III. *Researches in Physical Geology.—Third Series.* *By* W. Hopkins, *Esq., M.A., F.R.S., Fellow of the Royal Astronomical Society, of the Geological Society, and of the Cambridge Philosophical Society.*

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On the Thickness and Constitution of the Earth's Crust.

1. The result at which I arrived in my preceding memoir, respecting the precessional motion of the pole, on the hypothesis of the earth's interior fluidity, is the following:

$$P' - P_1 = \left(1 - \frac{\varepsilon}{\varepsilon_1}\right)\left(1 - \frac{\eta}{1 + \frac{h}{q^5 - 1}}\right) P_1, \quad \ldots \ldots \ldots \quad (1.)$$

where P_1 denotes the precession of a solid homogeneous spheroid of which the ellipticity $= \varepsilon_1$, that of the earth's exterior surface, and P' the precession of the earth, supposing it to consist of an interior heterogeneous fluid, contained in a heterogeneous spheroidal shell, of which the interior and exterior ellipticities are respectively ε and ε_1, the transition being immediate from the entire solidity of the shell to the perfect fluidity of the interior mass.

2. In the application of this result to the actual case of the earth, we must observe, in the first place, that, supposing the interior mass to be fluid, its fluidity cannot be quite *perfect*, as explained in the introductory observations to my first memoir. Consequently the assumption, made in our investigations, of the absence of all tangential action between the shell and fluid will not be accurately true. Moreover, it would seem extremely probable that the transition from the solidity of the shell to the fluidity of the interior mass, instead of being immediate, must be gradual. If, however, the thickness of the shell be not considerable with reference to the earth's radius, the fluidity of the portion not remote from the centre will be nearly perfect, and the whole of the interior mass, with the exception of that part near the actual solid portion, may, as a near approximation, be considered as perfectly fluid with reference to any mechanical action upon it. Again, supposing the change from the solidity of the exterior to the fluidity of the interior mass to be continuous, let us conceive a surface passing through all the lowest points of that portion of the mass which may be regarded as perfectly solid, and another surface through all the highest points of that portion which may be regarded as perfectly fluid. The first of these surfaces will be one of equal solidity and the second one of equal fluidity; the fluidity of the mass contained between them being imperfect. Now if we were to consider

the whole of this imperfectly fluid mass as entirely solid, we should manifestly take the thickness of the shell too large to represent the actual phenomena depending on that thickness; and if, on the contrary, we should consider the whole of the imperfectly fluid portion as perfectly fluid, we should take the thickness of the shell too small. Hence there must be some surface of equal fluidity (or, if we please so to term it, of equal *solidity*) intermediate to the above surfaces, such that if the whole mass superior to it were entirely solid, and that inferior to it entirely fluid, the phenomena of precession and nutation would be the same as in the actual case of a gradual transition from the solidity of the superior to the fluidity of the inferior portions of the mass. When, therefore, we speak of the interior surface of the solid shell with reference to our previous investigations as applicable to the case of the earth, it is this intermediate, or *effective surface*, which is always to be understood; and the thickness of the earth's crust, as defined by this surface, I shall term its *effective thickness*.

3. In order that the value of P' may agree with that determined by observation, we must have approximately

$$P' - P_1 = \frac{1}{8} P_1,$$

and, therefore, referring to the equation of Article 1, we must have

$$\left(1 - \frac{\varepsilon}{\varepsilon_1}\right)\left(1 - \frac{\eta}{1 + \frac{h}{q^5 - 1}}\right) = \frac{1}{8}. \quad \ldots\ldots\ldots\ldots \quad (2.)$$

An approximate value of $\frac{h}{q^5 - 1}$ will be obtained by making ϱ constant in the expression given for it in Article 5, Mem. II. We then have

$$\frac{h}{q^5 - 1} = \frac{2\,a^5}{a_1^{\,5} - a^5},$$

$$= \frac{2}{q^5 - 1}, \qquad \left(q^5 = \frac{a_1}{a}\right);$$

which gives

$$\left(1 - \frac{\varepsilon}{\varepsilon_1}\right)\left(1 - \eta \,.\, \frac{q^5 - 1}{q^5 + 1}\right) = \frac{1}{8}.$$

It has been stated (Art. 2. Mem. II.), that if η be ever negative, it can only be so when $a = a_1$ very nearly. But in this case $q^5 - 1$ is extremely small, and therefore the value of the second factor on the left-hand side of the above equation will be very nearly = unity. In all other cases it will be less than unity. Let it $= 1 - \beta$; then

$$1 - \frac{\varepsilon}{\varepsilon_1} = \frac{1}{8} \cdot \frac{1}{1 - \beta},$$

and

$$\frac{\varepsilon}{\varepsilon_1} = 1 - \frac{1}{8} \cdot \frac{1}{1 - \beta}.$$

If, as an approximation, we omit β (which will necessarily be considerably smaller

than unity for such values of $\frac{a_1}{a}$ as we shall be concerned with), we have

$$\frac{\varepsilon}{\varepsilon_1} = 1 - \frac{1}{8} = \frac{7}{8}, \quad \ldots \ldots \ldots \ldots \quad (3.)$$

which gives *too great* a value of ε. Now, that we may be able to satisfy equation (2.), ε must be less than ε_1, *i. e.* it must diminish as the thickness of the earth's crust increases; and, therefore, the thickness which corresponds to this approximate value of ε will be *too small*; or the actual thickness of the solid crust of the globe which would give the precession P′, must necessarily be *greater* than that for which the value of ε is $\frac{7}{8}\varepsilon_1$.

4. We must now proceed to determine the relation between the value of $\frac{\varepsilon}{\varepsilon_1}$ and that of $a_1 - a$, the thickness of the solid crust.

If we assume (as I shall now do) that the fusibility of the matter composing the earth is equal at equal depths*, it would seem that the only conceivable causes which can affect the degree of solidity or fluidity of the mass, are *temperature* and *pressure*. It may be doubted by some persons whether solidification be actually promoted by the latter cause or not; but there will be no corresponding uncertainty in our conclusions respecting the minimum thickness of the earth's crust consistent with the observed amount of precession; because, if they be true, this cause being effective, they will easily be seen to be so, *à fortiori*, if it produce no effect.

If temperature produced no effect in solidification, the surfaces of equal solidity (or fluidity) would be surfaces of equal pressure, and therefore of equal density; and if pressure did not promote solidification, the surfaces of equal solidity would be isothermal surfaces. Assuming both causes to be effective, conceive two surfaces of equal density and temperature respectively, passing through the same point (in the axis of the spheroid, for instance); then will the surface of equal solidity through the same point be intermediate to the two former, the ellipticities of which will therefore be *limits* to that of the surface of equal solidity. It is these limits which we must now proceed to determine.

The greatest difficulty in the determination of the temperature at any point of a body cooling by conduction, is that which arises from satisfying the conditions at the surface in each particular case. This has been effected only in the sphere, the circular cylinder, and a few other simple cases, but not including that of the spheroid, the *isothermal surfaces* of which, consequently, have never been completely determined†.

* This may admit of local exceptions, such as probably exist, without any sensible modifications in our general conclusions.

† An ingenious memoir on this subject by M. LAMÉ is contained in the fifth volume of the 'Mémoires des Savans Etrangers,' in which he has examined the conditions under which the isothermal surfaces within an ellipsoid will also be ellipsoids, when it has arrived at a permanent state of temperature. He has also made the general expression for the temperature at any time to depend on the integration of certain differential equa-

The approximate solution which I am about to offer is founded on the following assumption. Let r' be the distance of any point on the surface of the spheroid from the centre, and a_1 the polar or least distance; it is assumed that the temperature in the spheroid at a depth $= r' - a_1$ is the same as it would have been at that depth if the spheroid had been a sphere with radius $= r'$. If the ellipticity be small, and the process of cooling has been continued a sufficient length of time, this assumption will manifestly be almost accurately true. The solution of the problem is thus made to depend on formulæ which have been given by POISSON in his *Théorie de la Chaleur*. To this I now proceed.

§. *Determination of the Forms of Isothermal Surfaces within the Earth.*

5. Adopting POISSON's notation*, let u denote, at any time t, the temperature of any point of the earth, and let

$$u = u_1 + u',$$

where u_1 is such as to satisfy the general differential equation for the propagation of heat by conduction, the conditions relative to the original temperature, and that which would exist at the surface at any time if the earth were a sphere whose radius $= a_1$. Then when t becomes very great, as it is assumed to be in the case of the earth, taking the common expression for the temperature in a sphere of large radius, as a sufficient approximation to that of the actual case of the earth, we have

$$u_1 = \mathrm{C}\,\frac{a_1}{\pi r}\left(\sin\frac{r}{a_1}\pi - \frac{\pi r}{b\,a_1^2}\cos\frac{\pi r}{a_1}\right)\varepsilon^{-\frac{\alpha^2\pi^2}{a_1^2}t},$$

where C is the value of u_1 at the centre†. At the surface the first term vanishes, and the value of u_1 is reduced to the second term, which, however, is so small that it may here be altogether neglected. Consequently

$$u_1 = \mathrm{C}\,\frac{a_1}{\pi r}\sin\frac{r}{a_1}\pi\,\varepsilon^{-\frac{\alpha^2\pi^2}{a_1^2}t}.$$

Let ζ denote the value of u at any point for which $r = a_1$, ζ being a function of t and of θ, the angle which r makes with the axis of revolution of the spheroid; or since (t being very large) $u_1 = 0$, approximately when $r = a_1$, ζ may be taken as the value of u' for that value of r. It remains to find a value of u' which shall satisfy the general differential equation, and the particular condition $u' = \zeta$ when $r = a_1$‡.

Let

$$r\,u' = \mathrm{U}_0 + \mathrm{U}_1 + \ldots. + \mathrm{U}_n + \ldots.,$$

$\mathrm{U}_0 \ldots. \mathrm{U}_n \ldots.$ being a series of LAPLACE's coefficients, and functions of the polar coordinates of the proposed point. Also let

$$\zeta = (\mathrm{Z}_0 + \mathrm{Z}_1 + \ldots. + \mathrm{Z}_n + \ldots.)\,\varepsilon^{-mt},$$

tions of one independent variable, and of the second order. The complicated form of these equations, however, would seem to render them at present of little avail in the solution of the problem considered in the text.

* Théorie de la Chaleur, Art. 173. † Ibid. Art. 171. ‡ Ibid. Art. 173.

a series likewise of LAPLACE's coefficients, independent of r. Then shall we have*

$$U_n = R_n Z_n \varepsilon^{-mt},$$

where $R_n = M_n r^{n+1} \int_0^\pi \cos\left(\frac{r\sqrt{m}}{a}\cos\omega\right) \sin^{2n+1}\omega \, . \, d\omega$, M_n being an arbitrary constant. Hence if we denote the value of the definite integral by N_n, we shall have

$$u' = \left\{ M_0 N_0 Z_0 + \ldots + M_n N_n r^n Z_n + \ldots \right\} \varepsilon^{-mt}.$$

When $r = a_1$, u' must $= \zeta$; and therefore if the corresponding value of N_n be denoted by (N_n), we shall have

$$M_0 (N_0) Z_0 + \ldots + M_n (N_n) a^n_1 Z_n + \ldots = Z_0 + \ldots + Z_n + \ldots,$$

which gives for the determination of M_n,

$$M_n = \frac{1}{a_1^n (N_n)};$$

whence

$$u' = \left\{ \frac{N_0}{(N_0)} Z_0 + \ldots + \frac{N_n}{(N_n)} \cdot \frac{r^n}{a_1^n} Z_n + \ldots \right\} \varepsilon^{-mt}.$$

According to our assumptions for the value of ζ (Art. 4.), we must have ζ equal to the temperature (u_1) in a sphere, whose radius $= r'$, at a distance $r' - a_1$ from its surface. Therefore†

$$\zeta = C \, . \, \frac{1}{b\,r'} \left\{ 1 + b\,(r' - a_1) \right\} \varepsilon^{-\frac{\alpha^2 \pi^2}{r'^2} t};$$

or omitting $\frac{1}{b\,r'}$,

$$\zeta = C \, \frac{r' - a_1}{r'} \, \varepsilon^{-\frac{\alpha^2 \pi^2}{r'^2} t},$$

C being the temperature at the centre. Or since

$$r' = a_1 \, (1 + \varepsilon_1 \cos^2 \theta),$$

$$\zeta = C \, \varepsilon_1 \cos^2 \theta \, \varepsilon^{-\frac{\alpha^2 \pi^2}{a_1^2} t},$$

omitting smaller terms.

It is here supposed that the temperature of the surface of the earth is constant and equal to zero. If we take into account the variation of external temperature in passing from the pole to the equator, we have only to consider ε_1 as the ellipticity of that surface of equal temperature which touches the earth's surface at the equator, the temperature there being also assumed as zero. Then ε_1 will be rather greater than the ellipticity of the earth.

Putting the expression for ζ under the form of LAPLACE's coefficients, we have

$$\zeta = C \, \varepsilon_1 \left\{ \frac{1}{3} + \left(\cos^2 \theta - \frac{1}{3} \right) \right\} \varepsilon^{-\frac{\alpha^2 \pi^2}{a_1^2} t},$$

$$= \left\{ \frac{C \, \varepsilon_1}{3} + C \, \varepsilon_1 \left(\cos^2 \theta - \frac{1}{3} \right) \right\} \varepsilon^{-\frac{\alpha^2 \pi^2}{a_1^2} t},$$

* Théorie de la Chaleur, Art. 173.

† Ibid. Art. 171.

which gives $Z_n = 0$, except for $n = 0$, or $n = 2$; and

$$Z_0 = \frac{C\,\varepsilon_1}{3}, \quad Z_2 = C\,\varepsilon_1\left(\cos^2\theta - \frac{1}{3}\right);$$

also

$$m = \frac{\alpha^2\pi^2}{a_1^{\,2}}.$$

Hence

$$u^1 = C\,\varepsilon_1\left\{\frac{1}{3}\left(\frac{N_0}{(N_0)} - \frac{N_2}{(N_2)}\cdot\frac{r^2}{a_1^{\,2}}\right) + \frac{N_2}{(N_2)}\cdot\frac{r^2}{a_1^{\,2}}\cos^2\theta\right\}\varepsilon^{-\frac{\alpha^2\pi^2}{a_1^2}t}.$$

It remains to determine N_0, (N_0), N_2, and (N_2). We have

$$N_n = \int_0^{\pi}\cos\left(\frac{r}{a_1}\pi\cos\omega\right)\sin^{2n+1}\omega\,d\omega;$$

or putting $\cos\omega = x$,

$$N_n = \int_{-1}^{+1}\cos\left(\frac{r}{a_1}\pi\,.\,x\right)(1 - x^2)^n\,dx;$$

and performing the integrations for $n = 0$ and $n = 2$, we obtain

$$N_0 = \frac{2}{\pi}\cdot\frac{a_1}{r}\sin\frac{r}{a_1}\pi,$$

$$N_2 = \left(\frac{48}{\pi^5}\cdot\frac{a_1^{\,5}}{r^5} - \frac{16}{\pi^3}\cdot\frac{a_1^{\,3}}{r^3}\right)\sin\frac{r}{a_1}\pi - \frac{48}{\pi^4}\cdot\frac{a_1^{\,4}}{r^4}\cos\frac{r}{a_1}\pi;$$

and putting $r = a_1$, we have

$$(N_0) = 2,$$

$$(N_2) = \frac{48}{\pi^4}.$$

Hence

$$\frac{N_0}{(N_0)} = \frac{1}{\pi}\cdot\frac{a_1}{r}\sin\frac{r}{a_1}\pi,$$

$$\frac{N_2}{(N_2)} = \left(\frac{1}{\pi}\cdot\frac{a_1^{\,5}}{r^5} - \frac{\pi}{3}\cdot\frac{a_1^{\,3}}{r^3}\right)\sin\frac{r}{a_1}\pi - \frac{a_1^{\,4}}{r^4}\cos\frac{r}{a_1}\pi;$$

and by substitution, we have the complete value of u'; and for that of u we have

$$u = C\left\{\frac{a_1}{\pi r}\sin\frac{r}{a_1}\pi + \frac{\varepsilon_1}{3}\left(\frac{N_0}{(N_0)} - \frac{N_2}{(N_2)}\frac{r^2}{a_1^{\,2}}\right) + \varepsilon_1\frac{N_2}{(N_2)}\frac{r^2}{a_1^{\,2}}\cos^2\theta\right\}\varepsilon^{-\frac{\alpha^2\pi^2}{a_1^2}t};$$

or putting

$$\frac{u\,\varepsilon^{\frac{\alpha^2\pi^2}{a_1^2}t}}{C} = G,$$

we have for the equation to the isothermal surface, of which the temperature is u at the time t,

$$G = \frac{a_1}{\pi r}\sin\frac{r}{a}\pi + \frac{\varepsilon_1}{3}\left(\frac{N_0}{(N_0)} - \frac{N_2}{(N_2)}\frac{r^2}{a_1^{\,2}}\right) + \varepsilon_1\frac{N_2}{(N_2)}\frac{r^2}{a_1^{\,2}}\cos^2\theta.$$

Suppose

$$r = a_1 - x,$$
$$= a_1\left(1 - \frac{x}{a_1}\right),$$

where $\frac{x}{a_1}$ is small compared with unity. Then we have approximately

$$\frac{a_1}{\pi r}\sin\frac{r}{a_1}\pi = \frac{x}{a_1}\left(1 + \frac{x}{a_1}\right),$$
$$\frac{N_0}{(N_0)} = \frac{x}{a_1}\left(1 + \frac{x}{a_1}\right),$$
$$\frac{N_2}{(N_2)}\cdot\frac{r^2}{a_1^2} = 1 - \frac{\pi^2 - 9}{3}\cdot\frac{x}{a_1};$$

and therefore

$$G = \left(1 + \frac{\varepsilon_1}{3}\right)\frac{x}{a_1}\left(1 + \frac{x}{a_1}\right) - \frac{\varepsilon_1}{3}\left(1 - \frac{\pi^2 - 9}{3}\cdot\frac{x}{a_1}\right) + \varepsilon_1\left(1 - \frac{\pi^2 - 9}{3}\cdot\frac{x}{a_1}\right)\cos^2\theta.$$

We may here omit $\left(\frac{x}{a_1}\right)^2$ and the products of $\frac{x}{a_1}$ and ε_1, except in the last term, in which we may substitute for $\frac{x}{a_1}$ its approximate value, G. We have thus (putting $a_1 - r$ for x),

$$G = 1 - \frac{r}{a_1} + \left(1 - \frac{\pi^2 - 9}{3}G\right)\varepsilon_1\cos^2\theta;$$

whence

$$\frac{r}{a_1} = (1 - G)\left\{1 + \frac{1 - \frac{\pi^2 - 9}{3}G\,\varepsilon_1\cos^2\theta}{1 - G}\right\};$$

and

$$r = (1 - G)\,a_1\left[1 + \left\{1 + \left(1 - \frac{\pi^2 - 9}{3}\right)G\right\}\varepsilon_1\cos^2\theta\right],$$

the aproximate equation to the isothermal surface.

Hence it appears that the ellipticities of the isothermal surfaces within the earth are *greater* than that of the surface. Thus if $G = \frac{x}{a_1} = \frac{1}{10}$, we have

$$\text{ellipticity} = (1 + \cdot 07)\,\varepsilon_1 \text{ nearly.}$$

It will also be observed that it increases with G, i. e. with the depth. A further approximation gives a somewhat slower rate of increase, but the inference from the above formula is sufficient for our purpose.

§. *Ellipticity of any Surface of equal density within the Earth.*

6. If we assume the density of the earth (ϱ) at any distance (a) from its centre to be such that

$$\varrho = A\frac{\sin q' a}{a},$$

(A being constant), and take $q' a = 150°$, we obtain a value of ε_1 (the ellipticity of the surface) which coincides very nearly with its observed value, as shown in the common treatises on the figure of the earth. This expression for ϱ also gives us the ratio of the mean to the superficial density equal to 2·4225*, which agrees very nearly with the value determined by CAVENDISH. It therefore appears extremely probable that this formula represents very approximately the actual law of the earth's density.

The above expression for ϱ gives us

$$\frac{\varepsilon}{\varepsilon_1} = \frac{\tan q' a_1 - q' a_1}{\tan q' a - q' a} \cdot \frac{\left(1 - \frac{3}{q'^2 a^2}\right) \tan q' a + \frac{3}{q' a}}{\left(1 - \frac{3}{q'^2 a_1^2}\right) \tan q' a_1 + \frac{3}{q' a_1}}.$$

If we here substitute the above value of $q' a_1$, and put $a = \frac{3}{4} a_1$, we obtain a value of ε which nearly satisfies equation (3.) (Art. 1.).

If we take $q' a_1 = 160°$ we obtain the mean density more than three times the superficial density, and a value of ε_1 not so nearly coinciding with the observed value as in the former case. In this case the formula probably gives us a density increasing too rapidly with the depth, and therefore also a too rapid decrease of ellipticity in the surfaces of equal density. To obtain a value of ε, which will satisfy equation (3.), we must put a equal to about $\frac{4}{5} a_1$.

§. *Thickness of the Earth's Crust.*

7. If the surfaces of equal solidity were coincident with those of equal density, and we adopted the first value of $q' a_1$ mentioned in the preceding article, we should obtain the *effective thickness* of the crust $(= a_1 - a) = \frac{a_1}{4} = 1000$ miles; or if we adopt the other value of $q' a_1$ as less favourable to a great thickness of the crust, we shall have that thickness = 800 miles. But the surface of equal solidity through any point must be intermediate between those of equal density or pressure, and of equal temperature through the same point; and we have seen (Art. 5.) that the ellipticity of the latter increases with the distance from the external surface. Consequently the ellipticity of every surface of equal solidity must be greater than that of the corresponding surface of equal density, and, therefore, the effective thickness of the crust must be greater than that above determined, in order that it may be consistent with the observed amount of precession.

The thickness of the actually solid portion of the earth's crust will necessarily be less than what I have termed the effective thickness, but there cannot, I conceive, be any reasonable doubt that the difference between these quantities is small compared with either; for if τ_1 be the highest temperature at which any substance retains the property of solidity, and τ_2 the lowest at which it acquires that of fluidity, $\tau_2 - \tau_1$ is

* AIRY's Tracts, p. 178.

always found to be small compared with τ_1; and by analogy we conclude that such must be the case also with respect to the matter composing the earth, and under the pressure to which it is subjected at great depths. We may also remark, that the position of a surface of equal solidity or fluidity must necessarily so far incline to the corresponding surface of equal temperature as to differ materially from that of equal density, so that the real effective thickness of the crust is probably considerably greater than its inferior limit as above determined. Upon the whole, therefore, we may venture to assert that the minimum thickness of the crust of the globe which can be deemed consistent with the observed amount of precession, cannot be less than one-fourth or one-fifth of the earth's radius.

§. *Constitution of the Earth's Crust.*

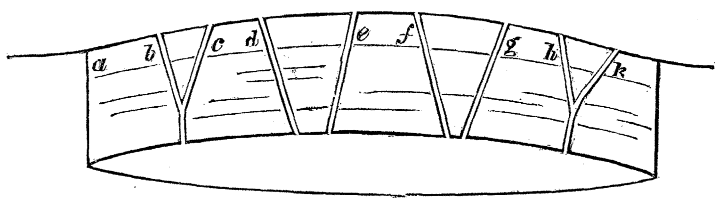
8. The results at which we have arrived respecting the thickness of the solid crust of the globe, have an important bearing on our physical theories of volcanic forces, and the mode in which they act, whether we consider the subject with reference to existing volcanos, or to that more general volcanic action to which we refer all the geological phenomena of elevation. Many speculations respecting actual volcanos have rested on the hypothesis of a direct communication, by means of the volcanic vent, between the surface and the fluid nucleus beneath, assuming the fluidity to commence at a depth little, if at all, greater than that at which the temperature may be fairly presumed to be such as would suffice, under merely the atmospheric pressure, to fuse the matter of the earth's crust*. When it is proved, however, that that crust must be several hundred miles in thickness, the hypothesis of this direct communication is placed, as I conceive, much too far beyond the bounds of all rational probability to be for an instant admitted as the basis of theoretical speculations. We are necessarily led, therefore, to the conclusion that the fluid matter of actual volcanos exists in subterranean reservoirs of limited extent, forming subterranean *lakes*, and not a subterranean *ocean*. Such also we conclude from the present thickness of the earth's crust, must have been the case for enormous periods of time; and, consequently, that there is a very high degree of probability that the same was true at the epochs of all the great elevations which we recognize, with the exception, perhaps, of the earliest. If, moreover, we find that the hypothesis of the existence of these subterranean lakes at no great depth beneath the surface, does enable us to account distinctly, by accurate investigations founded on mechanical principles, for the phenomena of elevation and the laws which they follow, then have we all the proof of the truth of our hypothesis which the nature of the case will admit of. These investigations I have given in my memoir on Physical Geology, published in the sixth volume of the Transactions of the Cambridge Philosophical Society. The fundamental

* Some of the most ingenious and determinate speculations of this kind are contained in a paper by Professor Bischoff, in the Edinburgh New Philosophical Journal for 1838–39. His views respecting the immediate agency by which volcanic action is produced will be equally applicable, whether the reservoir of volcanic matter beneath be of limited extent, or the central nucleus itself.

hypothesis of these investigations is thus found to be in perfect harmony with the results of the remoter researches contained in this and my two preceding memoirs.

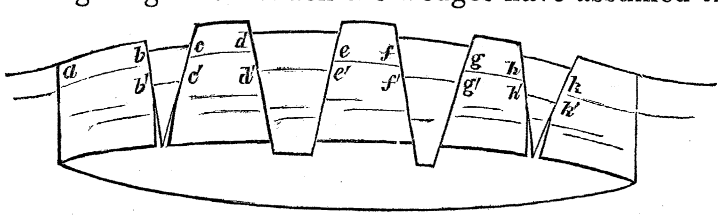
9. A question here arises as to the origin and continued existence of these insulated fluid masses enveloped in the solid portions of the earth. It would seem probable, I think, that their origin may be ascribed to the greater fusibility of the matter composing them; and their continuance in a state of fluidity may, I conceive, be accounted for partly by the same cause, and partly by another which I will proceed to explain.

Let us conceive an internal lake to have been formed as above supposed, or in any other manner, at a temperature which would just admit of the containing rock becoming solid, while it sufficed to preserve the fluid mass in a state of fusion. Let us then suppose an elevatory force, produced by the expansion of the fluid matter*, to raise the superincumbent solid mass, and to form in it a system of fissures. The plane of these fissures will scarcely ever be exactly parallel, and therefore will meet if sufficiently produced. Let the annexed diagram represent a transverse section of



(1.)

the system the instant after their formation, and before any relative displacement by further elevation, of the portions of the general mass contained between contiguous fissures, and forming so many complete or truncated wedges. The formation of these fissures will be completed at nearly the same instant of time†. Conceive the mass to be then still further uplifted. If every portion were raised equally, the width of the fissures would be increased, but such will not be the case. For the complete wedges, not reaching down to the fluid mass, will not be immediately acted upon by it at all, and the truncated wedges, whose narrower sides are downwards, will be acted on by the fluid pressure with less force in proportion to their masses, than those of which the broader sides are downwards. These latter portions, therefore, will be more elevated than the others, and the whole will assume a position like that represented in the following diagram. When the wedges have assumed these positions, it



(2.)

* Whatever difficulty there may be in fully explaining the causes of such expansion, there can be no doubt of the existence of such causes in aggregations of matter in a state of fusion as here supposed. The intensity with which they may act is attested by actual volcanos, as well as by the masses which must have been ejected at former epochs, in a state of fusion.

† See memoir on Physical Geology, above referred to.

will be impossible for the mass to subside, when the elevatory force shall cease, into that position which it originally occupied. It will be formed into an arch capable (if its abutments be sufficiently strong) of entirely or partially supporting itself. Consequently, if the cause producing the intumescence of the fluid mass cease to act, and that mass return nearly to its original dimensions, the pressure of the superincumbent solid mass may, in the manner now described, be entirely or partially removed from the fluid. Hence, assuming that solidification is promoted by great pressure, it evidently appears how a portion of the interior mass might be maintained in a state of fluidity by the removal of a superincumbent pressure, which would otherwise have brought it to a state of solidity.

It is not here essential to suppose that the arch shall entirely support itself. It may be partly supported by the fluid beneath, or it may break down in certain points, or along certain lines, and form there new supports, intermediate to the extreme ones. Instead of one continuous internal lake, a number may thus be formed, connected with each other by more or less obstructed channels of communication, as I have supposed in the exposition of my theoretical views on the Elevation of the Wealden District, recently laid before the Geological Society. It is the existence of subterranean lakes under this form, which best enables us, as I conceive, to account for the observed phenomena of elevation.

10. The above view of the relative displacements of the different portions of an uplifted and disrupted solid mass, as resulting from its geological elevation, is strongly confirmed by its enabling us to account so completely for the law, first recognized by Mr. Phillips, and which I have myself verified in numerous instances, in the relative displacement of the beds on opposite sides of a fault. In diagram (2.) we observe that the line $f' g'$ is relatively depressed below $e f$, with which it was originally continuous; *i. e.* the beds are *lowest* on that side of the fault *towards* which the plane of the fault inclines from the vertical in *descending*. This precisely accords with the law above alluded to. It will be observed to hold at each of the faults represented in the diagram, and probably admits of fewer exceptions than almost any other law observable in the phenomena of elevation.

§. *Permanence in the Inclination of the Earth's Axis.*

11. To the conclusions above deduced from the investigations of my two preceding memoirs, I may add that of the permanence in the mean inclination of the earth's axis to the plane of the ecliptic. This permanence has been frequently insisted on, and is highly important with reference to our speculations on the causes of those changes of superficial temperature which certain geological phenomena seem so unequivocally to indicate. The proof, however, which has hitherto been given of this constancy of inclination has rested on the hypothesis of the entire solidity of the globe, an assumption which, whatever may be the actual state of our planet, can never be admitted as necessarily applicable to it at all past epochs of time, at which organic

forms may have existed on its surface. My previous investigations have now demonstrated the truth of this conclusion as applicable to the earth from the first formation of its external solid shell. All such hypotheses, therefore, as have sometimes been made with respect to a change in the position of the earth's axis are entirely excluded, whether we suppose the interior of the earth to be now, or to have been heretofore, solid or fluid. A fact also, in itself not uninteresting, is thus established in the earliest history of our globe.

Nor would it have been possible, I conceive, to arrive at this result by any general considerations immediately derivable from the nature of our problem, and independent of its complete solution. The investigations contained in my two preceding memoirs were, in fact, commenced under the impression that the solution of this problem of the precession and nutation of the earth's axis on the hypothesis of the interior fluidity of the earth, would probably lead to results different from those which had been long before obtained on the supposition of the earth's entire solidity. This impression was founded on the consideration of the great difference between the direct action of a force on a solid, and that on a fluid mass, in its tendency to produce rotatory motion. We have seen, in fact, that the disturbing forces of the sun and moon do not tend to produce directly any motion in the interior fluid, in which the rotatory motion causing precession and nutation, is produced indirectly by the effect of the above forces on the position of the solid shell. A modification is thus produced in the effects of the centrifugal force, which (as appears from the results of our investigations) compensates for the want of any direct effect from the action of the disturbing forces. This compensation will scarcely, perhaps, be deemed less curious than many of those which have been recognized in the solar system, and by which, amidst apparently conflicting causes, its harmony and permanency are so beautifully preserved.

§. *Condition respecting the Temperature of Fusion for the matter composing the Earth, in order that its actual Temperature may be due to its original Heat.*

12. There is also another conclusion to be drawn from our investigations which it may be worth while to notice. It has been assumed in these memoirs that pressure is effective in producing solidification; it has been already remarked, however, that should that not be the case, our conclusions respecting the thickness of the earth's crust will still, *à fortiori*, be true. Our determination, therefore, of the least limit to that thickness is independent of this unknown effect of pressure, or, in other words, of the experimental determination of the temperatures of fusion for different substances under high pressures. With the aid of a proper series of experiments on this point, a direct method of arriving at an approximation to the thickness of the crust of the globe, or rather, to its least limit, might be easily explained. I shall not here, however, enter into any discussion on this subject. The conclusion to which I would now direct attention is this—the present temperature of the interior of the earth cannot be due to its original heat, if the temperature of fusion for the matter composing

it be independent of the pressure to which the fused matter is subjected. For if the terrestrial temperature be due to that source, it must undoubtedly be sufficient at the depth of one-fortieth or one-fiftieth of the earth's radius, to fuse all the rocks composing the superficial solid portion of the globe placed under the atmospheric pressure. Consequently matter must exist at such depths in a state of fusion, and the crust of the earth must be extremely thin, unless its solidification has been promoted by pressure. I have shown, however, that the crust of the globe cannot be very thin, and therefore the truth of our proposition is manifest.

There is also another mode, independent of our results respecting precession, by which we arrive at the same conclusion. Making the assumption just stated respecting the origin of the actual terrestrial heat, there is no doubt of its being immensely greater at the earth's centre than that which would be necessary to reduce the matter composing the earth's surface, under the atmospheric pressure, to a state of fusion. It would probably reduce a large portion of it to a state of vapour. Now this actual central temperature must necessarily be at least something less than that which existed at the time of the earth's incipient solidification, whether the solidification commenced at the centre or surface (Mem. I.). If it began at the centre it must have been owing to the predominance of pressure in promoting solidification over high temperature in opposing it, and the truth of our proposition is therefore involved in this hypothesis. Again, suppose the solidification to have commenced at the surface. In this case it has been shown (Mem. I.) that the whole mass would arrive at that state in which the fluidity would just become imperfect, at nearly the same time. The superficial temperature would then be just that of *perfect fusion* under the atmospheric pressure for the matter constituting the earth's surface, which, as just stated, must be small compared with the actual central temperature, and, *à fortiori*, small compared with the central temperature at the epoch referred to. Consequently, at that epoch, the central and superficial parts of the earth, under widely different temperatures, would have the same degree of fluidity, viz. that at which it just became *imperfect*, or that at which the component particles would cease to move among themselves in the process of cooling. If then τ_1 denote the temperature of *perfect fusion* for a point of the earth's mass at any depth beneath its surface (or the temperature at which the mass would there acquire perfect fluidity), τ_1 must be some function of the pressure at the proposed point. Also let τ_2 denote the temperature of *incipient fusion* at the same point (or that at which the matter then under the same pressure would just lose its property of solidity), then the question is, whether τ_2 be a function of τ_1 or not. Now that there should be some necessary relation between τ_1 and τ_2 would scarcely seem to admit the possibility of a doubt. But in such case τ_2, being a function of τ_1, must depend on the pressure. Hence it follows, as before, that the temperature of fusion of the earth's mass must depend on the pressure to which it is subjected, assuming always that the fusibility of the matter composing the central portion of the globe is not extremely different from that which constitutes its surface.