

XIV. *On the Resistance of the Air to the Motion of Elongated Projectiles having variously formed Heads.* By F. BASHFORTH, B.D., Professor of Applied Mathematics to the Advanced Class of Artillery Officers, Woolwich, and late Fellow of St. John's College, Cambridge. Communicated by Professor STOKES, Sec. R.S.

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THE famous theory of the parabolic motion of projectiles was at an early period found to give results not in accordance with practice. Manifestly, then, the air must offer a very sensible resistance to a body which is moving through it with a high velocity. This resistance will depend upon the *form* of the moving body, and upon the *velocity* with which it is moving. Hence, before the path of a projectile can be calculated, it will be necessary to determine experimentally the resistance opposed by the air to the motion of the projectile, corresponding to various velocities. According to NEWTON'S law, the resistance of the air varies as the square of the velocity. But the velocities were low in the experiments made under his direction. In 1719 JOHN BERNOULLI gave equations for finding by the method of Quadratures the path &c. of a projectile, when the resistance of the air was supposed to vary according to any power of the velocity. But in spite of grave doubts respecting the accuracy of NEWTON'S law, it has been adopted by most of the eminent mathematicians who have written on the subject, such as EULER (1753), LAMBERT (1765), BORDA (1769), BEZOUT (1789), TEMPELHOF (1788–9), D'EHRENMALM (1788), LOMBARD (1796), and POISSON.

The first good experiments made with a view to determine the resistance of the air to the motion of projectiles were those of ROBINS in 1742. The projectiles used were leaden bullets of small size. When we consider the great density of the material used, its liability to change its form in the barrel of the gun, and the smallness of the *solid* projectiles, it is truly wonderful that ROBINS was able to accomplish so much with his ballistic pendulum. Afterwards HUTTON carried on ROBINS' system of experimenting both with the whirling machine and ballistic pendulum, introducing additional precautions, and using iron projectiles of greater size. In recent times MM. DIDION, MORIN, and PIOBERT have carried on experiments in France with heavier spherical projectiles, by the help of an improved ballistic pendulum; but they have done little more than confirm the results of ROBINS and HUTTON, and extend them to spherical projectiles of larger diameter.

ROBINS came to the conclusions:—"First, That, till the velocity of the projectile surpasses that of 1100 feet in a second, the resistance may be esteemed to be in the duplicate proportion of the velocity; and its mean quantity may be taken to be nearly the

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same with that I have assigned in the former paper. *Second*, That, if the velocity be greater than that of 1100 or 1200 feet in a second, then the absolute quantity of that resistance in these greater velocities will be near three times as great as it should be by a comparison with the smaller velocities"*. HUTTON remarks in a note on these conclusions:—"These suppositions are not nearly correct. In fact, by more accurate experiments with cannon-balls, it appears that the law of the resistance begins to increase above the ratio of the square of the velocity, from the very slowest motions, and thence goes on increasing gradually more and more above what is assigned by that ratio, till we arrive at the velocity of 1600 or 1700 feet per second, where it is at the greatest, amounting in that maximum state to only $2\frac{1}{10}$ times the quantity resulting from the ratio of the square of the velocity. And at the velocity of 1100 feet, instead of answering to that law, it amounts to 1·86 times the same." EULER, in the remarks which accompany his translation of ROBINS' 'Gunnery,' states that, the greater the velocity of the shot, so much the more does theory deviate from the truth†. HUTTON's formula of resistance consisted of two terms, one varying as the velocity, and the other as the square of the velocity.

In the year 1836 M. PIOBERT reexamined HUTTON's experiments, and found that the resistance of the air for various velocities was sufficiently well represented by a formula of two terms, one of which varied as the square, and the other as the cube of the velocity. In 1839 and 1840 numerous experiments were made at Metz, under the direction of a commission, by means of an improved ballistic pendulum. The projectiles used were spherical solid shot of 24, 12, and 8, or 26·47 lbs., 13·38 lbs., and 8·86 lbs. in weight, and 5·85 inches, 4·66 inches, and 4·06 inches in diameter, and a shell 50·71 lbs. in weight and 8·67 inches in diameter. The distances from the gun at which the pendulum was placed were 49 feet, 82 feet, 164 feet, 246 feet, and 328 feet. The resistance of the air to these projectiles was found to be represented by the formula

$$\pi R^2 v^2 \times 0\cdot027(1 + 0\cdot0023v) \ddagger;$$

and the new calculation of HUTTON's experiments gave

$$\pi R^2 v^2 \times 0\cdot02786(1 + 0\cdot0023v) \ddagger.$$

When spherical balls and smooth-bored guns were used, it was only possible to strike the receiver properly when at a moderate distance from the gun; and thus the variation of velocity to be measured was confined within very narrow limits. There was also the disadvantage that, as the velocity of the ball had to be reduced to that of the receiver in order to determine the striking velocity of the ball, only one velocity could be measured for each round fired. It would therefore be quite impossible to employ ROBINS' ballistic pendulum to find the velocities of the heavy elongated projectiles in use at the present day.

* ROBINS' Tracts on Gunnery, by HUTTON, 1805, p. 181.

† Neue Grundsätze der Artillerie, 1745, p. 508.

‡ DIDION, Traité de Balistique, 1860, pp. 61 & 64.

Various attempts have been made to measure the velocities of cannon-balls by the aid of electricity. The machines with revolving cylinders were in general failures, because their inventors made their success depend upon the known uniform angular velocity of the cylinder. These failures opened the way for the introduction of Major NAVEZ' electro-ballistic pendulum, and others of the same class, which worked with two screens, and therefore furnished no means for testing the probable accuracy of the velocity determined. The apparent convenience and portability of these instruments led to their general use both in Europe and America. Major NAVEZ' instrument, in its original complicated form, is now out of fashion; whilst Colonel BENTON'S two-pendulum instrument and Colonel LEURS' modification of it are in common use, as they are simpler than Major NAVEZ' instrument, and give results quite as much to be relied upon. Even if the electro-ballistic pendulum were perfect in itself as a measurer of time, considerable errors might be expected to arise from the imperfections of the indications of two screens placed at moderate distances apart. A reference to a paper by Major NAVEZ, "*Considérations sur les expériences de balistique en ce qui concerne la mesure du temps*"*, will show how little had been accomplished when that was written (1865). And Colonel BENET has well remarked:—"Electro-ballistic machines heretofore used have been powerless to solve one of the most important problems in ballistics—the law of the movement of a projectile through the air,—and this because of the limited number of points of the trajectory that could be determined"†. From preliminary experiments already made, I feel certain that a simpler, cheaper, and better instrument might be substituted with advantage for electro-ballistic pendulums where such instruments can be used. The time occupied by a body in falling from rest through a given space, or the time occupied by the sound of the explosion in travelling over a given space, might be made the foundation of the measurement of a velocity; or the velocity of the shot might be directly compared with the velocity of the sound of the explosion.

In the spring of 1864, when I was appointed Professor of Applied Mathematics to the Advanced Class of Artillery Officers at Woolwich, and Referee of the Ordnance Select Committee, I strongly recommended the construction of a chronograph capable of recording the time occupied by a projectile in passing over *nine* or more *successive equal spaces*. The principle of the chronograph used at the Greenwich Observatory was plainly the one to be adopted. The chief difficulties to be overcome were found (1) in the arrangement of a proper system of screens, so that the ball in passing might merely cause a *momentary interruption* (not a rupture) of the galvanic current, and that the resistance of the circuit might be kept *perfectly constant* during the experiment, (2) in the arrangement of a system of marking, which should give definite records on the surface of the cylinder when moving with a velocity of about 10 inches per second, and (3) in the compensation for the want of uniformity in the angular velocity of the cylinder.

* *Revue de Technologie Militaire*, t. iv.

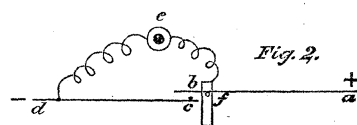
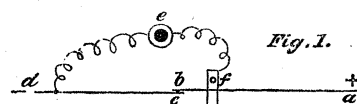
† *Electro-ballistic Machines*, 1866, p. 39.

The chronograph was completed in the summer of 1865, and in November and December of the same year it was tried with ten screens, placed at intervals of 120 feet. Satisfactory records were obtained for eleven out of eighteen rounds of elongated shot fired from a 12-pounder B.L. gun, which very plainly indicated that the resistance of the air varied as the cube of the velocity. A full account of the chronograph and of the method of using it, accompanied by a detailed statement of the eleven successful rounds, was printed in the Proceedings of the Royal Artillery Institution, Woolwich, for August 1866, and was also published in a separate form. At present it will be sufficient to state that the axis of the cylinder is vertical, and in a line with the axis of the horizontal fly-wheel, to which it is attached. The fly-wheel is spun by hand. When the gun is ready to be fired, the markers are brought into contact with the paper; and after the clock has recorded three or four seconds the gun is fired. The clock is allowed to record three or four seconds more, and then the markers are raised and the experiment is completed. The friction on the axis of the fly-wheel, the resistance of the air to the motion of the wheel, and the friction of the markers on the paper tend to reduce the angular velocity of the cylinder. But as the pendulum of a half-seconds' clock raises a lever once each double swing, it interrupts the clock galvanic current once a second, and thus the clock-records show what spaces have passed under the markers each second. The changing angular velocity of the cylinder thus becomes accurately known. For if we commence measuring from some arbitrary point taken two or three seconds before the screen-records, and measure along the spiral traced by the clock marker, noting each record of the clock, and continuing our measurements two or three seconds beyond the screen-records, and if we difference these quantities, we shall find whether the angular velocity has been sufficiently regular in its change during the experiment. If so, we can by interpolation find what would have been the records if one had been made every *tenth* of a second. It has always been found to be sufficient to suppose the angular velocity constant during each tenth of a second, and to calculate smaller intervals of time by proportional parts.

As the clock goes on all day breaking the current once a second, every record of the clock is made under precisely the same circumstances. If there be a loss of time between the breaking of the current and the making of the corresponding record, the loss of time may always be expected to remain the same for any single experiment, and therefore there can be no error; for in experiments on gunnery the exact length of a second, only, is required, and not the exact time of the day. The galvanic current which works the screen-marker is kept constantly circulating through all the screens, excepting during the momentary interruption caused by the breaking or repairing of a screen, or some accidental rupture of the conducting wire. Although there is no necessity for this arrangement, it is found to be practically convenient. The ordinary screens used for other instruments are formed of fine copper wire stretched across a frame repeatedly, and through which the galvanic current circulates. When, then, a shot passes through the screen the current is permanently broken, unless some of the broken wires happen

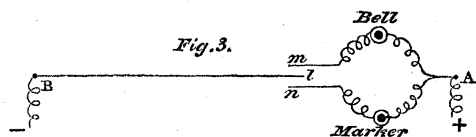
to rest in contact. But for my chronograph it was necessary to make provision for the breaking and immediate restoration of the current, and that without changing the resistance of the circuit. The galvanic current passes along the top of each of the ten screens simultaneously. Equal weights are attached by long pieces of sewing-cotton to certain wire springs which project through holes in sheet copper. When the shot cuts one or more threads, the corresponding springs are released and fly from the bottom to the top of their holes. So long as any single spring is not in contact with the side of the hole through which it projects, the current is interrupted. This kind of screen secures a perfectly constant resistance to the screen galvanic current. But it has been objected that if the galvanic current circulated about the screen electro-magnet for several minutes, which elapse between successive rounds, the loss of time between the breaking of the galvanic current and its corresponding registration would *not be equal* for all the ten screens. This error would be the more pernicious because it might be expected to *follow some law*, and therefore could not be eliminated. The following arrangements have been made partly to meet this difficulty, and partly for the sake of the convenience of keeping up a constant communication between the instrument and the range.

A self-acting contact-breaker and ringer (figs. 1 & 2) is placed by the side of the gun. Ordinarily the lever ab is down, as in fig. 1, in which case the galvanic current takes the direct course, $abc d$. When the lever ab is raised, the current is permanently interrupted, but the insertion of a metallic pin f (fig. 2) opens a passage through the contact-breaker, e , when all the bells in the circuit are continu-



ously rung. After the screens have been mended the lever ab is raised to try whether the current is good. If so, it is lowered and the gun is loaded. When the range is clear and all is ready, the lever ab is raised and the pin f is inserted. The fly-wheel of the chronograph is now put in motion. The rapid interruptions of the current by the contact-breaker are recorded on the cylinder, till a pull at the lanyard, to fire the gun, simultaneously withdraws also the pin f , and so shuts out the contact-breaker. The screen-records often follow so closely that it is difficult, before measurement, to say which is the first screen-record.

Another improvement, represented in fig. 3, has recently been introduced. The two ends of the circuit are at A and B, near the chronograph. When the markers are being raised from the paper



by a lever, the spring l is simultaneously brought into contact with m , which turns the screen-current from the marker to the bell. Thus every interruption of the screen-current caused by the repair of the screens is signaled by the bell. When the bell rings continuously, it is known that the lever ab (fig. 2) is raised ready for firing. The fly-wheel is caused to spin, the markers are brought down upon the paper, and *simultaneously* the spring l is brought into contact with n , when the bell is silenced and the

marker registers the breaks of the contact-breaker till the pin f is withdrawn and the gun is fired. The clock is allowed to make a few beats; and then the markers are raised from the paper, and contact is reestablished between l and m . Thus the galvanic current only circulates for eight or ten seconds about the screen electro-magnet for each experiment, and the current is always being rapidly interrupted quite up to the firing of the gun; so that there is no opportunity for the development of a varying strength of remaining magnetism. The whole arrangement is found to work so satisfactorily that on one occasion nine rounds (23 to 31) were fired in forty-five minutes.

After all possible precautions have been taken, it is found that there are small corrections required in order to make the successive records of both clock and screen consistently regular. The unit of the scale used in measuring is about half an inch, and the scale is read off to two places of decimals, or to the $\frac{1}{200}$ of an inch. The corrections are carried to three places of decimals of the scale. The final calculations are carried to four places of decimals of a second. This is done to secure accuracy to the nearest $\frac{1}{1000}$ of a second of time, giving an opening for an error of $\pm \frac{1}{2000}$ of a second of time, or 6 or 8 inches of space, in finding the time occupied by the shot in passing from the first to any succeeding screen. These corrections of the readings of the scale are rendered necessary because the screens cannot be practically maintained at perfectly equal distances. The point of the shot may strike fairly upon a thread at one screen, and between two threads at the next screen. One spring may act more promptly than another. One string may bend more than another before breaking. These corrections are often merely nominal, but there are some sufficiently large to warn us to beware of trusting implicitly to any measurement of a velocity by two screens only.

Shortly after the publication of the description of my chronograph, my attention was directed to a chronograph with a cylinder, the invention of Captain SCHULTZ*, of the French Artillery, which had been tried in France and America. The instrument is adapted for making any number of records; and, like my own, its success does not depend upon the uniformity of rotation of the cylinder. My instrument makes the clock- and screen-records side by side, on glazed paper which covers the cylinder, so that the original records of the experiments can be preserved for future reference. Captain SCHULTZ makes his records on the slightly smoked metal surface of his cylinder, which are effaced when they have been read off. Captain SCHULTZ uses a large tuning-fork, usually called a diapason, the vibrations of which are sustained by electro-magnetism, to effect the mechanical division of the second into 250 or more equal intervals. The diapason, vibrating as the cylinder turns, traces a sinuous spiral line. The pendulum in swinging interrupts a galvanic circuit once a second, and causes a spark from a Ruhmkorff's coil to strike the cylinder and make a record. Thus it is found how many vibrations the diapason makes per second. The clock is then taken out of the circuit, and the current is made to pass through the Ruhmkorff coil and the first screen. When the first screen is broken the coil gives a spark, and the galvanic current is made to pass through the

* Colonel BENET's 'Electro-ballistic Machines,' 1866, p. 32.

second screen*. When the second screen is broken, a recording spark is given by the coil, and the current is passed on to the third screen, and so on to the end. All the time, the diapason is tracing its sinuous spiral. Unless a careful system of compensation be provided, this method of working the screens would cause great variation in the resistance of the circuit. In the arrangement of my screens, I was careful to maintain a *constant resistance* to the current, which end is secured by making the current pass through all the screens simultaneously, and by providing for it to be interrupted, but not broken. I am not aware that Captain SCHULTZ' instrument has been tried with more than two screens; but Colonel BENET has given a Table showing the number of vibrations per second made by the diapason as determined at the Frankfort Arsenal. In this Table there are striking variations in the numbers of vibrations made in successive seconds, as in the second trial, extending to twenty-five seconds, we find 249·1, 252·0, 249·5, 248·5, 246·0, 249·0, &c. The result of the trials is stated as if the constancy of the *mean* number of vibrations per second was all that was required. This is not the case. A succession of *equal* intervals of time must be marked out by the clock, or diapason, so that, when the gun is fired, the time of passing the screens may be noted by the side of a *correct* scale of time. Now it would make an important difference in the resulting velocity, if the gun were fired when the diapason was making 252·0, 248·5, or 246·0 vibrations per second. If the vibrations of the diapason be maintained by the vibrations of a second fork which alternately makes and breaks contact, as described by HELMHOLTZ†, it is hardly to be expected that the number of vibrations per second can be maintained with sufficient constancy. There is another question. The point from which the spark is discharged cannot be allowed to touch the smoked surface of the cylinder; and it may be asked whether the spark is not liable to deviate in its passage.

After my chronograph had passed its first trial (in December 1865), it appeared to be desirable to institute experiments with a view to find the resistance of the air to various forms of heads of elongated shot, but such as were likely to be of practical utility.

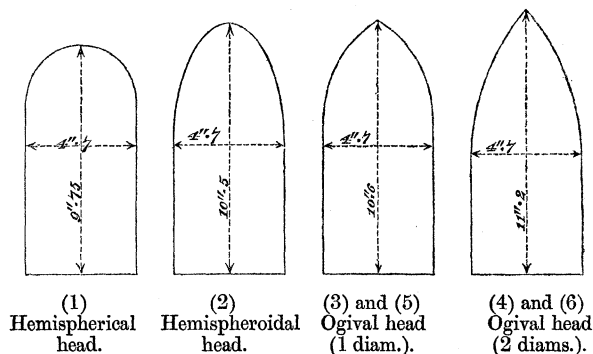
The proposed experiments received the sanction of the Right Honourable the Secretary of State for War, and ten of each kind of the following elongated shot were prepared for the 40-pounder M.L. gun:—

- (1) Solid. Hemispherical head.
- (2) Solid. Hemispheroidal head (ratio of diameters 1 : 2).
- (3) Solid. Ogival, struck with radius equal to a diameter of shot.
- (4) Solid. Ogival, struck with radius equal to two diameters of shot.
- (5) Hollow. Ogival, having precisely the same external form and length as (3).
- (6) Hollow. Ogival, having precisely the same external form and length as (4).

* "In working the instrument it is essential that the current pass only through one target at a time, there being but one coil and one battery no matter how many targets may be used. After the first target is ruptured, the current must be transferred to the succeeding one before the projectile reaches it, and so on throughout the series."—BENET, p. 36. See also the Practical Mechanic's Journal, Oct. 1, 1867, p. 195, to the same effect.

† Tonempfindung, p. 584.

Forms of Shot.



Rounds 1-13 were fired on the 25th September 1866.

14-31 „ „ „ 26th September 1866.

32-43 „ „ „ 27th September 1866.

A barge having anchored in the range, there was no opportunity of firing a single round during the whole of the following day, which interfered with the completeness of the experiment with the solid shot. As a uniform charge of 5 lbs. of powder was used for all the rounds fired, and as the solid were nearly double the weight of the hollow shot, the hollow shot had a much higher initial velocity than the solid. Hence the hollow shot had also a much higher initial angular velocity than the solid shot; and it is probable that the initial angular velocity would be preserved nearly unimpaired throughout the observed range. This is manifested in the greater steadiness of the hollow ogivals.

For the first round, the method of forming the Time-table, and the mode of using it in the calculation of the times of arrival of the shot at each successive screen, are indicated at full length. For further details I must refer the reader to the published description of the chronograph.

The screens were placed 150 feet apart, which distance is denoted by l . The first screen was 75 feet from the gun. t denotes the time occupied by the shot in passing from the first screen to a distance s feet, when $t = as + bs^2$; and $0, t_2, t_3, \dots t_n$ will denote the time when the shot passes the first, second, third, \dots n th screen, or the times corresponding to the particular values $0, l, 2l, \dots \overline{n-1} l$ of s .

(1) Hemispherical-headed Shot. Diameter 4·7 inches.

Round 1. Weight of shot 39·34 lbs.

Clock.			
	Reading.	Correc- tion.	Corrected reading.
"			
1	4·91	0	4·910 Δ_1
2	20·82	0	20·820 +15·910 Δ_2
3	36·65	+·010	36·660 15·840 67
4	52·44	-·007	52·433 15·773 64
5	68·14	+·002	68·142 15·709 61
6	83·79	0	83·790 +15·648

Interpolation.			
"			
2·0	20·820	Δ_1	Δ_2
2·5	28·748	+7·928	-16
3·0	36·660	7·912	-17
3·5	44·555	7·895	-17
4·0	52·433	+7·878	

Time-table obtained by interpolation.			
"			
2·5	28·748	+1·584	
2·6	30·332	1·583	
2·7	31·915	1·582	
2·8	33·497	1·582	
2·9	35·079	1·581	
3·0	36·660	1·580	
3·1	38·240	1·580	
3·2	39·820	1·579	
3·3	41·399	1·578	
3·4	42·977	1·578	
3·5	44·555	1·577	
3·6	46·132	1·576	
3·7	47·708	1·576	
3·8	49·284	1·575	
3·9	50·859	1·575	
4·0	52·433	+1·574	

Screens.				
No.	Reading.	Correction.	Corrected reading.	
1	28·26	0	28·260 Δ_1	Δ_2
2	30·26	+·002	30·262 +2·002	+28
3	32·30	-·008	32·292 2·030	28
4	34·35	0	34·350 2·058	28
5	36·43	+·006	36·436 2·086	28
6	38·54	+·010	38·550 2·114	27
7	40·76	*	40·691 2·141	26
8	42·86	-·002	42·858 2·167	26
9	45·06	-·009	45·051 2·193	26
10	47·26	+·010	47·270 +2·219	+26

Having thus obtained the space described by the clock marker at the end of every tenth of a second, from 2^h·5 up to 4^h·0, we can calculate the times when the screens were passed as follows:—

Screen	No.	Reading.	Correction.	Corrected reading.
1	1	28·26	0	28·260 Δ_1
2	2	30·26	+·002	30·262 +2·002 Δ_2
3	3	32·30	-·008	32·292 2·030 +28
4	4	34·35	0	34·350 2·058 28
5	5	36·43	+·006	36·436 2·086 28
6	6	38·54	+·010	38·550 2·114 27
7	7	40·76	*	40·691 2·141 26
8	8	42·86	-·002	42·858 2·167 26
9	9	45·06	-·009	45·051 2·193 +26
10	10	47·26	+·010	47·270 +2·219 +26

Round 13. Weight of shot 39.33 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
1	15.51	0	15.510 Δ_1	1	64.69	0	64.690 Δ_1	0.0000 Δ_1
2	39.56	0	39.560 +24.050 Δ_2	2	67.72	-.005	67.715 +3.025 Δ_2	.1266 +1266 Δ_2
3	63.52	-.007	63.513 23.953	3	70.77	+.006	70.776 3.061 36	.2548 1282 +16
4	87.37	0	87.370 23.857	4	73.87	+.003	73.873 3.097 34	.3846 1298 14
5	111.13	0	111.130 +23.760	5	77.01	-.006	77.004 3.131 35	.5158 1312 16
				6	80.17	0	80.170 3.166 35	.6486 1328 15
				7	* *	*	83.371 3.201 36	.7829 1343 16
				8	86.61	-.002	86.608 3.237 37	.9188 1359 16
				9	89.89	-.008	89.882 3.274 +37	1.0563 1375 16
				10	93.19	+.003	93.193 +3.311	1.1955 +1392 +17

Round 34. Weight of shot 39.34 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
4	89.69	0	89.690 Δ_1	1	104.33	0	104.330 Δ_1	0.0000 Δ_1
5	111.17	-.010	111.160 +21.470 Δ_2	2	107.05	-.002	107.048 +2.718 Δ_2	.1267 +1267 Δ_2
6	132.55	+.010	132.560 21.400 -70	3	109.80	-.001	109.799 2.751 +33	.2550 1283 +16
7	153.89	0	153.890 +21.330 -70	4	112.58	+.003	112.583 2.784 32	.3849 1299 16
				5	115.40	-.001	115.399 2.816 35	.5164 1315 16
				6	118.24	+.010	118.250 2.851 36	.6495 1331 17
				7	121.16	-.023	121.137 2.887 38	.7843 1348 19
				8	124.06	+.002	124.062 2.925 36	.9210 1367 17
				9	127.02	+.003	127.023 2.961 +36	1.0594 1384 17
				10	130.02	0	130.020 +2.997 +36	1.1996 +1402 +18

Round 43. Weight of shot 39.34 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
1	24.05	0	24.050 Δ_1	1	75.83	0	75.830 Δ_1	0.0000 Δ_1
2	45.51	-.005	45.505 +21.455 Δ_2	2	78.50	+.015	78.515 +2.685 Δ_2	.1260 +1260 Δ_2
3	66.88	+.010	66.890 21.385 70	3	81.24	-.005	81.235 2.720 35	.2536 1276 +16
4	88.21	-.005	88.205 21.315 70	4	83.99	0	83.990 2.755 35	.3829 1293 17
5	109.45	0	109.450 +21.245 -70	5	86.78	0	86.780 2.790 35	.5140 1311 18
				6	89.61	-.003	89.607 2.827 37	.6469 1329 18
				7	92.48	-.010	92.470 2.863 36	.7815 1346 17
				8	95.37	0	95.370 2.900 37	.9179 1364 18
				9	98.31	-.003	98.307 2.937 +36	1.0561 1382 18
				10	101.28	0	101.280 +2.973	1.1961 +1400 +18

Summary (1). Hemispherical-headed Shot.

Time occupied by shot in passing from the first to each of the other screens.

No. of round.	Screen 1.	Screen 2.	Screen 3.	Screen 4.	Screen 5.	Screen 6.	Screen 7.	Screen 8.	Screen 9.	Screen 10.
1	0.0000	0.1264	0.2546	0.3847	0.5166	0.6504	0.7860	0.9233	1.0623	1.2030
13	0.0000	.1266	.2548	.3846	.5158	.6486	.7829	.9188	1.0563	1.1955
34	0.0000	.1267	.2550	.3849	.5164	.6495	.7843	.9210	1.0594	1.1996
43	0.0000	.1260	.2536	.3829	.5140	.6469	.7815	.9179	1.0561	1.1961

Velocities at the following distances from the gun.

No. of round.	150 ft.	300 ft.	450 ft.	600 ft.	750 ft.	900 ft.	1050 ft.	1200 ft.	1350 ft.
	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>
1	1187	1170	1153	1137	1121	1106	1092	1079	1066
13	1185	1170	1156	1143	1130	1117	1104	1091	1078
34	1184	1169	1155	1141	1127	1113	1097	1084	1070
43	1190	1175	1160	1144	1129	1114	1100	1085	1071

No. of round.	Weight of shot.	Value of δl^2 .	Difference from mean.
	lbs.		
1	39.34	·00091	+·00007
13	39.33	·00078	-·00006
34	39.34	·00082	-·00002
43	39.34	·00085	+·00001
Means	39.34	·00084	·00004

(2) Hemispheroidal-headed Shot (Axes 1 : 2), Solid.

Round 2. Weight of shot 38.72 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
2	24.44	0	24.440	1	103.28	+·005	103.285	t 0.0000
3	48.19	0	48.190	2	106.28	+·002	106.282	Δ_1 ·1267
4	71.90	-·004	71.896	3	109.31	-·007	109.303	Δ_2 +1267
5	95.56	+·003	95.563	4	112.35	+·002	112.352	Δ_3 +11
6	119.20	-·004	119.196	5	115.43	+·002	115.432	29
7	142.80	0	142.800	6	118.55	-·006	118.544	31
				7	121.70	-·010	121.690	32
				8	124.87	+·001	124.871	34
								35
								·6457
								1318
								1332
								+15
								·9136
								+1347

Round 7. Weight of shot 38.69 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
3	60.24	0	60.240	1	82.59	+·003	82.593	t 0.0000
4	81.88	0	81.880	2	85.34	+·001	85.341	Δ_1 ·1273
5	103.45	-·010	103.440	3	88.13	-·013	88.117	Δ_2 +1273
6	124.91	+·010	124.920	4	90.92	+·001	90.921	Δ_3 +13
7	146.32	0	146.320	5	93.75	+·005	93.755	28
				6	96.62	-·001	96.619	29
				7	99.51	0	99.510	30
				8	102.43	-·001	102.429	31
				9	105.35	+·025	105.375	32
								33
								·2559
								1300
								1314
								1315
								1329
								1342
								1356
								+13
								·9200
								+1369

Round 35. Weight of shot 38.69 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
3	52.97	0	52.970	1	68.09	0	68.090	t 0.0000
4	71.20	0	71.200	2	**	*	70.399	Δ_1 ·1268
5	89.37	-·003	89.367	3	**	*	72.728	Δ_2 +1268
6	107.47	0	107.470	4	**	*	75.080	Δ_3 +13
				5	**	*	77.455	23
				6	79.85	+·003	79.853	23
				7	82.29	-·016	82.274	23
				8	84.72	-·001	84.719	24
				9	87.19	-·002	87.188	24
				10	89.68	0	89.680	23
								24
								·2548
								1293
								1306
								1319
								1333
								1347
								1361
								+13
								1.0507
								+1374

Round 40. Weight of shot 38.69 lbs..

Clock.	Reading.	Correc- tion.	Corrected reading.			Screen.	Reading.	Correc- tion.	Corrected reading.			Time of passing each screen.		
"				Δ_1	Δ_2					Δ_1	Δ_2	t	Δ_1	Δ_2
4	87.30	0	87.300	+19.750	-50	1	113.53	0	113.530	+2.491	+26	0.0000	+1264	+14
5	107.05	0	107.050	19.700	-50	2	116.02	+0.01	116.021	2.517	26	.1264	1278	13
6	126.75	0	126.750	+19.650	-50	3	118.55	-0.12	118.538	2.543	26	.2542	1291	14
7	146.40	0	146.400			4	121.08	+0.01	121.081	2.569	27	.3833	1305	14
						5	123.65	0	123.650	2.596	27	.5138	1319	14
						6	126.23	+0.16	126.246	2.623	27	.6457	1333	14
						7	128.87	-0.01	128.869	2.649	26	.7790	1347	14
						8	131.52	-0.02	131.518	2.676	27	.9137	1361	14
						9	134.19	+0.04	134.194	+2.702	+26	1.0498	+1375	+14
						10	136.90	-0.04	136.896			1.1873		

Summary (2). Hemispheroidal-headed Shot.

Time occupied by shot in passing from the first to each of the other screens.

No. of round.	Screen 1.	Screen 2.	Screen 3.	Screen 4.	Screen 5.	Screen 6.	Screen 7.	Screen 8.	Screen 9.	Screen 10.
2	0.0000	0.1267	0.2545	0.3836	0.5139	0.6457	0.7789	0.9136	" **	" **
7	0.0000	.1273	.2559	.3859	.5173	.6502	.7844	.9200	1.0569	**
35	0.0000	.1268	.2548	.3841	.5147	.6466	.7799	.9146	1.0507	1.1881
40	0.0000	.1264	.2542	.3833	.5138	.6457	.7790	.9137	1.0498	1.1873

Velocities at the following distances from the gun.

No. of round.	150 ft.	300 ft.	450 ft.	600 ft.	750 ft.	900 ft.	1050 ft.	1200 ft.	1350 ft.
	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>
2	1184	1174	1162	1151	1138	1126	1114	**	**
7	1178	1166	1154	1142	1129	1118	1106	1096	**
35	1183	1172	1160	1149	1137	1125	1114	1102	1092
40	1187	1174	1162	1149	1137	1125	1114	1102	1091

No. of round.	Weight of shot.	Value of b^2 .	Difference from mean value.
	lbs.		
2	38.72	.00064	-.00003
7	38.69	.00069	+.00002
35	38.69	.00065	-.00002
40	38.69	.00069	+.00002
Means	38.70	.00067	.00002

(3) Ogival-headed Shot (one diameter), Solid.

Round 3. Weight of shot 39.56 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.			Screen.	Reading.	Correc- tion.	Corrected reading.			Time of passing each screen.		
"				Δ_1	Δ_2					Δ_1	Δ_2	t	Δ_1	Δ_2
1	21.09	0	21.090	+25.850	-100	1	114.12	0	114.120	+3.310	+35	0.0000	+1293	+14
2	46.94	0	46.940	25.750	79	2	117.43	0	117.430	3.345	43	.1293	1307	18
3	72.69	0	72.690	25.671	62	3	120.78	-0.05	120.775	3.388	48	.2600	1325	18
4	98.36	+0.01	98.361	25.609	-49	4	124.15	+0.13	124.163	3.436	49	.3925	1343	20
5	123.97	0	123.970	+25.560		5	127.60	-0.01	127.599	3.485	46	.5268	1363	18
6	149.53	0	149.530			6	131.09	-0.06	131.084	3.531	+39	.6631	1381	16
						7	134.62	-0.05	134.615	+3.570		.8012	+1397	+16
						8	138.18	+0.05	138.185			.9409		

Round 36. Weight of shot 39.56 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.			Screen.	Reading.	Correc- tion.	Corrected reading.			Time of passing each screen.		
3	52.79	0	52.790	Δ_1	Δ_2	1	99.25	0	99.250	Δ_1	Δ_2	t	Δ_1	Δ_2
4	72.49	0	72.490	+19.700	-105	2	101.29	*	101.716	+2.466	+27	0.0000	+1264	+15
5	92.08	+005	92.085	19.595	100	3	104.21	-001	104.209	2.493	26	.1264	1279	15
6	111.58	0	111.580	19.495	-95	4	106.73	-002	106.728	2.519	25	.2543	1294	13
7	130.98	0	130.980	+19.400		5	109.27	+002	109.272	2.544	25	.3837	1307	13
						6	111.84	+001	111.841	2.569	25	.5144	1320	13
						7	114.44	-005	114.435	2.594	26	.6464	1335	15
						8	117.05	+005	117.055	2.620	+25	.7799	1349	14
						9	119.70	0	119.700	+2.645	+25	.9148	1362	+13
												1.0510		

Round 41. Weight of shot 39.56 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.			Screen.	Reading.	Correc- tion.	Corrected reading.			Time of passing each screen.		
2	53.16	-006	53.154	Δ_1	Δ_2	1	123.37	0	123.370	Δ_1	Δ_2	t	Δ_1	Δ_2
3	76.64	+008	76.648	+23.494	-90	2	126.30	+001	126.301	+2.931	+31	0.0000	+1260	+14
4	100.06	-008	100.052	23.404	90	3	129.26	+003	129.263	2.962	31	.1260	1274	14
5	123.36	+006	123.366	23.314	-90	4	132.26	-004	132.256	2.993	30	.2534	1288	13
6	146.59	0	146.590	+23.224		5	135.27	+009	135.279	3.023	29	.3822	1301	13
						6	138.33	+001	138.331	3.052	29	.5123	1314	13
						7	141.41	+002	141.412	3.081	29	.6437	1328	14
						8	144.51	+012	144.522	3.110	29	.7765	1341	13
						9	147.65	+011	147.661	3.139	+29	.9106	1354	13
						10	150.84	-011	150.829	+3.168	+29	1.0460	1367	+13
												1.1827		

Summary (3). Ogival-headed Shot (one diameter), Solid.

Time occupied by shot in passing from the first to each of the other screens.

No. of round.	Screen 1.	Screen 2.	Screen 3.	Screen 4.	Screen 5.	Screen 6.	Screen 7.	Screen 8.	Screen 9.	Screen 10.
3	0.0000	0.1293	0.2600	0.3925	0.5268	0.6631	0.8012	0.9409	" *	" *
36	0.0000	.1264	.2543	.3837	.5144	.6464	.7799	.9148	1.0510	*
41	0.0000	.1260	.2534	.3822	.5123	.6437	.7765	.9106	1.0460	1.1827

Velocities at the following distances from the gun.

No. of round.	150 ft.	300 ft.	450 ft.	600 ft.	750 ft.	900 ft.	1050 ft.	1200 ft.	1350 ft.
3	<i>f.s.</i> 1160	<i>f.s.</i> 1147	<i>f.s.</i> 1132	<i>f.s.</i> 1117	<i>f.s.</i> 1101	<i>f.s.</i> 1086	<i>f.s.</i> 1074	<i>f.s.</i> *	<i>f.s.</i> *
36	1187	1173	1159	1148	1136	1124	1112	1101	*
41	1190	1177	1165	1153	1142	1130	1119	1108	1097

No. of round.	Weight of shot.	Values of bt^2 .
3	lbs. 39.56	.00085
36	39.56	.00071
41	39.56	.00068
Mean...	39.56	Mean of 2 rounds .00070

(4) Ogival-headed Shot (two diameters), Solid.

Round 4. Weight of shot 38.56 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.			Screen.	Reading.	Correc- tion.	Corrected reading.			Time of passing each screen.		
1	17.50	0	17.500	Δ_1		1	63.78	-.001	63.779	Δ_1		0.0000	Δ_1	
2	40.62	-.005	40.615	+23.115	Δ_2	2	66.67	+.019	66.689	+2.910	Δ_2	.1266	+1266	Δ_2
3	63.64	+.010	63.650	23.035		3	69.63	+.002	69.632	2.943		.2546	1280	+15
4	86.63	-.012	86.618	22.968	67	4	72.61	-.002	72.608	2.976	32	.3841	1295	15
5	109.53	0	109.530	+22.912	-56	5	75.62	-.004	75.616	3.008	32	.5151	1310	14
						6	78.65	+.006	78.656	3.040	31	.6475	1324	13
						7	81.73	-.003	81.727	3.071	28	.7812	1337	14
						8	84.83	-.004	84.826	3.099	29	.9163	1351	13
						9	87.95	+.004	87.954	3.128	+28	1.0527	1364	+12
						10	91.11	0	91.110	+3.156		1.1903	+1376	

Round 37. Weight of shot 38.48 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.			Screen.	Reading.	Correc- tion.	Corrected reading.			Time of passing each screen.		
4	51.40	0	51.400	Δ_1		1	71.99	0	71.990	Δ_1		0.0000	Δ_1	
5	68.04	0	68.040	+16.640	Δ_2	2	74.10	+.003	74.103	+2.113	Δ_2	.1275	+1275	Δ_2
6	84.60	0	84.600	16.560	-80	3	76.24	-.001	76.239	2.136	+23	.2565	1290	+15
7	101.08	0	101.080	+16.480	-80	4	78.40	-.004	78.396	2.157	21	.3868	1303	13
						5	80.57	+.004	80.574	2.178	21	.5184	1316	13
						6	82.77	0	82.770	+2.196	+18	.6513	+1329	+13

Round 42. Weight of shot 38.47 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.			Screen.	Reading.	Correc- tion.	Corrected reading.			Time of passing each screen.		
4	69.77	0	69.770	Δ_1		1	81.34	-.004	81.336	Δ_1		0.0000	Δ_1	
5	84.74	-.010	84.730	+14.960	Δ_2	2	83.17	+.002	83.172	+1.836	Δ_2	.1230	+1230	Δ_2
6	99.61	0	99.610	14.880	-80	3	85.03	+.002	85.032	1.860	+24	.2476	1246	+16
7	114.41	0	114.410	+14.800	-80	4	86.91	+.005	86.915	1.883	23	.3739	1263	17
						5	88.82	+.001	88.821	1.906	23	.5018	1279	16
						6	90.75	0	90.750	1.929	23	.6313	1295	16
						7	92.71	-.008	92.702	1.952	22	.7625	1312	17
						8	94.68	-.004	94.676	1.974	22	.8952	1327	15
						9	96.66	+.012	96.672	1.996	22	1.0295	1343	16
						10	98.69	+ 0	98.690	+2.018	+22	1.1654	+1359	+16

Summary (4). Ogival-headed Shot (two diameters), Solid.

Time occupied by shot in passing from the first to each succeeding screen.

No. of round.	Screen 1.	Screen 2.	Screen 3.	Screen 4.	Screen 5.	Screen 6.	Screen 7.	Screen 8.	Screen 9.	Screen 10.
4	0.0000	0.1266	0.2546	0.3841	0.5151	0.6475	0.7812	0.9163	1.0527	1.1903
37	0.0000	.1275	.2565	.3868	.5184	.6513	*	*	*	*
42	0.0000	.1230	.2476	.3739	.5018	.6313	.7625	.8952	1.0295	1.1654

Velocities at the following distances from the gun.

No. of round.	150 ft.	300 ft.	450 ft.	600 ft.	750 ft.	900 ft.	1050 ft.	1200 ft.	1350 ft.
4	f.s. 1185	f.s. 1172	f.s. 1158	f.s. 1145	f.s. 1133	f.s. 1122	f.s. 1110	f.s. 1100	f.s. 1090
37	1176	1163	1151	1140	1129	*	*	*	*
42	1220	1204	1188	1173	1158	1143	1130	1117	1104

No. of round.	Weight of shot.	Values of $b\bar{b}^2$.
4	lbs. 38·56	·00071
37	38·48	·00069
42	38·47	·00081
Mean...	38·52	Mean of 2 rounds ·00070

(5) Ogival-headed Shot (one diameter), Hollow.

Round 14. Weight of shot **21.78** lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
1	27.48	−.001	27.479	1	96.37	+.002	96.372	$\begin{matrix} t. \\ 0.0000 \end{matrix}$
2	48.56	−.002	48.558	2	98.46	−.001	98.459	$\begin{matrix} \Delta_1. \\ +1.001 \end{matrix}$
3	69.55	+.007	69.557	3	100.61	−.012	100.598	$\begin{matrix} \Delta_2. \\ +2.25 \end{matrix}$
4	90.47	+.008	90.478	4	102.79	−.001	102.789	$\begin{matrix} \Delta_3. \\ +2.25 \end{matrix}$
5	111.33	−.011	111.319	5	105.02	+.012	105.032	$\begin{matrix} \Delta_4. \\ +2.25 \end{matrix}$
6	132.08	0	132.080	6	107.32	+.007	107.327	$\begin{matrix} \Delta_5. \\ +2.25 \end{matrix}$
				7	109.68	−.007	109.673	$\begin{matrix} \Delta_6. \\ +2.25 \end{matrix}$
				8	112.09	−.021	112.069	$\begin{matrix} \Delta_7. \\ +2.25 \end{matrix}$
				9	114.52	−.004	114.516	$\begin{matrix} \Delta_8. \\ +2.25 \end{matrix}$
				10	117.01	+.004	117.014	$\begin{matrix} \Delta_9. \\ +2.25 \end{matrix}$

Round 16. Weight of shot 21.81 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
1	5.14	0	5.140	1	58.75	+0.005	58.755	$\frac{t}{0.0000}$
2	31.63	0	31.630	2	61.37	-0.012	61.358	$\frac{\Delta_1}{+986}$
3	58.07	0	58.070	3	64.02	0	64.020	$\frac{\Delta_2}{+1008}$
4	84.45	-0.002	84.448	4	66.74	+0.004	66.744	$\frac{\Delta_3}{+1032}$
5	110.75	0	110.750	5	69.52	+0.014	69.534	$\frac{\Delta_4}{+1058}$
				6	72.39	+0.003	72.393	$\frac{\Delta_5}{+1084}$
				7	75.33	-0.010	75.320	$\frac{\Delta_6}{+1110}$
				8	78.33	-0.015	78.315	$\frac{\Delta_7}{+1136}$
				9	81.37	+0.008	81.378	$\frac{\Delta_8}{+1162}$
				10	84.50	+0.009	84.509	$\frac{\Delta_9}{+1188}$

Round 18. Weight of shot 21.81 lbs.

[illegible]

Round 20. Weight of shot 21·83 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
"								$t.$
1	11·95	0	11·950 Δ_1	1	94·50	0	94·500 Δ_1	0·0000 Δ_1
2	33·74	0	33·740 +21·790 Δ_2	2	96·64	−005	96·635 +2·135 Δ_2	0·0990 + 990 Δ_2
3	55·45	+·010	55·460 21·720 −70	3	98·82	+·006	98·826 2·191 +56	2·007 1017 +27
4	77·11	0	77·110 21·650 70	4	101·08	−007	101·073 2·247 54	3·050 1043 26
5	98·68	+·010	98·690 21·580 70	5	103·37	+·004	103·374 2·301 52	4·119 1069 26
6	120·20	0	120·200 21·510 70	6	105·73	−003	105·727 2·353 51	5·212 1093 24
7	141·65	−·010	141·640 +21·440 −70	7	108·15	−019	108·131 2·404 50	6·329 1117 24
				8	110·59	−005	110·585 2·454 50	7·470 1141 24
				9	113·08	+·009	113·089 2·504 +50	8·635 1165 +23
				10	115·64	+·003	115·643 +2·554 +50	9·823 +1188 +23

Round 22. Weight of shot 21·81 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
"								$t.$
3	66·77	0	66·770 Δ_1	1	97·44	0	97·440 Δ_1	0·0000 Δ_1
4	87·23	−005	87·225 +20·455 Δ_2	2	99·45	+·006	99·456 +2·016 Δ_2	0·0990 + 990 Δ_2
5	107·60	+·008	107·608 20·383 −72	3	101·52	−001	101·519 2·063 +47	2·002 1012 +22
6	127·92	0	127·920 +20·312 −71	4	103·63	−001	103·629 2·110 48	3·038 1036 24
				5	105·78	+·007	105·787 2·158 49	4·098 1060 25
				6	107·99	+·004	107·994 2·207 49	5·183 1085 24
				7	110·26	−010	110·250 2·256 49	6·292 1109 24
				8	112·55	+·005	112·555 2·305 50	7·425 1133 26
				9	114·90	+·010	114·910 2·355 50	8·584 1159 +25
				10	117·32	−005	117·315 +2·405 +50	9·768 +1184 +25

Round 24. Weight of shot 21·83 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
"								$t.$
2	36·69	0	36·690 Δ_1	1	71·78	0	71·780 Δ_1	0·0000 Δ_1
3	61·65	−010	61·640 +24·950 Δ_2	2	74·26	−006	74·254 +2·474 Δ_2	0·0995 + 995 Δ_2
4	86·49	+·002	86·492 24·852 −98	3	76·79	−001	76·789 2·535 +61	2·016 1021 +26
5	111·24	−002	111·238 24·746 106	4	79·39	−005	79·385 2·596 61	3·061 1045 24
6	135·87	+·002	135·872 +24·634 −112	5	82·04	+·003	82·043 2·658 62	4·132 1071 26
				6	84·75	+·013	84·763 2·720 62	5·228 1096 26
				7	87·56	−015	87·545 2·782 62	6·350 1122 25
				8	90·39	0	87·545 2·845 63	7·497 1147 27
				9	93·30	−002	90·390 2·908 63	8·671 1174 +25
				10	96·27	−001	93·298 +2·971 +63	9·870 +1199 +25

Round 26. Weight of shot 21·81 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
"								$t.$
3	38·40	+·001	38·401 Δ_1	1	78·16	−005	78·155 Δ_1	0·0000 Δ_1
4	56·51	−003	56·507 +18·106 Δ_2	2	79·94	+·010	79·950 +1·795 Δ_2	0·0997 + 997 Δ_2
5	74·54	+·004	74·544 18·037 −69	3	81·78	+·010	81·790 1·840 +45	2·021 1024 +27
6	92·52	−001	92·519 17·975 62	4	83·68	−005	83·675 1·885 44	3·069 1048 25
7	110·44	0	110·440 +17·921 −54	5	85·59	+·014	85·604 1·929 43	4·142 1073 24
				6	87·57	+·006	87·576 1·972 43	5·239 1097 26
				7	80·60	−009	80·600 2·015 42	6·362 1123 23
				8	91·65	−002	89·591 2·057 42	7·508 1146 23
				9	93·74	+·007	91·648 2·099 +42	8·677 1169 +24
				10	95·89	−002	93·747 +2·141 +42	9·870 +1193 +24

Round 28. Weight of shot 21·83 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
1	27·41	+·010	27·420	1	77·15	+·010	77·160	0·0000
2	47·93	-·005	47·925	2	79·19	-·010	79·180	+·0991
3	68·37	+·008	68·378	3	81·25	0	81·250	+·2005
4	88·78	0	88·780	4	83·37	0	83·370	+·3044
5	109·13	0	109·130	5	85·54	0	85·540	+·4169
				6	85·76	0	87·760	+·5198
				7	90·05	-·020	90·030	+·6312
				8	92·35	0	92·350	+·7452
				9	94·72	0	94·720	+·8616
				10	97·14	0	97·140	+·9804

Round 30. Weight of shot 21·81 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
1	27·54	0	27·540	1	91·67	0	91·670	0·0000
2	49·61	-·005	49·605	2	93·83	+·013	93·843	+·0995
3	71·58	0	71·580	3	96·07	0	96·070	+·2014
4	93·47	0	93·470	4	98·35	+·002	98·352	+·3059
5	115·28	0	115·280	5	100·69	-·001	100·689	+·4130
				6	103·08	0	103·080	+·5226
				7	105·53	-·004	105·526	+·6347
				8	108·04	-·014	108·026	+·7494
				9	110·59	-·011	110·579	+·8665
				10	113·18	+·004	113·184	+·9863

Round 32†. Weight of shot 21·81 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
3	44·45	0	44·450	1	65·19	0	65·190	0·0000
4	66·08	0	66·080	2	**	*	67·429	+·1036
5	87·68	+·003	87·683	3	69·72	+·002	69·722	+·2097
6	109·26	0	109·260	4	72·07	-·001	72·069	+·3183
				5	74·46	+·005	74·465	+·4292
				6	76·91	+·001	76·911	+·5424
				7	79·41	-·001	79·409	+·6581
				8	81·97	-·006	81·964	+·7764
				9	84·58	-·002	84·578	+·8974
				10	87·25	0	87·250	+·10211

Summary (5). Ogival-headed Shot (one diameter), Hollow.

Time occupied by shot in passing from the first to each succeeding screen.

No. of round.	Screen 1.	Screen 2.	Screen 3.	Screen 4.	Screen 5.	Screen 6.	Screen 7.	Screen 8.	Screen 9.	Screen 10.
14	0·0000	0·1001	0·2027	0·3078	0·4155	0·5258	0·6385	0·7537	0·8713	0·9911
16	0·0000	·0986	·1994	·3026	·4084	·5168	·6278	·7414	·8576	·9764
18	0·0000	·0993	·2011	·3055	·4124	·5218	·6338	·7483	·8653	·9848
20	0·0000	·0990	·2007	·3050	·4119	·5212	·6329	·7470	·8635	·9823
22	0·0000	·0990	·2002	·3038	·4098	·5183	·6292	·7425	·8584	·9768
24	0·0000	·0995	·2016	·3061	·4132	·5228	·6350	·7497	·8671	·9870
26	0·0000	·0997	·2021	·3069	·4142	·5239	·6362	·7508	·8677	·9870
28	0·0000	·0991	·2005	·3044	·4109	·5198	·6312	·7452	·8616	·9804
30	0·0000	·0995	·2014	·3059	·4130	·5226	·6347	·7494	·8665	·9863
32†	0·0000	·1036	·2097	·3183	·4292	·5424	·6581	·7764	·8974	1·0211

† Gun loaded all night. The initial velocity is therefore reduced.

Velocities at the following distances from the gun.

No. of round.	150 ft.	300 ft.	450 ft.	600 ft.	750 ft.	900 ft.	1050 ft.	1200 ft.	1350 ft.
	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>	<i>f.s.</i>
14	1499	1462	1427	1393	1360	1331	1302	1276	1252
16	1521	1488	1453	1418	1384	1351	1320	1291	1263
18	1511	1473	1437	1403	1371	1339	1310	1282	1255
20	1515	1475	1438	1403	1372	1343	1315	1288	1263
22	1515	1482	1448	1415	1382	1353	1324	1294	1267
24	1508	1469	1435	1401	1369	1337	1308	1278	1251
26	1505	1465	1431	1398	1367	1336	1309	1283	1257
28	1514	1479	1444	1408	1377	1347	1316	1289	1263
30	1508	1472	1435	1401	1369	1338	1308	1281	1252
32	1448	1414	1381	1353	1325	1296	1268	1240	1213

No. of round.	Weight of shot.	Values of b^2 .	Difference from mean value.
	lbs.		
14	21.78	.00125	.00000
16	21.81	.00124	-.00001
18	21.81	.00127	+.00002
20	21.83	.00127	+.00002
22	21.81	.00119	-.00006
24	21.83	.00127	+.00002
26	21.81	.00125	.00000
28	21.83	.00123	-.00002
30	21.81	.00126	+.00001
32	21.81	.00123	-.00002
Means	21.81	.00125	.00002

(6) Ogival-headed Shot (two diameters), Hollow.

Round 15. Weight of shot 21.92 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
"								<i>t.</i>
1	3.55	0	3.550 Δ_1	1	44.59	-.003	44.587 Δ_1	0.0000 Δ_2
2	26.00	0	26.000 +22.450 Δ_2	2	46.80	+.008	46.808 +2.221 Δ_2	.0993 +.993 Δ_2
3	48.39	0	48.390 22.390 -60	3	49.08	+.004	49.084 2.331 +55	.2012 1019 +26
4	70.69	0	70.690 22.300 90	4	51.43	-.015	51.415 2.386 55	.3055 1043 24
5	92.87	0	92.870 +22.180 -120	5	53.79	+.011	53.801 2.440 54	.4124 1069 24
				6	56.25	-.009	56.241 2.493 53	.5217 1093 24
				7	58.74	-.006	58.734 2.544 51	.6334 1117 24
				8	61.27	+.008	61.278 2.594 +50	.7475 1141 +24
				9	63.87	+.002	63.872 +2.594 +50	.8640 +1165 +24

Round 21. Weight of shot 21.89 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
"								<i>t.</i>
1	4.24	0	4.240 Δ_1	1	59.66	-.005	59.655 Δ_1	0.0000 Δ_2
2	24.66	0	24.660 +20.420 Δ_2	2	61.66	+.013	61.673 +2.018 Δ_2	.0994 +.994 Δ_2
3	45.02	-.009	45.011 20.351 -65	3	63.74	-.002	63.738 2.065 +47	.2014 1020 +26
4	65.29	+.007	65.297 20.286 65	4	65.85	+.001	65.851 2.113 48	.3058 1044 24
5	85.53	-.008	85.522 20.225 61	5	68.00	+.012	68.012 2.161 48	.4125 1067 23
6	105.69	0	105.690 +20.168 -57	6	70.23	-.009	70.221 2.209 48	.5216 1091 24
				7	72.48	-.003	72.477 2.256 47	.6331 1115 24
				8	74.78	-.001	74.779 2.302 46	.7469 1138 23
				9	77.13	-.003	77.127 2.348 46	.8630 1161 +24
				10	79.52	0	79.520 +2.393 +45	.9815 +1185 +24

Round 23. Weight of shot 21·94 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
1	21·66	0	21·660 Δ_1	1	92·55	0	92·550 Δ_1	0·0000 Δ_2
2	45·00	0	45·000 +23·340 Δ_2	2	94·84	-·002	94·838 +2·288 Δ_2	·0986 +986 Δ_2
3	68·28	+·010	68·290 23·290 -50	3	97·18	-·001	97·179 2·341 +53	·1995 1009 +23
4	91·53	0	91·530 23·240 50	4	99·57	+·003	99·573 2·394 53	·3028 1033 23
5	114·73	-·010	114·720 23·190 -50	5	102·01	+·012	102·022 2·449 55	·4084 1056 24
6	137·85	+·010	137·860 +23·140 -50	6	104·53	-·003	104·527 2·505 55	·5164 1080 24
				7	107·11	-·023	107·087 2·560 55	·6268 1104 24
				8	109·71	-·007	109·703 2·616 56	·7397 1129 25
				9	112·37	+·005	112·375 +2·672 +56	·8549 +1152 +23

Round 25. Weight of shot 21·97 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
1	28·14	0	28·140 Δ_1	1	97·88	-·002	97·878 Δ_1	0·0000 Δ_2
2	49·45	+·005	49·455 +21·315 Δ_2	2	99·95	+·015	99·965 +2·087 Δ_2	·0993 +993 Δ_2
3	70·67	-·002	70·668 21·213 -102	3	102·10	-·001	102·099 2·134 +47	·2008 1015 +22
4	91·78	0	91·780 21·112 101	4	104·28	0	104·280 2·181 47	·3047 1039 24
5	112·78	+·010	112·790 21·010 -100	5	106·50	+·007	106·507 2·227 46	·4108 1061 22
6	133·70	0	133·700 +20·910 -100	6	108·78	+·001	108·781 2·274 47	·5192 1084 23
				7	111·11	-·008	111·102 2·321 47	·6298 1106 22
				8	113·46	+·010	113·470 2·368 48	·7427 1129 24
				9	115·88	+·006	115·886 2·416 48	·8580 1153 24
				10	118·35	0	118·350 +2·464 +48	·9757 +1177 +24

Round 27. Weight of shot 21·97 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
1	1·40	0	1·400 Δ_1	1	45·61	0	45·610 Δ_1	0·0000 Δ_2
2	20·84	-·010	20·830 +19·430 Δ_2	2	47·52	+·006	47·526 +1·916 Δ_2	·0989 +989 Δ_2
3	40·21	+·010	40·220 19·390 -40	3	49·46	+·028	49·488 1·962 +46	·2003 1014 +24
4	59·57	0	59·570 19·350 40	4	51·50	-·004	51·496 2·008 46	·3041 1038 23
5	78·88	0	78·880 +19·310 -40	5	53·54	+·008	53·548 2·052 44	·4102 1061 23
				6	55·65	-·007	55·643 2·095 43	·5185 1083 22
				7	57·78	+·002	57·782 2·139 44	·6291 1106 23
				8	59·96	+·004	59·964 2·182 43	·7420 1129 23
				9	62·19	0	62·190 2·226 44	·8572 1152 +23
				10	64·46	0	64·460 +2·270 +44	·9747 +1175 +23

Round 29. Weight of shot 21·97 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
1	16·26	0	16·260 Δ_1	1	62·14	0	62·140 Δ_1	0·0000 Δ_2
2	39·53	0	39·530 +23·270 Δ_2	2	64·44	-·009	64·431 +2·291 Δ_2	·0989 +989 Δ_2
3	62·73	+·010	62·740 23·210 -60	3	66·77	+·008	66·778 2·347 +56	·2002 1013 +24
4	85·88	-·003	85·877 23·137 73	4	69·20	-·019	69·181 2·403 56	·3040 1038 25
5	108·93	-·001	108·929 +23·052 -85	5	71·64	-·002	71·638 2·457 54	·4101 1061 23
				6	74·15	-·002	74·148 2·510 53	·5186 1085 24
				7	76·81	-·009	76·711 2·563 53	·6294 1108 23
				8	79·32	+·006	79·326 2·615 52	·7425 1131 23
				9	* *	*	81·992 2·666 51	·8578 1153 22
				10	84·71	0	84·710 +2·718 +52	·9754 +1176 +23

Round 31. Weight of shot 21·91 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
1	13·22	−·010	13·210	1	57·06	0	57·060	0·0000
2	32·46	+·010	32·470	2	58·96	−·003	58·957	0·0993
3	51·66	−·010	51·650	3	60·89	+·005	60·895	2·008
4	70·73	+·010	70·740	4	62·87	+·004	62·874	3·044
5	89·73	0	89·730	5	64·89	+·006	64·896	4·104
				6	66·95	+·012	66·962	5·188
				7	69·07	+·004	69·074	6·296
				8	71·24	−·008	71·232	7·429
				9	73·44	−·004	73·436	8·587
				10	75·68	+·006	75·686	9·770

Round 33†. Weight of shot 21·94 lbs.

Clock.	Reading.	Correc- tion.	Corrected reading.	Screen.	Reading.	Correc- tion.	Corrected reading.	Time of passing each screen.
3	45·67	0	45·670	1	59·32	0	59·320	0·0000
4	64·25	0	64·250	2	61·21	+·003	61·213	1·018
5	82·82	0	82·820	3	63·14	+·006	63·146	2·058
				4	65·13	−·009	65·121	3·121
				5	67·14	0	67·140	4·207
				6	69·21	−·005	69·205	5·319
				7	71·33	−·015	71·315	6·456
				8	73·47	0	73·470	7·617
				9	75·67	+·002	75·672	8·804

Summary (6). Ogival-headed Shot (two diameters), Hollow.

Time occupied by shot in passing from the first to each succeeding screen.

No. of round.	Screen 1.	Screen 2.	Screen 3.	Screen 4.	Screen 5.	Screen 6.	Screen 7.	Screen 8.	Screen 9.	Screen 10.
15	0·0000	0·0993	0·2012	0·3055	0·4124	0·5217	0·6334	0·7475	0·8640	**
21	0·0000	·0994	·2014	·3058	·4125	·5216	·6331	·7469	·8630	·9815
23	0·0000	·0986	·1995	·3028	·4084	·5164	·6268	·7397	·8549	**
25	0·0000	·0993	·2008	·3047	·4108	·5192	·6298	·7427	·8580	·9757
27	0·0000	·0989	·2003	·3041	·4102	·5185	·6291	·7420	·8572	·9747
29	0·0000	·0989	·2002	·3040	·4101	·5186	·6294	·7425	·8578	·9754
31	0·0000	·0993	·2008	·3044	·4104	·5188	·6296	·7429	·8587	·9770
33	0·0000	·1018	·2058	·3121	·4207	·5319	·6456	·7617	·8804	**

Velocities at the following distances from the gun.

No. of round.	150 ft.	300 ft.	450 ft.	600 ft.	750 ft.	900 ft.	1050 ft.	1200 ft.	1350 ft.
15	f.s. 1511	f.s. 1472	f.s. 1438	f.s. 1403	f.s. 1372	f.s. 1343	f.s. 1315	f.s. 1288	f.s. 1266
21	1509	1471	1437	1406	1375	1345	1318	1292	**
23	1521	1487	1452	1420	1389	1359	1329	1302	**
25	1511	1478	1444	1414	1384	1356	1329	1301	1274
27	1517	1479	1445	1414	1385	1356	1329	1302	1277
29	1517	1481	1445	1414	1382	1354	1326	1301	1276
31	1511	1478	1448	1415	1384	1354	1324	1295	1268
33	1473	1442	1411	1381	1349	1319	1292	1264	**

† Round 32 had remained in the gun all night. This probably accounts for the reduced initial velocity of Round 33.

No. of round.	Weight of shot.	Value of δ^2 .	Difference from mean.
	lbs.		
15	21·92	·00124	+·00006
21	21·89	·00121	+·00003
23	21·94	·00118	·00000
25	21·97	·00114	−·00004
27	21·97	·00118	·00000
29	21·97	·00119	+·00001
31	21·91	·00116	−·00002
33	21·94	·00118	·00000
Means	21·94	·00118	·00002

Since the second differences of $0, t_2, t_3, \dots, t_{10}$ are nearly constant in the preceding experiments, we are led to the equation

$t=as+bs^2$, connecting space and time.

This shows that if v be the velocity of the shot at time t , and f the retarding force, then

$$v = \frac{ds}{dt} = \frac{1}{a + 2bs};$$

and if $v=V$ when $s=0$, we have $V=\frac{1}{a}$;

$$\therefore v = \frac{1}{\frac{1}{V} + 2bs}$$

and

$$f = \frac{d^2s}{dt^2} = -2bv^3.$$

The values of bl^2 for each experiment were obtained as follows:—

$$\begin{array}{ll} \text{Since} & t = as + bs^2 \\ \text{If } s=l, & \text{then } t_2=al+bl^2, \\ s=2l, & ,, \quad t_3=2al+4bl^2 \\ . & . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ s=\overline{n-1}l, & ,, \quad t_n=\overline{n-1}al+\overline{n-1}^2bl^2, \end{array}$$

or

$$\begin{aligned} \frac{t_2}{1} &= al + b l^2, \\ \frac{t_3}{2} &= al + 2b l^2, \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \frac{t_n}{n-1} &= al + \overline{n-1} b l^2. \end{aligned}$$

Finding, then, the numerical values of

$$\frac{t_2}{1}, \frac{t_3}{2}, \dots, \frac{t_n}{n-1}$$

for each experiment, and taking the difference of two of these quantities, we find

$$\frac{t_n}{n-1} - \frac{t_3}{2} = \overline{n-3bl^2},$$

where $l=150$ feet, and the mean value of bl^2 so determined from each round may be taken to correspond to the mean velocity of that round.

Since the retarding force is $f=-2bv^3$, acting upon an experimental shot of weight W' lbs., the resistance of the air measured in pounds

$$=-2bv^3 \frac{W'}{g};$$

and the resistance will always be the same against a shot of the same external form when moving with the velocity v . If W be the weight of a different shot, then the retarding force

$$f'=-2bv^3 \frac{W'}{g} \times \frac{g}{W} = -2 \left(b \frac{W'}{W} \right) v^3,$$

or b varies inversely as the weight of a shot of given external form. If we vary the diameter of the shot, it is usual to assume that the resistance of the air varies as the square of the diameter for similar forms of heads of shot. The values of $2b$ are given below in terms of R , the radius of the shot in feet, and W , its weight in pounds. But inasmuch as the diameters of shot are generally given in inches, the same values of $2b$ have been expressed in terms of d , the diameter of the shot in inches. Unfortunately there are only three successful rounds for each of the solid ogivals, and one value of bl^2 in each case is manifestly too large. This must have arisen from unsteadiness of the shot in its flight. If we reject these two, the values of bl^2 derived from the solid ogivals agree very well with those derived from the hollow shot of the like forms.

For the hemispherical head we have $bl^2=.00084$ for shot of mean weight 39.34 lbs., and 4.7 inches in diameter. To find the value of $2b$ adapted for hemispherical-headed shot of weight W lbs., and diameter $=d$ inches $=2R$ feet, we have

$$2b = \frac{2 \times .00084}{(150)^2} \times \left(\frac{d}{4.7} \right)^2 \left(\frac{39.34}{W} \right) = .0000001329 \frac{d^2}{W},$$

or

$$= \frac{2 \times .00084}{150} \left(\frac{24R}{4.7} \right)^2 \frac{39.34}{W} = .000077 \frac{R^2}{W}, \text{ and so on for the rest.}$$

Table of values of $2b$ for differently formed heads adapted for elongated shot of weight W lbs. and radius R feet, or diameter d inches.

Form of head.	Experimental value of bl^2 .	Mean weight of experimental shot.	Value of $2b$ when diameter $=2R$ feet.	Value of $2b$ when diameter $=d$ inches.
(1) Hemispherical00084	lbs. 39.34	.000077 $\frac{R^2}{W}$.0000001329 $\frac{d^2}{W}$
(2) Hemispheroidal.....	.00067	38.70	.000060 $\frac{R^2}{W}$.0000001043 $\frac{d^2}{W}$
(3) Ogival (1 diameter)00070	39.56	.000064 $\frac{R^2}{W}$.0000001114 $\frac{d^2}{W}$
(4) Ogival (2 diameters).....	.00070	38.52	.000062 $\frac{R^2}{W}$.0000001085 $\frac{d^2}{W}$
(5) Ogival (1 diameter)00125	21.81	.000063 $\frac{R^2}{W}$.0000001097 $\frac{d^2}{W}$
(6) Ogival (2 diameters)00118	21.94	.000060 $\frac{R^2}{W}$.0000001042 $\frac{d^2}{W}$

Although the motion of a shot may be well represented by supposing a retarding force $= -2bv^3$ to act through a range of 1400 feet, there is reason to suppose that for velocities ranging from 1500 to 900 feet per second the value of b will be less for the lower velocities with equal degrees of steadiness. It unfortunately happens, however, that the angular velocity imparted to a shot, which most probably remains little changed during the time of flight, depends directly upon the initial velocity of the shot. Hence, when shot are fired with low initial velocities with a view to determine the value of b for low velocities, the steadiness of the shot is diminished, and therefore there is an increase of the resistance of the air on this account. The only way to meet the difficulty is to place screens near the gun to find the initial velocity, and others at a distance of 2000 yards or more, and so compare theory and experiment.

It is worthy of notice that if a body move in a straight line under the action of a force varying as the velocity cubed, the mean velocity obtained by dividing space by time is exactly the actual velocity at the middle point of that space. Thus

$$\frac{\text{space } 2s}{\text{time of describing space } 2s} = \frac{2s}{2as + 4bs^2} = \frac{1}{a + 2bs} = \text{velocity at distance } s.$$

The date of the Report of the above experiments was October 23, 1866.

I have long been aware that Major OTTO had made trial of various laws of the resistance of the air in a work published in 1855. The law of the cube of the velocity was tried, but without any definite result*. It was in April 1867 that I first learnt that M. HÉLIE† had proposed the law of the cube of the velocity as the law of the resistance of the air to elongated projectiles in a work dated 1865, which law he had deduced from experiments made at Gâvre in 1859, 1860, and 1861‡. It will be convenient to quote his own statement of the best series of experiments made at Gâvre in 1859, in order to show the nature of the work done, and the perfect independence of my own methods and numerical results. M. HÉLIE used one of the electro-ballistic pendulums to measure his velocities, but he does not state distinctly which it was.

“Si la résistance de l'air est réellement proportionnelle au cube de la vitesse, on doit avoir un résultat sensiblement constant en substituant, dans l'expression $\frac{v' - v''}{v'v''x}$, les valeurs de v' , v'' et x correspondantes à la même charge.

* Hilfsmittel für ballistische Rechnungen, 1855, p. 12.

† Traité de Balistique, 1865.

‡ [In a Memoir, “Études de Balistique expérimentale,” presented to the Belgian Academy by Captain P. C. BOULENGÉ, June 12, 1867, the author, having deduced the cubic law of resistance of the air from his experiments, proceeds to remark :—

“Ce résultat est en accord complet avec les travaux les plus récents faits en France ; en effet, les expériences exécutées par la commission des principes du tir, en 1856 et 1857, ont conduit M. le capitaine WELTER, professeur à l'École d'application de l'artillerie et du génie de Metz, à reconnaître que la résistance de l'air sur les projectiles sphériques est simplement proportionnelle au cube de la vitesse.

“Cette loi, admise depuis 1862, comme base des études balistiques à cette école, a fourni des formules très-simples et très-facilement calculables sans l'intervention de tables, se prêtant à des recherches que les anciennes formules balistiques ne permettaient pas d'aborder, et donnant des résultats plus conformes à la pratique” (p. 84). —Aug. 1, 1868.]

“Résultats moyens des expériences.
(Chaque vitesse est déduite de 30 coups.)

Charge du canon.	Première vitesse, v' , à 33 mètres du canon.	Dernière vitesse, v'' .	Intervalle des points d'observation, x .	Valeur de $\frac{v' - v''}{v'v''x}$ ou c .
kilog.	mètr.	mètr.	mètr.	
1·5	225·1	215·6	464	0·000000422
2·0	263·7	252·5	467	0·000000360
2·5	291·9	275·9	467	0·000000425
3·0	309·6	291·8	467	0·000000422
3·5	326·9	306·4	467	0·000000438

“Sauf l'anomalie que présente la charge de $2^k \cdot 0$ la valeur de $\frac{v' - v''}{v'v''x}$ se montre sensiblement constante. La formule $r = cv^3$ est donc suffisamment justifiée”*. The diameter of the shot was $0^m \cdot 1623 = 6^{ins} \cdot 39$, its weight $30^k = 66^{lbs} \cdot 14$, and its form hollow ogival with head struck with a radius of nearly two diameters.

In order to facilitate the comparison of this experiment with my own, I have converted the French into English measures.

Charge.	v' .	v'' .	x .	$c = 2b$.
lbs.	<i>f.s.</i>	<i>f.s.</i>	ft.	
3·31	738·5	707·4	1522·4	0·0000000391
4·41	865·1	828·4	1532·2	0·0000000334
5·51	957·7	905·2	1532·2	0·0000000395
6·61	1015·8	957·4	1532·2	0·0000000392
7·72	1072·5	1005·3	1532·2	0·0000000407
Mean value of c or $2b =$				0·0000000384
				$= \cdot 0000358 \frac{R^2}{W}$ if R ft. = radius of shot.
				$= \cdot 000000069 \frac{d^2}{W}$ if d in. = diameter of shot.

The values of $2b$ given by my own experiments for shot of the same form were

$$\cdot 000060 \frac{R^2}{W} \text{ or } \cdot 0000001042 \frac{d^2}{W}$$

for velocities ranging from 1520 to 1270 *f.s.*; so that the value of $2b$ deduced from my own experiments might be expected to be sensibly greater than its value deduced from M. HÉLIE'S experiments made with lower velocities. There are three or four other tabular statements of less complete experiments given by M. HÉLIE. In the present state of the question it is impossible to make any more exact comparison of the two systems of experimenting†.

* HÉLIE, *Traité*, pp. 407, 408.

† [An extended series of experiments just completed at Shoeburyness, with 3, 5, 7, and 9-inch elongated shot, has shown conclusively that, although the motion of a shot may be well represented by supposing the resistance of the air to vary as the cube of the velocity, and to be equal to $-2bv^3$ for a range of 1200 or 1300 feet, still

Meteorological Register, Shoeburyness.

Date.	Hour.		Barometer.		Thermometer.		Wind.		Weather and Remarks.
	A.M.	P.M.	Reading.	Temperature.	Dry.	Wet.	Direction.	Force.	
1866.									
Sept. 25th	10	...	30.05	61	61	54	S.W. by S.	2	Clear.
	...	3	30.05	61	61	55	S.W. by W.	2	Clear.
Sept. 26th	10	...	30.05	61	61	59	S.S.W.	3	Hazy.
	...	3	30.00	61	60	59	S.W. by S.	3	Ditto.
Sept. 27th	10	...	29.95	59	56	56	N.W. by W.	2	Ditto with rain.
	...	3	29.95	60	59	58	W.	2	Hazy.

Direction of firing from the North to South.

b is subject to considerable variations for large variations in v . For ogival heads struck with a radius of $1\frac{1}{2}$ diameter of the shot, fired with great steadiness, the following values of v and b have been found.

v .	$2b \frac{W}{d^2}$.
<i>f.s.</i>	
900	0.000000604
950	0.000000640
1000	0.000000720
1050	0.000000868
1100	0.000001005
1200	0.000001085
1300	0.000001046
1400	0.000000977
1500	0.000000956
1600	0.000000925

It thus appears that M. HÉLIE's value of $2b$ or c is true only for velocities about 950 *f.s.*—Aug. 1, 1868.]