PHILOSOPHICAL TRANSACTIONS.

XIV. The Bakerian Lecture.—*Inquiries concerning the Elementary Laws of Electricity.—Third Series. By* W. Snow Harris, *Esq. F.R.S.*

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1. THERE is no department of science in which the perfection of quantitative measurement, and a clear perception of what we really measure, is more called for than in that of electricity. If we except the valuable researches of Professor Robison and of Coulombe, and the more recent investigations of Dr. Faraday, we can scarcely be said to possess, in common electricity at least, any connected series of experiments carrying with them a rigid numerical value. In the various inquiries into the elementary laws of electricity, which I have had the honour of submitting to the consideration of the Royal Society, it has been my endeavour to perfect our methods of electrical measurement, whether relating to the quantity of electricity, intensity, inductive power, or other element requiring an exact numerical value, and, by operating with large statical forces both attractive and repulsive, to avoid many sources of error inseparable from the employment of very small quantities of electricity, such as those affecting the delicate balance used by Coulombe.

The instruments resorted to in these further inquiries have been employed with this view; they have been already described*; I have only occasion to briefly mention some recent improvements in the hydrostatic electrometer mentioned in my first paper*, and represented in Plate III. fig. 1.

2. In the instrument here shown, the column M, carrying the graduated arc, and wheelwork W, consists of two cylindrical brass tubes t, $'t$, about an inch in diameter and fourteen inches high. That on which the wheelwork is placed moves freely within the other, so as to be readily elevated or depressed by means of a rack fixed in it, and a pinion attached to the upper part of the outer tube at M. The object of this motion is to enable the experimenter to vary the distance between the attracting or repelling discs m, f without disturbing the lower disc f, or otherwise to adjust the same

* Philosophical Transactions for 1834.

distance by changing the position of both the discs, manipulations which greatly simplify many intricate cases of experiment.

3. In order to estimate the distances when the position of the disc *m* is varied, a graduated sliding piece *t*, about three inches long, is placed upon the inner tube, free of the rack-work, and being moveable upon it with friction, may be set with any required altitude of the whole column M to zero of its scale. In this way all subsequent changes of distance produced by elevating or depressing the interior tube *t* are easily known.

Changes of distance attendant on the motion of the lower disc *f* are estimated, as before, by the graduated slide *g*, the fixed tube of which is attached to a foot-piece P, moveable in a bevelled groove in the base B; the whole may be hence withdrawn for a certain distance, if required, so as to place the disc *f* without the influence of the upper disc *m*.

The disc *m* is suspended from the fine silver thread passing over the balanced wheel W, by three threads of varnished silk, after the manner of a common scale-pan, so as to insulate it if requisite; it is connected with the ground in ordinary cases by a fine wire, terminating in a small hook loosely hung from the silver thread to the surface of the disc, as at $h\ h'$, fig. 1.

The glass vessel V containing the water, and counterpoise float, are here supported in a ring of brass, moveable in a brass tube attached to a sliding rod *q*. This rod is acted on by a nut and screw inclosed in a cylindrical piece *c* fixed to the horizontal plate carrying the wheel-work. Hence the water vessel may be elevated or depressed at pleasure, and the index *i* readily adjusted to zero, or any other required point on the arc.

4. These provisions enable us to operate with the instrument in the following way. Let it, for example, be required to estimate the attractive force between the plates *m*, *f* at any given distance, D, suppose ·6 of an inch.

We first bring the discs in contact so nearly as may be, and then set the graduated slider *t* at zero of its scale, by bringing it to coincide with the upper edge of the outer tube M. We then (having also set the slider supporting the insulated disc *f* at zero) either raise the tube *t* ·6 of an inch, or depress the slide *g* by the same quantity, or otherwise raise the upper and depress the lower disc by quantities making together ·6 of an inch; in either case the discs will be finally ·6 of an inch apart, measured between the opposed surfaces previously in contact. Under these conditions let either plate be taken insulated and charged, whilst the other is neutral and free. Suppose the lower disc *f* to be charged with a given quantity, and the suspended disc *m* free. Then the attractive force which ensues will cause the index to advance in the direction *o y* a given number of degrees, consequently the distances between the plates *m*, *f* will be diminished. Let the index be now brought again to zero by turning the milled head of the screw *c* so as to depress the water vessel V; then the force between the plates, whatever it be, is acting at ·6 of an inch. To discover the amount

of this in degrees, we discharge effectually the air and opposed plates *m*, *f* by touching them simultaneously with a bent wire. The force then vanishes, and the index declines in the direction *o x*. The amount of this declination is evidently the force in degrees at the given distance D = ·6 of an inch.

Do we require to obtain a force of any given number of degrees at a given distance, we first adjust the distance as before, the index being at zero of the arc, then depress the water vessel V until the index has declined the given number of degrees in the direction *o x*; we now continue to add small quantities of electricity to the insulated disc *f* until the index is again brought to zero, we have then in noting the quantity of electricity the required attractive force due to this quantity at the given distance*.

5. The general principles of this instrument, together with its mode of action, being now understood, I may merely observe, that although not available for the measurement of such minute forces as those applicable to the balance of torsion employed by Coulombe, it is still peculiarly delicate, and admirably well adapted to researches in statical electricity. Its indications depending on the force between two opposed planes operating on each other under given conditions, are reducible to simple laws, and are hence invariable and certain. The attractive force between the discs is not subject to any oblique action, is referable to any given distance, and may be estimated in terms of a known standard of weight.

6. It may not be unimportant to consider here the nature of the indications of this and similar instruments in electricity; in other words, what we really measure by means of such instruments.

This question is of considerable moment; for should the force between the discs be in any way influenced by a precarious distribution of the electricity, on or about their surfaces, and liable to become further complicated by the distribution on the surfaces of the conductors, and other bodies placed occasionally in tortuous connection with them, as in fig. 7, 8, 9, it would, perhaps, be impossible to say what we did really measure by any instrument such as that described; we should at least require the aid of a formidable analysis involving definite integrals of unknown functions to determine it: fortunately however such is not the case; the force between the attracting plates *m*, *f*, stands, as may be clearly shown by experiment, quite clear of any hypothetical distribution whatever of the electricity, with which the discs are charged, or with which any series of bodies may be charged in connection with them. Thus the general laws of electrical action arrived at in my former communications† remain uninfluenced by any new condition of the connecting rods, or other bodies of variable form, with which the attracting bodies may happen to be connected. The question, therefore, what do we measure? is very easily answered, as readily, in fact, as a similar

* There are some precautions essential to observe in the use of this and similar instruments, which will be apparent as we proceed.

† Transactions of the Royal Society for 1834.

question would be, if applied to the thermometer or any other instrument, the nature of whose indications are necessarily determined by experiment. Thus we observe that with double, treble, &c. quantities of electricity, either accumulated on the insulated plate *f*, or on any insulated conductor connected with it, we have invariably 4, 9, &c. times the attractive force indicated on the graduated arc, the distance between the discs remaining the same; no matter for the form of the charged bodies in connexion with the insulated plate *f*, or the form or disposition of any number of rods connecting them. Hence conversely, when the attractive forces are at distance unity, 4, 9, &c. times as great, then the respective quantities of electricity accumulated, under the existing disposition of the conducting surface, are 2, 3, &c. times as great; the quantity being as the square root of the indicated force. Similar observations apply to a variety of other quantitative measurements, of which this instrument is susceptible, as in the case of the attractive force communicated to the insulated plate *f*, when connected with a charged conductor of any given form, and which by experiment remains invariable, at whatever point of it the connection be made. Hence the general electrical intensity of such a conductor is always represented under any new disposition of its surface, as shown in my experiments on the capacity of rectangular plates, cylinders, spheres*, &c. These and similar facts deducible by experiment enable us, independently of all theory, to investigate by this electrometer certain electrical relations, such as that of quantity to surface, of intensity to figure, and the like, with accuracy and precision. This question then being disposed of, we may proceed to the further consideration of the elementary laws of electrical action, the subject of these inquiries.

7. Electrical attraction and repulsion, as commonly observed, are invariably attended, if not altogether preceded by other forces of a more elementary character, without the presence of which, neither of these interesting phenomena would probably ensue. These more primary actions we have, in accordance with the prevailing theories of electricity, classed under the general head of inductive actions. FARADAY has lately investigated the nature of these actions in the Eleventh Series of his admirable Researches in Electricity. The three following experiments exhibit some phenomena of induction not generally noticed, although the results are such as might be previously anticipated: the experiments however are still new of their kind, and very illustrative of what takes place, at the instant two bodies attract or repel each other by the agency of electricity.

Let a circular disc of gilded wood, about six inches in diameter, *a*, fig. 2, be suspended by an insulating thread of varnished silk from a common balance or the periphery of a light wheel W, the axis of which rests on friction wheels so as to allow it great freedom of motion; attach a delicate electroscope *r*, to this disc, and counterpoise the whole by a weight *p*. Let a similar disc *b*, insulated on the glass rod *g*, and having also an electroscope *r*' attached to it, be placed at any convenient distance

* Philosophical Transactions for 1834.

immediately under the former*: when the two discs are placed in certain electrical relations to each other, under a good insulating state of the air, the following phenomena may then be observed.

Exp. 1. The disc *b* being charged with either electricity, and opposed to *a*, insulated and neutral, we observe the electroscope *r* of the neutral disc begin to rise off its surface, whilst that of the charged disc *b*, already in a state of divergence, will tend to collapse. When these respective effects ensue, the suspended disc descends towards the charged disc.

Exp. 2. The two discs being both previously charged with opposite electricities, we observe, in opposing them as before, both the electroscopes, *r*, *r'*, begin to fall back, at which instant the discs appear to attract each other as before.

Exp. 3. The discs being now both charged with the same kind of electricity, we observe the divergence of the electroscopes *r*, *r'* increase; at this instant the suspended disc recedes from the fixed disc, being apparently repelled by it.

It may be further observed in the first experiment, that the operation of the neutral on the charged disc, as indicated by the electroscope *r'*, is greater when its superficial extent is increased. Thus, if it have a temporary connexion with an insulated conductor of twice or three times its surface; or otherwise, if it form the terminating surface of a hollow cylinder of the same diameter and of any given altitude, the collapsing of the excited electroscope *r'* attached to the charged disc will be more considerable. If it have a conducting communication with the ground, then, as is well known, its influence on the charged disc is, at a constant distance, the greatest possible. In many of these cases, however, the influence of the charged on the neutral disc is, on the contrary, less sensibly shown by the electroscope *r* attached to the latter, in consequence of the operation being extended to larger masses, and to more distant points.

8. The respective influences on the electroscopes just observed, and which precede, or *at least* accompany the attractive or repulsive forces between the discs, may be distinguished in the following way. In the case of the first experiment, the electroscopes indicate two inductive and simultaneous actions. One of these may be considered as a direct induction, the other as a reflected induction; supposing, as is not unlikely, that the induction of the neutral on the charged plate is entirely dependent on the electrical state excited in the former by the direct influence of the latter, by which an opposite force is induced, and which reacting on the charged disc, neutralizes a portion of its free electricity.

* The electroscopes *r*, *r'* may consist of light reeds, with pith balls at their extremities, counterpoised on a delicate axis of brass, set on points in metallic rings *r*, *r'*. These are fixed in the centre of the plates *a*, *b*. The reed and ball on the upper one is a little heavier than the counterpoise *r*; on the lower one the counterpoise is heavier than the reed and ball. Thus, in a neutral state, the balls lie parallel with the surfaces of the plates, and rise from them by repulsion, with the slightest force. It is only requisite to observe, that the pith balls should not in the neutral state actually *touch* the surface, but remain elevated within a very small distance of it, which may be readily effected by fixing a small slip of cork to the plate just under the reed. By this we avoid any adhesion of the ball to the plate, which interferes with the success of the experiment.

9. Pursuing this view we may further infer, as in fact may be shown experimentally, that similar inductions really tend to arise in both the subsequent experiments 2, 3, notwithstanding the presence of the similar or dissimilar electricities; that is to say, inductive forces tend to arise in each disc, similar to those which would ensue if either plate were reduced to a state of neutrality, thus increasing the amount of the attractive forces in the case of the plates being charged with opposite electricities, and decreasing that of the repulsive forces when the discs are charged with similar electricities*: the laws of electrical attraction and repulsion therefore are intimately associated with these inductive forces; hence, as observed by FARADAY, it is requisite to examine rigidly the nature and operation of this wonderful influence.

10. There are many striking phenomena of electrical induction, which lead us to conclude that it is in some way dependent on the presence of an exquisitely subtle form of matter pervading bodies, which may become disturbed in them, and assume new states or conditions of distribution. The following experimental analysis goes far to place this beyond the limit of a mere hypothesis, and to confirm it as an elementary principle in electricity.

M, N, fig. 3, are two cylinders of gilded wood, about four inches in diameter and six inches long; the extremities of these cylinders terminate in thin slices, *a, b, c, d*; all the different pieces are insulated on slender rods of varnished glass, fixed in separate stands; these slide upon the pieces E, F by means of a groove and steadying pins in the stands; thus the false ends *a, b, c, d* may be either placed in contact with the cylinders M, N, or be otherwise placed at any given distance from them. The pieces E, F also slide in a similar way upon the base P P, so as to admit of the two bodies, M, N, being placed at any given distance apart.

Exp. 4. Electrify the cylinder M, and slide it to within any given distance of N; the latter will, as is well known, become electrified by induction, and the cylindrical slices *b, d,* if removed on their insulating rods, will be in opposite electrical states. But if before removal the charged body M be withdrawn, then the whole system returns to its previous state, and exhibits no electrical sign whatever.

Now it may be inferred, that if the peculiar condition of the extremities *b* or *d*, considered as portions of the body under induction, depended merely upon some peculiar affection of the particles of common matter, and not on some agency associated with them, then on removing the slices *b, d* forming the extremities, the forced state should vanish; for it is difficult to conceive how any principle of return to quiescence applicable to the whole body N, when removed from the influence of M, should not also apply to any part of it placed under the same conditions, *e. g.* to the thin sections, *b, d,* forming its extremities. But on removing the extremity *d* we find, as just stated, that the forced state remains.

Exp. 5. Examine in a similar way the thin slices *a, c,* constituting the extremities of the charged body M, having first determined the intensity of the charge previously

* Philosophical Transactions for 1836.

to opposing it to the neutral cylinder N. Then we find the intensity of the distant extremity *c* considerably diminished, and that of the proximate extremity *a* considerably increased; and this effect becomes the more apparent as the distance between the bodies is less, and will be strikingly shown if N be connceted with the earth*.

Exp. 6. Electrify the cylinders M, N, one positively, the other negatively; then on examining each as before, similar results will ensue; the distant extremities will show on removal less accumulation than the proximate ones.

Exp. 7. Electrify the cylinders, both positively, or both negatively, we have then an opposite result. The distant extremities will exhibit a higher positive or negative accumulation than the near ones.

11. These results, therefore, all appear to show, that some extremely subtle form of matter pervades bodies, which may be caused to change its state in them in respect of quantity. To determine the respective states of accumulative change in the bodies, I employed the electrometer just described, and examined them, either by their inductive influence on an area of equal magnitude, or by simply opposing the removed slices to the suspended plate *m*, fig. 1. at a constant distance, or by other methods not necessary to dwell on here. I obtained in this way some interesting results in respect of the relative quantities of electricity displaced at given distances, which will be noticed in another place (32.).

If we place at the extremity *b*, of the cylinder N, three or more consecutive insulated slices, insulating them on glass rods, in any convenient way as in fig. 4., and then proceed to examine the electrical state of these slices whilst under the influence of the charged body, the electricity of the distant extremity *d*, will frequently be found extending up to the last section *b*; and, contrary perhaps to what we might expect, the point to which the electricity of the distant extremity *d* extends toward *b*, will be greater as the intensity of M is greater and its distance from N less, as if the displaced electricity not being enabled to pass freely off in the direction *b d*, supposing M positively charged, was continually, as it were, bounding back or reverberating upon the extremity *b*, a fact which may be further observed in the following experiment.

Exp. 8. Oppose a cylindrical conductor D, fig. 5., about three feet in length and four inches in diameter, to an electrified cylinder C, charged with positive electricity. Test the state of different points of this cylinder D by touching it with a tangent disc of very small thickness. If we ascertain in this way the first point at which the electricity evinced is negative, we shall find on bringing the charged cylinder C within a less distance, or otherwise increasing its intensity, that the same point will become positive. The same thing occurs in increasing the extent of the opposed areas at the extremities of the cylinders, until at last the points immediately in the vicinity of the opposed end will become positive.

* In these experiments it is better to detach first the central portions M or N whilst under induction, when we wish to examine the state of the remaining slices *a*, *b* or *c*, *d* forming the extremities.

Exp. 9. Detach in fig. 4. the central part N and extremity *d* of the cylinder N, and remove the charged body M. Then if the extremity *b* consist of two or more sections, *b*, *r*, *s*, these sections will evince negative electricity, M being positively charged; but if after detaching the central mass N we allow the charged body M to remain, and then take away the further slice *s*, this slice, which under the former arrangement evinced negative electricity, will now evince positive. There must consequently still remain a portion of electricity sufficient to electrify the remote section positively, although not equivalent to the negative state of the three slices taken together, and removed from without the influence of the charged body.

12. Since then the negative state of an indefinitely thin slice *b*, immediately opposed to the charged body, may be supposed to depend on the electricity displaced from it, and collected in the detached central mass N, we may conclude, in accordance with the well-known fact, that this negative state will be the greatest possible when the electrical capacity of N is indefinitely great, that is to say, when it is connected with the ground, and whilst the influence of the charged body M is still operating on it. The near surface *b*, therefore, of a free neutral body, N, fig. 3, under the induction of a body, M, positively charged, is greater than would be apparent upon the whole body after cutting off its connexion with the earth.

The inductive action, whatever it be, which thus takes place between a charged and neutral body, does not appear to be in any degree influenced by angular divergence, but is exerted equally in every direction from the point at which the induction first commences; this may be inferred from the following experiment.

Exp. 10. A cylindrical conductor, N, figs. 6, 7, was so constructed as either to constitute a single straight line, fig. 6, or assume a rectangular form, as in fig. 7. It consisted of two straight cylinders of gilded wood, each about a foot in length, and three-fourths of an inch in diameter, united to an intervening ball N by means of short brass pegs. In this way the two cylinders could be easily placed in one straight line, as in fig. 6, or at right angles to each other, as in fig. 7. Under this condition one extremity, *f*, fig. 6, was placed in contact with the insulated ball *f* and disc of the electrometer, fig. 1, and the opposite extremity united to a light disc *c*, of about three inches diameter. The inductive force was impressed upon this last by a charged cylinder, M. The distance, *c a*, and quantity with which M was charged being the same, it was easy to discover whether any difference arose in the attractive force on the electrometer at the extremity *f* when the conductor was bent at right angles, as in fig. 7, or otherwise allowed to form a straight line, as in fig. 6. No appreciable difference, however, was observable in these different dispositions of the conductor N. Thus the distance *c a* being one-fourth of an inch, and the charge the same, the electrometer indicated in each instance ten degrees, the discs *m*, *f* being also one-fourth of an inch apart. The quantity and distance *c a* was varied, but a similar result ensued. It may be hence inferred that the force impressed upon the disc *c* is propa-

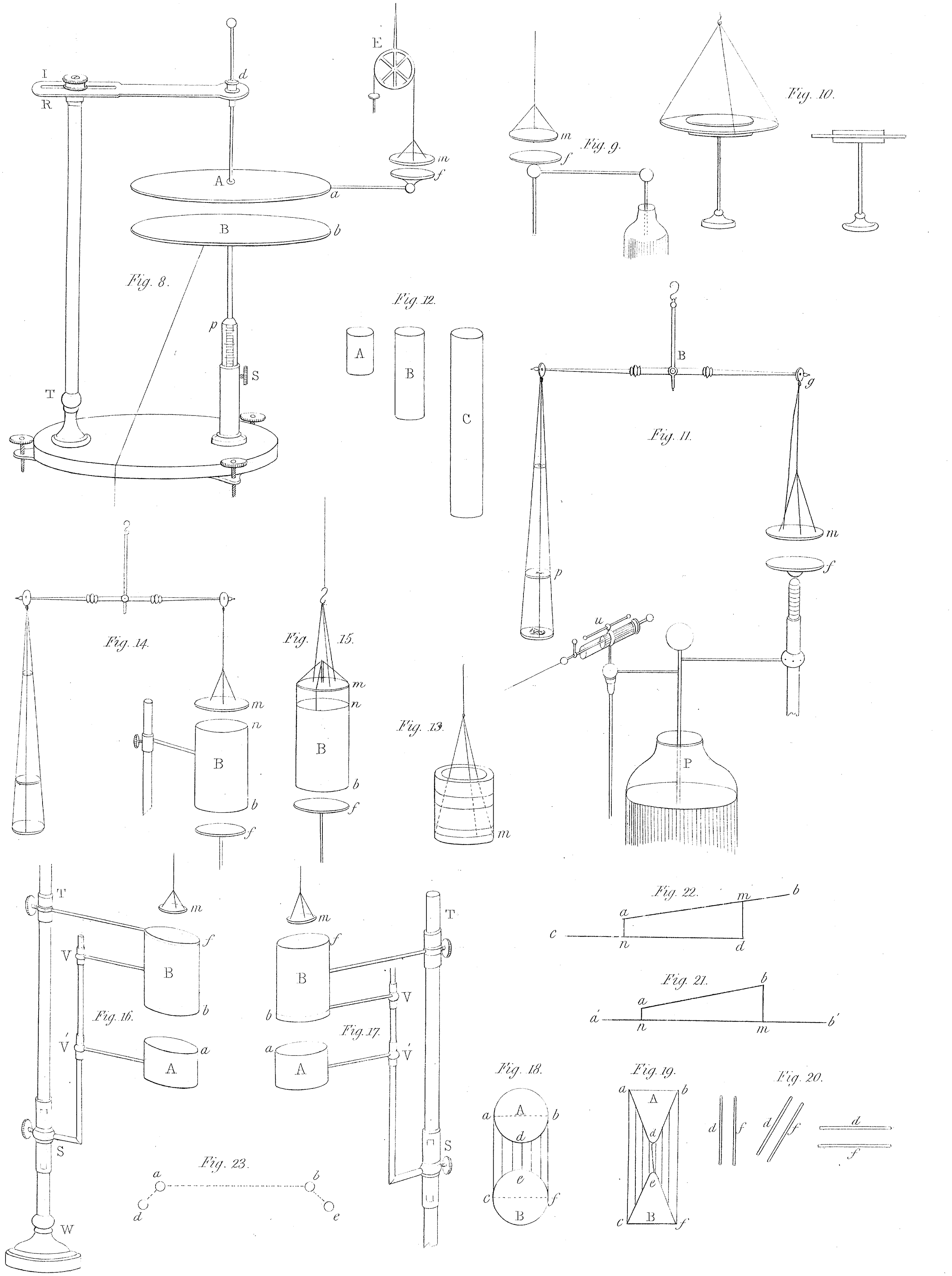
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12.
Fig. 13.
Fig. 14.
Fig. 15.
Fig. 16.
Fig. 17.
Fig. 18.
Fig. 19.
Fig. 20.
Fig. 21.
Fig. 22.
Fig. 23.

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gated in the conductor equally in all directions, thus exhibiting in a remarkable way one of the primary laws of fluids.

13. Since these facts furnish some evidence in favour of the conclusion that electrical phenomena result from certain changes in the distribution of a subtle kind of matter associated with the particles of bodies, it may not be unimportant to examine the laws of these changes in the cases of induction and attraction above mentioned.

It must be apparent by experiments 4, 5, 6, figs. 3, 4, that in the ordinary attractive force between a body positively charged with electricity, and a neutral body N in a free state, three actions arise claiming particular notice, viz. a receding of the natural electricity of the neutral body, from the points nearest the body charged positively; a passing of electricity from the remote points of the positively charged body toward the neutral body; lastly, a tendency of the opposite electrical forces to come together and enter into a species of union; which last condition seems to be the immediate cause of these two bodies approaching each other, all impediment to motion being removed.

14. With a view of discovering some of the laws of these induced changes, I resorted to the method represented in Plate IV. fig. 8, in which A, B are two flat discs of wood covered with tin foil, insulated on varnished glass rods, A *d*, B *p*. The upper disc A is connected with the insulated disc *f* of the electrometer E, and is supported by the arm R *d* fixed to the glass or wood column R T. The lower disc B can be set at any required distance from A by the graduated slider S, and retained there by a stop screw, S. The rod *d* A also is moveable with friction through a compressed collar of cork at *d*, by which it can be elevated or depressed for the more perfect adjustment of the contact of the plates A, B, when the slide S is at zero of its scale.

Exp. 11. The distance, *a b*, between the plates being made = ·3 of an inch, and the plate B connected with the earth, a charge was accumulated on A, indicating on the electrometer two degrees, the discs *m*, *f* being set at ·4. The plate B was now depressed to twice the distance, so as to make the distance *a b* = ·6 of an inch. The electrometer now indicated at the same distance of ·4, eight degrees. By making *a b* = ·9, or three times the first distance, the electrometer had advanced to eighteen degrees. The march of the electrometer therefore was directly as the squares of the distances between the plates.

15. Now it is clear in this experiment that the quantities of free electricity in the upper plate were inversely proportional to the distances between the opposed surfaces, the quantity by the demonstrated law of the electrometer (4.) being as the square root of the indicated attraction; hence the force or influence of the lower plate, that is the reflected induction (8.), is, if measured by the quantities of electricity it ceases to hold in equilibrio, as the distance *a b* between the plates A, B inversely; if measured by the indication of the electrometer, inversely as the squares of this distance.

16. Exp. 12. The former experiment relates to the influence of distance; it may

not be unimportant therefore to observe the effect of quantity under similar circumstances. In this experiment, then, the distance *a b* was first fixed at ·4 of an inch, and four measures of electricity deposited on the plate A. The indicated force with this quantity amounted to 3°. When eight charges were placed on A, that is double the former, the indicated force amounted to 12°, or four times the former, and so on in the same ratio of the square of the number of charges, up to the limit of the action of the electrometer, which is the law above mentioned, and shown for the coated jar in my first series of papers*.

17. I confirmed this result by making the distance *a b* = ·8, and communicating to the insulated plate the same number of charges. The electrometer now indicated 46°, being little different from 48°, the number which should appear by the former experiment. The plates and electrometer discs were now perfectly discharged, and the same eight charges deposited on A when the distance *a b* was reduced to ·2 or one quarter. The electrometer now indicated 3° only, or $\frac{1}{16}$th of the former force.

18. The reflected influence of the lower plate therefore is such, as to hold quantities of electricity in equilibrio directly proportional to the quantity with which the insulated body is charged.

Exp. 13. I extended these experiments to opposed plates whose areas were less than the former, but equal, and found, as in the former experiments with the Leyden jar, that the quantity and distance being constant, the indicated force was as the squares of the opposed areas inversely. Thus, when the areas of the plates were doubled, the force was only one-fourth as great.

19. We cannot by this method deduce any accurate result for plates of unequal area, as in my former experiments on attraction†, since the charge expanding over the whole of the upper disc, and also the electrometer, it does not admit of being neutralized throughout, hence the influence of the opposed portions only is not apparent; neither will the laws above mentioned be rigidly exact, if we charge the insulated plate A without the influence of the opposed plate B, since in this case also the electrometer discs receive a maximum of charge at once, which cannot be subsequently diminished so as to show the action of the neutral plate B. But in accumulating the charge under the influence of B, the electrometer charges gradually, and with the electricity not held in equilibrio by the lower plate.

20. In the three preceding experiments the neutral disc B, had a free connexion with the ground. We shall now take it insulated.

Exp. 14. The plate A being charged under the influence of B as before, at a distance of ·2 of an inch, indicated by 4° of the electrometer with the discs *m*, *f* at ·5, the conducting connexion of B with the ground was removed, so as to insulate it. In this case the induction upon B was the greatest possible at the given distance ·2. The plate B was now depressed ·2 and ·4 of an inch further, so as to obtain double and quadruple the first distance successively. The march of the electrometer in this

* Transactions of the Royal Society for 1834.

† Ibid.

case was no longer as the squares of the distances *a b*, as in Exp. 11, being now as the distance; thus at ·2 the force was 4°, at ·4 = 8°, at ·8 = 16°.

The following table exhibits the results of the comparison of the insulated and uninsulated state of the plate B at other distances and with other charges.

TABLE I.

Insulated at ·3.		Uninsulated.	
Dist.	Force.	Dist.	Force.
·3	3°	·3	3°
·6	7	·6	12
·9	10	·9	26

The approximation to the laws above mentioned are here very close.

It is then apparent from these results, that by limiting the electrical capacity of the opposed plate, we fix the direct inductive action up to a certain limit; hence the subsequent effect is due to change of distance only, and is in a simple inverse ratio of the distance. FARADAY has shown in the Eleventh Series of his valuable Researches in Electricity, since this experiment was first made, in what this effect of change of distance consists; he observes, (sec. 1303,) "there is perhaps no distance so great that induction cannot take place through it: but with the same constraining force, it takes place more easily, according as the extent of the dielectric is lessened," &c. &c.

21. Exp. 15. A plate of glass very dry and varnished being opposed to the plate A, and a charge accumulated as before, no change was perceptible on altering the distance, or on the removal of the glass; we may hence infer, that a perfectly non-conducting substance is insensible of any new electrical state, by simple induction under common circumstances.

Exp. 16. The plate B being insulated and opposed to A, the difference on the electrometer after removal was still very small, and at moderate distances of ·2, ·3 of an inch quite inappreciable. These differences, however, became greater by increasing the thickness of the plate, or by allowing a small conducting rod to project from it.

We may, therefore, further infer, that in an insulated conducting disc indefinitely thin, the inductive capacity is indefinitely diminished*.

22. Electrical attraction then, is evidently a complicated operation, and may not unfrequently give rise to results apparently of an anomalous character. The uniformity of the force depending on a perfect accomplishment of the inductive changes above mentioned, should these be in any way limited, or interfered with, the force may appear to vary, or be also limited in the law of its action to certain distances, as is apparent in the following experiments.

* The case must not be taken as identical with that described by FARADAY (1295.), we are here speaking of an insulated disc, not a disc connected with the earth: no sensible thickness is then required, as already shown by the suspended disc of the electrometer.

Exp. 17. When two circular discs *m f*, fig. 1, were opposed to each other, the suspended disc *m* being perfectly insulated and thin, and the lower disc charged positively, little or no attractive force was observable at ·3 of an inch distance. At ·2 of an inch the force amounted to about a degree. On touching the neutral disc with a conducting rod the force appeared to be indefinitely increased, not being under the same charge and distance measurable by the instrument.

Exp. 18. A varnished disc of glass being substituted for the disc *m*, the force was not appreciable at any distance.

Exp. 19. The opposed discs being charged, one positively, the other negatively, and to as nearly the same degree as possible, the forces were observed corresponding to various distances by the process above described (4.); they were as the simple distance inversely, or very nearly so. Now it is to be observed in this case, that the discs being thin, the positive and negative accumulations, as in Exp. 14., were already the greatest possible, or very nearly so: all subsequent inductive change was therefore precluded, and hence the increase of force was due to change of distance only, or according to FARADAY, to the intervening dielectric particles being lessened.

Exp. 20. The lower disc being charged positively and the upper plate allowed to remain neutral and free (4.), the force within given limits was as $\frac{1}{D^2}$, but at near distances as $\frac{1}{D}$.

In this case the lower disc *f* having a limited thickness and charge, is not fully susceptible of the reflected induction at all distances; hence this force is impeded as before, and the conditions at last approximate to those of the permanent positive and negative states in the preceding experiment.

Exp. 21. The lower disc *f* being connected with a charged jar or coated plate either of air or glass, as in figs. 8 and 9, and the suspended disc *m* placed in connexion with the ground, the forces were as the squares of the distances inversely at all the distances at which the experiment could be tried.

It is to be observed here, that the inductive capacity of each surface was indefinitely increased, whilst the quantity of electricity accumulated might also be considered as indefinitely great in respect of the charged surface.

23. The generally received law of electrical attraction would, by these phenomena, appear to be rather a result of the conjoint operation of two elementary actions than a simple law, such as that observable in the action of forces supposed to emanate from a centre, since it is not demonstrable, except under given conditions of induction peculiar to the attracting bodies. When the positive and negative states are fixed and invariable, the attraction between the plates, as found by experiment, is really in a simple inverse ratio of the distance.

Exp. 22. With a view of examining this result more rigidly, and disengaging the electrical particles so far as possible from all association with a conducting substance, I procured two thin circular plates of glass, about ·6 of an inch diameter, and having

given them temporary coatings of gilded wood, as in fig. 10, I charged each with a given quantity. The coatings were now removed, and the charged glass plates transferred to the electrometer with the positive and negative surfaces opposed to each other, as at *m f*, fig. 1. The lower plate in this case was supported on a slightly curved glass, similar to a common watch-glass, so as to avoid the presence of any conducting substance, and three silk lines of suspension attached to the upper plate by a little sealing-wax. In this experiment we may conceive the force to result purely from the action of the opposite electricities, which may in this case be considered as fixed and incapable of any further change, since by the law of the coated jar no electricity can be added or taken from one side without a simultaneous corresponding change on the other, hence one side only may exhibit free electricity. The glass plates themselves also, not being susceptible of induction (Exp. 15.), cannot be supposed to share in the attractive force, whilst the remote surfaces of each plate are virtually neutral. Under these circumstances the force varied very rigidly as the distances between the plates inversely, at all distances at which the experiment could be tried.

24. The relations of the inductive to the attractive force becomes under this view an interesting subject of inquiry in electricity. I endeavoured to examine still further by a careful series of experiments the general laws of these forces, and succeeded in arriving at many results calculated to throw additional light on the nature of electrical action.

Two discs *m*, *f*, fig. 11., were opposed to each other, as explained in the Society's Transactions for 1834, p. 220, that is to say, the disc *m* was suspended from one arm of the balance B, whilst the disc *f* was connected with a coated jar; the attractive force between the discs at various distances being measured by weights placed in the scale pan *p*, and the quantity of electricity estimated by the unit jar *u*.

Exp. 23. The suspended disc being very perfectly insulated by varnished silk lines, the force of attraction at a constant distance was examined with a given quantity of electricity in a very dry atmosphere, and subsequently compared with the force under the same conditions of quantity and distance, when placed in a free state by a small connecting wire accurately balanced and hung from the point *g*; the difference, as may be anticipated, was very great. In the insulated state, it required sixty charges before the force was equal to one grain at a distance of ·2 of an inch. In the free state, three charges only were requisite to raise one grain at ·2.

Now we have before seen, that the force between two attracting discs, is as the square of the quantity of electricity communicated to the charged body; the force, therefore, in the free state with sixty charges, could this quantity be accumulated and retained, would have amounted to 400 grains; or otherwise taking only three charges in the insulated state, it would have been only the $\frac{1}{400}$th of a grain. The force therefore in these two states may be taken as inversely proportional to the square of the number of charges. Hence the force between the plates was greater in the free state in this particular case, in the ratio of 400 : 1.

25. Exp. 24. With a view of observing the rate of increase of the attractive force as the capacity of the neutral body was caused to increase, I placed successively on the suspended disc *m* a series of rings of gilded wood, as in fig. 13, so as to increase the thickness from ·1 of an inch to two inches: the results are given in the following Table.

TABLE II.

Thickness	·1	·2	·4	·6	·9	1·2	2·0
Charges	60	50	40	30	24	20	16
Force compared with the force when free	400 : 1	277 : 1	177 : 1	100 : 1	64 : 1	44 : 1	28 : 1
Force of sixty measures reduced to grains	1—	1·44	2·25	4	6·2	9	14

The amount of charge for the first experiment, both in the insulated and free state, was as near as could be determined. The beam dropped in the latter case with something more than three charges, hence the force must be assumed as something less than a grain. In the insulated state it was not easy to arrive at the precise number of charges, although the variations were not considerable: sixty charges of the unit measure, however, corresponded upon a mean number of experiments, to the same force, about one grain. If we assume the force for thickness ·1 to be something less than a grain, let it for example be taken at ·7, then the force as expressed in the lower line would increase nearly as the thickness or altitude of the cylinder, which is not a little remarkable.

Exp. 25. I followed out this result by examining the force upon three cylindrical conductors, A, B, C, fig. 12, whose altitudes were 1·5 inches, three inches, and six inches, that is, as the numbers 1, 2, 4, and having terminating plane surfaces equal to that of the plane disc *m*, fig. 11. These being suspended successively from the balance, the force, taken in a free state, amounted as before to one grain, with rather more than three charges, being the same as the simple disc taken in a free state; when taken insulated, the forces varied, and were nearly as the altitude of the cylinders directly. Now on referring to the induced force, I found that all this time the inductive charges upon the disc and cylinders continually increased with the thickness. Thus when the two extremely thin slices, *b*, *r*, fig. 4, were employed alone, and the experiment taken as before in Experiment 4, the opposite electrical state of the near slice was extremely small; whereas on increasing the number of slices, and finally the extent of the body N, the induction continued to increase rapidly, and nearly in proportion to the length of N, up to a certain limit.

26. The attractive forces with these cylinders taken insulated, were found to be as the square of the number of charges accumulated, and inversely as the distance,—a result I subsequently confirmed, and found general in all cases of attractive force

between a charged and insulated neutral body. The following Table, abridged from experiments too numerous to detail here, exhibits numerical examples of this result, the charges and distances being given for forces of 1, 4, &c. grains.

TABLE III.

Distance of surfaces.	A. 1·5 inches.		B. 3 inches.		C. 6 inches.	
	Charges.	Force.	Charges.	Force.	Charges.	Force.
{·2	20	1	15−	1	9+	1
{·2	40	4	30−	4	19	4
·4	40	2	29	2	18+	2

It may be here seen that the force is as the square of the quantity divided by the distance, and may be hence represented by the general expression $F = \frac{Q^2}{D}$. We may further, by this result, deduce the force which would arise, in each case, with a unit of quantity at a unit of distance, and hence arrive at the comparative force for each altitude, A, B, C. Thus in the first line, if we take twenty charges as a unit of quantity throughout for A, B, C, we obtain, taking as before the force which would arise as the square of the number of charges (4.), the following result very nearly:—

Cylinder A. 1·5 inches high. Force 1.
Cylinder B. 3 inches high. Force 2.
Cylinder C. 6 inches high. Force 4.

By which we perceive that the forces are as the altitudes, or very nearly, the approximations being evidently very near, thus confirming the result already arrived at (25.). It is not unlikely that in this case we remove, by increasing the length of the cylinder, the similar electricity to a greater distance from the charged body; and hence the force between the dissimilar electricities is less disturbed, so that we may in this case be merely measuring the difference between the attraction of opposite electricities and the repulsion of similar ones.

27. Exp. 26. I endeavoured to observe the relation between the attractive and inductive forces more directly in the following way. Having interposed a cylindrical conductor, B, fig. 14, about three inches high, between the charged plate *f* and the suspended plate *m*, and noted the distances *n m* and *b f*, the number of charges was determined corresponding to a given induced force in B, as measured by the effect on *m*. The intermediate cylinder was now attached to the suspended disc *m* by varnished silk lines, as in fig. 15, and both suspended from the balance, so as to ascertain under precisely the same conditions of distances, *m n*, *b f*, and quantity of charge, the amount of the attraction between *b* and *f*.

By this method we obtain, 1° a measure of the induction, 2° of the corresponding attraction; and may hence compare these forces under the same or certain relative states; the distances *b f* and *m n* being either constant or variable. I examined in

this way the relative forces of induction and attraction in a great variety of cases, and obtained the following general result, viz. the attractive and induced force was either precisely the same or otherwise in the same ratio, or otherwise reducible by the application of the general laws above mentioned (4.) to the same numerical value. The following Table contains the respective forces of attraction and induction, at given distances, &c. between the opposed bodies *b*, *f*, which may perhaps be quite sufficient as an experimental illustration.

TABLE IV.

Distances.		Induction.		Attraction.	
Charged plate. Dist. *b f*.	Suspended disc. Dist. *m n*.	Quantity.	Force.	Quantity.	Force.
A. ·2	·2	9−	1	9	1
		17	4	17+	4
		27	9	26	9
B. ·2	·4	19+	1	10	1
		40	4	21	4
		60	9	32	9

28. We perceive in this Table that the first forces (A.) of attraction and induction correspond to the same quantity of electricity, or very nearly, and are therefore to be considered the same. In the second set of forces (B), where the distance of the suspended disc *m*, by which the induction is estimated, is increased, the number of charges corresponding to the inductive force is greater. Still the forces of induction and attraction are in the same ratio, as compared with the quantity of electricity. Thus we have, nearly,

Ind. quant. 19 + : Att. quant. 10 : : Ind. quant. 40 : Att. quant. 21

or

Ind. quant. 40 : Att. quant. 21 : : Ind. quant. 60 : Att. quant. 32.

But in comparing these numerical values we may reason thus: since the quantity and distance for the forces (B) differ, let them be reduced by calculation to a unit of distance and a unit of quantity; let the unit of distance be ·2, and let the unit of quantity be about nine or ten charges, as in the first line of the forces A; then taking quantity 19 + corresponding to inductive force 1, we should have for 9·5 charges, that is, half the number of charges, only one-fourth of a grain (4.); but in reducing the distance ·4 also to one half or ·2, this force would be again quadrupled, and would become one grain as before, supposing the force on the suspended disc *m* to vary as the square of the distance inversely, which by experiment it was found to do in this case sufficiently near. If we pursue the same course with the remaining experiments on the inductive forces (B), a similar result will be arrived at. Thus we may reduce the forty charges to twenty-one, the quantity for the attraction, and take the distance ·2 instead of ·4, we have then similar forces of four grains. The real state of this and similar cases, is simply this. In consequence of the increased

distance of the suspended plate, twice the number of charges accumulate before the same induced force is shown by the electrometer, although the induction is really doubled; hence in suspending B, fig. 15, the attraction takes place with half the number of charges, that is, half the induction, being the same as before (A).

29. These experiments require precision in the adjustment of the respective distances, and in the measurement of the respective quantities; the approximations may therefore be considered as being very close, especially when we take into the account the many delicate manipulations of a general character peculiar to electrical experiments. It is, however, easy to discover when the result is disturbed by errors of observation, or by accidental variations in some of the circumstances attendant on a long series of experiments. Thus in the preceding Table (B) it is quite apparent that the number of charges corresponding to the induced force would be double of those corresponding to the attractive force; and that the difference of a few sparks of the unit measure arises probably from some minute variation in some of the conditions of the experiment. I tabulated many hundred results; some of them were perfectly coincident and exact in the numerical values above given, others less so; but I had no difficulty in observing the laws above mentioned throughout.

30. It may not be amiss to observe, that, as a preliminary step, the influence of the upper disc *m*, fig. 14, was examined experimentally, since the active inductive force may be supposed to proceed more easily in proportion as this disc in a free state is placed nearer to the body B under induction. I had not, however, much trouble in simplifying the conditions of the experiment. The force between the interposed cylinder B and the disc *m*, being, within certain limits, as the squares of the distances *m n* inversely, and as the square of the number of charges directly; I was consequently enabled to select such distances and forces as were best adapted to the particular case. The influence of small variations in the distance *m n* on the inductive susceptibility of B was thus avoided. Thus in the experiments given in Table IV. (B), the intermediate cylinder underwent nearly as much inductive change with the given quantity, when the disc *m* was ·4 distance, as at ·2, as appears by the reductions just given, and by the attractive forces being the same, or very nearly, the difference in the number of charges being small. Thus in the attractive forces (A) the charges were 9, 17 +, and 26; in the attractive forces (B) they were 10, 21, 32. As the numbers refer to small measures of the unit jar, the differences upon the whole quantity are not greater than might be expected. I have obtained other results, in which the numbers were nearly alike. The experiments here recorded, however, better represent the average results.

31. The influence of a free neutral disc thus opposed to the terminating plane surface of an insulated cylinder, B, fig. 14, being such as to increase the inductive capacity of the cylinder, we cannot always estimate by this method the corresponding inductive and attractive forces. Thus in the case of the three cylinders, Exp. 25, Table III., in which the attractive force was as the altitude, we could not by this

method estimate the inductive change, since the influence of the suspended free disc would be such as to give each the same inductive susceptibility at the lower attracting surface. It is, however, to be observed, that the attractive forces are, there also, the same as in the case of taking the cylinders in a free state. In this case, therefore, the induction must be measured by a process similar to that resorted to (10.) Exp. 4.

32. Considering these results of consequence to a true theory of electricity, I thought it desirable to institute other methods of experiment, so as to expose more completely the operation of the inductive process, and at the same time verify the preceding deductions. With this view I resorted to the method represented in figs. 16 and 17, in which S T represents a cylindrical wood column attached to the foot-piece W, or substituted for the part P *g f*, fig. 1. This column carries the brass sliders S T. The slider S sustains the light tubular brass rod S V' V, and smaller sliders V V'; these support, by the glass rods V' A, V B, the conducting cylinders A, B. In like manner the sliding piece T sustains the thin slice *f*, fig. 16, forming a false upper end to B; or otherwise, if this slice be placed at the lower extremity of B, as in fig. 17, it is supported by the slider V, whilst T carries the body B. Now it is easy by this arrangement to remove, by the slider S, the bodies A and B, fig. 16, simultaneously, and without interfering with distance *a b*, so as to leave the thin slice *f* in operation on the electrometer disc *m*. Hence, if we suppose in this case, that A is charged with a given quantity, and B neutral, we may measure by the electrometer disc *m* the result of the direct inductive force upon *f*, apart from the bodies A and B, and this may be done without the result being influenced by the presence of the electrometer disc *m*, which may be temporarily turned aside during the previous process. We may also in fig. 17, supposing B charged and A neutral, examine the reflected induction by withdrawing A and the false end *b* simultaneously, and finally estimating by the electrometer disc *m*, the proportion of the whole charge abstracted by the influence of A at different distances. We may, in fact, obtain any required complicated mechanical arrangement peculiar to this kind of research, and arrive at a very complete experimental analysis of the reciprocal inductive action between the two opposed bodies, under a variety of new conditions*.

Exp. 27. The body A, fig. 16, being charged with a given quantity and placed within ·2 distance of B, the force upon the electrometer disc amounted to 20°; the distance *m f*, of the latter being ·5 of an inch. The cylinders A and B were now withdrawn simultaneously, leaving the false end *f* in place. The force amounted now to 8° only. This process was repeated with the distance $a\ b$ = ·4, in which case the remaining force was 4°, or one half the former. The induction, therefore, as expressed in degrees of the electrometer, was as the distance *a b* inversely, and, consequently, the respective quantities of electricity left on the false end *f* as $\sqrt{1} : \sqrt{2}$ (4.). The quantity of electricity displaced, therefore, varied as the square root of the distance

* To prevent the exposure of any additional surface on the removal of the false end, the cylinders were hollowed out for about an inch within the extremity, upon which the false end rested.

a b, being the law arrived at in the preceding experiments (29.). I extended this to distances ·6 and ·8, and still found the force in degrees as these distances inversely.

33. Exp. 28. The object of this experiment was to discover the resulting negative state induced in B under the influence of A, by touching it with a conducting wire and then removing A. With this view A was charged as before, and B rendered negative at the distance ·2; after this, A was withdrawn and discharged, and the negative force observed, which amounted to 20°, being the same as the previous induced positive state. This being repeated at distances ·4 and ·8, the respective negative forces were 10° and 7°, being in an inverse ratio to the distances, as before, and identical in this case with the previously induced positive forces, these last being observed upon the whole mass B whilst under the influence of A.

I verified this experiment by charging A so as to induce 5° and 20° of positive charge in B, that is to say, attractive forces in the ratio of 1 : 4, corresponding to quantities in the ratio of 1 : 2 (4.). On rendering B negative at the same constant distance *a b*, the forces were still 5° and 20° of negative charge. I found also on further repetition, that the same result ensued in multiplying the number of measures simply. Thus the negative force induced in B by three measures, being 10° at ·4 distance, six measures induced 40°, or very nearly; when therefore the charge in A is doubled, the induced negative state is also doubled in respect of quantity, since the corresponding degrees of the electrometer are quadrupled (4.). The following simple expressions may, therefore, be taken to represent this result: ind. $= \frac{Q^2}{D}$, if valued in degrees; or ind. $= \frac{Q}{\sqrt{D}}$, if valued in quantity.

34. Exp. 29. This experiment applies to the quantity of electricity influenced in B when charged by the opposed body A taken neutral and free, that is to the reflected induction (8.). The general arrangement is represented in fig. 17, in which the false end *f*, fig. 16, is placed at *b*, so as to detach it by the slider S, together with the neutral body A, by which we may discover how much of the whole quantity with which B is charged, considered as a unit of quantity, is determined as it were upon the near surface *b*, as also the respective quantities of electricity in operation between the opposed planes *a*, *b* at different distances. In conducting this experiment, the quantity with which B was charged $= m$, was first observed in degrees of the electrometer and taken as a unit of quantity, A being turned aside. Secondly, the false end *b* was removed, and the remaining quantity $= n$ also observed in degrees, so as to determine the decrease due to the removal of *b* alone $= t = m - n$. Thirdly, the false end *b* was replaced, the original charge made complete to 20° $= m$, and the body A in a free state opposed to B at a given distance. Lastly, *b* and A were under this condition withdrawn together and the remaining quantity $= p$ observed in degrees, so as to obtain the comparative quantity actually existing in *b* whilst under the influence of A $= m - p$, as also the comparative quantity determined upon *b* by induction $= (m - p) - t$: putting $m - p = S$, we have the reflected induction in quantity

$= \mathrm{S} - t$. Thus the whole quantity m being taken as unity or 1, the electrometer indicated 20° when the disc m was ·4 of an inch distant from the upper plane surface of B, the cylinder A being withdrawn. The false end b being now removed, the electrometer indicated 12°·5 : the quantity remaining therefore $= n$ was ·79 since (4.)

$$1 : n :: \sqrt{20} : \sqrt{12 \cdot 5}$$
$$:: 4 \cdot 472 : 3 \cdot 535$$

hence $n = \cdot 79$. The decrease, therefore, due to b alone is in this case $= 1 - \cdot 79 = \cdot 21 = t$, hence about $\frac{1}{5}$th of the whole of this particular charge was collected in the extremity b.

Now when b was withdrawn under the influence of A at distance ·2 then 3°·5 only remained: this corresponds to quantity ·42, nearly $= p$, since we have (4.)

$$1 : p :: \sqrt{20} : \sqrt{3^\circ \cdot 5}$$
$$:: 4 \cdot 472 : 1 \cdot 872.$$

The decrease, therefore, due to b and A together is $1 - \cdot 42 = \cdot 58 = m - n = \mathrm{S}$: hence about six-tenths of this particular charge was collected on the near surface b at distance ·2, and the quantity therefore determined there by the reflected induction is $\cdot 58 - \cdot 21 = \cdot 37 = \mathrm{S} - t$. These respective elements for different distances between the opposed surfaces of A and B determined as in the above example, are given in the next table.

Table V.

Original charge 20° = quantity 1 = m. Remaining deg. 12·5 = quantity ·79 = n. Quantity on b alone = ·21 = $m - n = t$.

a.	*b.*	*c.*	*d.*	*e.*
Distance.	Final Degrees.	Quant. = p.	Quant. = S due to A + b.	Quant. S − t due to Ind. of A.
·2	3·5	·42	·58+	·37+
·3	5·5	·53	·47	·26−
·4	7+	·59	·41	·20
·5	8	·63	·37	·16
·6	9+	·67	·33	·12+

35. It may be inferred from this table, columns d and e, that the quantity of electricity (column d) accumulated in the near extremity b of the charged cylinder, fig. 17, was as the square root of the distance from the opposed surface of the neutral body, whilst the quantity displaced by induction of A (column e) is nearly in the inverse ratio of the distance. The numerical results are not everywhere rigidly exact, but they evidently point out these laws, and are in some cases extremely close. Thus in column e we have, taking the distances ·2 and ·6, which are as 1 : 3, the inverse quantities ·12 + and ·37 +, which are as 1 : 3; so also in column d we have corresponding to the same distances the quantities ·58 and ·33, which are to each other as the $\sqrt{3} : \sqrt{1}$.

36. The result shown in column *d* is strikingly in accordance with that arrived at by a former and distinct method of experiment (15.); by which it was found, that the reflected induction, if measured by the quantities of electricity the neutral body ceases to hold in equilibrio at different distances, is as the simple distance inversely. The result also of column *d* corresponds with the law just found (Exp. 27.) for the direct induction on the neutral body. The experiments, therefore, are clearly consistent one with the other.

When different charges were taken, and the distance *a b*, fig. 17, made constant, the numerical values varied with the charge, the forces being as the square of the charge expressed in degrees of the electrometer. This was at least observed for all the charges which could be fairly brought within the experimental range of the instrument.

37. The application of these results to the phenomenon of attraction by electrical agency, is not a little interesting; they in fact help us to a more complete perception of this wonderful operation. We may perceive, for example, that when a conducting substance charged with electricity attracts another conducting body in a free neutral state, the electrical distribution is so disturbed in each as to cause an accumulation in the opposed parts *a*, *b*, fig. 17, inversely proportional to the square roots of the distances. By the laws of electrical action, therefore, before explained, (4.) and (22.), Exp. 18, we have eventually the whole force, as shown by the electrometer, as the squares of the distances inversely. For let a unit of force at a unit of distance be given; suppose, for example, at the distance one inch; the force between *a*, *b*, fig. 17, was five degrees: let the distance be now taken = ·5, or one half the former, then the quantities of electricity in the opposed surfaces will be as 1 : 1·414, that is, as $\sqrt{1} : \sqrt{2}$ inversely (32.) (35.); but the force is as the square of the quantity. The force therefore with this change would be twice as great at the distance unity; but it varies also with the distance (Exp. 18.); hence at the distance ·5, or one half the former, it is again doubled; that is to say, the forces are as 1 : 4 when the distances are as 2 : 1, being a result of two simple laws taken conjointly, as already noticed (23.): similar reasoning applies to distances $\frac{1}{3}$, $\frac{1}{4}$ . . . $\frac{1}{n}$th, &c. When, as we have before explained (22.), the inductive changes are small, and admit of the quantity at the distance unity being taken as constant, then the force is as the simple distance inversely, depending solely on the closer approximation of the electrical particles (Exp. 18), or according to Faraday, on the diminution of the number of particles of the dielectric through which the force operates. Thus in the case of the attractive force between a charged and insulated neutral conductor, the induction may be very inconsiderable in respect of the whole charge. We observe in the free state, Table V. (*e.*), where the inductive force is the greatest possible, that at the distance ·4 not above one-fifth of the whole charge was determined towards the opposed surface; now these additions by the reflected induction of an insulated body may not greatly influence the result,

especially within certain limits; we may, for example, have so little as the $\frac{1}{500}$th and $\frac{1}{250}$th only of the whole charge disturbed at distances ·4 and ·2, which slight addition to the already existing accumulation in the opposed surface may not materially affect the electrometer: hence the force may vary nearly as the distance inversely. This, together with the circumstances above given (23.), may perhaps account for the difference in the law of the force, as regards the distance, between the insulated and neutral state. There are, however, probably, other conditions of induction already mentioned (26.) applicable to this case, and which the masterly investigations of FARADAY bid fair to evolve: every one conversant with this interesting branch of science must necessarily allow, that never before has it been enriched by results so comprehensive and momentous as those contained in his several series of researches. It is also to be further considered, whether at small distances, although the force between two particles should be as $\frac{1}{D^2}$, still the force between the plates may be as $\frac{1}{D}$ simply, the whole attraction being found by a double integration, which sums all the forces, every particle of A being supposed to attract every particle of B.

38. The preceding facts lead us to refer every case of attraction in electricity, in a non-conducting or insulating medium, to the conditions of that peculiar combination of electrics and conductors, termed the Leyden jar, or coated pane, a combination consisting of an insulating body, interposed between two conducting surfaces. Now it is admitted that the charge which this combination can receive, is quite independent of the thickness of the opposed conductors, or of any hypothetical distribution upon them, or other bodies in connexion with them. Thus the charge which the electrical jar can receive under a given intensity, is as great when the coatings are mere films of metallic leaf, as when a solid mass of metal; the only condition essential to the perfect success of the experiment, is the free state of one of the coatings, and the complete insulation of the other, as also their close approximation. The action of the coatings in this case is reduced to one of these cases already given (16.), in which the distance between the bodies is constant, and the quantity of charge variable; the only difference being this, that in the case of the jar or coated pane, the intermediate insulator or dielectric is a solid, and the opposed conductors fixed, so that all motion by the resulting attraction is precluded, and all discharge between the conductors effectually prevented. The amount of charge which might be possibly collected on a small surface in this way, under a very dense atmosphere, is quite unknown. The charge might continue to accumulate until the resulting force between the opposed surfaces became so great, as to fracture the most impervious insulating substance placed between them.

39. The two following illustrations are conclusive of the general application of the laws above mentioned to the phenomena observable in accumulating electricity between two conducting surfaces under the conditions above mentioned.

1. Let A B, fig. 8, be two attracting plane areas, one of which is charged, and the

other, B, free; let the force between them with a unit of quantity = 1: suppose these areas to become now twice as great; then we have the charge distributed on twice the surface; and if we conceive it in each instance to be distributed equally, there would in the latter be then only one half the quantity in any given point, and hence, as found by experiment (16.) (18.), the indication of the force by the electrometer would become reduced to one fourth. In this state let the quantity be doubled, we have then by the same law (16.) the attractive force = 1, as before; that is to say, the charge which can be accumulated under a given attraction and distance between the plates, is as the opposed areas directly.

Now the indication of the charge by the electrometer, E, fig. 8, connected with the charged side, are, as we have seen (16.) (15.) (18.), proportional to the square of the quantity of electricity which the free surface ceases to hold in equilibrio. But the amount of charge and distance of the plates being constant, the quantity of electricity held in equilibrio will vary with the areas; and therefore if the area and quantity vary together, the electrometer will not change; hence it is, that a given number of degrees may, under this condition, correspond with any quantitative accumulation whatever.

If, then, we conceive in fig. 8. the two opposed plates to be the coatings of the intervening air, or any other dielectric, then, as just shown, the accumulation under a given intensity will be as the areas opposed.

2. Let the force between the plates A B, fig. 8, at a unit of distance *a b*, and with a unit of quantity = 1 as before, and suppose the distance *a b* to become now twice as great; then the force of attraction will be reduced to one-fourth, since it varies as $\frac{1}{D^2}$. Let the quantity under this condition be doubled, the attractive force will be = 1 as at first, since it is as the square of the quantity (4.); hence the accumulation between the opposed areas is under a constant attractive force directly as the distances between them.

Now the indications of intensity by an electrometer E, fig. 8, in connexion with the charged side, will be, as in the former instance, dependent on the reflected action of the free plate: this, taken in degrees of the electrometer, is as we have just seen (15.), as the squares of the distances inversely. If, therefore, with the quantity one, the distance *a b* be doubled, the intensity by the electrometer will be quadrupled: under this condition, let one half the quantity be again withdrawn, then the intensity by the electrometer will be the same as the first. If, therefore, we suppose, as in the preceding case, that the opposed plates are merely the coatings of the interposed dielectric, it follows that the charge under a given intensity will be as the distance between the plates inversely.

These deductions are in complete accordance with the many experiments made in this department of science by the learned Mr. Cavendish, who states in the 66th vol. of the Royal Society's Transactions, "that the quantity of electricity which coated glass can receive under the same degree of electrification is as the area of the coating directly, and as the thickness of the glass inversely."

It may be further remarked, that the force between two planes *m, f*, fig. 1, is not sensibly increased by increasing the area of one of the coatings only; is not influenced by the form or dimensions of the unopposed portions; and is the greatest possible when one of them is placed in a free state: circumstances which apply particularly to the case of charged glass. In considering the attractive force, therefore, between two conducting bodies of any form whatever, one of them being charged, the other free, it is only requisite to take into the calculation the opposed surfaces, and reason upon the inductive actions and distance according to the laws already given (27.) (32.) (34.).

40. Let for example the opposed bodies A, B be two spheres, as in fig. 18, or cones, or paraboloids, as in fig. 19, then the intervening dielectric will be of the form *a c e f b d a*, and the coatings *a d b c e f*, will be hemispherical or otherwise, according to the figures; and the unopposed portions, taking the bodies in a charged and free state, will not affect the result; they may be of any form, or have any connexion whatever with other bodies. We may always predict the attractive force, a unit of force at a unit of distance being given, on the supposition that it is as the opposed areas directly, and as the squares of the distances inversely; the electricity accumulated in the opposed points being as the square roots of the respective distances* (32.).

This general result is not vitiated by any oblique action which may appear to arise in consequence of change in the position of the opposed surfaces. Thus if the two opposed plane areas, *d, f*, fig. 20, be any how placed, provided they maintain their relative position with respect to each other, it is evident no difference can possibly arise; it would be in fact merely placing a coated pane in different angular positions. If we suppose one of the plates brought into the position *a b*, fig. 21, so as to be oblique to the other, still the same general principle applies: we may conceive the interval of air, *a n m b*, to be a solid dielectric of unequal thickness, the coatings of which are the opposed areas *a b*, *m n*; the attractive force, therefore, between the plates, *a b*, $a' b'$, would become diminished by the exposure of the unopposed portions, $n a'$, $m b'$, and by the general increase of distance. If the plates were so opposed as to cause a portion of one to project beyond the other, as in fig. 22, then the force would be reduced to the opposed portions, *a m*, *n d*, and would be diminished by the external unopposed parts *n c*, *m b*.

We suppose, however, in all these cases that the force is exerted between a neutral body in a free state, and a body charged with a quantity of electricity, considered indefinitely great with respect to the opposed surfaces; directly, however, we limit these conditions, either by insulating the neutral body or by narrowing the capacity of the charged body, then corresponding variations arise in the laws of the force, but which may be reduced to calculation on the general principles above stated (37.).

41. The influence of induction on the repulsive force evinced by similarly electrified substances is such as to merit very particular attention; I have shown in the

* Transactions of the Royal Society for 1834, p. 240.

second series of these inquiries, that the operation of electrical repulsion is subject occasionally to considerable variation, the result being dependent on quantity, intensity, distance, and a variety of other contingent circumstances*. Without taking into account, therefore, the attractive forces generated between the discs, *m*, *f*, fig. 11, I could not, by the method of experiment before employed (24.), obtain any uniform result; the repulsive force appeared irregular, and in many cases capricious, appearing sometimes as great at one distance as at another: but in calculating first the force of the disc *f* when charged, on *m* taken as neutral, and then the force of *m* similarly charged, on *f* taken as neutral, the results, with the corrections thus deduced, were uniform, and according to a given law, as may be seen in the following experiment.

Exp. 30. Two discs, *m*, *f*, fig. 11, being opposed to each other, one of them *m*, was charged with a given quantity, and the other placed in connexion with a coated jar, charged with the same electricity; the charges in the latter were estimated by the unit measure. The elementary measurements were, first, the attraction of the suspended disc *m*, charged with a given quantity, on *f*, considered as neutral $= a$; secondly, the attraction of *f*, charged with a given number of measures, on *m*, taken as neutral $= p$; lastly, the repulsive force between the bodies with a given number of measures $=$ R. The attractive forces were neutralized by small weights placed on the scale pan *p*; the arm *g* of the balance, up to the instant of the repulsion, rested on a small support projecting from the brass work supporting the beam B. The repulsive force was estimated by weights either placed on the disc *m*, or otherwise removed from the scale pan, and by which the whole had been previously brought to balance.

The annexed table comprises the numerical results of a few experiments conducted in this way; the attractive force between *m* and *f*, taking *m* charged, varied as the square of the distance inversely, or very nearly, for the quantities employed. The force, taking *f* charged, was in general so small as to admit of being neglected (Exp. 22.).

TABLE VI.

Distance.	Attr. Force.	Rep. Force.	Quantity.
·4	5	9	38
·8	1+	4·5	36
1·2	0·3	3+	36
1·6	..	2+	37

It may be seen here that the repulsive force was in the simple inverse ratio of the distance, and that the number of measures for each experiment did not greatly differ.

A similar result was obtained by means of the electrometer, fig. 1: the attractive forces were here very easily determined, and the number of corresponding degrees

* Transactions of the Royal Society for 1836.

noted, by employing charged glass, as in Experiment 22. The repulsive forces alone were obtained; they were, however, still in the simple inverse ratio of the distances.

42. These phenomena may not be unimportant to further advances in this department of science; they rather lead us to consider electrical attraction as essentially differing in its nature from forces emanating from a centre, and go far to assist us in elucidating many of its operations, hitherto considered of a complicated character. Thus the attractive force between spheres and bodies of other forms has given rise to a mathematical analysis of some difficulty*, which although displaying the highest order of talent is certainly not indispensable. We may on very simple principles determine, as already shown (40.), the laws of the force between bodies of any form, whether insulated or free, whether charged positively or negatively, or whether electrics or conductors.

43. The following are a few simple expressions which may be taken to represent some of the elementary laws of electrical induction and attraction, in which Q = quantity of charge, T the direct induction, q the quantity of electricity displaced, t its intensity, T' the reflected induction, q' the disturbed quantity, t' its intensity, q'' the total quantity in the opposed charged surface, A the surface, D the distance between the opposed points, F = force of attraction.

We have then for the direct induction

$$T = q = \frac{Q}{\sqrt{D}}, \quad t = \frac{Q^2}{D}.$$

For the reflected induction T' we have

$$T' = q' = \frac{Q}{D}, \quad t' = \frac{Q^2}{D^2}, \quad q'' = \frac{Q}{\sqrt{D}}.$$

We have for the attractive force between a charged and neutral free conductor

$$F = \frac{Q^2}{D^2}, \quad F = \frac{T}{A^2}.$$

For the force between an unchangeable positive and negative surface we have

$$F = \frac{Q^2}{D}.$$

44. In these inquiries I have not resorted to the view of electrical action I was led to entertain in the first series of these papers, in which a portion only of the whole charge is supposed to be appreciable by the electrometer, being unwilling to embarrass the inquiry with theoretical speculations not essential to a full development of the experimental facts. I may, however, still observe, that the present state of this department of science does not warrant any very perfect confidence in the common mechanical explanations of the mode of operation of electrical forces generally, and which after all seems to be of a cumbrous and difficult character. If we suppose a particle of the electricity = a on a charged body A, fig. 8, to attract every par-

* Supplement Encyclop. Britt., Article ELECTRICITY.

ticle of the opposite electricity = b on the opposed body B, instead of supposing the force to be confined to the near particle immediately opposed to it, we cannot take all the forces as equal, and the whole force to be as $a\ b$; it must still be only as some function of it, and we have to sum the forces under this condition. But in this case even, all the oblique actions may at last be indefinitely small in respect of the force exerted upon the opposite and nearest particle, which would still admit of the whole action being reduced to a system of parallel forces, such as represented in fig. 18, more especially when we take into consideration the fact that the force between two particles, *a*, *b*, fig. 23, is greatly diminished, and may become very small by placing a similar third particle, *d*, nearer either of them, supposing it charged with the opposite electricity; and this result will be again augmented by placing another particle, *e*, nearer the other in a similar way. In this case the action between the original particles *a*, *b* almost vanishes.

45. In concluding this communication it may not be improper to state, that the experiments were conducted in a good insulating atmosphere, generally in a room dried by an air stove: the late contrivance of Dr. Arnott is quite invaluable to the practical electrician for this purpose. The manipulations requiring especial care are, 1. Measure of quantity; 2. of distance; 3. Adjustment of the electrometer, especially in such experiments as No. 26; 4. Perfection of the insulators. The circumstances liable to interfere with a rigid numerical result are, slight changes in the position of the bodies under experiment,—the bodies should be firmly steadied; inaccuracy in the value of the unit measures, either by less perfect insulation of the air, or by other causes; a want of free connexion of the external coating of the jar, P, fig. 16, with the ground; small residuary charges in the discs of the electrometer; these should be always discharged by a bent wire at each experiment. A series of delicate manipulations of this kind, although apparently difficult, may yet by a little habit and attention be completely managed.

Plymouth,
April 10, 1839.

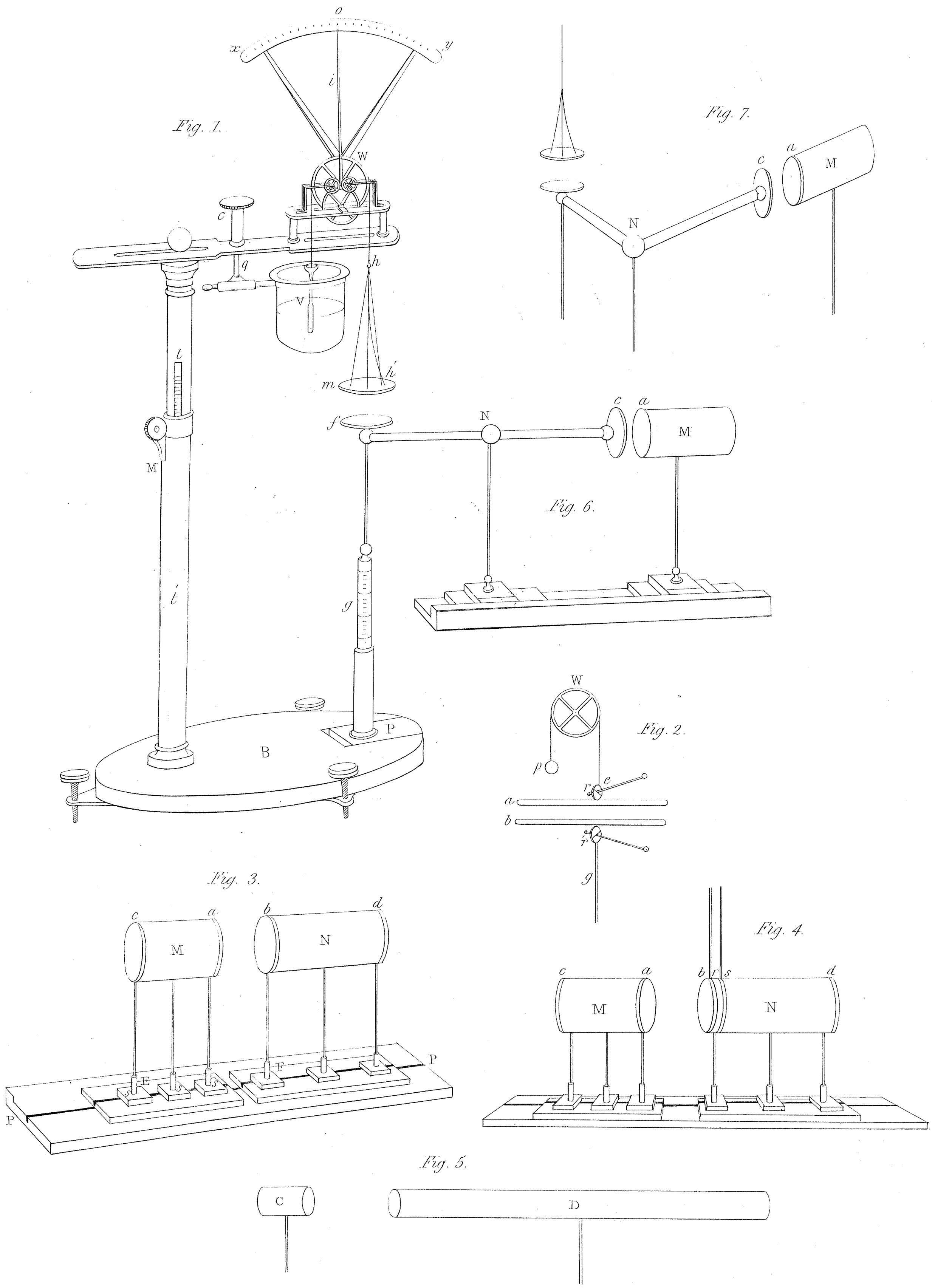
Fig. 1.
o
x
y
i
W
c
q
h
V
t
m
h'
M
t'
B
P
Fig. 7.
a
M
c
N
Fig. 6.
f
N
c
a
M
g
Fig. 2.
W
p
r
e
a
b
r'
g
Fig. 3.
c
a
b
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Fig. 4.
c
a
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Fig. 5.
C
D

J. Basire. sc.