X. *Abstract of the Results of the Comparisons of the Standards of Length of England, France, Belgium, Prussia, Russia, India, Australia, made at the Ordnance Survey Office, Southampton. By Captain* A. R. Clarke, *R.E., F.R.S., &c., under the Direction of Colonel Sir* Henry James, *R.E., F.R.S., &c., Director of the Ordnance Survey. With a Preface by Colonel Sir* Henry James, *R.E., F.R.S., &c.*

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The principal triangulation of the United Kingdom was finished in 1851; and the triangulations of France, Belgium, Prussia, and Russia were so far advanced in 1860 that, if connected, we should have a continuous triangulation from the Island of Valentia, on the south-west extremity of Ireland, in north latitude 51° 55′ 20″, and longitude 10° 20′ 40″ west of Greenwich, to Orsk, on the River Ural in Russia.

It was therefore possible to measure the length of an arc of parallel in latitude 52° of about 75°, and to determine, by the assistance of the electric telegraph, the exact difference of longitude between the extremities of this arc, and thus obtain a crucial test of the accuracy of the figure and dimensions of the earth, as derived from the measurement of arcs of meridian, or the data for modifying the results previously arrived at.

The Russian Government, therefore, at the instance of M. Otto Struve, Imperial Astronomer of Russia, invited (in 1860) the cooperation of the Governments of Prussia, Belgium, France, and England, to effect this most important object, and to their great honour they all consented, and granted the necessary funds for the execution of the work.

The portion of the work which was assigned to me was the connexion of the triangulation of England with that of France and Belgium, and I published the results of this operation in 1862*. But this work has been done in duplicate; for when application was made to the French Government to permit the necessary observations to be made in France, they not only consented to allow this, but at the same time volunteered to join in the labour and expense of the work itself.

It would obviously have been wrong to mix up observations made with different kinds of instruments and on different principles, and therefore it was agreed that the work should, in fact, be made in duplicate, both the French and English geometricians using the exact same stations.

The results obtained by the French geometricians is published in the Supplement to vol. ix. of the 'Mémorial du Dépôt Général de la Guerre,' 1865, and the agreement with the results obtained by the English is truly surprising.

But however accurately the trigonometrical observations might be performed, it is obvious that, without a precise knowledge of the relative lengths of the standards used

* Extension of the Triangulation of the Ordnance Survey into France and Belgium. London, 1863.

as the units of measure in the triangulation of the several countries, it would be impossible accurately to express the length of the arc of parallel in terms of any one of the standards.

It was therefore necessary that a comparison of the standards of length should be made; and as we had a building and apparatus expressly erected for the purpose of comparing standards at this office, the English Government, on my recommendation, invited the Governments of the several countries named to send their standards here; and we have had the following compared with the greatest accuracy:—

1. Russian Standard double Toise, P.
2. Prussian Standard Toise.
3. Belgian Standard Toise.
4. Platinum Metre of the Royal Society, compared with the Standard Metre of France by M. ARAGO.
5. English Standard Yards, A, B, C, 29, 47, 51, 55, 58.
6. Ordnance Survey 10-foot Standard Bar.
7. Indian 10-foot Standard Bars, new and old.
8. Australian 10-foot Standard Bars.
9. In addition to the above, the 10-foot Standard Bar of the Cape of Good Hope was compared here in 1844.

We have invited the Governments of Austria, Spain, and the United States of America, also to send their standards. We have been promised that of Austria, and but for the unfortunate war in which she has been lately engaged, we should have received it before this.

I have entrusted the execution of the work of comparison and the drawing up of the results to Captain ALEXANDER R. CLARKE, of the Royal Engineers, who designed the apparatus used. The numerous comparisons to be made entailed a great amount of labour upon him and his assistants, Quartermaster STEEL and Corporal COMPTON, of the Royal Engineers.

Before the connexion of the triangulations of the several countries into one great network of triangles extending across the entire breadth of Europe, and before the discovery of the electric telegraph and its extension from Valentia to the Ural Mountains, it was not possible to execute so vast an undertaking as that which is now in progress. It is, in fact, a work which could not possibly have been executed at any earlier period in the history of the world. The exact determination of the figure and dimensions of the earth has been the great aim of astronomers for upwards of two thousand years; and it is fortunate that we live in a time when men are so enlightened as to combine their labours to effect an object desired by all, and at the first moment when it was possible to execute it.

A full detailed account of the 'Comparisons of the Standards of Length,' with numerous plates, has just been published, and may be obtained from the agents for the sale of the publications of the Ordnance Survey.

HENRY JAMES, *Colonel R.E.*

Ordnance Survey Office,
Southampton, 14*th November*, 1866.

On the Comparison of English and Foreign Geodetical Standards with the English Standard Yard. By Captain A. R. Clarke, *R.E., F.R.S., &c.*

In the Philosophical Transactions, Part III., 1857, is the Astronomer Royal's "Account of the Construction of the New National Standard of Length and of its Principal Copies." Those who have looked carefully into this paper must have perceived that the difficulties attending the comparisons of standards, where results of a high order of precision are aimed at, are considerable; requiring the very best workmanship in nearly every part of the apparatus, and demanding the greatest patience and circumspection on the part of the observer. But the difficulties which were encountered and so successfully overcome by Mr. Sheepshanks are considerably enhanced, when, as in the operations which have been recently conducted at the Ordnance Survey Office, Southampton, the bars to be compared are of different and incommensurable lengths. It was therefore foreseen that without building a room especially for the purpose, and devising an apparatus that could be adapted to the measurements of all kinds of lengths up to 13 or 14 feet, the comparison of the geodetical standards with the yard could not be undertaken with any prospect of success.

The bar-room is 20 feet in length by 11 in breadth; the walls are double, the outer 2 feet thick, and the inner $4\frac{1}{2}$ inches with an interval of 3 inches. The foundations are very strongly built. The roof is flat, the walls being spanned by iron girders whose lower flanges support large slates, which again are covered uniformly by 9 inches of concrete. The direction of the length of the room is nearly east and west; in the north face are two small windows, sufficient to admit a moderate amount of daylight. The sash frames, which slide in the interval between the two walls, are further protected by wooden shutters, 3 inches in thickness, on the outside. The doorway is at the east end of the room and is closed by double doors, one on the outside of the room, the other on the inside, so that any one entering the room may close the first or outer door before he opens the second or inner door.

An outer building encloses this room, and so protects it from the variations of temperature of the external air. The outer building is of brick, and is 40 feet in length by 20 in breadth, with an ordinary slate roof. Thus shielded from external influences, the temperature of the inner room is exceedingly steady, leaving nothing to be desired in this respect.

Along the southern wall of the room are three stone piers for supporting the micrometer-microscopes. The centre pier, or block, measures on its upper surface 4 feet by 16 inches; the outer blocks are of the same breadth, but only 3 feet 6 inches in length; they are distant 5 feet from centre to centre from the middle block. These stones are close to the wall of the room, but are not actually in contact with it; they have separate

and deep foundations of brickwork; their upper surfaces are about 4 feet 6 inches above the floor of the room. Each stone is so cut as to present along its front horizontal edge a projecting ledge, the upper surface in fact projecting 3 inches beyond the lower part or face of the stone; the vertical depth of the projecting ledge is 3 inches.

Immediately in front of and close to the piers is placed a large mahogany beam, measuring 14 feet in length by 14 inches in breadth, and 9 inches in vertical depth. Its position is horizontal and parallel to the length of the room, and to the faces of the stone piers; to its upper surface (which is about 2 feet above the floor of the room) are fastened a pair of planed cast-iron rails, 11 inches from centre to centre, and extending the whole length of the beam. In the fixing of these rails to the beam, provision is made for any warping which the beam might undergo; so that the rails can always be kept straight and parallel.

This beam being intended to support the standard bars when under the microscopes, is not itself supported by the floor of the room, but has, like the stone piers, its own foundations. The flooring upon which the observer stands has no contact either with the stone piers, or with the foundations by which the beam is supported. Further, the foundations for the beam are entirely disconnected with the stone piers, thus (and it has been repeatedly and severely tested) no movement of the observer can disturb either the microscopes or the bars under observation. Before perfect immunity from disturbance, however, was obtained, it was found necessary to disconnect the wooden flooring entirely from the walls of the room; the flooring is framed in three separate pieces, each being supported by, or simply resting on, four large blocks of india-rubber.

Micrometer Microscopes.—The magnifying power of the microscopes is about sixty. The length of the tube from the diaphragm to the object-glass is 12 inches, and from the object-glass to its external focus 3 inches. The value of one division of the micrometer is about the 35,000th part of an inch. Each microscope is held immediately in a strong hollow gun-metal cylinder about 6 inches in length, the axis of which coincides with that of the microscope. At either extremity this cylinder is internally provided with circular Y's, into or against which the tube of the microscope is pushed by springs, the tube having two strong accurately turned collars for this purpose at one-fourth and three-fourths of its length. The upper collar has a flange which determines longitudinally the position of the microscope with respect to the gun-metal cylinder; while at the same time the microscope is free to revolve in the cylinder, but without anything approaching to a shake. From the cylinder, at its mid length, project three arms by which it is held and levelled, each arm having through its extremity a cylindrical hole bored ($\frac{1}{4}$ inch diameter) parallel to the cylinder itself. This gun-metal cylinder, again, is supported by and held firmly to a strong and heavy plate of cast iron, which, having three bosses on its under surface, rests on one of the stone piers, part of the plate projecting beyond the front of the stone towards the room. That part of the iron plate which rests immediately on the stone is a rectangle of 12 inches by 14 inches, and the projecting part may be described as something like an equilateral triangle of 8 inches

side: the plate is rather more than an inch thick. Towards the apex of the projecting triangle there is a circular hole 2 inches in diameter through the plate, through which the gun-metal cylinder holding the microscope passes. Equidistant from the centre of this hole, and equidistant from one another, are three vertical screws strongly bolted into the iron plate and projecting upwards about 2 inches in length. These screws pass freely through the holes in the arms of the gun-metal cylinder. Suppose now two nuts running upon each of these screws, one above and one below each of the arms, and it is clear that we have the means of rendering truly vertical the axis of the microscope, and also of holding the microscope very firm by clamping down the upper nuts. It will also be seen that the microscope is held without the least strain, and that it can be raised or lowered small quantities so as to bring to focus over a given object. The iron plate is not held down to the stone in any way; its own weight gives sufficient stability; it may be shifted to any position on any of the stones.

Illumination.—Much depends upon the proper illumination of the divided surfaces under observation. A candle, whose flame is mechanically kept in a constant position, stands behind the microscopes; and its light, condensed by a lens 3 inches in diameter, passes through an aperture in the projecting part of the cast-iron plate, being brought to a focus on the divided surface under observation. Abundance of light is thus obtained, and the candle being above the bar, the heated air is continually carried away from it; besides, the heat of a candle is the least practicable with a sufficiency of light*.

Carriages.—The box containing the bar, or bars under observation, is supported by two carriages which run upon the rails that have been described as fixed on the upper surface of the large mahogany beam; one of the rails is flat, the other triangular in section; each carriage runs on three wheels, two of which, being grooved, run on the angular rail, the third on the flat rail. Thus it will be seen that the motion of the carriage is without any possible jamming. Each of the two carriages is double, that is, consists of an upper and lower carriage; the upper carriage runs upon short rails on the surface of the lower carriage, and in a direction perpendicular to the motion of the latter. A slow-motion screw affords the means of communicating, when required, a small motion to the upper carriage. Without going into further details as to the construction of these carriages, it may be sufficient to say that the different parts are so put together that no shake exists, nor can it be introduced by wear. The box containing the bar or bars under comparison has therefore, when resting on the carriages, a perfectly steady bearing; while it can be moved in a longitudinal direction by the running of the carriages along the rails on the mahogany beam, or moved transversely by the movement of the upper carriages on the lower.

* This method of illumination was decided on after a considerable number of experiments with gas, oil lamps, &c., directed and condensed, or reflected in different ways. Had there been only a few different lengths to compare, the light might have been brought in from the outside of the room through horizontal holes in the wall and piers, and in this manner some of the earlier comparisons on different ten-foot bars were made. But this method could not have been applied to the comparisons generally, on account of the large number of holes that would have been required.

Suppose now we have two bars lying alongside one another in a box (the boxes are of a uniform breadth of 8 inches externally), their axes parallel, and about, as usual, 3 inches apart; then by the movement of the upper carriages, the one bar and the other may be brought alternately under the microscopes.

Bars.—The various copies of the standard yard are all 1 inch square in section, and about 38 inches in length. At about an inch from either extremity of the bar, a cylindrical well is drilled halfway through the metal; at the bottom of each well is a gold pin let into the bar. On the surfaces of these gold pins, which are, it will be observed, in the neutral axis of the bar, the lines defining the measure are drawn. The bar has also wells in its upper surface for the bulbs of two or four thermometers. That particular copy of the standard yard which has been compared with all the geodetical standards is No. 55 (Swedish Iron); it is supported on rollers at one-fourth and three-fourths of its length.

Ordnance Survey Standard O_1.—This is a bar of wrought iron 10 feet 2 inches in length, $1\frac{1}{2}$ inch broad, and $2\frac{1}{2}$ inches deep; supported on rollers at one-fourth and three-fourths of its length. The ends of the bar are cut away to half its depth, so that the dots marking the measure of 10 feet are in the neutral axis of the bar. There are two wells for thermometers.

Ordnance Intermediate Bar OI_1 is of wrought iron, in section having the form of a girder, with equal upper and lower flanges, the extreme breadth and depth being the same as in the last-mentioned bar. On the upper surface are seven disks, *a*, *b*, *c*, *d*, *e*, *f*, *g*: the spaces *a b* and *f g* are each one yard; *b c*, *c d*, *d e*, *e f* are each one foot. Each disk has one transverse, crossed by two parallel longitudinal lines. This bar is supported on a cradle system of eight rollers.

Indian Standards I_S, I_B are bars similar to one another and to the bar OI described above, differing only in this, that one is of cast steel, and the other of BAILY's metal or bronze.

Ordnance Toise (T_0) *and Metre* (M_0).—These bars are of cast steel, similar in section to the last-mentioned bars, but only an inch wide by one and a half deep. The toise has four disks, *a*, *b*, *c*, *d*; the spaces *a b*, *b c* are each one yard; *cd* is approximately 4·74 inches. It is supported on a cradle system of eight rollers. The metre has three disks; the first two are a yard apart, the second and third are 3·37 inches apart.

Prussian and Belgian Toises T_{10}, T_{11} are flat bars of cast steel, an inch and three-quarters in breadth and four-tenths of an inch thick, terminating in cylinders about half an inch in length, the axis of the cylinder coinciding with that of the bar, and the diameter of the cylinder coinciding with the depth of the bar. At the extremity of each of these cylinders is affixed a smaller (co-axial) cylinder of tempered steel, an eighth of an inch in diameter, and only a sixtieth of an inch long. The faces of these small cylinders, which are perfect planes beautifully polished, and at right angles to the axis of the bar, form the terminal planes of the measure.

Russian Double Toise P is also an end measure, but the terminal surfaces are not

planes, being slightly convex; it is a bar of wrought iron two toises in length, and an inch and a half square in section, supported at one-fourth and three-fourths of its length.

Standard Foot F is a bar 13 inches long by 1 inch square divided into inches. The extreme inches are further divided on inlaid strips of platinum, into tenths, and some of these tenths into hundredths.

Thermometers.—The standard thermometers, to which all others have been referred, have had their errors determined by calibration measurement to every fifth degree, by means of an apparatus constructed for this purpose. The apparatus consists essentially of three parts:—(1) as a base, a heavy rectangular plate of metal 2 feet long lying horizontally, having at its extremities upright pieces 10 inches high; (2) the top of these uprights are joined by a couple of steel rods parallel to one another and at the same height; along these rods there slides a platform carrying a vertical microscope between the rods; (3) a sliding frame, supported immediately by the lower plate, moving in the direction of the length of the lower plate or of the steel rod above, carries the thermometer to be examined. This sliding frame is moved by means of a micrometer screw, which therefore draws the thermometer along backwards or forwards in the direction of its own axis. The bed of the thermometer is rendered truly horizontal, and the micro scope has a level attached by means of which its axis may be always kept strictly vertical. The thermometer is protected, as far as possible, from variations of temperature by being closely surrounded (except its upper surface) by metal; and the detached column of mercury can be shifted from one position to another without touching the thermometer with the hand. The error of the mean length of a degree is determined by boiling the thermometer (in a horizontal position), and immediately after placing it in ice.

The thermometers which record the temperatures of the bars are only 5 inches in length, each showing 20° range of temperature; the degree is about a fifth of an inch in length, and subdivided to tenths. These thermometers are compared as often as necessary (and this is very often) with the standard thermometers. The apparatus for the comparison of thermometers consists of a water trough, 29 inches long by 9 inches broad and 9 inches deep (internal measurement), resting on three points, one of which is a levelling-screw. The thermometers rest on cross bars at the mid depth of the water, their tubes truly level. This trough stands on the ground, and the thermometers are read by means of a long microscope, which is mounted on a travelling platform, and of which the axis is made vertical. The degrees on the long standard thermometers are not generally subdivided; and in order to read them accurately the following arrangement was adopted: on a small strip of plate glass are drawn a system of eleven equidistant and slightly converging lines; this strip slides in the diaphragm of the microscope, and can be moved by the hand; then, if it be required to subdivide a degree seen in the centre of the field of the microscope, the glass slide is moved until the outer lines of the system coincide with the bounding lines of the degree. Thus tenths are immediately read, and the hundredths can be estimated.

In order to read the thermometers when lying in the bars, orifices are provided in the

covers of the boxes: the microscopes used here are 12 inches in length, they are mounted on sliding plates so as to traverse the whole length of the thermometer-tube; the verticality of their axes is also ensured.

Method of Comparing.—Whenever practicable, the two bars which have to be compared are mounted side by side in the same box. Each bar is capable of being levelled (by raising or lowering the cradles or rollers on which it rests), or brought to focus under the microscopes. Each microscope has attached to it a level whereby the verticality of the axis may be tested. It is usual to arrange a pair of bars for comparison on the afternoon of one day, and to commence observing the next day. The bars are visited three or four times each day; a *series* of comparisons has generally consisted of about ten visits or comparisons; and the bars are then dismounted, to be compared another time. All adjustments are frequently put out and renewed; there is little use in multiplying observations while none of the circumstances of the observations are changed. Consequently, as far as practicable, the comparisons of any two bars have been made in detached series; thus the fear of constant error is diminished. It is generally assumed that the temperatures of two bars lying together in the box are the same, the minute differences which are sometimes found in comparing the readings of the thermometers being attributed to the thermometers themselves, which certainly do not always immediately indicate changes of two or three hundredths of a degree of temperature. The two bars are also made to interchange places, so that either one of them is next to the observer about as often as it is next to the piers. A temporary constant error may also creep in, if great care is not taken that the divided surfaces are clean; minute particles of dust, almost indiscernible to the eye, or one such particle hanging about the edges of the line where it is to be bisected, will give a false result to all observations; and it is of course undesirable to clean the surfaces, or even rub them gently, oftener than can possibly be helped.

The observations made at any one visit to the bar-room are generally as follows:—(1) the two thermometers in each bar are read; (2) the bar A being adjusted to focus under the microscopes, three readings (bisections) of the microscope on the left are taken, and then three readings of the microscope on the right; (3) B being now adjusted under the microscopes, similar readings are made; (4) B is thrown out of focus by the levelling-screws, and being readjusted in focus under each microscope is observed as before; (5) A is observed a second time; (6) the thermometers are read again. A slight disturbance and rise of temperature is almost inevitably caused by the observer's presence and the heat of the two candles. On the average, the second readings of the thermometers are $0^\circ{\cdot}035$ above the first readings.

In comparing any two bars, the greater part of the comparisons have been made at temperatures differing not more than 2° from the standard temperature of 62°, and a small portion at a temperature as low as possible; thus the expansions of the bars are eliminated.

With respect to personal error in bisections, this has been found to exist in the case

of some particular lines, to the amount of one or sometimes two micrometer divisions. The only way of eliminating personal error is by the employment of as many expert observers as can be commanded.

Flexure.—It is of the utmost importance that a bar be supported invariably at the same points; if these points be altered, then (unless the divided surfaces be in the neutral axis) the length of the bar undergoes a change. The proper positions for the supporting rollers of a bar have been investigated by Mr. AIRY in the Memoirs of the Royal Astronomical Society. In order to test the theory of flexure of bars (considered as elastic rods) by actual experiment, a large number of observations as to changes of length corresponding to change of supports were made on three iron bars specially prepared for the purpose. Each bar was 40 inches long by an inch square. It will suffice here to explain generally the process and give the results for one of the bars. If a bar be supported by its extremities, it is clear that the whole of its upper surface will be compressed, while the lower surface will be correspondingly extended; therefore the length of the bar as measured by the distance between two dots at the extremities of the upper surface will be less than if the neutral axis were straight; in fact, if i be the inclination of the bar at either extremity to the horizon, and k the depth of the bar, its curvature will cause the dots to approach each other by the quantity ik. This effect of curvature will be greatly exaggerated if the dots be engraved, not on the surface of the bar, but on the tops of bits of strong wire inserted (in a vertical position) into the bar at its extremities; if h be the length of either wire, the approach of the dots to one another by the curvature of the bar will be $i(k+2h)$. Accordingly four such perpendiculars were erected on the upper surface of the bar, one at either extremity, and one 10 inches from either extremity. A box fitted with rollers was prepared to receive the bar; each roller, mounted in a frame, could be fixed in any required position, and could at the same time be moved in a vertical direction up or down by means of a slow-motion screw outside the box. Now suppose four rollers so fixed in the box that one is under each extremity and two more at 2 inches right and left of the centre of the bar. Suppose also four microscopes adjusted over the four dots, their axes vertical, and their outer foci ranging in a straight horizontal line; by the working of the slow-motion screw (which it is to be particularly observed does not require the opening of the box or the touching of the bar with the hand) the centre rollers can be withdrawn, that is lowered until they cease to have contact with the bar, and then the bar resting on the extreme rollers can be adjusted to focus under the microscopes. The microscopes are then read. The extreme rollers are then lowered, and the bar comes in contact with the rollers near its centre, which are then raised until the dots are in focus. The microscopes are now read a second time; and by these readings, compared with the former, are obtained the changes of length of the whole bar, and of its subdivisions resulting from the alteration in the positions of the supports.

The supporting-rollers were placed at different times in the following positions:—(1) at 20 inches right and left of the centre; when in this position the supports are

designated EE′; (2) at 2 inches right and left of the centre designated CC′; (3) at $\frac{40}{\sqrt{3}}$ inches right and left of the centre designated NN′; (4) at $\frac{40}{6}$ inches right and left of the centre designated SS′.

The following Table contains the observed and computed changes of length of one of the bars, of its whole length, and of its subdivisions; the dots are marked in order from left to right, m, n, n', m'; $\Delta(mn')$ is the alteration of the distance mn' due to the change of supports; the unit is the millionth of a yard.

Changes of supports.		$\Delta[mn']$.		$\Delta[mm']$.		$\Delta[nm']$.	
From	To	Observed.	Computed.	Observed.	Computed.	Observed.	Computed.
NN′	EE′	57·8	58·6	70·7	71·8	60·7	58·6
„	EN′	25·0	26·3	25·9	26·4	17·4	16·8
„	NE′	16·4	16·8	26·8	26·4	28·5	26·3
„	CC′	−36·2	−35·3	−35·8	−35·7	−35·7	−35·3
„	NC′	−31·8	−30·0	−31·8	−30·4	−32·2	−30·1
„	CN′	−31·7	−30·1	−31·8	−30·4	−32·4	−30·4
CC′	EE′	93·5	93·9	106·5	107·5	93·4	93·9

The modulus of elasticity by which the computed results are obtained is derived from the observations themselves.

Absolute Expansion.—The coefficients of expansion of the Indian standards I_S, I_B and of the two other 10-foot bars of iron, OI_1, OI_2, have been obtained by means of an apparatus constructed for the purpose. Theoretically it is a simple matter to determine the coefficients of expansion of two bars A, B; it may be done as follows:—Compare A hot, say at a steady temperature of 100°, with B at the temperature of, say 40°; next compare A at 40° with B at 100°; and lastly compare the bars when both at the same temperature. But the practical difficulty is to maintain a steady temperature for the hot bar, so that it shall not be cooling while under observation. This has been effected in the following manner:—Imagine two closed tanks of copper measuring 124 inches by 5 by 3 inches, and suppose them fixed to the upper surface of a stout mahogany plank of the same length and 8 inches broad; between the tanks there remains a vacant space 2 inches wide and 5 inches deep; into this space the bar goes with its supporting-rollers, which are capable of slight vertical movement for level or focus adjustment. A current of hot water at a steady temperature enters the bar-room from without by a flexible tube; this current is made to subdivide into four equal streams, entering each tank by two orifices in its upper surface, at one-fourth and three-fourths of the length. The water escapes from each tank by an orifice at the bottom of either extremity, and is conducted out of the bar-room through flexible tubes. The supply being purposely greater than can be carried away from the extremities, an overflow pipe is provided at the centre of the tanks, and this overflow is also carried away from the room by flexible tubes. Thus a constant circulation of water is maintained, and no part of the water in the tanks can be still or cooling, nor can the tanks empty or overflow. The whole is well wrapped up in

blankets. The other bar is similarly mounted between tanks which are full of cold water; but no current is required, as the observations are made in the cold weather; this is also carefully covered with blankets. With respect to the interchange of the bars under the microscopes, this is effected with all desirable rapidity by a piece of mechanism whereby each bar (with its appendage of plank, tanks, and water) is simply rolled away from or up to the microscopes; thus the observers do not have to encounter the weight of these masses. Thus arranged the comparisons of a hot and cold bar are effected with almost as much facility as the ordinary comparisons.

The coefficients of expansion obtained from 6500 micrometer and thermometer readings for the four bars are—

BAILY's metal	I_B	. . . .	$0{\cdot}0000098277 \pm 0{\cdot}0000000057$.
Steel	I_S	. . . .	$0{\cdot}0000063478 \pm 0{\cdot}0000000056$.
Wrought-iron	OI_1	. . . .	$0{\cdot}0000064729 \pm 0{\cdot}0000000031$.
Wrought-iron	OI_2	. . . .	$0{\cdot}0000064773 \pm 0{\cdot}0000000033$.

The Indian bars were heated up to very nearly, but not quite 100°.

Probable Errors of Observation.—The quantities measured by the micrometers in the observations just specified are large, and require a very accurate knowledge of the values of the screws. The values for the two microscopes H and K are, expressed in millionths of a yard,

H : one micrometer division . . . . $0{\cdot}79566 \pm {\cdot}00008$

K : „ „ . . . . $0{\cdot}79867 \pm {\cdot}00009$

These were obtained from repeated measurements of a space of $\frac{4}{100}$ of an inch on F, the scale being readjusted to focus each measurement. There is no appearance of personal error in the observations (of three observers) from whence these values are deduced. It appears that the probable error of a single measurement of a space of n thousand divisions, m bisections on each line being supposed, is

$$\text{for H} \ . \ . \ . \ \pm\sqrt{\frac{0{\cdot}20}{m}+0{\cdot}072n^2} \text{ micrometer divisions,}$$

$$\text{for K} \ . \ . \ . \ \pm\sqrt{\frac{0{\cdot}20}{m}+0{\cdot}116n^2} \quad „ \quad „$$

The greatest space measured by either of the microscopes in the expansion experiments was 1100 divisions; and m being $=2$, the probable error of the measure would be 0·43 or 0·49 of a micrometer division, according to the microscope used. The probable error of a single bisection by either of the observers is about $\pm 0^{d}{\cdot}316$.

The Standard Foot, and its subdivisions.—The length of this foot F in terms of Y_{55} was determined as follows:—Four microscopes, H, I, J, K, were mounted on the stone piers at the distance of 12 inches apart, their axes being vertical and their outer foci in a horizontal straight line. The two bars lying side by side in their box, F was brought successively under the microscopes H I, I J, J K; then Y_{55} under the microscopes H K

The resulting value of F from 900 micrometer and 180 thermometer readings, extending over twelve days, is

$$F=\tfrac{1}{3}Y_{55}-0\cdot36+0\cdot0066(t-62), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

the probable error, when $t=62°$, being $\pm0\cdot108$. The unit to which these small quantities are referred is the millionth of a yard.

The inch lines upon the foot are marked *a, b, c, d, e, f, g, h, k, l, m, n, p*; the inch [*a* . *b*] is divided into tenths by lines marked 1, 2, 3, 4, 5, 6, 7, 8, 9; the spaces [2 . 3], [6 . 7] are subdivided into hundredths; one of the subdividing lines in [2 . 3], called the toise-line, is indicated by the letter τ; the metre-line in [6 . 7] is known by the letter μ. The values of the different spaces, as derived from 8000 micrometer readings, are as follows:—

$$\left.\begin{array}{ll}
[a\,.\,2]=\tfrac{2}{10}\,\frac{F}{12}+3\cdot71\pm\cdot063, & [a\,.\,8]=\tfrac{8}{10}\,\frac{F}{12}+1\cdot65\pm\cdot070,\\
[a\,.\,\tau]=\tfrac{26}{100}\,\frac{F}{12}+5\cdot30\pm\cdot109, & [a\,.\,b]=\frac{F}{12}+0\cdot14\pm\cdot037,\\
[a\,.\,3]=\tfrac{3}{10}\,\frac{F}{12}-0\cdot20\pm\cdot064, & [a\,.\,c]=2\frac{F}{12}+1\cdot74\pm\cdot047,\\
[a\,.\,4]=\tfrac{4}{10}\,\frac{F}{12}-0\cdot77\pm\cdot066, & [a\,.\,d]=3\frac{F}{12}+1\cdot31\pm\cdot038,\\
[a\,.\,6]=\tfrac{6}{10}\,\frac{F}{12}+3\cdot00\pm\cdot068, & [a\,.\,e]=4\frac{F}{12}+0\cdot84\pm\cdot051,\\
[a\,.\,\mu]=\tfrac{62}{100}\,\frac{F}{12}+2\cdot08\pm\cdot086, & [a\,.\,f]=5\frac{F}{12}+2\cdot20\pm\cdot047,\\
[a\,.\,7]=\tfrac{7}{10}\,\frac{F}{12}+1\cdot58\pm\cdot068. & [a\,.\,g]=6\frac{F}{12}-0\cdot87\pm\cdot037.
\end{array}\right\} \quad . \quad . \quad . \quad (2)$$

Ten-feet Standards.—The length of the 10-foot bar OI_1 was obtained by comparing each of the yard spaces on its surface, [*a* . *b*], [*b* . *e*], [*c* . *f*], [*f* . *g*], with Y_{55}, and the two 12-inch spaces [*b* . *c*], [*e* . *f*] with the foot F. The results are, at 62°,

$$[a\,.\,b]=Y_{55}+54\cdot75\pm0\cdot130,$$
$$[b\,.\,f]=\tfrac{4}{3}Y_{55}-23\cdot44\pm0\cdot219,$$
$$[f\,.\,g]=Y_{55}-10\cdot23\pm0\cdot156,$$

whence the whole length of the bar, by adding these equations is

$$OI_1=\tfrac{10}{3}Y_{55}+21\cdot08\pm0\cdot299. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The length of the Indian steel standard, obtained in nearly the same manner, is

$$I_S=\tfrac{10}{3}Y_{55}+70\cdot62\pm0\cdot250. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Comparisons have also been made between the following 10-foot bars; O_1, OI_1, I_B, I_S, and I_b. (This last is the old Indian Standard B, of which the comparisons between it and O_1 in 1831 and 1846 are detailed in the "*Account of the Measurement of the Lough*

Foyle Base": it is a bar similar to O_1, but rather lighter. Since 1846 it was for some time at St. Petersburg in custody of M. STRUVE, who compared it with his own standard.) The results of the comparisons are these:

$$\left.\begin{aligned} OI_1-O_1&=\ 18{\cdot}38\pm0{\cdot}26,\\ I_S-O_1&=\ 63{\cdot}28\pm0{\cdot}26,\\ I_B-O_1&=195{\cdot}36\pm0{\cdot}26,\\ I_S-\ I_b&=\ 86{\cdot}50\pm0{\cdot}41,\\ I_B-\ I_b&=218{\cdot}58\pm0{\cdot}22.\end{aligned}\right\}\quad\ldots\ldots\quad(5)$$

If from the seven last equations we seek by the method of least squares the most probable lengths of the five 10-foot standards in terms of the yard, we get, at 62°,

$$\left.\begin{aligned} OI_1&=\tfrac{10}{3}Y_{55}+\ 22{\cdot}32,\\ I_S&=\tfrac{10}{3}Y_{55}+\ 69{\cdot}38,\\ O_1&=\tfrac{10}{3}Y_{55}+\ \ 5{\cdot}17,\\ I_B&=\tfrac{10}{3}Y_{55}+200{\cdot}84,\\ I_b&=\tfrac{10}{3}Y_{55}-\ 17{\cdot}43.\end{aligned}\right\}\quad\ldots\ldots\quad(6)$$

These values being substituted in the seven equations, the residual errors are the following:—

$$\begin{aligned}&+1{\cdot}24,\\&-1{\cdot}24,\\&-1{\cdot}23,\\&+0{\cdot}93,\\&+0{\cdot}31,\\&+0{\cdot}31,\\&-0{\cdot}31.\end{aligned}$$

Now these errors are considerably larger than the directly computed probable errors of the different sets of comparison; it is clear therefore that constant error has been influencing some or all the different series. After all, the residual errors are as small as could be well expected. As to I_b, it appears that at present its relation to O_1 is this:

$$I_b-O_1=-22{\cdot}60.$$

The difference of the same two bars, as determined in 1831, was $-22{\cdot}25$; and in 1847 it was $-24{\cdot}03$; an agreement most satisfactory.

It appears, then, from the above, that the Ordnance Survey Standard Bar is but very slightly in error, being only $\frac{1{\cdot}55}{1,000,000}$ths of its length too great; this corresponds to barely 6 feet in the length of this kingdom from Scilly to Shetland; or to 32 feet in the earth's radius.

Ordnance Toise and Metre.—In the toise, each of the yard spaces $[a\,.\,b]$, $[b\,.\,c]$ were

compared with the yard Y_{55}, and the space $[c\,.\,d]$ of 4·74 inches was compared with the very nearly equal space $[\tau f]$ on the foot F: the result of the comparisons is

$$[a\,.\,b] = Y_{55} + 5{\cdot}95,$$
$$[b\,.\,c] = Y_{55} + 2{\cdot}34,$$
$$[c\,.\,d] = \tfrac{474}{3600} Y_{55} + 0{\cdot}16,$$

or

$$T_0 = \tfrac{7674}{3600} Y_{55} + 8{\cdot}45 \pm 0{\cdot}21,$$

both bars being at 62° F. If both be at 61°·25 F., or 16·25 Centigrade,

$$T_0 = \tfrac{7674}{3600} Y_{55} + 9{\cdot}17 \pm 0{\cdot}21. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The yard space on the metre was compared with Y_{55}, and the small space with $[\mu\,.\,e]$ on F; with these results,

$$[a\,.\,b] = Y_{55} - 1{\cdot}20,$$
$$[b\,.\,c] = \tfrac{338}{3600} Y_{55} - 134{\cdot}25,$$

or

$$M_0 = \tfrac{3938}{3600} Y_{55} - 135{\cdot}45 \pm 0{\cdot}16,$$

both bars being at 62°. If both be at 61°·25 F., or 16·25 C.,

$$M_0 = \tfrac{3938}{3600} Y_{55} - 135{\cdot}11 \pm 0{\cdot}16. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The length of T_0 is the result of 3120 micrometer and 640 thermometer readings; for M_0 were made 1680 micrometer and 320 thermometer readings. The observations for the former bar occupied thirty-three days, for the latter sixteen days.

The Toise.

The unit of length in which by far the greater part of the European geodetical measurements are expressed, is the toise; the actual physical representative of this length being the *Toise of Peru**, at the temperature of 16°·25 C., or 61°·25 F. This bar, which was constructed in 1735 for Messrs. BOUGUER and LACONDAMINE as their standard of reference in the operations conducted by them in Peru, is an end measure of polished iron, 1·51 inch in width and 0·40 inch in thickness. In order to trace our direct connexion with this celebrated and important standard, we must refer to two direct copies of it made by FORTIN, of Paris, one in 1821 for M. STRUVE, and the other in 1823 for M. BESSEL. The authority of the former rests on a certificate of the *Bureau des Longitudes*, signed by M. ARAGO†, in which he states that he has compared the copy with the

* We have no precise information as to the present state of this bar, but from report it would appear that at least as far back as 1858 it was so far damaged that comparisons with it were worthless. For a description of the Toise of Peru see 'Base du Système Métrique Décimal,' tome iii. pp. 405, 680, and the work entitled 'Mesure des Trois premiers Degrés du Méridien dans l'Hémisphère Austral,' par M. de la CONDAMINE; à Paris, 1751, pp. 75, 85.

† See 'Arc du Méridien de 25° 20′ entre le Danube et la Mer Glaciale mesuré depuis 1816 jusqu'en 1855 . . . . .' par F. G. W. STRUVE, St. Pétersbourg, 1860, tome i. Introduction, p. lxxiv.

toise of PERU and has found them "perfectly equal" in length. The authority of BESSEL's toise is stated in a certificate signed by MM. ARAGO and ZAHRTMANN*, who assert that they have compared the toise constructed for M. BESSEL with the toise of PERU, and have found it too short by $\frac{1}{1278}$ of a "ligne" (the toise $=864$ lignes). If, therefore, we put F_s and F_b for the length of these two toises at 16°·25 C., and $\mathfrak{T}$ for the length of THE TOISE

$$F_s=\mathfrak{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$F_b=\mathfrak{T}-0^{l}\cdot00080 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

In Russia the standard to which all their geodetical measures are referred is a bar two toises in length, designated N†. Its length was determined from F_s: the comparisons are given in M. STRUVE's 'Account of the Russian Meridional Arc of 25° 20′,' the deduced length of N being, at 16°·25 C.,

$$N=2F_s+0^{l}\cdot01249\pm0\cdot00070. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The Russian double toise P, which has been compared at Southampton with two lengths of T_0, is a copy of N; the difference of the two bars at 16°·25 C. being

$$P=N-0\cdot01809\pm0\cdot00019. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

This bar P has been very extensively used in Russian geodesy. In 1850 it was used as the Standard in the Measurement of the Base of Alten, in Finmark, and the next year in the measurement of the Base of Öfver-Tornea, in Lapland. In 1852 it was used at the measurement of the Base at Taschbunar at the southern extremity of the Russian Arc in Bessarabia, and in various other bases for geodetical operations undertaken by the Topographical Depôt. Subsequently it was used in Eastern Russia.

M. STRUVE also compared the Prussian toise T_{10} described above, with his normal bar N, the result being, at 16°·25 C.,

$$2T_{10}=N-0\cdot01421\pm0\cdot00020. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The Prussian and Belgian toises T_{10}, T_{11} were compared in 1852 by General BAEYER with BESSEL's toise. The comparisons will be found in the work entitled "*Compte rendu des Opérations de la Commission instituée par M. le Ministre de la Guerre, pour étalonner les règles qui ont été employées...... à la mesure des bases géodésiques belges*," Bruxelles, 1855. The results are‡, at 16°·25 C.,

$$T_{10}=F_b-0^{l}\cdot00019\pm0\cdot00011, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

$$T_{11}=F_b-0^{l}\cdot00020\pm0\cdot00012. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The difference of these two toises, according to these comparisons, is a very minute and inappreciable quantity. It appears, then, that although the very valuable standards T_{10}, T_{11}, P, which have been compared at Southampton with T_0, have not been themselves immediately compared with the toise of Peru, yet the connexion with that standard is

* See 'Untersuchungen über die Länge des einfachen Secundenpendels,' von F. W. BESSEL, Berlin, 1828, p. 126.

† See 'Arc du Méridien de 25° 20′ . . . .' Introduction, pp. xl, lxxiii–lxxvi, also pp. 36–38. The results of M. STRUVE's comparisons among different bars are collected in the Table, pp. lxxiii, lxxiv.

‡ See pages 36–52 of the work named.

sufficiently direct, and judging by the expressed probable errors, should be very accurate. Unhappily, however, there is no information as to the precision of the comparisons made by M. ARAGO between either F_s or F_b and the toise of Peru. But it is to be remarked that, if we eliminate N and F_s between the equations (9), (11), and (13), we get

$$T_{10} = \mathfrak{T} - 0^{t}\cdot00086,$$

as the result of M. STRUVE's comparisons; while from General BAEYER's, (10) with (14),

$$T_{10} = \mathfrak{T} - 0^{t}\cdot00099,$$

the difference between these two entirely independent values of the Prussian toise is only $0^{t}\cdot00013$, or less than the six millionth part of a toise. This shows that ARAGO's assigned lengths of F_s F_b are at any rate admirably consistent.

We must now explain how the toises *à bouts* have been compared with the toise *à traits*. Suppose for a moment a cube of steel, one-eighth of an inch side, its faces polished, and a fine dot engraved on one of the faces at about one-hundredth of an inch from one of the edges and exactly opposite the middle point of that edge. Suppose the toise lying horizontally, and consequently its terminal planes in a vertical position, and let a cube as described above be applied against each end of the toise, the face carrying the dot being uppermost and horizontal; then the distance $T+\sigma$ between the dots when so held is about two-hundredths of an inch greater than the toise. Next let the cubes be placed in contact under the microscope, and the distance σ between the dots measured; we shall then, by subtracting this quantity, know the exact length of the toise. But the mechanical difficulties to be overcome in this theoretically simple arrangement are found to be very great. After numerous experiments in different ways, the following modification was adopted: suppose a sphere of steel, three-quarters of an inch in diameter, to be cut by two parallel planes, one-eighth of an inch apart, on opposite sides of and equidistant from the centre. Taking the central segment, let it be laid on a horizontal plane, and cut in two along a diameter, leaving two semicircles; next let these two pieces, without removing either of them from the horizontal plane, be placed so that their curved surfaces shall come in contact, while their bases or semidiameters are parallel and at the maximum distance apart; then the common tangent plane at the point of contact will be a vertical plane, even if there should have been any error in the cutting of the sphere, so that one of the planes was nearer the centre than the other. Next suppose each of these semicircles to be placed on and fastened to a carefully planed rectangular plate of steel, say 4 inches long, the diameter of the semicircle being perpendicular to the length of the rectangle, and the curved surface projecting slightly beyond the end of the plate: suppose we have the means of levelling this plate, of raising or lowering it small quantities, of giving it a small motion in the direction of its length, and also in the direction perpendicular to its length, and lastly of giving it an azimuthal movement; then it is clear that we have absolute command as to position over the semicircular pieces. On the upper surface of each semicircle suppose a fine line drawn parallel to the base (or perpendicular to the length of the plate), and as near

as possible to the curved edge. Next, let the two semicircles be placed in contact, the plates being in the same horizontal plane and their lengths parallel in direction; the semicircles being kept in contact by pressure of a spring. In this position the lines drawn on the semicircles will be parallel and very close to one another. If we now, by the transverse movement only, slightly alter the position of one of the plates, the distance of these parallel lines will vary, and there is obviously a certain position in which their distance is a maximum; this occurs when the (vertical) tangent plane to the curved surfaces at their point of contact is parallel to the lines. This distance, when measured, is that by which the length of the toise is increased when the contact pieces are adjusted to its extremities.

In the actual apparatus, this distance σ, from very numerous observations, repeated on various occasions, is found $=565{\cdot}85\pm0{\cdot}108$ millionths of a yard. The toise (Prussian or Belgian) has been invariably supported on four points 21·5 inches apart. For the comparisons at Southampton a stout bar of iron, rather more than a toise in length, was prepared, carrying on its upper surface four rollers fitted with the necessary adjustments for strict alignment; at each of its extremities it carried a horizontal brass plate to which the contact apparatus was attached. The iron bar itself was held at one-fourth and three-fourths of its length; either support being capable of vertical movement for focus or levelling. The four rollers on which the toise lay were adjusted to a horizontal plane by means of a spirit-level. The contact pieces were held in contact with the ends of the toise by spring pressure. To prevent any constant error in the comparisons, the contacts were renewed after each comparison, and *all* the adjustments thrown out and re-made as often as possible.

The number of comparisons between the Prussian toise and T_0 is very large; they extend over twenty-five days, involving 2340 micrometer and 520 thermometer readings. The resulting difference of length of the two bars at 61°·25 F. is

$$T_{10}=T_0-154{\cdot}52\pm0{\cdot}15. \quad \ldots \quad (16)$$

In the case of the Belgian toise, the comparisons extend over eight days, and with the following result:—

$$T_{11}=T_0-156{\cdot}33\pm0{\cdot}27. \quad \ldots \quad (17)$$

From these comparisons it would appear that there is a sensible difference between the Prussian and Belgian toises, amounting to 1·81 millionth of a yard; whereas General BAEYER found the difference only $0{\cdot}02\pm0{\cdot}40$.

The comparisons of the Russian double toise extend over 14 days, involving 960 micrometer and 480 thermometer readings. Its length in terms of T_0 is found to be, at 61°·25 F.,

$$P=2T_0-321{\cdot}52\pm0{\cdot}31. \quad \ldots \quad (18)$$

The ten equations (9) . . . (18) trace the connexion between the Ordnance toise and the toise of Peru through the intervention of six other bars. If we make the unit for

small quantities in the first seven equations the same as in the last three, and put

$$\left.\begin{aligned} F_s &= \mathfrak{T}+x_1, \\ F_b &= \mathfrak{T}+x_2, \\ N &= 2\mathfrak{T}+x_3, \\ P &= 2\mathfrak{T}+x_4, \\ T_{10} &= \mathfrak{T}+x_5, \\ T_{11} &= \mathfrak{T}+x_6, \\ T_0 &= \mathfrak{T}+x_7, \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (19)$$

these equations, substituted in (9) . . . (18), give the following:—

$$\left.\begin{aligned} x_1 \qquad\qquad &= 0, \\ x_2+\quad 1\cdot97 &= 0, \end{aligned}\right\} \text{Comparisons by M. ARAGO.}$$

$$\left.\begin{aligned} x_3-2x_1-\quad 30\cdot81 &= 0, \\ x_4-\ x_3+\ 44\cdot63 &= 0, \\ 2x_5-\ x_3+\ 35\cdot06 &= 0, \end{aligned}\right\} \quad \text{,,} \quad \text{M. STRUVE.}$$

$$\left.\begin{aligned} x_5-\ x_2+\quad 0\cdot47 &= 0, \\ x_6-\ x_2+\quad 0\cdot49 &= 0, \end{aligned}\right\} \quad \text{,,} \quad \text{General BAEYER.}$$

$$\left.\begin{aligned} x_5-\ x_7+154\cdot52 &= 0, \\ x_6-\ x_7+156\cdot33 &= 0, \\ x_4-2x_7+321\cdot52 &= 0. \end{aligned}\right\} \quad \text{,,} \quad \text{Southampton.}$$

From these ten equations the values of the seven quantities x have to be determined by least squares. In doing so we shall not make reference to the probable errors attaching to them, as indeed they are not all known, but regard them as of equal weight. The values of $x_1 \ldots x_7$ being found, and substituted in the preceding equations, give

$$\left.\begin{aligned} F_s &= \mathfrak{T}-\quad 0\cdot07, \\ F_b &= \mathfrak{T}-\quad 1\cdot09, \\ N &= 2\mathfrak{T}+\ 30\cdot65, \\ P &= 2\mathfrak{T}-\ 14\cdot33, \\ T_{10} &= \mathfrak{T}-\quad 2\cdot05, \\ T_{11} &= \mathfrak{T}-\quad 2\cdot65, \\ T_0 &= \mathfrak{T}+153\cdot42, \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (20)$$

the weight of x_7 being $\frac{74}{69}$. The residual errors of the ten equations are,

$$\left.\begin{matrix} -\cdot007 \\ +0\cdot07 \end{matrix}\right\} \qquad \left.\begin{matrix} -0\cdot03 \\ -0\cdot35 \\ +0\cdot31 \end{matrix}\right\} \qquad \left.\begin{matrix} +0\cdot32 \\ -0\cdot26 \end{matrix}\right\} \qquad \left.\begin{matrix} -0\cdot95 \\ +0\cdot26 \\ +0\cdot35 \end{matrix}\right\}$$

showing that the different series of comparisons are remarkably consistent, and the value of the toise most satisfactorily determined; in fact

$$T_0=\mathfrak{T}+153{\cdot}42\pm0{\cdot}30. \qquad (21)$$

Now let T_0 be eliminated between equations (7) and (21), and we get

$$\mathfrak{T}=\tfrac{7674}{3600}Y_{55}-144{\cdot}25\pm0{\cdot}37. \qquad (22)$$

Here Y_{55} is supposed at the temperature 61°·25. Now the expansion of the yard for 1° FAHRENHEIT is 6·5145; therefore if we wish $\mathfrak{T}$ in terms of Y_{55} at 62° we must substitute in the above equation instead of Y_{55}, $Y_{55}-4{\cdot}886$. Thus it becomes

$$\mathfrak{T}=(2{\cdot}13151201\pm0{\cdot}00000037)Y_{55}, \qquad (23)$$

which is the length of *the toise* in terms of Y_{55} at 62° FAHRENHEIT.

The Metre.

The metre being by definition 443·296 "lignes" of the toise of Peru, its true length as inferred from the above value of the toise, is

$$\mathfrak{M}=(1{\cdot}09362355\pm{\cdot}00000019)Y_{55}. \qquad (24)$$

The result of the comparisons made in August and December 1864, extending over eight days, is that

$$M=M_0+9{\cdot}98, \qquad (25)$$

where M is the length of the platinum metre, both bars being supposed at 61°·25 F. This equation, combined with (8), gives

$$M=\tfrac{3938}{3600}Y_{55}-125{\cdot}13,$$

where Y_{55} is at 61°·25; but if Y_{55} be at 62°,

$$M=\tfrac{3938}{3600}Y_{55}-130{\cdot}47. \qquad (26)$$

It would appear from what is stated in the '*Base du Système Métrique Décimal*'*, that the platinum bars which were to represent the metre at 32° F. were laid off from the toise of Peru at 16°·25 C. or 61°·25 F., allowance being made for the contraction of the bars according to the rate of expansion ascertained by BORDA. At page 326, tom. iii. BORDA states his results thus, that the expansion of platinum for one degree REAUMUR is $\frac{1}{92800}$. According to this, the correction to the length of the platinum metres at 61°·25 F. would be $\frac{13}{92800}$, or 153·20 millionths of a yard. Hence the supposed length of the Royal Society's platinum metre at 32° F. would be

$$\tfrac{3938}{3600}Y_{55}-283{\cdot}67.$$

Finally, according to M. ARAGO†, this particular metre is too short by 17·59 thousands

* See tom. iii. p. 681.

† The only authority on this point is the statement by Captain KATER in the Philosophical Transactions for 1818, pp. 103, 104.

of a millimetre, or 19·24 millionths of a yard. Adding this, we obtain for the length of the true metre

$$\mathfrak{M}=1\cdot09362446Y_{55},$$

the yard being at 62° F. The difference between this result and that derived through the Prussian, Belgian, and Russian toises is less than a millionth of a yard. We shall adopt the first-determined value, as the probable error of the second is not assignable.

The Yard.

From numerous comparisons among different copies of the Standard Yard, it has been found that at 62°, Y_{55} is too short by 0·40; or if $\mathfrak{Y}$ be the true *yard*,

$$Y_{55}=\mathfrak{Y}-0\cdot40.$$

If we substitute this value in our equations we have the following

Final Results.

Measures.	Expressed in Terms of the Standard Yard. $\mathfrak{Y}$.	Expressed in inches. Inch $=\frac{1}{36}\mathfrak{Y}$.	Expressed in lines of the Toise. Line $=\frac{1}{864}\mathfrak{T}$.	Expressed in Millimetres. Millimetre $=\frac{1}{1000}\mathfrak{M}$.
The Yard	1·00000000	36·000000	405·34622	914·39180
Copy No. 55 of the Yard at its Standard Temperature of 62°·00 F.	0·99999960	35·999986	405·34606	914·39143
Ordnance Standard Foot " " " 62·00	0·33333284	11·999982	135·11521	304·79681
Indian Standard Foot " " " 62·00	0·33333611	12·000100	135·11653	304·79980
Ordnance 10-foot Bar O_1 " " " 62·00	3·33333717	120·000138	1351·15563	3047·97616
Ordnance 10-foot Bar OI_1 " " " 62·00	3·33335432	120·000755	1351·16259	3047·99184
Indian 10-foot Bar I_S " " " 62·00	3·33340138	120·002450	1351·18166	3048·03488
Indian 10-foot Bar I_B " " " 62·00	3·33353284	120·007182	1351·23495	3048·15508
Indian 10-foot Bar I_b " " " 62·00	3·33331457	119·999324	1351·14647	3047·95550
Australian Standard OI_4 " " " 62·00	3·33330427	119·998954	1351·14230	3047·94608
Australian Standard OI_6 " " " 62·00	3·33333747	120·000149	1351·15576	3047·97644
Ordnance Toise " " " 61·25	2·13166458	76·739925	864·06219	1949·17660
Ordnance Metre " " " 61·25	1·09374800	39·374928	443·34662	1000·11420
Royal Society's Metre à traits " " " 32·00	1·09360478	39·369772	443·28857	999·98324
Prussian Toise No. 10 " " " 61·25	2·13150911	76·734328	863·99917	1949·03444
Belgian Toise No. 11 " " " 61·25	2·13150851	76·734306	863·99893	1949·03390
Russian Double Toise P " " " 61·25	4·26300798	153·468287	1727·99419	3898·05952
The Toise	2·13151116	76·734402	864·00000	1949·03632
The Metre	1·09362311	39·370432	443·29600	1000·00000

Table of Logarithms for converting Geodetical Distances.

Distances to be converted.	Logarithmic multipliers to convert into		
	Feet.	Metres.	Toises.
Distances given in the "Account of the Principal Triangulation of Great Britain"	0·00000050	9·48401156	9·19419163
Distances expressed in Metres	0·51598894	....	9·71018007
Distances expressed in Toises	0·80580887	0·28981993	....