 | 1.18 | −0.13 | | |
| | Severndroog Caſtle from { Hanger-hill Tower | | | | . . . . | 84376.68 |
| | Hundred Acres . | | | | . . . . | 74077.66 |
| VI. | Norwood . | 107 53 37 | | | 107 53 35.75 | |
| | Hanger-hill Tower | 26 12 22.5 | | | 26 12 23 | |
| | Severndroog Caſtle | 45 54 1.5 | | | 45 54 1.25 | |
| | | 180 0 1 | 0.44 | +0.56 | | |
| | Norwood from { Hanger-hill Tower | | | | . . . . | 63673.31 |
| | Severndroog Caſtle | | | | . . . . | 39155.15 |
| VII. | Norwood . | 88 5 58 | | | 88 5 58.07 | |
| | Hanger-hill Tower | 29 41 20.75 | | | 29 41 21 | |
| | Hundred Acres . | . . . . . | | | 62 12 40.93 | |
| | | | 0.53 | | | |
| | Norwood from Hundred Acres | | | | . . . . | 35648.21 |
| VIII. | Tranſit Room, Greenwich Obſervatory | 111 56 50 | | | 111 56 50 | |
| | Severndroog Caſtle | 47 48 14 | | | 47 48 13 | |
| | Norwood . | 20 14 58 | | | 20 14 57 | |
| | | 180 0 2 | 0.01 | + 1.9 | | |
| | Greenwich Obſervatory from { Severndroog Caſtle | | | | . . . . | 14610.58 |
| | Norwood . | | | | . . . . | 31274.48 |

N°

| N° of triangles. | Names of the stations. | Observed Angles. | Spherical excess. | Diff. or error. | Angles corrected for calculation. | Distances. |
|---|---|---|---|---|---|---|
| | | ° ′ ″ | ″ | ″ | ° ′ ″ | Feet. |
| IX. | Botley Hill | 74 37 17.5 | | | 74 37 18 | |
| | Hundred Acres | 66 0 56.2 | | | 66 0 56 | |
| | Severndroog Castle | 39 21 46.25 | | | 39 21 46 | |
| | | 179 59 59.95 | 0.78 | −0 88 | | |
| | Botley Hill from Hundred Acres | | | | . . . . . | 48726.75 |
| | Botley Hill from Severndroog Castle | | | | . . . . . | 70194.76 |
| X. | Wrotham Hill | 54 25 1 | | | 54 25 1.25 | |
| | Botley Hill | 67 53 11 | | | 67 53 10.25 | |
| | Severndroog Castle | 57 41 49 | | | 57 41 48.5 | |
| | | 180 0 1 | 1.12 | −0.12 | | |
| | Wrotham Hill from Botley Hill | | | | . . . . . | 72953.12 |
| | Wrotham Hill from Severndroog Castle | | | | . . . . . | 79962.13 |
| XI. | Frant | 50 19 19 | | | 50 19 18 | |
| | Botley Hill | 57 15 11.25 | | | 57 15 11 | |
| | Wrotham Hill | 72 25 31.2 | | | 72 25 31 | |
| | | 180 0 1.45 | 1.3 | +0.15 | | |
| | Frant from Botley Hill | | | | . . . . . | 90364.16 |
| | Frant from Wrotham Hill | | | | . . . . . | 79723.57 |
| XII. | Hollingborn Hill | . . . . . | | | 47 18 59 | |
| | Wrotham Hill | 84 12 24.5 | | | 84 12 23.5 | |
| | Frant | 48 28 37.5 | | | 48 28 37.5 | |
| | | | 1.52 | | | |
| | Hollingborn Hill from Wrotham Hill | | | | . . . . . | 81196.58 |
| | Hollingborn Hill from Frant | | | | . . . . . | 107897.5 |
| XIII. | Fairlight Down | 48 25 53.5 | | | 48 25 55 | |
| | Frant | 79 23 3 | | | 79 23 2 | |
| | Hollingborn Hill | . . . . . | | | 52 11 3 | |
| | | | 2.85 | | | |
| | Fairlight Down from Frant | | | | . . . . . | 113928.2 |
| | Fairlight Down from Hollingborn Hill | | | | . . . . . | 141747.1 |

N°

| N° of triangle. | Names of the stations. | Observed angles. | Spherical excess. | Diff. or error. | Angles corrected for calculation. | Distances. |
|---|---|---|---|---|---|---|
| | | ° ′ ″ | ″ | ″ | ° ′ ″ | Feet. |
| XIV. | Goudhurst . | 35 26 32.5 | | | 35 26 34.5 | |
| | Botley Hill . | 40 4 42 | | | 40 4 42 | |
| | Wrotham Hill . | 104 28 44 | | | 104 28 43.5 | |
| | | 179 59 58.5 | 1.35 | −2.85 | | |
| | Goudhurst from Botley Hill . | | | | . . . . . | 121809.3 |
| | Goudhurst from Wrotham Hill | | | | . . . . . | 80997.43 |
| XV. | Goudhurst . | 72 23 32.5 | | | 72 23 33.87 | |
| | Frant . | 75 33 16 | | | 75 33 13.63 | |
| | Wrotham Hill . | 32 3 12.8 | | | 32 3 12.5 | |
| | | 180 0 1.3 | 0.81 | +0.49 | | |
| | Goudhurst from Frant . | | | | . . . . . | 44389.68 |
| XVI. | Hollingborn Hill | 63 46 44 | | | 63 46 47 | |
| | Wrotham Hill . | 52 9 11.5 | | | 52 9 11 | |
| | Goudhurst . | 64 4 3.5 | | | 64 4 2 | |
| | | 179 59 59 | 1.22 | −2.22 | | |
| | Hollingborn Hill from Goudhurst . | | | | . . . . . | 71296.03 |
| XVII. | Tenterden . | 67 7 55 | | | 67 7 56.46 | |
| | Goudhurst . | 68 13 21 | | | 68 13 19.5 | |
| | Hollingborn Hill . | . . . . . | | | 44 38 44.04 | |
| | | | 0.85 | | | |
| | Tenterden from Goudhurst . | | | | . . . . . | 54374.66 |
| | Tenterden from Hollingborn Hill | | | | . . . . . | 71855.0 |
| XVIII. | Fairlight Down . | . . . . . | | | 35 20 58.42 | |
| | Goudhurst . | 49 39 34 | | | 49 39 35.77 | |
| | Tenterden . | 94 59 26 | | | 94 59 25.81 | |
| | | | 0.91 | | | |
| | Fairlight Down from Goudhurst . | | | | . . . . . | 93625.92 |
| | Fairlight Down from Tenterden . | | | | . . . . . | 71634.73 |

N°

| N° of triangles. | Names of the stations. | Observed angles. | Spherical excess. | Diff. or error. | Angles corrected for calculation. | Distances |
|---|---|---|---|---|---|---|
| | | ° ′ ″ | ″ | ″ | ° ′ ″ | Feet. |
| XIX. | Allington Knoll . | 48 24 38 | | | 48 24 39 | |
| | Hollingborn Hill | . . . . . | | | 40 0 58.96 | |
| | Tenterden . | 91 34 23 | | | 91 34 22.04 | |
| | | | 1.05 | | | |
| | Allington Knoll from { Hollingborn Hill | | | | . . . . . | 96036.45 |
| | Allington Knoll from { Tenterden . | | | | . . . . . | 61775.34 |
| XX. | Lydd . | . . . . . | | | 63 14 9.82 | |
| | Allington Knoll . | 73 0 27.5 | | | 73 0 27 | |
| | Tenterden . | 43 45 22 | | | 43 45 23.18 | |
| | | | 0.67 | | | |
| | Lydd from { Allington Knoll | | | | . . . . . | 47849.27 |
| | Lydd from { Tenterden . | | | | . . . . . | 66166.93 |
| XXI. | Fairlight Down . | 54 59 18.5 | | | 54 59 17.31 | |
| | Lydd . | . . . . . | | | 62 27 50.18 | |
| | Tenterden . | 62 32 53 | | | 62 32 52.51 | |
| | | | 0.99 | | | |
| | Fairlight Down from Lydd | | | | . . . . . | 71689.73 |
| XXII. | Allington Knoll . | 32 59 22.5 | | | 32 59 23 | |
| | Lydd . | 125 42 0.25 | | | 125 42 0 | |
| | Fairlight Down . | . . . . . | | | 21 18 37 | |
| | | | 0.33 | | | |
| | Allington Knoll from Fairlight Down | | | | . . . . . | 106922.5 |
| XXIII. | Lydd . | 43 20 48.25 | | | 43 20 48.5 | |
| | Ruckinge . | 48 58 49.75 | | | 48 58 49.5 | |
| | High Nook near Dymchurch . | 87 40 21.75 | | | 87 40 22 | |
| | | 179 59 59.75 | 0.21 | −0.26 | | |
| | The Base of Verification between High Nook and Ruckinge . . . . | | | | . . . . . | 28532.92 |
| | Lydd from { Ruckinge | | | | . . . . . | 41533.89 |
| | Lydd from { High Nook | | | | . . . . . | 31362.58 |

N°

| N° of triangles. | Names of the ſtations. | Obſerved angles. | Spherical exceſs. | Diff. or error. | Angles corrected for calculation. | Diſtances. |
|---|---|---|---|---|---|---|
| | | ° ′ ″ | ″ | ″ | ° ′ ″ | Feet. |
| XXIV. | Allington Knoll . | 91 27 20 | | | 91 27 19.5 | |
| | Ruckinge . | 54 19 17 | | | 54 19 18.5 | |
| | High Nook . | 34 13 21 | | | 34 13 22 | |
| | | 179 59 58 | 0.09 | − 2.09 | | |
| | Allington Knoll from | | High Nook | | . . . . | 23184.93 |
| | | | Ruckinge | | . . . . | 16052.44 |

Hence, in the quadrilateral formed by *High Nook*, *Ruckinge*, *Lydd*, and *Allington Knoll*, making uſe of the two obtuſe angles, as contained within their now reſpective known ſides, we have for the mean diſtance of the points of the acute angles, at Lydd and Allington Knoll, repreſented by a dotted line in the plan of the triangles, 47849.27 feet. This diſtance agrees accurately with the length of the ſame ſide in the XXth triangle, as given by the baſe meaſured on Hounſlow Heath. Here however it is to be remarked, that, in order to produce this agreement, the angle at Hollingborn Hill, between Allington Knoll and Fairlight Down, has been made $48^\circ\ 56'\ 28''$ inſtead of $48^\circ\ 56'\ 31''\frac{1}{2}$, being a difference of $3''\frac{1}{2}+$, which, according to obſervation, it ſhould have been. Had not this reduction been made, the diſtance between Allington Knoll and Fairlight Down, being one of the ſides of the XXIId triangle, would have been 106924 feet, that is to ſay, $1\frac{1}{2}$ foot longer. Now, ſince this ſide, compared with the baſe of verification, bears nearly the proportion of four to one, it follows, that the real difference between the meaſured length of that baſe, and its computed length deduced from that on Hounſlow Heath, ſeventy miles to the weſtward, or of

either base with respect to its opposite one, amounts only to about $4\frac{1}{2}$ inches.

| N° of triangles. | Names of the stations. | Observed angles. | Spherical excess. | Diff. or error. | Angles corrected for calculation. | Distances. |
|---|---|---|---|---|---|---|
| | | ° ′ ″ | ″ | ″ | ° ′ ″ | Feet. |
| XXV. | Folkstone Turnpike | 24 17 6.25 | | | 24 17 6.25 | |
| | Allington Knoll | 76 1 54 | | | 76 1 53.25 | |
| | High Nook . | 79 41 0.75 | | | 79 41 0.5 | |
| | | 180 0 1 | 0.29 | +0.71 | | |
| | Folkstone Turnpike from Allington Knoll | | | | . . . . . | 55461.7 |
| | Folkstone Turnpike from High Nook . | | | | . . . . . | 54706.0 |
| XXVI. | Folkstone Turnpike | . . . . . | | | 32 6 56.89 | |
| | Allington Knoll | 109 50 40 | | | 109 50 39.35 | |
| | Lydd . . | 38 2 24 | | | 38 2 23.76 | |
| | | | 0.59 | | | |
| | Folkstone Turnpike from Lydd | | | | . . . . . | 84659.88 |
| XXVII. | Padlesworth . | 108 9 34.5 | | | 108 9 34.5 | |
| | High Nook . | . . . . . | | | 14 48 25.5 | |
| | Folkstone Turnpike | 57 2 0 | | | 57 2 0 | |
| | | | 0.16 | | | |
| | Padlesworth from High Nook . | | | | . . . . . | 48303.7 |
| | Padlesworth from Folkstone Turnpike | | | | . . . . . | 14713.82 |
| XXVIII. | Padlesworth . | 105 29 40.5 | | | 105 29 40 | |
| | Lydd . | 9 38 29 | | | 9 38 29.36 | |
| | Folkstone Turnpike | . . . . . | | | 64 51 50.64 | |
| | | | 0.27 | | | |
| | Padlesworth from Lydd . . | | | | . . . . . | 79533.34 |
| | Padlesworth from Folkstone Turnpike | | | | . . . . . | 14713.82 |
| XXIX. | Padlesworth . | 12 16 3 | | | 12 16 2.65 | |
| | Lydd . | 154 5 54.75 | | | 154 5 54.4 | |
| | Fairlight Down | . . . . . | | | 13 38 2.95 | |
| | | | 0.59 | | | |
| | Padlesworth from Fairlight Down | | | | . . . . . | 186113.0 |

| N° of triangles. | Names of the ſtations. | Obſerved angles. | Spherical exceſs. | Diff. or error. | Angles corrected for calculation. | Diſtances. |
|---|---|---|---|---|---|---|
| | | ° ′ ″ | ″ | ″ | ° ′ ″ | Feet. |
| XXX. | Swingfield . | 48 38 15 | | | 48 38 15 | |
| | Padleſworth . | 70 54 5.5 | | | 70 54 5.5 | |
| | Folkſtone Turnpike | 60 27 39.5 | | | 60 27 39.5 | |
| | | 180 0 0 | 0.06 | –0.06 | | |
| | Swingfield from Padleſworth . | | | | . . . . | 17056.06 |
| | Swingfield from Folkſtone Turnpike | | | | . . . . | 18525.15 |
| XXXI. | Dover Caſtle, North Turret . | 34 39 26.5 | | | 34 39 26.5 | |
| | Swingfield . | 75 36 40 | | | 75 36 40 | |
| | Folkſtone . | 69 43 53.5 | | | 69 43 53.5 | |
| | | 180 0 0 | 0.13 | –0.13 | | |
| | Dover Caſtle from Swingfield . | | | | . . . . | 30559.32 |
| | Dover Caſtle from Folkſtone Turnpike | | | | . . . . | 31554.58 |

Hence, in the quadrilateral formed by *Folkſtone Turnpike, Swingfield, Padleſworth,* and the *North Turret of the Keep of Dover Caſtle,* making uſe of the two obtuſe angles, as contained within their reſpective known ſides, we have for the mean diſtance of the points of the acute angles at Padleſworth and Dover Caſtle 42561.18 feet; and hence, in the triangle, *Dover, Folkſtone Turnpike, Padleſworth,* we have the acute angle at Dover $15° 18' 44''\frac{1}{2}$, and that at Padleſworth $34° 29' 42''\frac{1}{2}$, as were repeatedly obſerved.

| | | | | | | |
|---|---|---|---|---|---|---|
| XXXII. | Dover Caſtle . | . . . . | | | 21 37 55.42 | |
| | Padleſworth . | 152 15 25.5 | | | 152 15 25.15 | |
| | Fairlight Down | . . . . | | | 6 6 39.43 | |
| | | | 0.69 | | | |
| | Dover Caſtle from Fairlight Down | | | | . . . . | 186113.0 |

| N° of triangles. | Names of the stations. | Observed angles. | Spherical excess. | Diff. or error. | Angles corrected for calculation. | Distances. |
|---|---|---|---|---|---|---|
| | | ° ′ ″ | ″ | ″ | ° ′ ″ | Feet. |
| | Dover Castle . | . . . . . | | | 87 30 29.58 | |
| | Fairlight Down | . . . . | | | 43 19 58.52 | |
| | Montlambert . | . . . . | | | 49 9 31.9 | |
| XXXIII. | | | 7.4 | | | |
| | Montlambert from Dover Castle . | | | | . . . . . | 168821.07 |
| | Montlambert from Fairlight Down | | | | . . . . | 245777.5 |
| | Fairlight Down | . . . . . | | | 25 33 55.02 | |
| | Dover Castle . | . . . . . | | | 110 55 29.83 | |
| | Blancnez . | . . . . . | | | 43 30 35.15 | |
| XXXIV. | | | 4.78 | | | |
| | Blancnez from Fairlight Down | | | | . . . . | 252469.9 |
| | Blancnez from Dover Castle . | | | | . . . . . | 116655.93 |
| | Dover Castle . | 23 25 0.25 | | | 23 25 0.25 | |
| | Montlambert . | . . . . . | | | 36 53 18.11 | |
| | Blancnez . | . . . . . | | | 119 41 41.64 | |
| XXXV. | | | 1.84 | | | |
| | Blancnez from Montlambert | | | | . . . . . | 77235.0 |

In this last triangle, the angle at Blancnez, as determined with great care from a mean of many obſervations, by the French Academicians, was found to be 119° 41′ 28″.9, that is to ſay, 12″.7 leſs than what reſults from our obſervations acroſs the Channel. This difference, which is the *maximum* of the error between us in the joint operation, being ſmall, and of no real importance one way or other, with regard to the main point in diſcuſſion, ſince it only varies the diſtance between Blancnez and Montlambert two or three feet, and the longeſt ſides of the triangles, which connect the two Coaſts, by eight or nine; it has therefore been judged beſt not

to

to make any alteration whatever on account of that difference (except as will be mentioned underneath), but to proceed with our own ſcale of diſtances for fixing the relative ſituation of Dunkirk; making uſe, nevertheleſs, in the firſt ſeven following triangles, from the XXXVIth to the XLIId incluſive, of the angles as ultimately ſettled by the late French operations, which Comte DE CASSINI has been ſo obliging as to ſupply us with for that purpoſe. The angles of the XLIIId and XLIVth triangles are taken from M. CASSINI DE THURY'S Book *(La Méridienne verifiée)*; and thoſe of the XLVth triangle reſult from the combined operation.

In conformity with the exception above alluded to, we have, in the XXXVIth triangle, added 3″ to our angle obſerved at Dover between Blancnez and the *flèche* of the ſpire of Notre Dame at Calais; that is to ſay, inſtead of 12° 46′ 39″ it has been made 12° 46′ 42″, on a ſuppoſition, that the ſpire may overhang ſo much from the perpendicular towards Blancnez: becauſe the ſpace between the poſition of the white light on the gallery, and the axis of the ſpire, being carefully meaſured by Dr. BLAGDEN, correſponds to an angle of 9″, whereas the obſervation gave only a difference of 6″.

| N° of triangles. | Names of the ſtations. | Angles corrected for calculation. | Diſtances. |
|---|---|---|---|
| | | ° ′ ″ | Feet. |
| XXXVI. | N.D. at Calais | 47 27 6 | |
| | Blancnez ſignal | 119 46 12 | |
| | Dover Caſtle 12° 46′ 39″ | 12 46 42 | |
| | N.D. at Calais from Dover Caſtle | | 137449.9 |
| | N.D. at Calais from Blancnez ſignal | | 35023.3 |
| | Exceſs above 180° = 0″.84. | | |

| N° of triangles. | Names of the ſtations. | Angles corrected for calculation. | Diſtances. |
|---|---|---|---|
| | | ° ′ ″ | Feet. |
| XXXVII. | Fiennes ſignal. . . . | 94 26 27.5 | |
| | Blancnez ſignal . . . | 51 18 27.3 | |
| | Montlambert ſignal . . | 34 15 5.2 | |
| | Fiennes ſignal from Blancnez ſignal . | . . . . . | 43600.8 |
| | Fiennes ſignal from Montlambert ſignal | . . . . . | 60464.4 |
| XXXVIII. | N.D. at Calais . . . | 64 21 43.1 | |
| | Fiennes . . . . | 46 24 25.2 | |
| | Blancnez . . . | 69 13 51.7 | |
| | N.D. at Calais from Fiennes . | . . . . . | 45219.6 |
| XXXIX. | Watten . . . | 27 37 14.8 | |
| | N.D. at Calais . . . | 66 30 36.2 | |
| | Fiennes . . . | 85 52 9 | |
| | Watten from Fiennes . | . . . . . | 89453.5 |
| | Watten from N.D. at Calais | . . . . . | 97283.0 |
| XL. | Dunkirk . . . | 51 39 12 | |
| | Watten . . . | 85 57 46.5 | |
| | N.D. at Calais . . . | 42 23 1.5 | |
| | Dunkirk from N.D. at Calais | . . . . | 123734.8 |
| | Dunkirk from Watten . | . . . . | 83616.2 |
| XLI. | Mont-Caſſel . . . | 63 24 50 | |
| | Dunkirk . . . | 42 7 12 | |
| | Watten . . . | 74 27 58 | |
| | Mont-Caſſel from Watten . | . . . . . | 62711.1 |
| | Mont-Caſſel from Dunkirk | . . . . . | 90087.5 |
| XLII. | Hondſcôte . . . | 93 31 34.1 | |
| | Dunkirk . . . | 51 7 4.5 | |
| | Mont-Caſſel . . . | 35 21 21.4 | |
| | Hondſcôte from Mont-Caſſel | . . . . . | 70260.7 |
| | Hondſcôte from Dunkirk . | . . . . . | 52228.4 |
| XLIII. | Dunes, ſignal at the eaſt end of the baſe . | 81 57 30 | |
| | Dunkirk . . . | 48 43 23 | |
| | Hondſcôte . . . | 49 19 7 | |
| | Signal on the Dunes from Hondſcôte | . . . . . | 39641.0 |
| | Signal on the Dunes from Dunkirk . | . . . . . | 40000.5 |
| XLIV. | Dunes ſignal . . . | 5 19 47 | |
| | Fort Revers, weſt end of the baſe . | 90 17 29 | |
| | Dunkirk . . . | 84 22 44 | |
| | Dunkirk from Signal on the Dunes | . . . . . | 40000.5 |
| | Dunkirk from Fort Revers . | . . . . . | 3715.6 |
| | Fort Revers from the Dunes, the meaſured baſe | . . . . . | 39808.7 |
| | But this baſe by meaſurement was . | . . . . . | 39801.7 |
| | The difference is . | . . . . . | 7.0 |

In

In order to complete the triangular connection between Greenwich and Paris, there remains yet one triangle more (the XLVth) to be given, whereby we ſhall be enabled to connect the point M near Dunkirk with Dover. For this purpoſe it is neceſſary to make ſome remarks on the Dunkirk baſe; and alſo to ſhew, from the French operations, how the point M is ſituated with reſpect to Paris, Dunkirk, and Calais.

M. Cassini de Thury, in his book already quoted (p. 23. and 54.) has informed us of the manner in which this baſe on the Strand near Dunkirk was meaſured; and that its mean length amounted to 6224.36 toiſes, which are equal to 39801.7 Engliſh feet. Thus it appears, that there is a difference as above ſtated of ſeven feet in defect, between the meaſured and computed length of the laſt ſide of a combined ſeries of 44 Britiſh and French triangles, depending on a baſe meaſured on Hounſlow Heath, and verified by another meaſured in Romney Marſh. But a ſeries of 24 French triangles, founded on a baſe meaſured near Paris, and corrected by another executed near Amiens, gives for the length of the ſame baſe near Dunkirk 39809.94 Engliſh feet, and conſequently only an exceſs of 15 inches with regard to our reſult. This very near agreement in the determination of the ſame length by two different ſerieſes of triangles, whoſe extremities are ſituated at ſo great a diſtance from each other, ſufficiently proves the excellency of trigonometrical meaſurement, and ſhews to what a wonderful degree of accuracy operations of this ſort may be brought when fine inſtruments are made uſe of, and great care beſtowed in the application of them. Doubtleſs ſmall errors may have ariſen in the progreſs of the work, unſeen on both ſides; but theſe falling ſometimes one way and ſometimes the other, they ſeem ſo far to have compenſated for, or deſtroyed each other,

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that their effects have almoſt wholly diſappeared. With regard to the deficiency of ſeven feet found between the actual meaſurement with the deal rods, on the Strand near Dunkirk, and the trigonometrical reſult, it is neceſſary to call to remembrance what would have happened, if the baſe on Hounſlow Heath had been meaſured with our deal rods, when in their greateſt ſtate of expanſion from the moiſture they had imbibed. In the volume of Philoſophical Tranſactions for 1785, p. 438. it has been ſhewn, that our baſe would thereby have been rendered more than ſeven feet and a half too ſhort. Now, although the French rods were covered with ſeveral coats of oil-paint to prevent their imbibing the ſalt water, which we are told reſted on the Strand at particular places ſix inches deep; yet it is preſumable, that it would be impoſſible to prevent it from entering by the extremities at the junction of the ferrules, and extending along the fibres, underneath the paint. Hence, in all likelihood, the intended remedy would prove worſe than the diſeaſe: for the paint might prevent the rods from drying ſo ſoon as they otherwiſe would have done, and thereby the meaſurement would be given ſtill ſhorter than if no paint had been applied. Whether this ſuppoſition may be thought to be well founded or not, is left to the determination of thoſe who are converſant in matters of this ſort. But the curious fact, one way or other, might be aſcertained by means of ſuch a ſteel chain as ours, in the ſpace of one or two days at moſt. For, ſuppoſing the extremities of the baſe between *Fort Revers* and the *Dunes* to be accurately known, and the *allignement* traced out on the Strand only with camp colours, placed at reaſonable diſtances from each other, and a moveable cord, by the ſimple application of the chain on the common ſurface, without any extraordinary apparatus whatever, for-

wards and again backwards, the diſtance might certainly be determined within a foot of the truth. And hence the importance is obvious of having at all times ſo accurate and eaſy a mode of meaſurement.

On due conſideration of all theſe circumſtances, it will not be thought ſurpriſing, that in fixing the ſituation of Dunkirk and the point M near it, where the meridian of the Royal Obſervatory at Paris interſects a line drawn from thence to N.D. at Calais, the *Dunkirk baſe*, with the corrections depending upon it, are here rejected; and that the ſcale of diſtances furniſhed by the Britiſh triangles is adhered to, as not differing ſenſibly from the mean reſult given by the other two French meaſurements.

From M. DE CASSINI's Book, *La Méridienne Vérifiée*, p. 51. 53. and 56. it appears, that Dunkirk (rejecting the corrections formerly alluded to) is north from the Royal Obſervatory at Paris 125522.2 toiſes, which are equal to 133775.3 fathoms. And from p. 51. and 57. it further appears that, by the mean of two ſuites of triangles, Dunkirk is eaſt from the meridian of Paris 1420.41 toiſes, which are equal to 1513.8 fathoms. Again, at p. 276. of the ſame Book, Dunkirk is ſaid to be north 125517 toiſes, and eaſt 1430 toiſes, which are reſpectively equal to 133769.7 and 1524.2 fathoms. And, laſtly, at p. 36, of the *Déſcription Géometrique de la France*, of the ſame Author, publiſhed in 1783, and which being the lateſt ſhould of courſe be the moſt correct work, Dunkirk is made north from the Royal Obſervatory 125495 toiſes, and eaſt from its meridian only 1416 toiſes, which are reſpectively equal to 133746.3 and 1509.1 fathoms. Now, without pretending here to enter into the inveſtigation of the various corrections + and − which have been applied to the angles of the tri-

angles, to bring out these different results, we shall abide by the first, that being immediately produced by the mean of the observations without any arbitrary correction whatever; and being, with regard to easting, nearly a mean between the two extremes; and since we have it in our power to settle with great precision the longitude of Dunkirk, and likewise the point M, with regard to Greenwich, we shall then be enabled to determine the difference of longitude between the two Royal Observatories within a mere trifle of the truth.

By Comte DE CASSINI's triangles, executed in the autumn of 1787, and communicated in January 1789, it appears, that Hondscôte is south-eastward from the meridian of Dunkirk 67° 53′ 20″; which angle being subtracted from the total angle between Hondscôte and Calais 144° 53′ 28″.5, being the sum of the three angles at Dunkirk in the XLth, XLIst, and XLIId triangles, there remains 77° 0′ 8″.5 for the angle that Calais is south-westward from the meridian of Dunkirk. And this last angle being again subtracted from 180°, we have 102° 59′ 51″.5 for the angle between the same meridian produced northward, and a line drawn from Dunkirk through M to Calais.

Now the two distances 133775.3 and 1513.8 fathoms being severally reduced in the proportion of 39809.94 to 39808.7 the two lengths assignable to the base on the Strand near Dunkirk, as formerly established, we have 133771.1 fathoms for the distance in British measures of the parallel of Dunkirk from that of the Royal Observatory at Paris; and 1513.75 fathoms for the distance of Dunkirk eastward from its meridian. Again, making use of the angle 77° 0′ 8″.5, and its complement to 90° = 12° 59′ 51″.5, we have 1553.56 fathoms for the direct distance between Dunkirk and the point M;

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alſo 358.6 fathoms for the ſpace that M is ſouthward from Dunkirk. But the diſtance of Dunkirk from the Royal Obſervatory at Paris, given in the 276th page of M. CASSINI DE THURY's Book, when reduced in the proportion of the two baſes becomes 133765.7 fathoms, and taking a mean between this number and that formerly found 133771.1 fathoms, we have for the mean diſtance of Dunkirk from the Obſervatory 133768.4 fathoms, from which ſubtracting 358.6 fathoms, the mean ſouthing of M from Dunkirk, there will ultimately remain 133409.8 fathoms for the diſtance between M and the Royal Obſervatory at Paris, meaſured on the meridian.

Now, ſince in the XLth triangle we have the diſtance of N.D. at Calais from Dunkirk 123734.8 feet, if from this number we ſubtract 1553.56 fathoms = 9321.36 feet, there will remain 114413.4 for the diſtance of the point M from Calais. Thus, with the ſupplemental angle to 360° at Calais, *viz.* 139° 17′ 33″.2, contained within its now known ſides, we are finally enabled to complete the XLVth triangle, and thereby to determine the ſituation of M with regard to Dover.

| | | | ° | ′ | ″ | |
|---|---|---|---|---|---|---|
| XLV. | N.D. at Calais | | 139 | 17 | 33.2 | The ſpherical exceſs above 180° = 2″.42. |
| | Dover Caſtle | | 18 | 24 | 37.3 | |
| | M near Dunkirk | | 22 | 17 | 49.5 | |
| | | | | | | Feet. |
| | N.D. at Calais from | Dover Caſtle | | | | 137449.9 |
| | | M near Dunkirk | | | | 114413.4 |
| | Dover Caſtle from M | | | | | 236273.7 |
| | Alſo, Dover is from Dunkirk | | | | | 243291.3 |

ART. III. *Reſult of the trigonometrical operation, in as far as relates to the geodetical ſituation of the different ſtations, with regard to the Royal Obſervatory at Greenwich.*

Having, by the preceding calculations of the lengths of the ſides and meaſures of the angles of a continued ſeries of

 forty-

forty-five triangles, determined the relative ſituation of every ſtation with regard to thoſe neareſt adjacent to it, we are next to ſhew, from theſe *data*, and the angles which Norwood and Severndroog Caſtle make with the meridian of Greenwich Obſervatory, the ſituation of each ſtation with reſpect to that meridian, to its perpendicular, and alſo the direct or diagonal diſtance with the bearing from the Obſervatory itſelf. Theſe various determinations are contained in the ſix firſt columns towards the left hand of the annexed table of reſults, wherein the ſtations are likewiſe diſtinguiſhed into two ſets, as ſituated to the weſtward or eaſtward of Greenwich.

By means of a ſcaffold, perfectly ſimilar in principle to that formerly deſcribed, but more ſlight as being made for the temporary purpoſe only, the ſtand of the inſtrument was raiſed to the height of thirty-eight feet above the floor of the tranſit-room of the Obſervatory. At this elevation all the ſurrounding objects which we wiſhed to obſerve (St. Paul's excepted, which is hidden by the camera-turret of the great room) could be diſtinctly ſeen, and the angles between them and the ſouth meridian mark accurately meaſured. As that mark is but at a ſhort diſtance, namely, about 1500 feet from the tranſit, and conſequently $\frac{1}{10}$th of an inch correſponding to about a ſecond of an angle on the mark, it was therefore very neceſſary that the center of the inſtrument ſhould be brought with great preciſion over the center of the axis of the tranſit-teleſcope underneath. In this operation, and indeed in every other while at Greenwich, the Aſtronomer Royal gave us his beſt aſſiſtance. In the firſt place, the central point of the axis was determined by the interſection of diagonal lines drawn acroſs the ſquare part in the middle. On this ſquare part, when the teleſcope was in its horizontal poſition, a baſon of quickſilver was placed,

placed, having a small cross made of two thin bits of wood fitted to the inside of the bason, and lying very near the surface of the quicksilver, in such a manner as to make the center of the cross co-incide with the intersection on the brass underneath. A small perspective glass being then fixed in a moveable board under the center of the instrument, this was made to slide at right angles to itself in the direction of the meridian and that of the axis of the transit, until the center of the cross coincided with the axis of vision in looking downwards. The board being there fastened, and the perspective removed, the intersection of silk threads stretched across the board, marked very accurately the point corresponding with the center of the transit, over which the center of our instrument was brought by the help of the plummet. The second method was still more direct. Dr. MASKELYNE had an object glass prepared for his transit telescope of a focus suited to the vertical height of the stand of our instrument above it. This glass being applied to the transit, and the aperture contracted by a piece of pasteboard with a circular hole in the middle, a very small pin-hole being likewise made in the board at top, the same was gradually moved by directions from the observer below, looking through the telescope in its vertical position, until the pin-hole nicely co-incided with the axis of vision. The instrument was then brought as before, by the help of the plummet, exactly over the pin-hole. In this manner, which was that adhered to, no doubt remained of more than about $\frac{1}{100}$th part of an inch, with respect to the center of the instrument being in the intersection of two vertical planes passing through the axis of vision and that of motion of the transit underneath. After having remained a week, the co-incidence of the pin-

 hole

hole with the axis of viſion of the teleſcope was tried, and found to have ſuffered no alteration.

In the VIIIth triangle, the angle at Greenwich, between Severndroog Caſtle on Shooter's Hill and the ſtation on Norwood heights, hath been ſhewn to be 111° 56′ 50″. By ſeveral obſervations on different parts of the circle, Norwood ſtation was found to be weſtward from the meridian 38° 7′ 16″, which of courſe leaves for the angle that Severndroog Caſtle is eaſtward from it 73° 49′ 34″; and either of theſe two angles is ſuppoſed to be within a very ſmall fraction of a ſecond of the truth.

Now, with the ſides and angles of the ſeries of triangles already known, and the angle 38° 7′ 16″ now given, which Norwood makes with the meridian of Greenwich towards the weſt, it will be ſufficiently obvious to thoſe who are in the leaſt acquainted with plane trigonometry, that the diſtances of that or of any other ſtation of the ſeries, from the meridian of Greenwich and from its perpendicular, are eaſily obtained. Nevertheleſs, that thoſe who are but little converſant with matters of this ſort may themſelves be able to examine the computations whereby the columns towards the left-hand of the annexed table have been ſupplied, we ſhall give one example, which will ſerve for the whole.

Suppoſe (Plate X. fig. 1.) GM to repreſent the meridian of the tranſit-room at Greenwich; GW the perpendicular to that meridian produced indefinitely towards the weſt; N the ſtation at Norwood, and H that at Hundred Acres, whoſe diſtances are required, that is to ſay, weſtward from the meridian, and ſouthward from the perpendicular: then through the ſtations N and H, let dotted lines be drawn parallel to the meridian and perpendicular reſpectively, whereby four parallelograms will

will be formed. In the firſt, or that which is neareſt to Greenwich, having GN in the VIIIth triangle given $= 31274.48$ feet, and the angle $NGm = 38° 7' 16''$, with its complement $NGp = 51° 52' 44''$, it follows, that $Nm$ repreſenting the diſtance of Norwood weſtward from the meridian is $= 19306.54$ feet; and $Np$ repreſenting its diſtance ſouthward from the perpendicular is $= 24603.86$ feet. Again, by attending to, and ſumming up the angles round the point N, we ſhall find the angle $GNH = 175° 44' 36''.82$, which wanting $4° 15' 23''.18$ of $180°$, ſhews that the direction of the ſide NH inclines ſo much more to the weſtward than the angle $NGm$. Wherefore $NGm = 38° 7' 16'' + 4° 15' 23''.18 = 42° 22' 39''.18 = HNs$, is the angle which the line NH makes with $Ns$, a parallel to the meridian of Greenwich drawn through the point N. Now, in the ſupplemental parallelogram, having the diagonal $NH = 35648.21$ given in the VIIth triangle, and the angle $HNs = 42° 22' 39''.18$, alſo its complement $= 47° 37' 20''.82$, making uſe of NH as radius, and theſe two laſt angles reſpectively, we have $sH = 24027.36$ feet for the ſpace that H is more weſtward than N; and $wH = 26334.04$ feet, that H is more ſouthward than N. Hence $mN + sH = 43333.9$ feet is the ſpace that H is to the weſtward of Greenwich; and $pN + wH = 50937.9$ feet is the ſpace that H is ſouthward from the perpendicular to the meridian of Greenwich. Laſtly, with theſe two given ſides, and the contained angle $90°$, we find the angle $MGH = 40° 23' 18''.54$, that Hundred Acres is ſouth-weſtward from the meridian of Greenwich; whence the direct or diagonal diſtance $GH = 66876.73$ feet. Now, by referring to the table of reſults, for the two firſt ſtations weſtward from Greenwich, the numbers brought out in this example will be found in the left-hand columns under their reſpective heads, and ſo it would be with

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the reſt. In another place we ſhall have occaſion to point out, how the columns towards the right-hand of the ſaid table have been filled up.

## SECTION FIFTH.

*On the difference between horizontal angles on a ſphere and ſpheroid. Plate X.*

IN the Paper of 1787, various computations were given concerning the figure and dimenſions of the earth, founded chiefly on the actual meaſurement of different arcs of the meridian in different latitudes, ſome of them very remote from each other. From the alternate compariſon of theſe reſults it appeared, that the figure aſſigned to the earth by M. Bouguer in his ſecond ſpheroid agreed better with theſe meaſured portions of the curve, as ſo many *data*, than any of the other *hypotheſes*. Hence it naturally occurred, that the trigonometrical operation which we were then about to commence might probably throw ſome further light on this intricate ſubject, which, for a great length of time, has engaged a conſiderable ſhare of the attention of the ſcientific world. In the conſideration of this matter, a new and curious point, not formerly attended to, and immediately connected with our operation, preſented itſelf for inveſtigation, viz. *ſuppoſing the earth to be a ſpheroid, ſuch as M.* Bouguer's, *conſiderably flattened at the poles, what might be the difference between horizontal angles obſerved with a fine inſtrument on that ſpheroid, and on a ſphere?*

The

The following ſolution * of that important problem, being the only unexceptionable one that I have received, is here given in the author's (Mr. DALBY's) own words.

Let CE and CP (Plate X. fig. 2.) repreſent the equatorial and polar ſemi-diameters of the earth, conſidered as a ſpheroid flattened at the poles; PE and PN two meridians; *pe* and *pn* two correſponding ones (that is, in the ſame planes) on a ſphere, having the ſame center C. Let the points *a*, *b*, A, B, on the ſphere and ſpheroid have the ſame latitudes reſpectively. Draw the radii *a*C, *b*C, and the verticals AG, BW.

Then, becauſe the angles AON, BDE, in the ſpheroid, are always equal to the latitudes of the points A, B, theſe angles are therefore reſpectively equal to the angles *a*C*n*, *b*C*e*, in the ſphere, and conſequently the verticals AG, BW, are parallel to the radii *a*C, *b*C.

Let the latitude of B or *b* be greater than that of A or *a*; and let it be required to make the horizontal angle PA*r* on the ſpheroid equal to the angle *pab*, or what the horizontal angle would be on the ſphere.

Becauſe the angle *pab* is meaſured by the inclination of the planes, *a*C*b*, *a*C*p*, and AG is the common interſection of all the planes of the vertical circles at A, and is parallel to *a*C, and in the ſame plane; therefore, when the horizontal angle PA*r* is equal to the angle *pab*, the planes GA*r*, CA*b*, muſt be parallel to each other; and conſequently G*r*, the line where the plane GA*r* interſects the plane of the meridian EP, is pa-

* From my correſpondence by letter, and otherwiſe, with Dr. MASKELYNE, I had reaſon to hope, that he would have favoured me with ſome communication on this ſubject. No doubt, he has been prevented by other buſineſs; but he will probably give his method of ſolving ſpheroidical triangles to the Royal Society on ſome future occaſion.

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rallel to C*b* in the ſphere, or WB in the ſpheroid. Hence, if from G, the point where the vertical AG meets the axis, we draw G*r* parallel to the vertical BW, it will give the point *r* in the meridian EP, making the horizontal angle PA*r* equal to the angle *pab*, or what the horizontal angle would be on the ſphere.

In like manner, if the angle PB*v* is to be made equal to the angle *pba*, W*v* muſt be drawn parallel to GA, and the plane *v*WB will be parallel to the plane AG*r*; and therefore the angles of the ſpheroidical triangles PA*r*, P*v*B, as meaſured by the inclination of the planes, are equal to each other reſpectively, and equal to the ſpherical angles of the triangle *pab*.

From hence it follows, that if A be the place of an inſtrument which meaſures horizontal angles in the meridian NP on a ſpheroid, and BT a flag-ſtaff ſet perpendicular to the ſurface of the earth on another meridian EP, the obſerved horizontal angle PAB, between the meridian PA and the flag-ſtaff BT, will be greater than it would be on a ſphere (the latitudes and longitudes being the ſame in both) as long as the latitude of the flag-ſtaff is greater than that of the inſtrument, the exceſs being the angle BA*r*; but if the latitude of the inſtrument is the greateſt, as ſuppoſe it was at B, and the flag-ſtaff at A, then the obſerved angle PBA will be leſs than it would be on the ſphere, the defect being the angle AB*v*, which, becauſe the planes W*v*B, GA*r*, are parallel, will be the ſame as the exceſs on the other ſide.

If the latitudes of the points A and B are the ſame, the planes W*v*B, GA*r*, will co-incide, or the verticals will meet in the ſame point in the axis, and therefore the obſerved angles will be equal to each other, and the ſame as they would be if obſerved on a ſphere.

Becauſe

Becaufe AG, *v*W, BW, *r*G, are parallel to *a*C, *b*C, the angles *v*WB, AG*r*, will be equal to the angle *a*C*b*, or arc *ab*, therefore the arcs *v*B, A*r*, will each be equal to the arc *ab*; that is, they are arcs of great circles of the fame value, intercepted between the meridians PN, PE, at B and A.

Draw GR perpendicular to the vertical BW; then, becaufe BW and *r*G are parallel, it will alfo be perpendicular to *r*G; and becaufe the axis PW is the common interfection of the planes of all the meridians, and BW, *r*W, are in the plane of the meridian PB, therefore GR is in that plane; and becaufe the angle WB*r*, made by the vertical and meridian, and the angle GRB, are right ones, therefore GR is equal to the arc B*r* nearly, and confequently is nearly equal to what fubtends the difference of the horizontal angles on the fphere and fpheroid.

And if GS be perpendicular to the vertical GA, it will be equal to the arc A*v* nearly, and therefore GR, GS, or the arcs B*r*, A*v*, will be as the cofines of the latitudes of B and A.

Draw AK the tangent to the meridian at A, to meet the axis CP produced; alfo draw AH perpendicular to the vertical AG, to meet G*r* produced; through H draw KHT, and join AT. Then, becaufe the points K, H, are in the plane of the horizon of A, the line KHT will be in that plane; and becaufe *r*H and BT are in the plane of the meridian BP, therefore HT is alfo in the fame plane, and is what fubtends the angle TAH, the true difference of the horizontal angles, which, when the fpheroid is given, may be determined as follows.

From the nature of the fpheroid, find the length of the vertical AG; alfo the points G and W, where the verticals meet the axis: then, becaufe the angle AKG is equal to the latitude of A, and AGH is its complement, GK and AK will be given.

 Let

Let $a$ and $b$ on the ſphere have the ſame latitudes and difference of longitude as A and B on the ſpheroid, and find the angles $pab$, $pba$, and the arc $ab$, or angle $aCb$; then becauſe AG is given, and the angle AGH equal to the angle $aCb$, AH will be given; with AH and AK, and the included angle HAK (equal to the ſpherical angle $bap$) find the angle AHK, and alſo KH; then, becauſe the triangles KHG, KTW, are in the ſame plane (that of the meridian BP) and GH is parallel to WT, theſe triangles will be ſimilar. Hence GK : HK :: WG : TH; now HA, HT, and the included angle AHT (the complement of AHK) being given, the angle TAH, the difference of the horizontal angles, will be given.

*Example.* Let the ſpheroid be that of M. BOUGUER; and let the latitude of A be 49° 40′, of B 50°, and their difference of longitude 0° 30′.

From the nature of the ſpheroid, the radii of curvature of the meridian at the equator and the pole, will be 3465507 and 3524069 fathoms nearly; their difference is 58562 fathoms, the length of the evolute of the meridian; and the vertical $AG = 3465507 + \frac{8}{15} \times 58562 + \frac{1}{5} \times 58562 \times \overline{\text{ſine } 49° \, 40'}|^4 + \frac{4}{15} \times 58562 \times \overline{\text{ſine } 49° \, 40'}|^2 = 3509769.5$ fathoms; alſo $OG = \frac{8}{15} \times 58562 + \frac{4}{15} \times 58562 \times \overline{\text{ſine } 49° \, 40'}|^2 = 40307.66$ fathoms; and $DW = \frac{8}{15} \times 58562 + \frac{4}{15} \times 58562 \times \overline{\text{ſine } 50°}|^2 = 40397.23$ fathoms. Now, the angles GOC, WDC, being = the latitudes of A and B, we get CG = 30726.16, and CW = 30946.08 fathoms, their difference being 219.92 fathoms = GW.

The ſides $pa$, $pb$, being equal to 40° 20′ and 40° reſpectively, and the included angle = 30′, will give the angle $pab$ = 43° 51′.48″.2, the angle $pba$ = 135° 45′ 16″.2, and $ab$, or the angle $aCb$, = 27′ 49″.7.

Now

Now, by proceeding as directed above, we get GK = 4604232.9, AK = 2980006.3, and AH = 28412.2 fathoms. Hence the angle AHK = 135° 45′ 20″.08, and KH = 2979745.4 fathoms; whence HT = 141.37 fathoms. This, with AH, and the included angle AHT = 44° 14′ 39″.92 (the complement of AHK) give the angle TAH = 11′ 58″.9, the difference between the horizontal angles on the ſphere and ſpheroid.

Hence the obſerved angles at A and B would be 43° 51′ 48″.2 + 11′ 58″.9 = 44° 3′ 47″.1, and 135 45′ 16″.2 − 11′ 58″.9 = 135° 33′ 17″.3.

If the figure is an ellipſoid having the ſame *axes*, the angle TAH will be found = 8′ 4″.4.

It may be remarked, that the angle TAH, or the horizontal angle TAK, diminiſhes or augments as the point obſerved in TB is elevated or depreſſed; this variation is however too ſmall to be worth attending to in practice, as may be ſhewn in the following manner.

Let the ſpheroid be M. Bouguer's (becauſe the difference will be greater than on an ellipſoid); and let the points A, B, fig. 3. have the ſame latitudes and difference of longitude as above; alſo, let BT be the flag-ſtaff, and through B draw GB*n*.

Now, if we ſuppoſe B to be in the horizontal line nearly, the horizontal angle at A, taken between the north part of the meridian AP and the flag-ſtaff at B, will be the angle BAP, the teleſcope in this caſe being pointed to B, and the vertical plane which it would then move in is the plane *n*BGA; but if the teleſcope is directed to ſome point T in the flag-ſtaff above B, the angle TAP in this caſe will evidently be leſs than it was in the former by the angle *n*AT nearly; and conſequently it diminiſhes as the obſerved point T is elevated; and it is alſo

4 evident,

evident, that it will be augmented as the point obſerved is below B.

The latitudes of A and B being 49° 40′ and 50°, and their difference of longitude 30′, BG will be nearly equal to AG, or 3509769 fathoms, and GR being equal to 141.36 fathoms, and the angle GRB a right one, we have BG (3509769) : rad. :: GR (141.36) : ſin. 8″, the angle GBW, or TB*n*. Now, ſuppoſing the point T to be a mile above the ſurface, this with the angle *n*BT = 8″, will give T*n* equal to about three inches; but T*n* is in the plane of the meridian PBE, and conſequently would be ſeen obliquely, if viewed from A, becauſe the angle ABT is about 135°, and therefore T*n* muſt ſubtend a very ſmall angle at the diſtance of 33½ miles, which is nearly the diſtance between A and B.

From the determination of the horizontal angles that would be obſerved at A and B (fig. 2.) on the ſpheroid, if AP, BP, the co-latitudes of A and B are known, and the angles ABP, BAP, are given by obſervation, it follows, that the greater of theſe obſerved angles muſt be augmented, and the leſſer diminiſhed, by the ſame quantity of a degree, till the ſum and difference give the oppoſite ſides AP, BP, accurately by ſpherical computation, and then the third angle, or difference of longitude, will be given: for the obſerved angles at B and A being reſpectively 135° 33′ 17″.3 and 44° 3′ 47″.1, we have ſine 135° 33′ 17″.3 + 11′ 58″.9 : ſine AP :: ſine 44° 3′ 47″.1 − 11′ 58″.9 : ſine BP accurately; but taking the angles that would be obſerved, ſine 135° 33′ 17″.3 : ſine AP :: ſine 44° 3′ 47″.1 : ſine of an arc greater than BP; and ſine 44° 3′ 47″.1 : ſine BP :: ſine 135° 33′ 17″.3 : ſine of an arc leſs than AP; and this will ſhew if the obſerved angles are conſiſtent, as angles that

2 ought

onght to be found by obſervation on a ſpheroid flattened at the poles.

Becauſe the ſum of the obſerved angles at A and B on the ſpheroid are equal to the ſum that would be obſerved on a ſphere, the latitudes and difference of longitude being the ſame on both, and the differences equal, therefore the ſum for computation is the ſame for both, and the quantity of each for computation on the ſpheroid may be found from the following

*Theorem.*

In any ſpherical triangle BPA (fig. 4.) if two of the ſides PB, PA, and the ſum of the oppoſite angles, PBA+PAB, are given, it will be,

*As the tangent of half the ſum of the ſides,*
*Is to the tangent of half their difference;*
*So is the tangent of half the ſum of the angles,*
*To the tangent of half their difference.*

In the ſpherical triangle *abp* (fig. 2.), as ſine *bap* : ſine *bp* :: ſine *abp* : ſine *ap*; that is, on the ſpheroid, ſine B*v*P : ſine BP :: ſine *v*BP : ſine AP. Now, the arc B*v* being = the arc *ba*, conſidered as an arc of a great circle, it follows, that in the ſpheroidical triangle *v*BP, if *v*B, BP, and the included angle *v*PB are given, the other angles at P and *v* may be found by ſpherical computation, but not the third ſide *v*P. Suppoſe BP, B*v*, are given, and the included angle *v*BP a right one; then rad. : ſine BP :: cotang. B*v* : cotang. angle BP*v*; therefore, if the latitude of the point B, and the angle BW*v*, or the quantity of the arc B*v*, as an arc of a great circle perpendicular to the meridian at that point, are given on a ſpheroid, the difference of longitude may be found by ſpherical computation, but not the latitude of the point *v*.

But

But if the ſpheroid is known, the latitude of a given point ($v$) in a great circle perpendicular to the meridian, may be found nearly from what has been delivered above. Thus, as rad. : coſine BP :: coſine B$v$ : coſine of an arc (PA) leſs than P$v$, the co-latitude of $v$. Now, with the latitude (ſuppoſe of the point A) thus found, and the given latitude of B, find GS (fig. 2.) which will be very nearly equal to the arc A$v$, and the value of this, as an arc of the meridian, being added to PA, will give P$v$, the co-latitude of $v$.

## SECTION SIXTH.

*Manner of determining the latitudes of the ſtations. Application of the pole-ſtar obſervations to computations on different ſpheres, and alſo on M.* BOUGUER's *ſpheroid, for the determination of the differences of longitude. Ultimate reſult of the trigonometrical operation, whereby the difference of the meridians of the Royal Obſervatories of Greenwich and Paris is determined. Plate X.*

### ARTICLE I. *Preamble, ſhewing the general principles adopted for ſettling the latitudes of the ſtations.*

In the Paper of 1787, ſo often quoted, and which was intended only as a ſketch of the mode then propoſed to be followed in conducting the recent trigonometrical operation, we had occaſion to ſhew, that the meaſured arc of the meridian between the point M near Dunkirk, and Perpignan ſituated at the

the bottom of the Pyrenean mountains, correſponding to an arc in the heavens of nearly 8°⅓ of latitude, differed but little from what ſhould be its true length, ſuppoſing the earth to have the figure and dimenſions aſſigned to it by M. BOUGUER in his ſecond ſpheroid. Here, however, it is become neceſſary to take notice of ſome miſtakes * that, through inadvertency, were fallen into in the computed lengths of the arcs, which, although they affect in a certain degree the accuracy of the numbers brought into compariſon, do not invalidate the general reaſoning there advanced, and the only thing meant to be eſtabliſhed, namely, that M. BOUGUER's hypotheſis agreed better with actual meaſurement on different parts of the ſurface of

* The miſtakes adverted to in the text were of three kinds. Firſt, an erroneous mode of ſumming up the lengths of the arcs from the lengths of the degrees, although theſe taken ſeparately were very accurately computed: for inſtance, the 43d was taken as that extending from 42 to 43, whereas it ſhould have been taken for the middle point, that is, from 42½ to 43½, and ſo on in regard to others. Hence the arcs are all made ſomewhat too long. The ſecond was the omiſſion of the value of 93½ toiſes in eſtimating the length of the celeſtial arc between Greenwich and Perpignan, the ſector with which the ſtars were obſerved having ſtood ſo much to the northward of the church of St. Jaumes, the point to which the triangular meaſurement correſponded. The third was fallen into from not knowing that the French obſervations of the ſtars had been corrected for the nutation of the earth's axis, in a Paper of M. DE LA CAILLE's, inſerted in the Memoirs of the Academy of Sciences for the year 1758, whereby all the lengths of the celeſtial arcs were thereby in ſome degree changed from what had been aſſigned to them reſpectively in the Book, *La Méridienne vérifiée*, publiſhed in 1744. From the ſame Paper it further appears, that they rejected altogether their obſervations at Perpignan, as being probably affected by the attraction of the Pyrenées. With regard to that part of the Table of Compariſon in the Paper of 1787, which is affected by theſe errors, the only thing that now can be done is to annex to this paper a corrected ſlip, which may be referred to occaſionally, or cut off and paſted over the former.

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the earth, than any of the others with which it was compared.

In proof of this, we need only for the prefent remark, what will be made fully to appear hereafter, that the diftance between the parallels of Greenwich and Dunkirk, or Greenwich and M, being now added (by our trigonometrical operation) to the meafured length of the meridian of France, the meafured and computed fections of the united meridian will be found to agree almoft exactly at Paris; that the excefs of the meafurement is but of the value of 3″ or 4″ at Bourges; only of 6″ at Rodés; aud even as low down as Perpignan, comprehending in the whole an arc of the heavens of more than 8°¾, the excefs is not greater than what would anfwer to between 16″ and 17″, the chief part of which is probably owing to the attraction of the plummet of the fector by the Pyrenees. In the Paper of 1787, the effect had been affumed at a quantity equal to about 10″. But every thing on this head muft be confidered as merely matter of fuppofition, which cannot be determined one way or other until triangular meafurements fhall have been extended beyond the Pyrenean mountains into Spain, and correfponding obfervations of the ftars made on both fides with the fame inftrument, which fhould be one of the beft that could poffibly be invented for the purpofe. In the mean time, fince the French have rejected their own obfervations at Perpignan, we fhall avoid drawing any conclufions with regard to latitudes from the obfervations to the fouthward, and confine ourfelves to thofe immediately connected with our operation, made at the northern ftations of the meridian.

In carrying on the trigonometrical operation, it never was propofed that we fhould attempt to determine the latitudes of

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the ſtations, by actual obſervations of the zenith diſtances of ſtars, which, with the very beſt inſtruments hitherto uſed for that purpoſe, could not have been done nearer than about 1″ of an angle in the heavens, anſwering in theſe parts to 101 feet on the ſurface of the earth. Even if we could have been ſupplied with a ſector ſo far ſurpaſſing the old ones (ſuch perhaps as Mr. RAMSDEN may hereafter invent) that would have given zenith diſtances to one-tenth part of a ſecond, or about ten feet on the ſurface of the earth, the application of it in our operation would have been mere loſs of time: for the Aſtronomer Royal having ſettled the latitude of Greenwich 51° 28′ 40″, to within leſs than half a ſecond of the truth; and the geodetical ſituation of each ſtation of our ſeries being determined ſo accurately with regard to that point, as to leave no where an uncertainty of more than one or two feet; we have thereby been able to determine the relative latitudes to a ſmall fraction of a ſecond. Here, however, it is to be underſtood, that we have adhered to M. BOUGUER's ſcale, as anſwering almoſt exactly in the narrow ſpace of 26′ 51″, or thereabout, of latitude between Greenwich and M, to which our operations have been confined.

That this mode of ſettling the latitudes of our ſtations is extremely accurate, will more fully appear from the following conſiderations. In the general computation of ſpherical triangles, a ſphere whoſe diameter is a mean between the longeſt and ſhorteſt of M. BOUGUER's ſpheroid has been adopted, becauſe it was obvious, that in our latitudes the degree of ſuch a ſphere could not differ ſenſibly from the mean degree of the ſpheroid. Thus the degree of the ſphere 60859.1 fathoms anſwers (as may be perceived by conſulting the table in the Paper of 1787) to the degree of the meridian on the ſpheroid in

the latitude of 51° 5′. Again, if the total length of one-fourth part of the spheroidical meridian of the earth, between the equator and the pole, 5478094.4 fathoms be divided by 90° (*Fig. de la Terre de* Bouguer, p. 310. and 311.), we shall have 60867.72 fathoms for the mean degree of the meridian, which in the same table will be seen not to differ sensibly from that answering to the latitude of Greenwich; in or near which parallel the curves of such a sphere and M. Bouguer's spheroid intersect each other, as will be readily conceived by referring to and considering the representation of them, in Plate X. fig. 3.

### Art. II. *Of the pole-star observations in general.*

It became necessary, in the preceding article, to point out in what manner the latitudes of our stations have been deduced from their relative situation with regard to Greenwich; because the method adhered to of settling the differences of longitude by the observations of the pole-star, which could rarely be made except on one side, that is to say, at night, when the star was eastward from the pole, implied as a matter of course, that the latitude of the station should be accurately known, for the computation of the star's azimuth. With the declination of the star, settled to so great a nicety as it has been by the Astronomer Royal, and the latitude of the place given, a single azimuth was sufficient for obtaining immediately the true direction of the meridian. Much time would have been uselessly lost in attempting to get observations of the star in day-light when on the west side of the pole, whereby the double azimuth would have been obtained; and in that case the bisection of the angle would have given the true meridian of the place, without the knowledge of its latitude.

For the purpoſe of the pole-ſtar obſervations a ſmall table had been previouſly computed, of the exact times of the ſtar's being in the eaſt and weſt; whence the moments of its greateſt elongation were readily known. On theſe occaſions the Board of Longitude's *præmium watch*, by the late Mr. HARRISON, was made uſe of. Its rate of going all the time that it was in the field in 1787, was very uniformly 9½ ſeconds a day faſter than mean time. But in the winter months the watch gradually changed its rate from *plus* to *minus*, and when it was carried into the field in 1788, and, during the five weeks that it continued there, it regularly loſt on mean time from 3½ to 4 ſeconds each day; having in that ſhort interim been twice compared in Argyll-ſtreet, with an excellent clock made by CUMMING, with an improved ELLICOT's pendulum.

With regard to theſe pole-ſtar obſervations, whereby the differences of longitude, or the angles of convergence of the meridians towards each other, have been determined, it is neceſſary to remark, that although ſome few were made to the weſtward of Greenwich, yet theſe were not at ſufficient diſtance from it, and alſo taken of too ſhort ſides, to afford reſults that were perfectly ſatisfactory and concluſive. It is on the obſervations to the eaſtward only, and chiefly on thoſe made at Goudhurſt and Botley Hill, which are upwards of twenty-three miles from each other, and reciprocally viſible, that we have relied for the ſcale of degrees of a great circle perpendicular to the meridian in theſe latitudes; whence thoſe of longitude have been obtained. The obſervations made at Folkſtone Turnpike, which is upwards of fifty-eight miles in direct diſtance from Greenwich, and where, fortunately, the double azimuth of the pole-ſtar was obtained, are perfectly conſiſtent with thoſe taken at Goudhurſt and Botley Hill. But

when

when at Fairlight Down we had no obſervations of the ſtar, being at that time ſo much engaged with the other eſſential buſineſs of the triangles, and particularly with the interſection of the lights on the Coaſt of France, as to render it impoſſible to attend to any thing elſe, even if the weather had proved leſs unfavourable than it was at the period alluded to, for celeſtial obſervations.

ART. III. *Pole-ſtar obſervations at Goudhurſt and Botley Hill applied to computations on the mean ſphere.*

Let B (Plate X. fig. 5.) be Botley Hill; PBR its meridian; G Goudhurſt; W Wrotham Hill; T Tenterden; RG an arc of a great circle paſſing through G, and falling perpendicularly on the meridian BR; alſo let ⁎ ⁎ repreſent the circle of the pole-ſtar's apparent declination; and B⁎, G⁎, be two azimuth circles touching that circle.

| | ° | ′ | ″ |
|---|---|---|---|
| Auguſt 14, 1788, at Goudhurſt, the angle ⁎GT, or that between the pole-ſtar, when at its greateſt apparent diſtance from the pole on the eaſt ſide of the meridian, and the reverberatory lamp at Tenterden was obſerved †, . . . | 104 | 32 | 19½ |
| The angle BGT, between the lamp at Botley Hill and Tenterden, was repeatedly obſerved, | 167 | 43 | 56 |
| Their difference = angle ⁎GB is, . | 63 | 11 | 36½ |

† The obſervations of the pole-ſtar at Goudhurſt and Botley Hill were repeated for ſeveral nights at each place; but theſe here given are the moſt exact. At Goudhurſt the angle which the ſtar made with the lamp being noted, the teleſcope removed, and the plane of the inſtrument being turned 180°, or half round, the teleſcope replaced and directed again to the ſtar, the difference on the circle was found to be only 1″¼. The ſame method was univerſally adhered to in all places where obſervations of the ſtar were obtained. At Botley Hill, in particular, the difference between the readings was no more than 1″½.

Auguſt

| | ° | ′ | ″ |
|---|---|---|---|
| August 23, 1788, at Botley Hill, the angle ∗BW, or that between the pole-star at its greatest apparent elongation and the lamp at Wrotham Hill, was observed, . . . | 76 | 21 | 37 |
| The angle WBG, by repeated observations, was, | 40 | 4 | 42 |
| Their sum = angle ∗BG is, . . | 116 | 26 | 19 |

In order to obtain the star's azimuth at each place, we may take, without producing any sensible error, the latitudes of G and B, as they would be found on M. BOUGUER's figure, which we have already announced, and will hereafter prove to be consistent with observation. Thus B, or Botley Hill, is south from Greenwich 72882½ feet, and nearly on the same meridian; wherefore its latitude will be $51° 16' 41''.54$, and its co-latitude BP of course is $= 38° 43' 18''.46$. Now, P∗ the apparent distance of the star from the pole at that time being $= 1° 49' 22''.84$, in the right-angled spherical triangle P∗B, we have sine BP : rad. :: P∗ : sine $2° 54' 54''.2$ equal to the angle ∗BP, the star's azimuth from the north. And this being added to the angle ∗BG observed $116° 26' 19''$, we have the angle ∗BP $= 119° 21' 13''.2$ for that comprehended between the meridian and Goudhurst.

The distance of Goudhurst from the perpendicular to the meridian of Greenwich is 132592 feet, and its distance from the meridian of Botley Hill, on a perpendicular to that meridian, is 106171 feet nearly = GR. Hence the latitude of the point R is $51° 6' 52''.89$; therefore RP $= 38° 53' 7''.11$, and RG $= 106171$ feet $= 17' 19''.7$ nearly. Hence, as rad. : cosine RP :: cosine RG : sine $51° 6' 49''.7$, the latitude of G nearly; therefore GP $= 38° 53' 10''.3$; and P∗ the star's apparent distance at the time being $1° 49' 25''.34$, we have the angle PG

PG$*$, the ſtar's azimuth $= 2° \ 54' \ 20''.8$, which being ſubtracted from the angle BG$*$ obſerved at Goudhurſt between the lamp on Botley Hill and the ſtar, there remains the angle BGP $= 60° \ 17' \ 15''.7$ comprehended at Goudhurſt, between Botley Hill and the meridian.

Now with theſe *data* let us ſuppoſe, in the firſt place, the earth to be a ſphere, whoſe diameter is a mean between the longeſt and ſhorteſt of M. Bouguer's ſpheroids, the latitude of B, and of courſe its co-latitude BP, given; alſo the angles PBG and PGB reſpectively $119° \ 21' \ 13''.2$ and $60° \ 17' \ 15''.7$, we ſhall then have PG the co-latitude of G, and the angle BPG or difference of longitude of B and G. And becauſe the degree of ſuch a ſphere contains 60859.1 fathoms, the latitude of Botley Hill will then be $51° \ 16' \ 41''.45$, and BP its co-latitude $= 38° \ 43' \ 18''.55$. This laſt ſide, with the former angles PBG and PGB reſpectively $119° \ 21' \ 13''.2$ and $60° \ 17' \ 15''.7$, give PG $= 38° \ 53' \ 6''.72$ the co-latitude of G; and alſo the angle BPG, the difference of longitude of the points B and G equal to $27' \ 36''.7$. Again, in the right-angled ſpherical triangle PRG, rad. : tang. GP :: coſine of the angle RPG : tang. $38° \ 53' \ 3''.47 =$ RP. But the point R is 22094 fathoms ſouth from Greenwich, and nearly on its meridian, therefore its latitude will be $51° \ 6' \ 52''.8$; and hence PR the co-latitude will be $38° \ 53' \ 7''.2$, which exceeds PR formerly found by ſpherical computation to be $38° \ 53' \ 3''.47$ by $3''.73$, an arc equal to 63 fathoms. Alſo RG, the diſtance of Goudhurſt from the meridian of Botley Hill, on a perpendicular to that meridian, is equal nearly to 17695 fathoms, which, allowing 60859.1 fathoms for a degree, correſponds to an arc of $17' \ 26''.7$. But ſpherical computation formerly gave RG $= 17' \ 20''$, the difference conſequently is $6''.7 = 113\frac{1}{4}$ fathoms; therefore the earth cannot be this mean ſphere, which was

aſſumed

aſſumed for the purpoſe of exemplification, becauſe its degrees, in the direction of the meridian, differ ſo little in theſe latitudes from thoſe of M. BOUGUER's ſpheroid.

ART. IV. *The ſame pole-ſtar obſervations applied to computations on a ſphere of greater dimenſions.*

Let us ſuppoſe, in the ſecond place, the earth to be a ſphere of ſuch magnitude as to have degrees of a great circle containing $61253\frac{1}{2}$ or 61254 fathoms, we ſhall then get the latitude of B or Botley Hill $= 51° 16' 46''$, the latitude of $R = 51° 7' 1''.2$, and $PR = 38° 52' 58''.8$; alſo $RG = 17' 19''.9$. Now, $BP = 38° 43' 13''.9$, and the obſerved angles will give the angle BPG, or the difference of longitude $= 27' 36''.7$, the ſame as before, and the arc PG or co-latitude of $G = 38° 53' 2''.05$ *. This laſt ſide, with the angle $RPG = 27' 36''.7$ of the right-angled ſpherical triangle PRG, will give $PR = 38° 52' 58''.8$, and $RG = 17' 19''.9$; that is to ſay, the obſerved angles PBG and PGB, at Botley Hill and Goudhurſt reſpectively, are nearly the ſame as they would be found on a ſphere of ſuch magnitude as to have degrees containing $61253\frac{1}{2}$ or 61254 fathoms. But ſince the value of RG as an arc of a great circle was before found by the triangles BPG and RPG to be $17' 20''$, when the latitude of B was taken as belonging to a ſphere whoſe degrees contained 60859.1 fathoms; and the ſame arc as now determined, *viz.* $17' 19''.9$, agrees very nearly

* It is evident, that as the latitude of B increaſes, the ſtar's azimuth, or the angle ✱BP, and conſequently the angle PBG, increaſe likewiſe. But at G the angle PGB is diminiſhed by the increaſe of the angle ✱GP, or the azimuth; and therefore if the difference of the latitudes of B and G remains the ſame, or nearly the ſame, the ſum of the angles PBG, PGB, will alſo be nearly the ſame; wherefore no ſenſible difference in the angle BPG, or difference of longitude, will be found on this account.

 with

with the former, although the latitude of B be now taken on a sphere whose degrees contain 61253½ fathoms, it obviously follows, from these recent observations, *that whatever the precise figure of the earth may be, or the ratio between its diameters, the degree of a great circle upon it perpendicular to the meridian, cannot in these latitudes differ much in length from 61253½ fathoms.*

ART. V. *The pole-star observations at Folkstone Turnpike applied to computations on the same greater sphere.*

Let G (Plate X. fig. 6.) be Greenwich; PR its meridian; F, H, and T, the stations at Fairlight Down, High Nook, and Folkstone Turnpike, respectively; also let PF and PT be meridians passing through F and T; and FR and T*r* great circles cutting the meridian of Greenwich PR at right angles in R and *r*.

| | ° | ′ | ″ |
|---|---|---|---|
| At the station T, on the 7th of September, 1788, at night, the angle between the pole-star, when at its greatest apparent elongation from the pole on the east side of the meridian, and the reverberatory lamp at H, was observed, . | 123 | 19 | 3¼ |
| On the following morning, Sept. 8th, the angle between the star, when at its greatest distance on the west side, and the flag-staff at H, was observed, . . . | 117 | 30 | 52½ |
| The difference or double azimuth is, . | 5 | 48 | 10¾ |
| And the half sum is, . . | 120 | 24 | 57.87 |

This half sum $120° 24' 57''.87$ * is the angle PTH, or that comprehended between the meridian PT and H. The angle HTF,

* By taking the latitude of T as determined on M. BOUGUER's spheroid, = $51° 5' 45''.3$ nearly, the co-latitude or TP is equal to $38° 54' 14''.7$, and the star's apparent distance at the time being $1° 48' 18''.03$, we have, as sine

38°

HTF, or that between the lamp at H and the white lights repeatedly fired at F, was twice obſerved 22′ 48″; therefore 120° 24′ 57″.87 + 22′ 48″ = 120° 47′ 45.87 is the angle PTF, that the ſtation on Fairlight Down makes with the meridian of Folkſtone Turnpike.

Now, *r*T being equal to 45827.88 fathoms, and RF = 23884.68 fathoms, if we take 61253½ fathoms = 1°, we ſhall have G*r* = 22871 fathoms = 22′ 24″.18; GR = 36436.1 fathoms = 35′ 41″.42; *r*T = 44′ 53″.4; and RF = 23′ 23″.75; therefore PR, the co-latitude of R, will be 39° 7′ 1″.42; and that of *r* or P*r* will be 38° 53′ 44″.18. Hence, in the right-angled ſpherical triangle PRF, we ſhall have the angle RPF = 37′ 4″.901, and PF = 39° 7′ 7″.294. Further, the triangle P*r*T gives the angle *r*PT = 1° 11′ 29″.143, and PT = 38° 54′ 5″.98. Now, 1° 11′ 29″.143 − 37′ 4″.901 = 34′ 24″.242 = the angle FPT. This laſt angle, with the two containing ſides PT and PF, give the angle PTF = 120° 47′ 44″.75, the ſame as it was actually obſerved very nearly. And hence we have another ſtrong proof, *that on this part of our earth the degree of a great circle, perpendicular to the meridian, cannot differ much in length from 61253½ fathoms, whatever may be its real figure, which cannot be determined until theſe obſervations ſhall have been compared with others that may hereafter be made in the ſame way, and with equal care, in latitudes remote from each other.*

38° 54′ 14″.7 : rad. :: ſine 1° 49′ 18″.03 : ſine 2° 54′ 5″.12 the ſtar's azimuth. Twice this angle, or 5° 48′ 10″.24, agrees very nearly with the double azimuth 5° 48′ 10″¾, found by the obſervations on the 7th and 8th of September. This near agreement, at the ſame time that it ſerves to ſhew the accuracy of theſe obſervations in particular, and the goodneſs of the mode that was adopted in general, ſerves alſo to prove, that Dr. MASKELYNE has ſettled the declination of the pole-ſtar to great preciſion.

ART. VI. *The latitude of the point* M *near Dunkirk, and consequently the distance between the parallels of Greenwich and* M, *deduced from the same length of a degree perpendicular to the meridian. Also the comparison of its length with that of the meridional degree.*

Again, let us suppose G (Plate X, fig. 7.) to be Greenwich; P*r* its meridian; M the point near Dunkirk, supposed to be in the meridian of Paris; M*r* a great circle passing through that point, and falling perpendicularly on P*r*. Then, if we take $61253\frac{1}{2}$ fathoms $= 1°$, we shall have *r*M ($= 89674.7$ fathoms) $= 1° 27' 50''.37$, and G*r* ($= 25831.43$ fathoms) $= 25' 18''.17$. Hence P*r* will be $38° 56' 38''.17$; and therefore as rad. : cosine P*r* :: cosine *r*M : sine $51° 1' 58''.5$ the latitude of M; and $51° 28' 40'' - 51° 1' 58''.5 = 26' 41''.5$ is the difference of latitude between Greenwich and M, or the distance of their parallels. Now, as $3600''$ : $61253\frac{1}{2}$ :: $26' 41''.5$ ($= 1601''.5$) : 27249.3 fathoms; and this being added to 133409.8 fathoms, the measured arc of the meridian between M and the Royal Observatory at Paris, we have 160659.1 fathoms for the length of the terrestrial arc of the meridian comprehended between the parallels of the two Royal Observatories nearly. But the length of the celestial arc between them being $2° 38' 26''$ would, at the rate of $61253\frac{1}{2}$ fathoms to a degree, give $=$ 161743.3 fathoms, which exceeds the measured arc by 1084.2 fathoms. *Therefore it is sufficiently obvious, that the earth cannot be a sphere of these dimensions; but it must be an oblate spheroid, on which a degree of a great circle, perpendicular to the meridian, in this way of considering it, exceeds in length the mean degree of the meridian between Greenwich and Paris in the proportion of* $61253\frac{1}{2}$ *to* 60842, *or* $411\frac{1}{2}$ *fathoms.*

ART.

ART. VII. *Application of the results of the pole-star observations to computations on M.* BOUGUER's *spheroid, for the distance of the parallels of Greenwich and* M.

Hitherto the results obtained by the geodetical measurement and pole-star observations have been applied to spherical computations on two spheres suited to the different lengths of degrees found in two opposite directions, at right angles to each other, the meridian and its perpendicular; and from these computations it has been clearly proved, that the earth cannot be either of the assumed spheres.

Let us therefore, in the next place, suppose the earth to have the figure and the dimensions of M. BOUGUER's spheroid, and by way of comparison apply the same results to computations on that figure. Thus the latitude of the point *r* will be found 51° 3′ 12″.09, and the arc $rM = 1° 27' 49''.03$. Hence, as rad. : cosine $rP$ :: cosine $rM$ : sine 51° 1′ 48″.85 = the latitude of M nearly. Now, let the points *r* and M be represented by B and *v* (Plate X. fig. 2.) then will A represent the point whose latitude is 51° 1′ 48″.85; and by proceeding in the manner formerly directed for a spheroid, we get GW = 15.12 fathoms = to the distance in the axis between the points where the verticals from the latitudes 51° 3′ 12″.09 and 51° 1′ 48″.85 meet the said axis. Hence, as rad. : 15.12 (GW) :: cosine 51° 1′ 48″.85 (angle SWG) : 9.509 fathoms = GS, or the arc A*v* extremely near. Now the value of A*v*, as an arc of the meridian, is = 0″.56, which being added to 38° 58′ 11″.15 (AP), gives 38° 58′ 11″.71 = P*v*, the co-latitude of *v*; and hence the true latitude of *v*, or M (fig. 7.), is 51° 1′ 48″.29, which being subtracted from 51° 28′ 40″, the latitude of Greenwich, there remains 26′ 51″.71 for the arc between them, or distance of their

their parallels, which on this ſpheroid correſponds to 27248.2 fathoms, leſs only by 1.1 fathom than the ſpace found, in the laſt article, to anſwer to an arc of 26′ 41″.5, being the diſtance of the ſame parallels on the greater ſphere.

Thus the meaſured length of the arc between Greenwich and M, 27248.2 fathoms, being added to the meaſured diſtance of M from the Royal Obſervatory at Paris, we have for the total length of the arc between Greenwich and Paris 160658 fathoms, which exceeds the computed length of the ſame arc on M. BOUGUER's hypotheſis by no more than $7\frac{1}{2}$ fathoms.

But it hath been already ſhewn, that whatever the preciſe figure of the earth may be, a degree of a great circle upon it, perpendicular to the meridian, cannot in theſe latitudes differ much in length from $61253\frac{1}{2}$ fathoms, being but $16\frac{1}{2}$ fathoms leſs than 61270 fathoms, the length of the correſponding degree on M. BOUGUER's ſpheroid.

As far therefore as we are enabled to judge from the reſult of theſe obſervations, the earth differs but little either in its latitudinal or longitudinal dimenſions from what hath been aſſigned to it by M. BOUGUER.

ART. VIII. *Application of the pole-ſtar obſervations at Botley Hill and Goudhurſt, for determining the length of the degree of a great circle, perpendicular to the meridian.*

Since M. BOUGUER's ſcale for the degrees of the meridian hath been found to agree almoſt exactly with obſerved latitudes in this part of the earth, let us take the latitudes of B and R (fig. 5.) as they would be found on his ſpheroid nearly, and apply the pole-ſtar obſervations at B and G, in order to find the length of the degree of a great circle, perpendicular to

7 the

the meridian of Botley Hill, paſſing through Goudhurſt. We ſhall then have PB, the co-latitude of Botley Hill, $= 38° 43' 18''.46$, and PR, the co-latitude of R, $= 38° 53' 7''.14$. Now, if the latitudes of B and R are nearly true, it follows, *that the point* G *muſt be ſomewhere in the great circle* RG, *whatever may be its longitude.* Therefore the angle BPG, or the difference of longitude between B and G, will be found in the following manner.

*Augment the obſerved angle* PBG $= 119° 21' 13''.2$, *and diminiſh the obſerved angle* PGB $= 60° 17' 15''.7$ *by the ſame quantity of a degree, until* PR *determined from the triangle* BPG *becomes* $= 38° 53' 7''.14$ *nearly; which will be when that quantity is* $9' 21''$. Thus the angles for computation will be $119° 21' 13''.2 + 9' 21'' = 119° 30' 34''.2$, and $60° 17' 15''.7 - 9' 21'' = 60° 7' 54''.7$; whence the angle RPG, or difference of longitude between B and G, will be found $= 27' 36''.75$, and the arc RG $= 17' 20''.06$ nearly $= 17695$ fathoms. *And hence the degree of a great circle, perpendicular to the meridian, of this new ſpheroid, will, in the latitude of* R, *contain* 61248 *fathoms nearly.*

*This follows as a corollary from what hath been already ſaid concerning ſpheroidical triangles.*

But ſince the difference of longitude between B and G was formerly determined to be nearly the ſame, *viz.* $27' 36''.7$, when the obſerved angles at theſe two ſtations, and alſo the latitude of B, were ſuppoſed to be on a figure different from this new ſpheroid; it therefore follows, *that the difference of longitude between any two ſtations* B *and* G, *diſtant in the preſent caſe from each other twenty-three miles* (and they ſhould never be much leſs remote) *may be found with ſufficient exactneſs, by having the*

*horizontal*

*horizontal angles at each ſtation obſerved very accurately, and the latitude of one of the ſtations given nearly.*

The difference of longitude between Botley Hill and Goudhurſt, found as above, 27′ 36″.75, being augmented by the value of the ſmall arc comprehended between the meridians of Greenwich and Botley Hill = 2″.7, we have ultimately 27′ 39″.45 for the longitude of Goudhurſt, eaſtward from Greenwich.

Art. IX. *Difference between obſerved angles on the new ſpheroid and that of M.* Bouguer.

Laſtly, on the ſubject of theſe compariſons, let us ſee what would be the difference between the obſerved angles at B and G, as determined on the new ſpheroid and on that of M. Bouguer?

The latitudes of B and G on M. Bouguer's ſpheroid would reſpectively be 51° 16′ 41″.54 and 51° 6′ 49″.66 nearly, and the angle BPG, or difference of longitude, would be = 27′ 36″.18. Now, this laſt angle, with the two co-latitudes PB and PG, as containing ſides, and ſuppoſed to form a ſpherical triangle, will give the angles at B and G reſpectively 119° 31′ 26″.47 and 60° 7′ 3″.18. But the obſerved angles at theſe ſtations would be 119° 21′ 32″.97 and 60° 16′ 56″.68, the common difference between them being 9′ 53″.5, which is 32″.5 greater than 9′ 21″, as was before determined. Hence we may conclude, that in this new ſpheroid, founded immediately on the recent geodetical meaſurements and obſervations of the pole-ſtar made at Botley Hill and Goudhurſt, the verticals from B and G meet the earth's axis at a leſs diſtance from each other than they would in M. Bouguer's ſpheroid. The length of the vertical is ſhorter as well as the radius of the parallel, whereby Goudhurſt,

hurst, or the point R, is less removed from the earth's axis than it would be on the former figure; and consequently it is probable, that the spheroid is less oblate.

From the preceding determinations it is further evident, that supposing the latitudes of B and G, with the horizontal angles PBG and PGB to be given by observation, not only the difference of longitude, or the angle BPG, will be obtained, but also the arc BR of the meridian, the arc RG of a great circle perpendicular to it, and the oblique arc BG, all considered as arcs of great circles of the spheroid.

ART. X. *Further illustration of the manner of settling the latitudes and longitudes of the stations comprehended in the general table of results.*

Having shewn, in the preceding part of this section, how the length of the degree of a great circle, perpendicular to the meridian, and also the differences of latitude and longitude, have been obtained by very accurate observations of the pole-star made at certain stations to the eastward of Greenwich, whereby we have been furnished with a scale for settling the longitudes of all the other stations where no observations of the pole-star could be had, or only such as were not to be depended upon; we shall, by way of further illustration of this matter, give another example of the calculations for the point M near Dunkirk, which will suffice for all the other stations comprehended in the general table of results placed at the end of this section, where the respective columns have been filled up by the same or a similar mode of computation.

Let G (Plate X. fig. 8.) be Greenwich; GR*r* its meridian; G*g* the perpendicular to that meridian, produced eastward; MR a parallel to that perpendicular drawn through the point

M; and let M$g$ be a portion of a ſmall circle of the ſpheroid, or parallel to the meridian of Greenwich, produced from M northward, until it interſects the perpendicular in the point $g$. Alſo, let MP repreſent the meridian of the Royal Obſervatory at Paris, paſſing through the point M, and interſecting the parallel of Greenwich in P. Further, let C repreſent the church of *Notre Dame* at Calais, and making, as appears by the triangles, an angle RMC of 14° 51′ 3″.9 with the parallel to the perpendicular of the meridian of Greenwich drawn through the point M.

From the annexed general table of the reſults of the triangles, it appears, that MR = $g$G contains 538048 feet = 89674.7 fathoms; and that GR = $g$M contains 154938 feet = 25823 fathoms. Now, ſince great circles, perpendicular to any meridian of the ſpheroid, converge towards each other, as they depart from that meridian, in the ſame manner as the meridians themſelves do in departing from the equator, but by a ſlower rate, it is obvious, that the perpendicular to the meridian of Greenwich, paſſing through the point M, muſt fall below or to the ſouthward of R on that meridian, ſo as that G$r$ : GR :: rad. : coſine MR = 1° 27′ 51″, conſidered as a portion of a great circle of the ſpheroid, perpendicular to the meridian of Greenwich. Hence, G$r$ will contain 25831.43 fathoms = 25′ 27″.9 of latitude, and therefore the latitude of $r$ will be 51° 3′ 12″.1, and its co-latitude 38° 56′ 47″.9. Alſo, R$r$ meaſures 8.43 fathoms, and ſubtends an angle RM$r$ = 19″.42.

In the right-angled ſpherical triangle, pole $r$M, right-angled at $r$, making uſe of the half ſum and half difference of the containing ſides, $r$ pole and $r$M, with the co-tangent

gent of 45° we have the angle of longitude *r* pole M, . . . . . $= 2°\ 19'\ 42''.5$

The angle, pole M*r*, . . . $= 88°\ 11'\ 21''.4$

And the complement of this laſt, or the convergence of the meridian of M (ſuppoſed to coincide with that of Paris) to the meridian of Greenwich, . . . . . . $= 1°\ 48'\ 38''.6$

Alſo, as ſine 88° 11′ 21″.4 : ſine 38° 56′ 47″.9 :: rad. : ſine 38° 58′ 11″.2 the co-latitude of M; whence its latitude becomes 51° 1′ 48″.8, from which, deducting the ſpheroidical correction 0″.5, we have the true latitude of M = 51° 1′ 48″.3. The difference between this and the latitude of *r* = 51° 3′ 12″.1 will be 1′ 23″.8, anſwering on the ſpheroid to 1416.77 fathoms; and this laſt number being added to the value of the arc G*r* = 25831.43 fathoms, we have 27248.2 fathoms for MP, or the diſtance of the parallels of Greenwich and M on this new ſpheroid.

Laſtly, if to this mean diſtance of the parallel of M from Greenwich, we add the mean diſtance of the parallel of M from Paris = 133398.8 fathoms, we ſhall then have 160647 fathoms for the mean diſtance of the parallels of Greenwich and Paris, anſwering to the celeſtial arc of 2° 38′ 26″. Hence the mean degree of the meridian between Greenwich and Paris correſponding to the latitude of 50° 9′½, contains 60838.3 fathoms, or about 1½ fathom leſs than M. Bouguer's degree for the ſame latitude.

Art. XI. *Comparison of the angle between the meridian of the point* M *and a line drawn from thence to Calais, as approximately deduced from the British and French observations.*

In the spheroidical quadrilateral $GgrM$ (fig. 8.), formed by three arcs of three great circles, and one of a small circle of the spheroid, we have two right angles at G and $r$, and two others at $g$ and M, each greater than a right angle by $9''.7$; therefore the angle RMC, resulting from the triangles $= 14° 51' 3''.9 - RMr (19''.42) = 14° 50' 44''.5 + 90° 0' 9''.7 (CMg) = 104° 50' 54''.2$, is the angle $gMC$, or that which Calais makes with a parallel to the meridian of Greenwich drawn through the point M. From this last angle subtracting the angle $PMg = 1° 48' 38''.6$, or the quantity by which the meridian of M (supposed to co-incide with that of Paris) converges towards that of Greenwich, there remains the angle $PMC = 103° 2' 15''.6$ for the angle that the meridian of M should make with a line drawn from D, or Dunkirk, through that point to Calais, according to the British observations.

By the late French operations, the meridian of Dunkirk makes, with a line drawn through M to Calais, an angle of $102° 59' 51''.5$. The convergence of the meridian of M to that of Dunkirk, on a difference of longitude of $2' 21''.54$, is $1' 49''.94$, which being added to $102° 59' 51''.5$, we have $103° 1' 41''.44$ for the angle that the meridian of M, or of Paris, makes with a line drawn from Dunkirk through that point to Calais. The difference between the two results $34''.16$ is nearly equal to the mean of two extremes $\left(\frac{1' 15''}{2}\right) = 37''\frac{1}{2}$, the apparent uncertainty, in the determination of that angle

by two sets of angles given in the *Méridienne vérifiée*, as adverted to in the Paper of 1787, Phil. Transf. Vol. LXXVII. p. 195, 196.

ART. XII. *The longitudes of Dunkirk and Paris, eastward from Greenwich, determined by the sum of four differences of meridians.*

In fig. 9. let PA be the meridian of Greenwich; G Goudhurst, PR its meridian; T the station at Folkstone Turnpike, PS its meridian; C Calais, PC its meridian; D Dunkirk, and PB its meridian. Also, let AG, RT, SC, and BC, be arcs of great circles, making the angles PAG, PRT, PSC, and PBC, right ones.

The angle at Goudhurst, between its meridian and Tenterden, is 107° 26′ 40″.3; hence, by drawing parallels to this meridian through Tenterden and the station at Allington Knoll (see the plan of the triangles) we shall get 946.6 fathoms for what the station at the Turnpike is southward, and 28098.8 fathoms for what it is eastward from the meridian of Goudhurst. Now, 60859.4 fathoms being nearly = 1° of the meridian in the latitude of Goudhurst, we have 946.6 fathoms = 56″ nearly = the arc GR; and the latitude of Goudhurst being 51° 6′ 49″.6, that of the point R is 51° 5′ 53″.6; hence the co-latitude RP = 38° 54′ 6″.4: and since the degree of a great circle, perpendicular to the meridian, in this latitude has been shewn to contain 61248 fathoms nearly; therefore RT = 28098.8 fathoms will be 27′ 31″.6. This arc and RP give the angle RPS = 0° 43′ 49″.86 for the difference between the meridians of Goudhurst and Folkstone Turnpike.

The angle at Folkstone Turnpike between its meridian and Dover was observed 66° 48′ 35″, and if we draw a parallel to this meridian through Dover, we shall find, that Calais is

25284.2

25284.2 fathoms eaſtward from the meridian of the Turnpike. Now, the latitude of Calais being 50° 57′ 30″ nearly (which is accurate enough for computation) the length of the degree of a great circle, perpendicular to the meridian in that latitude, will be 61246 fathoms nearly. Hence, 25284.2 fathoms = 24′ 46″.8 = the arc CS; this, with the co-latitude CP (39° 2′ 30″), give the angle CPS = 39′ 19″.48, for the difference of longitude between the Turnpike and Calais.

By Comte de Cassini's Paper, communicated in January 1789, it appears, that the angle at Dunkirk, between its meridian and Broulezele, is 10° 18′ 25″; and that between Broulezele and Calais 66° 41′ 46″½, the ſum is 77° 0′ 11″½ for the angle at Dunkirk, between its meridian and Calais. In the ſame Paper we have 19349.34 toiſes for the diſtance of Dunkirk from Calais; this, with the angle 77° 0′ 11″½, give 18853.7 toiſes or 20093.3 fathoms for the diſtance of Calais weſtward from the meridian of Dunkirk, which, by taking 61246 fathoms = 1° (that of a great circle perpendicular to the meridian in the latitude of Calais), is equal to 19′ 41″.1 = the arc BC; and this arc, with CP the co-latitude of Calais, give the angle CPB = 31′ 15″.11 for the difference between the meridians of Calais and Dunkirk.

| | | ° | ′ | ″ |
|---|---|---|---|---|
| The angle APR, or the difference of the meridians of Greeenwich and Goudhurſt, has already been found (ſee the end of the 8th article, and alſo the table of general reſults), . | = | 0 | 27 | 39.45 |
| The angles RPS . . . . . | = | 0 | 43 | 49.86 |
| SPC . . . . . | = | 0 | 39 | 19.48 |
| CPB . . . . . | = | 0 | 31 | 15.11 |
| Hence the total angle APB, or long. of Dunkirk, is | = | 2 | 22 | 3.9 |

It

It hath been already remarked, that, from p. 276. *Méridienne vérifiée*, Dunkirk is 1430 toises eastward from the meridian of Paris; and that in p. 36. of the *Déscription Géometrique de la France*, we find it only 1416 toises. Now, these will give $2' \, 22''.6$ and $2' \, 21''.2$ respectively for the difference of meridians of Dunkirk and Paris; the mean is $2' \, 21''.9$ for the longitude of Dunkirk east from Paris; therefore $2^\circ \, 22' \, 3''.9 - 2' \, 21''.9 = 2^\circ \, 19' \, 42''$, or $9' \, 18''.8$ in time, will be the longitude of Paris east from Greenwich nearly.

Again, in fig. 10. let P be the pole; G Greenwich, PW its meridian; RD an arc of a great circle making the angle at R a right one, and passing through D; and DW an arc of the parallel of latitude of Dunkirk.

By p. 240. of the *Mem. de l'Acad.* 1758, the celestial arc between Paris and the station of the sector near Dunkirk is $2^\circ \, 11' \, 50''$; to which adding $5''.3$ ($=84\frac{1}{4}$ toises) for what the tower is north from the station, we have $2^\circ \, 11' \, 55''.3$ for the arc between Paris and Dunkirk; therefore, if the latitude of Paris is $48^\circ \, 50' \, 14''$, that of Dunkirk will be $51^\circ \, 2' \, 9''.3$, whence its co-latitude becomes $38^\circ \, 57' \, 50''.7 = DP$.

From what has been said concerning spheroidical triangles, it follows, by way of corollary, that to find RD by spherical computation, when DP and the angle at P are given, it is necessary to diminish DP by a certain quantity determinable from the nature of the spheroid; this quantity is about $0''.5$ when the spheroid is M. Bouguer's; therefore DP may be taken $= 38^\circ \, 57' \, 50''.2$, which is sufficiently accurate for computation.

Hence, as rad. : sine DP :: sine $2^\circ \, 22' \, 3''.9 = WPD$ : sine $1^\circ \, 29' \, 19''.17 =$ the arc DR. Now, 61247 fathoms being equal to $1^\circ$ of a great circle perpendicular to the meridian in the

the latitude of Dunkirk nearly, we have, as $1^\circ : 61247 ::$ $1^\circ\ 29'\ 19''.17 : 91175.8$ fathoms = the arc DR.

But the length of this arc DR has been found nearly the ſame, that is, 91176.3 fathoms (ſee the table of general reſults) by continually drawing parallels to the meridian of Greenwich through the different ſtations between it and Dunkirk; therefore, although that method in general is not ſtrictly accurate, having a tendency to give the reſults in exceſs; yet it is evident, that the length of the arc of a great circle ſo determined will differ very little from the truth, when the ſeries of triangles employed for that purpoſe are contiguous to it, and follow its direction nearly.

Art. XIII. *For the diſtance between the parallels of latitude of Greenwich and Paris.*

The diſtance between the parallels of Greenwich and Paris has already been determined in Art. X. of this ſection, by taking M (1420.41 toiſes weſtward from Dunkirk) as the intermediate point. Let us next ſee what will be the reſult when Dunkirk is made uſe of inſtead of M?

In fig. 10. as coſ. RD : rad. :: coſ. DP : coſ. PR $= 38^\circ\ 56'\ 24''.07$; but DP by obſervation is $= 38^\circ\ 57'\ 50''.7 =$ PW; hence PW − PR $= 38^\circ\ 57'\ 50''.7 - 38^\circ\ 56'\ 24''.07 = 1'\ 26''.63$ = RW = 1464.5 fathoms nearly, by taking 60858 fathoms for a degree of the meridian, that being nearly its value in the latitude of Dunkirk.

By our operation Dunkirk is 25425 fathoms ſouthward from Greenwich; but the great circle DR meets the meridian of Greenwich about $8\frac{1}{2}$ fathoms further ſouth, that is to ſay, GR is $25425 + 8.5 = 25433.5$ fathoms, which being added to 1464.5

gives

gives 26898 for the diſtance between the parallels of latitude of Greenwich and Dunkirk.

Becauſe Dunkirk is ſituated near the meridian of Paris, the diſtance between the parallels of latitude of theſe places will be nearly equal to what Dunkirk is north from Paris, namely, $125517\frac{1}{2}$, or 125495 toiſes (ſee the pages formerly quoted). Theſe numbers give reſpectively $133770 + 26898 = 160668$, and $133746.3 + 26898 = 160644.3$ fathoms for the diſtances between the parallels of Greenwich and Paris, a mean between which will nearly be 160656 fathoms.

If therefore the celeſtial arc of the meridian between Greenwich and Paris is 2° 38′ 26″, we get $60846\frac{1}{4}$ or $60837\frac{1}{2}$ fathoms for a degree of the meridian in latitude 50° 9′$\frac{1}{2}$, the middle point between Greenwich and Paris; and a mean of theſe two reſults $60841\frac{3}{4}$ only exceeds M. Bouguer's degree for the ſame latitude about $1\frac{3}{4}$ fathom, a quantity not differing ſenſibly from the defect that was brought out by the computation in Art. X. Finally, therefore, by taking a mean between this and the former length $60838\frac{1}{4}$, we ſhall have 60840 fathoms for the degree of the meridian in latitude 50° 9′ 27″, agreeing almoſt exactly with that of M. Bouguer.

Art. XIV. *Compariſon of the length of a degree of a great circle, perpendicular to the meridian in Kent, with that in the South of France.*

M. Cassini de Thury, in his Book *La Méridienne vérifiée*, has given us the detail of an operation carried on in the South of France in latitude 43° 32′, for the determination of the length of a degree of longitude, by marking, at the extremities of a long and well aſcertained diſtance, the inſtantaneous exploſion of gunpowder in the open air. For this purpoſe a

 ſeries

ſeries of triangles was extended along the ſhore of the Mediterranean Sea, between Cette and St. Victoire, the extreme ſtations from whence the light was repeatedly obſerved, as fired at the church of St. Maries, nearly in a central ſituation, at the mouth of the leſſer branch of the river Rhone.

From the reſult of this operation, the beſt of the kind that has ever been executed in any country, it appears, that a degree of longitude in that latitude meaſures 44355.7 fathoms; whence it follows, that the degree of a great circle, perpendicular to the meridian there, muſt contain 61182½ fathoms, being 65½ fathoms leſs than the degree in the middle of Kent, latitude 51° 6′ 50″.

Now, if we compare this difference with that found between the correſponding degrees of great circles for the ſame latitudes on M. BOUGUER'S hypotheſis, we ſhall find them perfectly conſiſtent with each other in their rate of diminution: for, by conſulting the table, it will be ſeen, that this degree in latitude 51° 6′ 50″ exceeds that in latitude 43° 32′ by 64.7 fathoms, agreeing within leſs than a fathom with the former difference.

On due conſideration of ſo many corroborating circumſtances as have been adduced in the courſe of this ſection, there ſeems, therefore, to be ſufficient room to conclude, that the earth differs but little either in its figure or dimenſions from what hath been aſſigned to it by M. BOUGUER. It is true, indeed, that a new ſpheroid has been here preſented, ſomewhat leſs flat than the former, founded immediately on the Britiſh obſervations; and theſe being again compared with the reſult of the above-mentioned operation, whereby the degree of longitude in the South of France was determined, it is from the combination of both reſults that the annexed table of the

lengths

lengths of degrees of great circles and of longitude has been conſtructed for middle latitudes only, extending from 42° to 52°. Without the help of ſuch a table, the new longitudes of ſome intermediate places, which we ſhall have occaſion hereafter to compare with the old, could not have been ſo accurately computed as was wiſhed. Now, although it is believed, that this table will be found to anſwer nearly in that zone of the earth for which it is intended; yet it is only offered for temporary uſe, until future obſervations of the pole-ſtar in the ſame parallel, but on longer diſtances than our recent ſeries of triangles afforded, or the extenſion of operations of the ſame nature with ours into remoter latitudes, ſhall have furniſhed *data* for one more correct.

Table of the degrees of great circles and of longitude for middle latitudes.

| Places. | Latitudes. | | | Deg. of great circles, perp. to the meridian in fath. | | Degrees of longitude in fathoms. | |
|---|---|---|---|---|---|---|---|
| | ° | ′ | ″ | | Diff. | | Diff. |
| | 42 | 0 | 0 | 61170.5 | | 45458.5 | |
| | 43 | 0 | 0 | 61178. | 7.5 | 44742.8 | 715.7 |
| South of France | 43 | 32 | 0 | 61182.5 | 4.5 | 44355.7 | 387.1 |
| | 44 | 0 | 0 | 61186.5 | 4. | 44013.9 | 341.8 |
| | 45 | 0 | 0 | 61195. | 8.5 | 43271.4 | 742.5 |
| | 46 | 0 | 0 | 61203. | 8. | 42515.2 | 756.2 |
| | 47 | 0 | 0 | 61211.5 | 8.5 | 41746.1 | 769.1 |
| | 48 | 0 | 0 | 61220. | 8.5 | 40964.2 | 781.9 |
| Paris Royal Obſervatory | 48 | 50 | 14 | 61227.5 | 7.5 | 40303.2 | 662.0 |
| | 49 | 0 | 0 | 61229. | 1.5 | 40169.8 | 132.4 |
| | 50 | 0 | 0 | 61237.5 | 8.5 | 39362.8 | 807.0 |
| | 51 | 0 | 0 | 61246.5 | 9. | 38542.7 | 820.1 |
| M near Dunkirk | 51 | 1 | 48.3 | 61246.7 | 0.2 | 38518.8 | 23.9 |
| Dunkirk | 51 | 2 | 9.3 | 61246.75 | 0.05 | 38514.1 | 4 7 |
| Middle of Kent | 51 | 6 | 49.6 | 61248. | 1.25 | 38450.0 | 64.1 |
| Greenwich Royal Obſerv. | 51 | 28 | 40 | 61251.1 | 3.1 | 38148.7 | 301.3 |
| | 52 | 0 | 0 | 61255. | 3.9 | 37712.3 | 436.4 |

Art. XV. *Comparison of the old longitudes of some places on the skirts of the kingdom of France with what they will be when computed by the new* data.

If the preceding determinations of the longitudes of the several stations between Greenwich and Dunkirk are accurate, or nearly so, as founded immediately on the British observations, and ultimately combined with the result of the operation in latitude 43° 32′, it follows, that all the longitudes of the great map of France, the labour of more than half a century, will be considerably affected thereby, in proportion to the distances of the places, eastward or westward, from the meridian of the Royal Observatory at Paris respectively.

To shew the effect produced by the new *data*, we shall collect, in the following table, the latitudes and old longitudes of a few noted places on the skirts of that great kingdom, and annex to them the new longitudes resulting from computations made with new lengths of degrees of great circles, perpendicular to the meridian, corresponding to their latitudes respectively. It will readily be conceived, that the object here in view is solely this; namely, that astronomers who live near those places, and who have their time, that is to say, the directions of their meridians very accurately ascertained, may, by their future corresponding observations (which should only be occultations of the fixed stars behind the moon's dark limb) compare the old with the new longitude, and thus be enabled to satisfy the curious world, which of the two comes nearest to the truth.

Com-

Comparative table of the old and new longitudes of ſome noted places on the ſkirts of the kingdom of France.

| | Places. | Latitudes. | Longitudes, Old. | Longitudes, New. | Diff. of old and new long. in degs &c. | Diff. of old and new long. in time. |
|---|---|---|---|---|---|---|
| | | ° ′ ″ | ° ′ ″ | ° ′ ″ | ′ ″ | Sec. Thirds. |
| East from Paris. | Pilier de Cette . . | 43 24 6 | 1 21 7 | 1 20 37 | 0 23 | 1 32 |
| | Tour de Planier, near Marſeilles | 43 11 58 | 2 54 8 | 2 53 6 | 1 2 | 4 8 |
| | Signal of St. Victoire . | 43 31 52 | 3 15 8 | 3 13 58 | 1 10 | 4 40 |
| | Straſbourg (Conn. de T. 1788) | 48 34 35 | 5 26 18 | 5 24 6 | 2 12 | 8 48 |
| | Ditto (Déſcrip. Géomét. 1783, p. 171. . . | 48 34 50 | 5 25 0 | 5 23 33 | 1 27 | 5 48 |
| West from Paris. | Tour de Cordouan at the mouth of the Garonne, Conn. de T. 1788 | 45 35 15 | 3 30 38 | 3 29 18 | 1 20 | 5 20 |
| | St. Malo . . . . . | 43 39 0 | 4 22 22 | 4 20 37 | 1 45 | 7 0 |
| | Fort du Pilier, at the mouth of the Loire . . . . . | 47 2 29 | 4 42 20 | 4 40 30 | 1 50 | 7 20 |
| | Uſhant Light-houſe . . | 48 28 30 | 7 24 33 | 7 21 41 | 2 52 | 11 28 |
| | The greateſt difference between Straſbourg and Uſhant . | | | | 5 4 | 20 16 |
| | The leaſt difference . . . . . . | | | | 4 19 | 17 36 |
| | The mean difference . . . . . | | | | 4 41½ | 18 46 |

With regard to the longitudes in the preceding table, it is only neceſſary to obſerve, that the two books of 1744 and 1783, ſo often quoted, are not always conſiſtent with each other, and both do ſometimes diſagree with what has been placed on the margin of the map of France. It would ſeem, that in the *Déſcription Géométrique* a ſcale for degrees of longitude has been uſed conſiderably greater than that correſponding with the ſpherical hypotheſis adhered to in the conſtruction of the map, yet ſtill too ſmall for what we have found to be their meaſure in Kent, or that reſulting from their own operations in the South of France. But if a ſimilar mode to that which they practiſed with ſo much ſucceſs in the South had

had been employed in the North of France, the ſame ſort of reſult as we have obtained in Kent would probably have been the conſequence; in which caſe it cannot be doubted, that the ſpherical hypotheſis would have been entirely rejected, and their lengths of degrees of longitude would have been ſuited to an oblate ſpheroid, whoſe degrees of the meridian and of great circles perpendicular to it had nearly the proportion to each other of 60840 to 61239 for the middle latitude between Greenwich and Paris, being an exceſs of 399 fathoms on each degree in the longitudinal direction.

On the whole, therefore, as matters ſtand at preſent, it is ſufficiently obvious, that, in the total extent of the kingdom of France from Straſbourg on the eaſt to Uſhant on the weſt, the difference between the old and new longitude amounts to between 17 and 20 ſeconds of time; that is to ſay, the real difference between the meridians of thoſe places, it is preſumed, will not be found by future obſervations made on the occultations of the fixed ſtars, to be ſo great as it was formerly ſuppoſed to be by that quantity, or ſomething approaching it very nearly.

ART. XVI. *The obſervations of eclipſes cannot be depended upon for determining with ſufficient accuracy the difference of longitude in vicinal ſituations.*

Finally, with regard to differences of longitude, it may not be improper in this place to remark, that, in vicinal ſituations, ſuch as Greenwich and Paris, the eclipſes of the ſun and moon and Jupiter's ſatellites do not, in general *, give reſults

* The reſult deduced by the Profeſſor PIAZZI, of the Univerſity of Palermo, from the obſervations of the eclipſe of the ſun on the 3d of June, 1788, made at Greenwich, in company with Dr. MASKELYNE and M. D'ARQUIER, as given in the Phil. Tranſ. for 1789, p. 58. is an exception well worthy of notice.

 ſuffi-

ſufficiently near the truth to deſerve even the name of an approximation. This will inconteſtably appear by comparing the aſtronomical reſult produced in that way with ours obtained by actual meaſurement on the ſurface of the earth, and angular obſervations of the pole-ſtar. Thus, by taking a mean of a multitude of the beſt of theſe obſervations of eclipſes, &c. collected and corrected with great care for the purpoſe, the difference in time between Greenwich and Paris amounts to 9′ 30″½ *, inſtead of being only 9′ 19″ nearly, which our operation makes it. Now, if the difference in time between theſe two Royal Obſervatories was really ſo much, the degree of a great circle perpendicular to the meridian in theſe latitudes (51° 6′ 50″ that of Goudhurſt, or 51° 1′ 48″ that of the point M, it matters little which of the two is taken) would be between 1200 and 1300 fathoms ſhorter than the degree of the meridian in the ſame latitudes. Hence the earth, inſtead of being an oblate ſpheroid conſiderably flattened at the poles, would be one extremely prolate, in proportion with regard to the former figure of more than three to one, or between 1200 and 1300 – to about 400 +.

In remote ſituations, ſuch as Europe and America, Europe and the eaſtern parts of Aſia, ſeparated from each other by wide oceans, the differences of longitude can only be obtained by means of aſtronomical obſervations. And as theſe will always be liable to ſome error, which may be as great on a difference of one or two, or a few degrees, as on the whole 180°, it is ſufficiently obvious that, to render the effect of ſuch

* From Dr. MASKELYNE's Paper of 1787, Phil. Tranſ. p. 183. it appears, that the eclipſes of the 1ſt ſatellite of Jupiter give immediately for the difference of meridians of the two Obſervatories 9′ 30″½, without being combined with obſervations made in other parts of Paris.

error

error as ſmall as poſſible, occultations of fixed ſtars ſhould only be made uſe of, for obtaining concluſive determinations.

In vicinal ſituations, the next beſt mode to angular meaſurement is no doubt that of marking, by means of well-regulated clocks, as was done in the South of France, the repeated inſtantaneous exploſion of light, obſerved at ſtations as far diſtant to the eaſtward and weſtward of the place of exploſion as the circumſtances will permit in practice, theſe diſtances having been for the purpoſe accurately ſettled by trigonometrical operation. The preferable ſtations for experiments of this ſort will be pointed out in the concluſion to the preſent Memoir.

## TABLE containing the GENERAL RESULTS of the

| | Stations. | By Plane Trigonometry. | | | |
|---|---|---|---|---|---|
| | | Diſtances in Feet. | | Bearing or Angle with the Meridian. | |
| | | from the Merid. of Greenwich. | from the Perp. to the Merid. | ° ′ ″ | |
| | Greenwich R. Ob. Tranſit Room - - | . . . . | . . . . | . . . . | . . |
| Weſt from Greenwich | Norwood - - - - | 19306.54 | 24603.86 | 38. 7.16 | SW |
| | Hundred Acres - - - | 43333.9 | 50937.9 | 40.23.18.5 | SW |
| | Hanger Hill Tower - - - | 67740.69 | 16729.21 | 76. 7 40.2 | NW |
| | Hampton Poor-houſe - - - | 83086.16 | 18537.98 | 77.25.20.3 | SW |
| | King's Arbour - - - - | 102264.55 | 1037.69 | 89.25. 7.7 | NW |
| | St. Ann's Hill - - - - | 119404.04 | 28852.77 | 76.24.55.7 | SW |
| | Wardrobe Tower of Windſor Caſtle - - | 137050.54 | 2562.72 | 88.55.43.5 | NW |
| Eaſt from Greenwich | Botley Hill - - - | 171.6 | 72882.5 | 0. 8. 5.6 | SE |
| | Severndroog Caſtle on Shooter's Hill - - | 14032.3 | 4069.8 | 73.49.34 | SE |
| | Frant - - - - - | 62342.5 | 138460.4 | 24.14.23.6 | SE |
| | Wrotham Hill - - - - | 71850.3 | 59305.8 | 50.27.48.5 | SE |
| | Goudhurſt - - - - | 106342.5 | 132592. | 38 43.50.1 | SE |
| | Fairlight Down - - - - | 143308.1 | 218611.6 | 33.14.46.8 | SE |
| | Hollingborn Hill - - - | 151078.1 | 77077.6 | 62.58.12.2 | SE |
| | Tenterden - - - - | 158317.6 | 148567. | 46.49.11.4 | SE |
| | Ruckinge - - - - - - | 204801.6 | 149410.3 | 53.53.16.3 | SE |
| | Lydd - - - - | 209339.3 | 190695.6 | 47.40.6 | SE |
| | Allington Knoll - - - | 219926.3 | 144032.3 | 56.46.43.5 | SE |
| | High Nook near Dymchurch - - - | 228246.4 | 165672.9 | 54. 1.33.4 | SE |
| | Padleſworth - - - - | 261707.6 | 130836.4 | 63.26.16.8 | SE |
| | Swingfield - - - - | 273722.7 | 118730.9 | 66.33.2 | SE |
| | Folkſtone Turnpike - - - | 274967.3 | 137214.2 | 63.28.47.6 | SE |
| | Dover Caſtle, north Turret of the Keep - | 303766.8 | 124319.1 | 67.44.34 | SE |
| | Montlambert near Boulogne - - - | 382889.6 | 273450.3 | 54.27.59.4 | SE |
| | Blancnez - - - - | 394891.7 | 197153.5 | 63.28.8 | SE |
| | N. D. at Calais - - - - | 427456.6 | 184263.3 | 66.40.50.2 | SE |
| | Point M near Dunkirk, Merid. R. Ob. Paris | 538048.2 | 154938.2 | 73.56. 7.2 | SE |
| | Dunkirk - - - - | 547058.1 | 152549.1 | 74.25. 7.2 | SE |

[To face page 232.]

of the Trigonometrical Operation.

metry.

| g or ith the lian. | | Direct Dist. from Greenwich. | Latitude. | Longitude. in Degrees, &c. | Longitude. in Time. | Vertical Heights. Ground above the Sea. | Vertical Heights. Telescope above Ground. | Vertical Heights. Total. |
|---|---|---|---|---|---|---|---|---|
| | | Feet. | ° ′ ″ | ° ′ ″ | m. s. th. | Feet. | Feet. | Feet. |
| . | . . | . . . | 51.28.40 | | | 170.5 | 43.5 | 214 |
| ) | SW | 31274.48 | 51.24.37.34 | 0. 5. 2.4 | 0.20. 9.6 | 380.3 | 9.2 | 389.5 |
| 8.5 | SW | 66876.73 | 51.20.17.3 | 0.11.19.5 | 0.45.18. | 433.8 | 9.2 | 443 |
| 0.2 | NW | 69775.8 | 51.31.24.16 | 0.17.46.5 | 1.11. 6. | 213. | 38. | 251 |
| 0.3 | SW | 85129.1 | 51.25.35.2 | 0.21.45.3 | 1.27. 1.2 | 63.5 | 37.5 | 101 |
| 7.7 | NW | 102269.8 | 51.28.47.16 | 0.26.48.5 | 1.47.14. | 94.8 | 37.5 | 132.3 |
| 5.7 | SW | 122840.6 | 51.23.51.4 | 0.31.14.7 | 2. 4.58.8 | 321.3 | 20.7 | 342 |
| 3.5 | NW | 137074.5 | 51.28.59.7 | 0.35.55.8 | 2.23.43.2 | | | |
| 5.6 | SE | 72882.7 | 51.16.41.54 | 0. 0. 2.7 | 0. 0.10.8 | 859.3 | 20.7 | 880 |
| 4 | SE | 14610.58 | 51.27.59.8 | 0. 3.40.65 | 0.14.42.6 | 417.8 | 64.2 | 482 |
| 3.6 | SE | 151848.2 | 51. 5.53.9 | 0.16.12.5 | 1. 4.50. | 604.1 | 54.9 | 659 |
| 8.5 | SE | 93164.6 | 51.18.53.86 | 0.19.12.3 | 1.16.49.2 | 761.8 | 9.2 | 771 |
| 0.1 | SE | 169968.8 | 51. 6.49.62 | 0.27.39.45 | 1.50.37.8 | 437.9 | 59.1 | 497 |
| 6.8 | SE | 261396.7 | 50.52.38.84 | 0.37. 5. | 2.28.20. | 539.5 | 5.5 | 599 |
| 2.2 | SE | 169604.1 | 51.15.53.5 | 0.39.25.2 | 2.37.40.8 | 611 | 5.5 | 616.5 |
| 1.4 | SE | 217109.7 | 51. 4. 8.15 | 0.41. 8.15 | 2.44.32.6 | 220.6 | 101.7 | 322.3 |
| 6.3 | SE | 253509.6 | 51. 3.54.9 | 0.53.12.6 | 3.32.50.4 | 31.9 | 5.5 | 37.4 |
| | SE | 283174.5 | 50.57. 7.4 | 0.54.15.4 | 3.37. 1.6 | 31.7 | 98.7 | 130.4 |
| 3.5 | SE | 262893.3 | 51. 4.46.1 | 0.57. 9.4 | 3.48.37.6 | 323.5 | 5.5 | 329 |
| 3.4 | SE | 282035.3 | 51. 1.11.7 | 0.59.14.6 | 3.56.58.4 | 22.1 | 5.5 | 27.6 |
| 6.8 | SE | 292590.2 | 51. 6.50.36 | 1. 8. 9. | 4.32.36. | 621.3 | 20.7 | 642 |
| | SE | 298364.1 | 51. 8.47.8 | 1.11.14.6 | 4.44.58.4 | 473.1 | 56.9 | 530 |
| 7.6 | SE | 307302.3 | 51. 5.45.3 | 1.11.29.3 | 4.45.57.2 | 569.8 | 5.5 | 575.3 |
| 4 | SE | 328222 | 51. 7.47.7 | 1.19. 2.1 | 5.16. 8.4 | 373.9 | 95.1 | 469 |
| 9.4 | SE | 470509.8 | 50.43. 2.3 | 1.38.45. | 6.35. 0. | 639.5 | | |
| | SE | 441371.7 | 50.55.31.3 | 1.42.17.9 | 6.49.11.6 | 444.5 | | |
| 0.2 | SE | 465480.5 | 50.57.30.67 | 1.50.48.8 | 7.23.15.8 | 140.5 | | |
| 7.2 | SE | 559912.8 | 51. 1.48.3 | 2.19.42.5 | 9.18.50. | | | |
| 7.2 | SE | 567929.4 | 51. 2. 9.3 | 2.22. 4. | 9.28.16. | | | |

SECTION

| | | ° | ′ | ″ | ‴ | | ° | ′ | ″ | ‴ | ° | Pts. | | | | Mean error on each deg. + 62.53 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Greenwich Rl. Ob. | 51 | 28 | 40 | 00 | | 8 | 46 | 48 | 00 | 8. | 78 | | 534343. | | | 535848. | |
| | Perpignan - - | 42 | 41 | 52 | 00 | | | | | | | | | | | | | |
| | Greenwich and M | MP | — | — | — | diff. | 0 | 26 | 50 | 52 | 0. | 4475 | | 27235. | | | 27342. | |
| 2 | Point M near Dunkirk | 51 | 1 | 49 | 8 | | 8 | 19 | 57 | 8 | 8. | 3325 | 506687 | 507108. | +521 | | 508506. | +181 |
| | Perpignan - - | 42 | 41 | 52 | 00 | | | | | | | | | | | | | |
| | Point M and Paris - | — | — | — | — | diff. | 2 | 11 | 35 | 8 | 2. | 1931 | 133409 | 133467. | + 58 | | 133957. | + 584 |
| 3 | Paris Rl. Ob. - | 48 | 50 | 14 | 00 | | 6 | 8 | 22 | 00 | 6. | 1394 | 373278 | 373641. | +363 | | 374549. | +1271 |
| | Perpignan - - | 42 | 41 | 52 | 00 | | | | | | | | | | | | | |
| | Paris and Bourges | — | — | — | — | diff. | 1 | 45 | 9 | 19 | 1. | 7525 | 106512 | 106658. | + 46 | | 106989. | + 377 |
| 4 | Bourges - - | 47 | 5 | 4 | 41 | | 4 | 23 | 12 | 41 | 4. | 3869 | 266666 | 266983. | +317 | | 267560. | + 894 |
| | Perpignan - - | 42 | 41 | 52 | 00 | | | | | | | | | | | | | |
| | Bourges and Rodés | — | — | — | — | diff. | 2 | 43 | 51 | 5 | 2. | 6309 | .65961 | 166200. | +239 | | 166599. | + 638 |
| 5 | Rodés - - - | 44 | 21 | 13 | 36 | | 1 | 39 | 21 | 36 | 1. | 656 | 100705 | 100783. | + 78 | | 100961. | + 256 |
| | Perpignan - - | 42 | 41 | 52 | 00 | | | | | | | | | | | | | |

| Observed Celestial Arc of the Meridian between Greenwich and Perpignan. | | | | | | | | | Meas[d] Terrest[l] Arc. | on a Sphere. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Latitudes. | | | Differences. | | | | | Semidiam. | 3486975. | 1st R.S.D. | { 179. / 178. |
| | | ° | ′ | ″ | ° | ′ | ″ | | Fath[s] | Fath[s] | | Fath[s] | |
| 1 | Greenwich Rl. Ob. - / Tower of Dunkirk - | 51. | 28. | 40. | 0. | 26. | 30.7 | | 26898 | 26891 | − 7 | 27000 | + 1 |
| 2 | Tower of Dunkirk - / Paris Rl. Ob. - - | 51. | 2. | 9.3 | 2. | 11. | 55.3 | | 133758 | 133811 | + 53 | 134299 | + 5 |
| 3 | Paris Rl. Ob. - - / Tower of Bourges - | 48. | 50. | 14. | 1. | 45. | 11.4 | | 106647 | 106696 | + 49 | 1070 24 | + 3 |
| 4 | Tower of Bourges - / N. D. at Rodés - | 47. | 5. | 2.6 | 2. | 44. | 0.2 | | 166115 | 166352 | +237 | 166753 | + 6 |
| 5 | N. D. at Rodés - / St Jaumes at Perpignan | 44. 21. 2.4 / 42. 41. 56. | | | 1. | 39. | 6.4 | | 100448 | 100526 | + 78 | 100704 | + 2 |
| | Greenwich & Perpignan | | | | 8. | 46. | 44. | | 533866 | 534276 | +410 | 535780 | + 19 |

The measured Arc between Paris and Dunkirk is taken a Mean between 125517½ and 125495 Toises,
The Latitudes of Dunkirk, Bourges, Rodés, and Perpignan, are deduced from the Celestial Arcs giver
*Mérid. vérifiée.*

For correcting the TABLE in the PAPER of 1787. [To

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 535758. | | | 535024. | | | 534909. | | | 533880. | | | 533801. |
| | | 27337. | | | 27294. | | | 27288. | | | 27234. | | | 27224. |
| +1819 | | 508421. | +1734 | | 507730. | +1043 | | 507621. | +934 | | 506646. | −41 | | 506577. |
| + 584 | | 133932. | + 523 | | 133732. | + 323 | | 133700. | +291 | | 133440. | +31 | | 133398. |
| +1271 | | 374489. | +1211 | | 373998. | + 720 | | 373921. | +643 | | 373206. | −72 | | 373179. |
| + 377 | + 218.3 | 106971. | + 359 | + 208.1 | 106820. | + 208 | + 125.17 | 106796. | +184 | +112.09 | 106590. | −22 | −4.92 | 106569. |
| + 894 | | 267518. | + 852 | | 267178. | + 512 | | 267125. | +459 | | 266616. | −50 | | 266610. |
| + 638 | | 166572. | + 611 | | 166354. | + 393 | | 166320. | +359 | | 166002. | +41 | | 165990. |
| + 256 | | 100946. | + 241 | | 100824. | + 137 | | 100805. | +100 | | 100614. | −91 | | 100620. |

Computed Terrestrial Arc of the Meridian between Geenwich a[...]

on different Ellipsoids.

| 179.047 to 178.047 | 2d.RS.D. | 192.483 to 191.483 | 3d R.S.D. | 216.06 to 215.06 | 4th R.S.D. | 222.55 to 221.55 | 5th R.S.D. | 230. to 229. | 6th R.S.D. |
|---|---|---|---|---|---|---|---|---|---|
| | Fath^s | | Fath^s | | Fath^s | | Fath^s | | Fath^s |
| +102 | 26995 | + 97 | 26953 | + 55 | 26946 | + 48 | 26893 | − 5 | 26883 |
| +541 | 134274 | +516 | 134073 | + 315 | 134042 | +284 | 133782 | +24 | 133739 |
| +377 | 107006 | +359 | 106847 | + 200 | 106831 | +184 | 106625 | −22 | 106604 |
| +638 | 166727 | +612 | 166516 | + 401 | 166474 | +359 | 166156 | +41 | 166144 |
| +256 | 100689 | +241 | 100567 | + 119 | 100548 | +100 | 100358 | −90 | 100364 |
| +1914 | 535691 | +1825 | 534956 | +1090 | 534841 | +975 | 533814 | −52 | 533734 |

Toises, or 133770 and 133746 Fathoms. See Sect. VI. Art. XII.

rcs given by M. DE LA CAILLE, in pag. 240, 241, *Mém. de l'Acad.* 1758, and making the proper Reductions for

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01. | | | 533734. | | | 534928. | | | 533652.4 | | |
| 24. | | | 27212. | | | 27288. | | | 27234. | | |
| 77. | −110 | −13.2 | 506522. | −165 | 9.8 | 507640. | +953 | +114.37 | 506418.4 | −268.6 | −32.23 |
| 98. | − 11 | | 133354. | − 55 | | 133705. | +296 | | 133416.5 | + 7.5 | |
| 79. | − 99 | | 373168. | −110 | | 373935. | +657 | | 373001.9 | −276.1 | |
| 69. | − 43 | | 106549. | − 63 | | 106796. | +184 | | 106550.4 | − 61.6 | |
| 10. | − 56 | | 266619. | − 47 | | 267139. | +473 | | 266451.5 | −214.5 | |
| 90. | + 29 | | 165986. | + 25 | | 166335. | +374 | | 165908.5 | − 52.5 | |
| 20. | − 85 | | 100633. | − 72 | | 100804. | + 99 | | 100543. | −162. | |

nwich and Perpignan.

| | | | | on different Spheroids. | | | |
|---|---|---|---|---|---|---|---|
| . D. | 310.3 to 309.3 | 7th R. S. D. | 540. to 539. | 1ſt R. S. D. | 222.55 to 221.55 | 2d R. S. D. | 179.4 to 178.4 |
| 1ˢ | | Fath | | Fathˢ | | Fathˢ | |
| 83 | − 15 | 26872 | − 26 | 26946 | + 48 | 26893 | − 5 |
| 39 | − 19 | 133695 | − 63 | 134047 | +289 | 133758 | 0 |
| 04 | − 43 | 106584 | − 63 | 106831 | +184 | 106585 | − 62 |
| 44 | + 29 | 166140 | + 25 | 166489 | +374 | 166062 | − 62 |
| 64 | − 84 | 100377 | − 71 | 100547 | + 99 | 100287 | −161 |
| 34 | −132 | 532668 | −195 | 534860 | +994 | 533585 | −281 |

ns for the Diſtances of the Stations of the Sector at each Place, as given in the

SECTION

# SECTION SEVENTH.

*An Account of the observations made during the course of the trigonometrical operation for the determination of terrestrial refraction. Plate X.*

## ARTICLE I. *Preamble.*

ASTRONOMICAL refraction, or that which the rays of light suffer in passing from the heavenly bodies to our earth, hath, by the investigations of different philosophers, been nearly ascertained. From the theory of dioptrics, as well as experience, it hath been proved, that the rays, in coming from a very rare into a very dense medium, are gradually bent downwards, out of their rectilinear direction, into lines more or less curved in proportion to the angular distance of the objects from the zenith, where obliquity ceasing, refraction ceases likewise; since from that point light takes the shortest route through the refracting medium to the eye of the observer. Hence it follows, that the apparent altitudes of celestial objects are greater than they otherwise would be by the quantity of this refraction, which is greatest at the horizon, amounting there to 33′.

The late Dr. BRADLEY from his experience has shewn, that in the mean state of the barometer taken at 29.6 inches, and of FAHRENHEIT's thermometer at 50°, the refraction at 45° of altitude is 57″ (according to Dr. MASKELYNE only 56″½).

 In

In other ſtates of the atmoſphere, it varies with the height of the barometer, and inverſely with that of the thermometer augmented by the number 350 to the number 400.

It is therefore obvious, that terreſtrial objects as well as celeſtial, muſt ſuffer a refraction greater or leſs, according as they are leſs or more removed from the horizon; and that ſuppoſing celeſtial refraction to be perfectly aſcertained, the meaſure of the lower part of any of its curves, co-inciding with a particular object on the ſurface of the earth, ſhould give the quantity by which the apparent altitude of that object would exceed its real altitude, or what would be its angle of elevation, if no ſuch effect as refraction did exiſt in nature.

The inſtrument made uſe of in the triangular operation was extremely well calculated, as will be remembered from its deſcription, for meaſuring with much exactneſs ſmall angles of elevation or depreſſion, and conſequently was in that reſpect very fit for the purpoſe, if the multiplicity of other buſineſs we had on our hands at the time had permitted refraction to become a primary, inſtead of being only a ſecondary object. This will readily be conceived by thoſe who have any idea of the trouble of conducting, eſpecially at a late ſeaſon of the year, an operation of the nature of that in which we were engaged. Along with the lights on the French ſide of the Channel, we had by day as well as by night our own inland obſervations to attend to; the very circulation of orders to the men poſted at the different ſtations from twelve to fifteen or twenty miles off, in different directions around the horizon, when any part of the arrangements failed, ſo as to render a repetition of lights neceſſary, was not a matter of ſmall detail.

2 But

But befides the important bufinefs of the triangles, which engroffed almoft our whole attention, it is fufficiently obvious, that, in order to have been enabled to make conclufive obfervations, the relative heights of the ftations fhould in ftrictnefs all have been afcertained by levelling: for purpofes of this fort geometrical determinations, however good in other refpects they may be, fhould not here be admitted, becaufe they involve the very point in queftion, that is, the height, which fhould be obtained independently of angular meafurement. Befides barometers and thermometers at both ftations, two obfervers and two inftruments of the fame kind would have been neceffary, for taking at the fame inftant the reciprocal angle of elevation and depreffion *.

Although, therefore, in our fituation the circumftances did not admit of conclufive obfervations on terreftrial refraction, confidered either by the mean or its extremes; neverthelefs, fince in a variety of cafes angles both of elevation and depreffion were reciprocally obtained at the fame ftations, but at different times, it is hoped, fuch new light will be thrown on the

* Dr. MASKELYNE, in a letter that I lately received from him, remarks, that it would be of ufe to have a perfon to note the thermometer at the object as well as at the ftation of the obferver, whereby (if niceties of this fort were of confequence) the refraction might be more accurately computed by the application of a new correction. Thus, calling $r = \frac{a}{10} = \frac{1}{10}$th of the arc of diftance; $h$ = the height of the uniform atmofphere; $t$ = the difference of the thermometers at the two ftations; $x$ = the difference of altitude of the two ftations above a common level; the correction would then be $= \frac{rth}{400x}$; and the true or whole refraction would be $= r \mp \frac{rth}{400x}$, according as the thermometer ftood lower or higher at the upper ftation.

 matter

matter as may poſſibly hereafter lead to further inveſtigations of this curious, but at preſent vague and indetermined ſubject: for from theſe obſervations it will appear, that terreſtrial refraction, inſtead of being $\frac{1}{9}$th of the comprehended arc, according to M. BOUGUER, $\frac{1}{10}$th according to Dr. MASKELYNE, $\frac{1}{14}$th according to M. LAMBERT, varies from $\frac{1}{5}$ to $\frac{1}{24}$th part of that arc; and perhaps, if it had been poſſible for us to have tried it on heights conſiderably more elevated, we ſhould have found it almoſt wholly to vaniſh.

## ART. II. *Relative heights.*

Before we proceed to give any account of the obſerved angles of elevation or depreſſion, at the ſtations reciprocally, for trying the quantity of terreſtrial refraction, it will be proper to call to remembrance, that, in the meaſurement of the baſe on Hounſlow Heath, the mouth of the pipe at Hampton Poorhouſe was ſhewn to be elevated about 60 feet above low-water ſpring tides at the ſea, as far as could then be determined by referring it to the ſurface of high water at Iſleworth; and that the extremity of the baſe near King's Arbour was found by levelling to be higher than the former end by 31 feet 3 inches.

The mouth of the pipe at the ſouth-eaſt end of the baſe of verification at High Nook near Dymchurch, in Romney Marſh, Lieut. FIDDES found by levelling to be above low-water mark at ſpring tides 22.1 feet.

The top of the parapet of the north turret of the Keep of Dover Caſtle was found by Lieut. HAY, of the Royal Engineers (by levelling from the top of the cliff at Queen Elizabeth's gun downwards, and adding to that the height of the

7 ground

ground and Caſtle above the ſaid gun) to be 465.8 * feet above low water at ſpring tides. Having alſo meaſured a baſe for the purpoſe, he determined the height of the cliff geometrically, which agreed within leſs than a foot of the reſult by levelling. In juſtice to this very meritorious Officer it is incumbent on me to ſay, that not only on this occaſion, but on every other during the progreſs of our operations near Dover, his aſſiſtance was moſt eſſentially uſeful.

The height of the ball of St. Paul's above the Thames at Paul's Wharf, and the height of Shooter's Hill Inn above the Gun Wharf in Woolwich Warren, were ſeverally determined in 1773, at which time the experiments were carrying on for the purpoſe of finding a theory for the meaſurement of altitudes by the barometer.

The height of Severndroog Caſtle, lately built on Shooter's Hill, has ſince been deduced from that of the Inn.

Laſtly, the altitudes of all the intermediate ſtations, as expreſſed in the three colums towards the right hand of the general table of reſults, placed at the end of the preceding ſection, have been eſtabliſhed by the reciprocal angles of elevation or depreſſion, gradually carried on from ſtation to ſtation, throughout the whole ſeries of triangles, whereby the two extremities are connected together; and no greater uncertainty has been found at Hampton Poor-houſe than a few feet, occaſioned no doubt by the uncertainty of terreſtrial refraction: for it is to be remarked, that, to the weſtward of Greenwich, no double but only ſingle obſervations were obtained; wherefore, the

* Sir THOMAS HYDE PAGE, when engineer at Dover, at my requeſt, had been ſo obliging as to order his workmen (he himſelf being ill at the time) to determine the height of the turret of the Keep, which, by a miſtake of about nine feet in the height of the cliff, they made 475 feet above low-water mark.

rela-

relative heights of thefe ftations have been determined by taking $\frac{1}{10}$th of the arc of diftance for the effect of terreftrial refraction.

### ART. III. *General Theorem.*

Let C (Plate X. fig. 11.) be the center of the earth confidered as a fphere; S*s* the furface; H*h* two places at the fame height above the furface; HO the horizontal line, or apparent level at the place H; and *ho* the horizontal line, or apparent level at the place *h*; alfo, let C*m* bifect the angle at C.

Then, becaufe the angles *m*HC and *mn*H are right ones, the angle *m*H*n*, or *mhn*, is equal to the angle *m*CH or *m*C*h*; that is to fay, if two places H, *h*, are of equal heights, the one as feen from the other is depreffed below the horizontal line of the place of obfervation, by an angle equal to half of the arc of the great circle contained between them, or half the angle at C. Hence it follows, that any diftant object is higher or lower than the place of obfervation, according as the depreffion is lefs or greater than half the contained arc, fuppofing no fuch effect as refraction to exift in nature.

### ART. IV. *Determination of the refraction between Dover Caftle and Folkftone Turnpike.*

Let D (fig. 12.) be the place of the axis of the telefcope on the north turret of the Keep in Dover Caftle; T the ground at the ftation near Folkftone Turnpike; DO the horizontal line; and SL=CD.

The diftance of the ftations is 31554.6 feet, which, taking the obliquity of the direction into confideration, gives 61188 fathoms = 1°; and confequently 5′ 9″.4 for the length of the contained

contained arc of diſtance nearly, one-half of which, or 2′ 34″.7, is equal to the angle ODL.

At the ſtation D, the ground at T was, by obſervation, elevated 8′ 37″ equal to the angle TDO, to which adding ODL = 2′ 34″.7, we have for the angle TDL 11′ 11″.7.

Now, if the diſtance of the ſtations be taken as radius, the lines TO, TL, &c. will be nearly as the tangents of their oppoſite angles; therefore the angle TDL, with the diſtance 31554.6 give TL = 102.7 feet, or what the ground at the ſtation T would have been higher than the axis of the teleſcope at D, if there had been no refraction.

But the axis of the teleſcope, when at the ſtation T, was 5.5 feet above the ground; therefore 102.7 + 5.5 = 108.2 feet would be the height of the axis at T above the axis at D.

Now, let T (fig. 12.) be the place of the axis of the teleſcope, when the inſtrument ſtood at Folkſtone Turnpike; D the top of the parapet of the north turret in the Keep of Dover Caſtle; TO the horizontal line; and CL = ST.

At the ſtation T, the parapet of the turret was by obſervation depreſſed 14′ 17″.5 = OTD, from which ſubtracting half the arc of diſtance 2′ 34″.7, there remains for the angle LTD 11′ 42″.8. This laſt angle, with the former known diſtance, give LD = 107.5 feet, or what the parapet was lower than the axis at T, if there had been no refraction.

But the axis of the teleſcope when at D was 3.2 feet above the parapet; hence 107.5 − 3.2 = 104.3 is what the axis at D would be lower than the axis at T.

In this caſe it is evident, that half the ſum of 108.2 and 104.3, or 106.25 feet, is the difference of the relative heights of the axis at the two ſtations by a mean refraction; and that this mean refraction is ſubtended by half the difference, or

108.2

$\frac{108.2-104.3}{2}=1.95$ feet. Hence, as the diftance $=31554.6$ : rad :: $1.95$ : tang. $12''.8$ the mean refraction.

For fuppofe $t$ (fig. 12.) to be the true place of the ground, then the elevation TDO − the refraction $=$ TD$t$, or $8'\ 37''-12''.8=8'\ 24''.2=$ the angle $t$DO; therefore $t$DO $+$ ODL $=10'\ 58''.9=t$DL; whence $t$L $=100.8$ feet, to which adding 5.5 feet, the height of the axis above the ground, we have 106.3 feet, the height of the axis at T above the axis at D as before.

Alfo, if $d$ (fig. 13.) be the true place of the parapet, we fhall have the depreffion $+$ the refraction, or OTD $+$ DT$d=$ OT$d=14'\ 30''.3$, and OT$d-$OTL, or $14'\ 30''.3-2'\ 34''.7=11'\ 55'',6$ $=$ the angle LT$d$. Hence, L$d=109.5$ feet is what the parapet of the turret would be lower than the axis at T, from which taking 3.2 feet, the height of the axis above the parapet, there remains, as before, 106.3 feet, for the difference of the heights of the two ftations.

| | Feet. |
|---|---|
| The axis of the telefcope in Dover Caftle being above low-water fpring tides . . | 469. |
| To this add the height of the axis at the turnpike above that at Dover . . | 106.3 |
| We then have, for the height of the axis at the turnpike above low water, nearly . . | 575.3 |

And $5'\ 9''.4$ the contained arc of a great circle or arc of diftance being divided by $12''.8$, the mean refraction at the two ftations, we have in this inftance *about* $\frac{1}{24}$*th part for the quantity of terreftrial refraction.*

ART.

ART. V. *Refraction on the distance between Dover Castle and Calais.*

Let D (fig. 14.) be the place of the axis of the telescope, on the north turret of the Keep in Dover Castle, as before; G the top of the great balustrade of the steeple of Notre Dame Church at Calais; DO the horizontal line; and CL = SD.

The distance of Dover from Calais by the triangles is 137450 feet, which, allowing 61169 fathoms for a degree, gives 22′ 28″.2 for the length of the contained arc nearly; half of which 11′ 14″.1 is equal to the angle ODL.

| | |
|---|---|
| The height of D above low water at spring tides, as before, is . . . . | 469 feet. |
| The height of G is . . . | 140.5 |
| Therefore the difference is . . | 328.5 = GL. |
| Then, as 137450 : rad :: 328.5 : tang. | ′ ″<br>8 13 = LDG. |
| To which adding the angle ODL . . | = 11 14.1 |
| We have . . . . . | 19 27.1 for the |

angle ODG, or what G would be depressed below the place of observation, if there was no such effect as refraction. But the depression by observation was found to be 17′ 59″; wherefore the difference is 1′ 28″.1, by which, dividing the length of the arc of distance 22′. 28″.2, *we have in this instance* 15.3, *or between* $\frac{1}{15}$*th and* $\frac{1}{16}$*th part, for the quantity of terrestrial refraction.*

ART. VI. *Refraction on the distance between Allington Knoll and Tenterden.*

Let K (fig. 15.) be the place of the axis of the telescope at Allington Knoll; T the top of the flag-staff on Tenterden Steeple;

Steeple; SC the earth's ſurface; KO the horizontal line at right angles to KC; and SL = CK.

The diſtance between Allington Knoll and Tenterden has by the triangles been found to be 61775.3 feet, which, allowing 61234 fathoms = 1° gives 10′ 5″.3 for the length of the contained arc CS nearly. Half of this arc = 5′ 2″.6 is the angle OKL, from which ſubtracting the obſerved angle of depreſſion of T as ſeen from K = OKT = 3′ 51″, there remains the angle TKL = 1′ 11″.6, and conſequently this angle will be ſubtended by 21.4 feet = LT, or what the top of the flag-ſtaff at Tenterden would have been higher than the axis of the teleſcope at the Knoll, if there had been no refraction.

But the top of the flag-ſtaff on Tenterden ſteeple was 3.1 feet higher than the axis of the teleſcope when the inſtrument ſtood at that ſtation; therefore, 21.4 − 3.1 = 18.3 is what the axis at Tenterden would have been higher than the axis at the Knoll, if there had been no refraction.

Again, let T (fig. 16.) be the place of the axis of the teleſcope on Tenterden Steeple; K the ground at the ſtation on Allington Knoll; TO the horizontal line; and CL = ST.

At the ſtation T, the depreſſion of the ground at K, or the angle OTK, was obſerved 3′ 55″, which being ſubtracted from 5′ 2″.6 = OTL = half the arc of diſtance, there remains 1′ 27″.6 = the angle KTL. This laſt angle, with the diſtance between the ſtations, 61775.3 feet, give KL = 26.3 feet, for what the ground at K would have been higher than the axis at Tenterden, if there had been no refraction.

But the axis of the teleſcope, when at the ſtation K, was 5.5 feet above the ground; therefore, 26.3 + 5.5 = 31.8 is what the axis at the Knoll would be higher than the axis at Tenterden.

 Hence

Hence it follows, that ſuppoſing the refraction to have been the ſame at each of the ſtations when the obſervations were made, half the difference of theſe heights, or $\frac{31.8-18.3}{2}=6.7$ feet, would be the difference between the relative heights of the axis at the two ſtations; and that the quantity of refraction would be ſubtended by half the ſum, or $\frac{31.8+18.3}{2}=25.05$ feet; therefore, to find the mean refraction, as the diſtance of the ſtations : rad. :: 25.05 feet : tang. $1'\ 23''\frac{1}{2}$ the mean refraction.

For ſuppoſing $t$ (fig. 15.) to be the true place of the top of the flag-ſtaff, we ſhall then have the angle OKT the depreſſion + the angle TK$t$ the refraction, or $3'\ 51''+1'\ 23''\frac{1}{2}=5'\ 14''\frac{1}{2}$ = the angle OK$t$. Hence the angle OKT − the angle OKL = $5'\ 14''\frac{1}{2}-5'\ 2''.6=0'\ 11''.9$ = the angle LK$t$. Now, this laſt angle, with the diſtance of the ſtations 61775.3, give L$t$ = 3.6 feet, or what the top of the flag-ſtaff at Tenterden would be lower than the axis at the Knoll; and this being added to 3.1 feet (what the axis at Tenterden was lower than the top of the flag-ſtaff), we get, as before, 6.7 feet for the height of the axis at the Knoll above the axis at Tenterden.

In like manner, ſuppoſing $k$ (fig. 16.) to be the place of the ground at the Knoll, we have the ſum of the depreſſion and refraction, or OTK + KT$k$ = $3'\ 35''+1'\ 23''\frac{1}{2}=4'\ 58''\frac{1}{2}$ = OT$k$; and OTL − OT$k$ = $5'\ 2''.6-4'\ 58''\frac{1}{2}=0'\ 4''.1$ = the angle $k$TL. Hence, $kt$ = 1.2 feet is the height of the ground at the Knoll above the axis at Tenterden, which being added to 5.5 feet the height of the axis at the Knoll above the ground, we have as before 6.7 feet for the difference of the heights.

 The

| | Feet. |
|---|---|
| The height of the axis of the telescope at Allington Knoll above low-water mark at spring tides, as determined by the observations there, and at the station of High Nook, is . . . . | 329 |
| The axis on Tenterden Steeple has been shewn to be lower than the Knoll . . . . | 6.7 |
| Therefore, the axis on Tenterden Steeple is higher than low water . . . . | 322.3 |

The arc of distance of the two stations = 10′ 5″.3 being divided by 1′ 23″½ the mean refraction, *we have in this case* 7″¼, *or between* $\frac{1}{7}$*th and* $\frac{1}{8}$*th part for terrestrial refraction.*

The example in Art. IV. and this last are given at large, because, if the points where the axis of the telescope was at the respective stations had been observed, in the first, one would have been a depression, and the other an elevation; but in this both would have been depressed by observation.

### Art. VII. *General Remarks.*

The three preceding examples being sufficient to shew the mode that has been invariably adhered to in computing the effect of terrestrial refraction, we have, in the following table, collected the whole of the results together, beginning with those distances where it has been found the greatest, and ending with these where it has been found the least.

The titles at the tops of the columns respectively fully explain the nature of the table, which contains more double observations, made on a greater variety of very accurate distances, and with a better instrument for determining small angles of elevation and depression, than perhaps were ever obtained before.

fore. Theſe reſults are not however offered as being free from error; on the contrary, if the circumſtances had permitted this to become a principal object in our operation, the ſucceſſive repetition of the obſervations for many times would, no doubt, have furniſhed ſtill more ſatisfactory concluſions. It is hoped, nevertheleſs, that theſe, ſuch as they are, may have their uſe, were it only by ſhewing the variableneſs of terreſtrial refraction, to induce to the making of others, which, as has been already obſerved, would ultimately lead to a much more minute inveſtigation of this curious and intereſting ſubject.

The heights of the barometer and thermometer are inſerted on the days on which the obſervations were made, merely to ſhew what was nearly the ſtate of the atmoſphere at the reſpective times. But we have not attempted to apply any correction on that account, becauſe it could not be done in a ſatisfactory manner, and conſequently could not be uſeful, unleſs the circumſtances had permitted reciprocal obſervations to have been made at correſponding times with double ſets of inſtruments, which in our ſituation was impoſſible.

By attending to the reſults in the table, it will in general be ſeen, that terreſtrial as well as celeſtial refraction certainly diminiſhes as the heights of the ſtations above the ſea increaſe; and that, at particular times at leaſt, it is much greater than has hitherto been ſuppoſed, even to between $\frac{1}{2}$ and $\frac{1}{3}$ part of the arc of diſtance, inſtead of being only $\frac{1}{9}$th or $\frac{1}{14}$th part. Beſides the inſtance of this extraordinary effect inſerted in the table, between Allington Knoll and Ruckinge, where the diſtance of the ſtations is but ſmall, and one of them little higher than the ſea, we could have given another on a diſtance as well as on heights ſtill more conſiderable, namely, Shooter's Hill and the ball of St. Paul's Church: for, ſuppoſing the

firſt to be 482, and the laſt 403 feet, above low water at the ſea, the refraction on the morning of the 1ſt of September, 1787, as obſerved at Shooter's Hill, was 1′ 47″, which is between $\frac{1}{3}$d and $\frac{1}{4}$th part of the contained arc.

If the circumſtances had permitted the refraction on the diſtance between Dover and Calais to have been repeatedly tried, at the bottom of the cliff, at the top of the cliff, and again at the top of the caſtle, we ſhould probably have found the refraction at the three ſtations conſiderably different, with the ſame length of arc, or one only varying inſenſibly.

But, in order to be enabled to make concluſive obſervations of this ſort, the operation ſhould become a diſtinct one, or at moſt only comprehend ſuch others as are connected with the modifications of the atmoſphere. For purpoſes of this kind very fine levels would be requiſite; and ſome of the higheſt mountains of Scotland, ſituated near the ſea, ſuch as *Ben Nevis* and *Cruachan Ben*, where the relative heights of the ſtations might be accurately aſcertained by levelling, would ſeem to be eligible ſituations.

TABLE

TABLE containing the RESULTS of the OBSERVATIONS for the EFFECT

| Dates of the Obfervations. | Places. | Bar. | Therm. | Stations. | He Tel the |
|---|---|---|---|---|---|
| | | In. | ° | | |
| 1787, Oct. 21. | Allington Knoll - | 29.61 | 56 | Allington Knoll and Ruckinge — — | A. |
| 23. | Ruckinge - - | 29.82 | $51\frac{1}{2}$ | | R. |
| 19. | Dymchurch Inn - | 29.9 | $55\frac{1}{2}$ | High Nook and Lydd — — | H. L. |
| 21. | Allington Knoll - | 29.61 | 56 | Allington Knoll and High Nook — | A. |
| 19. | Dymchurch Inn - | 29.9 | $55\frac{1}{2}$ | | H. |
| 21. | Allington Knoll - | 29.61 | 56 | Allington Knoll and Tenterden — — | A. |
| 26. | Tenterden Inn - | 29.54 | $56\frac{1}{2}$ | | T. |
| 7. | Padlefworth - - | 29.6 | 70 | Padlefworth and Lydd — — | P. L. |
| 1788, Aug. 18. | Frant Inn - - | 29.36 | 58 | Frant and Botley Hill — — — | F. |
| 23. | Botley Hill - - | 28.89 | $62\frac{1}{2}$ | | B. |
| 1787, Sept. 28. | Dover Caftle - | 29.62 | $58\frac{1}{2}$ | Dover Caftle and Padlefworth — | D. |
| Oct. 7. | Padlefworth - - | 29.6 | 70 | | P. |
| 13. | Fairlight Down - | 28.81 | $55\frac{1}{2}$ | Fairlight Down and Tenterden — | F. |
| 26. | Tenterden Inn - | 29.54 | $56\frac{1}{2}$ | | T. |
| 26. | Tenterden Inn - | 29.54 | $56\frac{1}{2}$ | Tenterden and Lydd — — | T. L. |
| 1788, Aug. 11. | Goudhurft Churchyard | 29.74 | $58\frac{1}{4}$ | Goudhurft and Frant — — | G. |
| 18. | Frant Inn - - | 29.36 | 58 | | F. |
| 1787, Oct. 13. | Fairlight Down - | 28.81 | $55\frac{1}{2}$ | Fairlight Down and Lydd — — | F. L. |
| 1788, Aug. 11. | Goudhurft Churchyard | 29.74 | $58\frac{1}{4}$ | Goudhurft and Tenterden — — | G |
| 1787, Oct. 26. | Tenterden Inn - | 29.54 | $56\frac{1}{2}$ | | T. |
| 21. | Allington Knoll - | 29.61 | 56 | Allington Knoll and Lydd — — | A L. |
| Sept. 28. | Dover Caftle - - - | 29.62 | $58\frac{1}{2}$ | Dover Caftle and Folkftone Turnpike | D |
| 1788, Sept. 2. | Folkftone Turnpike | 29.55 | 64 | | F. |

RESULTS of SINGLE OBSERVATIONS depending on the Heights of Dover Caftle and F Coaft of France.

| | | | | | |
|---|---|---|---|---|---|
| 1788, Sept. 2. | — — — | 29.55 | 64 | Folkftone Turnpike and N. D. Church at Calais | F C |
| 1787, Sept. 28. | — — — | 29.62 | $58\frac{1}{2}$ | Dover Caftle and N. D. Church at Calais | D C |
| 1788, Sept. 2. | — — — | 29.55 | 64 | Folkftone Turnpike and Station at Montlambert — — — | F M |

## ꞉FFECT of TERRESTRIAL REFRACTION.

| | Height of the Telefcopeabove the Sea in Feet. | | Diftance of the Station in Fathoms. | Contain-ed Arc nearly | Mean Refraction. | Proportion-able Part. |
|---|---|---|---|---|---|---|
| | | | | ′ ″ | ′ ″ | |
| | A.<br>R. | 329.<br>37.4 | 2675.4 | 2 38 | 1 8 | $\frac{1}{2}$ and $\frac{1}{3}$ |
| | H.<br>L. | 27.6<br>130.4 | 5227.1 | 5 8 | 0 55 | $\frac{1}{5}$ and $\frac{1}{6}$ |
| | A.<br>H. | 329.<br>27.6 | 3864.1 | 3 48 | 0 38 | $\frac{1}{6}$ |
| | A.<br>T. | 329.<br>322.3 | 10296. | 10 5 | 1 23½ | $\frac{1}{7}$ and $\frac{1}{8}$ |
| | P.<br>L. | 624.<br>130.4 | 13255.5 | 13 2 | 1 31 | $\frac{1}{8}$ |
| | F.<br>B. | 659.<br>880. | 15060.7 | 14 48 | 2 1 | $\frac{1}{8}$ |
| | D.<br>P. | 469.<br>642. | 7093.5 | 6 57 | 0 42 | $\frac{1}{10}$ |
| | F.<br>T. | 599.<br>322.3 | 11939.1 | 11 46 | 1 12 | $\frac{1}{10}$ |
| | T.<br>L. | 322.3<br>130.4 | 11027.8 | 10 50 | 0 55½ | $\frac{1}{11}$ and $\frac{1}{12}$ |
| | G.<br>F. | 497.<br>659. | 7398.3 | 7 15 | 0 38½ | $\frac{1}{11}$ and $\frac{1}{12}$ |
| – | F.<br>L. | 599.<br>130.4 | 11948.3 | 11 43 | 0 55 | $\frac{1}{13}$ |
| | G.<br>T. | 497.<br>322.3 | 9062.4 | 8 53 | 0 35 | $\frac{1}{13}$ |
| | A.<br>L. | 329.<br>130.4 | 7975. | 7 51 | 0 21 | $\frac{1}{22}$ |
| | D.<br>F. | 469.<br>575.3 | 5259.1 | 5 9.4 | 0 12.8 | $\frac{1}{24}$ |

and Folkftone Turnpike, combined with thofe on the

| | | | | | | |
|---|---|---|---|---|---|---|
| ꞉a- | F.<br>C. | 575.3<br>140.5 | 26697. | 26 4 | 3 54.8 | $\frac{1}{6}$ and $\frac{1}{7}$ |
| | D.<br>C. | 469.<br>140.5 | 22908. | 22 28.2 | 0 15.4 | $\frac{1}{15}$ and $\frac{1}{16}$ |
| m- | F.<br>M | 575.3<br>639.5 | 28967. | 28 29 | 1 45. | $\frac{1}{17}$ |

SECTION

## SECTION EIGHTH.

*Secondary triangles, ſubdivided into two ſets, for the improvement of the maps of the country, and the plan of the City of London and its Environs. Plate XI.*

IN the ſeries of great triangles, whereby the diſtance between the meridians of the Royal Obſervatories of Greenwich and Paris has been determined, the ſame excellent inſtrument having been placed at every ſtation on our ſide of the Channel, and all the angles obſerved with the utmoſt care, it hath conſequently followed, that the baſe on Hounſlow Heath, and that in Romney Marſh, reciprocally meaſure each other within a few inches of the truth, which is an inſtance of ſuch exactneſs as probably never occurred in any former operation of this ſort. The extreme ſmallneſs of the error on the ſum of the three angles of each triangle ſufficiently proves that the general reſult would not have differed greatly, if only two of the angles had actually been obſerved. But in an operation of ſo much importance, this could not have been depended upon; nothing was to be left doubtful; and therefore, in the execution of the various parts, the moſt minute attention was paid to every circumſtance whereby the accuracy might be affected, and particularly to the placing of the lights and inſtrument reciprocally over the ſame point marking the ſtation, that no poſſible error might ariſe from parallax or excentricity.

From

From this mode of conducting the operation, it will readily be ſeen, that, if time had permitted, the ſituation of a multitude of other points in the country might have been very accurately determined, beſides thoſe actually marking the points of the triangles, whereby the ordinary maps would have been greatly improved by ſuch as choſe at any time hereafter to make uſe of theſe as ſo many given diſtances. But the circumſtances not having permitted us to multiply thoſe points to the extent that might have been wiſhed, and that would have been eaſily practicable, if the operation had commenced at an earlier ſeaſon of the year; we have therefore been obliged to limit the number to a few of the moſt conſpicuous and beſt defined objects.

Theſe ſecondary triangles are ſubdivided into two ſets. The firſt ſet conſiſts of thirty-five, whereby the relative diſtances of ſo many points have been determined from certain ſtations of the principal ſeries, beginning with thoſe objects that have been interſected from the moſt weſterly ſtations, and ſo on, proceeding gradually with the others towards the eaſt. Two angles only of each of thoſe triangles being obſerved, the third is that at the interſected object, or the ſupplement to 180°. Although the diſtances thus obtained cannot be quite ſo accurate as the ſides of the principal ſeries; yet there is no reaſon to apprehend, that they will be found to differ widely from the truth, when they come to be proved in the courſe of any ſubſequent operation, by which alone they can be put to the teſt.

Com-

## Computation of the firſt ſet of ſecondary triangles.

| N° | Triangles. | Angles. | Diſtances of the ſtations from the interſected object in feet. | |
|---|---|---|---|---|
| 1. | King's Arbour . | 8° 52′ 57″ | from Stanwell | 10927 |
| | St. Ann's Hill . | 4 4 44 | | 23720 |
| | *Stanwell Church* . | 167 2 19 | | |
| 2. | King's Arbour . | 28 35 34 | from Harrow on the Hill | 42944 |
| | Hanger-hill Tower . | 89 23 52 | | 20553.3 |
| | *Harrow on the Hill* . | 62 0 34 | | |
| 3. | King's Arbour . | 70 1 47 | from Banſtead Church | 80994 |
| | Hanger-hill Tower . | 82 19 25.1 | | 76807.5 |
| | *Banſtead Church* . | 27 38 47.9 | | |
| 4. | Hampton Poor-houſe . | 88 58 23 | from Kew Pagoda | 22849 |
| | King's Arbour . | 40 14 25 | | 35364.5 |
| | *Kew Pagoda* . . | 50 47 12 | | |
| 5. † | Harrow on the Hill . | 69 43 8 | from Spring Grove Houſe | 35851 |
| | St. Paul's Church . | 35 58 9 | | 57253.9 |
| | Spring Grove Houſe, *Sir* Jo. BANKS * . . | 74 18 43 | | |
| 6. | Hanger hill Tower . | 19 33 4.3 | from Richmond Royal Obſervatory . | 20164.4 |
| | Spring Grove Houſe * . | 82 46 15.9 | | 6802.1 |
| | *Richmond Royal Obſervatory* | 77 40 39.8 | Hanger Hill from Spring Grove . . | 19857.8 |

† The Royal Obſervatory in Richmond lower Park could not be ſeen from any of the ſtations of the great ſeries of triangles, except Hanger-hill Tower, from whence the bearing of it was taken. In order to interſect this bearing, the aſſiſtance of certain operations made with the aſtronomical quadrant in 1783 at Spring Grove Houſe has been called in, by the help of which the ſituations of the Obſervatory and of Spring Grove Houſe have been determined. In like manner, the bearings of Batterſea and Stretham, taken from Hundred Acres, have been interſected with the quadrant from St. Paul's and Fulham. The ſtations where the quadrant was uſed are diſtinguiſhed with aſteriſks.

| N° | Triangles. | Angles. | Distances of the stations from the intersected object in feet. | |
|---|---|---|---|---|
| | | ° ′ ″ | | |
| 7. | Hundred Acres . . | 14 13 27 | from Battersea Church | 50664.5 |
| | St. Paul's * . . | 34 3 49.2 | | 22226 |
| | *Battersea Church* . . | 131 42 43.8 | | |
| 8. | Hundred Acres . | 27 51 55.6 | from Stretham Church | 35957.3 |
| | Fulham Church * . | 46 12 54.4 | | 23279.3 |
| | *Stretham Church* . | 105 55 10 | | |
| 9. | Hundred Acres . | 36 59 35.8 | from Clapham Common | 43351.7 |
| | Severndroog Castle . | 33 28 20.5 | | 47296.4 |
| | Clapham Common, Mr. CAVENDISH . . | 109 32 3.7 | | |
| 10. | Norwood . . | 76 19 14.5 | from Argyll Street Observatory | 40083.2 |
| | Severndroog Castle . | 52 41 37 | | 39963 |
| | *Argyll Street Ob. Maj. Gen.* ROY | 50 59 8.5 | | |
| 11. | Norwood . . | 62 30 23.5 | from St. Paul's Church | 37840.9 |
| | Severndroog Castle . | 57 8 8.5 | | 39963 |
| | *St. Paul's Church* . | 60 21 28 | | |
| | N. B. By combining the results of these two last triangles a third is formed, which gives for the distance of Argyll Street from St. Paul's | | | 9632 |
| 12. | Norwood . . | 36 36 32 | from Bromley College | 22695.4 |
| | Severndroog Castle . | 32 52 48 | | 24930.6 |
| | *Bromley College* . | 110 30 40 | | |
| 13. | Norwood . . | 31 53 3 | from Chislehurst Church | 35777.9 |
| | Severndroog Castle . | 67 48 12.5 | | 20981.1 |
| | *Chislehurst Church* . | 80 18 44.5 | | |
| 14. | Greenwich Royal Observatory | 92 38 13.5 | from Wanstead House | 34413.6 |
| | Severndroog Castle . | 64 46 33.5 | | 37999.7 |
| | *West Pediment of Wanstead Ho.* | 22 35 13 | | |
| 15. | Greenwich Royal Observatory | 131 45 43 | from Loampit Hill | 6352.6 |
| | Severndroog Castle . | 14 7 0 | | 19428.4 |
| | *Loampit Hill* . : | 34 7 17 | | |
| 16. | Greenwich Royal Observatory | 85 49 9 | from Beckenham Church | 25622 |
| | Severndroog Castle . | 63 29 48 | | 28555 |
| | *Beckenham Church* . | 30 41 3 | | |

| N° | Triangles. | Angles. | Distances of the stations from the intersected object in feet. | |
|---|---|---|---|---|
| | | ° ′ ″ | | |
| 17. | Greenwich Royal Observatory<br>Severndroog Castle .<br>*Eltham Church* . | 22 41 33<br>87 18 31.5<br>69 59 55.5 | from Eltham Church | 15531<br>5998.3 |
| 18. | Severndroog Castle .<br>Botley Hill . .<br>*Knockholt Beeches* . | 21 56 44<br>54 48 27<br>103 14 49 | from Knockholt Beeches | 58933<br>26951 |
| 19. | Severndroog Castle .<br>Botley Hill . .<br>*Leeth Hill Tower* . | 31 40 29.4<br>124 53 14<br>23 26 16.6 | from Leeth Hill Tower | 144761<br>92668 |
| 20. | Botley Hill . .<br>Frant Church . .<br>*Firedean Tower* . | 39 17 6.5<br>26 58 39<br>113 44 4.5 | from Firedean Tower | 44780.4<br>62507 |
| 21. | Botley Hill . .<br>Frant Church .<br>*Crowborough Beacon* . | 19 51 19.5<br>77 32 33<br>82 36 7.5 | from Crowborough Beacon | 88977<br>30949.7 |
| 22. | Botley Hill . .<br>Wrotham Hill .<br>*Sevenoaks Windmill* . | 24 22 7<br>28 57 42<br>126 40 11 | from Sevenoaks Windmill | 44032.4<br>37519.8 |
| 23. | Frant Church .<br>Goudhurst Church .<br>*Wadhurst Church* . | 46 5 9<br>26 21 46.5<br>107 33 4.5 | from Wadhurst Church | 20674<br>33538.7 |
| 24. | Goudhurst Church .<br>Fairlight Down .<br>*Brightling Windmill* . | 42 6 25<br>38 5 33<br>99 48 2 | from Brightling Windmill | 58616.3<br>63707.4 |
| 25. | Fairlight Down .<br>Lydd Church .<br>*Rye Church* . . | 22 40 17<br>21 23 25<br>135 56 18 | from Rye Church | 37598<br>39734 |
| 26. | Fairlight Down .<br>Dover Castle, north turret<br>*Dengeness Light-house* . | 19 34 30<br>13 54 24.6<br>146 31 5.4 | from Dengeness Light-house | 81082.7<br>113030 |
| 27. | Fairlight Down .<br>Goudhurst Church .<br>*Ore Church* . | 4 12 42<br>60 29 28<br>115 17 50 | from Ore Church | 7605.3<br>90123.2 |

| Nº | Triangles. | Angles. ° ′ ″ | Distances of the stations from the intersected object in feet. | |
|---|---|---|---|---|
| 28. | Fairlight Down . | 23 32 23 | from Fairlight Church | 5384 |
| | Lydd Church . | 1 50 43 | | 66787.7 |
| | *Fairlight Church* . | 154 36 54 | | |
| 29. | Tenterden Church . | 30 42 37 | from Ashford Church | 46096 |
| | Allington Knoll . | 46 45 7 | | 32319.2 |
| | *Ashford Church* . | 102 32 16 | | |
| 30. | Lydd Church . . | 43 34 50.5 | from Ruckinge Church | 41682.2 |
| | High Nook near Dymchurch | 87 40 22 | | 28758.4 |
| | *Ruckinge Church* . | 48 44 47.5 | | |
| 31. | High Nook . . | 83 44 33.5 | from New Romney Ch. | 16965.4 |
| | Ruckinge . . | 32 17 35.5 | | 31566.3 |
| | *New Romney Church* . | 63 57 51 | | |
| 32. | High Nook . . | 42 44 44.5 | from Lymne Castle | 23741.6 |
| | Allington Knoll . | 70 21 48 | | 17109.6 |
| | *Lymne Castle* . . | 66 53 27.5 | | |
| 33. | Lydd Church . . | 2 10 29.2 | from Folkstone Church | 78946.9 |
| | Folkstone Turnpike . | 27 26 22 | | 6501.3 |
| | *Folkstone Church* . | 150 23 8.8 | | |
| 34. | Folkstone Turnpike . | 24 35 59 | from Beachborough Summer House | 23325.2 |
| | Padlesworth . . | 123 46 35.2 | | 11681.6 |
| | *Beachborough Summer House* | 31 37 25.8 | | |
| 35. | Padlesworth . . | 32 36 0 | from Waldershare Monument | 37862.5 |
| | Dover Castle . . | 62 24 5 | | 23081.4 |
| | *Waldershare Monument* . | 84 59 55 | | |

*Second set of secondary triangles.*

In the Paper of 1787, sufficient reasons have been given for avoiding St. Paul's as a station in the series of great triangles. Indeed, if no other objection had existed, the smoke of the capital alone would have been found extremely inconvenient. This was experienced at Shooter's Hill, where we were

were detained a whole week, before the white lights, notwithſtanding their extraordinary brilliancy, could be ſeen at Hanger-hill Tower, or even at Argyll Street, the north-eaſt wind, which then prevailed, having brought the impenetrable maſs of ſmoke between the ſtation of the inſtrument and the points to be obſerved; and at laſt we were obliged to watch all night, till towards the morning the fires of London being extinguiſhed, the white lights could then be interſected.

It is not therefore ſurpriſing, that from the ſtations of Norwood, Greenwich, and Shooter's Hill, we ſhould only be able to fix, in a ſatisfactory manner, two points in London, namely, St. Paul's and Argyll Street. Bearings, it is true, of others were taken; but that theſe might be interſected by angles not too acute, it became neceſſary to make uſe of obſervations that had been formerly obtained at Argyll Street with my own inſtrument in its vertical poſition, and at St. Paul's with the aſtronomical quadrant. Moreover, by way of finiſhing the operation, and furniſhing ſuch part of the inhabitants of the metropolis as may be curious in matters of this ſort with a ſet of diſtances that cannot fail to be uſeful to them, two new ſtations were choſen for the great inſtrument to the northward of London, one on *Hornſey Hill*, and the other on *Primroſe Hill*. Thus, from the combined operations at theſe ſeveral places, we have been able to determine the ſituation of thirty-ſeven conſpicuous points, conſiſting chiefly of the moſt remarkable ſteeples in and near the capital.

By referring to Plate XI. which is in fact the ſkeleton, but on a very ſmall ſcale, for an improved plan of London and its environs, the relative ſituation of theſe points with regard to St. Paul's, and the four neareſt ſtations of the great ſeries, will be ſeen. Some of the principal of theſe ſecondary tri-

angles have been repreſented by dotted lines in the plan. To have expreſſed more of them in that way would only have occaſioned confuſion. Here it is to be remarked, that the diſtance of Argyll Street from St. Paul's, 9632 feet, reſulting from the 10th and 11th ſecondary triangles of the firſt ſet, becomes a baſe in the quadrilateral formed by *St. Paul's*, *Argyll Street*, *Hornſey Hill*, and *Primroſe Hill*. Hence, by the obſerved angles at theſe two laſt ſtations, and the aſſumed length of one of the unknown ſides, all the angles of the quadrilateral are computed, by which means, and the true length of one ſide given, the true lengths of all the others are readily obtained.

### Computation of the ſecond ſet of ſecondary triangles.

Situations determined by the great inſtrument from Hornſey Hill and Primroſe Hill.

| N° | Triangles. | Angles. ° ′ ″ | Diſtances of the ſtations from the interſected object in feet. | |
|---|---|---|---|---|
| 1. | Hornſey Hill . | 46 42 41 | from St. Paul's Church | 23297.1 |
| | Primroſe Hill . | 83 21 27.5 | | 17072.8 |
| | *St. Paul's* . . | 49 55 51.5 | Hornſey Hill from Primroſe Hill | 17949.05 |
| 2. | Hornſey Hill . | 23 8 34 | Argyll Street Obſervatory | 23803.4 |
| | Primroſe Hill. . | 112 49 57 | | 10150.7 |
| | *Argyll Street Obſervatory* | 44 1 29 | | |
| 3. | Hornſey Hill . | 23 33 59 | from Hampſtead Church | 17972 |
| | Primroſe Hill . | 78 23 43 | | 7335.5 |
| | *Hampſtead Church* . | 78 2 18 | | |
| 4. | Hornſey Hill . | 29 11 3.5 | from Mr. Duveluz's Cupola | 12181.2 |
| | Primroſe Hill . | 16 44 50 | | 7198.2 |
| | *Mr.* Duveluz's *Cupola,* Hornſey Lane, Highgate | 134 4 6.5 | | |
| 5. | Hornſey Hill . | 47 30 42 | from Iſlington Church | 14272.5 |
| | Primroſe Hill . | 51 42 39 | | 13409 |
| | *Iſlington Church* . | 80 46 39 | | |

N°

| | N° | Triangles. | Angles. | Diſtances of the ſtations from the interſected object in feet. | |
|---|---|---|---|---|---|
| With the great inſtrument from Hornſey Hill and Primroſe Hill. | | | ° ′ ″ | | |
| | 6. | Hornſey Hill .<br>Primroſe Hill .<br>*Highbury Houſe, Mr.* AUBERT | 55 28 32<br>29 28 52<br>95 2 36 | from Highbury Houſe | 8867.7<br>14845.4 |
| | 7. | Hornſey Hill .<br>Primroſe Hill .<br>*St. Luke's Church, Old Street* | 50 52 33<br>68 59 37<br>60 7 50 | from St. Luke's Church | 19323<br>16057.5 |
| | 8. | Hornſey Hill .<br>Primroſe Hill .<br>*St. Leonard's Ch. Shoreditch* | 62 9 30<br>63 36 33.5<br>54 13 56.5 | from St. Leonard's Church | 19816<br>19560.5 |
| | 9. | Hornſey Hill .<br>Primroſe Hill .<br>*Chriſt Church, Spitalfields* | 61 19 7.5<br>70 33 39<br>48 7 13.5 | from Chriſt Church, Spit. | 22733<br>21149.3 |
| | 10. | Hornſey Hill .<br>Primroſe Hill .<br>*Bow Church, Cheapſide* | 49 20 38.5<br>81 21 4.5<br>49 18 17 | from Bow Ch. | 23404.4<br>17959.6 |
| | 11. | Hornſey Hill .<br>Primroſe Hill .<br>*St. Bride's Ch. Fleet Street* | 42 33 43<br>86 44 24<br>50 41 53 | from St. Bride's Church | 23158<br>15689.1 |
| | 12. | Hornſey Hill .<br>Primroſe Hill .<br>*St. George's Ch. Bloomſbury* | 31 16 11<br>94 11 25<br>54 32 24 | from St. George's Church | 21977<br>11438 |
| | 13. | Hornſey Hill .<br>Primroſe Hill .<br>*St. Giles's Church* . | 29 32 8.5<br>100 13 30.5<br>50 14 21 | from St. Giles's Church | 22978.4<br>11510.4 |
| | 14. | Hornſey Hill .<br>Primroſe Hill .<br>*St. Ann's, Soho* . | 28 2 38<br>106 40 20<br>45 17 2 | from St. Ann's Church | 24197.1<br>11875.4 |
| One angle taken with the great inſtrument, and the other with that in Argyll Street Obſ. | 15. | Hornſey Hill .<br>Argyll Street Obſervatory<br>*Highgate Chapel* . | 59 52 55.5<br>22 37 47.6<br>97 29 16.9 | from Highgate Chapel | 9237.8<br>20766.9 |
| | 16. | Primroſe Hill .<br>Argyll Street Obſervatory<br>*St. Clement's Church* . | 20 43 50<br>123 0 9.4<br>36 16 0.6 | from St. Clement's Church | 14390.2<br>6073.8 |

N°

| | Nº | Triangles. | Angles. ° ′ ″ | Diſtances of the ſtations from the interſected object in feet. | |
|---|---|---|---|---|---|
| One angle taken with the great inſtrument, and the other with that in Argyll Street Obſervat. | 17. | Primroſe Hill<br>Argyll Street .<br>*St. Mary's Ch. in the Strand* | 17 52 31<br>127 21 15<br>34 46 14 | from St. Mary's Church | 14148.5<br>5463.4 |
| | 18. | Primroſe Hill .<br>Argyll Street Obſervatory<br>*St. Martin's Ch. in the Fields* | 7 32 8.5<br>152 0 27<br>20 27 24.5 | from St. Martin's Church | 13631.6<br>3808.8 |
| | 19. | Primroſe Hill .<br>Argyll Street Obſervatory<br>*Pantheon* . | 3 12 59.5<br>102 32 39.4<br>74 14 21.1 | from the Pantheon | 10295.5<br>591.8 |
| A ſmall HADLEY's ſextant uſed in Argyll Street. | 20. | Primroſe Hill .<br>Argyll Street Obſervatory<br>*St. George's Ch. Hanover Sq.* | 5 35 34<br>120 13 56<br>54 10 30 | from St. George's Church | 10816.5<br>1220.1 |
| | 21. | Primroſe Hill .<br>Argyll Street Obſervatory<br>*South Audley Chapel* . | 16 7 10<br>103 34 59<br>60 17 51 | from South Audley Chapel | 11359.3<br>3244.5 |
| A ſmall HADLEY's ſextant uſed at St. Paul's. | 22. | Hornſey Hill .<br>St. Paul's Church .<br>*Newington Church* . | 38 14 6<br>16 35 7<br>125 10 47 | from Newington Church | 8136.4<br>17641 |
| | 23. | Hornſey Hill .<br>St. Paul's Church .<br>*St. Matthew's Church, Bethnal Green* . | 20 29 59<br>66 7 5<br>92 22 56 | from St. Matthew's Church | 21321<br>8165.8 |
| The aſtronomical quadrant uſed at St. Paul's in 1783. | 24. | Hornſey Hill .<br>St. Paul's Church .<br>*St. George's, Ratcliff* . | 18 22 9<br>105 32 24<br>56 5 27 | from St. George's Church, Ratcliff | 27045.2<br>8846.4 |
| | 25. | Primroſe Hill .<br>St. Paul's Church .<br>*St. James's Church* . | 30 44 17<br>45 39 31<br>103 36 12 | from St. James's Church | 12562.7<br>8978 |
| | 26. | Greenwich Royal Obſervat.<br>St. Paul's Church .<br>*Limehouſe Church* . | 31 5 38<br>27 52 40<br>121 1 42 | from Limehouſe Church | 13999.3<br>15462 |

Nº

| | N° | Triangles. | Angles. | Diſtance of the ſtations from the interſected object in feet. | |
|---|---|---|---|---|---|
| The aſtronomical quadrant uſed at St. Paul's in 1783. | 27. | Argyll Street Obſervatory | 61° 47′ 27″ | from St. Peter's Church, Weſtm. | 6279.5 |
| | | St. Paul's Church | 39 42 24.5 | | 8661.8 |
| | | S.W. pinnacle of the S. tower of St. Peter's Ch. Weſtm. | 78 30 8.5 | | |
| Great inſt. and Argyll Street inſtrument. | 28. | Norwood | 18 5 5 | from the Monument | 36409.7 |
| | | Argyll Street Obſervatory | 64 9 55.7 | | 12557.5 |
| | | *The Monument* | 97 44 59.3 | | |
| Both with the aſtronomical quadrant. | 29. | Jew's Harp ſtation | 52 52 53 | from St. Paul's Church | 13522.0 |
| | | Black Lane ſtation | 92 12 30 | | 10790.3 |
| | | *St. Paul's Church* | 34 54 37 | from Jew's Harp to Black Lane, the baſe of 1783 | 7744.3 |
| One angle with the Argyll Str. inſtrument and the other with the aſtronomical quadrant. | 30. | Jew's Harp ſtation | 89 56 55.9 | from Argyll Str. | 5656.8 |
| | | Black Lane ſtation | 36 9 50 | | 9586.2 |
| | | *Argyll Street Obſervatory* | 53 53 14.1 | | |
| | 31. | Argyll Street Obſervatory | 95 30 56.5 | from the Britiſh Muſeum | 3488.3 |
| | | Jew's Harp ſtation | 30 5 26 | | 6511 |
| | | Wind Vane *of the Britiſh Muſeum* | 54 23 37.5 | | |
| | 32. | Argyll Street Obſervatory | 74 26 16.6 | from Charlotte Street Chapel | 1848.1 |
| | | Jew's Harp ſtation | 19 1 55.9 | | 5459.4 |
| | | *Charlotte Street Chapel* | 86 31 47.5 | | |
| Both angles obſerved with the aſtronomical quadrant in 1783. | 33. | Jew's Harp ſtation | 85 27 45 | from Portland Chapel | 4097.7 |
| | | Black Lane ſtation | 28 53 30 | | 8474.2 |
| | | *Portland Chapel* | 65 38 45 | | |
| | 34. | Jew's Harp ſtation | 60 43 55 | from Fitzroy Chapel | 4015.5 |
| | | Black Lane ſtation | 31 12 45 | | 6759.6 |
| | | *Fitzroy Chapel* | 88 3 20 | | |
| | 35. | Jew's Harp ſtation | 63 25 50 | from the Tabernacle | 4780.8 |
| | | Black Lane ſtation | 37 14 40 | | 7048.4 |
| | | *Tabernacle* | 79 19 30 | | |
| | 36 | Jew's Harp ſtation | 19 45 45 | from the Small-Pox Hoſpital | 6670.5 |
| | | Black Lane ſtation | 56 57 45 | | 2690.4 |
| | | *Small-Pox Hoſpital* | 103 16 30 | | |

| | Nº | Triangles. | Angles. | Distances of the stations from the intersected object in feet. | |
|---|---|---|---|---|---|
| Both angles observed with the ast. quadrant in 1783. | 37. | Jew's Harp station . | ° ′ ″ 4 41 45 | from St. Pancras Church | 5728.1 |
| | | Black Lane station . | 12 58 25 | | 2088.8 |
| | | *St. Pancras Church* . | 162 19 50 | | |

That these secondary triangles may be more generally useful to the inhabitants of London and its environs, the angles, which the 53 points comprehended in Plate XI. respectively form with each other at the center of the dome of St. Paul's, are collected in the annexed table, together with their several distances from that central point. The objects are arranged into two classes, according as they are situated to the eastward or westward of the meridian of St. Paul's. Those of the first class commence at the north meridian, and proceed by the east to 180°. These of the second commence at the south meridian, and proceed, in like manner, by the west to 180°. From this table the total angle between any two objects being had by simple subtraction, and the distances from St. Paul's given, the distances of the objects from each other are readily obtained. Whoever, therefore, should be desirous of knowing accurately his own situation in this great metropolis may easily satisfy himself, by taking two angles from the top of his house, with a good HADLEY's sextant or theodelet, between any known objects near to him, and the best disposed for the purpose. By the help of these *data*, and a very simple trigonometrical computation, he will obtain what he wants; and he may even satisfy another curiosity which will probably occur, namely, that of putting to the test our original operation, by trying how nearly different triangles bring out the same result. It will readily be conceived that, for trials of this sort, the

 points

points whofe fituations have been determined by the great inftrument fhould be chofen preferably to the others; and next to thefe, the objects that have been fixed by one angle, taken with the Argyll-Street inftrument, as more to be relied upon, than thofe obferved with the aftronomical quadrant or fextant. Thus an excellent foundation is laid for the improvement of the plan of London and its environs, which may by thefe triangles be rendered more accurate than would have been poffible by any other mode.

Table fhewing the bearings and diftances of objects fituated in and near London, from the center of the Dome of St. Paul's.

| | Objects. | Bearings from the north merid. eaftward. ° ′ ″ | Diftances in feet. |
|---|---|---|---|
| Eaftward of the meridian of St. Paul's. | Newington Church | 9 59 39·3 | 17641 |
| | St. Luke's Church, Old Street | 12 57 53·7 | 4262.7 |
| | St. Leonard's Church, Shoreditch | 44 57 39·8 | 6746.4 |
| | The weft pediment of Wanftead Houfe | 55 53 46·4 | 36308.4 |
| | St. Matthew's Church, Bethnal Green | 59 31 37·3 | 8165.8 |
| | Chrift's Church, Spitalfields | 70 38 37·3 | 5878.4 |
| | Bow Church, Cheapfide | 87 48 4.1 | 1078.1 |
| | Limehoufe Church | 92 51 6 | 15462 |
| | St. George's Church, Ratcliff Highway | 98 56 56.3 | 8846.4 |
| | The Monument | 115 15 45·7 | 3114.2 |
| | Severndroog Caftle, Shooter's Hill | 115 28 50.4 | 39963 |
| | Tranfit-room of Greenwich Royal Obferv. | 120 43 46 | 25655·5 |
| | Eltham Church | 123 46 4.2 | 41091 |
| | Loampit Hill | 134 40 48.7 | 23450.1 |
| | Station at Norwood | 175 47 18.4 | 37841 |
| | South Meridian of St. Paul's | 180 0 0 | -- -- |

 Weft-

| | Objects. | Bearings from the south merid. westward. | Distances in feet. |
|---|---|---|---|
| | | ° ′ ″ | |
| Westward of the meridian of St. Paul's. | Stretham Church | 13 57 7.8 | 31793.5 |
| | Clapham Common, Mr. CAVENDISH | 26 29 56.1 | 24563.5 |
| | Battersea Church | 52 22 27.6 | 22226 |
| | St. Peter's Ch. Westm. SW pinnacle of the S.T. | 52 32 13.2 | 8661.8 |
| | Fulham Church | 57 39 44.6 | 30746.3 |
| | Kew Pagoda | 71 2 36 | 47577.7 |
| | Richmond, Royal Observatory | 71 42 0.1 | 51941.1 |
| | St. Martin's Church, in the Strand | 74 28 59.2 | 6748.6 |
| | Spring Grove House, Sir Jos. BANKS, Bt. | 76 9 49.3 | 57253.9 |
| | St. James's Church | 77 49 9.8 | 8978 |
| | South Audley Chapel | 81 49 42.7 | 12211.1 |
| | St. Mary's, New Church in the Strand | 81 57 27.7 | 4291.6 |
| | St. Clement's Church | 85 57 36.7 | 3592.4 |
| | St. Ann's Church, Soho | 86 9 58.7 | 7753.9 |
| | St. George's Church, Hanover Square | 86 23 12.9 | 10304.5 |
| | St. Bride's Church, Fleet Street | 90 12 42.8 | 1771.7 |
| | Argyll Street Observatory, Maj. Gen. ROY | 92 14 37.7 | 9632 |
| | The Pantheon | 93 19 17.2 | 9066.8 |
| | Hanger-hill Tower | 94 24 59.6 | 45845.2 |
| | St. Giles's Church | 94 36 28.3 | 6917.3 |
| | Portland Chapel | 100 34 38.7 | 10301.4 |
| | Charlotte Street Chapel | 101 30 23.9 | 8500.4 |
| | St. George's Church, Bloomsbury Square | 103 15 20.6 | 6221.1 |
| | Wind Vane of the British Museum | 105 45 45.7 | 6701.5 |
| | The Tabernacle, Tottenham Court Road | 107 20 47 | 8876.2 |
| | Fitzroy Chapel | 109 41 10.2 | 9559.9 |
| | Harrow on the Hill Church | 112 7 58.3 | 58764.2 |
| | Jew's Harp station of 1783 | 112 58 30.7 | 13522 |
| | Primrose Hill station of 1788 | 123 28 40.8 | 17072.8 |
| | Hampstead Church | 128 56 11.8 | 24148.7 |
| | St. Pancras Church | 136 43 25.8 | 10600.7 |
| | The Small-Pox Hospital | 137 38 37.8 | 8732.3 |
| | Black Lane station of 1783 | 147 53 7.7 | 10790.3 |
| | Highgate Chapel | 150 59 18.1 | 24062.4 |
| | Mr. DUVELUZ's Cupola, Hornsey Lane, High. | 155 27 12.8 | 22646 |
| | Hornsey Hill station of 1788 | 173 24 32.3 | 23297.1 |
| | Islington Church | 174 40 21.4 | 9028.2 |
| | Highbury House, Mr. AUBERT | 178 43 11.6 | 14595.7 |
| | Ditto, the Transit-room of his Observatory | 179 1 56.6 | 14561.4 |
| | North meridian of St. Paul's | 180 0 0 | — — |

Com-

| Computed latitudes and longitudes of some of the places in the above table. | | | |
|---|---|---|---|
| Places. | Latitudes. | Longitude from Greenwich | |
| | | in degr. &c. | in time. |
| | ° ′ ″ | ° ′ ″ | h m sec. |
| St. Paul's . . | 51 30 49.43 | 0 5 46 8 | 0 0 23.12 |
| Highbury House, Transit-room . | 51 33 12.8 | 0 5 50.5 | 0 0 23.37 |
| St. James's Church . . | 51 30 30.7 | 0 8 5 | 0 0 32.33 |
| Argyll Street Observatory . | 51 30 53.05 | 0 8 18.36 | 0 0 33 224 |
| Clapham Common, Transit-room | 51 27 12.7 | 0 8 39.2 | 0 0 34.613 |
| Richmond Royal Observatory . | 51 28 7.9 | 0 18 42.3 | 0 1 14.82 |

## CONCLUSION.

IN the course of this Paper, an account has been given of the commencement, progress, and completion of an operation, the first of its kind in this country, undertaken by the command, and executed under the auspices, of a most gracious and beneficent Sovereign, the Patron of the Sciences.

From a liberal supply of much better instruments than ever were used for purposes of this sort on any former occasion, and every other assistance that could contribute towards success, the operation has undoubtedly derived some peculiar advantages: for, besides a more accurate mode of measuring the bases than has heretofore been practised, the angles of the triangles have been observed so truly, that the relative geodetical situations of the stations, as determined by plane trigonometrical computation, may be said to be free from sensible error.

The instrument too, by means of its transit telescope, being admirably calculated for determining, with great precision, the true direction of the meridians, their convergence to each other, and consequently the differences of longitude, have

thereby been obtained by angular meaſurement alone, without any regard to difference of time, more or leſs erroneous even with the very beſt time-keepers, and not perhaps to be depended upon to nearer than half a ſecond, after taking a mean of a number of compariſons. This mode, by angular meaſurement, was ſuggeſted in the Paper of 1787; and we preſume to think, that the reſult of the operation has fully verified the goodneſs of the method by the conſiſtency of the pole-ſtar obſervations among themſelves. It may be ſaid to be a new mode of ſurveying, by the help of the pole-ſtar as a fixed point, for preſerving the accuracy of the operation, ſucceſſively carried on from meridian to meridian; and the ſame mode ſhould be adhered to in future.

Another circumſtance muſt likewiſe be noticed, as having been propoſed at the ſame time, namely, the uſe of white lights for the diſtant ſtations: for without the help of theſe, obſerved with ſuch an inſtrument as ours, it would have been utterly impoſſible to have determined accurately the diſtances of Montlambert and Blancnez, the firſt nearly forty-ſeven, and the laſt nearly forty-eight miles from Fairlight Down.

Without farther recapitulation, the Writer of this Account cannot help conſidering it as being incumbent on him to recommend, that the trigonometrical operation, ſo ſucceſsfully begun, ſhould certainly be continued, and gradually extended over the whole iſland. Compared with the greatneſs of the object, the annual expence to the publick would be a mere trifle not worthy of being mentioned. In reality, a chief part of the expence, namely, that of fine inſtruments, has already been incurred; and it would be a pity indeed to ſuffer them to be laid up and remain uſeleſs. The honour of the nation is concerned in having at leaſt as good a map of this as there is of

any

any other country. But, by proceeding with the work in the ſame manner as it has been begun, with more perfect inſtruments than have heretofore been uſed, and ſome of theſe applied in a new way, a map of the Britiſh iſlands will at length be obtained, greatly ſuperior in point of accuracy to any that is now extant.

One additional inſtrument would certainly be wanted, that is, a zenith ſector for the determination of the latitudes, when the operation came to be extended to any conſiderable diſtance from the parallel of Greenwich. But this would not be neceſſary at firſt; while ſuch a one is preparing by Mr. RAMSDEN, and which he will no doubt render the compleateſt thing of the kind, the operation ſhould be continued in the parallel of Greenwich, or in the perpendicular of its meridian, quite to the weſtern ſide of the iſland in the manner following.

In more than one place of this Paper we have had occaſion to expreſs our regret, that the recent ſeries of triangles did not afford diſtances ſufficiently great between points reciprocally viſible, for the beſt application of the pole-ſtar obſervations, to the determination of the differences of longitude. It is believed, that the obſervations themſelves are extremely near the truth, but not wholly free from error; therefore, whatever this may amount to, on double or triple the diſtance it would certainly be reduced to one-half or one-third part.

Shooter's Hill, and Nettlebed Heights on the eaſtern ſkirt of Oxfordſhire, are reciprocally viſible at the diſtance of about forty-ſix or forty-ſeven miles from each other. Nettlebed Heights, and a thin clump of trees on the Glouceſterſhire range of hills, called *Paul's Epiſtle*, about two miles weſtward from *Frog Mill*, on the left hand ſide of the road leading from thence to Glouceſter, may likewiſe be ſeen from each other at the

the diſtance of fifty or fifty-two miles. This laſt commands a moſt extenſive proſpect over the plain of the Severn and the Welch mountains to a great diſtance beyond it. *Pen-y-Voel Hill*, called alſo the Sugar-loaf of Abergavenny, in Monmouthſhire, would become the third ſtation to the weſtward; and two, or at moſt three, ſtations more would reach St. David's Head, oppoſite to Wexford in Ireland.

But let us ſuppoſe, in the firſt place, the ſeries of triangles to be extended only to the third ſtation, in all which ſpace it would be wholly unneceſſary to obſerve any latitudes; by the pole-ſtar obſervations, repeated a ſufficient number of times on both ſides of the pole, at each of the ſtations, the length of the degree of a great circle, perpendicular to the meridian, and conſequently the differences of longitude, would thereby be obtained to the utmoſt preciſion. A determination of this ſort would abſolutely be concluſive, with regard to the length of the vertical and radius of the parallel in the latitudes of the reſpective ſtations, aſcertainable by their diſtances from the perpendicular to the meridian of Greenwich.

The ſecond part of the operation would be that of carrying a ſeries of triangles ſouthward from *Pen y-Voel Hill*, in the direction of its meridian to the Britiſh Channel; and afterwards extending theſe triangles in the uſual manner over the whole ſouth part of the iſland between Kent and the Land's-End.

If, beſides the zenith ſector, another circular inſtrument was provided, and ſome additional annual expence allowed, in order to accompliſh more ſpeedily ſo great and uſeful a work, at the ſame time that the operations to the ſouthward were carrying on, the ſeries of triangles, in the direction of the meridian of *Pen-y-Voel Hill*, ſhould be continued to the northward through-

out

out the extent of the iſland till it fell into the Murray Frith. A new meridian might then be taken more to the weſtward, perhaps that of Invernefs, or ſome hill near it, whereby the ſeries would be extended to the North Sea, bounding the coaſts of Sutherland and Caithneſs.

It is unneceſſary here to enter into any minute detail of what ſhould be the ſucceeding parts to be carried preferably into execution, as things of this ſort would naturally preſent themſelves, in the courſe of ſuch important operations, to thoſe entruſted with the direction. It is however ſufficiently obvious, that having, as above ſuppoſed, obtained the meaſure of a portion of the meridian amounting nearly to ſixteen degrees of latitude in continuity, between the Pyrenean mountains and the northern extremity of Britain, or more than one-ſixth part of the diſtance between the equator and the pole; the things of the next conſequence to be obtained would be, the meaſures of the *radii* of the vertical and parallel in the lowlands of Scotland, that is, in the latitude of Edinburgh, and again at the northern coaſt. In each of theſe ſituations it is evident, that about three degrees of longitude might be meaſured with great exactneſs. At the north, for inſtance, *Cape Wrath* being made the central ſtation, from thence the *Orkney Iſlands* to the eaſtward and *Butt of the Iſland of Lewes* to the weſtward, being diſtinctly ſeen, would conſequently become the ſtations to the right and left.

With regard to the uſe of white lights, ſo indiſpenſably neceſſary in all operations of this ſort, no opportunities have yet offered of aſcertaining with preciſion the immenſe diſtance to which they may be ſeen in favourable circumſtances of the weather, and with ſufficient elevation of the ſtations above the ſea. Thoſe commonly uſed in the recent operation were only

 three

three or four inches in diameter, and the largeſt but ſix or ſeven. Augmented to nine or ten inches, exhibited on the top of one high hill, and obſerved from the top of another, when there is no moonlight, and no rain or fog, they would probably be ſeen eighty or a hundred miles. In ſhort, wherever the moſt faint looming of the land in a very clear day can be diſcerned, the lights, from their extraordinary brilliancy, would undoubtedly be ſeen in a dark night, when the air was perfectly clear.

Hence, it will readily be conceived, how eaſily and accurately any trigonometrical operations that might be carrying on in England and Ireland at the ſame time might be connected with each other, by means of theſe lights, alternately exhibited and obſerved, for inſtance, on *Brach-y-pwl Point*, *Holyhead Hill*, and the *Iſle of Man*, on one ſide; and again on the *mountains of Wicklow*, *hill of Howth*, and *mountains of Mourne*, on the other.

In the Paper of 1787, and again in this, we have had occaſion to remark on the improbability of being able to determine the differences of longitude, by the inſtantaneous exploſion of light, ſo accurately as by angular meaſurement with a fine inſtrument, applied as it has been in the recent operation. But ſince, undoubtedly, there will be different opinions on this head, it will be very proper that both modes ſhould be tried, that the reſults may be compared.

To the eaſtward of Greenwich, the ſtation for exploſion might be taken at *Montlambert*, *Fienne Windmill*, or at *Folkſtone Turnpike*, in order to render the diſtance of the extreme ſtations as great as poſſible. Any of theſe points would be viſible from *Crowborough Beacon*, which would become the ſtation of the Engliſh aſtronomer with his clock and inſtruments. That of the French aſtronomer would of courſe be

taken in the moſt convenient place inland on the range of chalk hills, viſible from the place of exploſion, and the eaſieſt connected with the triangles of the meridian of Paris in the neighbourhood of *Helfaut* and *Bouvigny*. *Crowborough* is about 70 miles diſtant from *Montlambert*, and a point in the direction of theſe two near *Helfaut* would be about 32 miles inland from *Montlambert*, which would give for the extreme diſtance about 102 miles.

If experiments of the ſame kind were to be made to the weſtward of Greenwich, thoſe very ſtations, already propoſed for the continuation of the triangular operation, would be the fitteſt that could be choſen for the purpoſe.

Now, ſuppoſing the operations already mentioned in the parallel of Greenwich to be executed, the meridian of Pen-y-Voel Hill extended to the northern extremity of Scotland, and three degreees of longitude meaſured in that latitude, while the Eaſt India Company were carrying on operations of the ſame nature on the coaſt of Coromandel and in Bengal, every thing would then be done that Britain could do within her own dominions, in regard to the determination of the figure and dimenſions of the earth. If, after this, any doubts remained, theſe might eaſily be removed, by Portugal's meaſuring a degree or two of the meridian under the equator, and alſo a portion of the earth's equatorial circumference; while ſome other nation repeated the operations at the polar circle, or made new ones ſtill nearer to the pole, if ſuch ſhould be found practicable, all which has been ſuggeſted in the Paper of 1787. For the farther illuſtration of this ſubject, it will be proper to refer to fig. 3. in Plate X.

With regard to the execution of any future operation that may, and which it is hoped will, be hereafter undertaken in

Britain, there remains but one point more to be mentioned, that is, the meaſurement of new baſes. In the execution of the great map of France, no fewer than ſeveenteen were meaſured. But, with ſuch inſtruments as have been uſed in this country, a ſmaller number would ſuffice; and the beſt ſituations for the purpoſe will naturally preſent themſelves in the courſe of the operations.

Thoſe that immediately occur to the Writer of this Memoir, as likely to be found the moſt proper, are the following, *viz.*

1. On Sedgemoor in Somerſetſhire.
2. On Boſton Fens in Lincolnſhire.

<table>
<tr><td>3. On the ſands on the coaſt of North Wales between Pen man Mawr and Beaumaris, .<br>4. On the ſands between Holy Iſland and Berwick upon Tweed, . . .</td><td>at low water ſpring tides.</td></tr>
</table>

5. On Kincairden and Flanders Moſs, weſtward from Stirling.

<table>
<tr><td>6. On the ſands on the coaſt of Aberdeenſhire between the mouth of the Don and Newburgh,<br>7. The ſands on the coaſt of Murray, between the mouths of the rivers Findhorn and Nairn,</td><td>at low water ſpring tides.</td></tr>
</table>

8. On the Moan moraſs, inland from the Whiten Head, on the coaſt of Sutherland.

In the meaſurement of theſe baſes, which ſhould not be leſs than ſix, but as often as poſſible even eight or ten miles in length, there would not be any neceſſity for that wonderful exactneſs that was requiſite for determining the length of the firſt and ſecond baſes on Hounſlow Heath and Romney Marſh. Suppoſing them to be executed with the ſteel chain alone within a few feet of the truth, it would be perfectly ſufficient to ſhew

that no error of any confequence had accumulated in carrying on the operation to thefe diftant points refpectively, even as far as the remote fhore of the Northern Ocean.

Finally, in order to preferve the primitive fcale of diftances, whereon the accuracy of the recent operation, and all future ones that may hereafter be connected with it, muft always be fuppofed to depend, it is indifpenfably neceffary to eftablifh, without lofs of time, fome permanent marks at the extremities of the bafe on Hounflow Heath *. Thefe fhould be low circular buildings, rifing but a few feet above the furface of the Heath, compofed of the hardeft materials, fuch as granite, and conftructed in the moft durable manner by dove-tailing the ftones into each other. They would refemble thofe bafements of ancient croffes we often meet with, formed into regular fteps, whereby the afcent is rendered eafy to the top of a circular table or platform, of fufficient dimenfions for the reception of the great inftrument on any future occafion.

In the interior part of thefe little buildings, metal tablets would be inferted, containing the name of that much-beloved Monarch in whofe reign the operation was begun, and thefe buildings executed; the diftance from one to the other; the angle of the bafe with the meridian; and alfo the magnetical variation.

It is not to be doubted, that the refpective lords of the manors will readily veft in the Royal Society, the property of the two fmall fpots of the Heath fufficient for the erection of thefe

* Soon after the meafurement of the bafe, Mr. MYLNE, F. R. S. at my defire, was fo obliging as to give a defign for a building of this kind, which, being conftructed nearly in the way of the Eddyftone Light-houfe, executed by the ingenious Mr. SMEATON, would anfwer very well.

*termini.*

*termini.* They ſhould be carried into immediate execution; for, if this buſineſs ſhould be poſtponed for any length of time, there will be danger of its being altogether forgotten. In a few years the wooden pipes ſunk in the earth will decay; and thus the primitive ſcale of diſtances, which coſt ſo much labour and expence to obtain, will be irretrievably loſt.

# APPENDIX.

OUR late much respected Colleague, Major-General ROY, having finished, in September 1788, the trigonometrical measurement described in the first Part of this Volume, returned to London in a very indifferent state of health. From this time he employed all the leisure that his illness, and his various official avocations, allowed, in preparing the account of his operations, to be laid before the Royal Society. But toward the autumn of 1789 his infirmities encreased so much, that the medical Gentlemen he consulted advised him to spend the following winter at Lisbon, for which place he accordingly embarked in the beginning of November. Previous to this, however, he finished the first Copy of his Paper; but it was much hurried toward the latter part, and not rendered so perfect as the General would undoubtedly have made it with more time and better health. He returned to England in April 1790, and the Paper was sent to the press before the end of the same month. Unfortunately the General did not live to see the printing quite completed; he corrected, indeed, all the sheets except the three last; but without comparing his manuscript copy with the original papers and observations. Several errors which had been discovered in the course of the printing, together

ther with the obſcurity of the account in certain parts, induced ſome of the General's Friends, Members of the Royal Society, to requeſt, after his deceaſe, that the whole might be reviſed by a competent perſon, who ſhould compare it with the original documents, correct ſuch miſtakes as might be diſcovered, and illuſtrate whatever required further explanation. No one could be found ſo proper for this taſk as Mr. DALBY, the Gentleman of whom the General makes ſuch honourable mention in his Paper, and who, having aſſiſted in all the operations, was as well acquainted with every part of them as the General himſelf. The reſult of Mr. DALBY's examination is the following Remarks; which being much too long for inſertion in the liſt of *errata* (where only the errors of the preſs are noticed) is here added ſeparately, by way of Appendix.

C. BLAGDEN.

*Remarks on Major-General* Roy's *Account of the Trigonometrical Operation, from* Page 111. *to* Page 270. *of this Volume. By Mr.* Isaac Dalby.

PAGE 134. l. 20, &c. The inclinations of the bases with the meridians were determined by spherical computation, and therefore can only be considered as *nearly* true.

P. 171. l. 6. from bottom, *for* we have area in feet *put* we have log. area in feet.

P. 173. in the VIII triangle, *for* 0.01 *put* 0.1.

P. 174. in the IX triangle, *for* 0.88 *put* 0.83.

P. 175. in the XVII triangle, *for* 71855 *put* 71885.

P. 177. This method of making the comparison on a long distance, when the measured bases are short, and nearly of the same length, seems preferable to that of carrying the computation directly from one base to the other. Determinations of this kind, however, must always be uncertain to particular limits on account of the inaccuracy of instruments and observations combined with the unknown figure and dimensions of the earth. Was the earth a sphere of a known magnitude, the most natural method of computation would have been by spherical trigonometry, after the observed angles had been corrected for that purpose; which method (supposing the angles wanted no correction, or each had been accurately observed) would shew which base was measured nearest the truth. Or the same thing might also be obtained by plane trigonome-

try, uſing the chords of the meaſured baſes, and the angles formed by the chords of the other ſides of the triangles (which angles might then be found from the horizontal ones) inſtead of the obſerved angles.

In the application of plane trigonometry to the obſervations, that part of the earth's ſurface to which the operation has been confined, is conſidered as a plane, and the meaſured baſes as right lines on that plane; but whether the computations are made on this principle, or attempted on that of taking the baſes and the other ſides of the triangles as chords, there ſeems to be no certain rule for reducing the obſerved angles of each triangle to 180°, ſo as to give the diſtances the moſt correct, which would alſo be the caſe if the angles of each triangle had been taken in the ſame plane; hence it is evident, that the method of correction has been in ſome degree arbitrary; for, though the ſum of the three obſerved angles of each triangle is in general very near what it ought to be (taking the earth as a ſphere); yet, when that ſum is not exactly 180°, in reducing them to plane ones, each obſerved angle may be taken as a plane one, and the other two augmented in caſe of a defect; but each ought to be diminiſhed when there is an exceſs in the obſerved ſum. In making theſe reductions, however, it is evidently neceſſary to conſider whether each of the obſerved angles is equally well aſcertained, and correct them accordingly; but this muſt be left to the judgement of the obſerver.

From the foregoing conſiderations, it follows, that the angles of the triangles taken as plane ones may be varied to certain limits, and conſequently the oppoſite ſides deduced therefrom muſt vary to certain limits alſo; but it is evident, that a mean of the extreme reſults, obtained in this manner, will be very near the truth; and therefore this method of making the compariſon

comparifon feems lefs liable to objection than that by a fingle correction of the fame angles. Accordingly, if we vary the angles (in reducing them to 180°) from Hounflow Heath to the XIII triangle, fo as to produce the greateft and leaft effects on the lengths of the oppofite fides, there will refult 141750½ and 141746½ feet, nearly, for the greateft and leaft, and 141748½ for the mean diftance of Fairlight and Hollingborn. In like manner, the bafe on Romney Marfh will give 141745.6 and 141744.4 feet, nearly, and the mean 141745 feet, for the diftance of the fame ftations; the difference in the mean refults is 3½ feet on a diftance of near 27 miles; and therefore the bafe on Hounflow Heath meafures the other, by thefe determinations, to about 9 inches; and, becaufe the latter bafe is the longeft, it would meafure the former on Hounflow Heath to fomething lefs.

The diftance of the ftations of Fairlight and Hollingborn in the XIII triangle is 141747.1 feet, and from this all the diftances to the eaftward are computed; but if the bafes are meafured equally exact, the diftance of the above ftations, or 141745 feet, determined from the bafe on Romney Marfh, muft be more correct than the other, becaufe the connection of Fairlight and Hollingborn with this bafe is formed by three or four triangles only, whereas on the other fide, the computation runs through eight or nine. The difference, however, is but 2 feet, and that in an extent of almoft 27 miles, which will make about 7½ feet lefs for the diftance between the meridians of Greenwich and Paris.

Among the angles corrected for computation, it will be perceived, that fometimes the quantity of an angle feems not to be exactly what the obferved angle ought to give. In thefe cafes the obferved angle is lefs to be depended on than the

 others

others of the ſame triangle: for inſtance, in the firſt triangle the obſerved angle at Hanger Hill is 42° 2′ 32″, and that for computation 42° 2′ 34″; now this angle was not taken ſo accurately as could be wiſhed, but the others were repeatedly obſerved.

P. 178. in the XXIX triangle, *for* 186113 *put* 147386.9.

P. 180. in the XXXIV. triangle, *for* 252469.9 *put* 252496.9.

As the obſervations at Fairlight and Dover are omitted, upon which the angles of the XXXIII and XXXIV triangles depend, it will be neceſſary to give them, and alſo ſhew the manner of obtaining theſe angles.

| | ° | ′ | ″ |
|---|---|---|---|
| At Dover Caſtle, the angle between the white light at Montlambert and the lamp at Padleſworth was obſerved . . . . | 109 | 8 | 25.5 |
| Corrected for computation . . . | 109 | 8 | 25 |
| At Fairlight, the angle between the white lights at Montlambert and Blancnez was obſerved . . . . . . . | 17 | 46 | 5 |
| For computation . . . | 17 | 46 | 3.5 |
| Between the lamp at Lydd and white light at Blancnez . . . . . | 18 | 2 | 31 |
| For computation . . . | 18 | 2 | 31.5 |

The acute angles in the XXXII triangle reſult from the other angle and including ſides 147386.9 and 42561.18.

| | | ° | ′ | ″ |
|---|---|---|---|---|
| Angle at Fairlight | in the XXIX triangle | 13 | 38 | 2.95 |
| | in the XXXII triangle | 6 | 6 | 39.43 |
| Angle at Fairlight between Dover and Lydd | | 7 | 31 | 23.52 |
| Angle at Fairlight between Lydd and Montlambert (17° 46′ 3″½ + 18° 2′ 31″½) . | | 35 | 48 | 35 |
| Angle at Fairlight in the XXXIII triangle | | 43 | 19 | 58.52 |

| | ° | ′ | ″ |
|---|---|---|---|
| Angle at Dover between Padlesworth and Montlambert . . . | 109 | 8 | 25 |
| —— at Dover in the XXXII triangle | 21 | 37 | 55.42 |
| Angle at Dover in the XXXIII triangle . | 87 | 30 | 29.58 |

The third angle, or 49° 9′ 31″.9 at Montlambert, is the supplemental one.

If from the angle at Fairlight in this triangle we take 17° 46′ 3″½ we have 25° 33′ 55″.02 for the angle at Fairlight in the XXXIV triangle; and if to 87° 30′ 29″.58 we add 23° 25′ 0″.25 (the angle at Dover in the XXXV triangle) it gives 110° 55′ 29″.83 for the angle at Dover; that at Blancnez is the supplemental one.

The situation of the station at Montlambert, as determined by the observations made on this side of the Channel, has not however totally depended on those made at Fairlight and Dover; another observation at Padlesworth has been used by way of check, or verification. This was made in a very favourable state of the air, when the angle between the flag-staff at Dover and mast at Montlambert was found to be 58° 27′ 11″½, which is nearly what results from computation; for 42561.18 and 168821.07 feet, the respective distances of Dover from Padlesworth and Montlambert, with the included angle 109° 8′ 25″, give this angle 58° 27′ 10″.9.

It ought to be remarked, that the angle at Blancnez 119° 41′ 28″.9, communicated by M. Cassini, is an horizontal one; that of the XXXV triangle, or 119° 41′ 41″.6, is the result of a computation by plane trigonometry, which, if accurate, should be less than the horizontal one at the same point, and therefore the *maximum* of the difference must be somewhat greater than 12″.7.

P.

P. 182. &c. The triangles after the XXXVI, and what follows to the end of the article, do not feem neceffary, on our part, to complete the triangular connection between Greenwich and Paris, becaufe it may be done in the following manner from the triangle formed by Dover, Calais, and Dunkirk. In this triangle, the fide between Dover and Calais is 137449.9 feet (fee the XXXVI triangle); and by M. CASSINI's Paper, communicated in 1789, it appears, that the fide between Calais and Dunkirk is 19349.34 toifes (= 123729.3 feet), and the included angle at Calais 139° 17′ 35″.6; thefe give 19° 14′ 13″.1 and 21° 28′ 11″.3 for the other two angles, and 244919 feet for the diftance of Dover and Dunkirk; alfo 28232.7 for what Dunkirk is fouthward, and 243287 feet for what it is eaftward, from Dover: this laft added to 307366.8, the diftance of Dover from the meridian of Greenwich, gives 547053.8 feet for what Dunkirk is eaft from the meridian of Greenwich on a parallel to the perpendicular; but the length of the arc of the great circle which paffes through Dunkirk, and is perpendicular to the meridian of Greenwich, is very near the fame as the length of this parallel, or 547053.8 feet (though accurately fomewhat lefs); hence, if we take 61247½ fath. = 1°, we have 1° 29′ 19″.1 for that arc, and the latitude of Dunkirk being 51° 2′ 9″.3 (See Sect. VI. Art. 12.); therefore, cofine 51° 2′ 9″.3 : rad. :: fine 1° 29′ 19″.1 : fine 2° 22′ 3″.8 the longitude of Dunkirk (agreeing with that in Art. 12.); and rad. : tang. 51° 2′ 9″.3 :: tang. 1° 29′ 19″.1 : cofine 88° 9′ 34″ for the other angle of this triangle of which the co-latitude of Dunkirk is the hypothenufe.

Dunkirk is eaft from the meridian of Paris 1416, or 1430 toifes (fee Art. 12. Sect. VI.), a mean of thefe give 1′ 29″.14 as an arc of a great circle; this, with the co-latitude of Dun-

kirk for the hypothenuſe, will give 89° 58′ 10″ for the angle between the meridian and this arc; hence 89° 58′ 10″ − 88° 9′ 34″ = 1° 48′ 36″ is the angle at Dunkirk between the two arcs; one perpendicular to the meridian of Greenwich, and the other to that of Paris; therefore, if we take 1416, or 1430 toiſes, as the leg of a plane triangle adjacent to this angle, we get 9059, or 9148½ feet, for the diſtance of the meridian of Paris from Dunkirk on a great circle perpendicular to the meridian of Greenwich; theſe taken from 547053.8 will leave 537994.8 or 537905.3 feet, for the diſtance between the meridians of Greenwich and Paris on that circle, according as Dunkirk is 1416 or 1430 toiſes from the meridian of Paris.

Should it be thought more accurate to make uſe of the diſtance between Calais and Dunkirk, according to the ſcale in the XXXV triangle deduced from the Engliſh obſervations acroſs the Channel, it is had at one proportion thus; as 12077.85 toiſes (the French diſtance between Blancnez and Montlambert) is to 77235 feet (the Engliſh diſtance) ſo is 19349.34 toiſes (the French diſtance of Calais and Dunkirk) to 123734.5 feet, as in the XL triangle, which exceeds the French diſtance about 5 feet: this will give the diſtance between the meridians of Grenwich and Paris 4.4 feet more than the above determination. But was the baſe on Romney Marſh adhered to, it would give the diſtance about 3 feet leſs; and therefore the reſults of the French triangles on their coaſt would agree nearer with the deductions from this baſe than from the other on Hounſlow Heath.

P. 187. in lines 1. and 8. *for* 358.6 *put* 349.4; and conſequently in line 10. *for* 133409.8 *put* 133419. This error is the cauſe of the difference in the diſtances of the parallels of latitude of Greenwich and Paris as given in Art. 10. and 13. Sect. VI.: for,

for, in pages 185. 187. we have 133746.3 and 133768.4 fath. the distances of Dunkirk north from Paris, the mean is 133757.4; if from this we take 358.6, we have 133398.8, as in Art. 10.; but subtracting 349.4, gives 133408 for what M is north of Paris, which added to 27248.2 there results 160656.2 agreeing with that in Art. 13.

P. 190, 191. It will immediately be perceived, that the columns in the table of general results here alluded to, have been filled by a method similar to that of working a *traverse*. The following table, however, was previously drawn up to facilitate the computations, and by which the numbers in the two first columns of distances may at any time be easily examined.

This table will readily be understood; for, if we suppose parallels to the meridian of Greenwich to be drawn through the stations on the left, opposite on the right are the angles which the adjacent stations make with these parallels.

| | | ° | ′ | ″ | |
|---|---|---|---|---|---|
| Greenwich Obs. | Norwood . | 38 | 7 | 16 | SW |
| Norwood . | Greenwich Obs. | 38 | 7 | 16 | NE |
| | Hundred Acres | 42 | 22 | 39.2 | SW |
| | Hanger Hill . | 49 | 31 | 22.7 | NW |
| Hundred Acres . | Norwood . | 42 | 22 | 39.2 | NE |
| | Hanger Hill . | 19 | 50 | 1.7 | NW |
| | St. Ann's Hill . | 73 | 48 | 38.3 | NW |
| Hanger Hill . | Norwood . | 49 | 31 | 22.7 | SE |
| | Hundred Acres | 19 | 50 | 1.7 | SE |
| | Hampton Poor-house | 23 | 30 | 53.4 | SE |
| | St. Ann's Hill . | 48 | 34 | 42.2 | SW |
| | King's Arbour . | 65 | 33 | 27.4 | SW |

| | | ° | ′ | ″ | |
|---|---|---|---|---|---|
| Hampton Poor-houſe | Hanger Hill . | 23 | 30 | 53.4 | NE |
| | King's Arbour . | 44 | 24 | 45.6 | NW |
| | St. Ann's Hill . | 74 | 8 | 40.9 | SW |
| King's Arbour . | Hanger Hill . | 65 | 33 | 27.4 | NE |
| | Hampton Poor-houſe | 44 | 24 | 45.6 | SE |
| | St. Ann's Hill . | 29 | 49 | 49.4 | NW |
| | Windſor . | 87 | 29 | 43.1 | NW |
| St. Ann's Hill . | King's Arbour . | 29 | 49 | 49.4 | NE |
| | Hanger Hill . | 48 | 34 | 42.2 | NE |
| | Hampton Poor-houſe | 74 | 8 | 40.9 | NE |
| | Hundred Acres . | 73 | 48 | 38.3 | SE |
| | Windſor . | 29 | 19 | 24.6 | NW |
| Greenwich Obſ. . | Severndroog Caſtle | 73 | 49 | 34 | SE |
| Severndroog Caſtle | Greenwich Obſ. | 73 | 49 | 34 | NW |
| | Botley Hill . | 11 | 23 | 18.5 | SW |
| | Wrotham Hill . | 46 | 18 | 30 | SE |
| Botley Hill . | Severndroog Caſtle | 11 | 23 | 18.5 | NE |
| | Wrotham Hill . | 79 | 16 | 28.7 | NE |
| | Goudhurſt . | 60 | 38 | 49.3 | SE |
| | Frant . | 43 | 28 | 20.3 | SE |
| Frant . | Botley Hill . | 43 | 28 | 20.3 | NW |
| | Wrotham Hill . | 6 | 50 | 57.7 | NE |
| | Hollingborn Hill | 55 | 19 | 35.3 | NE |
| | Goudhurſt . | 82 | 24 | 11.4 | NE |
| | Fairlight Down | 45 | 17 | 22.7 | SE |
| Wrotham Hill . | Severndroog Caſtle | 46 | 18 | 30 | NW |
| | Botley Hill . | 79 | 16 | 28.7 | SW |
| | Frant . | 6 | 50 | 57.7 | SW |
| | Goudhurſt . | 25 | 12 | 14.7 | SE |
| | Hollingborn Hill | 77 | 21 | 25.7 | SE |

| | | ° | ′ | ″ | |
|---|---|---|---|---|---|
| Goudhurſt . | Hollingborn Hill | 38 | 51 | 47.3 | NE |
| | Tenterden . | 72 | 54 | 53.3 | SE |
| | Fairlight Down | 23 | 15 | 17.5 | SE |
| | Frant . | 82 | 24 | 11.4 | SW |
| | Botley Hill . | 60 | 38 | 49.3 | NW |
| | Wrotham Hill . | 25 | 12 | 14.7 | NW |
| Fairlight Down . | Frant . | 45 | 17 | 22.7 | NW |
| | Goudhurſt . | 23 | 15 | 17.5 | NW |
| | Hollingborn Hill | 3 | 8 | 32.3 | NE |
| | Tenterden . | 12 | 5 | 40.9 | NE |
| | Allington Knoll | 45 | 46 | 21.3 | NE |
| | Lydd . | 67 | 4 | 58.3 | NE |
| | Blancnez . | 85 | 7 | 29.7 | NE |
| | Montlambert . | 77 | 6 | 26.7 | SE |
| Hollingborn Hill | Wrotham Hill | 77 | 21 | 25.7 | NW |
| | Frant . | 55 | 19 | 35.3 | SW |
| | Goudhurſt . | 38 | 51 | 47.3 | SW |
| | Fairlight Down . | 3 | 8 | 32.3 | SW |
| | Tenterden . | 5 | 46 | 56.8 | SE |
| | Allington Knoll | 45 | 47 | 55.7 | SE |
| Tenterden Church | Hollingborn Hill | 5 | 46 | 56.8 | NW |
| | Goudhurſt . | 72 | 54 | 53.3 | NW |
| | Fairlight Down | 12 | 5 | 40.9 | SW |
| | Lydd . | 50 | 27 | 11.6 | SE |
| | Allington Knoll | 85 | 47 | 25.3 | NE |
| Lydd Church . | Fairlight Down | 67 | 4 | 58.3 | SW |
| | Tenterden . | 50 | 27 | 11.6 | NW |
| | Ruckinge . | 6 | 16 | 20.4 | NW |
| | Allington Knoll | 12 | 46 | 58.3 | NE |
| | High Nook . | 37 | 4 | 28.1 | NE |
| | Padleſworth . | 41 | 10 | 52.6 | NE |
| | Folkſtone Turnpike | 50 | 49 | 22 | NE |

Ruckinge

| | | ° | ′ | ″ | |
|---|---|---|---|---|---|
| Ruckinge . | Lydd . | 6 | 16 | 20.4 | SE |
| | High Nook . | 55 | 15 | 9.8 | SE |
| | Allington Knoll | 70 | 25 | 31.6 | NE |
| Allington Knoll . | Hollingborn Hill | 45 | 47 | 55.7 | NW |
| | Tenterden . | 85 | 47 | 25.3 | SW |
| | Fairlight Down | 45 | 46 | 21.3 | SW |
| | Lydd . | 12 | 46 | 58.3 | SW |
| | High Nook . | 21 | 1 | 47.8 | SE |
| | Folkſtone Turnpike | 82 | 56 | 18.9 | NE |
| High Nook . | Allington Knoll | 21 | 1 | 47.8 | NW |
| | Ruckinge . | 55 | 15 | 9.8 | NW |
| | Lydd . | 37 | 4 | 28.1 | SW |
| | Folkſtone Turnpike | 58 | 39 | 12.6 | NE |
| | Padleſworth . | 43 | 50 | 47.1 | NE |
| Padleſworth . | High Nook . | 43 | 50 | 47.1 | SW |
| | Lydd . | 41 | 10 | 52.6 | SW |
| | Folkſtone Turnpike | 64 | 18 | 47.4 | SE |
| | Dover Caſtle . | 81 | 11 | 30.1 | NE |
| | Swingfield . | 44 | 47 | 7.1 | NE |
| Folkſtone Turnpike | Lydd . | 50 | 49 | 22 | SW |
| | High Nook . | 58 | 39 | 12.6 | SW |
| | Allington Knoll | 82 | 56 | 18.9 | SW |
| | Padleſworth . | 64 | 18 | 47.4 | NE |
| | Swingfield . | 3 | 51 | 7.8 | NW |
| | Dover Caſtle . | 65 | 52 | 45.6 | NE |
| Swingfield . | Padleſworth . | 44 | 47 | 7.2 | SW |
| | Folkſtone Turnpike | 3 | 51 | 7.8 | SE |
| | Dover Caſtle . | 79 | 27 | 47.8 | SE |

| | | ° | ′ | ″ | |
|---|---|---|---|---|---|
| Dover Castle | Swingfield | 79 | 27 | 47.8 | NW |
| | Padlesworth | 81 | 11 | 30.1 | SW |
| | Folkstone Turnpike | 65 | 52 | 45.6 | SW |
| | Montlambert | 27 | 56 | 54.8 | SE |
| | Blancnez . | 51 | 21 | 55.1 | SE |
| | Calais . | 64 | 8 | 37.1 | SE |
| | Dunkirk . | 83 | 22 | 52.9 | SE |
| | ——— by M. CASSINI's distance of Dunkirk and Calais | 83 | 22 | 50.2 | |
| | Point M . | 82 | 33 | 14.4 | SE |

The difference between the complement of 82° 33′ 14″.4 and the angle at M in the XL triangle is 14° 51′ 3″.9, the angle RMC referred to in Art. 10. and 11. Sect. VI.

P. 194. It is said, that the angle AB*v* (Pl. X. fig. 2.) is equal to the angle BA*r*, and consequently at p. 199. that the sum of the observed angles PAB, PBA, are equal to the sum that would be found on a sphere. This (though extremely near in any of the spheroids hitherto assumed for the figure of the earth) is not geometrically true when the points of observation are *on* the surface of the spheroid, and each angle taken exactly in the plane of the horizon. For, it is evident, that to have the sum *accurately* the same, the points A, B (the places of observation) must be at equal distances from G and W; and therefore, if at any two points thus taken in the verticals GA, WB, the angles are supposed to be in planes parallel to the respective horizons at A and B, their sum will always be the same. Hence, because the vertical WB is greater than GA, if the angles are accurately horizontal ones at A and B, their sum must be *greater* on the north side, and *less* on the south, than

than on a ſphere, except the latitudes of A and B are the ſame. The difference, however, is ſo minute, that for practical purpoſes they may be conſidered as equal (as in this Section and the corollary, p. 215.), without ſenſible error. In the example at p. 196. the difference in the ſum of the horizontal angles at A and B on this ſpheroid, and on a ſphere, is a ſmall fraction of a ſecond; but it requires a nice computation to diſcover the exact quantity. The method, however, is to compute the angle at B in the ſame manner as that is done at A; or by taking the point of obſervation in the vertical GA produced, 168 fathoms (the difference of the verticals WB, GA) above the ſurface at A, and determining the diminution in the horizontal angle by a re-computation.

By purſuing a method of computation ſimilar to that for the point A at p. 196. it is evident, that the three horizontal angles of any triangle on a known ſpheroid may be determined.

P. 195. bottom line, *for* AGH *put* AGK.

P. 203. There ſeems a miſtake towards the latter part of this page; becauſe it will be ſeen, that no ſuch ſpherical triangles have been uſed in the computations but in Art. III. p. 206.

P. 205. l. 13. from bottom. This muſt allude to one place of obſervation only; becauſe in this operation (where the latitudes have not been obſerved) a principal advantage lies in having one of the ſtations (like Botley Hill) on, or near the meridian of Greenwich, on account of obtaining its latitude pretty exact; but the farther off the other place of obſervation is, the better it is for the purpoſe.

P. 207. In l. 14. from bottom, *for* :: P∗ *put* :: ſine P∗.

P. 208. from l. 5. to the period in l. 12. from bottom, ſhould run thus:

If

If the latitude of the point B was given, and the earth a ſphere, the co-latitude BP and the obſerved angles PBG = 119° 21′ 13″.2, and PGB = 60° 17′ 15″.7, would give PG the co-latitude of G, and the angle BPG the difference of longitude of B and G.

Taking a ſphere whoſe diameter is nearly a mean between thoſe in M. BOUGUER's ſpheroid, the length of a degree of a great circle is 60859.1 fathoms, and the latitude of B will be 51° 16′ 41″.45; therefore BP = 38° 43′ 18″.54; this, with the obſerved angles at B and G, give PG = 38° 53′ 6″.72, and the angle BPG, or difference of longitude = 27′ 36″.7; therefore in the right-angled ſpherical triangle PRG, rad. : tang. GP :: coſine angle RPG : tang. 38° 53′ 3″.47 = RP; and rad. : ſine GP :: ſine RPG : ſine 17′ 20″ = RG.

P. 209. l. 9. *for* 51° 16′ 46″ *put* 51° 16′ 46″.1.

P. 213. Correct the title of this Article, by reading *geodetical meaſurement* for "*pole-ſtar obſervations*," in the firſt line.

P. 217. After the word "meridian", in the third line of Art. X. inſtead of "and alſo the differences of latitude and longitude have been obtained by very accurate obſervations of the pole-ſtar made at certain ſtations to the eaſtward of Greenwich,", *read* and alſo the difference of longitude between Botley Hill and Goudhurſt have been obtained by obſervations of the pole-ſtar.—A correction of this kind ſeems neceſſary, becauſe the pole-ſtar obſervations have not been uſed in finding the differences of latitude. From the directions of the meridians at the above ſtations, the value (in parts of a degree) of the meaſured arc of a great circle, perpendicular to the meridian, has been determined; hence the lengths of the degrees in the Table, p. 227. have been inferred. The diſtances from

the meridian of Greenwich (in the Table of General Refults) have been converted into degrees, &c. according to this Table; and the others from the perpendicular in the next column, according to M. BOUGUER's fcale on the meridian (which is had fufficiently accurate from the Table, p. 298. *Fig. de la T.* or that at p. 228. Phil. Tranf. 1787, by an eafy approximation) thefe meridional arcs applied to the co-latitude of Greenwich, with the other arcs perpendicular to the meridian, form the legs of the triangles by which the latitudes and longitudes of the ftations have been computed. The meridional arcs, however, have been corrected, as in the example in this Article (where the value of R*r* has been added) when the diftances of the ftations from the meridian of Greenwich are confiderable.

In determining the latitude of M in this Article, a fpheroidical correction has been applied to the refult by fpherical trigonometry, as in Art. VII. but that computation is made on a figure of known dimenfions, and confequently the latitude of *r* (fig. 7.) is given; but it does not follow, that the true latitude of *r* (fig. 10.) would exactly correfpond with M. BOUGUER's hypothefis, though the length of the whole meridional arc between Greenwich and Paris is found to agree extremely near; and therefore no correction of this kind is applied to the other latitudes in the Table of General Refults.

The greateft accuracy, however, is abfolutely neceffary in determining the directions of the meridians if we would derive fatisfactory conclufions therefrom, when the places of obfervation are obliquely fituated with refpect to the meridian, and at a diftance from each other not greater than that between Botley Hill and Goudhurft, becaufe an error of 1″ in the horizontal angle at either of thefe places will produce an error in their

difference

difference of longitude of 1″.2 of a degree, and consequently a variation of about 6″ in the longitude of Dunkirk or Paris.

Was the distance of the stations about 36 miles, the error in longitude would be the same as that in the horizontal angle, or 1″.

The length of the arc RG (Pl. X. fig. 5.) is 17695 fathoms, and its value 17′ 20″.06 as an arc of a great circle perpendicular to the meridian. Now, was the earth a sphere, the length of any arc, would be to the number of degrees it contained, as 17695 to 17′ 20″.06; but this is not accurately the case on a spheroid; though, on this account only, the error in longitude (which is in defect) deduced from an arc of a great circle obtained in the above manner, must be small to the extent of 3 or 4 degrees (in the latitudes of the places of observation) on a spheroid not more oblate than the earth.

It may be observed, that in determining the differences of longitude by the pole-star observations, the stations should be as nearly east and west from each other as the nature of the country will permit, because in that direction, any errors which may be thought to arise from the uncertain inclination of the verticals on the sphéroid, will vanish; and, what is of more consequence, a longer arc of a great circle perpendicular to the meridian will thereby be determined than could be in any other direction with the same distance. On this account the stations at Botley Hill and Hollingborn Hill (for one is seen from the other) are eligible. Their distance is about 28½ miles, which would measure near 24′ of a degree of a great circle perpendicular to the meridian.

P. 220. Art. XI. seems to want correction: for, if M*g* is a lesser circle parallel to the meridian GR, it will cut the great circles *r*M, G*g*, at right angles. Hence, RMC – RM*r*

(14.

(14° 51′ 3″.9 − 19″.42) = 14° 50′ 44″.5 = $r$MC, which added to 90° ($r$M$g$) gives 104° 50′ 44″.5 for the angle $g$MC; from this take 1° 48′ 38″.6 (PM$g$), and we have 103° 2′ 5″.9 for the angle PMC deduced according to this method. But it cannot be ſaid to reſult from the Britiſh obſervations, becauſe the French angles were made uſe of to the eaſtward of Dover for obtaining the angle RMC, on which it depends.

P. 227. The lengths of the degrees of longitude in the Table were found thus: as radius : coſine of the latitude :: length of a degree of a great circle perpendicular to the meridian : length of a degree of longitude. This proportion is true on a ſphere, but not accurately ſo on a ſpheroid.

P. 229. *for* 43° 39′ *put* 48° 39′, the latitude of St. Malo.

As the new longitudes in this Table have not all been obtained in the ſame manner, it may not be improper to give the methods of computation.

The latitude of Straſbourg (*Deſcrip. Geom.*) is 48° 34′ 50″, and its diſtance from the meridian of Paris 204779 toiſes (=218243.17 fathoms) which, if we take 61225 fathoms = 1° (ſee the Table, p. 227.) gives 3° 33′ 52″.6; hence, as coſine 48° 34′ 50″ : rad. :: ſine 3° 33′ 52″.6 : ſine 5° 23′ 33″ the longitude.

In the *Connoiſſance des Temps* 1788, the latitude of Straſbourg is 48° 34′ 35″, longitude 5° 26′ 18″; therefore, as rad. : coſine 48° 34′ 35″ :: ſine 5° 26′ 18″ : ſine 3° 35′ 42″.3, the arc of the great circle (from which its longitude was computed) paſſing through Straſbourg, and falling perpendicular on the meridian of Paris.

According to the Advertiſement in the Map of France, the French computations have been made with a degree containing 57060 toiſes (= 60811.7 fathoms); therefore, if we reduce

$3^\circ\ 35'\ 42''.3$ in the proportion of 61225 to 60811.7, we have $3^\circ\ 34'\ 14''.9$ for the value of that arc when 61225 fathoms is $= 1^\circ$; hence cosine $48^\circ\ 34'\ 35''$ : rad. :: sine $3^\circ\ 34'\ 14''.9$ : sine $5^\circ\ 24'\ 6''$, the other longitude of Strasbourg. By the latter method the longitude of Cordouan was computed; but the other longitudes according to the former. The distances from the meridian of Paris are to be found in the publications alluded to in the above page.

P. 232. In the Table of General Results, *for* $1^\circ\ 8'\ 9''$ and 4 m. 32 s. 36th. *put* $1^\circ\ 8'\ 4''$ and 4 m. 32 s. 16th. the longitude of Padlesworth.

Against Calais, *for* 7 m. 23 s. 15.8th. *put* 7 m. 23 s. 15.2th.

Against Fairlight Down, in the last column but two, *for* 539.5 *put* 593.5.

P. 239. l. 15. *for* fig. 12. *put* fig. 13.

P. 242. l. 10. from bottom, *for* $3'\ 55''$ *put* fig. $3'\ 35''$.

P. 243. l. 13. *for* OKT *put* OK*t*. l. 5. from bottom, *for* *kt* put *kL*.

P. 224. l. 11. *for* $7''\frac{1}{4}$ *put* $7\frac{1}{4}$.

In the Table facing p. 246. in the column of *mean refraction*, for $0'\ 15''.4$ *put* $1'\ 28''.1$.

In addition to the examples of refraction in Sect. VII. the following (which was overlooked when this part was drawn up) may not improperly be given, as being of a different kind. It shews, that terrestrial refraction (though often much *greater*) must, at particular times, be much *less* than is generally supposed.

Oct. 7, 1787, at the station near Padlesworth, the depression of the horizon of the sea, in a SW direction nearly, was observed $26'\ 27''$. A degree of a great circle in this direction is about 61000 fathoms, and therefore $61000 \times 6 \times 57.2957795$

2 =

$=20970255$ feet, will be the radius of curvature nearly. The height of the ſtation above low-water ſpring-tides (as determined by alternate obſervations at this place and Dover Caſtle) was 642 feet; hence $\frac{20970255}{20970255+642} = ,9999693861$ the natural coſine of $26' 54''$ the *dip*; therefore $26' 54'' - 26' 27'' = 27''$ is what the horizon was elevated by refraction. The ſtate of the tide however, is not taken into conſideration; but the time was about noon.

The weather was calm and cloudy, and the horizon clear. Barometer 29.6. Thermometer 70°, at 1 P.M.

The above obſervation was made with great care and attention.

P. 249. in the 3. triangle, *for* 76807.5 *put* 76812.4.

The operations alluded to in the note at the botttom of this page were,

| | | ° | ′ | ″ |
|---|---|---|---|---|
| Angles obſerved at Spring-Grove Houſe between | Richmond Ob. and the Pagoda | 13 | 10 | 13 |
| | The Pagoda and St. Paul's | 23 | 17 | 15 |
| | St Paul's and Harrow Spire | 74 | 18 | 43 |
| Angles taken on Fulham Church between | Stretham Church and St. Paul's | 70 | 24 | 52 |
| | St. Paul's and Hampſtead Church | 44 | 50 | 46 |
| | Hamp. Ch. and Hanger H. Tower | 57 | 27 | 43 |

In 1787, at Hanger Hill, the angle between Richmond Obſervatory and the Pagoda was obſerved 12° 26′ 42″. At the Hundred Acres, that between Hanger Hill and Batterſea Church 23° 59′ 44″; and that between Hanger Hill and Stretham Church 42° 3′ 57″. The angle at St. Paul's between Batterſea Church and the SW Pinnacle of Weſtminſter Abbey is 9′ 45″.6.

The reſults from the obſervations made on Fulham Church cannot be conſidered as very exact, becauſe Hanger Hill Tower

 itſelf

itſelf was the object, inſtead of the Flagſtaff placed on it in 1787.

P. 250. in the 10. triangle, *for* 39963 *put* 48964.

In the 14. triangle, *for* 34413.6 *put* 34412.8.

P. 251. in the 20. triangle, *for* 39° 17′ 6″.5 *put* 39° 17′ 16″.5.

In the 27. triangle, there is a tranſpoſition of the two firſt angles, that oppoſite Fairlight ſhould ſtand oppoſite Goudhurſt.

P. 252. in the 28. triangle, *for* 5384 *put* 5385.

In the 35. triangle, *for* 23081.4 *put* 23018.4.

P. 254. in the 4. triangle, there is a tranſpoſition of the diſtances, 7198.2 ſhould ſtand oppoſite Hornſey Hill.

P. 255. in the 16. triangle, *for* 14390.2 and 6073.8 *put* 14390.8 and 6074.

P. 256. in the 22. triangle, *for* 8136.4 and 17641 *put* 8136 and 17640.3.

Neither of the angles of the 26. triangle was obſerved, becauſe St. Paul's is not ſeen from Greenwich Obſervatory. The diſtance of St. Paul's from Greenwich Obſervatory is alſo omitted. This diſtance is had from the VIII triangle, p. 173. and the 11. triangle, p. 250; from theſe we have,

| | | ° | ′ | ″ | |
|---|---|---|---|---|---|
| the angle at | Norwood | 42 | 15 | 26.5 | hence Greenwich Ob. from St. Paul's 25655½ feet. |
| | Greenwich Ob. | 82 | 41 | 1.1 | |
| | St. Paul's | 55 | 3 | 32.4 | |

The angles were determined thus:

| | ° | ′ | ″ | |
|---|---|---|---|---|
| Severndroog Caſtle | 22 | 20 | 46 | obſerved angles. |
| Greenwich Obſervatory | 134 | 16 | 31 | |
| *Limehouſe Church* | 33 | 22 | 43 | |

4 Hence

Hence Greenwich Obſervatory from Limehouſe Church is 13999 feet.

Taking the ſum of 111° 56′ 50″ (in the VIII triangle, p. 173.) 82° 41′ 1″ and 134° 16′ 31″ from 360° there remains 31° 5′ 38″ for the angle at Greenwich Obſervatory between Limehouſe Church and St. Paul's; this, with the including ſides 13999 and 25655 (neglecting the fractions) will give 121° 1′ 42″ and 27° 52′ 40″, the other two angles.

P. 257. in the 31. triangle, *for* 6511 *put* 6925.4.

P. 259. The bearing of Greenwich from the meridian of St. Paul's, on which the other bearings in the Table depend, was found as follows:

| | ° | ′ | ″ |
|---|---|---|---|
| Angle at Greenwich Obſervatory between its meridian and Norwood . | 38 | 7 | 16 |
| between Norwood and St. Paul's | 82 | 41 | 1.1 |
| Hence the angle at Greenwich Obſervatory between the north meridian and St. Paul's . | 59 | 11 | 42.9 |

This laſt angle, and its complement, with $25655\frac{1}{2}$ feet the diſtance of Greenwich Obſervatory and St. Paul's, give 13138.5 for what St. Paul's is north from Greenwich, and 22036 for what it is weſt from the meridian.

On M. Bouguer's ſpheroid 13138.5 feet anſwer to 2′ 9″.5 on the meridian; hence the latitude of the point on the meridian of Greenwich, where a great circle, paſſing through St. Paul's, falls perpendicular on that meridian, will be 51° 30′ 49″$\frac{1}{2}$: and taking 61251 fathoms for a degree perpendicular to the meridian (ſee Tab. p. 227.) we have (22036 feet) 3′ 35″.86 for the intercepted arc of that circle; this, and the complement of 51° 30′ 49″$\frac{1}{2}$ (as the legs of a ſpherical triangle) give 89° 55′ 29″ for the angle at St. Paul's, which, added

added to 30° 48′ 17″ (the complement of 59° 11′ 43″) gives 120° 43′ 46″, for the bearing of Greenwich Obſervatory, as in the Table.

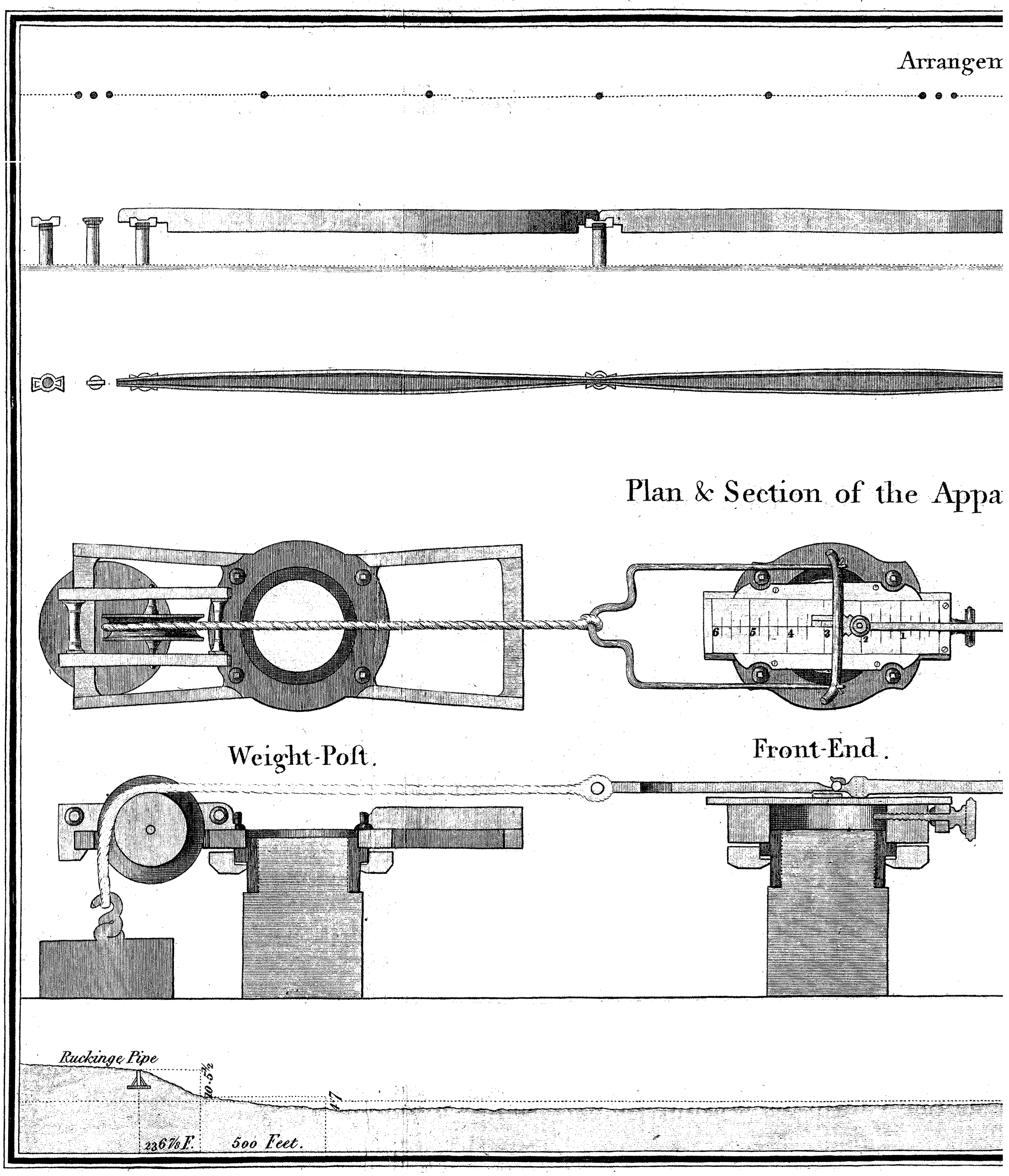
P. 259. againſt St. Luke's, *for* 12° 57′ 53″.7 *put* 12° 37′ 37″.
Againſt Shoreditch Church, *for* 44° 57′ 39″.8 and 6746.4 *put* 44° 54′ 58″.8 and 6743.2.
Againſt Severndroog Caſtle, *for* 115° 28′ 50″.4 *put* 115° 25′ 50″.4.
Againſt Eltham Church, *for* 123° 46′ 4″.2 *put* 123° 46′ 4″.

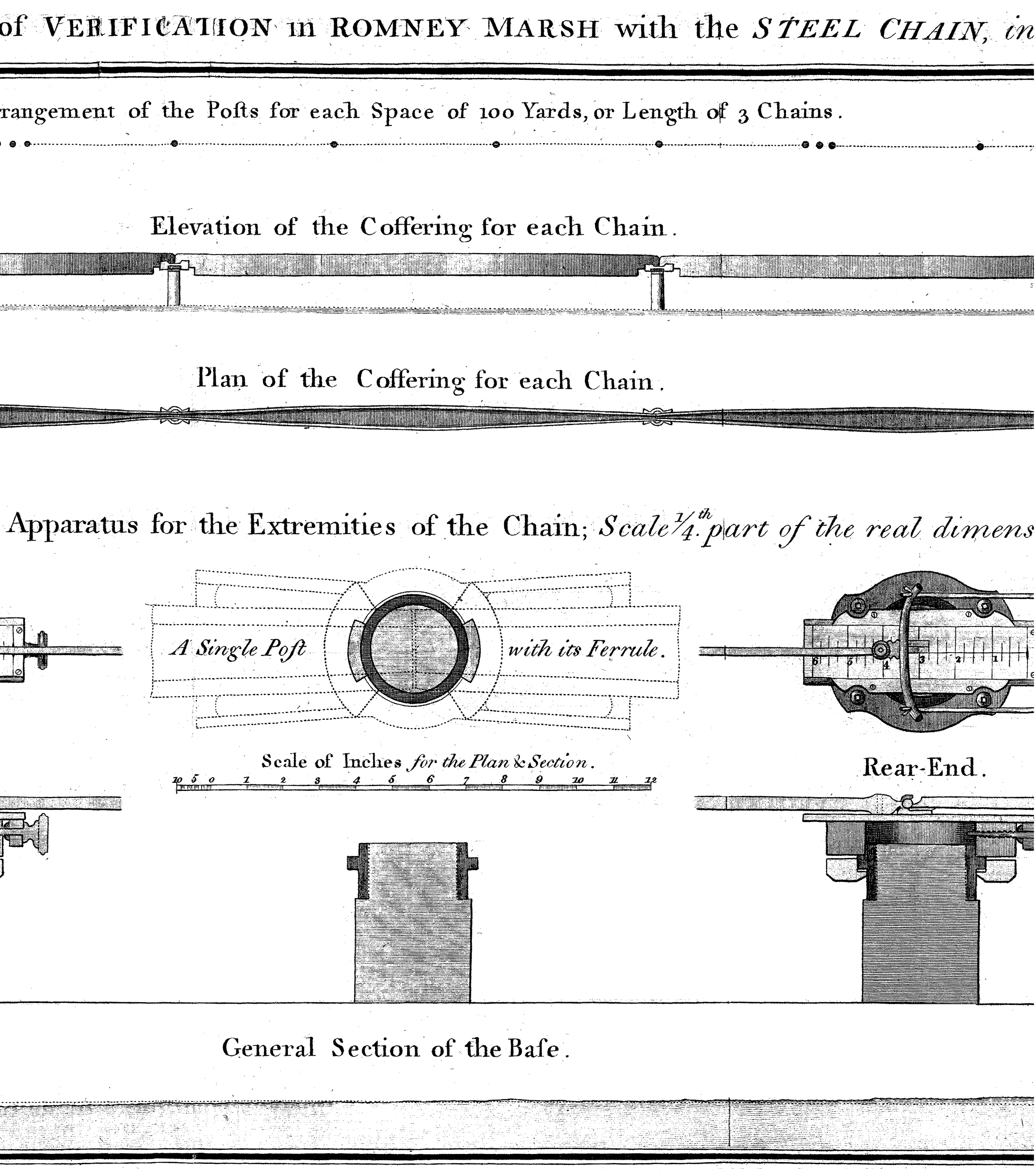
P. 260. againſt Stretham Church, *for* 31793.5 *put* 31739.5.
Againſt Clapham Common, *for* 26° 29′ 56″.1 *put* 26° 29′ 52″.
Againſt St. Bride's Church, *for* 1771.7 *put* 1687.6.
Againſt St. George's Bloomſbury, *for* 103° 15′ 20″.6 *put* 103° 15′ 50″.
Againſt the Tabernacle, *for* 107° 20′ 47″ *put* 107° 19′ 47″.
Againſt Highbury Houſe, *for* 178° 43′ 11″.6 *put* 178° 43′ 14″.6.

In Plate X. fig. 2. *p* is omitted at the concourſe of the meridians *eb*, *na*. Alſo a line from A to T in fig. 3. ib. *for* F *put* R.

Plate XI. Eltham Church is laid down too near the meridian of St. Paul's; and St. Bride's Church, Fleet-Street, ſhould ſtand on the north ſide of the weſt line. There are a few other corrections neceſſary on account of the errors in Tab. p. 259, 260.

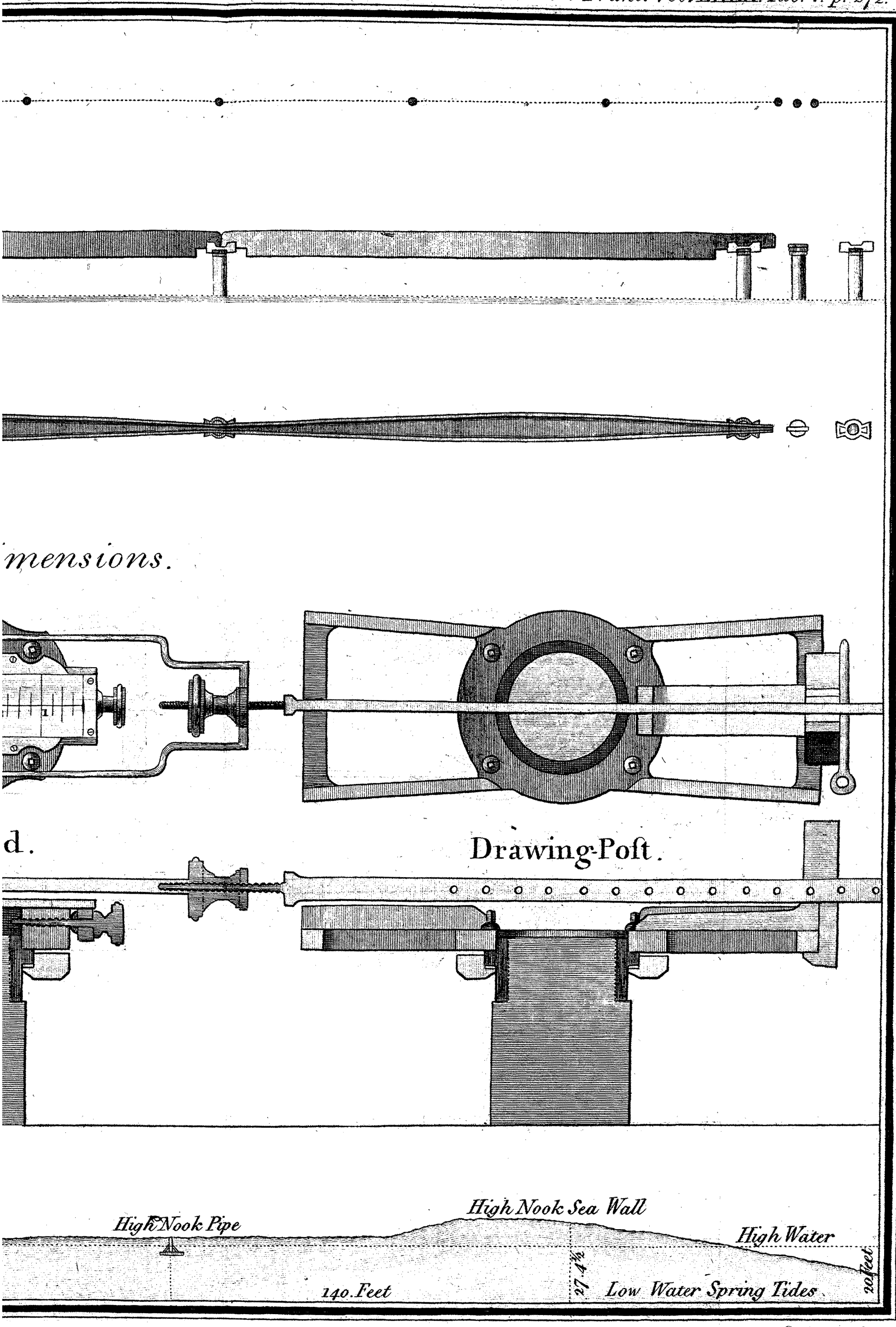
Pl.I. *FOR THE ACCOUNT of the MEASUREMENT of the* BASE of V




of VERIFICATION in ROMNEY MARSH with the *STEEL CHAIN, in*
rangement of the Poſts for each Space of 100 Yards, or Length of 3 Chains.
Elevation of the Coffering for each Chain.
Plan of the Coffering for each Chain.
Apparatus for the Extremities of the Chain; *Scale 1/4th part of the real dimens*
*A Single Poſt*
*with its Ferrule.*
Scale of Inches *for the Plan & Section.*
10 5 0 1 2 3 4 5 6 7 8 9 10 11 12
Rear-End.
General Section of the Baſe.

*N, in the Autumn of 1787.*

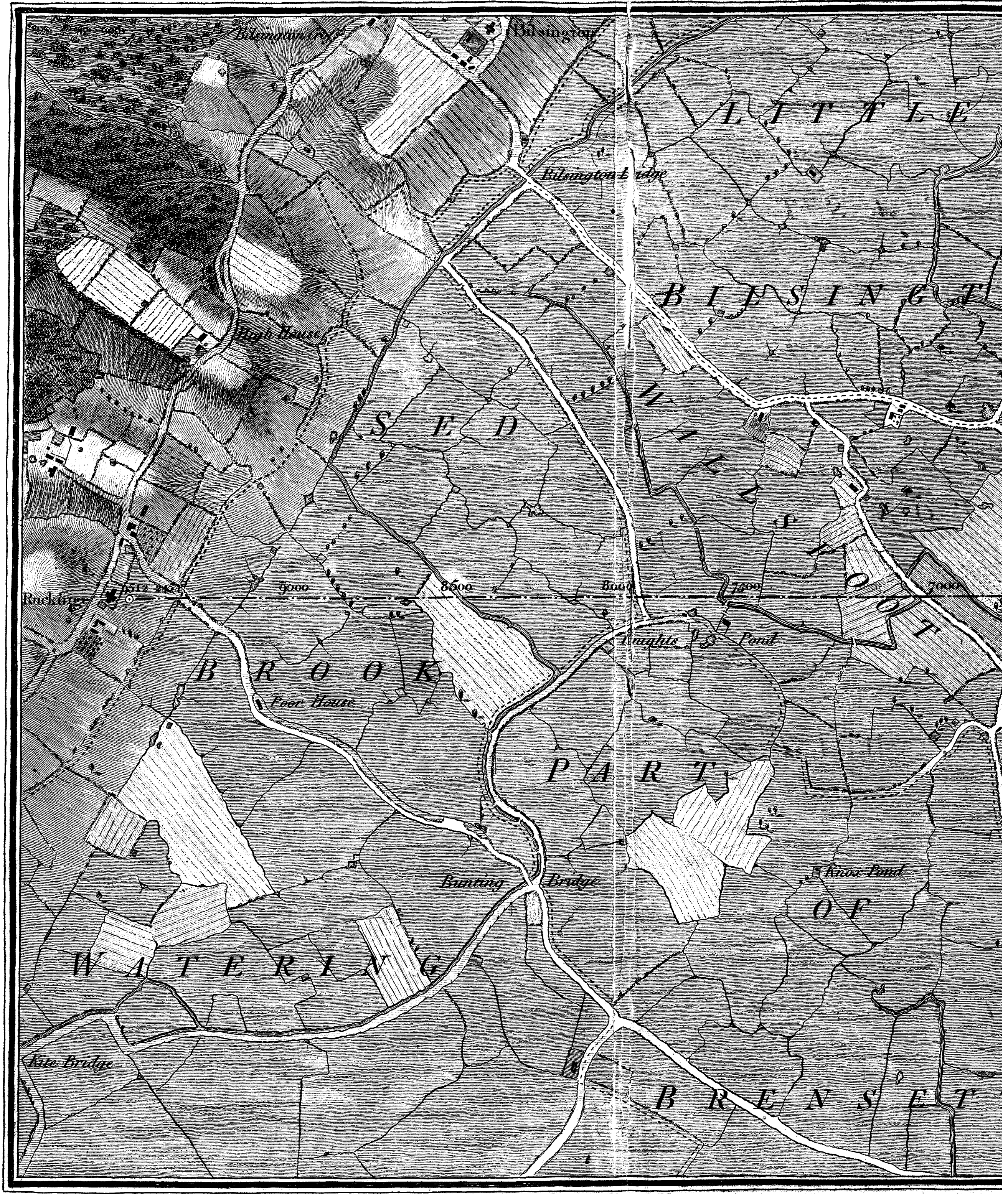
*Philos. Trans. Vol. LXXX. Tab. V. p. 272.*




*Basire Sc.*

Pl. II.

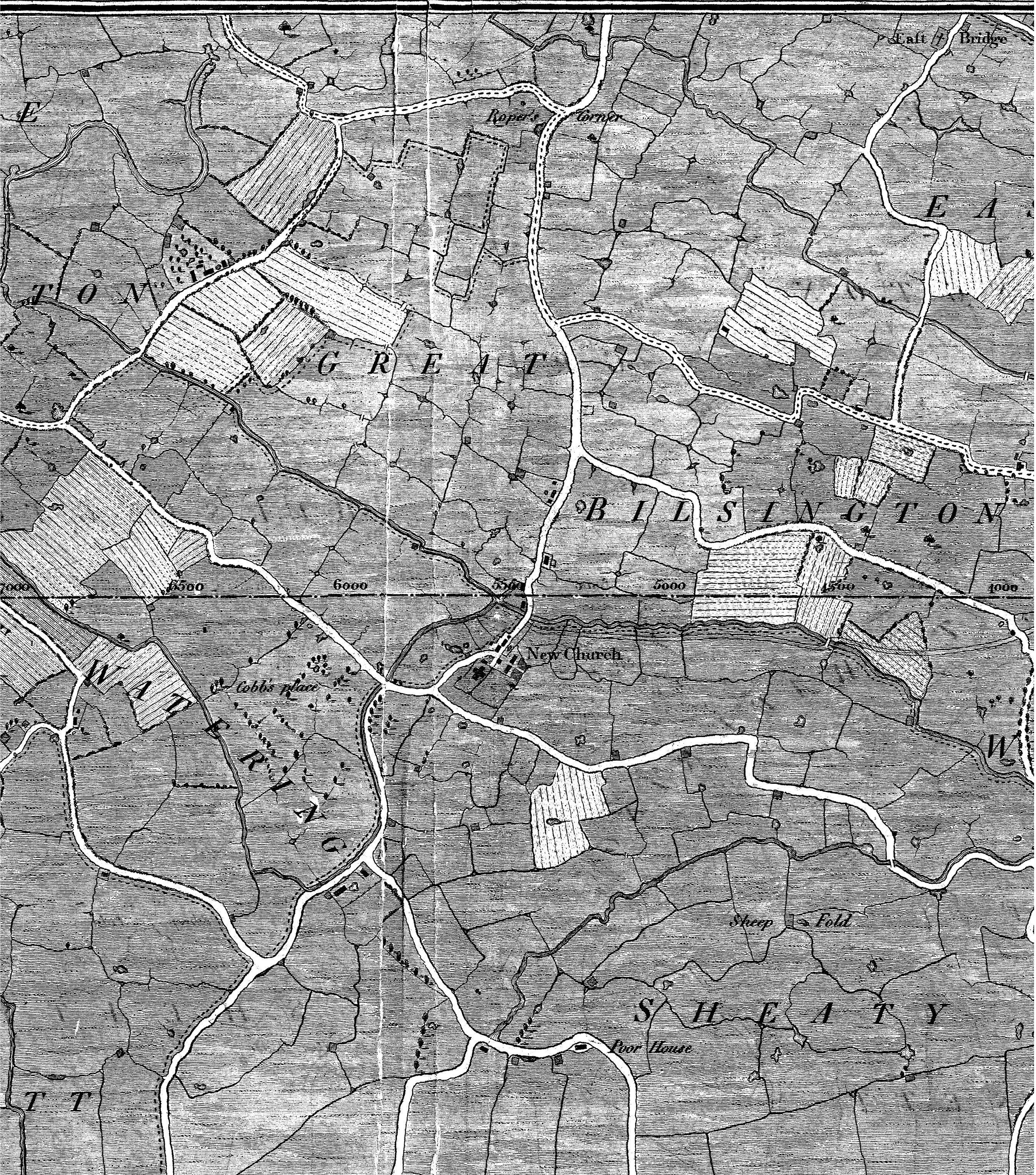
PLAN shewing th




Scale of Two Miles.

1 ¾ ½ ¼ 0

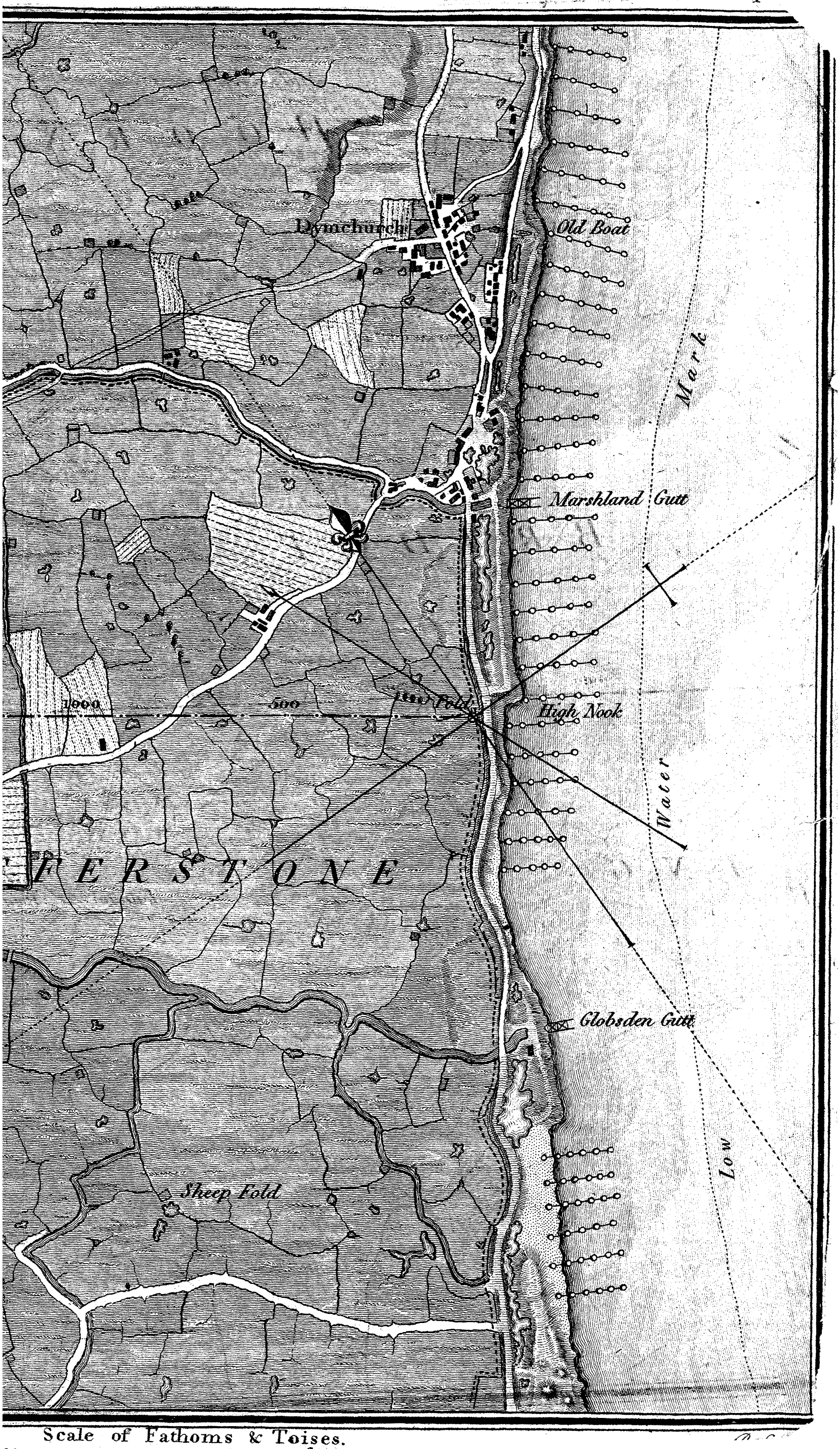
ng the situation of the BASE of VERIFICATION measured

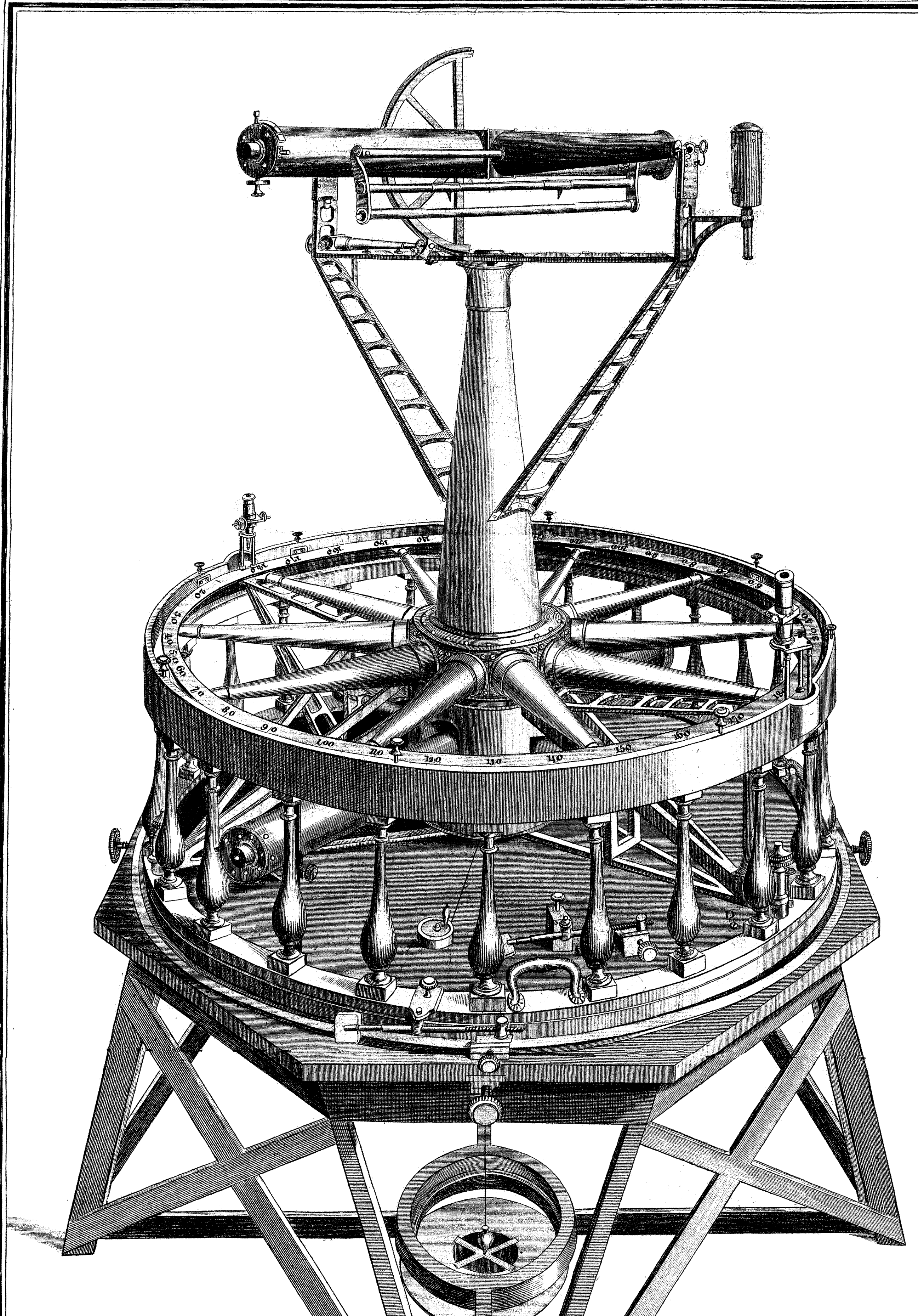


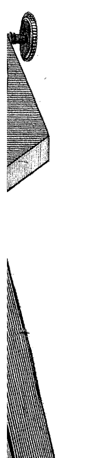

Scale of Ten Thousand Feet

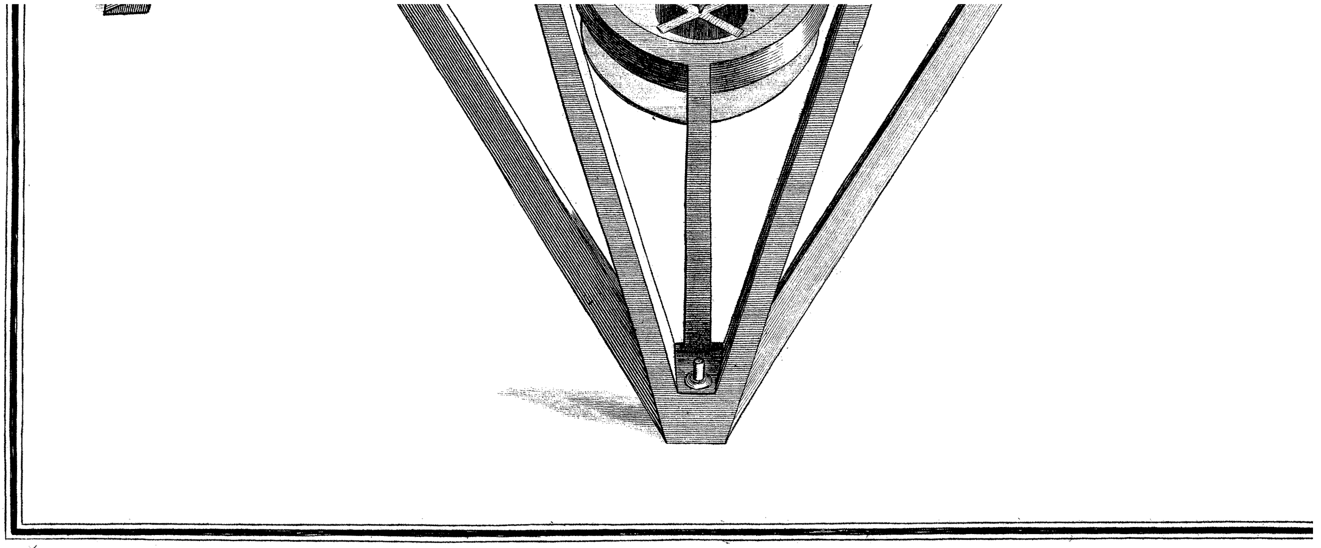
1 1000 500 0 1 2 3 4 5

red in ROMNEY MARSH in the Autumn of 1787.


Dymchurch
Old Boat
Mark
Marshland Gutt
1000
500
Fold
High Nook
Water
E R S T O N E
Globsden Gutt
Low
Sheep Fold
Scale of Fathoms & Toises.





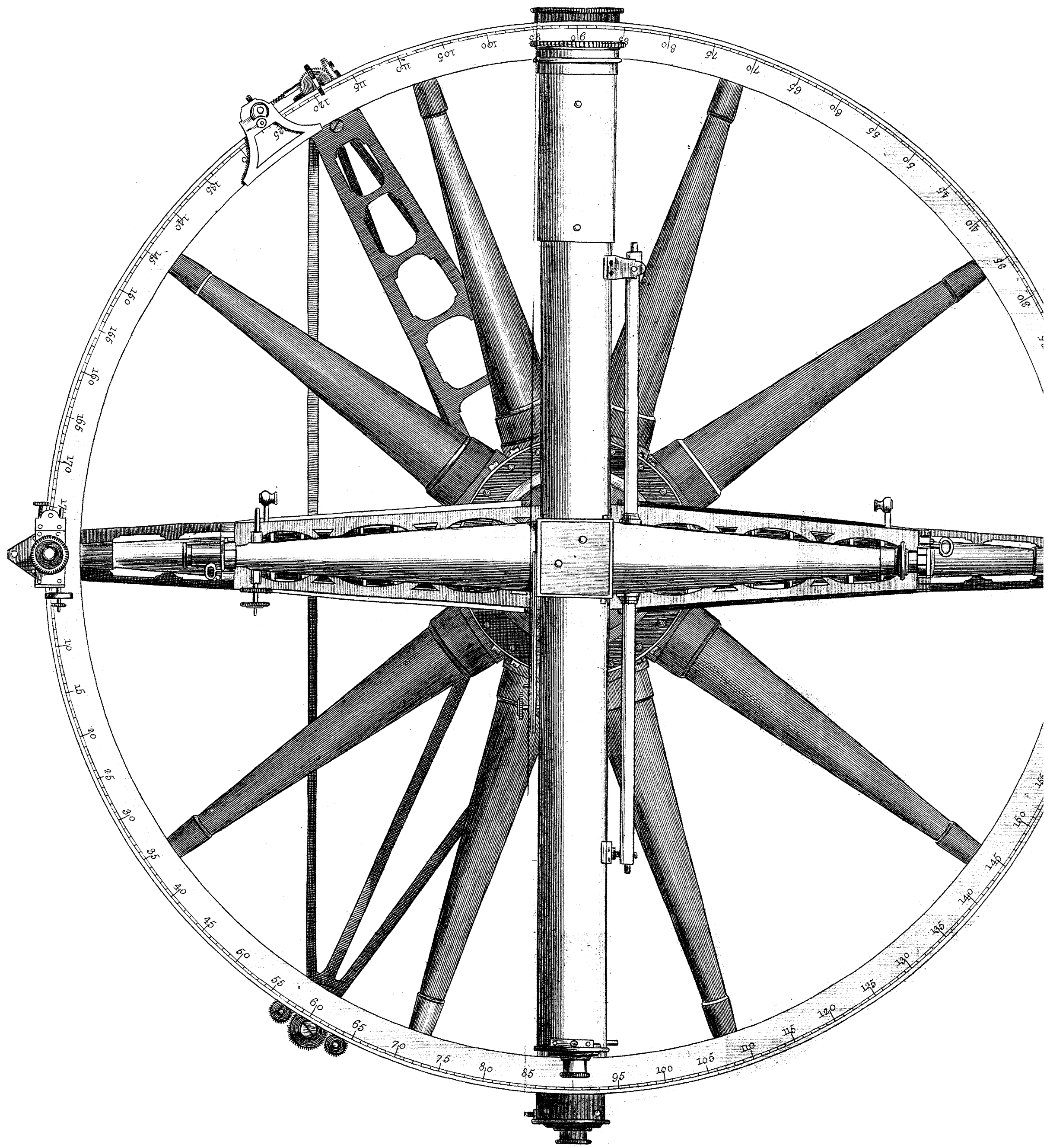


T. Milne del. Basi

# General View of the Instrument.

*Basire Sc.*

Plan *of the Instrument.*

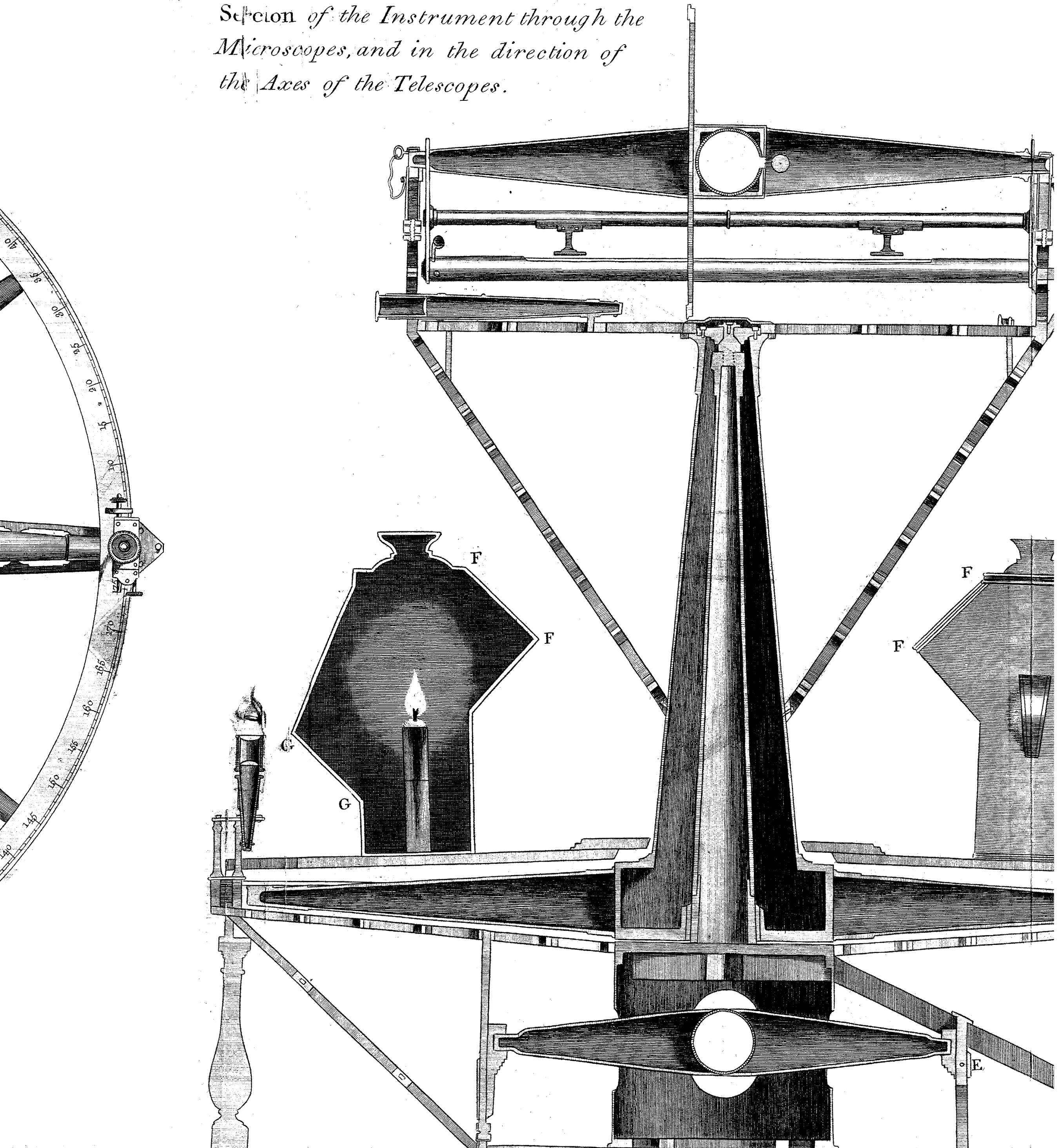


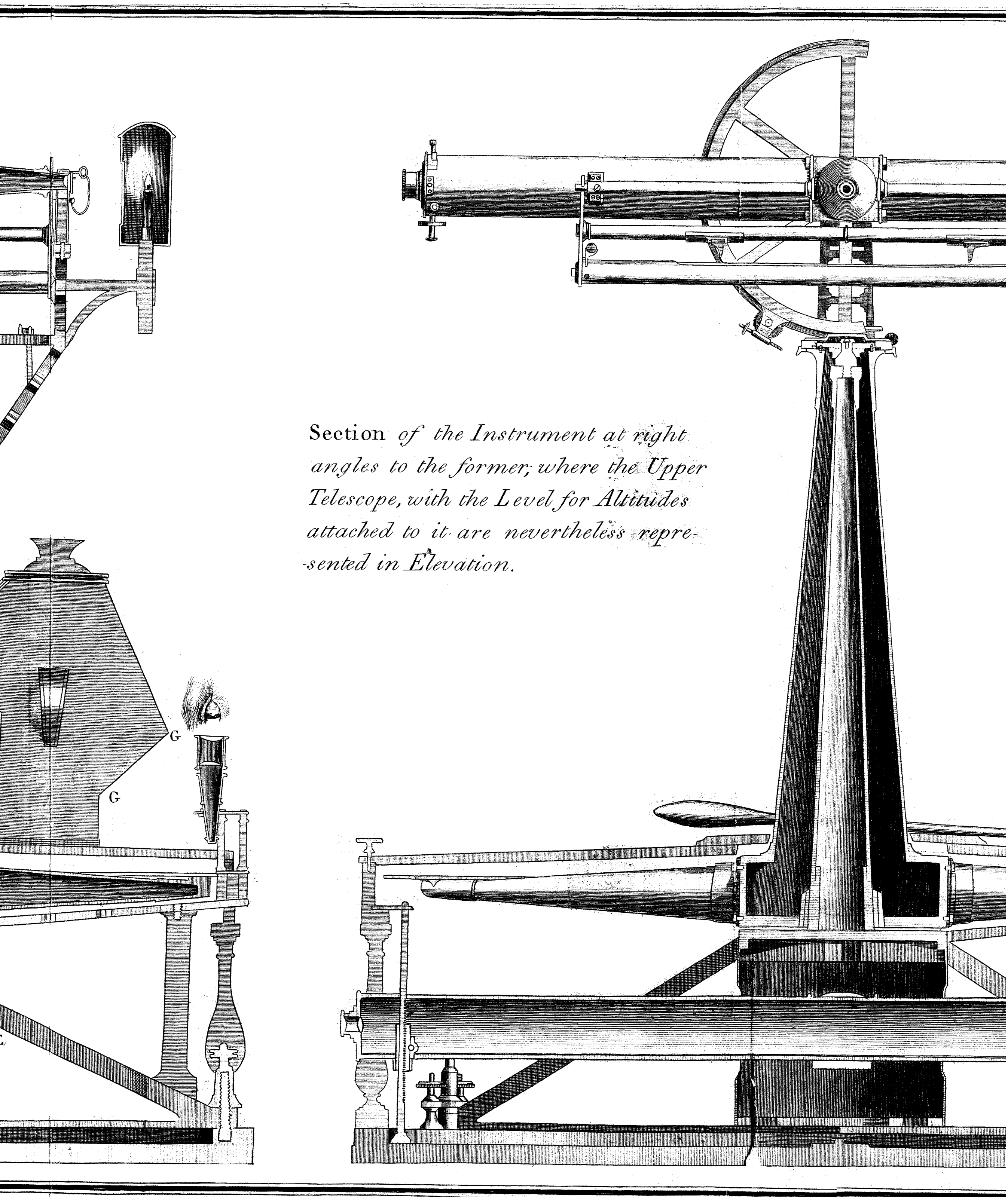
Scale of Inches.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37

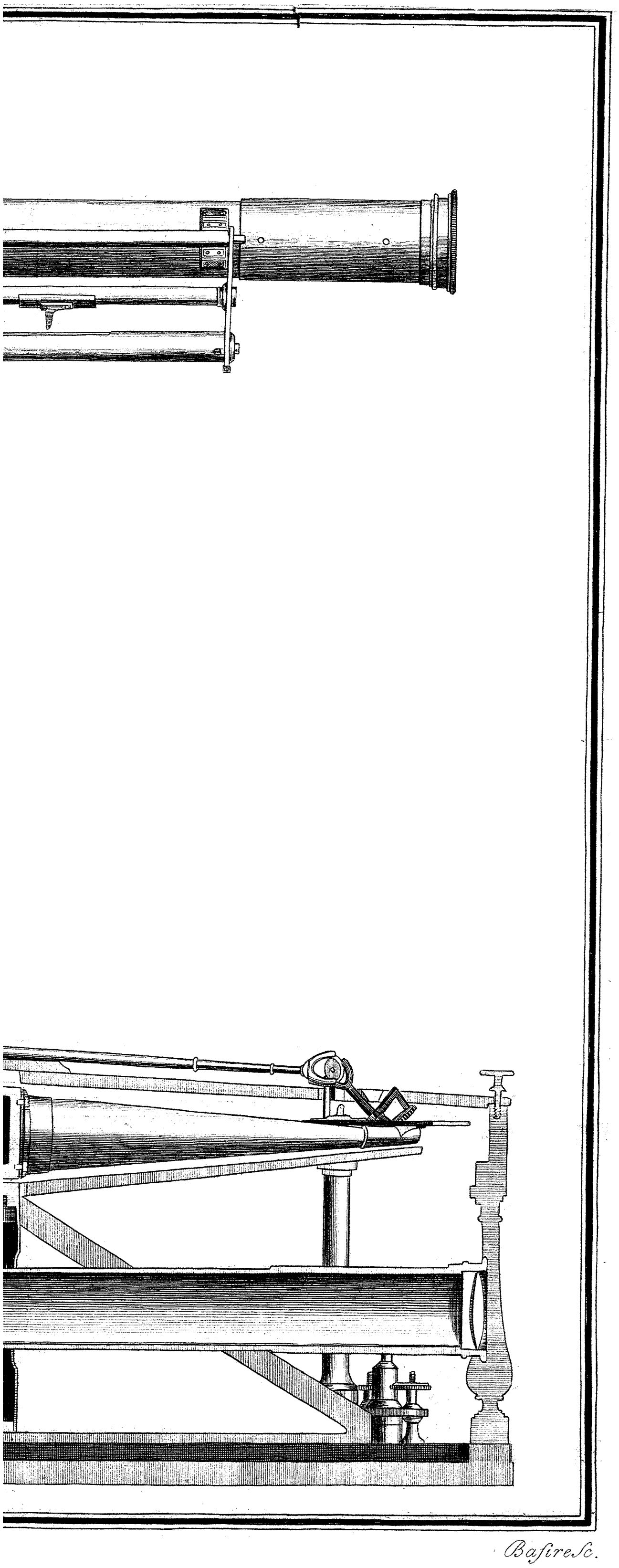
*J. Milne del.*

Section *of the Instrument through the Microscopes, and in the direction of the Axes of the Telescopes.*



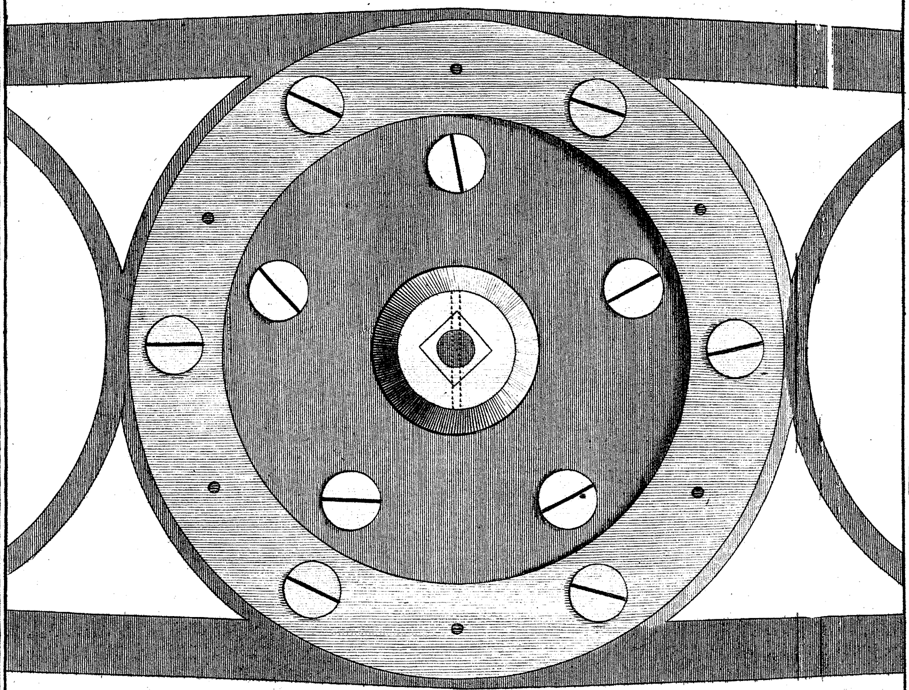



Section *of the Instrument at right angles to the former; where the Upper Telescope, with the Level for Altitudes attached to it are nevertheless represented in Elevation.*

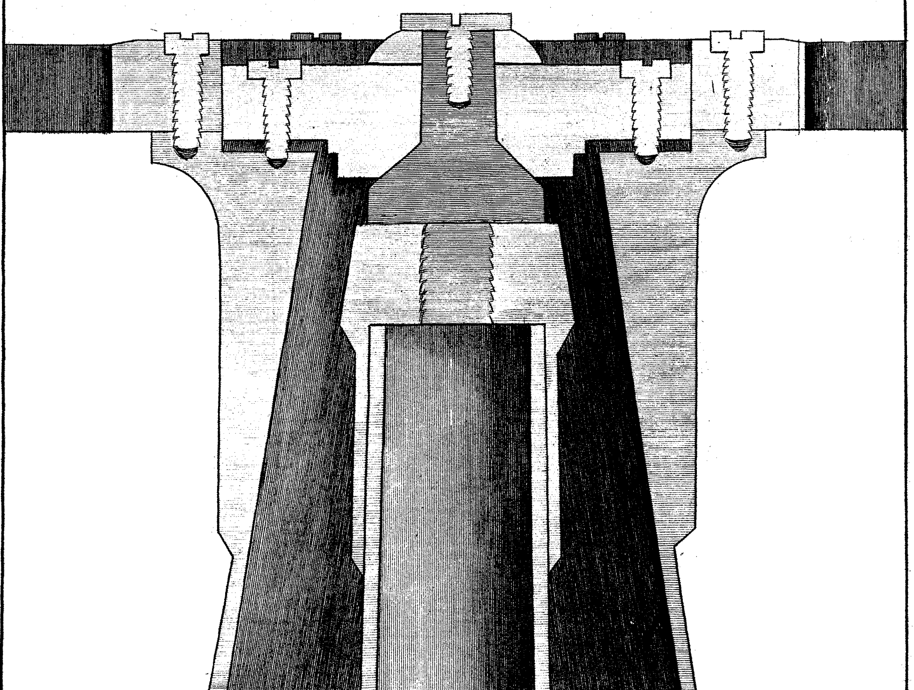


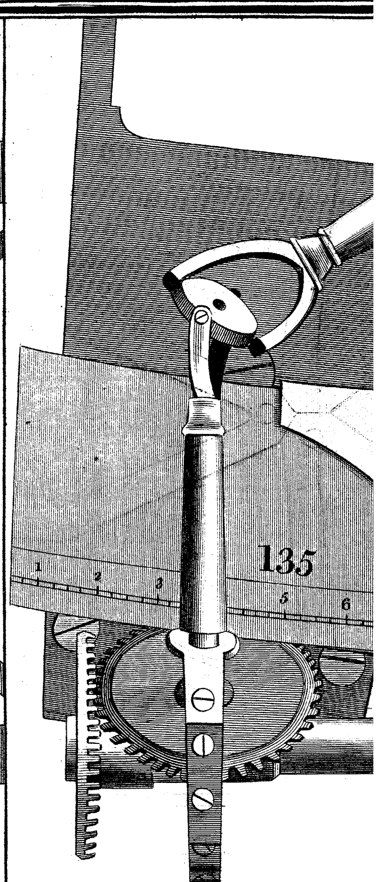
*Basire Sc.*

Plan *of the Top of the vertical Axis, with part of the horizontal Bar for carrying the Upper Telescope; in their real dimensions.*



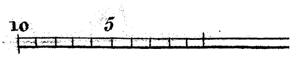
Section *of the Top of the vertical Axis.*

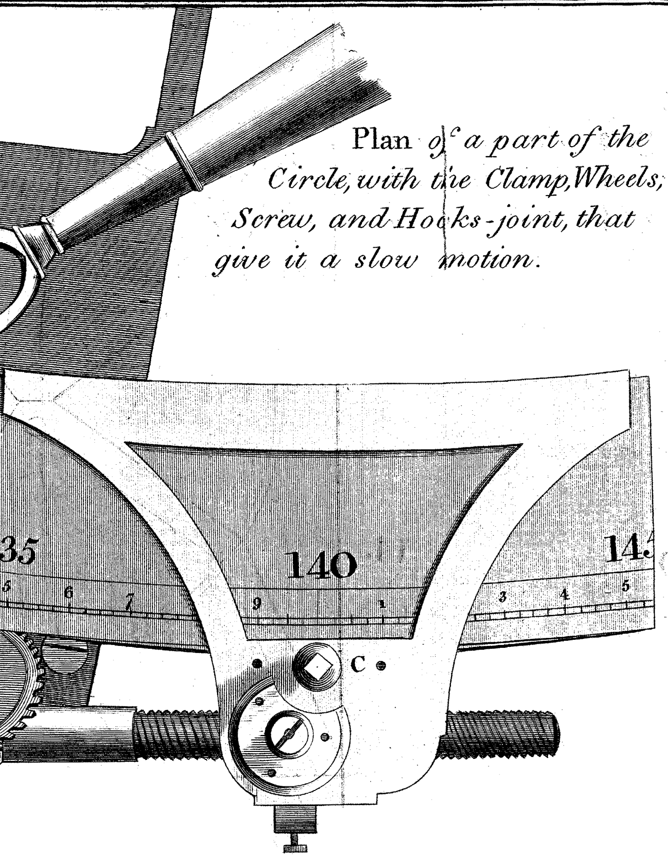




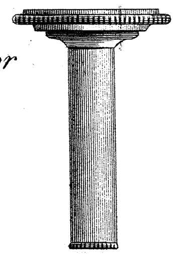

Milled-head-Key *plac*
*clamping the Ins*

Scale *of Inches*



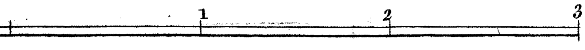


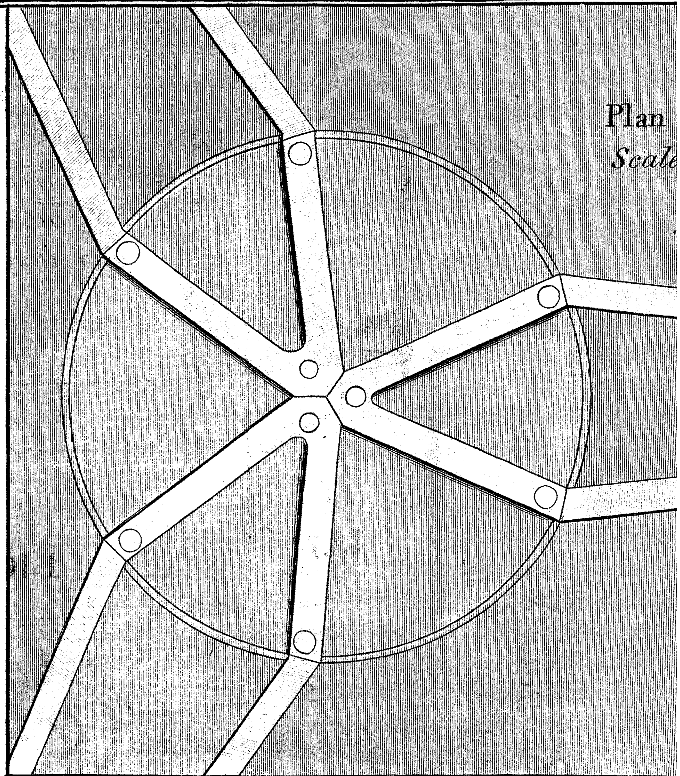

Plan *of a part of the Circle, with the Clamp, Wheels, Screw, and Hooks-joint, that give it a slow motion.*



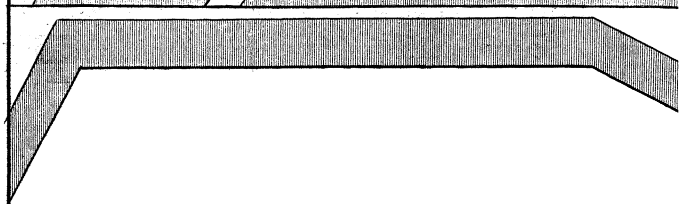
*...y placed at* C *for ...he Instrument.*

*Inches for parts represented in their real dimensions.*

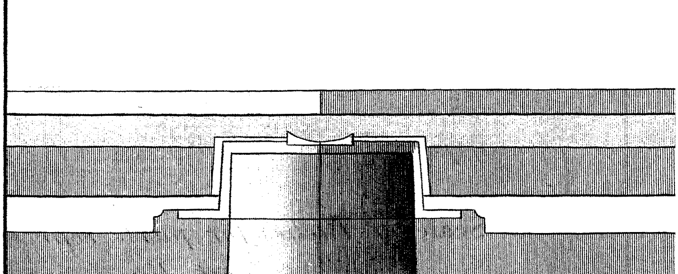


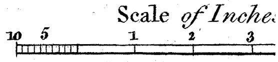
Plan ...
*Scal...*

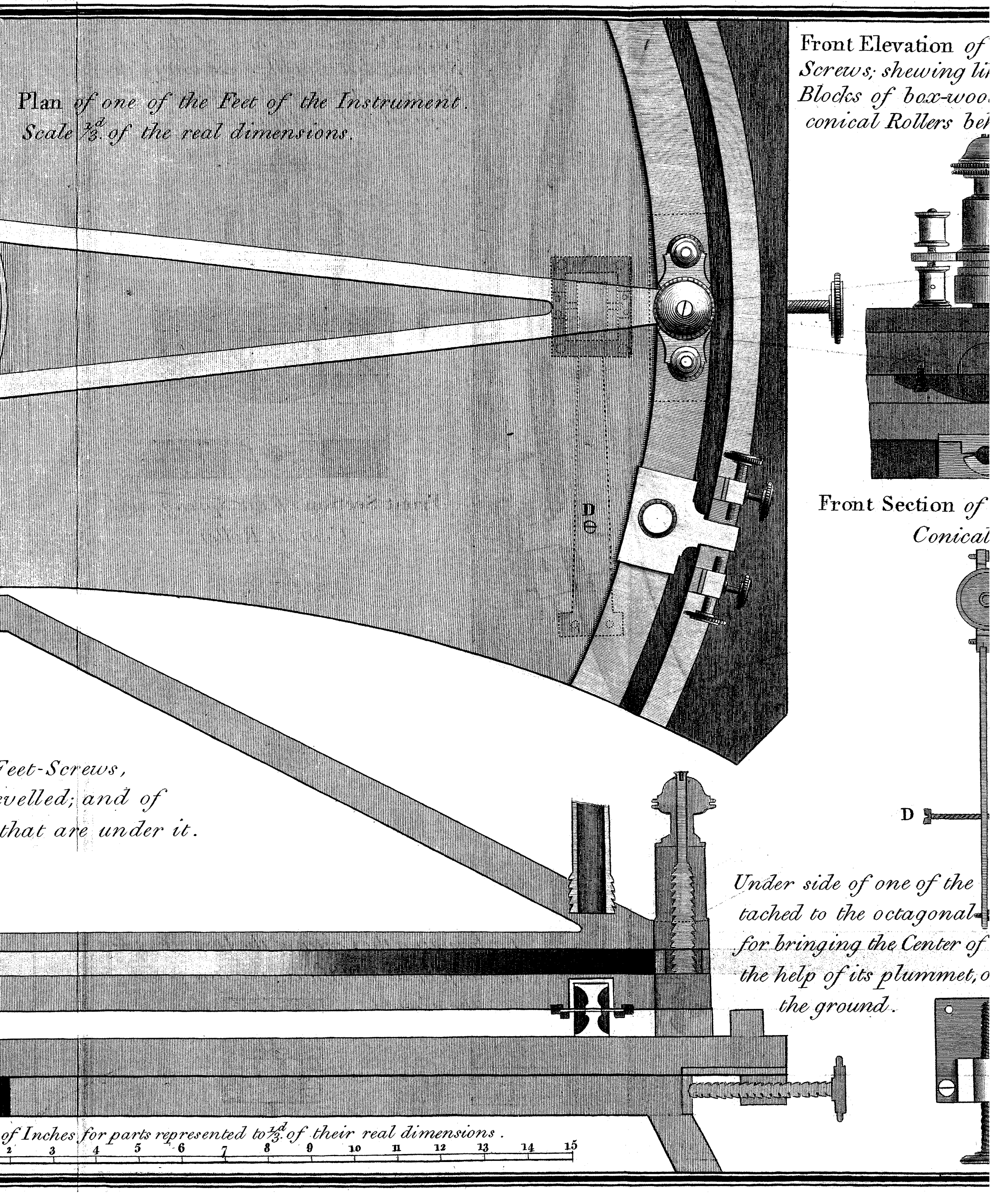


Section *through one of the Feet-Sc... whereby the Instrument is levelled, ... the three Mahogany Planes that a...*

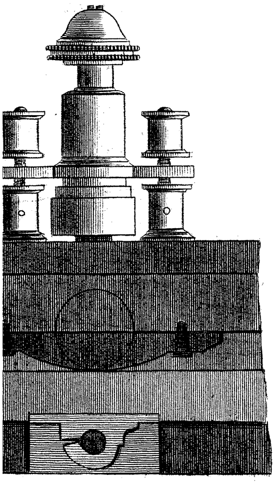


Scale *of Inche...*

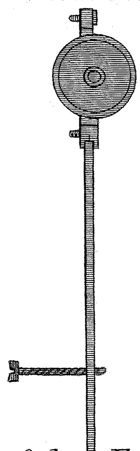



Plan of one of the Feet of the Instrument.
Scale ⅓d. of the real dimensions.
Front Elevation of
Screws; shewing li
Blocks of box-woo
conical Rollers be
Front Section of
Conical
D
D
Feet-Screws,
velled; and of
that are under it.
Under side of one of the
tached to the octagonal
for bringing the Center of
the help of its plummet, o
the ground.
of Inches for parts represented to ⅓d. of their real dimensions.
2 3 4 5 6 7 8 9 10 11 12 13 14 15

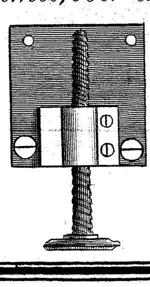
tion *of one of the Feet-*
*ving likewise one of the*
*ox-wood, and one of the*
*llers behind it.*

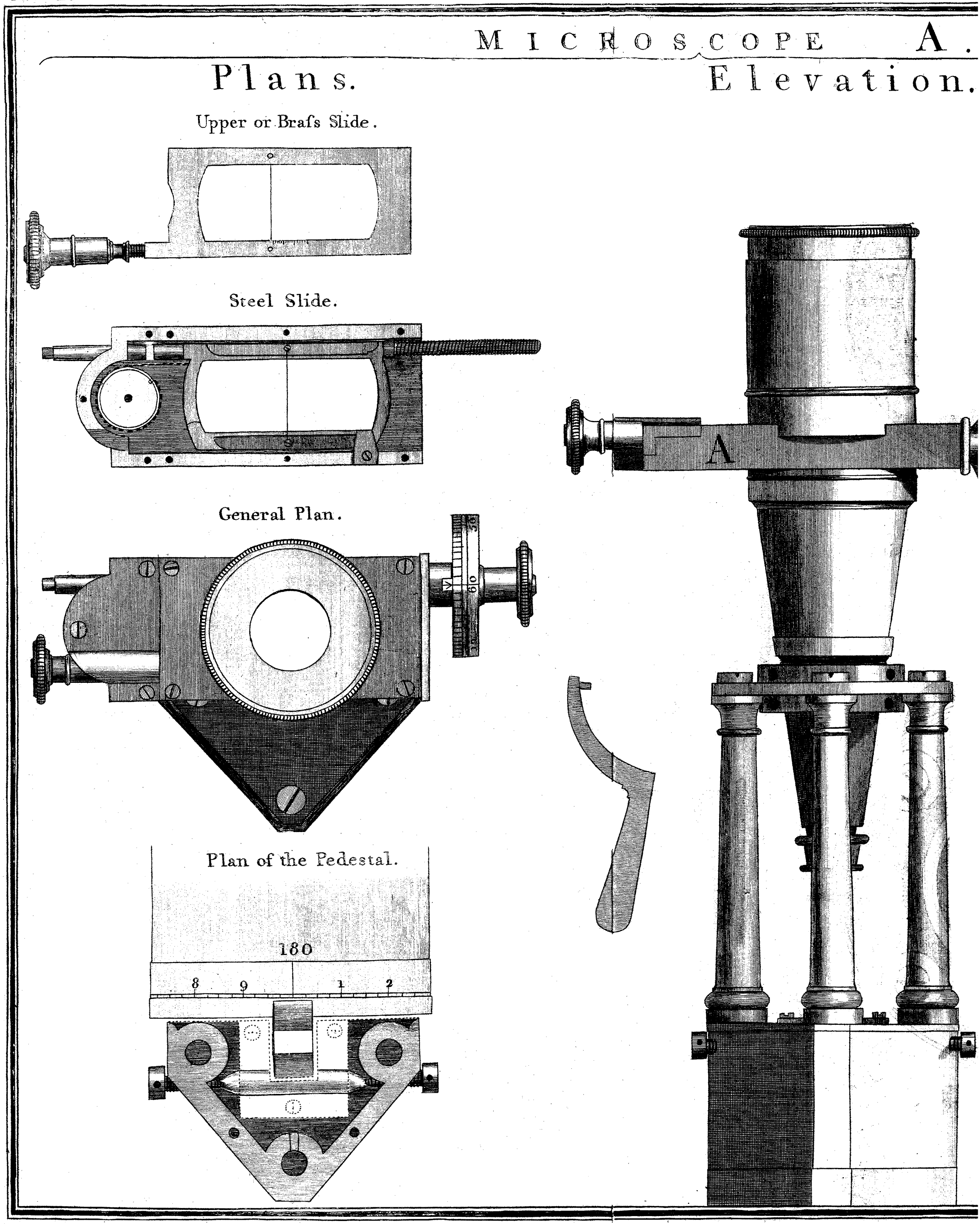


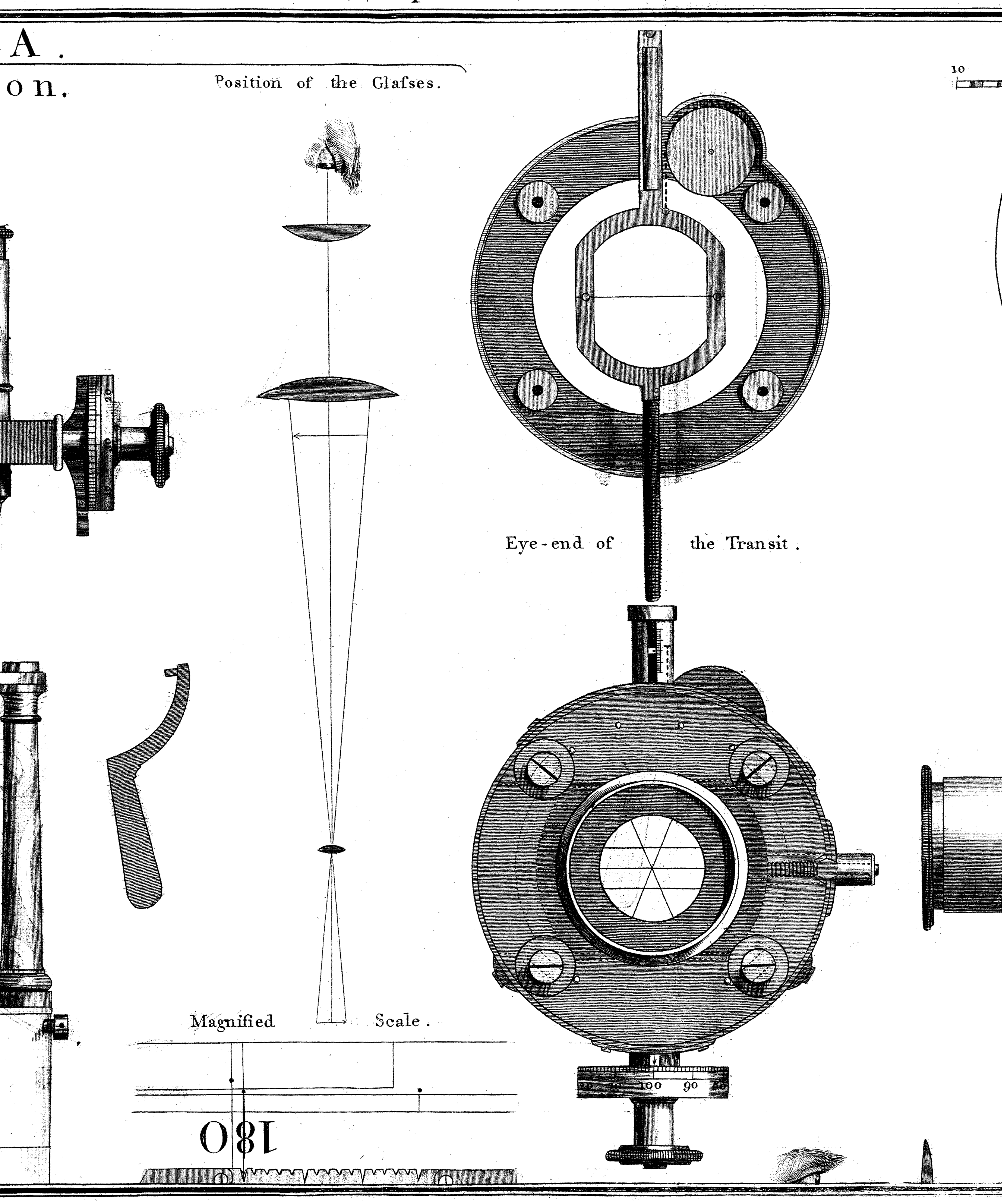
tion *of the Spring, and*
*Conical Roller.*



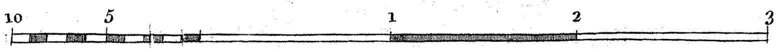
*of the Four Screws at-*
*gonal mahogany Plane*
*nter of the Instrument by*
*mmet, over any point on*



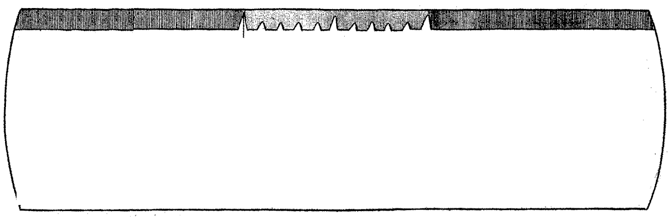

MICROSCOPE A.
Plans.
Elevation.
Upper or Braſs Slide.
Steel Slide.
General Plan.
Plan of the Pedestal.
180
8 9 1 2
A
30
60

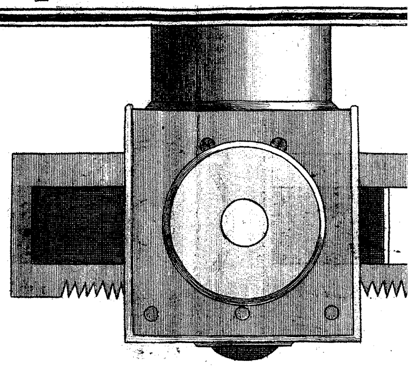

A .
on.
Position of the Glafses.
Eye-end of the Transit.
Magnified Scale.

Scale of Inches.



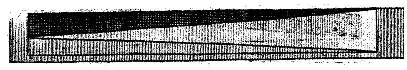

Magnified Scale of the Horizontal Microscope.



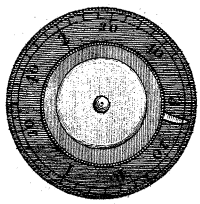


Front of the Prism Eye Tub

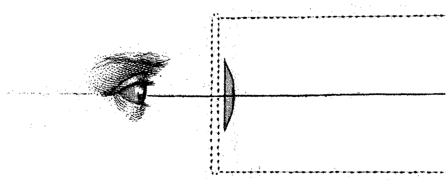
Plan of the Slide with the Dark Gl

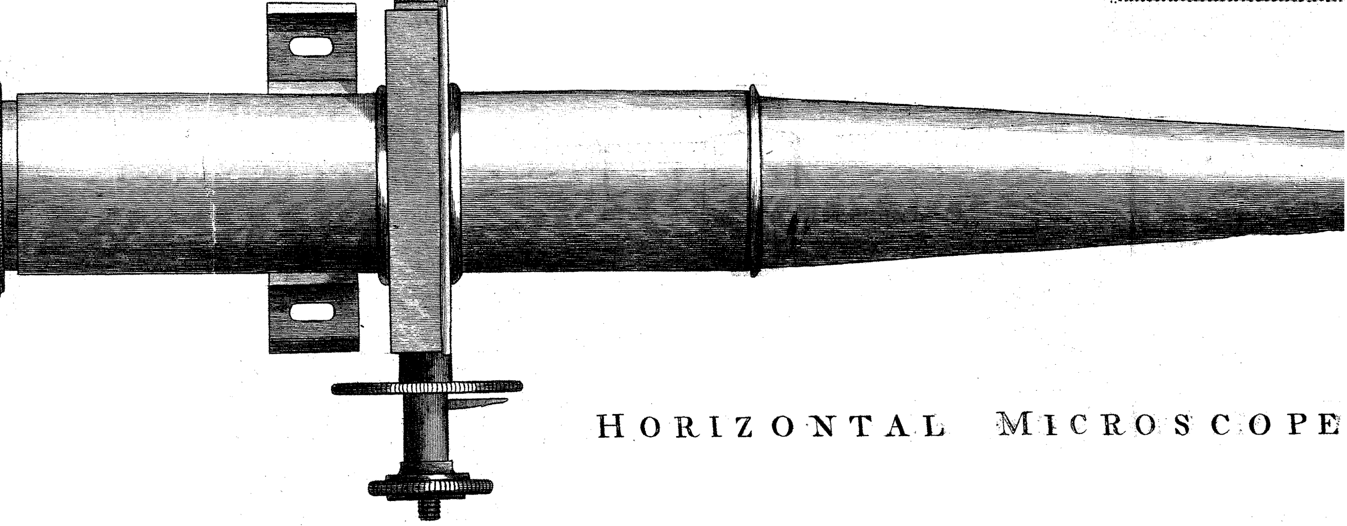


Micrometer Head.

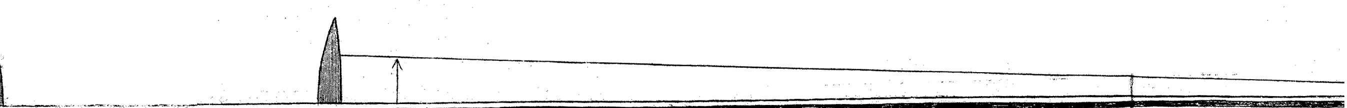


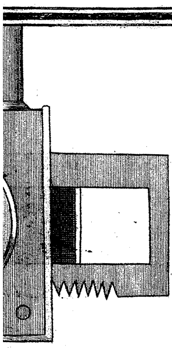
Prism Eye Tub

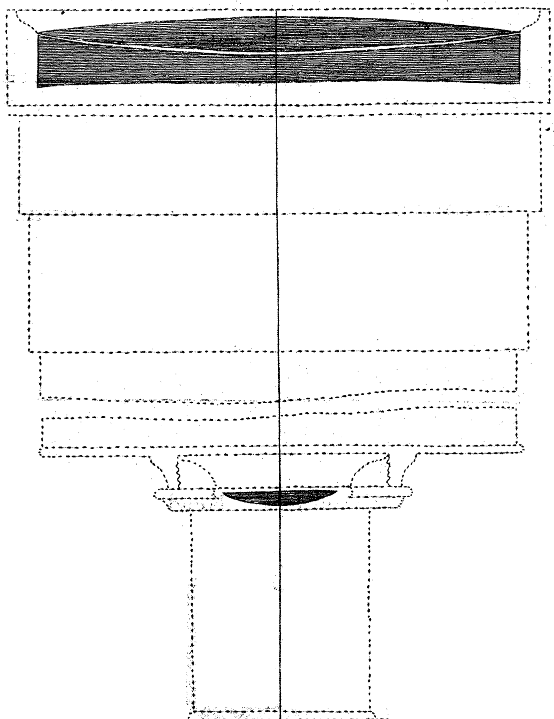




HORIZONTAL MICROSCOPE

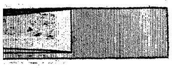




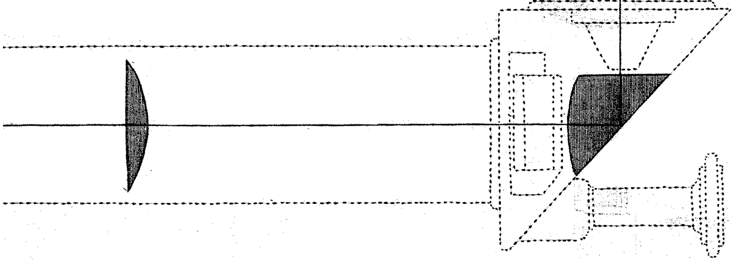


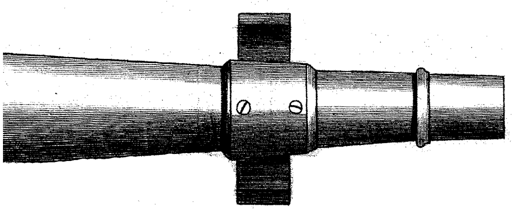
Eye Tube.

Dark Glafses.



Eye Tube of the Telescope.



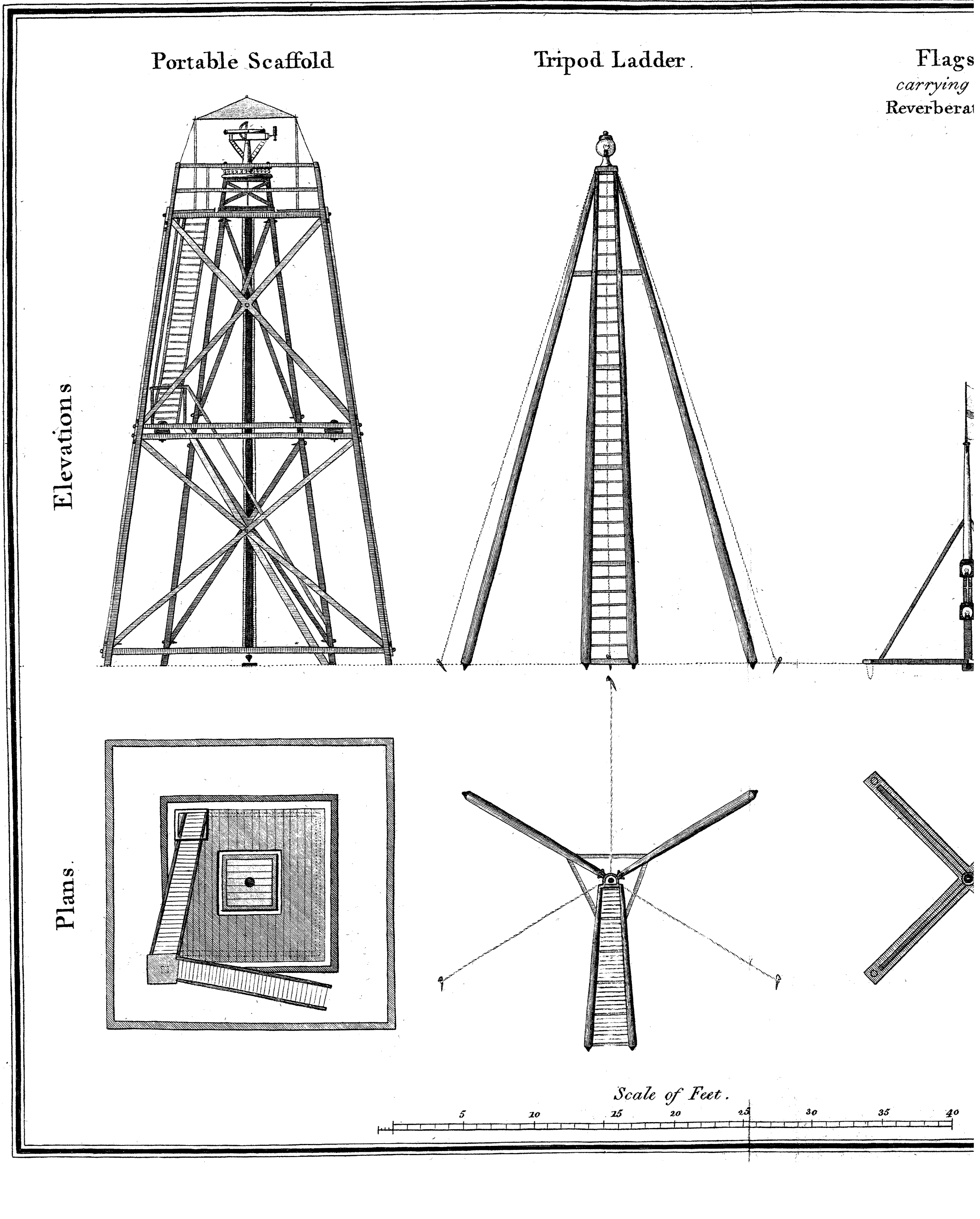


OPE.

Basire Sc.

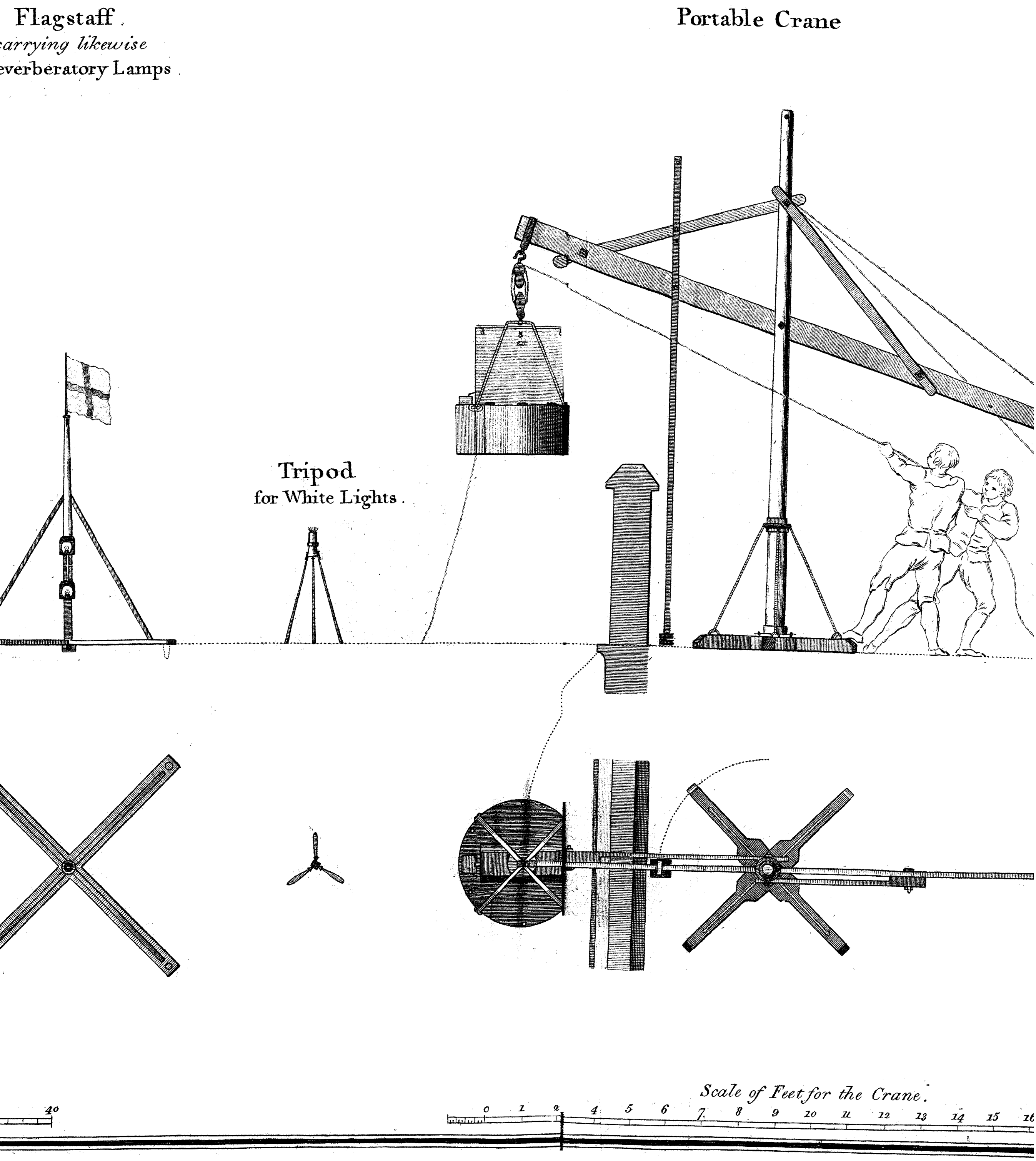
Pl. VII.

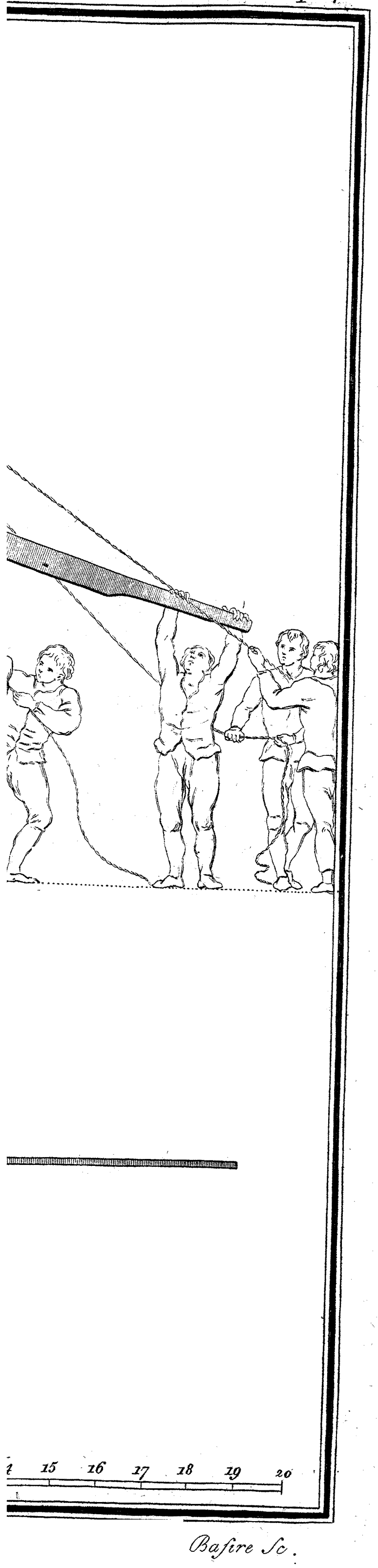
VARIOUS



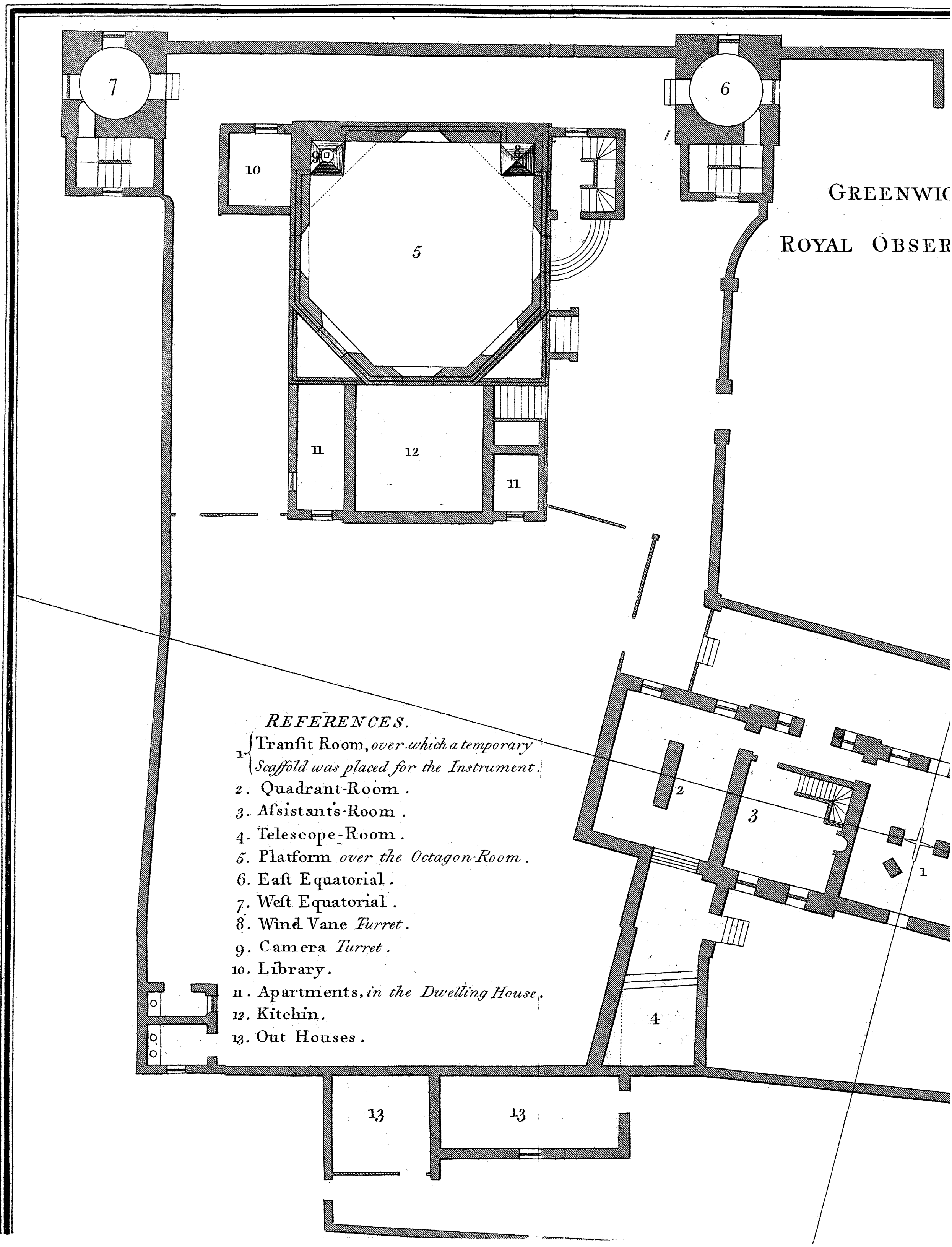
USED IN THE COURSE OF THE OPERATION .

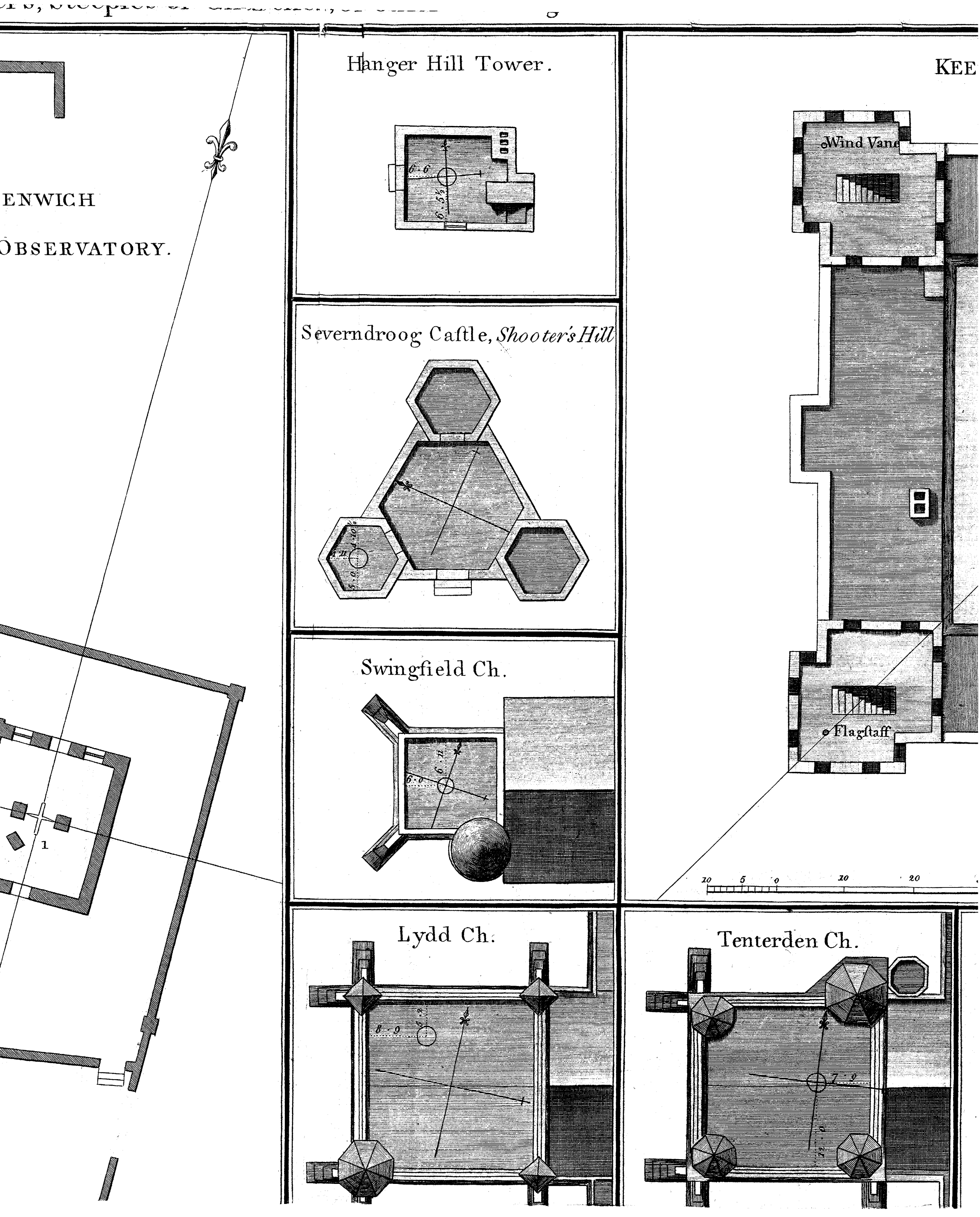


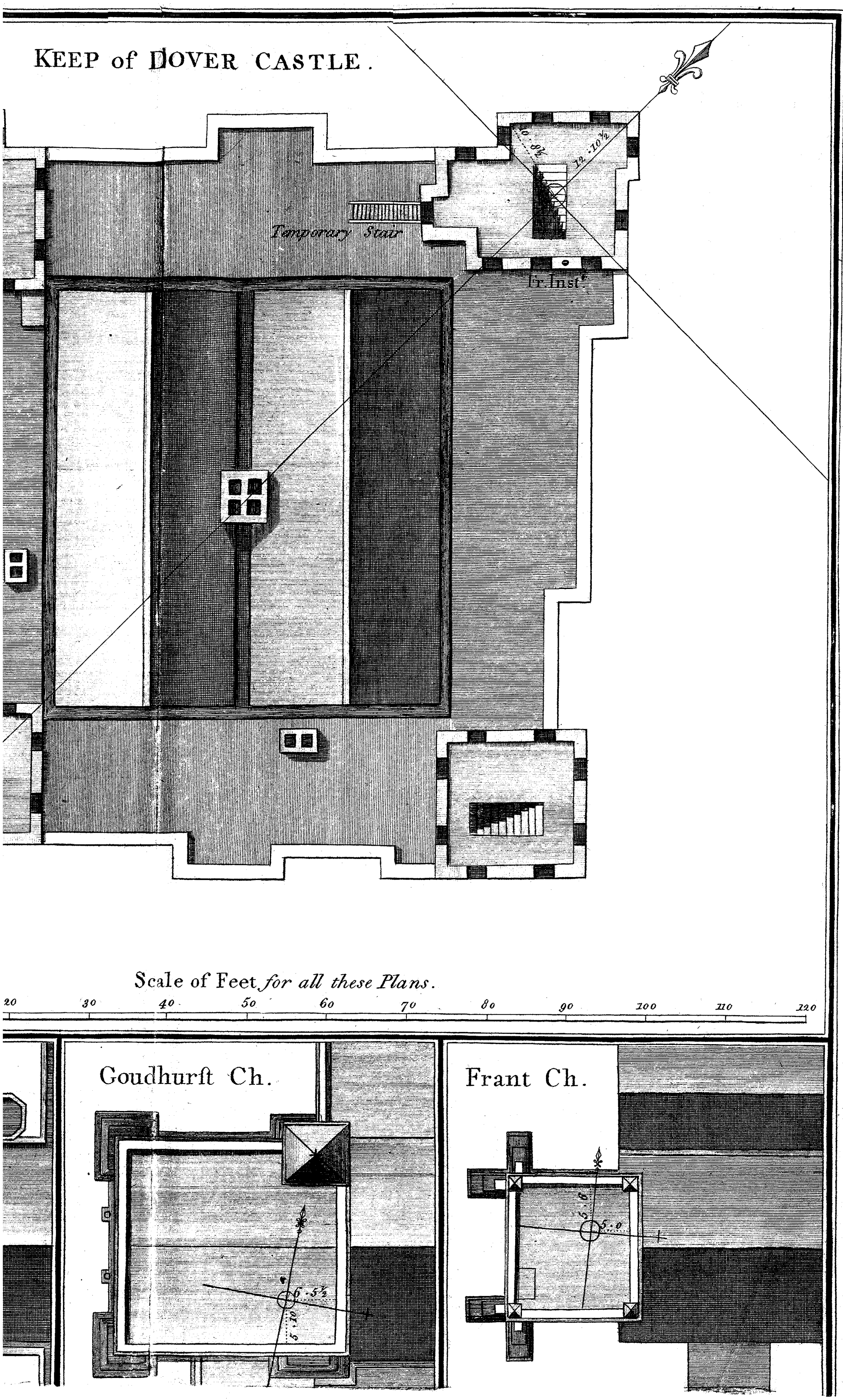





*Basire Sc.*

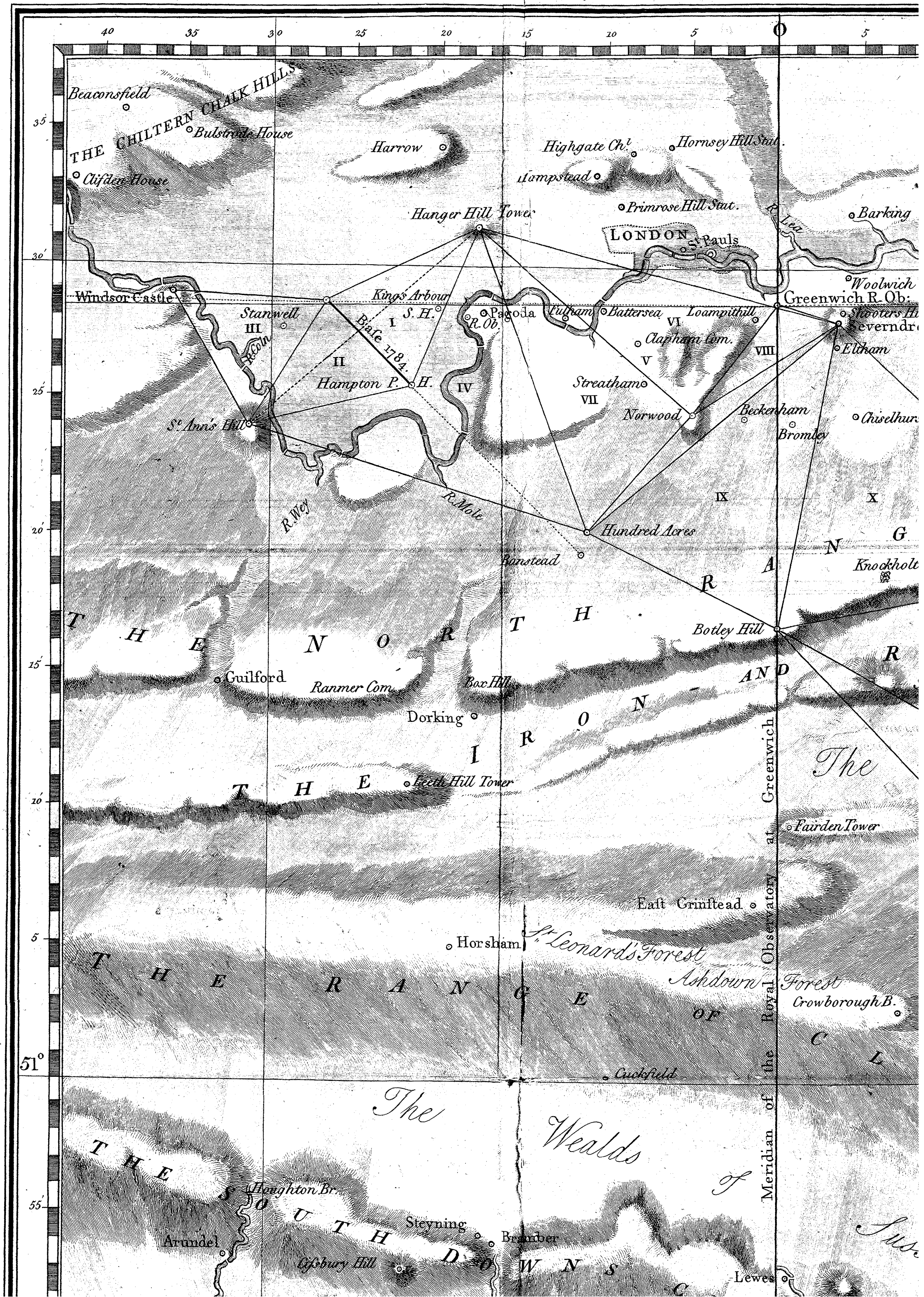

GREENWIC
ROYAL OBSER
7
6
10
9
8
5
11
12
11
2
3
1
4
13
13
REFERENCES.
1. Tranfit Room, *over which a temporary Scaffold was placed for the Instrument.*
2. Quadrant-Room.
3. Afsistant's-Room.
4. Telescope-Room.
5. Platform *over the Octagon-Room.*
6. Eaft Equatorial.
7. Weft Equatorial.
8. Wind Vane *Turret.*
9. Camera *Turret.*
10. Library.
11. Apartments, *in the Dwelling House.*
12. Kitchin.
13. Out Houses.


ENWICH
OBSERVATORY.
Hanger Hill Tower.
Severndroog Castle, *Shooter's Hill*
Swingfield Ch.
Lydd Ch.
Tenterden Ch.
KEE
Wind Vane
Flagstaff

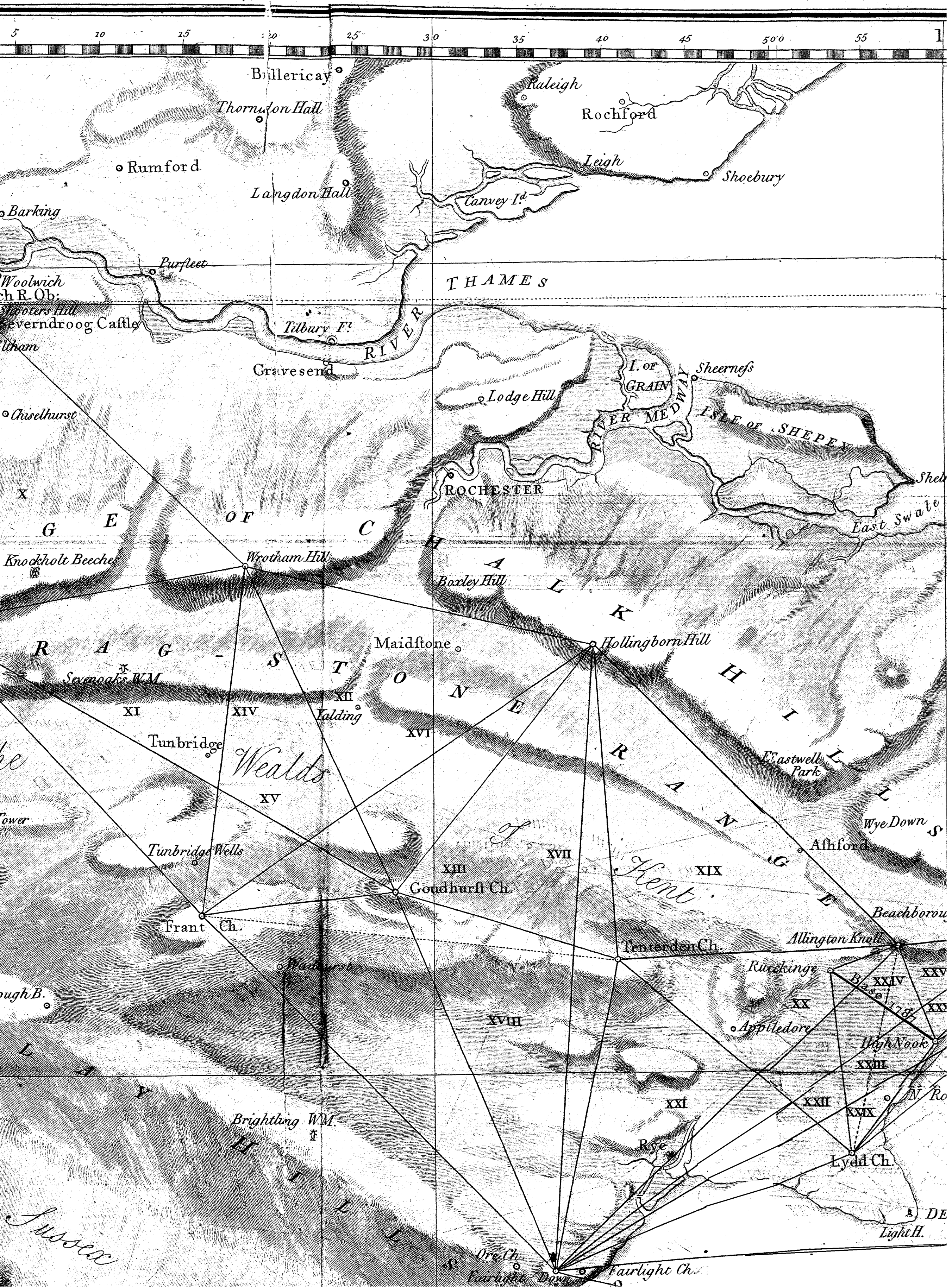

KEEP of DOVER CASTLE.
Temporary Stair
Fr. Inst.
12 . 10½
0 . 8½
Scale of Feet for all these Plans.
20 30 40 50 60 70 80 90 100 110 120
Goudhurst Ch.
6 . 5½
Frant Ch.
5 . 8
5 . 0

Pl. IX.

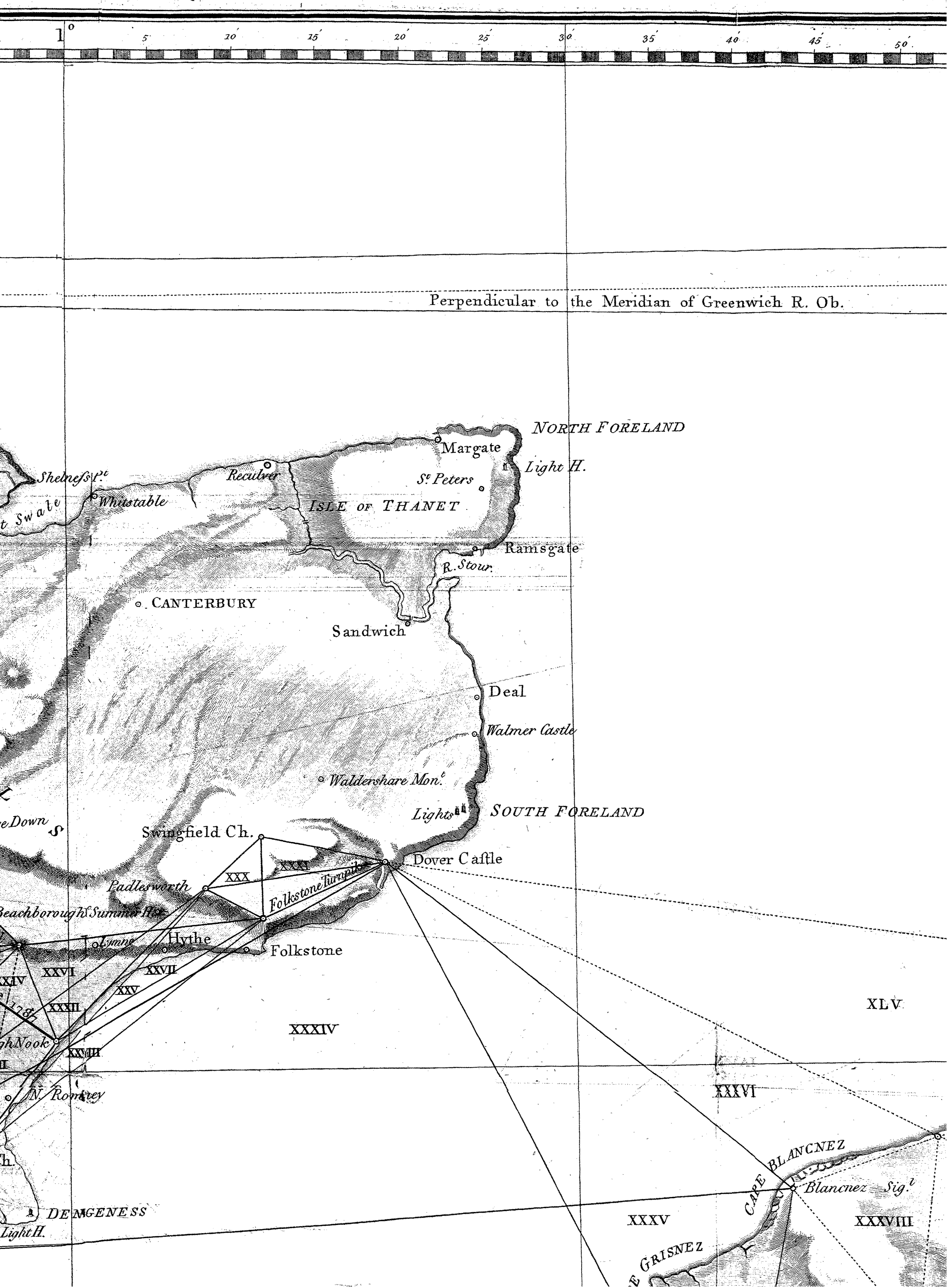
# PLAN of the TRIANGLE



GLES whereby the DISTANCE between the ROYAL O

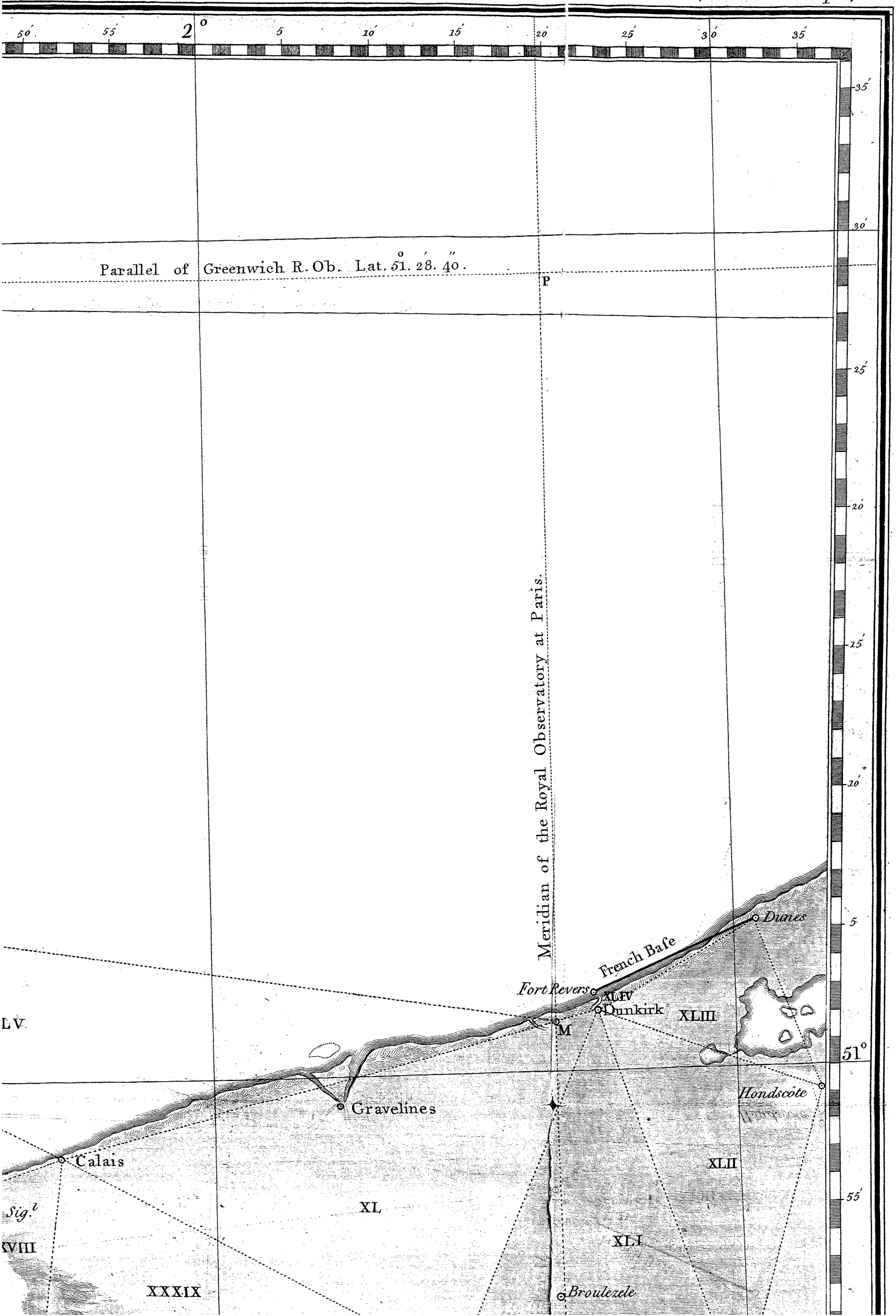


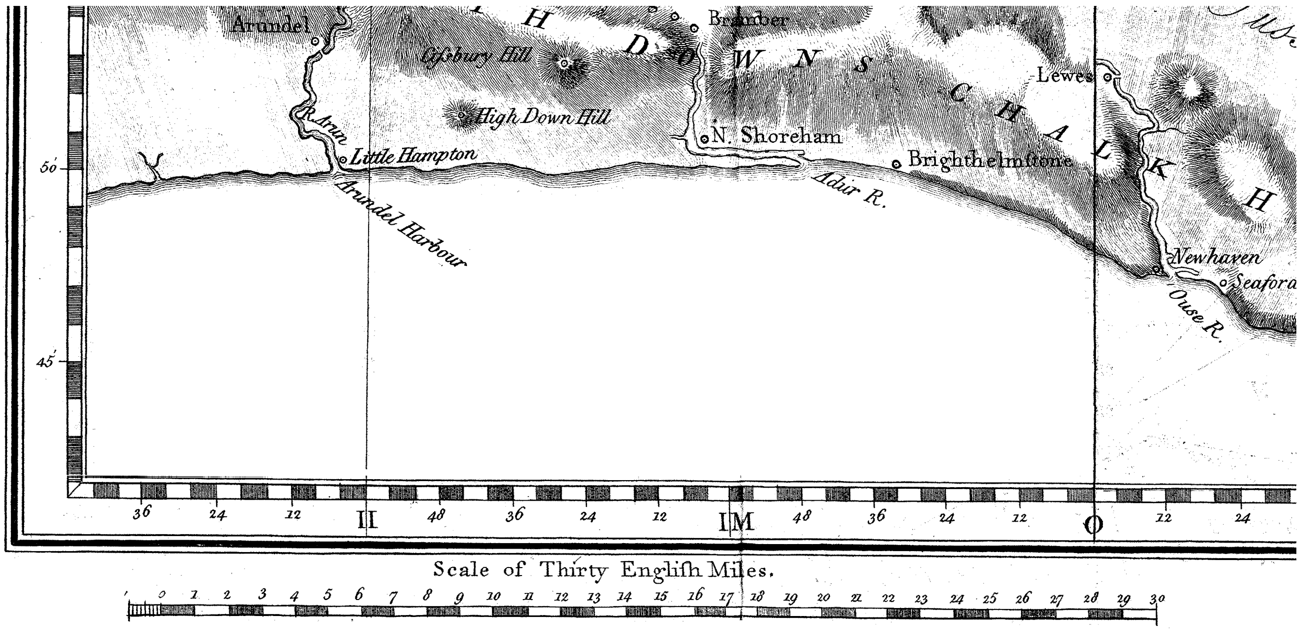
L OBSERVATORIES of GREENWICH and PARIS



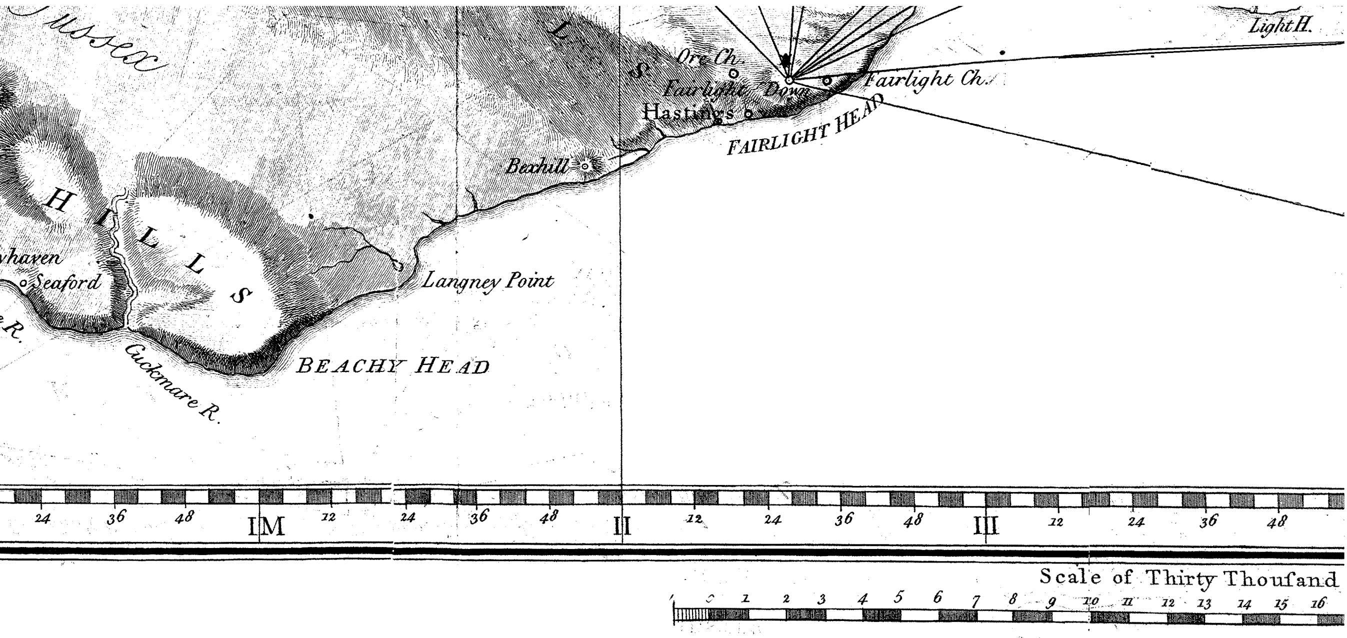
IS has been determined

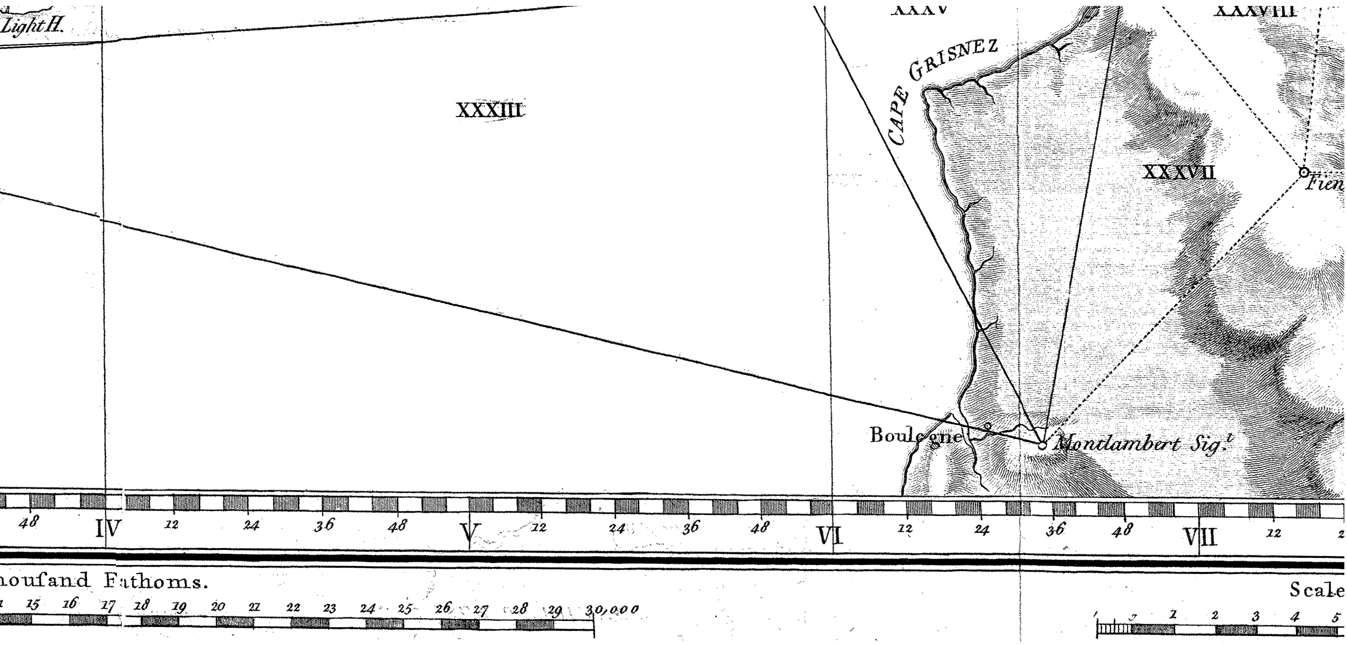




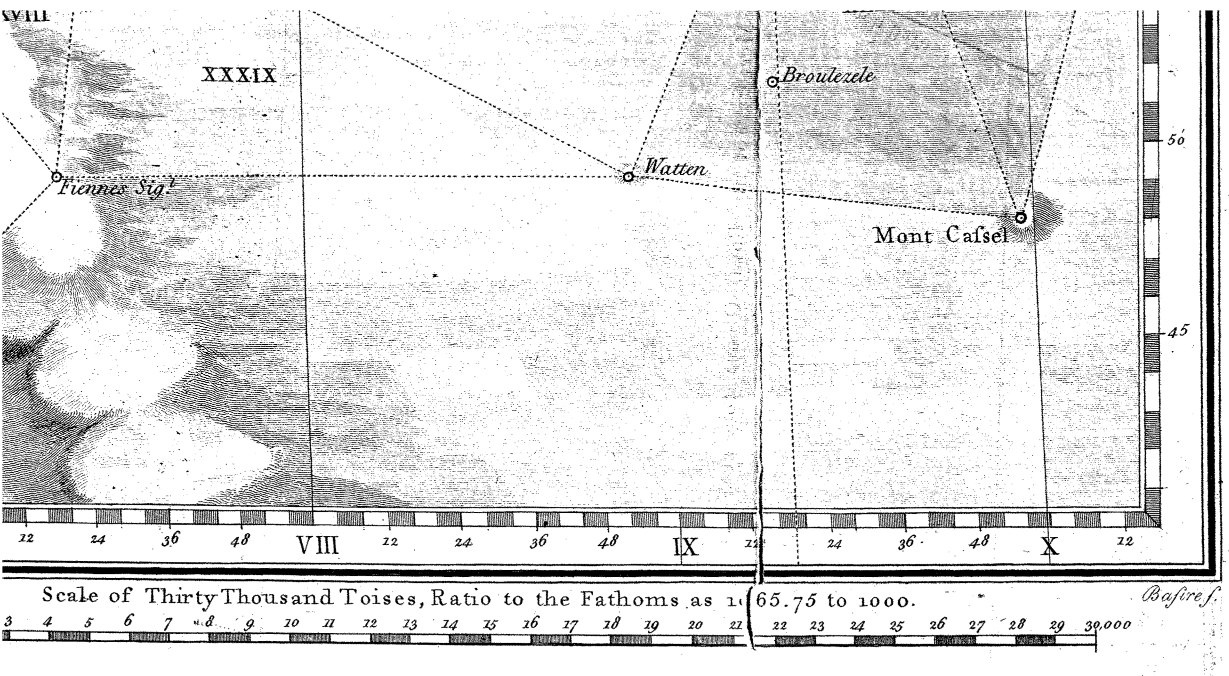


Arundel
Bramber
Gissbury Hill
High Down Hill
D O W N S
Lewes
C H A L K H
R. Arun
Little Hampton
N. Shoreham
Brighthelmstone
Adur R.
Arundel Harbour
Newhaven
Seaford
Ouse R.
50'
45'
36
24
12
II
48
36
24
12
IM
48
36
24
12
O
12
24
Scale of Thirty English Miles.
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30


Sussex
Light H.
Ore Ch.
Fairlight Down
Fairlight Ch.
Hastings
FAIRLIGHT HEAD
Bexhill
HILLS
Seaford
Langney Point
BEACHY HEAD
Cuckmare R.
24 36 48 I M 12 24 36 48 II 12 24 36 48 III 12 24 36 48
Scale of Thirty Thousand
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16



Light H.
XXXIII
XXXV
CAPE GRISNEZ
XXXVII
XXXVIII
Boulogne
Montlambert Sig.t
48
IV
12
24
36
48
V
12
24
36
48
VI
12
24
36
48
VII
12
Thousand Fathoms.
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30,000
Scale
1
2
3
4
5


XXXIX
Broulezele
Watten
Fiennes Sig.l
Mont Caſsel
50
45
12 24 36 48 VIII 12 24 36 48 IX 12 24 36 48 X 12
Scale of Thirty Thousand Toises, Ratio to the Fathoms as 1065.75 to 1000.
3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30,000
Baſire ſ.

# *For the Figure of the Earth, Pole-Star Observations*

Fig. 1.

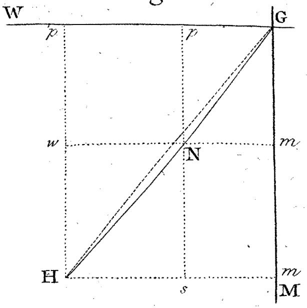



Fig. 2.

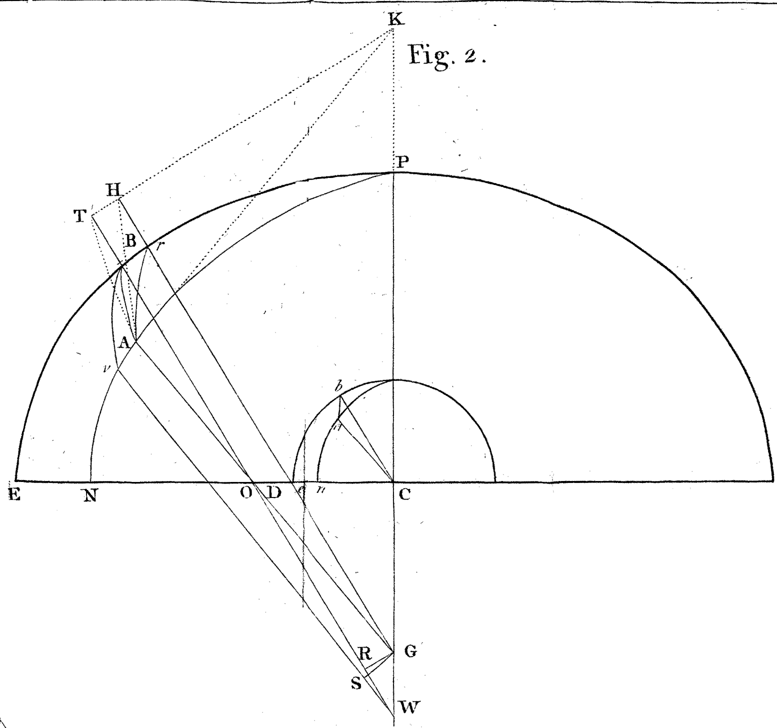



Fig. 6.

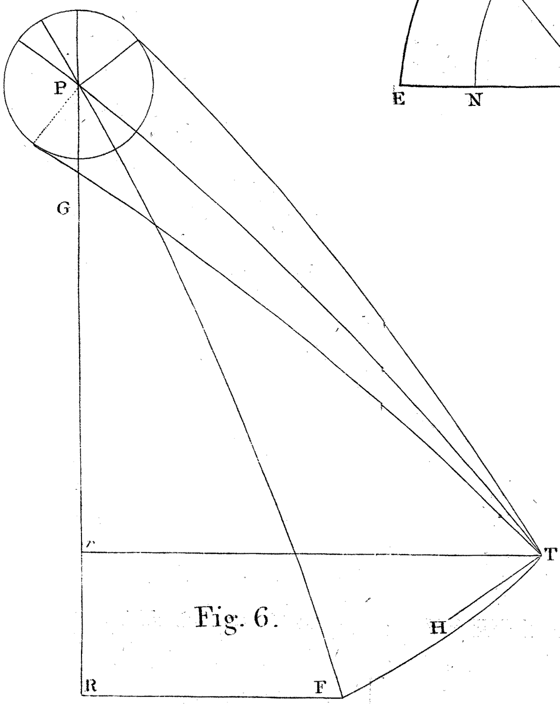
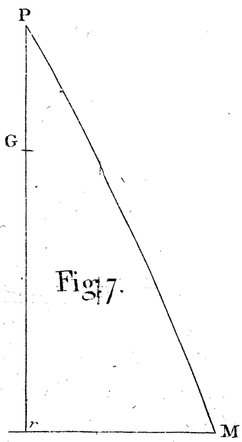



Fig. 7.



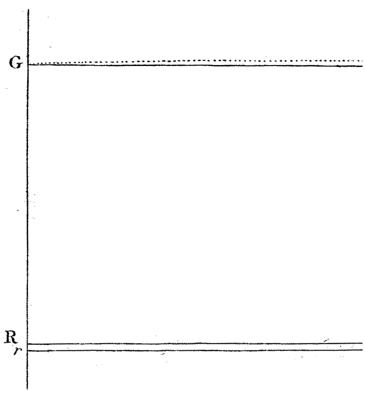




Fig. 11.

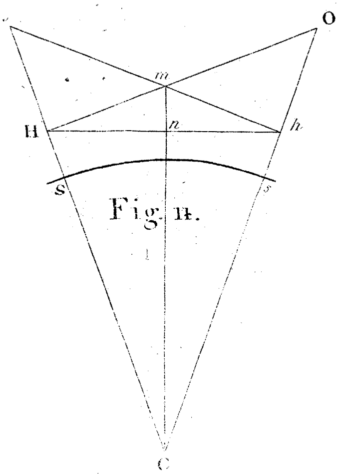



Fig. 12.

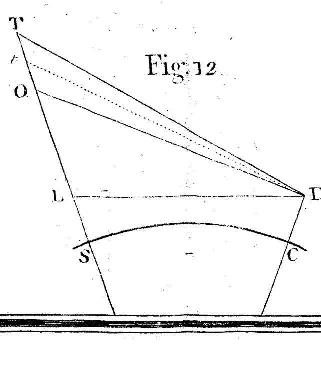



Fig. 1

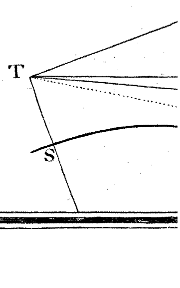


Fig. 3.

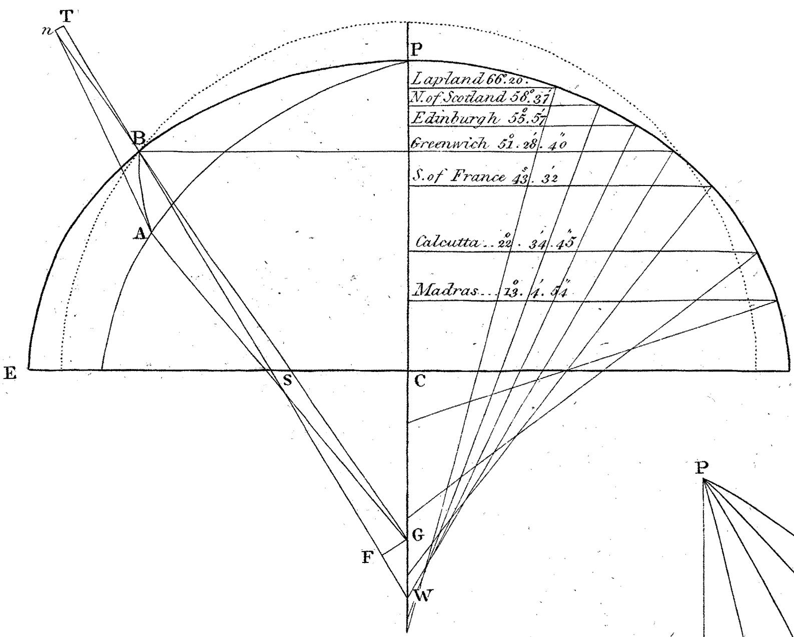
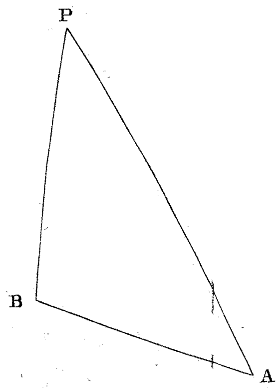



Fig. 4.



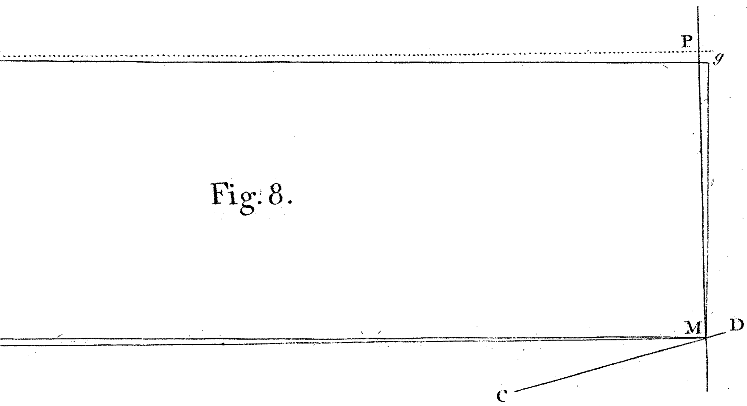




Fig. 8.

Fig. 9.

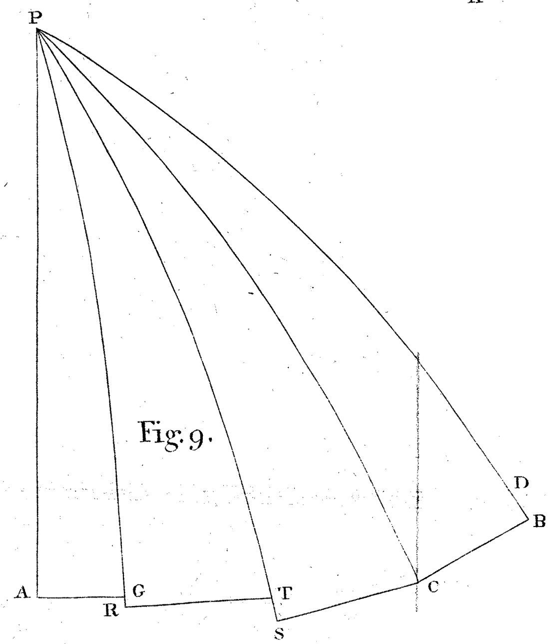



Fig. 13.

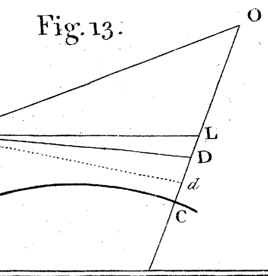



Fig. 14.

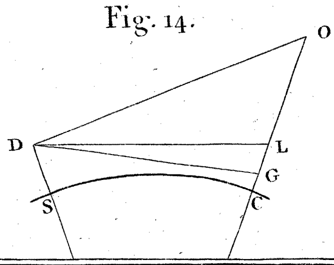



Fig. 15.

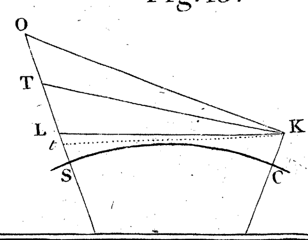
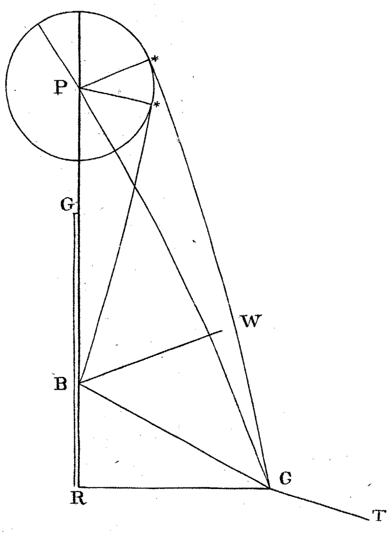


Fig. 5.





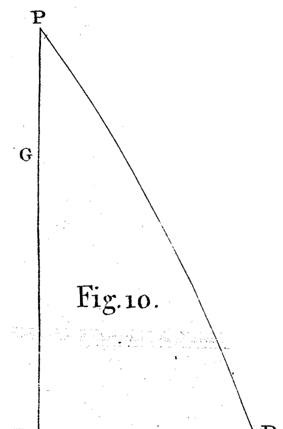



Fig. 10.

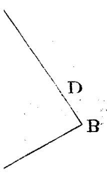
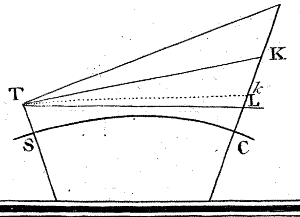


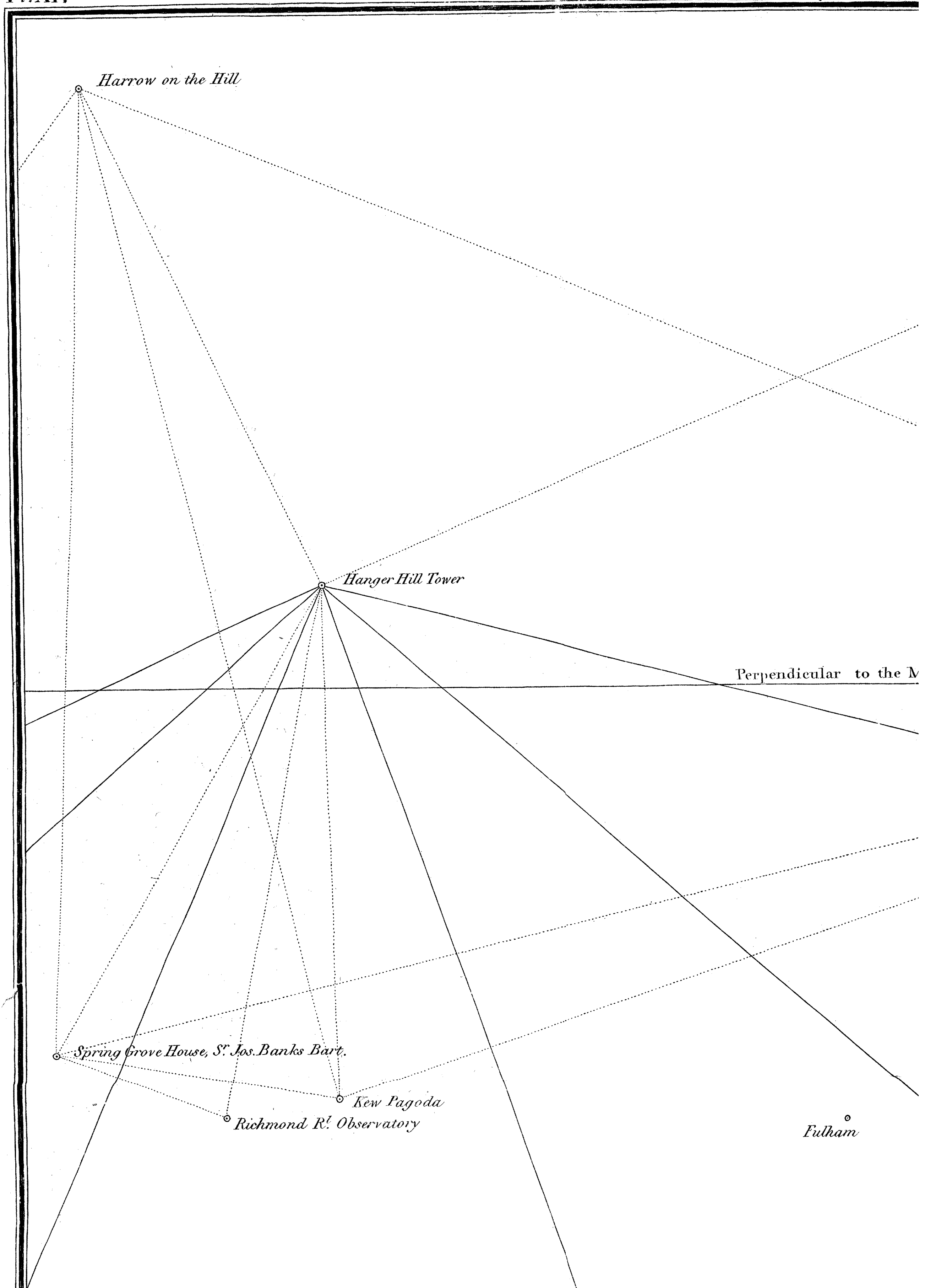

Fig. 16.



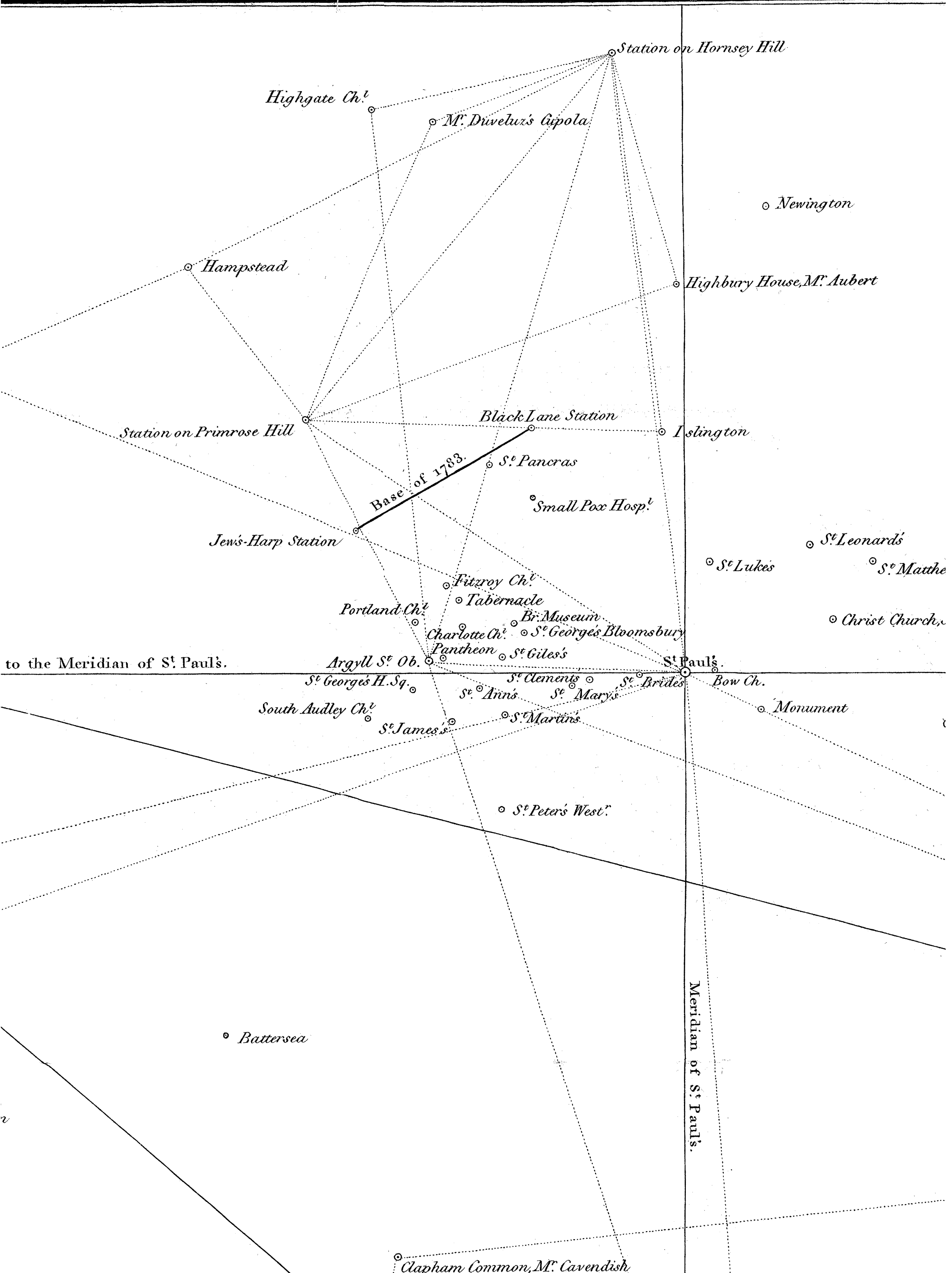

Basire Sc.

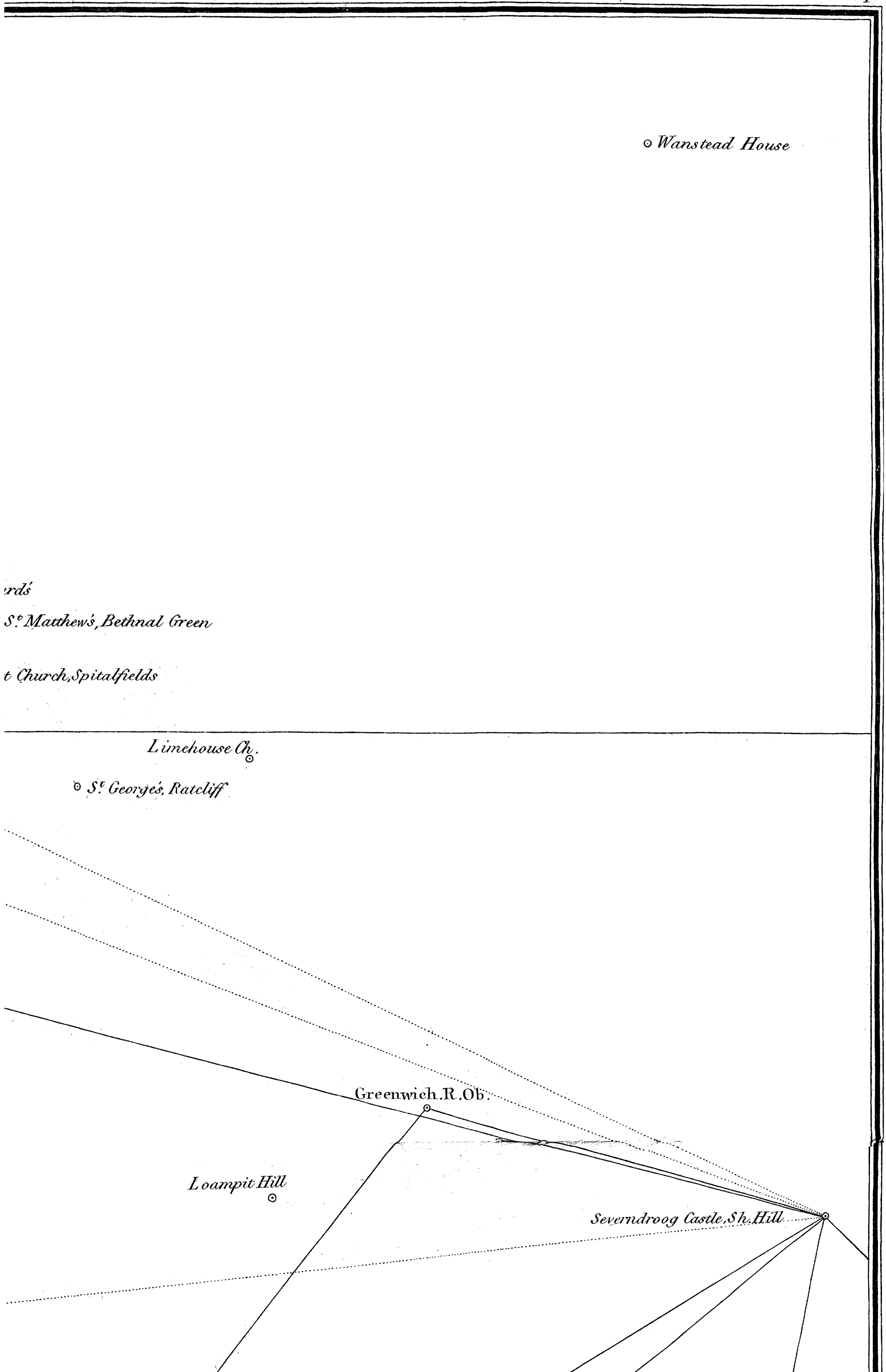
Pl.XI.

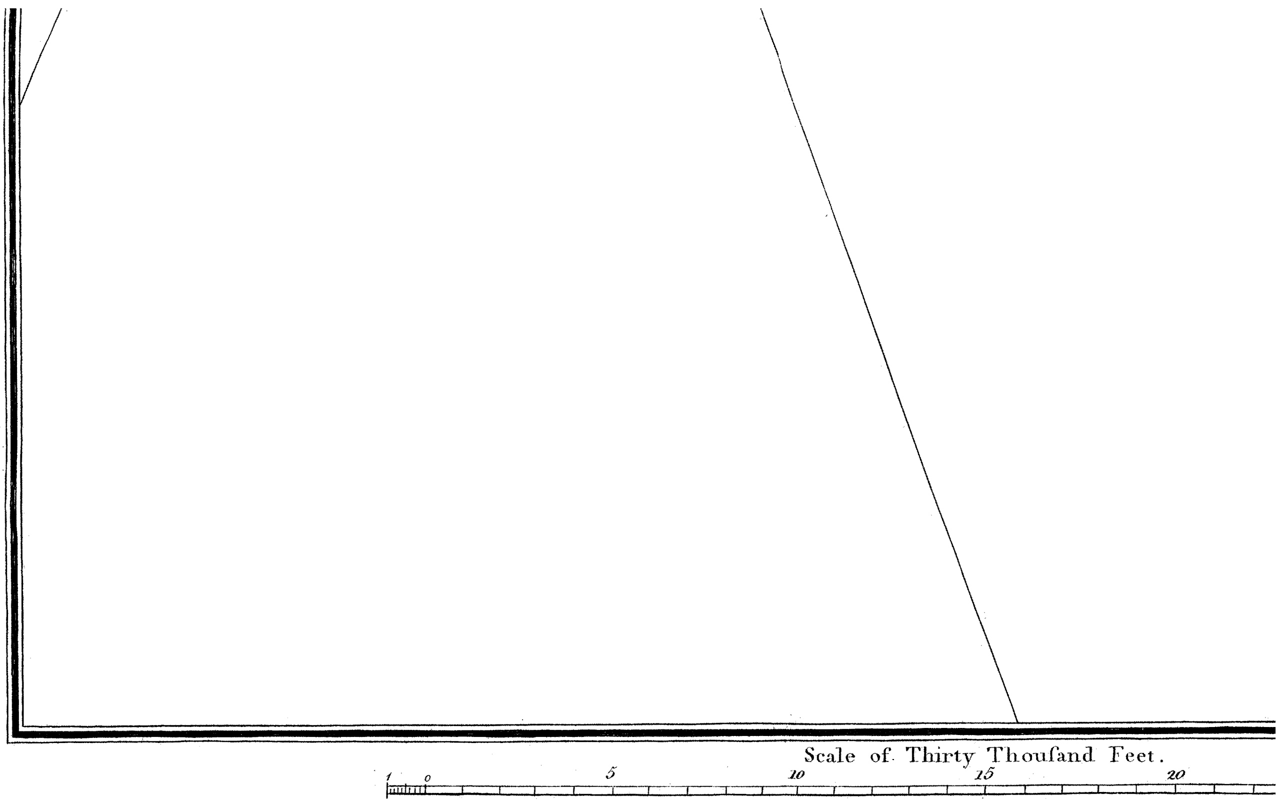
FOR THE IM

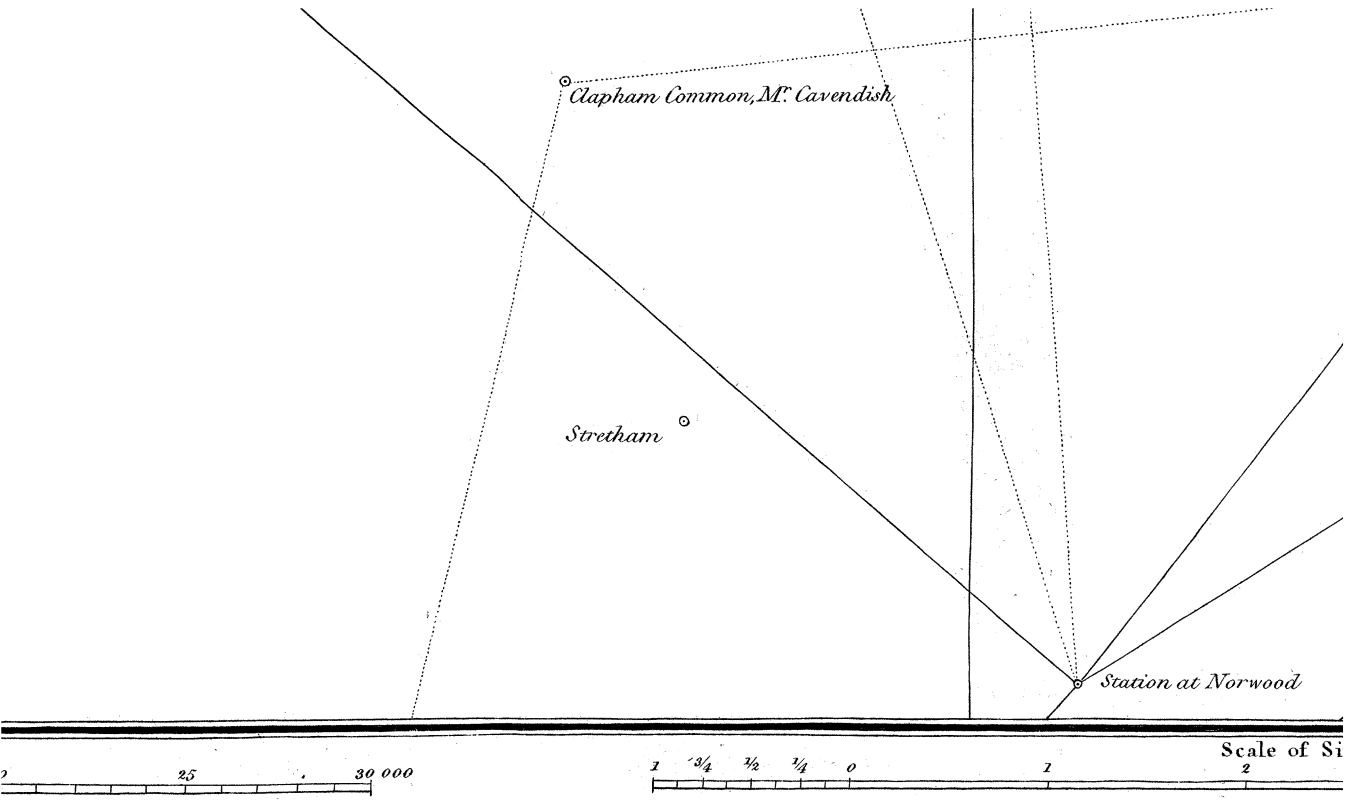


IMPROVEMENT OF THE PLAN OF LONDON and its ENVIRON




Wanstead House
rds
S.t Matthew's, Bethnal Green
t Church, Spitalfields
Limehouse Ch.
S.t George's, Ratcliff
Greenwich R. Ob.
Loampit Hill
Severndroog Castle, Sh. Hill


Scale of Thirty Thousand Feet.
1 0 5 10 15 20


Clapham Common, Mr. Cavendish
Stretham
Station at Norwood
Scale of Si
25
30 000
1
3/4
1/2
1/4
0
1
2


Eltham
wood
ale of Six English Miles.
3
4
5
6

GENERAL TABLE of the Measurement of the BASE of VERIFICATION in ROMNEY MARSH, executed in the Autumn of 1787, whereby the apparent Length is found to be $9512\frac{2454}{10000}$ Yards, and the true, or corrected Length in the Temperature of 62°, $28532\frac{92}{100}$ Feet.

| Days. | Spaces measured. Yards. | Temperature. Mean by 15Therm. | Temperature. diff. from 62°. | Correction for the difference. |
|---|---|---|---|---|
| Oct | | ° | ° | In. Parts. |
| 15 | 100 | 54.7 | − 7.3 | 0.16710 |
| 16 | 200 | 62.7 | + 0.7 | +0.01602 |
| | 300 | 61.3 | − 0.7 | 0.01602 |
| 17 | 400 | 57.0 | − 5.0 | 0.11445 |
| | 500 | 52.2 | − 9.8 | 0.22432 |
| | 600 | 53.6 | − 8.4 | 0.19228 |
| 20 | 700 | 46.8 | −15.2 | 0.34793 |
| | 800 | 58.9 | − 3.1 | 0.07096 |
| 23 | 900 | 53.9 | − 8.1 | 0.18541 |
| | 1000 | 55.3 | − 6.7 | 0.15336 |
| 24 | 1100 | 55.7 | − 6.3 | 0.14421 |
| | 1200 | 50.0 | −12.0 | 0.27468 |
| | 1300 | 55.2 | − 6.8 | 0.15565 |
| 26 | 1400 | 59.1 | − 2.9 | 0.06638 |
| | 1500 | 60.0 | − 2.0 | 0.04578 |
| 27 | 1600 | 59.1 | − 2.9 | 0.06638 |
| | 1700 | 63.1 | + 1.1 | +0.02518 |
| | 1800 | 68.1 | + 6.1 | +0.13963 |
| | 1900 | 57.9 | − 4.1 | 0.09385 |
| 29 | 2000 | 60.8 | − 1.2 | 0.02747 |
| | | | | −2.16540 |

| Days. | Spaces measured. Yards. | Temperature. Mean by 15Therm. | Temperature. diff. from 62°. | Correction for the difference. |
|---|---|---|---|---|
| Oct. | | ° | ° | In. Parts. |
| | 2100 | 65.0 | + 3.0 | +0.06867 |
| | 2200 | 64.1 | + 2.1 | +0.04807 |
| | 2300 | 56.7 | − 5.3 | 0.12132 |
| 30 | 2400 | 58.7 | − 3.3 | 0.07554 |
| | 2500 | 59.5 | − 2.5 | 0.05722 |
| 31 | 2600 | 57.3 | − 4.7 | 0.10758 |
| | 2700 | 54.6 | − 7.4 | 0.16939 |
| Nov. | 2800 | 53.9 | − 8.1 | 0.18541 |
| 1 | 2900 | 49.0 | −13.0 | 0.29757 |
| | 3000 | 54.0 | − 8.0 | 0.18312 |
| | 3100 | 50.9 | −11.1 | 0.25408 |
| 2 | 3200 | 49.1 | −12.9 | 0.29528 |
| | 3300 | 50.4 | −11.6 | 0.26552 |
| | 3400 | 48.5 | −13.5 | 0.30901 |
| | 3500 | 42.6 | −19.4 | 0.44407 |
| 5 | 3600 | 52.3 | − 9.7 | 0.22203 |
| | 3700 | 53.0 | − 9.0 | 0.20601 |
| | 3800 | 52.4 | − 9.6 | 0.21974 |
| | 3900 | 47.3 | −14.7 | 0.33648 |
| 7 | 4000 | 55.6 | − 6.4 | 0.14650 |
| | | | | −3.77913 |

| Days. | Spaces measured. Yards. | Temperature. Mean by 15Therm. | Temperature. diff. from 62°. | Correction for the difference. |
|---|---|---|---|---|
| Nov. | | ° | ° | In. Parts. |
| | 4100 | 55.2 | − 6.8 | 0.15565 |
| 10 | 4200 | 55.3 | − 6.7 | 0.15336 |
| | 4300 | 53.6 | − 8.4 | 0.19228 |
| | 4400 | 49.0 | −13.0 | 0.29757 |
| 12 | 4500 | 50.1 | −11.9 | 0.27239 |
| | 4600 | 47.9 | −14.1 | 0.32275 |
| 13 | 4700 | 44.7 | −17.3 | 0.39600 |
| | 4800 | 44.8 | −17.2 | 0.39371 |
| | 4900 | 41.3 | −20.7 | 0.47382 |
| 14 | 5000 | 41.8 | −20.2 | 0.46238 |
| | 5100 | 42.9 | −19.1 | 0.43720 |
| 15 | 5200 | 45.3 | −16.7 | 0.38226 |
| | 5300 | 44.1 | −17.9 | 0.40973 |
| | 5400 | 40.4 | −21.6 | 0.49442 |
| 16 | 5500 | 41.5 | −20.5 | 0.46924 |
| | 5600 | 44.8 | −17.2 | 0.39371 |
| | 5700 | 44.6 | −17.4 | 0.39829 |
| 17 | 5800 | 40.6 | −21.4 | 0.48985 |
| | 5900 | 39.4 | −22.6 | 0.51731 |
| | 6000 | 41.3 | 20.7 | 0.47382 |
| | | | | −7.58574 |

| Days. | Spaces measured. Yards. | Temperature. Mean by 15Therm. | Temperature. diff. from 62°. | Correction for the difference. |
|---|---|---|---|---|
| | | ° | ° | In. Parts. |
| | 6100 | 42.1 | −19.9 | 0.45551 |
| Nov. | 6200 | 39.3 | −22.7 | 0.51960 |
| 21 | 6300 | 43.3 | −18.7 | 0.42804 |
| | 6400 | 46.5 | −15.5 | 0.35479 |
| | 6500 | 45.6 | −16.4 | 0.37540 |
| 22 | 6600 | 42.5 | −19.5 | 0.44635 |
| | 6700 | 42.2 | −19.8 | 0.45322 |
| | 6800 | 41.2 | −20.8 | 0.47611 |
| 23 | 6900 | 39.8 | −22.2 | 0.50816 |
| | 7000 | 39.0 | −23.0 | 0.52647 |
| | 7100 | 37.7 | −24.3 | 0.55623 |
| 24 | 7200 | 36.2 | −25.8 | 0.59056 |
| | 7300 | 42.1 | −19.9 | 0.45551 |
| | 7400 | 40.5 | −21.5 | 0.49213 |
| 26 | 7500 | 35.2 | −26.8 | 0.61345 |
| | 7600 | 39.8 | −22.2 | 0.50816 |
| | 7700 | 38.5 | −23.5 | 0.53791 |
| 27 | 7800 | 33.6 | −28.4 | 0.65098 |
| | 7900 | 38.9 | −23.1 | 0.52876 |
| 28 | 8000 | 32.7 | −29.3 | 0.67068 |
| | | | | −10.14802 |

| Days. | Spaces measured. Yards. | Temperature. Mean by 15 Therm. | Temperature. diff. from 62°. | Correction for the difference. |
|---|---|---|---|---|
| | | ° | ° | In. Parts. |
| | 8100 | 36.3 | −25.7 | 0.58827 |
| | 8200 | 39.2 | −22.8 | 0.52189 |
| Nov. | 8300 | 39.5 | −22.5 | 0.51502 |
| 29 | 8400 | 34.8 | −27.2 | 0.62261 |
| | 8500 | 40.5 | −21.5 | 0.49213 |
| | 8600 | 42.3 | −19.7 | 0.45093 |
| 30 | 8700 | 30.5 | −31.5 | 0.72103 |
| | 8800 | 44.3 | −17.7 | 0.40515 |
| Dec. | 8900 | 46.0 | −16.0 | 0.36624 |
| 1 | 9000 | 43.0 | −19.0 | 0.43491 |
| | 9100 | 45.6 | −16.4 | 0.37540 |
| | 9200 | 48.7 | −13.3 | 0.30444 |
| 3 | 9300 | 41.1 | −20.9 | 0.47840 |
| | 9400 | 46.9 | −15.1 | 0.34564 |
| 4 | 9512.2454 | 48.4 | −13.6 | 0.34942 |
| | | | | − 6.97148 |
| | | | | −10.14802 |
| | | | | − 7.58574 |
| | | | | − 3.77913 |
| | | | | − 2.16540 |
| | | Total Correction | | −30.64977 |

| | Feet. | In. |
|---|---|---|
| From the above Table it appears, that the total apparent length of the Base, as given immediately by the Steel Chain, was $9512\frac{2454}{10000}$ yards, which are equal to | 28536 | 8.835 |
| The corrections in the above Table, and others specified in the Text, being subtracted from the apparent length, — — — | 3 | 9.799 |
| There remains, for the true length of the base in the temperature of 62° of Fahrenheit, — — — — — — | 28532 | 11.036 |
| equal to — | $28532\frac{92}{100}$ Feet. | |

SECTION

TABLE containing the GENERAL RESULTS of the TRIGONOMETRICAL OPERATION.

| | Stations. | By Plane Trigonometry. Distances in Feet. from the Merid. of Greenwich. | By Plane Trigonometry. Distances in Feet. from the Perp. to the Merid. | By Plane Trigonometry. Bearing or Angle with the Meridian. | | By Plane Trigonometry. Direct Dist. from Greenwich. | Latitude. | Longitude. in Degrees, &c. | Longitude. in Time. | Vertical Heights. Ground above the Sea. | Vertical Heights. Telescope above Ground. | Vertical Heights. Total. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | ° ′ ″ | | Feet. | ° ′ ″ | ° ′ ″ | m. s. th. | Feet. | Feet. | Feet. |
| | Greenwich R. Ob. Transit Room - - | . . . . | . . . . | . . . . | . . | . . . | 51.28.40 | | | 170.5 | 43.5 | 214 |
| West from Greenwich | Norwood - - - - - | 19306.54 | 24603.86 | 38. 7.16 | SW | 31274.48 | 51.24.37.34 | 0. 5. 2.4 | 0.20. 9.6 | 380.3 | 9.2 | 389.5 |
| | Hundred Acres - - - - | 43333.9 | 50937.9 | 40.23.18.5 | SW | 66876.73 | 51.20.17.3 | 0.11.19.5 | 0.45.18. | 433.8 | 9.2 | 443 |
| | Hanger Hill Tower - - - | 67740.69 | 16729.21 | 76. 7 40.2 | NW | 69775.8 | 51.31.24.16 | 0.17 46.5 | 1.11. 6. | 213. | 38. | 251 |
| | Hampton Poor-house - - - | 83086.16 | 18537.98 | 77.25.20.3 | SW | 85129.1 | 51.25.35.2 | 0.21.45.3 | 1.27. 1.2 | 63.5 | 37.5 | 101 |
| | King's Arbour - - - - | 102264.55 | 1037.69 | 89.25. 7.7 | NW | 102269.8 | 51.28 47.16 | 0.26.48.5 | 1.47.14. | 94.8 | 37.5 | 132.3 |
| | St. Ann's Hill - - - - | 119404.04 | 28852.77 | 76.24.55.7 | SW | 122840.6 | 51.23 51.4 | 0.31.14.7 | 2. 4.58.8 | 321.3 | 20.7 | 342 |
| | Wardrobe Tower of Windsor Castle - - | 137050.54 | 2562.72 | 88.55.43.5 | NW | 137074.5 | 51.28.59.7 | 0.35.55.8 | 2.23.43.2 | | | |
| East from Greenwich | Botley Hill - - - - | 171.6 | 72882.5 | 0. 8. 5.6 | SE | 72882.7 | 51.16.41.54 | 0. 0 2.7 | 0. 0.10.8 | 859.3 | 20.7 | 880 |
| | Severndroog Castle on Shooter's Hill - - | 14032.3 | 4069.8 | 73.49.34 | SE | 14610.58 | 51.27 59.8 | 0. 3.40.65 | 0.14.42.6 | 417.8 | 64.2 | 482 |
| | Frant - - - - - - | 62342.5 | 138460.4 | 24.14.23.6 | SE | 151848.2 | 51. 5.53.9 | 0.16.12.5 | 1. 4.50. | 604.1 | 54.9 | 659 |
| | Wrotham Hill - - - - - | 71850.3 | 59305.8 | 50.27.48.5 | SE | 93164.6 | 51.18.53.86 | 0.19.11.3 | 1.16.49.2 | 761.8 | 9.2 | 771 |
| | Goudhurst - - - - - | 106342.5 | 132592. | 38 43.50.1 | SE | 169968.8 | 51. 6.49.62 | 0.27.39.45 | 1.50.37.8 | 437.9 | 59.1 | 497 |
| | Fairlight Down - - - - | 143308.1 | 218611.6 | 33.14.46.8 | SE | 261396.7 | 50.52.38.84 | 0.37. 5. | 2.28.20. | 539.5 | 5.5 | 599 |
| | Hollingborn Hill - - - | 151078.1 | 77077.6 | 62.58.12.2 | SE | 169604.1 | 51.15.53.5 | 0.39.25.2 | 2.37.40.8 | 611 | 5.5 | 616.5 |
| | Tenterden - - - - - | 158317.6 | 148567. | 46.49.11.4 | SE | 217109.7 | 51. 4. 8.15 | 0.41. 8.15 | 2.44.32.6 | 220.6 | 101.7 | 322.3 |
| | Ruckinge - - - - - - | 204801.6 | 149410.3 | 53.53.16.3 | SE | 253509.6 | 51. 3.54.9 | 0.53.12.6 | 3.32.50.4 | 31.9 | 5.5 | 37.4 |
| | Lydd - - - - - | 209339.3 | 190695.6 | 47.40.6 | SE | 283174.5 | 50.57. 7.4 | 0.54 15.4 | 3.37. 1.6 | 31.7 | 98.7 | 130.4 |
| | Allington Knoll - - - | 219926.3 | 144032.3 | 56.46.43.5 | SE | 262893.3 | 51. 4.46.1 | 0.57. 9.4 | 3.48.37.6 | 323.5 | 5.5 | 329 |
| | High Nook near Dymchurch - - - | 228246.4 | 165672.9 | 54. 1.33.4 | SE | 282035.3 | 51. 1.11.7 | 0.59.14.6 | 3.56.58.4 | 22.1 | 5.5 | 27.6 |
| | Padlesworth - - - - | 261707.6 | 130836.4 | 63.26.16.8 | SE | 292590.2 | 51. 6.50.36 | 1. 8. 9. | 4.32.36. | 621.3 | 20.7 | 642 |
| | Swingfield - - - - - | 273722.7 | 118730.9 | 66.33.2 | SE | 298364.1 | 51. 8.47.8 | 1.11.14.6 | 4.44.58.4 | 473.1 | 56.9 | 530 |
| | Folkstone Turnpike - - - - | 274967.3 | 137214.2 | 63.28.47.6 | SE | 307302.3 | 51. 5.45.3 | 1.11.29.3 | 4.45.57.2 | 569.8 | 5.5 | 575.3 |
| | Dover Castle, north Turret of the Keep - | 303766.8 | 124319.1 | 67.44.34 | SE | 328222 | 51. 7.47.7 | 1.19. 2.1 | 5.16. 8.4 | 373.9 | 95.1 | 409 |
| | Montlambert near Boulogne - - | 382889.6 | 273450.3 | 54.27.59.4 | SE | 470509.8 | 50.43. 2.3 | 1.38.45. | 6.35. 0. | 639.5 | | |
| | Blancnez - - - - - | 394891.7 | 197153.5 | 63.28.8 | SE | 441371.7 | 50.55.31.3 | 1.42 17.9 | 6.49.11.6 | 444.5 | | |
| | N. D. at Calais - - - - | 427456.6 | 184263.3 | 66.40.50.2 | SE | 465480.5 | 50.57.30.67 | 1.50.48.8 | 7.23.15.8 | 140.5 | | |
| | Point M near Dunkirk, Merid. R. Ob. Paris | 538048.2 | 154938.2 | 73.56. 7.2 | SE | 559912.8 | 51. 1.48.3 | 2.19.42.5 | 9.18.50. | | | |
| | Dunkirk - - - - - | 547058.1 | 152549.1 | 74.25. 7.2 | SE | 567929.4 | 51. 2. 9.3 | 2.22. 4. | 9.28.16. | | | |

SECTION

| | ° | ′ | ″ | ‴ | | ° | ′ | ″ | ‴ | ° | Pts. | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 Greenwich Rl. Ob. | 51 | 28 | 40 | 00 | } | 8 | 46 | 48 | 00 | 8. | 78 | | 534343. | | | 535848. | | | 535758. | | | 535024. | | | 534909. | | | 533880. | | | 533801. | | | 533734. | | | 534928. | | | 533652.4 | | |
| Perpignan - - | 42 | 41 | 52 | 00 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Greenwich and M | MP | — | — | — | diff. | 0 | 26 | 50 | 52 | 0. | 4475 | | 27235. | | | 27342. | | | 27337. | | | 27294. | | | 27288. | | | 27234. | | | 27224. | | | 27212. | | | 27288. | | | 27234. | | |
| 2 Point M near Dunkirk | 51 | 1 | 49 | 8 | } | 8 | 19 | 57 | 8 | 8. | 3325 | 506687 | 507108. | +521 | | 508506. | +1819 | | 508421. | +1734 | | 507730. | +1043 | | 507621. | +934 | | 506646. | −41 | | 506577. | −110 | | 506522. | −165 | | 507640. | +953 | | 506418.4 | −268.6 | |
| Perpignan - - | 42 | 41 | 52 | 00 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Point M and Paris - | | | | | diff. | 2 | 11 | 35 | 8 | 2. | 1931 | 133409 | 133467. | + 58 | | 133957. | + 584 | | 133932. | + 523 | | 133732. | + 323 | | 133700. | +291 | | 133440. | +31 | | 133398. | − 11 | | 133354. | − 55 | | 133705. | +296 | | 133416.5 | + 7.5 | |
| 3 Paris Rl. Ob. - | 48 | 50 | 14 | 00 | } | 6 | 8 | 22 | 00 | 6. | 1394 | 373278 | 373641. | +363 | | 374549. | +1271 | | 374489. | +1211 | | 373998. | + 720 | | 373921. | +643 | | 373206. | −72 | | 373179. | − 99 | | 373168. | −110 | | 373935. | +657 | | 373001.9 | −276.1 | |
| Perpignan - - | 42 | 41 | 52 | 00 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Paris and Bourges | | | | | diff. | 1 | 45 | 9 | 19 | 1. | 7525 | 106312 | 106658. | + 46 | Mean error on each deg. + 62.53 | 106989. | + 377 | + 218.3 | 106971. | + 359 | + 208.1 | 106820. | + 208 | + 125.17 | 106796. | +184 | + 112.09 | 106590. | −22 | −4.92 | 106569. | − 43 | −13.2 | 106549. | − 63 | 9.8 | 106796. | +184 | + 114.37 | 106550.4 | − 61.6 | −32.23 |
| 4 Bourges - - | 47 | 5 | 4 | 41 | } | 4 | 23 | 12 | 41 | 4. | 3869 | 266666 | 266983. | +317 | | 267560. | + 894 | | 267518. | + 852 | | 267178. | + 512 | | 267125. | +459 | | 266616. | −50 | | 266610. | − 56 | | 266619. | − 47 | | 267139. | +473 | | 266451.5 | −214.5 | |
| Perpignan - - | 42 | 41 | 52 | 00 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Bourges and Rodés | | | | | diff. | 2 | 43 | 51 | 5 | 2. | 6309 | 165961 | 166200. | +239 | | 166599. | + 638 | | 166572. | + 611 | | 166354. | + 393 | | 166320. | +359 | | 166002. | +41 | | 165990. | + 29 | | 165986. | + 25 | | 166335. | +374 | | 165908.5 | − 52.5 | |
| 5 Rodés - - - | 44 | 21 | 13 | 36 | } | 1 | 39 | 21 | 36 | 1. | 656 | 100705 | 100783. | + 78 | | 100961. | + 256 | | 100946. | + 241 | | 100824. | + 137 | | 100805. | +100 | | 100614. | −91 | | 100620. | − 85 | | 100633. | − 72 | | 100804. | + 99 | | 100543. | −162. | |
| Perpignan - - | 42 | 41 | 52 | 00 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

| Observed Celestial Arc of the Meridian between Greenwich and Perpignan. | | | Measd. Terresl. Arc. | Computed Terrestrial Arc of the Meridian between Geenwich and Perpignan. | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Latitudes. | Differences. | | on a Sphere. | | on different Ellipsoids. | | | | | | | | | | | | | | on different Spheroids. | | | |
| | | | | Semidiam. | 3486975. | 1st R.S.D. | 179.047. to 178.047 | 2d. R.S.D. | 192.483 to 191.483 | 3d R.S.D. | 216.06 to 215.06 | 4th R.S.D | 222.55 to 221.55 | 5th R.S.D. | 230. to 229. | 6th R.S.D. | 310.3 to 309.3 | 7th R.S.D. | 540. to 539. | 1st R.S.D. | 222.55 to 221.55 | 2d R.S.D. | 179.4 to 178.4 |
| | ° ′ ″ | ° ′ ″ | Fath. | Fath. | | Fath. | | Fath. | | Fath. | | Fath. | | Fath. | | Fath. | | Fath | | Fath. | | Fath. | |
| 1 Greenwich Rl. Ob. -<br>Tower of Dunkirk - | 51. 28. 40. | 0. 26. 30.7 | 26898 | 26891 | − 7 | 27000 | +102 | 26995 | + 97 | 26953 | + 55 | 26946 | + 48 | 26893 | − 5 | 26883 | − 15 | 26872 | − 26 | 26946 | + 48 | 26893 | − 5 |
| 2 Tower of Dunkirk -<br>Paris Rl. Ob. - - | 51. 2. 9.3 | 2. 11. 55.3 | 133758 | 133811 | + 53 | 134299 | +541 | 134274 | +516 | 134073 | + 315 | 134042 | +284 | 133782 | +24 | 133739 | − 19 | 133695 | − 63 | 134047 | +289 | 133758 | 0 |
| 3 Paris Rl. Ob. - -<br>Tower of Bourges - | 48. 50. 14. | 1. 45. 11.4 | 106647 | 106696 | + 49 | 1070 24 | +377 | 107006 | +359 | 106847 | + 200 | 106831 | +184 | 106625 | −22 | 106604 | − 43 | 106584 | − 63 | 106831 | +184 | 106585 | − 62 |
| 4 Tower of Bourges -<br>N. D. at Rodés - | 47. 5. 2.6 | 2. 44. 0.2 | 166115 | 166352 | +237 | 166753 | +638 | 166727 | +612 | 166516 | + 401 | 166474 | +359 | 166156 | +41 | 166144 | + 29 | 166140 | + 25 | 166489 | +374 | 166062 | − 62 |
| 5 N. D. at Rodés -<br>St. Jaumes at Perpignan | 44. 21. 2.4<br>42. 41. 56. | 1. 39. 6.4 | 100448 | 100526 | + 78 | 100704 | +256 | 100689 | +241 | 100567 | + 119 | 100548 | +100 | 100358 | −90 | 100364 | − 84 | 100377 | − 71 | 100547 | + 99 | 100287 | −161 |
| Greenwich & Perpignan | | 8. 46. 44. | 533866 | 534276 | +410 | 535780 | +1914 | 535691 | +1825 | 534956 | +1090 | 534841 | +975 | 533814 | −52 | 533734 | −132 | 533668 | −195 | 534860 | +994 | 533585 | −281 |

The measured Arc between Paris and Dunkirk is taken a Mean between 125517½ and 125495 Toises, or 133770 and 133746 Fathoms. See Sect. VI. Art. XII.
The Latitudes of Dunkirk, Bourges, Rodés, and Perpignan, are deduced from the Celestial Arcs given by M. DE LA CAILLE, in pg. 240, 241, *Mém. de l'Acad.* 1758, and making the proper Reductions for the Distances of the Stations of the Sector at each Place, as given in the *Mérid. vérifiée*.

SECTION

## Table containing the Results of the Observations for the Effect of Terrestrial Refraction.

| Dates of the Obfervations. | Places. | Bar. | Therm. | Stations. | Height of the Telefcope above the Sea in Feet. | | Diftance of the Station in Fathoms. | Contained Arc nearly | Mean Refraction. | Proportionable Part. |
|---|---|---|---|---|---|---|---|---|---|---|
| | | In. | ° | | | | | ′ ″ | ′ ″ | |
| 1787, Oct. 21.<br>23. | Allington Knoll -<br>Ruckinge - - | 29.61<br>29.82 | 56<br>51½ | Allington Knoll and Ruckinge — — | A.<br>R. | 329.<br>37.4 | 2675.4 | 2 38 | 1 8 | 1/8 and 1/7 |
| 19. | Dymchurch Inn - | 29.9 | 55½ | High Nook and Lydd — — | H.<br>L. | 27.6<br>130.4 | 5227.1 | 5 8 | 0 55 | 1/5 and 1/6 |
| 21.<br>19. | Allington Knoll -<br>Dymchurch Inn - | 29.61<br>29.9 | 56<br>55½ | Allington Knoll and High Nook — | A.<br>H. | 329.<br>27.6 | 3864.1 | 3 48 | 0 38 | 1/6 |
| 21.<br>26. | Allington Knoll -<br>Tenterden Inn - | 29.61<br>29.54 | 56<br>56½ | Allington Knoll and Tenterden — — | A.<br>T. | 329.<br>322.3 | 10296. | 10 5 | 1 23½ | 1/7 and 1/8 |
| 7. | Padlefworth - - | 29.6 | 70 | Padlefworth and Lydd — — | P.<br>L. | 624.<br>130.4 | 13255.5 | 13 2 | 1 31 | 1/8 |
| 1788, Aug. 18.<br>23. | Frant Inn - -<br>Botley Hill - - | 29.36<br>28.89 | 58<br>62½ | Frant and Botley Hill — — — | F.<br>B. | 659.<br>880. | 15060.7 | 14 48 | 2 1 | 1/8 |
| 1787, Sept. 28.<br>Oct. 7. | Dover Caftle -<br>Padlefworth - - | 29.62<br>29.6 | 58½<br>70 | Dover Caftle and Padlefworth — | D.<br>P. | 469.<br>642. | 7093.5 | 6 57 | 0 42 | 1/10 |
| 13.<br>26. | Fairlight Down -<br>Tenterden Inn - | 28.81<br>29.54 | 55½<br>56½ | Fairlight Down and Tenterden — | F.<br>T. | 599.<br>322.3 | 11939.1 | 11 46 | 1 12 | 1/10 |
| 26. | Tenterden Inn - | 29.54 | 56½ | Tenterden and Lydd — — | T.<br>L. | 322.3<br>130.4 | 11027.8 | 10 50 | 0 55½ | 1/11 and 1/12 |
| 1788, Aug. 11.<br>18. | Goudhurft Churchyard<br>Frant Inn - - | 29.74<br>29.36 | 58½<br>58 | Goudhurft and Frant — — | G.<br>F. | 497.<br>659. | 7398.3 | 7 15 | 0 38½ | 1/11 and 1/12 |
| 1787, Oct. 13. | Fairlight Down - | 28.81 | 55½ | Fairlight Down and Lydd — — | F.<br>L. | 599.<br>130.4 | 11948.3 | 11 43 | 0 55 | 1/11 |
| 1788, Aug. 11.<br>1787, Oct. 26. | Goudhurft Churchyard<br>Tenterden Inn - | 29.74<br>29.54 | 58½<br>56½ | Goudhurft and Tenterden — — | G.<br>T. | 497.<br>322.3 | 9062.4 | 8 53 | 0 35 | 1/11 |
| 21. | Allington Knoll - | 29.61 | 56 | Allington Knoll and Lydd — — | A.<br>L. | 329.<br>130.4 | 7975. | 7 51 | 0 21 | 1/12 |
| Sept. 28.<br>1788, Sept. 2. | Dover Caftle - -<br>Folkftone Turnpike | 29.62<br>29.55 | 58½<br>64 | Dover Caftle and Folkftone Turnpike | D.<br>F. | 469.<br>575.3 | 5259.1 | 5 9.4 | 0 12.8 | 1/14 |

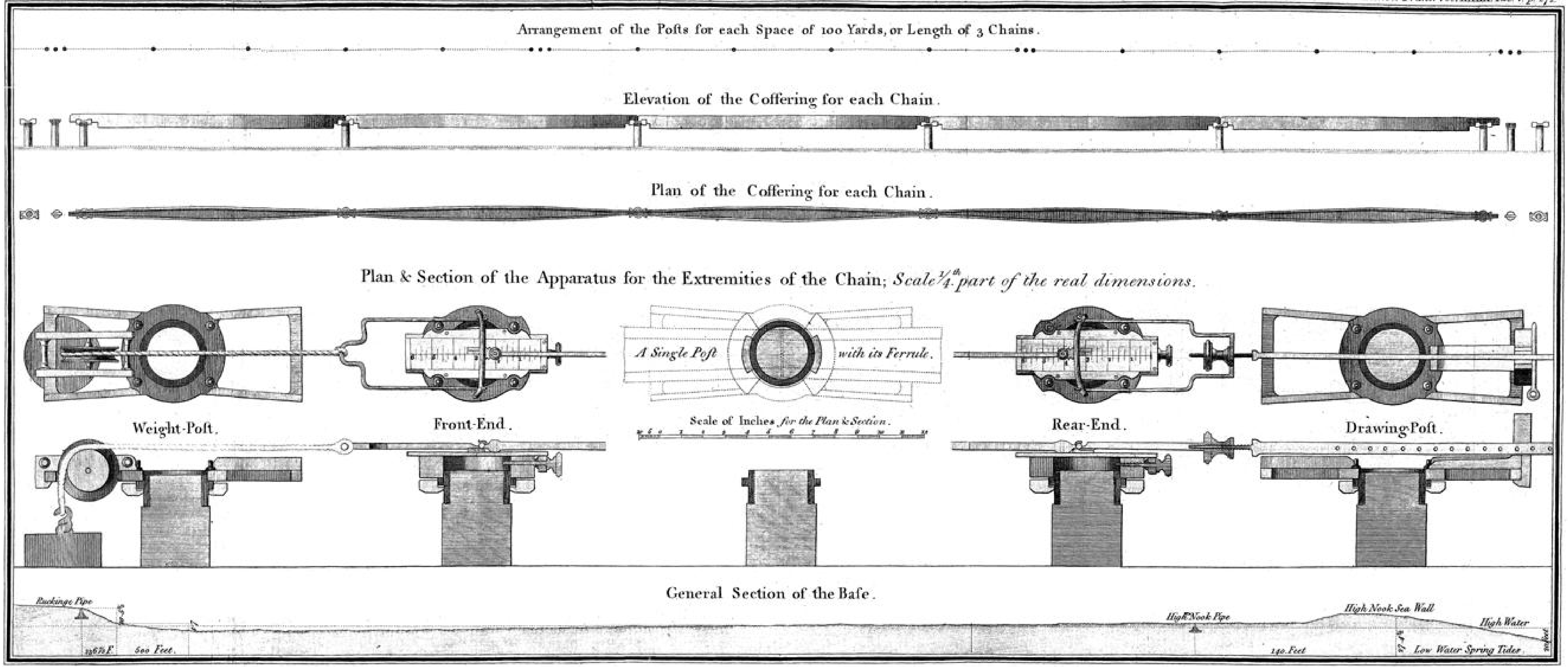
## Results of Single Observations depending on the Heights of Dover Caftle and Folkftone Turnpike, combined with thofe on the Coaft of France.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1788, Sept. 2. | — — — | 29.55 | 64 | Folkftone Turnpike and N. D. Church at Calais | F.<br>C. | 575.3<br>140.5 | 26697. | 26 4 | 3 54.8 | 1/6 and 1/7 |
| 1787, Sept. 28. | — — — | 29.62 | 58½ | Dover Caftle and N. D. Church at Calais | D.<br>C. | 469.<br>140.5 | 22908. | 22 28.2 | 0 15.4 | 1/17 and 1/18 |
| 1788, Sept. 2. | — — — | 29.55 | 64 | Folkftone Turnpike and Station at Montlambert — — — | F.<br>M | 575.3<br>639.5 | 28967. | 28 29 | 1 45. | 1/13 |

SECTION

Pl. I. *FOR THE ACCOUNT of the MEASUREMENT of the* BASE of VERIFICATION in ROMNEY MARSH with the *STEEL CHAIN, in the Autumn of 1787.*

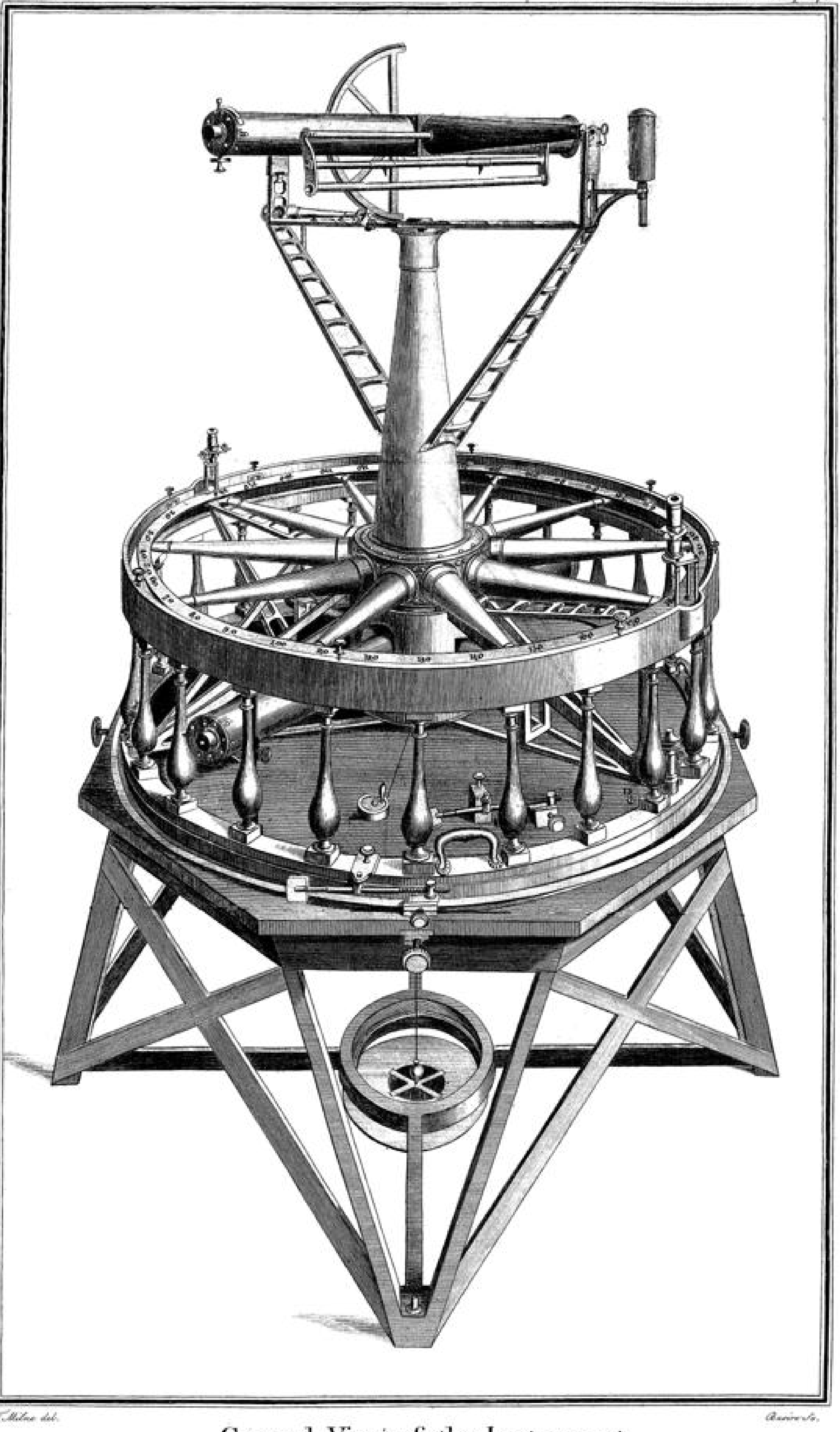
*Philos. Trans. Vol. LXXX. Tab. V. p. 172.*




*Basire Sc.*

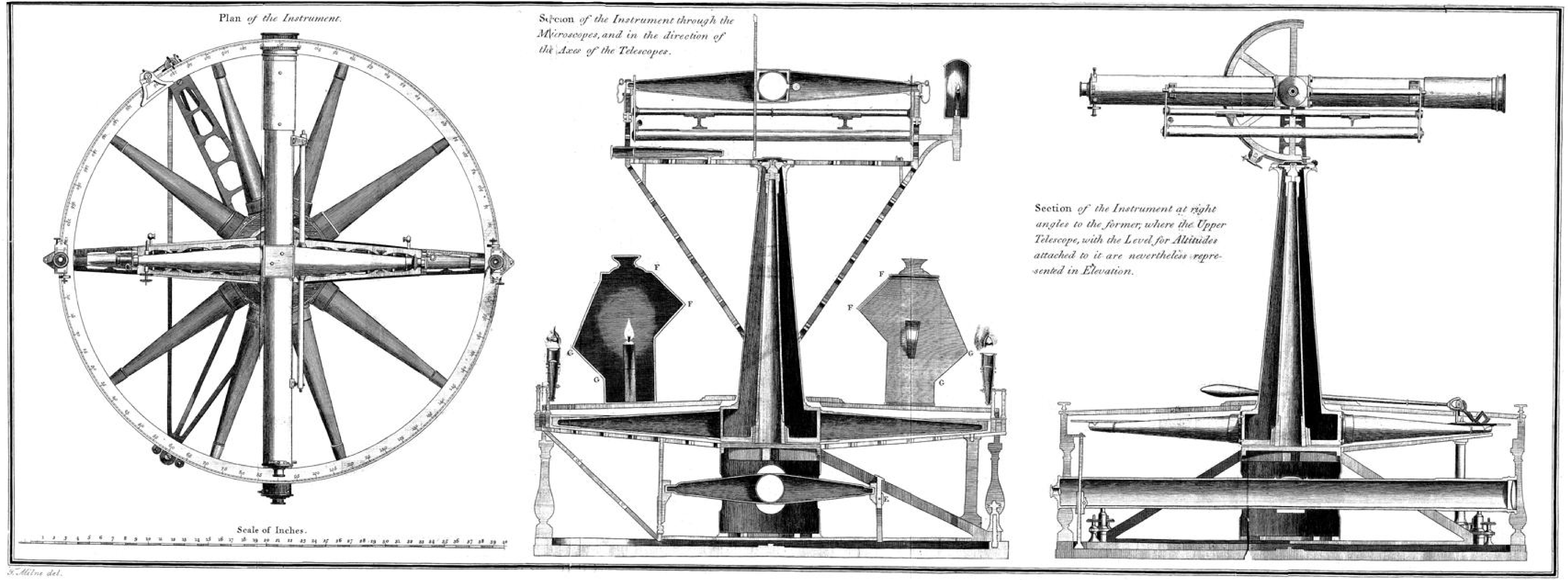
PLAN shewing the situation of the BASE of VERIFICATION measured in ROMNEY MARSH in the Autumn of 1787.





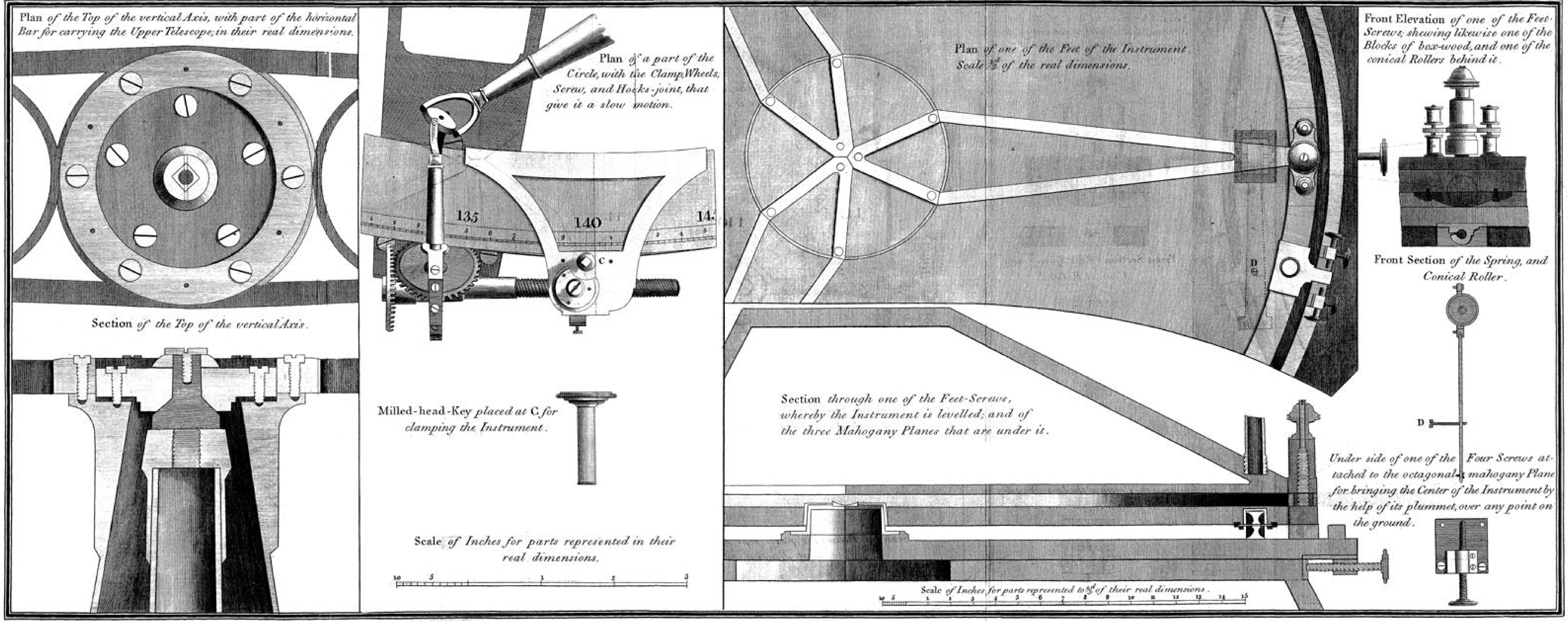
J. Milne del.
Basire sc.

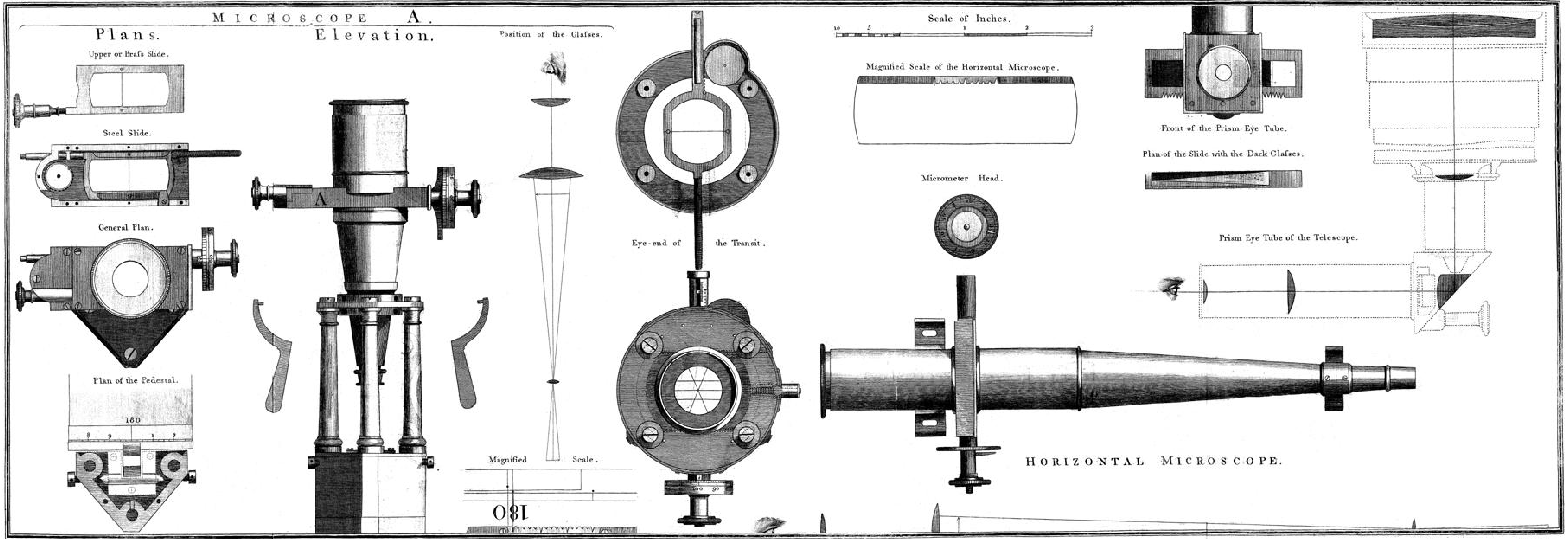
General View of the Instrument.




J. Milne del.

Basire sc.

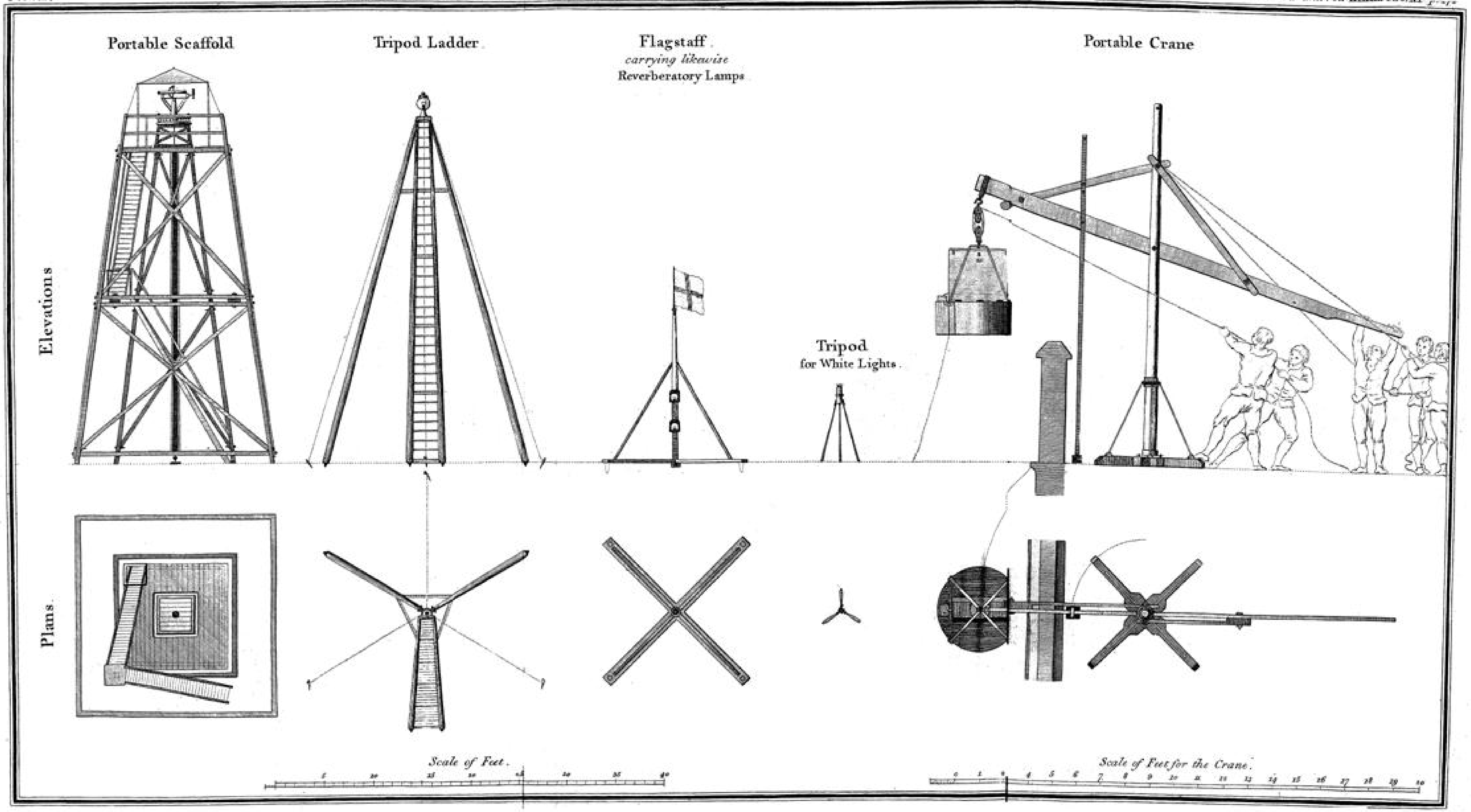

Plan of the Top of the vertical Axis, with part of the horizontal Bar for carrying the Upper Telescope; in their real dimensions.
Section of the Top of the vertical Axis.
Plan of a part of the Circle, with the Clamp, Wheels, Screw, and Hooks-joint, that give it a slow motion.
135
140
C
Milled-head-Key placed at C for clamping the Instrument.
Scale of Inches for parts represented in their real dimensions.
Plan of one of the Feet of the Instrument. Scale ⅓ of the real dimensions.
D
Section through one of the Feet-Screws, whereby the Instrument is levelled; and of the three Mahogany Planes that are under it.
Scale of Inches for parts represented to ⅓d of their real dimensions.
Front Elevation of one of the Feet-Screws; shewing likewise one of the Blocks of box-wood, and one of the conical Rollers behind it.
Front Section of the Spring, and Conical Roller.
D
Under side of one of the Four Screws attached to the octagonal mahogany Plane for bringing the Center of the Instrument by the help of its plummet, over any point on the ground.


MICROSCOPE A.
Plans.
Elevation.
Position of the Glafses.
Upper or Brafs Slide.
Steel Slide.
General Plan.
Plan of the Pedestal.
180
A
Magnified Scale.
180
Eye-end of the Transit.
Scale of Inches.
Magnified Scale of the Horizontal Microscope.
Micrometer Head.
HORIZONTAL MICROSCOPE.
Front of the Prism Eye Tube.
Plan of the Slide with the Dark Glafses.
Prism Eye Tube of the Telescope.

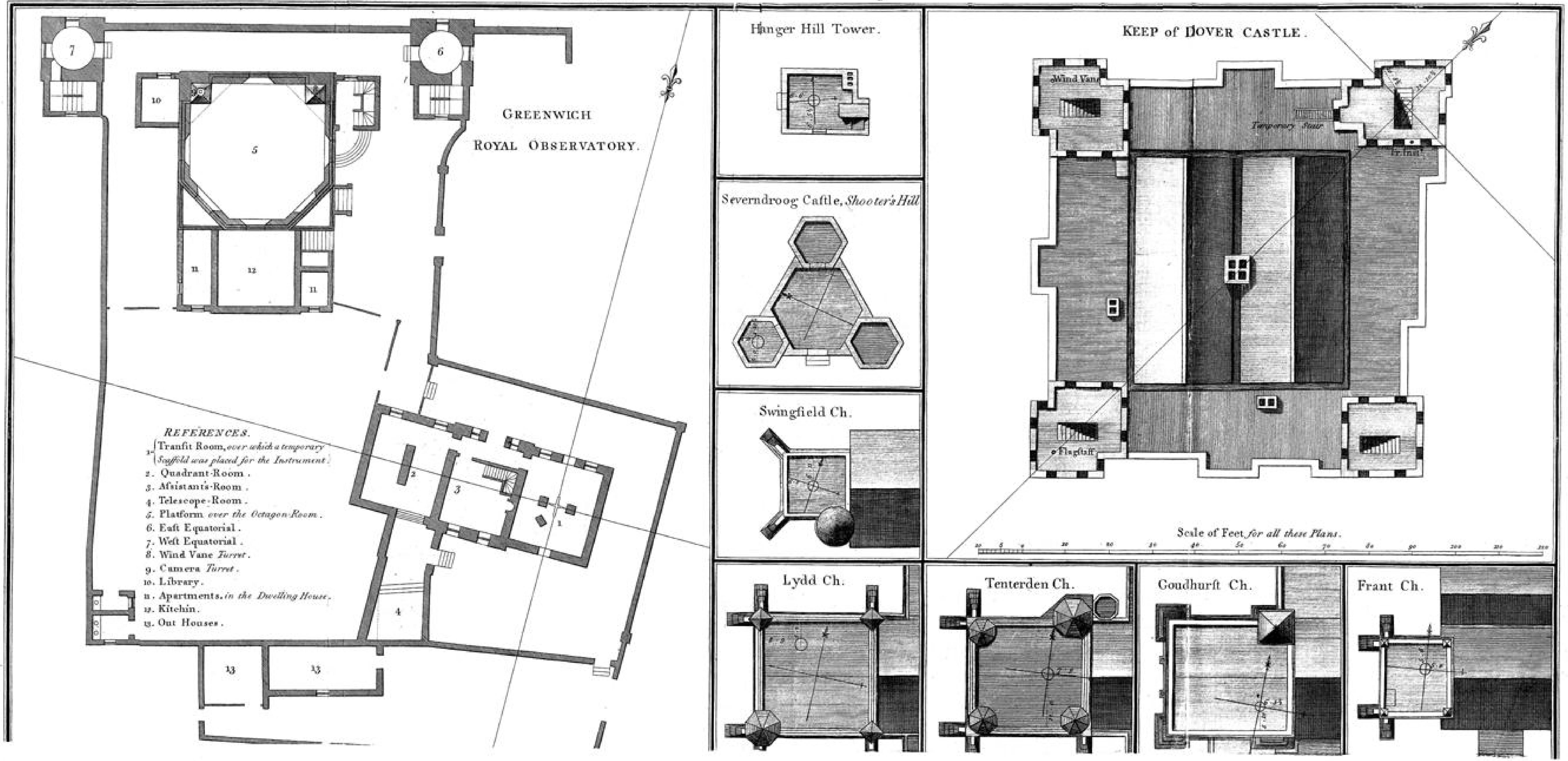
Pl. VII.

VARIOUS ARTICLES ... USED IN THE COURSE OF THE OPERATION.






Basire Sc.



GREENWICH ROYAL OBSERVATORY.
REFERENCES.
1. Tranfit Room, over which a temporary Scaffold was placed for the Instrument.
2. Quadrant-Room.
3. Afsistant's-Room.
4. Telescope-Room.
5. Platform over the Octagon-Room.
6. Eaft Equatorial.
7. Weft Equatorial.
8. Wind Vane Turret.
9. Camera Turret.
10. Library.
11. Apartments. in the Dwelling House.
12. Kitchin.
13. Out Houses.
Hanger Hill Tower.
Severndroog Caftle, Shooter's Hill
Swingfield Ch.
KEEP of DOVER CASTLE.
Wind Vane
Temporary Stair
Fr. Inst.
Flagstaff
Scale of Feet, for all these Plans.
Lydd Ch.
Tenterden Ch.
Goudhurft Ch.
Frant Ch.

Pl. IX.

# PLAN of the TRIANGLES whereby the DISTANCE between the ROYAL OBSERVATORIES of GREENWICH and PARIS has been determined

Philos. Trans. Vol. LXXX. Tab. XIII. p. 272.

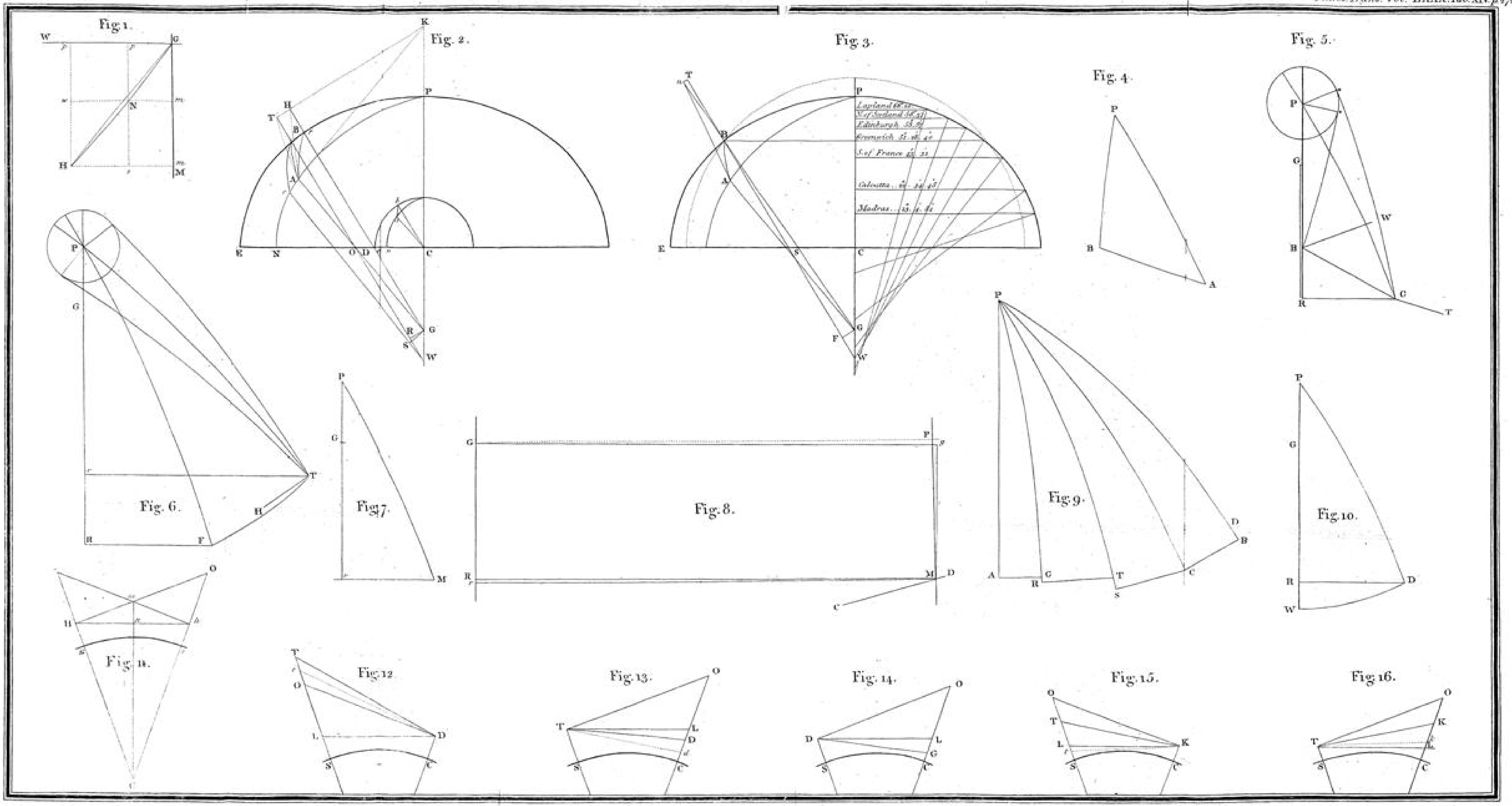

Scale of Thirty English Miles.

Scale of Thirty Thousand Fathoms.

Scale of Thirty Thousand Toises, Ratio to the Fathoms as 1065.75 to 1000.

Basire f.

Philos. Trans. Vol. LXXX. Tab. XIV. p. 272.



FOR THE IMPROVEMENT OF THE PLAN OF LONDON and its ENVIRONS.