ERRATUM.

Page 618, line 19, *for* Makerstown *read* Brisbane.

XXIX. *On the Figure, Dimensions, and Mean Specific Gravity of the Earth, as derived from the Ordnance Trigonometrical Survey of Great Britain and Ireland. Communicated by Lieut.-Colonel* JAMES, *R.E., F.R.S. &c., Superintendent of the Ordnance Survey.*

Received April 30,—Read May 8, 1856.

THE Trigonometrical Survey of the United Kingdom commenced in the year 1784, under the immediate auspices of the Royal Society; the first base was traced by General ROY, of the Royal Engineers, on the 16th of April of that year, on Hounslow Heath, in presence of Sir JOSEPH BANKS, the then President of the Society, and some of its most distinguished Fellows.

The principal object which the Government had then in view, was the connexion of the Observatories of Paris and Greenwich by means of a triangulation, for the purpose of determining the difference of longitude between the two observatories.

A detailed account of the operations then carried on is given in the first volume of the 'Trigonometrical Survey,' which is a revised account of that which was first published in the 'Philosophical Transactions' for 1785 and three following years.

At the time when these operations were in progress, the Survey of several counties in the south-east of England, including Kent, Sussex, Surrey, and Hampshire, was also in progress, under the direction of the Master-General of the Ordnance, for the purpose of making military maps of the most important parts of the kingdom in a military point of view; and it was then decided to make the triangulation which extended from Hounslow to Dover the basis of a triangulation for these surveys.

It is extremely to be regretted that a more enlarged view of the subject had not then been taken, and a proper geometrical projection made for the map of the whole kingdom. As it is, the south-eastern counties were first drawn and published in reference to the meridian of Greenwich, then Devonshire in reference to the meridian of Butterton in that county, and thirdly the northern counties in reference to the meridian of Delamere in Cheshire; but there is a large intermediate space, the maps of which are made of various sizes to accommodate them to the convergence of the meridian.

In 1799 the Royal Society gave further proof of the interest it took in the progress of the Survey, by lending to the Ordnance its great 3-foot Theodolite, made by RAMSDEN, for the purpose of expediting the work of the Survey; and I have great pleasure in stating, that although this instrument has been in almost constant use for the last sixty-seven years, during which time it has been placed on the highest

church towers and the loftiest mountains in the kingdom, from the Shetlands to the Scilly Islands, it is at this day in perfect working order, and probably one of the very best instruments that was ever made.

The great Trigonometrical operations of the Survey have been carried on under so many officers, from the time of their commencement under General ROY down to the present time, that it would be quite impossible, in this short notice, to mention more than the names of several Superintendents who have succeeded General ROY, viz. Colonel WILLIAMS, Major-General MUDGE, Major-General COLBY, and Colonel HALL; but in justice to the highly meritorious body of non-commissioned officers of the Corps of Royal Sappers and Miners, I should state, that whilst in the early part of the Survey the most important and delicate observations were entrusted solely to the commissioned officers, these duties have of late years been performed by the non-commissioned officers with the greatest skill and accuracy.

In the Historical sketch of the Survey which I purpose publishing shortly, I hope to be able to do justice to the individual merits of all employed.

The computations connected with the corrections of the observed angles, to make the whole triangulation as nearly as possible perfectly consistent, have been most voluminous, and have been made under the direction of Lieut.-Colonel YOLLAND, Captain CAMERON, and Captain ALEXANDER R. CLARKE; but I gladly avail myself of this opportunity to acknowledge the great and important assistance and advice which, both as regards the instruments and the calculations, we have at all times received from the Astronomer Royal.

The triangulation by the methods which will be explained, is now made consistent in every part, so that any side of any triangle being taken as a base, the same distance will be reproduced when it is computed through any portion or the whole series of triangles; and when the five measured bases on which we rely are incorporated in this triangulation, the greatest difference between their measured and computed lengths is not as much as 3 inches, and yet some of the bases are upwards of 400 miles apart.

Several bases of from five to seven miles long have been measured, but those upon which the chief reliance has been placed are the Lough Foyle and Salisbury Plain bases, which were measured with General COLBY's compensation bars. The difference between the measured and computed length of the one base from the other through the triangulation is 0·4178 ft., or about 5 inches.

This difference has been divided in proportion to the square root of the lengths of the measured bases, by which we have obtained the mean base which has been used in the triangulation; there is therefore a difference of $+$ or $-0\cdot2$ ft., or $2\frac{1}{2}$ inches between the measured and computed length of these bases from the mean base.

The Hounslow Heath base was measured with RAMSDEN's 100-ft. steel chains, and only differs 0·173 ft., or about 2 inches, from its computed length from the mean base.

The Belhelvie base in Aberdeenshire, also measured with the steel chains, differs only 0·24 ft., or less than 3 inches, from the computed length.

The difference between the measured and computed length of the Misterton Carr base, near Doncaster, also measured with the steel chains, is only 0·191 ft., or about 2 inches; and it will be observed that the difference between the computed and measured lengths of these three bases (measured with chains) is not greater than the difference between the measured and computed length of the Lough Foyle and Salisbury Plain bases (measured with the compensation bars), from which it may be inferred that bases measured with steel chains are deserving of the greatest confidence; and when the great simplicity, portability, and cheapness of the chains is compared with the complex, heavy and expensive apparatus of the compensation bars, I should anticipate that they would be more generally employed than they have been of late years, especially in the colonies, and in countries where the transport of heavy articles is effected with difficulty.

The length of the base on Rhuddlan Marsh in North Wales, which was measured with steel chains, differs 1·596 ft. from the computed length; but from the circumstance that the extremities of the base are very badly situated with reference to the surrounding Trigonometrical stations, the angles being very acute, and not well observed, we have placed little confidence in the result of the comparison of its computed and measured length.

One of the first practical results arising from the completion of the triangulation is, that we are now able to engrave the latitude and longitude on the marginal lines of the old sheets of the 1-inch map of England, and this is now being done.

The following account of the Trigonometrical operations and calculations has been drawn up by Captain ALEXANDER R. CLARKE, R.E.: this account may be considered as an abridgement of that more detailed account which is now in the press, and will be shortly published.

It will be seen that the equatorial diameter of the earth, as derived from the Ordnance Survey, is 7926·610 miles, or about one mile greater than it is given by the Astronomer Royal in his 'Figure of the Earth,' and that the ellipticity is $\frac{1}{299,33}$, or as the Astronomer Royal conjectured, something "greater than $\frac{1}{300}$," which he gives in the same paper.

The mean specific gravity of the earth, as derived from the observations at Arthur's Seat, was stated in a former paper to be 5·14; the calculations have since been revised, and we now find it to be 5·316.

The mean specific gravity of the earth, as derived from the only other observations on the attraction of mountain masses on which any reliance has been placed, viz. the Schehallien observations, give, as finally corrected by HUTTON, $\frac{99}{20}$, or almost 5·0.

From the experiments with balls we have the following results:—

By CAVENDISH, as corrected by BAILY	5·448
By BAILY	5·67
By REICH	5·44

From the pendulum experiments at a great depth and on the surface, the Astronomer Royal obtained 6·566.

I have recently received, through the Astronomer Royal, two copies of the new National Standard Yard: it is obviously necessary that our geodetic measures should be given in reference to the standard; but not knowing from what scale the standard has been taken, I am unable to say at present in what way the reduction is to be made; that is, whether by reference to the comparison of the old standards which have been already made, or by the mechanical process of a direct comparison of the Ordnance Standard with the new National Standard.

H. JAMES, *Lieut.-Colonel R.E.*

The Principal Triangulation of the Ordnance Survey of Great Britain and Ireland, extending from the Scilly Isles, in latitude 49° 53′, to the Shetland Isles, in latitude 60° 50′, and embracing at its widest extent about 12° of longitude, consists of about 250 stations.

The observations for the connexion of the trigonometrical stations have been made with four large theodolites, two of 3 feet, one of 2 feet, and the other of 18 inches in diameter. The first two instruments (one of which is the property of the Royal Society), and the 18-inch theodolite, were constructed by RAMSDEN at the commencement of the trigonometrical operations in England in 1798: the 2-feet theodolite was constructed by Messrs. TROUGHTON and SIMMS at the commencement of the Irish Survey in 1824.

The latitudes of thirty-two of the stations of the principal triangulation have been determined by observations made with RAMSDEN's zenith sector, and since the destruction of that instrument in the great fire at the Tower of London, with AIRY's zenith sector. All the observations made with these instruments have been published in detail*.

The mode of observing with the theodolite may be shortly described as follows:— The instrument being first placed very carefully over the precise centre of the station, an object having a fine vertical line of light, with a breadth of about 10″, is set up in a convenient position within a mile or two of the station; this object, called the "referring-object," serves as a point of reference from which all angles are measured. The lower limb of the instrument being clamped, the observer intersects the referring-object and then each of the principal points in succession, concluding with a second observation of the referring-object, which should be identical, within the limits of errors of observation, with the first reading of that object: the instrument is then unclamped and the bearings read again on different parts of the divided circle. The method by which these observations are reduced to the most probable

* 'Astronomical Observations made with RAMSDEN's Zenith Sector,' 1842. 'Astronomical Observations made with AIRY's Zenith Sector,' 1852.

results, is an approximate solution of the equations resulting from the method of least squares.

The direction of the meridian has been determined by observations of the elongations of α, δ, λ Urs. min. and 51 Cephei; at six of the stations at which these observations have been made, the probable error of the result is under $0''\cdot40$, at twelve under $0''\cdot50$, at thirty-four under $0''\cdot70$, and at fifty-one under $1''\cdot00$.

Measured Base Lines.

The account of the measurement with RAMSDEN's steel chain of the base lines on Hounslow Heath in 1791, on Salisbury Plain in 1794, on Misterton Carr in 1801, and on Rhuddlan Marsh in 1806, will be found in the 'Account of the Trigonometrical Survey.' These base lines are all expressed in terms of RAMSDEN's brass scale at the temperature of 62° FAHRENHEIT. The chains were compared with RAMSDEN's prismatic bar (20 feet in length), which was laid off from the brass scale at the temperature of 54° FAHRENHEIT. By a series of comparisons of the Ordnance 10-foot standard iron bar (designated O_1) with RAMSDEN's 20-foot standard, made at Southampton, it was found that

$$\text{RAMSDEN's bar} = 20\cdot0007656 \text{ feet of } O_1,$$

so that any measurement expressed in terms of feet of RAMSDEN's bar at 62° must be multiplied by 1·0000383 to give feet of O_1, at the same temperature. Also to reduce a measurement expressed in terms of RAMSDEN's brass scale to the same in terms of RAMSDEN's bar, it must be increased by a quantity corresponding to the difference of the expansions of brass and iron for 8°; and taking these quantities as used in the reduction of the bases, it will be found that the multiplier is 1·0000328, and hence to reduce the old bases to feet of O_1, they must be multiplied by 1·0000711.

In 1816 a base line of five miles in length was measured by Major-General COLBY on Belhelvie Sands, Aberdeenshire: the measurement was effected with RAMSDEN's steel chains, and in precisely the same manner as the previous bases. The chains were compared with RAMSDEN's bar by Mr. BERGE both before and after the measurement.

In 1826–27 the Lough Foyle Base was measured in the north of Ireland with Major-General COLBY's compensation bars. Of this measurement a description in detail has been published*.

In 1849 the old base line on Salisbury Plain was remeasured. This measurement exceeded the old measure when reduced to the same standard by a foot. The guns marking the extremities of the old line were found imbedded very firmly in the earth, and in all probability in exactly their original positions.

By a series of comparisons instituted in 1834† between the Royal Astronomical

* 'Account of the Measurement of the Lough Foyle Base,' by Captain YOLLAND, R.E., 1847.

† The observations which were made by the late Lieut. MURPHY, Royal Engineers, are given in the 'Account of the Measurement of the Lough Foyle Base.'

Society's Scale and the Ordnance Standard O_2, it was determined that

Ord. Standard $O_1=119\cdot997508$ mean inches of the centre yard of the Royal Astronomical Society's Scale;

also from Mr. BAILY's comparison of this scale with the standard metre he determined*

Standard Metre $=39\cdot369678$ mean inches of the centre yard of the Royal Astronomical Society's Scale.

From more recent observations, it appears that the Royal Astronomical Society's scale has undergone a permanent alteration of length; the interval however between the two series of observations above quoted was not sufficiently long to vitiate the connexion thus established between the Ordnance Standard and the metre. The resulting value of the metre in terms of O_1 is therefore

Standard Metre $=3\cdot2808746$ mean feet of O_1,

and hence since the metre $=443\cdot296$ lines of the toise of Peru,

Toise $=6\cdot3945438$ mean feet of O_1.

Reduction of the Triangulation.

If u represent the true ratio of the distance between any two points in a network of triangulation to the base line; A B C...A'... the true angles, whose observed values are $A+\alpha$, $B+\beta$, $C+\gamma$...$A'+\alpha'$..., then if u_1 be the calculated value of u obtained by using the series of observed angles ABC..., u'_1 the value obtained by using the series of observed angles A'B'C'....

$$u=f(ABC\ldots)=f(A'B'C'\ldots)=\ldots.$$
$$u_1=u+a\alpha+b\beta+c\gamma+\ldots.$$
$$u'_1=u+a'\alpha'+b'\beta'+c'\gamma'+\ldots.$$

and so on. Each different calculation of u will therefore give a different value for that quantity.

In the necessary existence of these discrepancies among the calculated values of u, it becomes of much importance to obtain the most probable value. In ordinary calculations this has been generally effected by assuming it to be the mean of all the calculated values $u_1u'_1u''_1$..... This might be improved upon by assigning to each value of u its proper weight by means of the weights of the observed angles, but the method would still be imperfect and discrepancies would still exist in other parts of the work.

From the above equations, though we cannot determine the precise value of u, yet we can obtain some precise information respecting the errors of observation; for we have evidently, since the quantities $abc\ldots a'b'c'\ldots$ are numerical, certain equations of condition between the unknown errors.

But the number of such equations of condition for the whole figure being necessarily less than the actual number of errors, an indefinite number of systems of corrections

* Memoirs of the Royal Astronomical Society, vol. ix.

might be obtained that would satisfy all the geometrical relations of the triangulation. The question then is to determine that system which is the most probable, and the solution derived from the theory of probabilities is, that the most probable system of corrections $x\,x'\ldots$ is that which makes the function $\Sigma(wx^2)$* a minimum.

If n be the number of observed angles in a network of triangulation, m the number of points, then $2(m-2)$ will be the number of angles absolutely required to fix all the points, consequently the geometrical figure must supply $n-2m+4$ equations of condition amongst the true angles or amongst the corrections to the observed angles: we have therefore $n-2+4$ equations of the form

$$0=a_1+b_1x+c_1x'+d_1x''+\ldots\ldots$$
$$0=a_2+b_2x+c_2x'+d_2x''+\ldots\ldots,$$

which are to be determined so that the quantity

$$U=wx^2+w'x'^2+w''x''^2+\ldots\ldots$$

shall be a minimum. Multiplying the equations of condition by unknown quantities $\lambda_1\,\lambda_2\,\lambda_3\ldots.$, we obtain by the theory of maxima and minima of functions of many variables,

$$\begin{aligned} wx &= b_1\lambda_1+b_2\lambda_2+b_3\lambda_3+\ldots. \\ w'x' &= c_1\lambda_1+c_2\lambda_2+c_3\lambda_3+\ldots. \\ w''x'' &= d_1\lambda_1+d_2\lambda_2+d_3\lambda_3+\ldots. \end{aligned}$$

Substituting the values of $xx'\ldots$ in terms of $\lambda_1\lambda_2\ldots.$ as obtained from these equations, in the equations of condition we get a system of equations from which $\lambda_1\lambda_2\ldots$ may be determined; and having obtained the numerical values of these quantities, the last set of equations will give the numerical values of the required corrections $xx'\ldots$

In order that the results of the triangulation as applied to the determination of the figure of the earth might have the greatest weight possible, the most probable system of corrections has been calculated according to BESSEL's method, shortly described above. The principal and only objection to the application of this method of obtaining the most trustworthy results, is the extremely voluminous and tedious nature of the calculations. The total number of equations of condition for the triangulation is 920; if therefore the whole were to be reduced in one mass, it would involve as a small part of the work the solution of an equation of 920 unknown quantities. The following method of approximation therefore was adopted: the triangulation was divided into twenty-one parts or figures, each affording a not unmanageable number of equations of condition; four of these parts, not adjacent, were first adjusted by the method just explained. The corrections determined in these figures were substituted, so far as they entered, in the equations of condition of the adjacent figure, and the sum of the squares of the remaining corrections in that figure made a minimum. Thus each division of the triangulation, with the exception of the four specified above, is dependent upon one or more of the figures to which it is adjacent.

* Where w is the weight of the observation corresponding to the error x.

The average number of equations in each figure is about 44; the greatest number of equations in any one figure is 77. Each figure was worked by two independent computers. This of itself alone would have been insufficient to secure freedom from error, but the final working of every possible triangle, after the corrections were applied to the observed angles, secured perfect accuracy.

Corrections to all the observed bearings having been obtained in the manner explained, it is clear that since all the geometrical relations of the figure are satisfied, no discrepancies can present themselves between the calculated values of the distance between any two points by whatever series of angles it may be obtained. The triangles are calculated by LEGENDRE's theorem. This theorem may be applied to triangles of any magnitude, up to two or three hundred miles, without fear of error. The greatest errors that can result in the values of the sides a, b of a spheroidal triangle as calculated from the side c by spherical trigonometry, using the geometric mean of the principal radii of curvature of the surface for the mean latitude of the triangle as the radius of the sphere, are (the position of the triangle in *azimuth* being the variable quantity with respect to which the errors are the greatest possible),

$$\varepsilon_a=\mu(4l^2+m^2+4n^2)^{\frac{1}{2}}$$

$$\varepsilon_b=\mu(l^2+4m^2+4n^2)^{\frac{1}{2}}$$

$$\mu=\frac{e^2\cos^2\lambda\,.\,abc}{12\mathrm{R}^2\sqrt{1-n^2}},$$

where R is the radius of the sphere, e the eccentricity of the earth's surface, lmn the cosines of the angles of the triangle opposite abc respectively, and λ its mean latitude.

Comparison of Bases.

The absolute length of any side, or the linear scale of the triangulation, is made to depend on the bases measured with the compensation bars at Lough Foyle and on Salisbury Plain. The discrepancy between the measured and calculated length of these bases is about 5 inches; this discrepancy is divided so that each of the two bases shall exhibit an error proportional to the square root of its length: the comparison of all the bases is then as follows:—

Date.	Bases.	Length in terms of RAMSDEN's Scale.	Length in terms of Ordnance Standard.	Length in Triangulation.	Difference.
		ft.	ft.	ft.	ft.
1791.	Hounslow Heath............	27404·24	27406·190	27406·363	+0·173
1794.	Salisbury Plain	36574·23	36576·830	36577·656	+0·826
1801.	Misterton Carr	26342·19	26344·060	26343·869	−0·191
1806.	Rhuddlan Marsh............	24514·26	24516·000	24517·596	+1·596
1817.	Belhelvie	26515·65	26517·530	26517·770	+0·240
1827.	Lough Foyle	....	41640·887	41641·103	+0·216
1849.	Salisbury Plain	....	36577·858	36577·656	−0·202

Latitudes and Longitudes.

For short distances the ordinary formulæ are sufficient, but in the case of distances above 80 or 100 miles the following formulæ are used, A being the given point, and B that whose latitude and longitude are required:—

Let s = distance AB measured on the surface of the earth
ν = normal to minor axis at A
θ = angle subtended at the foot of this normal by the curve s
α = azimuth of B at A
α' = azimuth of A at B, both measured from the north
λ = latitude of A; $\varkappa = 90 - \lambda$
λ' = latitude of B
ω = difference of longitude
ϱ = radius of curvature of the meridian for the latitude $\frac{1}{2}(\lambda+\lambda')$

$$\tan\tfrac{1}{2}(\alpha'+\zeta-\omega)=\frac{\sin\frac{1}{2}(\varkappa-\theta)}{\sin\frac{1}{2}(\varkappa+\theta)}\cot\tfrac{1}{2}\alpha$$

$$\tan\tfrac{1}{2}(\alpha'+\zeta+\omega)=\frac{\cos\frac{1}{2}(\varkappa-\theta)}{\cos\frac{1}{2}(\varkappa+\theta)}\cot\tfrac{1}{2}\alpha$$

$$\lambda'-\lambda=\frac{s}{\varrho}\,\frac{\sin\frac{1}{2}(\alpha'+\zeta-\alpha)}{\sin\frac{1}{2}(\alpha'+\zeta+\alpha)}\left(1+\frac{\theta^2}{12}\cos^2\tfrac{1}{2}(\alpha'-\alpha)\right)$$

$$\theta=\frac{s}{\nu}+\frac{e^2\theta^3}{6(1-e^2)}\cos^2\lambda\cos^2\alpha$$

$$\zeta=\frac{e^2\theta^2}{4(1-e^2)}\cos^2\lambda\sin 2\alpha,$$

ζ being a minute angular correction here expressed, as also θ, in angular measure.

In the calculation of latitudes and longitudes we must suppose all the points to be projected on a regular spheroidal surface (A), very nearly agreeing with the actual surface covered by the triangulation, then by equations of condition between the observed and calculated latitudes, longitudes and azimuths, small alterations to the elements of the assumed spheroid and its position must be determined: this new spheroid (B) will be that most nearly representing the actual surface under consideration.

For the first spheroid, that determined by the Astronomer Royal as most nearly representing the earth's surface; namely,

$$a=20923713 \text{ feet}$$
$$b=20853810 \text{ feet}$$

was used as the spheroid of reference (A).

If we resolve the inclination of the actual surface at any point to that of the spheroid (B) at the same point, or rather its projection, into two inclinations, one north and the other east, and call these inclinations ξ_n and η_n, these quantities being positive when the actual surface, as compared with that of the regular surface, rises to the

north and to the east, and if we put ξ and η for the same quantities at Greenwich, from which point the calculations of latitude and longitude and azimuths had their commencement, then each point at which the latitude has been observed will give an equation of the form

$$\xi_n = a + b\xi + c\eta + m\delta a + n\delta e,$$

and each point at which the longitude or direction of the meridian has been determined will give an equation of the form

$$\eta_n = a' + b'\xi + c'\eta + m'\delta a + n'\delta e,$$

in which δa and δe are the increments to the semiaxis major and eccentricity of the spheroid (A).

In consequence of the smallness of the coefficients $n\, n'$ in the latitude of Great Britain, the quantity δe would have very little weight as determined from these equations.

Surface of Great Britain.

The approximate results derived from the above equations are, assuming $\delta e = 0$,

$$\begin{aligned} \delta a &= 2536 \\ \eta &= 0 \\ \xi &= +1''\cdot 4, \end{aligned}$$

so that the semiaxes of the spheroid most nearly representing the surface of Great Britain are

$$\left.\begin{aligned} a &= 20926249 \\ b &= 20856337 \end{aligned}\right\} \text{feet of Ordnance Standard } O_1.$$

$$\frac{a-b}{a} = \frac{1}{299\cdot 33}$$

Most probable Deflections.

The last column of the following Table contains the most probable deflections at the various stations as resulting from a comparison of the actual observed latitudes with those of spheroid (B), or of the *apparent* and *mean* latitudes.

Stations.	Latitudes.				
	Observed.	Spheroid (A).	Diff. Obs.—Cal.	Spheroid (B).	Diff. Obs.—Cal.
	° ′ ″	″	″	″	″
Saint Agnes	49 53 33·94	30·78	−3·16	32·93	−1·01
Goonhilly	50 2 50·07	45·35	−4·72	47·42	−2·65
Hensbarrow	50 22 61·84	58·81	−3·03	60·73	−1·11
Port Valley	50 35 43·20	43·17	−0·03	44·95	+1·75
Week Down	50 35 51·42	50·28	−1·14	52·06	+0·64
Boniface Down	50 36 10·55	9·63	−0·92	11·41	+0·86
Dunnose	50 37 7·15	3·75	−3·40	5·53	−1·62
Black Down	50 41 8·89	10·29	+1·40	12·04	+3·15
Southampton	50 54 46·97	47·23	+0·26	48·88	+1·91
Greenwich	51 28 38·30	38·30	0·00	39·70	+1·40
Hungry Hill	51 41 10·26	11·47	+1·21	12·94	+2·68
Feaghmaan	51 55 22·85	20·30	−2·55	21·68	−1·17
Precelly	51 56 45·18	44·76	−0·42	45·99	+0·81
Arbury Hill	52 13 27·01	27·29	+0·28	28·36	+1·35
Forth Mountain	52 18 57·91	56·83	−1·08	57·93	+0·02
Delamere	53 13 18·56	17·27	−1·29	17·93	−0·63
Clifton Beacon	53 27 30·27	27·02	−3·25	27·56	−2·71
South Berule	54 8 56·40	57·60	+1·20	57·87	+1·47
Tawnaghmore	54 17 41·34	39·61	−1·73	39·93	−1·41
Burleigh Moor	54 34 19·75	15·74	−4·01	15·80	−3·95
S. end of Lough Foyle Base	55 2 38·74	33·93	−4·81	33·86	−4·88
Ben Lomond	56 11 26·27	25·26	−1·01	24·64	−1·63
Kellie Law	56 14 51·43	53·69	+2·26	53·02	+1·59
Ben Heynish	56 27 16·88	19·64	+2·76	18·95	+2·07
Great Stirling	57 27 49·12	50·14	+1·02	48·94	−0·18
Cowhythe	57 40 68·92	60·49	−8·43	59·20	−9·72
Monach	58 21 20·84	23·54	+2·70	22·01	+1·17
Ben Hutig	58 33 6·47	5·12	−1·35	3·47	−3·00
North Rona	59 7 15·19	17·67	+2·48	15·80	+0·61
Balta	60 45 1·75	7·16	+5·41	4·51	+2·76
Gerth of Scaw	60 48 56·43	61·82	+5·39	59·15	+2·72
Saxavord	60 49 38·58	41·99	+3·41	39·31	+0·73

Arcs of Meridian.

The two longest meridional lines in the triangulation are from Dunnose, in the Isle of Wight, to Saxavord in the Shetland Isles, and from St. Agnes' Lighthouse, in the Scilly Islands, to North Rona; the lengths and amplitudes are as follows:—

	Length.	Astronomical amplitude.
	feet	° ′ ″
Dunnose to Saxavord	3729335·8	10 12 31·43
St. Agnes to North Rona	3370394·2	9 13 41·25

Arc of Parallel.

The volume of the Greenwich Observations for 1845 contains the account of the determination of the longitude of Feaghmaan, a station in the Island of Valencia on the west coast of Ireland, by the transmission of chronometers.

The whole arc was subdivided by two intermediate stations, of which one was the

Liverpool Observatory, the other a temporary observatory erected at Kingstown. The following Table contains the comparison of the astronomical and geodetical determinations:—

Stations.	Observed Longitude W.	Geodetic Longitude.		Difference.	
		Spheroid (A).	Spheroid (B).	(A).	(B).
	m s	m s	m s	s	s
Liverpool	12 0·05	12 0·35	12 0·26	+0·30	+0·21
Kingstown	24 31·20	24 31·48	24 31·26	+0·28	+0·06
Feaghmaan......................	41 23·23	41 23·02	41 22·74	−0·21	−0·49

Observatories.

The positions of the principal observatories, as calculated by their connexion with the Triangulation of the Ordnance Survey, are contained in the following Table:—

Names.	Latitudes			Longitudes		
	On Spheroid (B).	As observed.	Diff.	On Spheroid (B).	As observed.	Diff.
	° ′ ″	° ′ ″	″	m s	m s	s
Greenwich	51 28 39·70	51 28 38·30	+1·40	0 0·00	0 0·00	0·00
Edinburgh	55 57 17·57	55 57 23·20	−5·63	12 43·61	12 43·00	+0·61
Dublin	53 23 14·21	53 23 13·46	+0·75	25 20·87	25 22·00	−1·13
Cambridge	52 12 51·90	52 12 51·63	+0·27	0 23·26	0 22·69	+0·57
Oxford	51 45 38·56	51 45 36·00	+2·56	5 2·91	5 2·60	+0·31
Durham	54 46 5·27	54 46 6·20	−0·93	6 20·25	6 19·75	+0·50
Liverpool	53 24 47·06	53 24 47·80	−0·74	12 0·26	12 0·05	+0·21
Makerstown	55 49 1·83	55 49 6·00	−4·17	19 26·20	19 28·00	−1·80
Armagh	54 21 10·76	54 21 12·67	−1·91	26 35·52	26 35·50	+0·02

Figure of the Earth.

In obtaining the spheroid most nearly representing the measured arcs of meridian, we shall follow the method given by Bessel in his determination of the figure of the earth in Nos. 333 and 438 of the 'Astronomische Nachrichten;' substituting for the English arc as used by him, the data of the present results, and for the Indian arc as used by him, the data contained at page 427 of Colonel Everest's 'Account of the Measurement of Two Sections of the Meridional Arc of India.'

Colonel Everest's measurements are expressed in terms of his standard 10-foot iron bar A, and the standard 6-inch scale A, twenty parts of any linear result being in terms of feet of the iron standard and one part in terms of feet of the 6-inch scale. By means of the comparisons contained at page 436 of Colonel Everest's work and at pages 101 and [40] of the 'Account of the Measurement of the Lough Foyle Base,' it will be found, that to reduce the results contained in the former work to feet of O_1, they must be multiplied by ·99999026. There is, however, some uncertainty in the unit of measure of the earlier portion of the second East Indian Arc.

Peruvian Arc.—The data are a mean between the reduced results obtained by Delambre and Zach (Base du Syst. Metr. iii. 133, and Mon. Corresp. xxvi. 52).

Indian Arc.—The data for the first Indian Arc are given in vol. viii. of the Asiatic Researches, page 137.

French Arc.—The data for this arc will be found in the Base du Syst. Metr. ii. 565, 615, and iii. 548, 549.

Hanoverian Arc.—From Gauss' Breitenunterscheid, &c., p. 71.

Danish and Russian Arcs, as used by Bessel, Astronomische Nachrichten, No. 333.

Prussian Arc.—Gradmessung in Ostpreussen: Bessel, 1838.

Swedish Arc.—Exposition des Opérations faites en Lapponie: Svanberg.

	Latitude.	Amplitude.	Distance between the parallels.
1. Peruvian Arc.			
Tarqui	−3° 4′ 32″·068		toise.
Cotchesqui	+0 2 31·387	3° 7′ 3″·455	176875·5
2. First East Indian Arc.			
Trivandeporum	11 44 52·590		feet.
Pandree	13 19 49·018	1 34 56·428	574327·9
3. Second East Indian Arc.			
Punnæ	8 9 31·722		feet.
Damargida	18 3 15·292	9 53 43·570	3591744·06
Kalianpur	24 7 11·262	15 57 39·540	5794648·82
Kaliana	29 30 48·322	21 21 16·600	7755786·84
4. French Arc.			
Formentera	38 39 56·11		toise.
Mountjouy	41 21 44·96	2 41 48·85	153673·61
Barcelona	41 22 47·90	2 42 51·79	154616·74
Carcassonne	43 12 54·30	4 32 58·19	259172·61
Evaux	46 10 42·54	7 30 46·43	428019·31
Panthéon	48 50 49·37	10 10 53·26	580312·41
Dunkirk	51 2 8·85	12 22 12·74	705257·21
5. English Arc.			
Dunnose	50 37 7·15		feet.
Southampton	50 54 46·97	0 17 39·82	107807·19
Greenwich	51 28 38·30	0 51 31·15	313716·97
Arbury Hill	52 13 27·01	1 36 19·86	586356·34
Clifton	53 27 30·27	2 50 23·12	1036581·55
Kellie Law	56 14 51·43	5 37 44·28	2055737·20
Great Stirling	57 27 49·12	6 50 41·97	2499839·45
Saxavord	60 49 38·58	10 12 31·43	3729335·78
6. Hanoverian Arc.			
Göttingen	51 31 47·85		toise
Altona	53 32 45·27	2 0 57·42	115163·725

	Latitude.	Amplitude.	Distance between the parallels.
7. Danish Arc.			
Lauenburg	53° 22′ 17″·046		toise.
Lyssabbel	54 54 10·352	1° 31′ 53″·306	87436·538
8. Prussian Arc.			
Trunz	54 13 11·466		toise.
Königsberg	54 42 50·500	0 29 39·034	28211·629
Memel	55 43 40·446	1 30 28·980	86176·975
9. Russian Arc.			
Bélin	52 2 40·864		toise.
Nemesch	54 39 4·519	2 36 23·655	148811·418
Jacobstadt	56 30 4·562	4 27 23·698	254543·454
Bristen	56 34 51·550	4 32 10·686	259110·085
Dorpat	58 22 47·280	6 20 6·416	361824·461
Hochland	60 5 9·771	8 2 28·907	459363·008
10. Swedish Arc.			
Malörn	65 31 30·265		toise.
Pahtawara	67 8 49·830	1 37 19·565	92777·981

If ϱ be the radius of curvature of the meridian at a point whose latitude is λ,

$$\varrho=\frac{a(1-e^2)}{(1-e^2\sin^2\lambda)^{\frac{3}{2}}};$$

or if we make

$$\frac{a-b}{a+b}=n,\quad e^2=\frac{4n}{(1+n)^2},$$

$$\therefore\quad \varrho=\frac{a(1-n)(1-n^2)}{(1+2n\cos 2\lambda+n^2)^{\frac{3}{2}}},$$

which being expanded becomes

$$\varrho=a(1-n)(1-n^2)\mathrm{N}\{1-2\alpha\cos 2\lambda+2\alpha'\cos 4\lambda-\dots\}$$

$$\mathrm{N}\;=1+\left(\frac{3}{2}\right)^2 n^2+\left(\frac{3.5}{2\cdot 2}\right)^2 n^4+\dots$$

$$\mathrm{N}\alpha=\frac{3}{2}n+\frac{3.5}{2.4}\cdot\frac{3}{2}n^3+\dots$$

$$\mathrm{N}\alpha'=\frac{3.5}{2.4}n^2+\frac{3.5.7}{2.4.6}\cdot\frac{3}{2}n^4+\dots$$

If s be the meridian distance between the points whose latitudes are $\lambda_1\lambda_2$

$$s=\int_{\lambda_1}^{\lambda_2}\varrho.d\lambda;$$

and putting $\lambda_2-\lambda_1=\phi$, $2\lambda=\lambda_1+\lambda_2$,

$$s=\frac{180g}{\pi}\left\{\phi-2\alpha\sin\phi\cos 2\lambda+\alpha'\sin 2\phi\cos 4\lambda-\dots\right\},$$

where g is the length of a mean degree of the meridian determined by the relation

$$g=\frac{\pi}{180}a(1-n)(1-n^2)\mathrm{N}.$$

Hence if s be the meridian distance of two points whose latitudes are λ_1+x_1 and λ_2+x_2, we must substitute in this equation $\phi+x_2-x_1$ for ϕ, neglecting the influence of the small quantities x on the mean latitude of the arc; after this substitution we obtain

$$x_2-x_1=\mu\left(\frac{s\pi}{180g}-\phi+2\alpha\sin\phi\cos 2\lambda-\alpha'\sin 2\phi\cos 4\lambda\right)$$

$$\mu=1+2\alpha\cos\phi\cos 2\lambda.$$

Now let g_1 α_1 be approximate values of g and α, so that

$$\frac{1}{g}=\frac{1+i}{g_1},\quad \alpha=\alpha_1(1+k).$$

Then substituting in the preceding equation, we have finally

$$x_2-x_1=\mu\left\{\frac{3600}{g_1}s-\phi+\mathrm{P}_1-\frac{1}{6}\mathrm{P}_2\right\}$$

$$+\mu\left\{\frac{3600}{g_1}s\right\}i+\mu\left\{\mathrm{P}_1-\frac{1}{3}\mathrm{P}_2\right\}k,$$

where x_1 x_2 and ϕ are expressed in seconds, and

$$\mathrm{P}_1=\frac{2\alpha_1}{\sin 1''}\sin\phi\cos 2\lambda$$

$$\mathrm{P}_2=\frac{5\alpha_1^2}{\sin 1''}\sin 2\phi\cos 4\lambda.$$

This equation contains the relation between the corrections to the terminal latitudes of a measured line s required to bring them into accordance with the measured distance, the elements of the spheroid of reference being as expressed above in g_1 α_1 i k. An arc in which there are n observed latitudes will therefore afford $n-1$ equations of the form

$$x_2-x_1=m+ai+bk;$$

the quantities ik must then be determined so as to make the function

$$x_1^2+x_2^2+x_3^2+\ldots.$$

a minimum.

The final equations thus deduced are

$$0=\mathrm{M}+\mathrm{A}i+\mathrm{B}k$$

$$0=\mathrm{M}'+\mathrm{B}i+\mathrm{B}'k$$

$$\mathrm{M}=\Sigma\left\{(am)-\frac{(a)(m)}{r}\right\}\quad \mathrm{M}'=\Sigma\left\{(bm)-\frac{(b)(m)}{r}\right\}$$

$$\mathrm{A}=\Sigma\left\{(a^2)-\frac{(a)^2}{r}\right\}\qquad \mathrm{B}=\Sigma\left\{(ab)-\frac{(a)(b)}{r}\right\}\quad \mathrm{B}'=\Sigma\left\{(b^2)-\frac{(b)^2}{r}\right\},$$

where r is the number of observed latitudes in any arc, the symbol Σ signifying summation with respect to the different arcs.

BESSEL adopts the approximate elements $g_1=57008$ toises $\alpha_1=\frac{1}{400}$; the equations for the different arcs are then as follows, putting $10000\,i=p$ and $10\,k=q$.

1. Peruvian Arc.

$$x_1^{(1)}-x_1=+1{\cdot}966+1{\cdot}1225\,p+5{\cdot}6059\,q.$$

2. First East Indian Arc.

$$x_2^{(1)}-x_2=+0{\cdot}937+0{\cdot}5697\,p+2{\cdot}5835\,q.$$

3. Second East Indian Arc.

$$x_3^{(1)}-x_3=+\ 5{\cdot}346+3{\cdot}5628\,p+15{\cdot}9269\,q.$$
$$x_3^{(2)}-x_3=+\ 4{\cdot}801+5{\cdot}7458\,p+24{\cdot}0257\,q.$$
$$x_3^{(3)}-x_3=+12{\cdot}440+7{\cdot}6875\,p+29{\cdot}7981\,q.$$

4. French Arc.

$$x_4^{(1)}-x_4=+3{\cdot}991+0{\cdot}9713\,p+0{\cdot}8601\,q.$$
$$x_4^{(2)}-x_4=+0{\cdot}646+0{\cdot}9772\,p+0{\cdot}8642\,q.$$
$$x_4^{(3)}-x_4=+0{\cdot}026+1{\cdot}6378\,p+1{\cdot}1889\,q.$$
$$x_4^{(4)}-x_4=-5{\cdot}035+2{\cdot}7041\,p+1{\cdot}2671\,q.$$
$$x_4^{(5)}-x_4=+1{\cdot}191+3{\cdot}6655\,p+0{\cdot}8659\,q.$$
$$x_4^{(6)}-x_4=+5{\cdot}171+4{\cdot}4537\,p+0{\cdot}2051\,q.$$

5. English Arc.

$$x_5^{(1)}-x_5=+\ 3{\cdot}772+0{\cdot}1064\,p-0{\cdot}1038\,q.$$
$$x_5^{(2)}-x_5=+\ 3{\cdot}735+0{\cdot}3095\,p-0{\cdot}3176\,q.$$
$$x_5^{(3)}-x_5=+\ 4{\cdot}302+0{\cdot}5784\,p-0{\cdot}6308\,q.$$
$$x_5^{(4)}-x_5=+\ 1{\cdot}258+1{\cdot}0224\,p-1{\cdot}2226\,q.$$
$$x_5^{(5)}-x_5=+\ 7{\cdot}874+2{\cdot}0272\,p-2{\cdot}8959\,q.$$
$$x_5^{(6)}-x_5=+\ 7{\cdot}127+2{\cdot}4649\,p-3{\cdot}7683\,q.$$
$$x_5^{(7)}-x_5=+10{\cdot}883+3{\cdot}6763\,p-6{\cdot}6163\,q.$$

6. Hanoverian Arc.

$$x_6^{(1)}-x_6=+5{\cdot}679+0{\cdot}7263\,p-0{\cdot}9224\,q.$$

7. Danish Arc.

$$x_7^{(1)}-x_7=-0\cdot369+0\cdot5513\,p-0\cdot8537\,q.$$

8. Prussian Arc.

$$x_8^{(1)}-x_8=-0\cdot368+0\cdot1779\,p-0\cdot2852\,q.$$

$$x_8^{(2)}-x_8=+3\cdot790+0\cdot5433\,p-0\cdot9157\,q.$$

9. Russian Arc.

$$x_9^{(1)}-x_9=+0\cdot248+0\cdot9384\,p-1\cdot3293\,q.$$

$$x_9^{(2)}-x_9=+5\cdot110+1\cdot6049\,p-2\cdot5184\,q.$$

$$x_9^{(3)}-x_9=+5\cdot939+1\cdot6337\,p-2\cdot5741\,q.$$

$$x_9^{(4)}-x_9=+2\cdot909+2\cdot2809\,p-3\cdot9289\,q.$$

$$x_9^{(5)}-x_9=+5\cdot276+2\cdot8953\,p-5\cdot3824\,q.$$

10. Swedish Arc.

$$x_{10}^{(1)}-x_{10}=-0\cdot507+0\cdot5839\,p-1\cdot9711\,q.$$

From which we obtain the following quantities for the different arcs:—

No. of Arc.	(*m*)	(*a*)	(*b*)	(*am*)	(*aa*)	(*ab*)	(*bm*)	(*bb*)
1.	+ 1·996	+1·1225	+ 5·6059	+ 2·2068	1·2600	+ 6·2926	+ 11·0211	31·4261
2.	+ 0·937	0·5697	+ 2·5835	+ 0·5338	0·3246	+ 1·4718	+ 2·4207	6·6745
3.	+22·587	16·9961	+69·7507	+142·2645	104·8052	+423·8645	+581·1810	1718·8272
4.	+ 5·990	14·4096	+ 5·2513	+ 18·3309	45·1641	+ 11·1409	− 0·2661	5·2976
5.	+38·951	10·1851	−15·5553	+ 78·8502	25·1874	− 41·2067	−127·4937	68·3661
6.	+ 5·679	0·7263	− 0·9294	+ 4·1247	0·5275	− 0·6750	− 5·2780	0·8638
7.	− 0·369	0·5513	− 0·8537	− 0·2034	0·3039	− 0·4706	+ 0·3150	0·7288
8.	+ 3·422	0·7212	− 1·2009	+ 1·9936	0·3268	− 0·5482	− 3·3655	0·9198
9.	+19·482	9·3532	−15·7331	+ 40·0469	19·7106	− 34·0396	− 68·3130	59·1418
10.	− 0·507	+ 0·5839	− 1·9711	− 0·2960	0·3409	− 1·1509	+ 0·9994	3·8852

No. of Arc.	M.	A.	B.	M′.	B′.
1.	+ 1·1034	0·6300	+ 3·1463	+ 5·5106	15·7131
2.	+ 0·2669	0·1623	+ 0·7359	+ 1·2104	3·3373
3.	+46·2918	32·5885	+127·4920	+177·3163	502·5376
4.	+ 6·0003	15·5021	+ 0·3313	− 4·7597	1·3583
5.	+29·2602	12·2204	− 21·4027	− 51·7569	38·1202
6.	+ 2·0624	0·2638	− 0·3375	− 2·6390	0·4319
7.	− 0·1017	0·1519	− 0·2353	+ 0·1575	0·3644
8.	+ 1·1710	0·1534	− 0·2595	− 1·9957	0·4391
9.	+ 9·6768	5·1302	− 9·5138	− 17·2270	17·8868
10.	− 0·1480	0·1705	− 0·5755	+ 0·4997	1·9426
Sums	+95·5831	66·9731	+ 99·3812	+106·3162	582·1313

The final equations are therefore—

$$0=+\ 95{\cdot}5831+66{\cdot}9731\,p+\ 99{\cdot}3812\,q$$
$$0=+106{\cdot}3162+99{\cdot}3812\,p+582{\cdot}1313\,q,$$

from which

$$p=-1{\cdot}548450$$
$$q=+0{\cdot}081718.$$

These quantities, when substituted in the equations of condition, give the following values of the corrections required:—

1. Peruvian Arc.	
Tarqui	−0″·360
Cotchesqui	+0·360
2. First Indian Arc.	
Trivandeporum	−0·134
Pandree	+0·134
3. Second Indian Arc.	
Punnæ	−0·492
Damargida	+0·638
Kalianpur	−2·625
Kaliana	+2·479
4. French Arc.	
Formentera	+2·270
Mountjouy	+4·828
Barcelona	+1·474
Carcassonne	−0·142
Evaux	−6·848
Panthéon	−2·144
Dunkirk	+0·562
5. English Arc.	
Dunnose	−2·738
Southampton	+0·860
Greenwich	+0·491
Arbury Hill	+0·616
Clifton	−3·164
Kellie Law	+1·760
Great Stirling	+0·264
Saxavord	+1·911
6. Hanoverian Arc.	
Göttingen	−2″·239
Altona	+2·239
7. Danish Arc.	
Lauenburg	+0·646
Lyssabbel	−0·646
8. Prussian Arc.	
Trunz	−0·736
Königsberg	−1·402
Memel	+2·138
9. Russian Arc.	
Bélin	−0·619
Nemesch	−1·932
Jacobstadt	+1·800
Bristen	+2·579
Dorpat	−1·563
Hochland	−0·260
10. Swedish Arc.	
Malörn	+0·786
Pahtawara	−0·786

The values of the axes are—

$$a=\frac{180g}{\pi N(1-n^2)^2}(1+n)$$

$$b=\frac{180g}{\pi N(1-n^2)^2}(1-n),$$

which in terms of g_1 p and q, are—

$$a=\frac{601}{600}\cdot\frac{180g_1}{\pi}\left(1-\frac{n^2}{4}\right)-\frac{180g_1}{10000\pi}\left(p-\frac{10}{6}q\right)+\cdots$$

$$b=\frac{599}{600}\cdot\frac{180g_1}{\pi}\left(1-\frac{n^2}{4}\right)-\frac{180g_1}{10000\pi}\left(p+\frac{10}{6}q\right)+\cdots$$

If we put ε for the mean error of an equation,

$$\text{Mean error of } p\pm\lambda q=\frac{\varepsilon}{291\cdot07}\sqrt{1267\cdot2\mp258\cdot7\lambda+115\cdot3\lambda^2}.$$

Now the sum of the squares of the errors, or quantities x, is 160·26;

$$\therefore\ \varepsilon=\sqrt{\frac{160\cdot26}{38-12}}=\pm2\cdot48.$$

The values of a and b, and their mean errors, are consequently

$a=20924933$; mean error ±800

$b=20854731$; mean error ±606.

The ratio of the axes is expressed by the relation

$$a:b::\frac{1}{2n}+\frac{1}{2}:\frac{1}{2n}-\frac{1}{2}$$

$$\frac{1}{2n}=300-30q+3q^2.$$

Consequently the compression is

$$\frac{a-b}{a}=\frac{1}{298\cdot07};\ \text{mean error of denominator } \pm2\cdot70.$$

The length of a degree of the meridian whose mean latitude is λ, is consequently

$$=364596\cdot61-1837\cdot79\cos 2\lambda+3\cdot85\cos 4\lambda,$$

and the length of a degree of longitude in latitude λ

$$=365515\cdot56\cos\lambda-306\cdot96\cos 3\lambda+0\cdot39\cos 5\lambda.$$

Had the point Evaux in the French Arc, at which there is obviously some peculiar local disturbance, been omitted, we should have obtained $p=-1\cdot65000$, $q=+\cdot09341$: these values would have increased the values of the semiaxes as obtained above by about 200 feet each, and increased the compression to

$$\frac{a-b}{a}=\frac{1}{297\cdot72}.$$

The corrections in the French Arc would then stand thus:—

Formentera	+1·319
Mountjouy	+3·788
Barcelona	+0·434
Carcassonne	−1·246
Panthéon	−3·457
Dunkirk	−0·839

and the correction for Evaux is increased to 8·059. The corrections for the other arcs are not materially altered, but are in general diminished: the mean error of the

equations is $\pm 2{\cdot}05$, which would diminish the probable error of the results in the proportion of 5 : 4.

Summary.

We may state the results of this paper briefly as follows:—

1st. The four bases of verification, when their measured lengths are compared with their lengths as calculated from a mean of the Lough Foyle and Salisbury Plain bases, show the following discrepancies, expressed in feet:

Hounslow.	Misterton Carr.	Rhuddlan Marsh.	Belhelvie.
+0·173	−0·191	+1·596	+0·240

2nd. The elements of the spheroid (B) most nearly representing the surface of Great Britain are—

$$\left.\begin{array}{lll} & \text{feet } O_1. & \text{miles.} \\ \text{Equatorial semidiameter} & =20926249 & =3963{\cdot}305 \\ \text{Polar semidiameter} \ldots\ldots & =20856337 & =3950{\cdot}064 \end{array}\right\} \text{compression} = \frac{1}{299{\cdot}33}.$$

3rd. The elements of the spheroid (C) most nearly representing the whole of the measured arcs considered in this paper are—

$$\left.\begin{array}{lll} & \text{feet } O_1. & \text{miles.} \\ \text{Equatorial semidiameter} & =20924933 & =3963{\cdot}057 \\ \text{Polar semidiameter} \ldots\ldots & =20854731 & =3949{\cdot}760 \end{array}\right\} \text{compression} = \frac{1}{298{\cdot}07}.$$

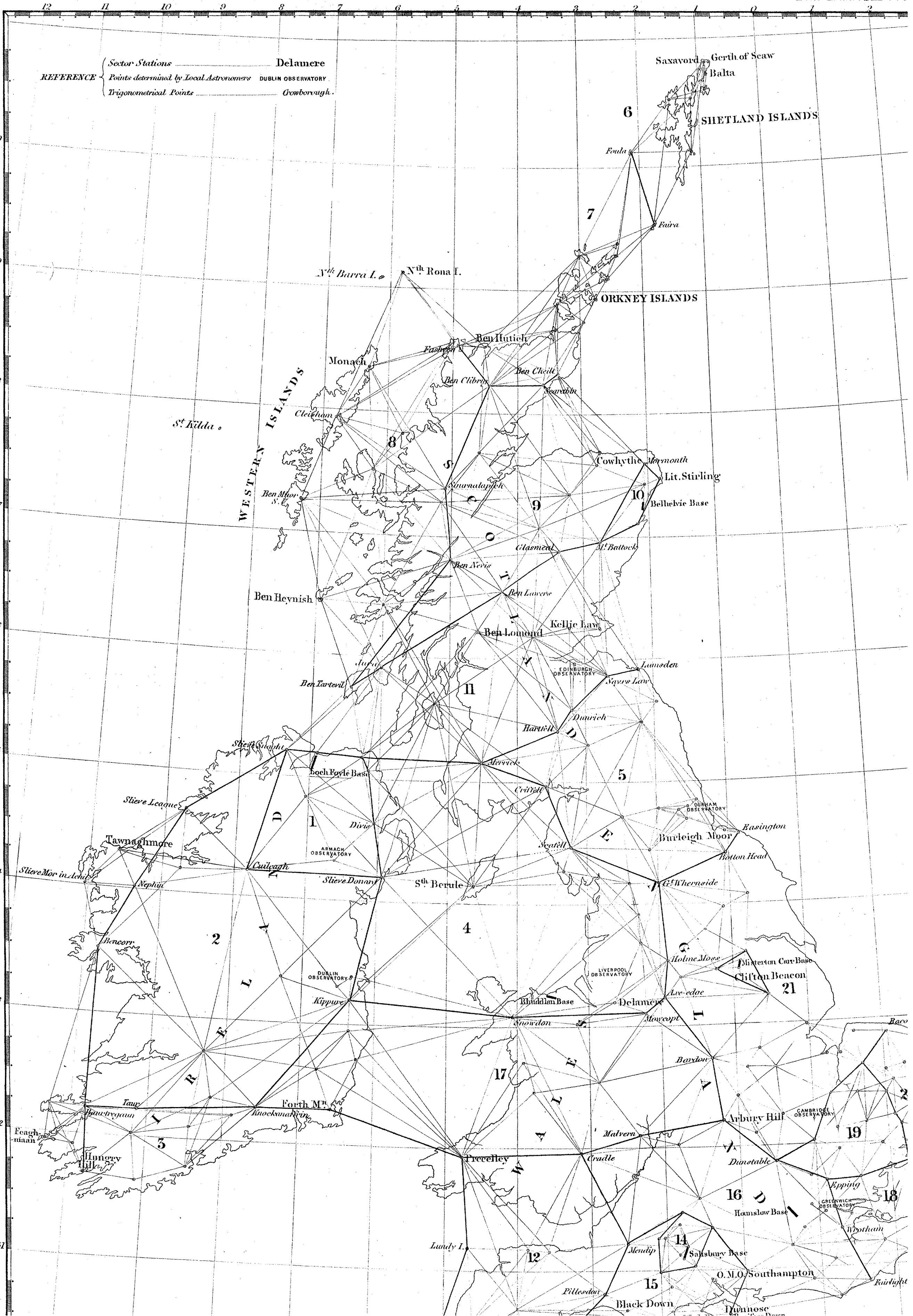
4th. The lengths of the degrees of latitude and longitude in Great Britain are as in the following Table:—

Mean latitude.	From Ord. Survey, Spheroid (B).		From Spheroid (C).	
	Length in feet of 1° of latitude.	Length in feet of 1° of longitude.	Length in feet of 1° of latitude.	Length in feet of 1° of longitude.
50°	364936·33	235227·42	364912·12	235214·58
51	364999·14	230312·27	364975·19	230299·77
52	365061·50	225326·39	365037·81	225314·19
53	365123·34	220271·15	365099·92	220259·23
54	365184·58	215148·11	365161·41	215136·58
55	365245·15	209958·83	365222·23	209947·61
56	365304·96	204704·93	365282·29	204694·04
57	365363·96	199387·90	365341·53	199377·33
58	365422·06	194009·37	365399·88	193999·13
59	365479·20	188571·00	365457·26	188561·08
60	365535·30	183074·50	365513·59	183064·93

Explanation of the Plate.

PLATE XXXIV.

Is a diagram of the triangulation of the United Kingdom, and is one of the plates which has been engraved at the Ordnance Survey Office to illustrate the Account of the Trigonometrical Survey of the United Kingdom which is now in the press.



REFERENCE
Sector Stations .... Delamere
Points determined by Local Astronomers DUBLIN OBSERVATORY
Trigonometrical Points .... Growborough
Saxavord
Gerth of Scaw
Balta
SHETLAND ISLANDS
Foula
Faira
ORKNEY ISLANDS
Nth Barra I.
Nth Rona I.
Ben Hutich
Fashven
Monach
Ben Clibrig
Ben Cheilt
Scaraben
WESTERN ISLANDS
St. Kilda
Cleisham
SCOTLAND
Cowhythe
Mormonth
Lit. Stirling
Belhelvie Base
Ben Mhor S. U.
Sournalapich
Glasmeal
Mt. Battock
Ben Nevis
Ben Lawers
Ben Heynish
Kellie Law
Ben Lomond
EDINBURGH OBSERVATORY
Lumsden
Jura
Ben Tartevil
Sayrs Law
Hartfell
Dunrich
Slieve Snaght
Loch Foyle Base
Merrick
Criffell
Slieve League
Divis
ARMACH OBSERVATORY
DURHAM OBSERVATORY
Easington
Tawnaghmore
Burleigh Moor
Scafell
Botton Head
Slieve Mor in Achil
Cuilcagh
Slieve Donard
Nephin
St. Berule
Gt. Whernside
IRELAND
ENGLAND
Bencorr
Holme Moss
Misterton Carr Base
Clifton Beacon
DUBLIN OBSERVATORY
LIVERPOOL OBSERVATORY
Kippure
Rhuddlan Base
Delamere
Axe-edge
Snowdon
Mowcopt
Barton
WALES
Forth Mt.
Taur
Baurtregaun
Knockanaffrin
Feagh-niaan
Arbury Hill
CAMBRIDGE OBSERVATORY
Malvern
Hungry Hill
Precelley
Cradle
Dunstable
Epping
GREENWICH OBSERVATORY
Hounslow Base
Wrotham
Lundy I.
Mendip
Salisbury Base
O. M. O. Southampton
Fairlight
Pillesdon
Black Down
Dunnose
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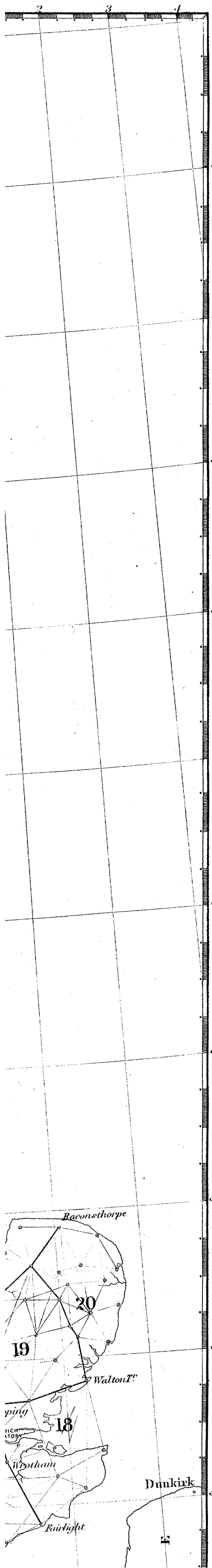
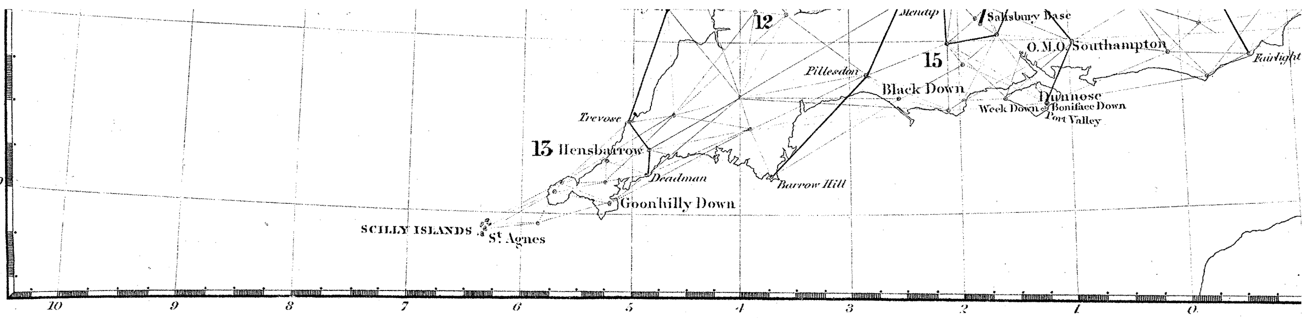
19
21

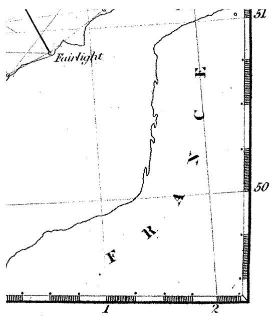

2
3
4
60
59
58
57
56
55
54
53
52
51
Baconsthorpe
20
19
Walton T.r
18
Wrotham
Fairlight
Dunkirk



Engraved at the Ord. Map Office, Southampton.

Scale of Geographical Miles.

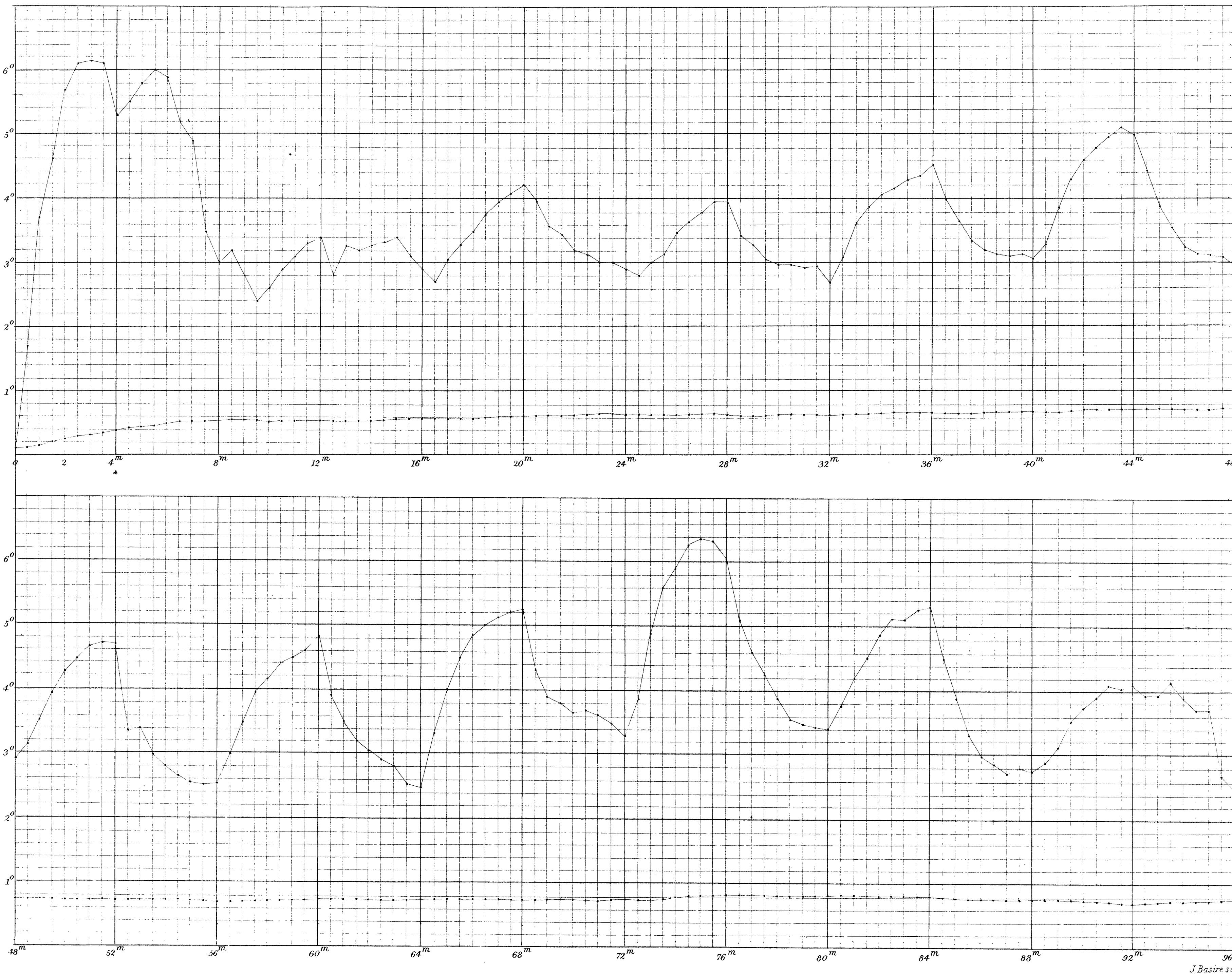
10 5 0 10 20 30 40 50 60 70 80 90 100



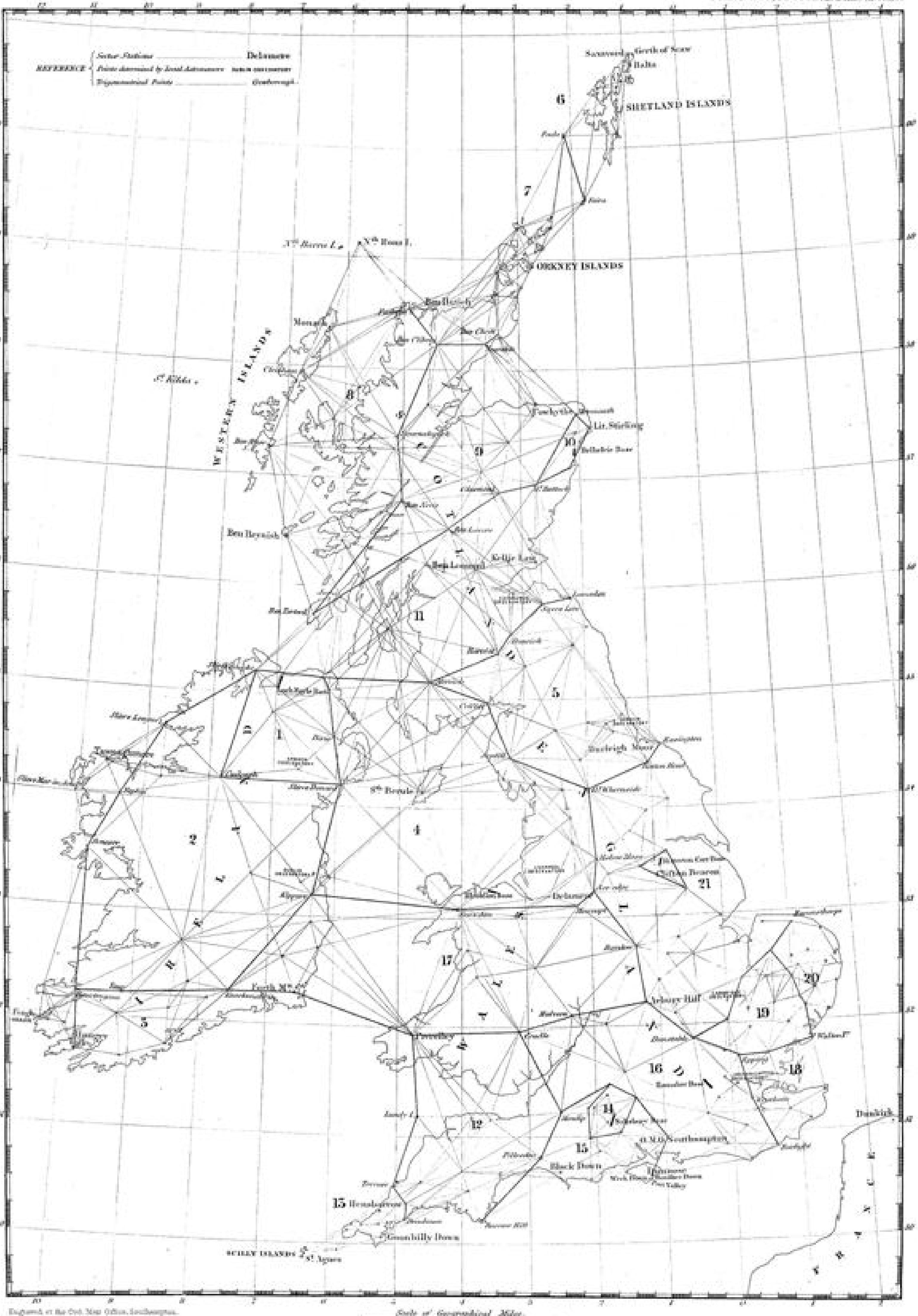
Fairlight
F R A N C E
51
50
1
2

March 25 1854 Iron conductor (two slips) heated by current.

The current flowing twelve times four minutes in each direction; temperatures observed every half minute. The ordinates of the upper curve show the differences of temperature at the times corresponding to the abscissas, and those of the lower curve show one one-hundredth of half the sums of the same temperatures.



J. Basire sc.



Engraved at the Ord. Map Office, Southampton.