

XIII. THE BAKERIAN LECTURE.—*An Account of several new Instruments and Processes for determining the Constants of a Voltaic Circuit.* By CHARLES WHEATSTONE, Esq., F.R.S., Professor of Experimental Philosophy in King's College, London, Corresponding Member of the Academy of Sciences at Paris, &c.

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§ 1.

I INTEND in the present communication to give an account of various instruments and processes which I have devised and employed during several years past for the purpose of investigating the laws of electric currents. The practical object to which my attention has been principally directed, and for which these instruments were originally constructed, was to ascertain the most advantageous conditions for the production of electric effects through circuits of great extent, in order to determine the practicability of communicating signals by means of electric currents to more considerable distances than had hitherto been attempted. In this endeavour, guided by the theory of OHM and assisted by the instruments I am about to describe, I have completely succeeded. But the use of the new instruments is not limited to this especial object; they will, I trust, be found of great assistance in all inquiries relating to the laws of electric currents, and to the various and daily increasing practical applications of this wonderful agent. An energetic source of light, of heat, of chemical action and of mechanical power, we only require to know the conditions under which its various effects may be most economically and energetically manifested, to enable us to determine whether the high expectations formed in many quarters of some of these applications are founded on reasonable hope, or on fallacious conjecture. The theory we now possess is amply sufficient to direct us rightly in this inquiry, but experiments have not yet been sufficiently multiplied to enable us to obtain, except in a few cases, the numerical values of the constants which enter into various voltaic circuits; and without this knowledge we can arrive at no accurate conclusions.

§ 2.

The instruments and processes I am about to describe being all founded on the principles established by OHM in his theory of the voltaic circuit, and this beautiful and comprehensive theory being not yet generally understood and admitted, even by many persons engaged in original research, I could scarcely hope to make my descriptions and explanations understood without prefacing them with a short account

of the principal results which have been deduced from it. It will soon be perceived how the clear ideas of electro-motive forces and resistances, substituted for the vague notions of intensity and quantity which have been so long prevalent, enable us to give satisfactory explanations of most important phenomena, the laws of which have hitherto been involved in obscurity and doubt. Viewing the laws of the electric circuit from the point at which the labours of OHM has placed us, there is scarcely any branch of experimental science in which so many and such various phenomena are expressed by formulæ of such simplicity and generality; in most of the physical sciences the facts of observation and experiment have kept pace with theoretical generalization, in this science alone they had gone on accumulating in prolific abundance without any successful attempt having been made to reduce them to mathematical expression. But this is now happily effected, and what has hitherto been mere matter of speculative conjecture is removed into the domain of positive philosophy.

By *electro-motive force* is meant the cause which in a closed circuit originates an electric current, or in an unclosed one gives rise to an electroscopic tension. By *resistance* is signified the obstacle opposed to the passage of the electric current by the bodies through which it has to pass; it is the inverse of what is usually called their conducting power.

When the activity of any portion of the circuit is increased or diminished, either by a change in the electro-motive force or in the resistance of that portion, the activity of all the other parts of the circuit increases or decreases in a corresponding degree, so that the same quantity of electricity always passes in the same instant of time through every transverse section of the circuit.

The force of the current is directly proportional to the sum of the electro-motive forces which are active in the circuit, and inversely proportional to the total resistance of all its parts, or in other words the force of the current is equal to the sum of the electro-motive forces divided by the sum of the resistances.

Let  $F$  denote the force of the current,  $E$  the electro-motive forces, and  $R$  the resistances: then

$$F = \frac{E}{R}.$$

The length of a copper wire of a given thickness, the resistance of which is equivalent to the sum of the resistances in a circuit, OHM calls its *reduced length*, an expression which it will frequently be found convenient to employ.

If the electro-motive forces and resistances in a circuit are proportionately increased or diminished the force of the current remains the same, or  $\frac{E}{R} = \frac{nE}{nR}$ . Hence a single voltaic element, or a battery consisting of any number of exactly similar elements, if no additional resistance be interposed in the circuit, produces the same effect. Also a thermo-electric element and a voltaic element will produce the same effect when the greatly inferior electro-motive force of the former is compensated by a corre-

sponding decrease in its resistance; in a thermo-electric arrangement the resistance is in general small, because the circuit is entirely metallic, while in a voltaic element the resistance of the liquid is always considerable.

Any interposed resistance weakens the force of the current, but less so as it is smaller in proportion to the other resistances in the circuit. Hence in two circuits, both producing currents of equal force, when the same resistance is introduced, the strength of the two currents may be weakened in very different proportions. A single voltaic element,  $\frac{E}{R}$ , and a series consisting of any number of such elements,  $\frac{nE}{nR}$ , form circuits in which the currents have the same force, but very different results will be obtained according as the added resistance is great or small compared with the original resistances in the circuits; if it be small, the effects of the two circuits will remain sensibly the same; but if it be large, the resistance that weakens to a very great extent the current in the circuit of the single element produces but a trifling diminution in that of the series. This explains the necessity of employing a series to overcome considerable resistances. The same remarks will apply to the comparison of a thermo-electric with a voltaic circuit.

The following is the general formula for the force of the current in a voltaic circuit when completed by a connecting wire; the metallic plates of the voltaic elements being parallel to each other and of equal size:

$$F = \frac{nE}{\frac{nRD}{S} + \frac{rl}{s}}$$

$F$  is the force of the current,  $E$  the electro-motive force of a single element,  $n$  the number of elements,  $R$  the specific resistance of the liquid,  $D$  the thickness of the liquid stratum or distance of the plates,  $S$  the section of the plates in contact with the liquid,  $r$  the specific resistance of the connecting wire,  $l$  its length,  $s$  its section.

Expressed in words we have the following laws:—

The electro-motive force of a voltaic circuit varies with the number of the elements, and the nature of the metals and liquids which constitute each element, but is in no degree dependent on the dimensions of any of their parts.

The resistance of each element is directly proportional to the distance of the plates from each other in the liquid, and to the specific resistance of the liquid, and is also inversely proportional to the surface of the plates in contact with the liquid.

The resistance of the connecting wire of the circuit is directly proportional to its length and to its specific resistance, and inversely proportional to its section.

The limits of this communication will not allow me to dwell longer on the consequences of OHM's theory of the electric circuit; for further developments I must refer to the author's work, 'Die Galvanische Kette mathematisch bearbeitet,' Berlin 1827, a translation of which has appeared in TAYLOR's Scientific Memoirs, vol. ii.; to his various other memoirs published in SCHWEIGER's 'Jahrbuch der Physik;' and to the

more recent applications of the theory made by FECHNER, LENZ, JACOBI, POGGENDORFF, POUILLET, &c.

There is, however, one class of considerations which it is indispensable I should bring forward, because upon it are founded many of the instruments and processes which I shall have occasion hereafter to mention,—I allude to the laws of the distribution of the electric current in the various parts of a circuit, when a branch conductor is placed to divert a portion of the current from a limited extent thereof.

Let  $\lambda$  be the reduced length of the portion of the circuit from which the current is partially diverted,  $\lambda'$  that of the wire which diverts the current, and  $L$  that of the undivided part of the circuit. The force of the current in each of the adjacent conductors,  $\lambda$  and  $\lambda'$ , can be shown to be in the inverse ratio of their reduced lengths, and the reduced length of a single wire, which substituted for both would not alter the force of the current, to be  $\frac{\lambda \lambda'}{\lambda + \lambda'}$ , which we will designate by  $\Lambda$ .

The force of the current in the original circuit before the introduction of the branch wire will then be expressed thus :

$$F = \frac{E}{L + \lambda},$$

and the strength of the current in the three different portions of the altered circuit by the following expressions :—

In the principal or undivided portion  $L$ ,

$$F_1 = \frac{E}{L + \Lambda} = \frac{E(\lambda + \lambda')}{L(\lambda + \lambda') + \lambda \lambda'}.$$

In the portion from which the current has been partially diverted, or  $\lambda$ ,

$$F_2 = \frac{E}{L + \Lambda} \cdot \frac{\Lambda}{\lambda} = \frac{E \lambda'}{L(\lambda + \lambda') + \lambda \lambda'}.$$

In the portion which partially diverts the current, or  $\lambda'$ ,

$$F_3 = \frac{E}{L + \Lambda} \cdot \frac{\Lambda}{\lambda'} = \frac{E \lambda}{L(\lambda + \lambda') + \lambda \lambda'}.$$

### § 3.

It is seldom that any real advance is made in a scientific theory without a corresponding change in its terminology being required. Now that it is proved beyond doubt that the various sources of continued electric action differ from each other only in the amount of their electro-motive forces, modified by the resistance of the circuit of which they form part, it becomes of importance, in order to give precision to our statements and to avoid circumlocutions otherwise inevitable, to adopt general terms to express the source of a current without reference to the peculiar mode of its production ; I shall therefore employ the word *Rheomotor* to denote any apparatus which originates an electric current, whether it be a voltaic element or a voltaic battery, a thermo-electric element or a thermo-electric battery, or any other source

whatever of an electric current; when speaking of a single element I shall term it a rheomotive element, and what is usually called a voltaic or thermo-electric pile or battery I shall term a rheomotive series. I shall still use the ordinary expressions when I have to refer to the specific sources of the production of electric currents, but when I employ the general terms they must be understood to apply to all these sources indifferently.

The want of a general term to designate an instrument to measure the force of an electric current without reference to its particular construction has been long felt. I shall use the word *Rheometer* for this purpose, continuing occasionally to employ galvanometer, voltameter, &c. to distinguish the particular instruments to which these names have been applied, though perhaps the terms Magnetic, Chemical, Calorific, &c. Rheometer would be more appropriate.

This may be the proper place to explain a few other terms which I have frequent occasion to use, though not in the course of the present communication. By *Rheotome* is meant an instrument which periodically interrupts a current, and by *Rheotrope* an instrument which alternately inverts it. A *Rheoscope* is an instrument for ascertaining merely the existence of an electric current. The word *Rheostat* will be hereafter explained.

I have not introduced these terms, which will be found greatly convenient and will enable us to state general propositions much more clearly, without good authority. The word Rheophore was employed by AMPÈRE to designate the connecting wire of a voltaic apparatus, as being the carrier or transmitter of the current; and the word Rheometer, first proposed by PECLET as a synonym for galvanometer, has been generally adopted by the French writers on physics.

#### § 4.

The method of obtaining the constants of a rheophoric circuit adopted by FECHNER, LENZ, POUILLET, &c., in their experimental verifications of OHM's theory, is essentially the following:—

The resistance of a circuit is determined by observing the force of the current, first without any extra interposed resistance in the circuit, and afterwards when a known resistance is added. Then

$$F = \frac{E}{R}, \text{ and } F' = \frac{E}{R+r} \therefore \frac{F}{F'} = \frac{R+r}{R},$$

from which equation the value of  $R$ , all the others being known quantities, is easily deduced.  $R = \frac{F'}{F - F'} r$ . The electro-motive force of a circuit is ascertained by multiplying the force of the current into the total resistance; for since  $F = \frac{E}{R} \therefore E = F R$ .

The principle of this method is extremely simple, but the difficulty of determining immediately the force of a current by means of a galvanometer is an obstacle to its

general employment. FECHNER\* measured the force of the current by the number of oscillations of the needle when placed at right angles to the coils, a very tedious operation; and others have employed the deviations of the needle, the corresponding degrees of force having been previously determined by some peculiar process, or inferred from some rule depending on the particular construction of the instrument. Another impediment to the use of a galvanometer to measure the force of a current arises from the changes in the magnetic intensity of the needle which frequently occur, especially when it has been acted upon by too strong a current.

The principle of my method is that of employing variable instead of constant resistances, bringing thereby the currents in the circuits compared to equality, and inferring from the amount of the resistance measured out between two deviations of the needle, the electro-motive forces and resistances of the circuit according to the particular conditions of the experiment. This method requires no knowledge of the forces corresponding to different deviations of the needle.

To apply this principle it is requisite to have a means of varying the interposed resistance so that it may be gradually changed within any required limits. I have contrived two instruments for effecting this purpose, one intended for circuits in which the resistance is considerable, the other for circuits where the resistance is small†.

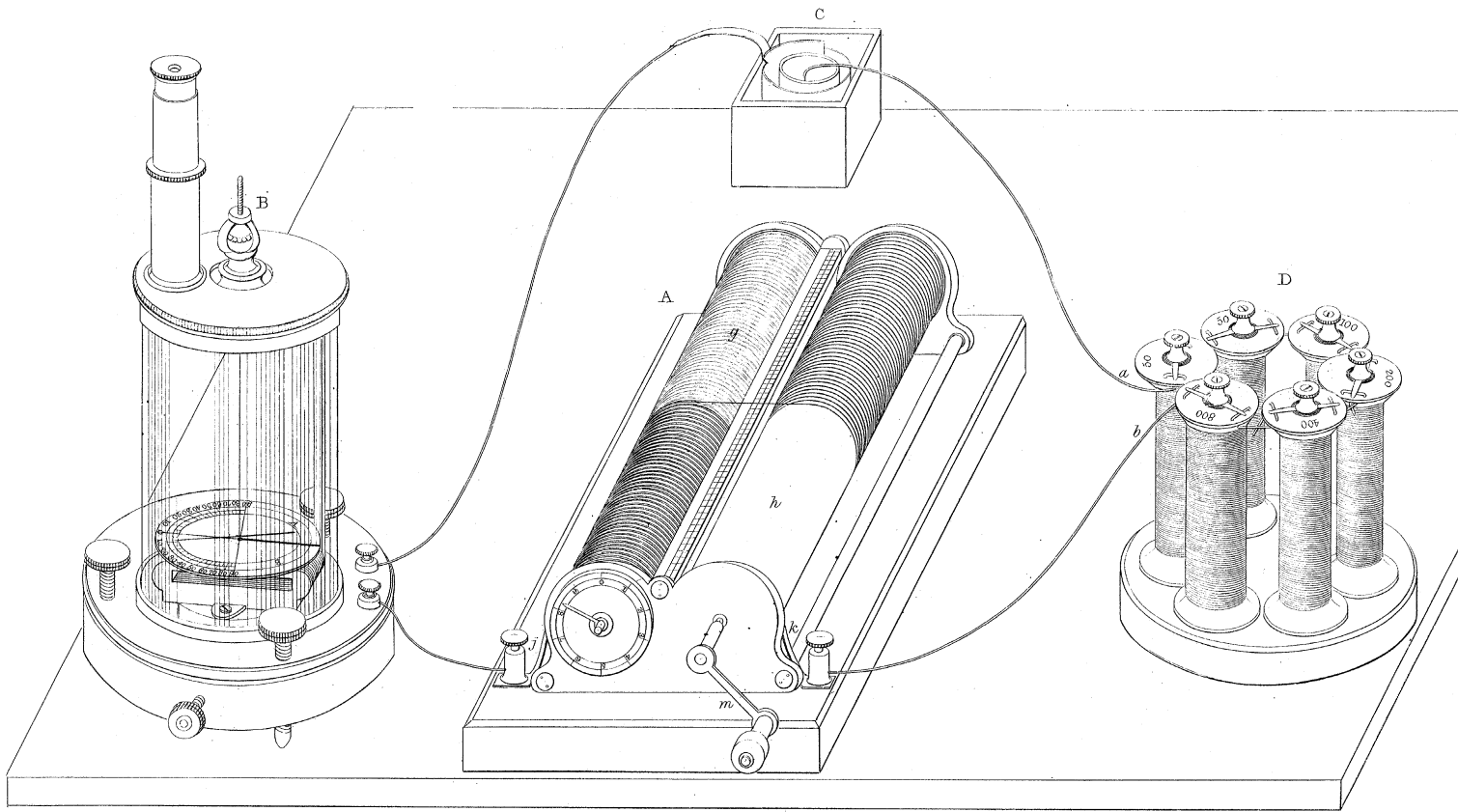
\* *Massbestimmungen über die Galvanische Kette.* Leipzig, 1831, p. 5.

† It appears that the idea of constructing an instrument of this kind had also occurred to Professor JACOBI of St. Petersburg. When I explained to this eminent experimentalist my instruments and processes in the beginning of August 1840, he informed me that he had himself constructed a similar instrument which he had exhibited to the Academy of Sciences at St. Petersburg, though no description of it had yet been published, and he at the same time showed me a drawing of it. This instrument, which he has since called an Agometer, differs in mechanical construction from either of mine, and is less convenient to manipulate; but its principle is the same. In a communication which Professor JACOBI made in the following month to the Meeting of the British Association at Glasgow, and which was published in the *Athenæum* of No. 678, 1840, he thus alludes to the subject:—

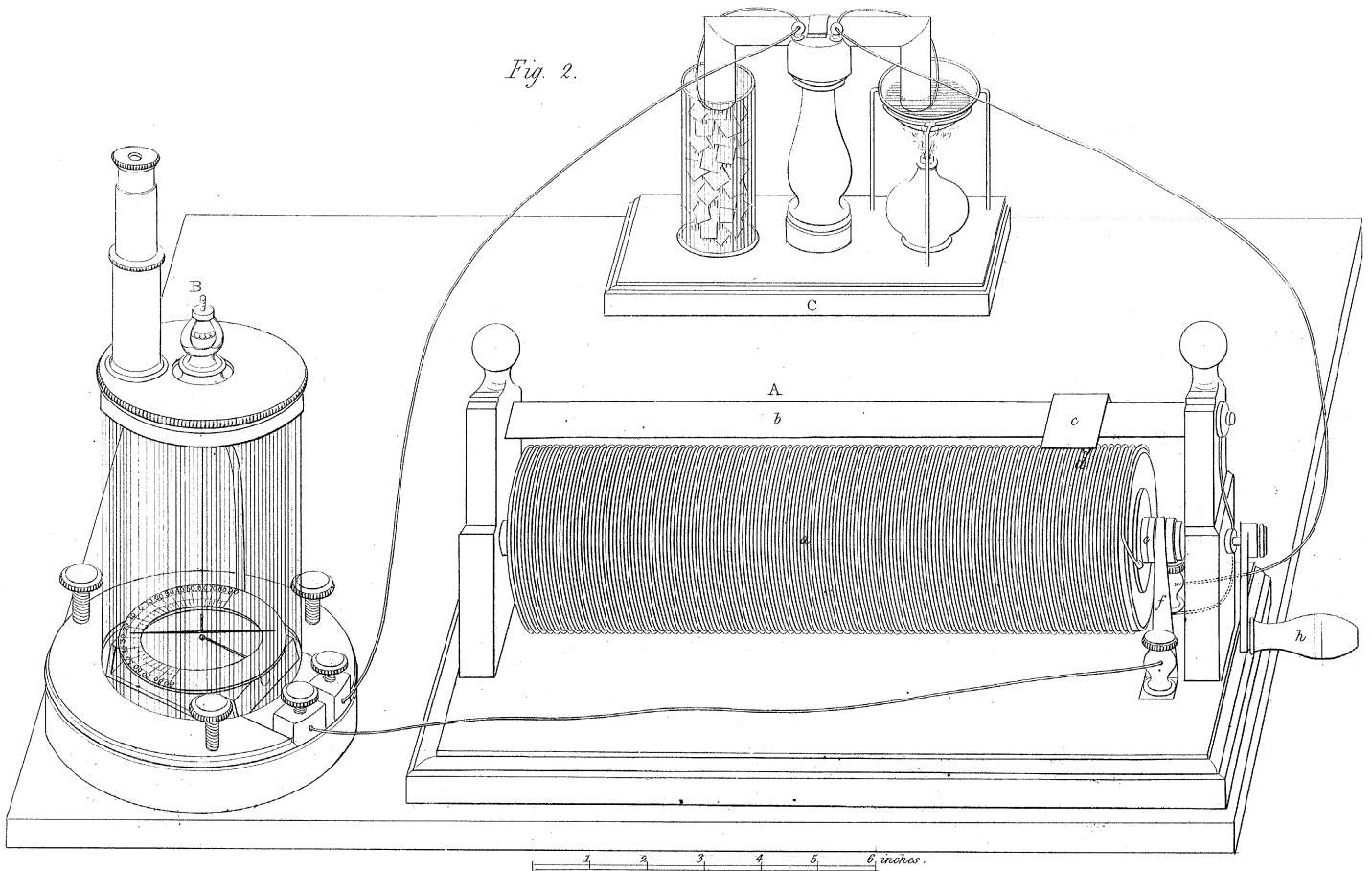
“Before proceeding, I may be permitted to make some remarks concerning an instrument which I laid before the Academy of Sciences in the commencement of this year. It is destined to regulate the galvanic current, and is of value in many investigations of this kind. During my sojourn in London, Professor WHEATSTONE has shown me an instrument, founded exactly on the same principles as mine, and with very insignificant modifications and differences. Now, it is quite impossible that he should have had the least notice of my instrument; but as it is probable that its use may be greatly extended, I must add, that while I have only used this instrument for regulating the force of the currents, he has founded upon it a new method of measuring these currents, and of determining the different elements or constants which enter into the analytical expressions, and on which depends the action of any galvanic combination. It is principally to the measure of the electro-motive force, by those means, that Mr. WHEATSTONE has directed his attention; and he has shown me, in his unpublished papers, very valuable results which he has obtained by this method.”

Professor JACOBI has since his return employed my method of determining the constants of a voltaic circuit. The memoirs in which his results were given were republished in Poggendorff's '*Annalen der Physik*,' vol. liv. No. 2. for 1841, and vol. lxii. No. 9. for 1842. To the latter the learned editor, who has made most valuable researches himself in the same path, has appended (p. 89) the following note:—"I will take this opportunity to call to mind that I applied the same method (or at least one identical to it in principle) before it was com-

*Fig. 1.*



*Fig. 2.*



## § 5.

The first instrument is represented in Plate XVI. fig. 1. A.  $g$  is a cylinder of wood, and  $h$  is a cylinder of brass, both of the same diameter, and having their axes parallel to each other. On the wood cylinder a spiral groove is cut, and at one of its extremities a brass ring is fixed, to which is attached one of the ends of a long wire of very small diameter, which when coiled round the wood cylinder fills the entire groove, and is fixed at its other end to the remote extremity of the brass cylinder. Two springs,  $j$  and  $k$ , pressing one against the brass ring on the wood cylinder, and the other against the extremity of the brass cylinder  $h$ , are connected with two binding screws for the purpose of receiving the wires of the circuit. The moveable handle  $m$  is for turning the cylinders on their axes. When it is placed on the cylinder  $h$  and is turned to the right, the wire is uncoiled from the wood cylinder and coiled on the brass cylinder, but when it is applied to the cylinder  $g$  and is turned to the left, the reverse is effected. The coils on the wood cylinder being insulated and kept separate from each other by the groove, the current passes through the entire length of wire coiled upon that cylinder, but the coils on the brass cylinder not being insulated the current passes immediately from the point of the wire which is in contact with the cylinder to the spring  $k$ . The effective part of the length of the wire is therefore the variable portion which is on the wood cylinder.

In the instrument I usually employ the cylinders are six inches in length and  $1\frac{1}{2}$  inch diameter, the threads of the screw are forty to the inch, and the wire is of brass the  $\frac{1}{100}$ th of an inch in diameter. I employ a very thin wire and a badly conducting metal, in order that I may introduce a greater resistance into the circuit.

A scale is placed to measure the number of coils unwound; and the fractions of a coil are determined by an index which is fixed to the axis of one of the cylinders and points to the divisions of a graduated circle.

As the principal use of this instrument is to adjust or regulate the circuit so that any constant degree of force may be obtained, I have called it a *Rheostat*.

Plate XVI. fig. 1 shows the arrangement of the circuit when prepared for an experiment. B is a delicate galvanometer with an astatic needle furnished with a microscope for reading off the divisions of the circle, which greatly facilitates the observations. C is the rheomotor.

I must here digress for a moment to describe the voltaic element which I have employed in most of my rheometric researches, and which I have found to be very

municated to the author by Mr. WHEATSTONE. See the Annals, vol. lii. p. 526." I have referred to this volume and find it was published in the latter part of 1841, while my communication to Professor JACOBI was, as above stated, made in August 1840. I may also mention, that the experimental process employed by Professor POGGENDORFF had no resemblance whatever to mine, and the result he sought was likewise different; the mathematical principle of the method was however in the single case he investigated undoubtedly the same.



constant in its action, and convenient to manipulate with. It is quite unnecessary to use large elements in such investigations, for when considerable resistances are introduced in the circuits, which is most frequently the case, they produce no perceptibly greater effect than smaller ones, and in all cases the measures may be as accurately determined by employing small elements as large ones.

The voltaic element C consists of a glazed porcelain cell, two inches square and one inch and a half high, in the centre of which is placed a small porous cylinder of earthenware or wood, filled with a liquid amalgam of zinc, the space between the two cells being charged with a solution of sulphate of copper; a slip of thin sheet copper bent round, and having one of its edges cut and turned over so that the wire of the circuit may be attached to it, or that it may dip into the amalgam of another similar cell, is placed in the solution. Fig. 3, Plate XVII. represents several such elements combined to form a series. It will be seen that, in principle, this is but a slight modification of Professor DANIELL'S constant battery, liquid amalgam of zinc being employed, as in Mr. KEMP'S first experiment, instead of amalgamated zinc bars or plates, and the acid solution being dispensed with. This arrangement is, besides being very constant in its action, extremely economical and easy to manipulate. Any negative metal may be substituted for copper provided a solution of a salt of that metal be employed as the interposed liquid.

### § 6.

The rheostat which I employ for circuits in which the resistance is comparatively small is represented at fig. 2. A. *a* is a cylinder of well-seasoned wood, on the surface of which a spiral groove is cut; a thick copper wire is wound round the cylinder occupying the groove, forming as it were the thread of a screw. Immediately above the cylinder and parallel with its axis is a triangular metal bar *b*, carrying a rider or slide *c*; to this rider a spring *d* is attached, which constantly presses against the spiral wire, yielding to any slight inequality. One end of the spiral wire is attached to a brass ring *e*, against which a spring *f* presses, which is connected by means of a binding screw to one end of the circuit, the other end of the circuit is held by the binding screw which is in metallic connection with the triangular metal bar. On turning the handle *h* the cylinder is caused to move on its axis in either direction, and the rider *c* guided by the wire moves along the bar, advancing or receding according as the cylinder is moved to the right or to the left; the rider coming in contact with a different point of the spiral wire, a different resistance is introduced into the circuit, consisting of that portion of the wire only which is included between the rider and the end of the wire connected with the spring *f*. The cylinder of the instrument I have constructed is  $10\frac{1}{2}$  inches in length, and  $3\frac{1}{4}$  inches in diameter; the wire is of copper the 16th of an inch thick, and it makes 108 coils round the cylinder. The dimensions of the instrument, and the thickness, length, and material of the wire, may be varied according to the limits of the variable resistance required

to be introduced into the circuit, and the degree of accuracy with which these changes are required to be measured.

Fig. 2 represents the arrangement of a thermo-electric circuit in which this instrument is interposed. C is the thermo-electric element ; B the galvanometer, which in this case must not have numerous coils of fine wire as in the preceding arrangement, for this would introduce too great a resistance into the circuit, but must consist of a single thick plate or wire making a single convolution ; or, which I think is preferable, the method of diverting a portion of the current from the wire of a delicate galvanometer described in § 15. may be adopted. Any rheomotor in which the resistance is small may be employed in conjunction with this form of the rheostat, instead of a thermo-electric element, as represented.

The rheostat, especially under the form last described, may be usefully employed as a regulator of a voltaic current in order to maintain for any required length of time precisely the same degree of force, or to change it in any desired proportion. Interposed in the circuit of an electro-magnetic engine, however the rheomotor may vary in its energy, the same velocity may be constantly restored by turning the cylinder of the regulator to the left or to the right, according as the velocity increases or decreases ; or any different velocity, within given limits, may be obtained by adjusting the rheostat accordingly. Since the consumption of materials in a voltaic battery in which there is no local action decreases in the same proportion as the increase of the resistance in the circuit, this method of altering the velocity has an advantage which no other possesses, the effective force is always strictly proportional to the quantity of materials consumed in producing the power, a point which, if further improvements should ever render the electro-magnetic engine an available source of mechanical power, will be of considerable importance.

In volta-typing operations the advantage of using the rheostat is obvious. By varying the rheostat from time to time so as to keep the needle of a galvanometer to the same point, a current of any required degree of energy may be maintained, without any notable increase or diminution, for any length of time ; and, as the nature of the deposit, when the solution from which it is made remains the same, varies only with the force of the current and the magnitude of the surface on which the metal is reduced, when once a good effect has been obtained the same circumstances may be reproduced with ease and certainty, and the effects of chance entirely eliminated.

In the operations of voltatyping, electro-gilding, &c., and in the production of NOBILI'S colours, the advantage of using the rheostat is obvious.

This however is not the place to dilate on this subject.

### § 7. *Standard of Resistance.*

It is of the highest importance to have a correct standard of resistance, and one that can easily be reproduced for the purpose of comparison. A copper wire of a given length and diameter might be employed, but as very small differences of dia-

meter are attended with considerable differences in the resistances of wires, it is more convenient to assume for the unit of resistance a wire of a given length and weight, which allows small differences to be very accurately determined. I shall in all my experiments, therefore, take for the unit of resistance a copper wire one foot in length, and weighing 100 grains. The diameter of this wire is the  $\cdot 071$  of an inch, and it is intermediate to the numbers designated in commerce as fifteen and sixteen.

### § 8. *The Resistance Coils.*

It is frequently required to measure resistances much greater than can be effected by means of the rheostat, though the reduced length of its wire is considerable. I may wish to know, for instance, the resistance of the wire of the electro-magnets of my telegraphic apparatus, which is sometimes many hundred yards in length; or that afforded by an extensive telegraphic line, or the resistance of a certain extent of an imperfectly conducting liquid. In all these cases and a variety of others I employ another instrument, which enables me to interpose in the circuit resistances to any amount, and yet to obtain by the conjoined use of the rheostat, which serves as its fine adjustment, any required degree of accuracy. This instrument is represented fig. 1. D; it consists of six coils of fine silk-covered copper wire, about the  $\frac{1}{200}$ th of an inch in diameter; two of these coils are fifty feet in length, the others are respectively 100, 200, 400, and 800 feet in length. The two ends of each coil are attached to short thick wires fixed to the upper faces of the cylinders, which serve to combine all the coils into one continued length; the two wires *a*, *b* form the extremities of the coils by which they are united to the circuit. On the upper face of each cylinder is a double brass spring moveable round a centre, so that its ends may rest at pleasure either on the ends of the thick connecting wires, or may be removed from them and rest only on the wood. In the latter position, the current of the circuit must pass through the coil, but in the former position, the current passes through the spring, and removes the entire resistance of the coil from the circuit. When all the springs rest on the wires, the resistance of the whole series of coils is removed, but by turning the springs so as to introduce different coils into the circuit, any multiple of 50 feet up to 1600 may be brought into it.

As the measurement of these long lengths of wire cannot be accurately depended upon, it is advisable to ascertain the number of units of resistance in each coil, which, with the aid of the rheostat, may be easily effected. I find the resistance of the entire 1600 feet to be equivalent to 218,880 units of resistance, or feet of the standard wire. I occasionally employ an auxiliary series of coils combined in the same way as the preceding, consisting of six coils of the same wire, each 500 yards in length. The reduced length of this series is above 233 miles of the standard wire. By combining it with the preceding, I am able to measure resistances equal to  $274\frac{1}{2}$  miles.

## § 9.

When a perfectly constant element, a galvanometer and a rheostat are placed in a circuit as in fig. 1, the resistance of any interposed body may be ascertained in the following way. Observe the point at which the needle stands; then remove the body, the resistance of which is to be measured, from the circuit, and, by means of the rheostat, add a sufficient length of wire to bring the needle again to the same point. The number of standard units corresponding to this added length will be the measure.

It is a point of importance to determine the resistance of the wire of the galvanometer employed in the experiments; to ascertain this by the above method an auxiliary galvanometer would be required, but when a second galvanometer is not at hand, recourse may be had to the following process. Take two rheomotive elements exactly equal both in electro-motive force and resistance; place one of them in the circuit fig. 1, and observe accurately the deviation of the needle; then interpose also the other element and bring the needle again to the same point by means of the rheostat. The equivalent of the wire uncoiled  $\lambda$ , will be the measure of the resistance of the galvanometer wire  $g$  plus that of the connecting wires  $r$ . Subtracting  $r$  from  $\lambda$ , the resistance of  $g$  will be determined,

$$\frac{E}{R + r + g} = \frac{2E}{2R + r + g + \lambda} \quad \therefore g = \lambda - r.$$

The resistance of a galvanometer wire or any other interposed resistance may be still more accurately ascertained by means of the instruments described in § 16.

§ 10. *Process to ascertain the Sum of the Electro-motive Forces in a Voltaic Circuit.*

The rheostat affords a most ready means of ascertaining the sum of the electro-motive forces active in a voltaic circuit, without requiring for this purpose the aid of a rheometer graduated to indicate proportional forces, or having recourse to the tedious process of counting the oscillations of a needle, employed by FECHNER in his investigations. To save time and trouble in this operation will be of great importance to the future progress of electro-chemistry, on account of the great number of experiments of this kind which yet remain to be made, and also from the fluctuations in the electro-motive forces of many circuits from chemical and other actions, which render observations requiring considerable time to make completely valueless.

The principle of my process is as follows:—In two circuits, producing equal rheometric effects, the sum of the electro-motive forces divided by the sum of the resistances is a constant quantity, i. e.  $\frac{E}{R} = \frac{nE}{nR}$ ; if  $E$  and  $R$  be proportionately increased or diminished,  $F$  will obviously remain unchanged. Knowing therefore the proportion of the resistances in two circuits producing the same effect, we are able immediately to infer that of the electro-motive forces. But as it is difficult in many cases to

determine the total resistance, consisting of the partial resistances of the rheomotor itself, the galvanometer, the rheostat, &c., I have recourse to the following simple process. Increasing the resistance of the first circuit by a known quantity

$r$ , the expression becomes  $\frac{E}{R+r}$ ; in order that the effect in the second circuit shall be rendered equal to this, it is evident that the added resistance must be multiplied by the same factor as that by which the electro-motive forces and original resistances are multiplied, for  $\frac{E}{R+r} = \frac{nE}{nR+nr}$ . The relations of the lengths of the added resistances  $r$  and  $n r$ , which are known immediately, give therefore those of the electro-motive forces.

Experimentally I proceed thus :—I interpose the rheostat and the galvanometer in the circuit, and then add, by means of the former, assisted if necessary by the resistance coils, a sufficient resistance to bring the needle exactly to  $45^\circ$ ; I then ascertain the length of wire uncoiled from the brass cylinder of the regulator necessary to reduce the deviation of the needle to  $40^\circ$ . The number of turns is the measure of the electro-motive force, the number corresponding to that of a standard element having been previously determined.

### § 11.

I subjoin a few measures of electro-motive forces obtained by the preceding process.

1. Three elements of different sizes, consisting of copper, a solution of sulphate of copper, and a liquid amalgam of zinc, were successively placed in the circuit. The number of turns of the rheostat requisite to reduce the needle from  $45^\circ$  to  $40^\circ$  were,

Small element described in § 5. . . . . 30 turns.

Copper cylinder  $3\frac{1}{2}$  inches high and  $2\frac{1}{2}$  inches diameter . . 30 turns.

Copper cylinder 6 inches high and  $3\frac{1}{2}$  inches in diameter . . 30 turns.

Hence, conformably to the theory, the magnitude of an element occasions no difference in its electro-motive force.

2. Five small elements of copper and amalgam of zinc were charged respectively with the following five solutions of copper, the sulphate, the ammonia sulphate, the acetate, the per-muriate and the nitrate. Though the force of the current produced by each element separately was very different, owing to the different conductivity of the solutions, yet, with the exception of the nitrate, all required the same number of turns, indicating equal electro-motive forces; the latter fluctuated between 23 and 29, owing to some disturbing action probably of the nitric acid on the mercury of the amalgam.

3. The electro-motive forces of a circuit in which 1, 2, 3, 4, 5 similar elements were successively placed, were measured.

1 element required	. . . . .	30 turns.
2 elements	. . . . .	61 turns.
3 elements	. . . . .	91 turns.
4 elements	. . . . .	120 turns.
5 elements	. . . . .	150 turns.

The electro-motive force of a circuit is therefore, as theory indicates, proportional to the number of similar elements of which it is formed, arranged in series.

4. The next experiments were made to determine the amount of the contrary electro-motive force which is introduced into a circuit when a voltameter or decomposing cell is interposed. The liquid in contact with the platinum electrodes was dilute sulphuric acid. The measure of this contrary electro-motive force is obtained by subtracting the actual number of turns from the number corresponding with the electro-motive force of the circuit when the decomposing cell is removed from it.

3 elements with decomposing cell	21 turns	90 —	Contrary electro- motive force.	21 = 69
4 elements with decomposing cell	50 turns	120 —		50 = 70
5 elements with decomposing cell	79 turns	150 —		79 = 71
6 elements with decomposing cell	109 turns	180 —		109 = 70
Mean				<hr/> 70

The contrary electro-motive force may be considered therefore in this case to be constant, and to be to that of a single standard element as 7 : 3. It is hence obvious why three such elements are necessary to decompose water in a cell with platinum electrodes of a certain size, and charged with dilute sulphuric acid. The amount of this contrary force varies with different liquids, and according to the nature of the electrodes employed: as it is not my present object to investigate this subject, but merely to give a few examples of the measures which may be obtained by the above-mentioned method, I shall not enter on the consideration of these interesting but intricate modifications.

5. The highest electro-motive force which a voltaic element consisting of two metals and one interposed liquid can manifest, is when the liquid is a solution of a salt of the negative metal, so that by the continual deposition of this metal the negative surface is kept free from the contact of heterogeneous substances which would tend to give rise to a reverse current. When, in consequence of the chemical action, any heterogeneous solid matter is deposited on, or any evolved gas adheres to, the negative surface, the electro-motive force of the element is reduced. The following measures will show the reduction in electro-motive force of a zinc and copper, and of a zinc and platinum element, by substituting dilute sulphuric acid for the metallic salt; the changes in these cases are effected by the adhesion of hydrogen to the surface of the negative metal.

Amalgam of zinc . .	Sulphate of copper . .	Copper . .	30 turns.
Amalgam of zinc . .	Dilute sulphuric acid .	Copper . .	20 turns.
Amalgam of zinc . .	Chloride of platinum .	Platinum .	40 turns.
Amalgam of zinc . .	Dilute sulphuric acid .	Platinum .	27 turns.

6. The proportion of zinc in the liquid amalgam does not appear to affect the electro-motive force of the voltaic element of which it forms part; the number of turns of the rheostat remains the same although the quantity of zinc may vary very considerably. I was therefore led to think that tolerably accurate measures might be made of the comparative electro-motive forces of the alkaline and earthy bases. An element was formed of liquid amalgam of potassium, sulphate of zinc, and zinc; the potassium was in proportion to the mercury less than 2 per cent.; there was no apparent local action, and the current was remarkably constant and continuous.

The following were ascertained to be the electro-motive forces of different elements in which the positive metal was amalgam of potassium, and the negative metals respectively were zinc, copper and platinum.

Amalgam of potassium . .	Sulphate of zinc . . .	Zinc . . .	29 turns.
Amalgam of potassium . .	Sulphate of copper . .	Copper . .	59 turns.
Amalgam of potassium . .	Chloride of platinum .	Platinum .	69 turns.

The electro-motive force of the first combination nearly corresponds with that of zinc and copper, and when the resistance in the circuit is equivalent, produces a current having nearly the same degree of force. The third combination is one of great electro-motive energy, and when a voltmeter with small electrodes is interposed in the circuit, decomposes the water in it abundantly.

It would not be difficult to submit to experiments of this kind all the alkaline and earthy bases; as the proportion in the amalgam does not seem to be of importance, they might be easily prepared by means of a voltaic battery. It would be interesting to know what rank the hypothetical base of ammonia would hold in this scale of electro-motive forces.

7. A still higher electro-motive force may be obtained by employing, in conjunction with the amalgam of potassium, a platinum plate covered with a film of peroxide of lead\*. Such a plate is easily prepared by making it the positive electrode in a decomposing cell, charged with a solution of acetate of lead. The films thus formed exhibit, as NOBILI has shown, according to their thickness, the colours of NEWTON's rings.

Amalgam of zinc . .	Dilute sulphuric acid . .	Peroxide of lead .	68 turns.
Amalgam of potassium	Dilute sulphuric acid . .	Peroxide of lead .	98 turns.

\* A rheomotive series of ten such elements will have an electro-motive force equal to thirty-three elements of DANIELL's battery, or fifty of WOLLASTON's arrangement in good action. Voltaic combinations, in which peroxide of lead is substituted for the negative metal, have been experimented with by Professors SCHÖNBEIN (Phil. Mag., 3rd Series, vol. xii. p. 225, March 1838) and DE LA RIVE (Archives de l'Electricité, No. 7, April 1843).

The following measures were obtained when peroxide of manganese was substituted for the peroxide of lead. The peroxide of manganese was deposited on a platinum plate which formed the positive electrode of a decomposing cell containing a solution of chloride of manganese.

Amalgam of zinc . . Diluted sulphuric acid. Peroxide of manganese . 54 turns.  
Amalgam of potassium Diluted sulphuric acid. Peroxide of manganese . 84 turns.

A weak current is produced by employing a clean platinum plate in conjunction with one covered with the peroxide, in which combination the former acts the part of zinc. In this case the positive metal undergoes no chemical action, but on the negative side the peroxide is reduced by the evolved hydrogen.

8. The following measures conclusively show, that if three metals be taken in their electro-motive order, the electro-motive force of a voltaic element, formed of the two extreme metals, is equivalent to the sum of the electro-motive forces of the two elements formed of the adjacent metals.

1.			
Amalgam of potassium	Sulphate of zinc	Amalgam of zinc	29 turns.
Amalgam of zinc	Sulphate of copper	Copper	<u>30</u> turns.
Amalgam of potassium	Sulphate of copper	Copper	<u>59</u> turns.

2.			
Amalgam of potassium	Sulphate of zinc	Amalgam of zinc	29 turns.
Amalgam of zinc	Chloride of platinum	Platinum	<u>40</u> turns.
Amalgam of potassium	Chloride of platinum	Platinum	<u>69</u> turns.

9. I wished to compare the electro-motive force of a thermo-electric element, the two metals of which were bismuth and copper, and whose opposite joints were exposed to the fixed temperatures of  $32^{\circ}$  and  $212^{\circ}$ , with that of a standard voltaic element. As the interposition of the galvanometer greatly reduced the force of the current in the thermo-electric circuit, so that I could not advance the needle to  $45^{\circ}$ , I employed, instead, the reduction of the needle from  $10^{\circ}$  to  $5^{\circ}$ . The ratios of the measures of the electro-motive forces remain the same between whatever two points the needle is made to vary, provided they do not change during the same series of experiments.

Thermo-electric element of bismuth and copper, the temperatures of the joints being $32^{\circ}$ and $212^{\circ}$ . . . . .	} 8 turns.
Standard voltaic element of amalgam of zinc, sulphate of copper, and copper . . . . .	
	} 757 turns.

The relative electro-motive forces are therefore as 1 : 94.6\*.

\* POUILLET, by a very different process, has ascertained this proportion to be as 1 : 95. See *Éléments de Physique Expérimentale*, 3<sup>ième</sup> ed. tom. i. p. 631.



## § 12.

The resistance or reduced length of a rheomotor may be ascertained by either of the following processes :—

*First Method.*—Place the galvanometer and the rheostat in the circuit, and adjust the latter until the needle of the galvanometer stands at a determined point. Then divide the current which passes through the wire of the galvanometer, by placing an equal resistance by its side ; the needle will recede. The reduced length, measured by the number of turns of the rheostat, required to be taken out of the circuit in order to make the needle stand at its former point, will be equal to half the total resistance of the undivided portion of the original circuit. The resistance of the galvanometer and connecting wires, and of the coils of the rheostat in the circuit before the experiment, having previously been determined, that of the rheomotor is easily obtained by subtracting the former from the total resistance measured.

Let  $E$  be the electro-motive force,  $g$  the resistance of the galvanometer wire, and  $R$  all the other resistances in the circuit. The force of the current acting upon the needle will be  $F = \frac{E}{R+g}$  ; adding by the side of the galvanometer wire another wire having the same resistance, is equivalent to substituting for it a wire of double section, and the expression for the resistance of the circuit becomes  $R + \frac{g}{2}$  ; but since, in consequence of the division of the current, only one-half its force acts upon the needle, this action may be represented by  $\frac{\frac{1}{2}E}{R + \frac{1}{2}g}$ . To render this expression equivalent to the first, the resistance  $R$  must be reduced one-half, for  $\frac{E}{R+g} = \frac{\frac{1}{2}E}{\frac{1}{2}R + \frac{1}{2}g}$  ; the resistance taken out of the circuit to effect this reduction is obviously equal to half the resistance of the undivided portion of the original circuit ;

$$\text{or} \quad \frac{E}{R+g} = \frac{\frac{1}{2}E}{R + \frac{1}{2}g - \lambda} \quad \therefore \lambda = \frac{R}{2}.$$

*Second Method.*—Bring the needle of the galvanometer, by means of the rheostat, to a determined point which we will call  $b$ . Ascertain the resistance  $r$  requisite to reduce the needle to a lower point  $a$ . Restore it to  $b$  ; then place a wire to divide the current with the galvanometer, and alter this wire until the needle again stands at  $a$ . When the needle stands at  $b$ ,  $F = \frac{E}{R+g}$  ; when it stands at  $a$  in the first case

$$F' = \frac{E}{R+g+r}, \text{ in the second case } F' = \frac{E r'}{R(g+r') + g r'}.$$

Equating these two expressions,

$$\frac{E}{R+g+r} = \frac{E r'}{R(g+r') + g r'} \quad \therefore R = \frac{r r'}{g},$$

and as these factors are known,  $R$  may be readily determined. The resistance of the rheomotor may be obtained from this as before.

If  $r' = g$ , that is if the resistance of the galvanometer wire be equal to that of the wire which diverts a portion of the current from it, then  $R = r$ .

*Third Method.*—Bring the needle to any determined point, and ascertain by means of the instrument described at § 18. what degree corresponds to one-half the intensity thus indicated. Since, when the electro-motive force remains the same, the force of the current is simply inversely as the total resistance, to reduce the needle from  $a$  to  $\frac{a}{2}$  a resistance exactly equal to that previously existing in the circuit must be added; therefore the number of turns of the rheostat required to reduce the needle from  $a$  to  $\frac{a}{2}$  will be the measure of the total resistance of the circuit when the needle stood at  $a$ . The total resistance being thus measured, that of the rheomotor is determined by subtracting from it the other known resistances, including that of the galvanometer.

More generally, if the forces of two currents,  $a$  and  $b$ , corresponding to two stationary positions of the needle, are known (§ 19.), the total resistance of the circuit will be  $R = \frac{br}{a-b}$ ,  $r$  being the resistance added to reduce the current from  $a$  to  $b$ . If  $a = 2b$ , then  $R = r$  as before.

*Fourth Method.*—For this and the following process two exactly equal rheomotors must be employed; their equality may be tested by successively interposing them in the same circuit, when one and the other should deflect the needle of the galvanometer precisely to the same degree.

Place one rheomotor in the circuit and adjust the rheostat until the needle points to any degree arbitrarily fixed upon; then add the second element by the side of the first, and increase the reduced length of the circuit by turning the rheostat until the needle again points to the same division. The known quantity, measured by the number of turns of the rheostat, by which the reduced length of the circuit is increased, is equal to one-half the resistance of a single rheomotor. By placing the second rheomotor by the side of the first, the resistance of that portion of the circuit is reduced one-half; therefore, to restore the former condition of the circuit, a resistance equal to one-half that of the rheomotor must be added. For

$$\frac{E}{R+r} = \frac{E}{\frac{R}{2} + r + \lambda} \quad \therefore \lambda = \frac{R}{2},$$

$R$  being the resistance of the rheomotor, and  $r$  the other resistances in the first circuit.

*Fifth Method.*—Place both the rheomotors in series, and vary the resistance until the needle stands at any determined degree. Then place them side by side, and increase the resistance, by turning the rheostat until the needle again stands as before. The resistance of a single rheomotor is equal to twice the resistance required to be added, plus all the resistances in the first circuit except that of the rheomotor,

$$\frac{2E}{2R+r} = \frac{E}{\frac{R}{2} + r + \lambda} \quad \therefore R = r + 2\lambda,$$

$R$  being the resistance of the rheomotor,  $r$  the other resistances in the first circuit, and  $\lambda$  the resistance added by the rheostat to make the force of the current in the second circuit equal to that in the first.

The resistance of one of the elements of the battery described in § 5, I have found to be equal to 2128 standard units.

### § 13.

The resistance of a standard rheomotor having been accurately determined by either of the processes above described, the resistance of any other rheomotor, in which the electro-motive force is the same, may be obtained by a still more expeditious method. The needle of the galvanometer being brought to a determined point when the standard rheomotor is interposed in the circuit; if this be removed, and the rheomotor to be measured be substituted in its place, the number of coils of the rheostat, added to or subtracted from the circuit, to make the current in the latter case equal to that in the former, when added to or subtracted from the resistance of the standard rheomotor, will give that of the rheomotor to be measured. If  $R'$  be greater than  $R$ ,  $R' = R + r$ ; but if  $R'$  be less than  $R$ ,  $R' = R - r$ . By this simple process the resistances of voltaic elements of different forms, magnitudes, &c. may be readily compared.

### § 14. *Instrument for Measuring the Resistance of Liquids.*

We do not at present possess any accurate measures of the conductibilities of liquids, nor have there yet been formed any tables which show the even real order of their conducting powers. In the experiments having this object in view which have hitherto been made, the contrary electro-motive force, which generally arises when the electric current passes through a liquid capable of undergoing decomposition (§ 11, 4.), has been left entirely out of consideration, and the results therefore have widely deviated from the truth. By the simple instrument represented at Plate XVII. fig. 4, I have been able to eliminate completely this source of error, and to obtain perfectly constant results. A is a glass tube about two inches long and half an inch in internal diameter; a portion of the tube is ground away for an inch and a quarter of its length, so as to leave a segment of  $270^\circ$ ; at one extremity of this aperture is fixed a metal plug terminated by a platinum plate, and at the other end is a moveable piston, terminated also by a plate of platinum, capable of being advanced to within a quarter of an inch from the fixed plate; the range of its motion is thus limited to one inch, and an attached micrometric apparatus enables any portion of this distance to be accurately measured. To obtain the measure of the resistance of a liquid I proceed in the following way:—I interpose in the circuit a small constant battery, consisting of about three elements, with the rheostat, the resistance-coils, the galvanometer, and the measuring tube just described. The end of the piston being a quarter of an inch distant from the fixed plate, I fill the intervening space with the liquid, the resistance

Fig. 5.

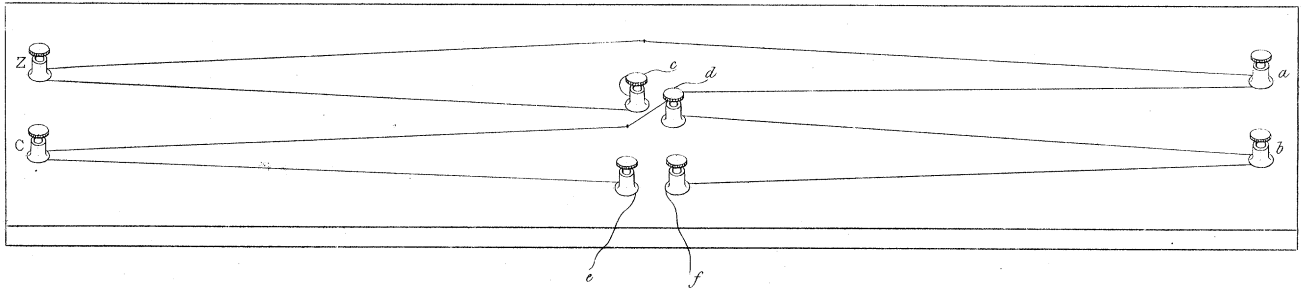


Fig. 8.

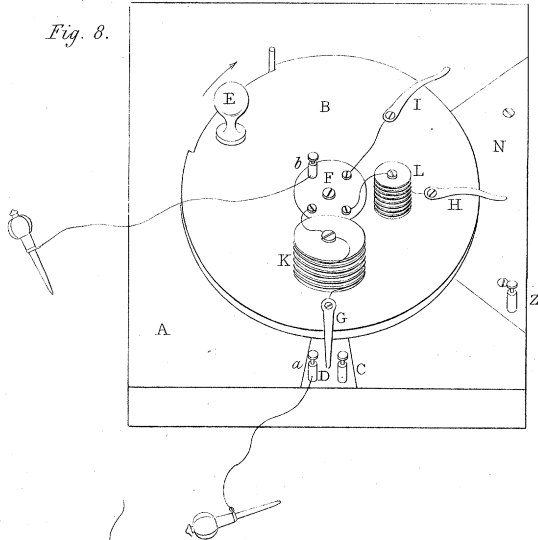


Fig. 7.

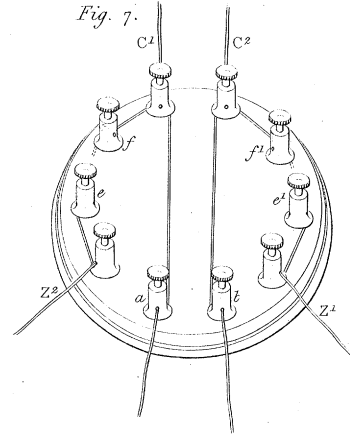


Fig. 3.

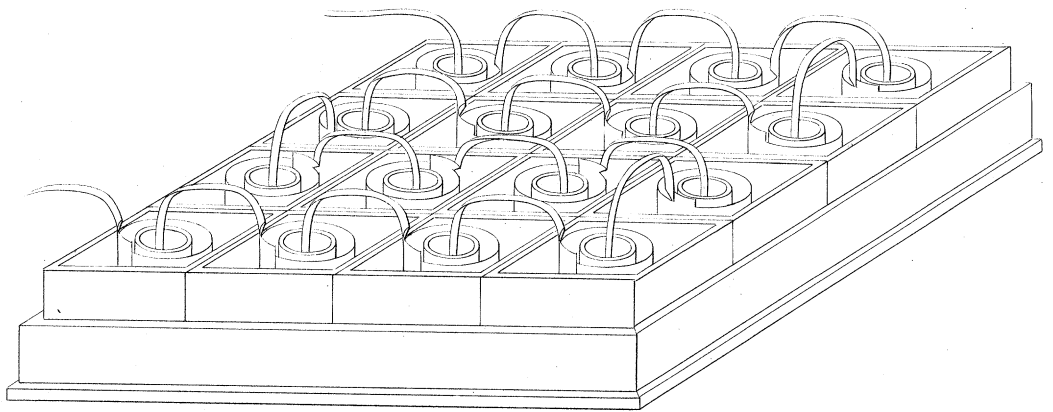
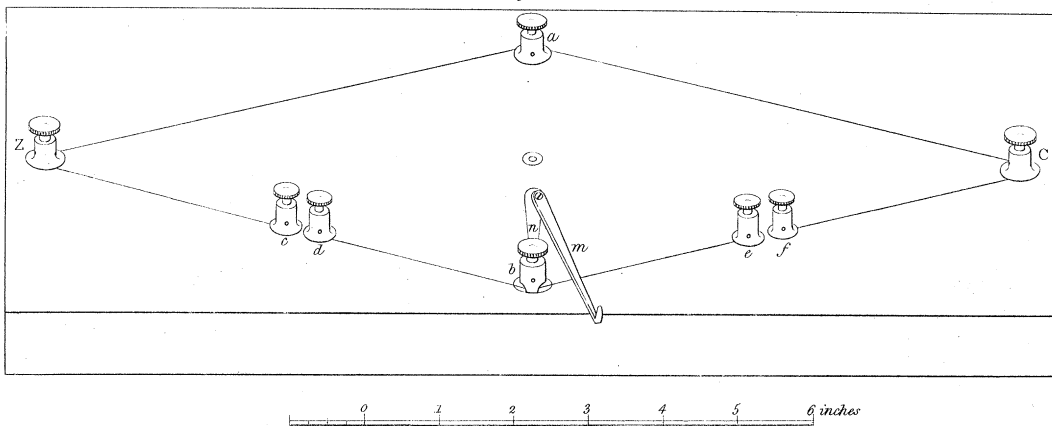


Fig. 6.



of which is to be measured. I then adjust the rheostat to bring the needle of the galvanometer to a determined point; this having been noted, I draw the piston back through the entire remaining space of one inch, and fill the vacancy with the same liquid; the needle will recede towards zero. I then diminish the resistance of the circuit by means of the rheostat and the resistance-coils, until the needle stands at the same point that it did when only a quarter of an inch of the liquid column was interposed. The reduced length of the wire thus taken out of the circuit will be the measure of the resistance of one inch of the liquid. The contrary electro-motive force arising from the decomposition of the liquid exists in the circuit during the whole process, and therefore does not affect the result.

The measure of the resistance of a liquid must be made immediately after it is placed in the circuit, because if a current be allowed to act upon it for any length of time the nature of the solution changes. In the case of sulphuric acid, for instance, the solution is rendered stronger by the decomposition and consequent diminution of the water, while, in the case of a metallic salt, not only is the water decomposed, but the metal is reduced, and free acid is liberated. Under the conditions, however, of my experiments, the chemical action is so slow, and the time of operation is so short, that no sensible changes of this kind take place.

The resistance of liquids to the transmission of electricity is, no doubt, one of their most important physical properties. An investigation of all the circumstances which occasion changes in this property, especially if accompanied with accurate quantitative determinations, must necessarily lead to important and hitherto unobserved relations. To investigate the changes due to different degrees of dilution and temperature alone will be a task requiring considerable patience. I have made many measures of the specific resistances of different conducting liquids, by the aid of the preceding process, but as they have not been sufficiently numerous to enable any general conclusions to be drawn, and as I am at present engaged in a more extensive series of experiments in which strict attention will be paid to all the known influencing circumstances, I shall defer an account of them to a future occasion.

As bodies differ so much from each other in their specific resistances, and as the means of determining this property are so easy, it cannot be doubted that hereafter this process will be extensively employed to detect the purity of substances and to distinguish them from each other.

Another method of measuring the resistance of a conducting liquid is the following :

—Prepare a circuit the electro-motive force and resistance of which is known,  $\frac{E}{R} = F$ . Interpose the liquid which is to be the subject of experiment in a small cell with two parallel platinum electrodes; the expression for the circuit will then be  $\frac{E - e}{R + x} = F'$ ;  $e$  being the contrary electro-motive force, and  $x$  the resistance of the liquid which is to be determined. Having ascertained the value of  $e$  by the process described in § 10, sub-

tract, by means of the rheostat and coils, a resistance which shall make the force again equal to  $F$ ; the expression will then become  $\frac{E - e}{R + x - \lambda} = \frac{E}{R}$ , whence  $x = \lambda - \frac{e}{E} R$ . Therefore the resistance  $x$  of the liquid is equal to the resistance  $\lambda$  taken out of the circuit by the rheostat, minus the total resistance of the original circuit multiplied by the ratio  $\frac{e}{E}$ .

### § 15.

When a galvanometer is employed to measure the force of a current, its wire is usually interposed in the circuit. But it is impossible, in this way, to make use of the same galvanometer to measure the force of the current in circuits of different kinds. A galvanometer with numerous coils of thin wire adds a very considerable resistance to a circuit in which the electro-motive force is great and the resistance small; while, on the other hand, a galvanometer with a short thick wire will give scarcely any indication in a circuit in which the resistance is very great, though the electro-motive force may be considerable. Besides, a delicate galvanometer is incapable of indicating energetic forces.

But by the following simple means the same delicate galvanometer may be employed to measure forces of every degree of energy, and in all kinds of circuits, without introducing any inconvenient resistance into them.

If the current be caused to pass simultaneously through two paths, one being the wire of the galvanometer, and the other another wire connected with its two ends, the current will be divided in the inverse proportion of the resistances of the two paths. The action upon the needle of the galvanometer may hereby, by employing different wires to divert a portion of the current, be reduced to any degree. If the proportionate forces are known for the galvanometer without the reducing wire, they will remain equally proportionate whatever the resistance of the latter may be; but measures made with the same instrument, with different reducing wires applied, will not be comparable unless the changed resistance of the galvanometer thus modified be taken into account.

But strictly comparable measures may be obtained, if the precaution be taken of adding, to the principal portion of the circuit, a resistance which will compensate for the diminution of resistance occasioned by placing the reducing wire. Let  $g$  be the reduced length of the galvanometer wire, and  $ng$  that of the reducing wire. The force of the current in the principal portion of the circuit will be to that in the galvanometer wire as  $1 : \frac{n}{n+1}$ . The resistance to be added to the principal portion of the circuit, in order to maintain the current the same as when no reducing wire is added, is  $\frac{g}{n+1}$ .

When the measures of energetic currents are required to be determined by means of a delicate galvanometer, it is sufficient to attach its two ends to two points of the

conducting wire\*. The distance between these points must remain the same in all comparative experiments, but the absolute deviations of the needle will be greater as these points are further from each other. In the case of the circuit of a powerful electro-magnetic engine, or of a volta-typing apparatus, the diminution of resistance occasioned by connecting the galvanometer wire in the manner above described is so trifling that it would be useless to take it into account, and the compensation above alluded to is, therefore, unnecessary.

### § 16. *The Differential Resistance Measurer.*

The method of determining the resistance of metal wires and other conductors of electricity by means of the rheostat, described in § 9, is inapplicable when small differences are to be observed. If, for instance, a short length of wire has to be examined, its resistance is so small compared with the other resistances in the circuit, including that of the battery, that whether it be interposed or not, no change is observable in the deviation of the needle; and, even if greater lengths of the conducting substance be employed, fluctuations in the power of the battery frequently render the observation uncertain.

The differential galvanometer proposed by M. BECQUEREL, had it been an instrument as practically as it is theoretically perfect, would have enabled us to ascertain very minute differences of resistance with great facility. But it is almost impossible so to arrange the two coils that currents of equal energy circulating through them shall produce equal deviations of the needle in opposite directions, the consequence of which is that the standing of the needle at zero is no indication of equality in the currents. This and other defects have prevented the differential galvanometer from coming into use.

All the advantages, however, which were expected from this instrument may be obtained, without any of its accompanying defects, by means of the simple arrangement I am about to describe, which, moreover, has the advantage of being immediately applicable to any galvanometer, instead of requiring, as in the former case, the instrument to be peculiarly constructed.

Fig. 5 represents a board on which are placed four copper wires,  $Zb$ ,  $Za$ ,  $Ca$ ,  $Cb$ , the extremities of which are fixed to brass binding screws. The binding screws  $Z$ ,  $C$  are for the purpose of receiving wires proceeding from the two poles of a rheomotor, and those marked  $a$ ,  $b$  are for holding the ends of the wire of a galvanometer. By this arrangement a wire from each pole of the rheomotor proceeds to each end of the gal-

\* Professor PETRINA of Linz has proposed (POGGENDORFF'S 'Annalen,' vol. lxii. 1842, No. 9) a similar means of measuring and comparing electric currents of every degree of force. He interposes in the circuit a canal of mercury, the section of which is four square lines, and plunges into it, at various distances from each other, the ends of the wire of a sensitive galvanometer. He shows that if the resistance in the galvanometer wire be very considerable, and that of the mercury in the canal be small in comparison, the force acting on the galvanometer needle will be sensibly proportional to the distance between the ends of the wire, and he has founded on this principle a ready approximative method of graduating the galvanometer.

vanometer wire, and if the four wires be of equal length and thickness, and of the same material, perfect equilibrium is established, so that a rheomotor however powerful will not produce the least deviation of the needle of the galvanometer from zero. The circuits  $Z b a C Z$ , and  $Z a b C Z$ , are in this case precisely equal, but as both currents tend to pass in opposite directions through the galvanometer, which is a common part of both circuits, no effect is produced on the needle. Currents are however established in  $Z b C Z$ , and  $Z a C Z$ , which would exist were the galvanometer entirely removed. But if a resistance be interposed in either of the four wires, the equilibrium of the galvanometer will be disturbed; if the resistance be interposed in  $Z b$  or  $C a$ , the current  $Z a b C Z$  will acquire a preponderance; if it be inserted either in  $Z a$  or  $C b$ , the opposite current,  $Z b a C Z$ , will become the most energetic. If the resistance interposed in the wire be infinite, or which is the same thing, if the wire (which we will suppose to be  $C b$ ) be removed, the energy of the current passing through the galvanometer will be that of a partial current  $Z b a$  passing through one of the wires plus the galvanometer wire; the path of the diverted portion of the current being  $Z a$ . According to this disposition, the force of the original

current  $= \frac{E}{R + 2r + g}$ , and that of the partial current acting on the galvanometer  $= \frac{Er}{R(3r + g) + 2r^2 + rg}$ ;  $R$  being the resistance of the rheomotor,  $r$  that of a single wire, and  $g$  that of the galvanometer.

The equilibrium having been disturbed by the introduction of a resistance in one of the wires, it may be restored by placing an equal resistance in either of the adjacent wires. For the purpose of interposing the measuring resistance and the resistance to be measured, the wires  $Z b$  and  $C b$  are interrupted, and binding screws,  $c, d$  and  $e, f$ , are fixed for the reception of the ends of the wires. The equilibrium when once established is not in any degree affected by fluctuations in the energy of the rheomotor.

Fig. 6 represents a different and, in some respects, a more convenient arrangement of the wires to produce the same result; the same reference letters are employed, and the preceding observations apply to it equally.

Slight differences in the lengths, and even in the tensions of the wires, are sufficient to disturb the equilibrium; it is therefore necessary to have an adjustment, by means of which, when two exactly equal wires are placed in  $C a$  and  $Z a$ , the equilibrium may be perfectly established. For this purpose, in the instrument, fig. 6, a piece of metal  $n$ , connected with the binding-screw  $b$ , is inlaid in the board, and another piece of metal  $m$  moves round  $n$  as a centre, whilst its free extremity always rests on the wire. According as the moveable piece of metal makes a greater angle with the fixed piece, the resistance of the path  $Z b$  is diminished; if, however, the equilibrium is disturbed because the resistance in  $C b$  is too great, the moveable piece of metal must be placed on the opposite side of the fixed piece.



No fixed dimensions can be assigned to these instruments. The boards of those I employ are fourteen inches long and four inches wide, and the wire is copper  $\frac{1}{20}$ th of an inch in diameter. A single voltaic element of large surface will produce a more considerable effect than a battery of small elements\*. A thermo-electric arrangement, or a magneto-electric machine may be substituted for the voltaic element or battery; and a voltameter or any other description of rheometer may in some cases supply the place of the galvanometer. It is scarcely necessary to state that these instruments are not adapted to measure the resistances of substances capable of undergoing chemical changes from the action of an electric current, on account of the contrary electro-motive forces which arise under such circumstances†.

### § 17.

Another differential arrangement, which will be found useful in some circumstances, may be worth mentioning; it is much more sensible than the preceding, but as the equilibrium indicated is that between two currents generated by independent rheomotors, instead of diverted portions of the same current as in the instruments previously described, the state of equilibrium will be disturbed by every fluctuation, whether of the electro-motive force, or resistance of either of the rheomotors; it can therefore only be safely employed when these are perfectly constant, or when the object is not to measure resistances, but to observe the comparative changes in two rheomotors.

Fig. 7 represents a circular board on which are fixed ten binding screws; the wires proceeding from one of the rheomotors are to be attached to  $C^1$  and  $Z^1$ , those from the other to  $C^2$ ,  $Z^2$ , and the ends of the galvanometer wire are to be fixed to  $a$  and  $b$ . The two currents,  $C^1 a b Z^1$  and  $Z^2 a b C^2$ , tend to pass through the galvanometer wire in opposite directions. When two equal wires are interposed between  $e f$  and  $e' f'$ , if the opposing currents be equal, perfect equilibrium is established in the galvanometer wire, and the needle remains at zero. But if the force of the current in either of the rheomotors varies, or, if while the force of the two rheomotors remains constant, the slightest difference is occasioned in the resistance of either of the wires interposed between  $e f$  or  $e' f'$ , the equilibrium in the galvanometer wire is disturbed and the needle is deflected.

\* When a single element of DANIELL's battery, 6 inches high and  $3\frac{1}{2}$  inches diameter, is employed, and two copper wires two feet long and  $\frac{1}{40}$ th of an inch diameter are interposed in the instrument, an augmentation of the tenth of an inch in one occasions a deviation of  $2^\circ$  in the galvanometer needle. This will suffice to show the accuracy with which resistances may be measured by this instrument.

† Mr. CHRISTIE, in his "Experimental determination of the laws of magneto-electric induction" printed in the Philosophical Transactions for 1833, has described a differential arrangement of which the principle is the same as that on which the instruments described in this section have been devised. To Mr. CHRISTIE must, therefore, be attributed the first idea of this useful and accurate method of measuring resistances. Another differential arrangement, proposed also in the same memoir, is analogous to that which forms the subject of the following section.

## § 18.

It would greatly facilitate our quantitative investigations if we had a certain and ready means of ascertaining what degree of the galvanometric scale indicated half the intensity corresponding to any other given degree. The properties of diverted currents, established by the theory of OHM, and fully confirmed by experiment, enables me to propose a simple method by which this object may be completely attained.

If a wire of the same length, thickness and conductivity as that of the galvanometer be placed so as to divert a portion of the current from it, it is obvious that one-half of the current will pass through the galvanometer wire, and the other half through the diverting path. Though it simplifies the consideration to suppose the extra wire to have the same length, diameter and conducting power, it is easy to see that the same result follows if the two wires present the same resistance which they do whenever  $s'c'l = sc'l$ . If the added wire produced no alteration in the intensity of the principal current, one-half of the former force would act upon the galvanometer; but this is not the case, the addition of the wire produces the same effect as doubling the section of the galvanometer wire would do, and the total resistance of the circuit is therefore diminished. If the strength of the original current when it passes wholly through the galvanometer  $= \frac{E}{R + r}$  ( $r$  being the resistance of the galvanometer wire, and  $R$  all the other resistances in the circuit),  $\frac{E}{R + \frac{r}{2}}$  will be the strength

of the principal current when the extra wire is added; if now an additional resistance  $= \frac{r}{2}$ , that is to say, a wire whose resistance is equal to half that of the galvanometer wire, be added to the principal portion of the circuit, the intensity will be again  $\frac{E}{R + \frac{r}{2} + \frac{r}{2}}$ , and the force acting on the galvanometer will be exactly half what it was at first.

The construction and use of the instrument (fig. 8) will now be easily understood. *A* is a square piece of wood, having two insulated pieces of brass, *D*, *N*, inlaid on its surface, on which are fixed the binding screws *C*, *Z* and *a*; *B* is a circle also of wood, moveable round its centre; upon this moveable circle are fixed the insulated piece of brass *F*, with the binding screw *b* upon it, and three springs *G*, *H*, *I*, the free ends of which press on the board *A*. A coil of wire *K*, the equivalent resistance to the wire of the galvanometer, measured by the process described in § 16, is connected by its two ends with the brass plate *F* and the spring *G*; and another coil, *L*, the resistance of which is one-half that of the former, is similarly interposed between the brass plate and the spring *H*. A short wire immediately connects the plate *F* with the spring *I*. *E* is a nut or pin by which the moveable circle is moved through a small arc.

The wires proceeding from the poles of a rheomotor being connected with the

binding screws C, Z, and the ends of the galvanometer wire being attached to the screws *a* and *b*; in the position of the instrument represented in the figure, the springs G and H resting respectively on the insulated pieces of brass D and N, the principal portion of the current passes through the resistance coil L, and the current is afterwards equally divided between the coil of the galvanometer and the resistance coil K. But when the circle is moved in the direction of the arrow, the springs G, H leave the brass plates, and rest on the wood, while the spring I is brought into contact with the plate E; both of the resistance coils are now thrown out of the circuit, and the current passes wholly through the wire of the galvanometer.

It is almost unnecessary to state that this instrument can only be used in conjunction with the galvanometer to which its resistance coils K and L have been adjusted.

In some cases, when an experiment has been performed with a current of a certain degree of intensity, it is required to repeat it with currents of other degrees of strength, the proportions of which to the first current shall have been accurately determined. The instrument above described readily affords the means of doing this. It may thus be ascertained whether the electro-motive force in any particular combination varies or remains constant when the energy of the current changes.

§ 19. *Process to determine the Degrees of Deviation of the Needle of a Galvanometer corresponding to the Degrees of Force; and the Converse.*

When the electro-motive force in the circuit remains constant, the force of the current is simply proportional to the resistance or reduced length of the circuit. If therefore the total resistance of the circuit, when the needle stands at  $1^\circ$ , be determined, and if then, by means of the rheostat and resistance coils, the resistance be successively reduced to  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ , &c., the corresponding forces of the current will be 2, 3, 4, 5, &c. Conversely, if the reduced lengths *a*, *b*, *c*, *d*, &c. necessary to be removed from the circuit in order to advance the needle from each degree to the one next above it be successively ascertained, the forces corresponding to these successive degrees will be

$$\frac{1}{R}, \frac{1}{R-a}, \frac{1}{R-(a+b)}, \frac{1}{R-(a+b+c)}, \text{ \&c.}$$

By the above processes, the relations between the degrees of force and those of the galvanometric scale may be far more readily determined than by either of the ingenious methods of NOBILI, BECQUEREL or MELLONI. When we consider the changes to which the needle of a delicate galvanometer, especially if it be astatic, is subject from the influence of strong currents, the vicinity of magnets, and, in a less degree, from changes of temperature, and in the intensity of the earth's magnetism, the importance of having an easy means of re-graduating the instrument, and of detecting the changes it has undergone, will not be esteemed too lightly.