XVI. *On the Unequal Distribution of Weight and Support in Ships, and its Effects in Still Water, in Waves, and in Exceptional Positions on Shore.* *By* E. J. Reed, *C.B., Vice-President of the Institution of Naval Architects.* *Communicated by Professor* G. G. Stokes, *Sec. R.S.*

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The object of the present paper is to bring within the grasp of calculation a much neglected division of ship building scienceand art. Many writers of great ability (French, Spanish, Dutch, Swedish, and English) have studied and explained the forces brought into action upon a ship by her own weight and stability, and by the action of the wind upon her sails and of the waves upon her hull; and the result of their investigations has been to encourage the construction of ships of such forms and such dispositions of weights as conduce to moderate and easy motions in the waves of the sea. The relative positions of the centre of gravity and the metacentre, the excursions of the centres of gravity and buoyancy, the inclinations of the axis of rotation, and many other like questions have been very fully and thoroughly discussed, especially by modern English naval architects; in some cases, I venture to say, with even more elaboration and minuteness of inquiry than their intrinsic importance demands. But while the means of securing ease and moderation of movement of the ship at sea have been thus elaborately studied, in order, mainly, as we have been told, to save the fabric of the ship and its fastenings from excessive strains, comparatively few writers upon naval architecture have pursued the subject to its legitimate and necessary development, by seeking to investigate the actual longitudinal bending- and shearing-strains to which the fabric is in fact exposed in ships of various forms under the various circumstances to which every ship is liable.

But more than this: not only has the question of internal strain and strength in the ship been left undeveloped, but a serious fallacy has underlain many of the writings even of men of the greatest eminence upon this subject, viz. the fallacy of considering *ease of motion* identical with *moderation of strain.* No doubt ease of motion is very desirable in all ships, and violence of motion tends to distress any given fabric; but at the same time it is quite practicable, as will clearly appear hereafter, to so design and build two ships, that in a sea-way the easier of the two shall be the more distressed even with precisely the same structural arrangements, and therefore it is obviously very desirable to examine the actual strains, both static and dynamic, with which we have to deal. The weakness exhibited by many ships, notwithstanding the greatest care on the part of the designers, has long pointed to the necessity of further investigation in this direction; but two modern events—the introduction of armoured ships, and the use of iron and

steel in shipbuilding—have added much to the urgency of the inquiry. A long armoured ship, say, like the 'Minotaur' or 'Agincourt' (400 feet in length, and with fine tapering extremities burdened with towering masses of armour), when pitching in Atlantic waves, undergoes a succession of stresses of great magnitude, undoubtedly requiring to be brought as much as possible within the grasp of calculation, the more so as these stresses undergo continual changes, sweeping through the fabric, so to speak, with prodigious velocity. The employment of iron and steel, and the improvements which the manufacture of both is undergoing, fortunately facilitate the concentration of the strength of the ship in those parts which are subject to the greatest stresses; and to further this object, a closer knowledge of these stresses than has hitherto been possessed is much needed.

BOUGUER, the author of the famous 'Traité du Navire' (1746), was one of the earliest authors who gave consideration to the strains brought upon ships by the unequal distribution of weight and buoyancy. The approximate and imperfect character of his theoretical investigations will readily be seen from the fact that he assumes the immersed portion of the ship to be formed of the halves of two equal right cones, set base to base, with their common axis in the water-line. He then constructs a curve the ordinates of which represent the areas of the corresponding sections of the cones, and this he calls the curve of buoyancy for the body. It necessarily comes out a parabola for each cone. He next assumes that the weight of the ship and her lading is uniformly distributed throughout the length, so that it can be represented by a rectangle which stands on the same base as the parabolic curve, and includes an equal area, the latter condition being fixed by the necessity for the displacement equalling the total weight. His investigations have, however, the merit of exhibiting an early example of that graphical or geometrical method of illustrating the distribution of weight and buoyancy which has since been employed by many authors, which Professor RANKINE in particular has recently used with great advantage, and which I have adopted throughout the following investigations.

About ten years after the publication of BOUGUER's work, DANIEL BERNOULLI wrote his celebrated memoir "Principes Hydrostatiques et Méchaniques," which obtained the prize offered by the Academy of Sciences in 1757 for the best essay on the means of preventing pitching and rolling. Being limited to the discussion of the one branch of the subject proposed by the Academy, BERNOULLI did not discuss, as the title of his work might lead us to expect, the strength and strains of ships; but two years afterwards the Academy, in order to supply this want, supplemented their previous question by another having reference to the strains caused by pitching and rolling, and this elicited EULER's well-known memoir*. Although even this treatise says little of statical strains, and contains but a cursory notice of the longitudinal bending-strains of a ship afloat in still water, it is remarkable on the ground that it presents us with the first instance in which we find account taken of the bending effect of the longitudinal pressure of the

* "Examen des efforts qui ont à soutenir toutes les parties d'un vaisseau dans le roulis et dans le tangage; ou recherches sur la diminution de ces mouvements." Par M. L. EULER, Directeur de l'Académie Royale des Sciences et Belles-Lettres de Prusse.

water on the immersed part of a ship. EULER seems to consider that it will tend to reduce hogging; the reverse is really true, as we shall see hereafter. In his later work (the 'Théorie complette') EULER omits all consideration of the effect of this pressure. No quantitative or practical results are arrived at in this division of EULER'S work.

In DON JUAN'S celebrated 'Examen Maritime' (1771), the graphical method of investigating the forces which result from the varying distribution of weight and buoyancy is again adopted, and the author attempts to make an approximation to the magnitude of the resulting bending-moments. He assumes that the immersed part of a ship may be represented by two surfaces of revolution, the fore body being generated "by the revolution of a semiellipse, and the after body by the revolution of a parabola," about a common axis coinciding with the middle line of the section made by the plane of floatation. No practical importance attaches to his results, since they rest upon so many assumptions; but the work is interesting on account of the grasp of the general conditions of the problem which DON JUAN displays.

ROMME, in 'L'art de la Marine' (1787), gives some attention to this subject, in the main following BOUGUER; but the chief interest of his observation lies in the information which he gives respecting the extent to which the ships of his day yielded to the bending-strains, and the practical methods resorted to for mitigating the evil.

This rapid glance at what the earlier writers had to say upon the subject brings us down to the present century, when our own countrymen took it up. Sir ROBERT SEPPINGS was the first who applied himself to it, but he did little or nothing to advance the science of the question. Dr. YOUNG was, however, fortunately called upon to report on SEPPINGS'S practical improvements, and brought great ability to bear upon the whole subject of the strains of ships, giving the valuable results of his investigations, first in a Report to the Admiralty (1811), and afterwards in a paper submitted to the Royal Society, and published in its Transactions (1814). A perusal of Dr. YOUNG'S paper clearly shows how very necessary it is, in discussing this subject, to place one's self in possession of such detailed calculations as I have now had carried out for several typical ships, the results of which I shall presently place on record. The "causes of arching" are first considered, the unequal distribution of weight and buoyancy being assigned as the chief cause, and hogging being named as the ordinary effect, although Dr. YOUNG thinks that there are many cases "in which a strain of a very different kind is produced." Alterations in stowage are also mentioned as causes of variation in the strains. As a type of ordinary ships, an English 74-gun ship is taken, for which the details of weight and buoyancy had been calculated for various parts of the length. Dr. YOUNG makes various arbitrary assumptions with respect to the positions of the centres of action of the resultant vertical forces of which he thus knows the amounts, and by this means obtains a balance of the moments of the upward and downward forces about either end of the ship, which is obviously necessary in order to satisfy the hydrostatical conditions of equilibrium for the ship. The result at which he arrives is graphically represented. Dr. YOUNG indicates a correct method of calculating the bending-moments at various parts of the ship, and makes an

approximation to the maximum bending-moment, taking account in each case of the action of the forces on one side only of the station considered. He also attempts, but with less success, to find expressions for the deflections corresponding to the bending-moments, in order to estimate the amount of hogging. Next he shows that the longitudinal fluid pressure has a considerable bending-effect on a ship afloat, following, no doubt unconsciously, in the path of EULER, but not falling into the error which EULER had made in supposing that this pressure tended to *reduce* the hogging caused by the vertical forces. The great merit of Dr. YOUNG'S paper consists in the fact that, throughout the investigations of the strains of ships, quantitative results based upon an actual ship are given. No preceding writer, to whom I have referred, followed this course, although BOUGUER and DON JUAN took hypothetical cases to illustrate their methods. Dr. YOUNG also takes a much more complete review of the principal causes of straining than any of his predecessors. In dealing with the question of the strength of ships, however, he is not so successful.

About two years after the publication of Dr. YOUNG'S paper in the Philosophical Transactions, appeared another written by the eminent French geometrician DUPIN. It was professedly "A Theoretical Examination of the Structure of English Vessels," meaning thereby vessels built on the diagonal system. Although DUPIN followed Dr. YOUNG and SEPPINGS in the discussion of this subject, he contributed several additional features possessing both interest and importance.

Without further preface I shall pass, first, to the consideration of the actual distribution of weight and buoyancy in various classes of ships when floating in still water. The testimony of BOUGUER, EULER, and DON JUAN puts it almost beyond doubt that in the older types of wood sailing-ships there was generally a great excess of buoyancy in the middle, and deficiencies of buoyancy at the ends only. In later sailing-ships, such as that referred to by Dr. YOUNG, there were portions of the amidship length (in wake of water, ballast, and other concentrated weights) of which the weight exceeded the buoyancy; and this excess, as well as that due to the heavy extremities, was counterbalanced by the surplus buoyancy of the portions of the ship intermediate between the middle and the extremities. With the introduction of steam as a propelling agent, and of very largely increased lengths and proportions for ships, a vastly different state of things has been brought about in the distribution of weight and buoyancy. At the ends of ships there still remains an excess of weight, exaggerated in many cases by the adoption of very fine under-water lines in combination with heavy bows and sterns above water; but the distribution of weights in the fuller parts of the ship becomes much changed. How great the change has been we may infer from the fact that at present merchant steam-ships are in actual employment of which the length is 400 feet, and the proportion of length to breadth exceeds 10 to 1, both length and proportion having been more than doubled since the introduction of iron into ship-construction and steam into ship-propulsion. Ships of even greater length, both actually and proportionately, to breadth are being constructed for trading between Europe and the East through the

Suez Canal. We usually find the weights of engines, boilers, and coals concentrated at some part of a ship. In a paddle-steamer they are found near the middle of the length, in full-powered screw-steamers rather abaft the middle, and in auxiliary screw-steamers very far aft. Wherever they come their weight obviously increases the downward pressure at that part very considerably; in some cases they cause, while in others they exaggerate, an excess of weight over buoyancy, and in others they bring up the weight very nearly to an equality with the buoyancy. No general law can now be laid down for the strains of all ships, and no general statement can be made to include all the conditions in which any particular ship may be placed by means of variations in her stowage or in the weights she has on board. Having given the details of the weights and buoyancies of various parts, however, the calculation of the resulting still-water strains is practicable, but involves considerable labour. I have taken the cases of three or four typical ships, and have had the DISTRIBUTION OF THE WEIGHT AND BUOYANCY very carefully calculated and graphically recorded. Each example is a ship of modern type, and the results are wholly unlike any which have before been published. In fact, owing to the great labour involved, or to some other cause, only the most meagre and unsatisfactory attempts to discover and exhibit the actual strains of ships have previously been made and recorded.

The first case represents the conditions of long fine paddle-steamers, of high speed, employed as yachts, or blockade-runners, or on other services where great cargo-carrying power is of comparatively minor moment. The case I have selected is that of the Royal Yacht 'Victoria and Albert,' and the diagram in Plate XVI. fig. 1 has been prepared in order to indicate the distribution of weight and buoyancy. In making the calculations required for this purpose, the total length (300 feet) has been divided into 20-feet spaces, and transverse planes of division have been supposed to be drawn in order to form the foremost and aftermost boundaries of the spaces. For each division of the ship the buoyancy, the weight of the hull, and the weight of the equipment have been determined; and the sum of the two latter qualities of course gives the total weight of ship and lading for any particular 20-feet space. A base-line A B (fig. 1) has been taken to represent the ship's length, and a series of equidistant ordinates has been erected, each ordinate representing, in position, the centre plane of a 20-feet space. The positions of the imaginary planes of division in the ship are indicated in the figure at the middle points of the parts of A B lying between the feet of the ordinates; and the distance between consecutive ordinates is, I need hardly say, 20 feet on the scale by which A B is set off. Upon these ordinates there have been set off on a certain scale of tons per inch*,—(1) a length representing the buoyancy of the division of the ship with which the ordinate corresponds, divided by the length of the division; the ordinate will therefore represent the average buoyancy of the division per unit of length: (2) a length representing in a similar way, and on a similar scale, the average weight of hull per unit of length for that division: (3) a length similarly representing the weight of hull and equipment for that division. Through the

* The various scales employed in constructing the diagrams are specified on the respective Plates.

three sets of points thus obtained three curves have been drawn. The curve D D represents the displacement or buoyancy, the curve H H represents the weight of hull, and the curve W W represents the total weight of hull and equipment. From this explanation it will be obvious that, by choosing a proper scale, the areas lying above the line A B, and enclosed by the various curves as well as by any two ordinates, may be taken as representatives of the buoyancy, total weight, and weight of hull, respectively, for the corresponding part of the ship. Hereafter it will appear preferable to adopt the latter mode of representation, and in the various diagrams of a character similar to fig. 1 this plan is followed.

These curves are not minutely accurate representations of the distribution of weight and buoyancy; but for our present purpose they are sufficiently close approximations to such representations. Our chief interest centres in the comparison of the curve of buoyancy with the curve of total weight of hull and equipment; but the curve H H of weight of hull has an interest attaching to it also, as it enables us to determine the straining-effect of the equipment, and to illustrate the importance of careful stowage of the weights carried. For the present I shall only make an examination of the distribution of the weight and buoyancy, and for this purpose shall compare the curves W W and D D. These curves, it will be noticed, cross each other at four points marked R^1, R^2, R^3, R^4 in fig. 1; at these stations the weight equals the buoyancy, and the ship is there "water-borne." Before the foremost water-borne section $R^1 R^1$, which is 50 feet from the bow, the weight exceeds the buoyancy by 85 tons; between this section and the water-borne section $R^2 R^2$ next abaft it, a length of about 68 feet, the buoyancy exceeds the weight by 225 tons; between the two water-borne sections $R^2 R^2$ and $R^3 R^3$, a length of 82 feet of the midship length (in which come the engines, boilers, and coals), the weight exceeds the buoyancy by 210 tons; and from $R^3 R^3$ to $R^4 R^4$, a length of 70 feet, the buoyancy exceeds the weight by 130 tons; while abaft $R^4 R^4$, which is 30 feet from the stern, the weight exceeds the buoyancy by 60 tons. These excesses and defects of buoyancy are graphically represented by the areas of the spaces enclosed by the two curves D D D and W W W between their various points of intersection. The hydrostatical conditions of equilibrium are, of course, satisfied by the distribution of the weight and buoyancy.

These figures will show the vastly different condition of many modern steam-ships as compared with the older types of sailing-ships, which had an excess of weight only at the extremities.

Some modern ships, however, have a distribution of weight and buoyancy similar in kind, although extremely different in degree, to that of their predecessors; and as an example of these I have taken the iron-clad frigate 'Minotaur.' This ship is armoured throughout the length, or, to use a more common phrase, is "completely protected," and may be considered a fair representative of extremely long fine ships so protected, with V-shaped vertical transverse sections at the bow. Her length is 400 feet; the heavy weights of engines, boilers, water, powder, and provisions are distributed over a consi-

derable portion of the length; the guns are also distributed along the broadside; and the weight of hull is nearly uniform, except at the extremities. We should naturally expect, therefore, that the weight would considerably exceed the buoyancy at the bow and stern, and that the buoyancy would exceed the weight throughout the amidship section. The curves in Plate XVI. fig. 2 show that this is actually the case. They are constructed and marked similarly to those of the 'Victoria and Albert.' In this case there are only two water-borne sections $R^1 R^1$, $R^2 R^2$. The first is about 80 feet from the stem, and before it the weight exceeds the buoyancy by about 420 tons; the second is 70 feet from the stern, and on this length there is an excess of weight of about 450 tons; between $R^1 R^1$ and $R^2 R^2$, a length of 250 feet, the buoyancy exceeds the weight by the sum of these excesses—870 tons. It will be observed that at the stern the curve of buoyancy D D in fig. 2 is ended at some distance before the curve of total weight W W; the same thing, although in a less prominent degree, is observable in fig. 1. The overhang of the stern above water is the cause of this method of ending the curves; and in the 'Minotaur' the distance between the points where they terminate is greater than in the 'Victoria and Albert,' because she is a larger ship and has a screw-propeller.

My third illustration is taken from the 'Bellerophon,' for which ship the curves of total weight, weight of hull, and buoyancy shown in fig. 3 have been constructed in the manner previously described for the 'Victoria and Albert.' This case may be taken as a representative of the distribution of weight and buoyancy in iron-clads of moderate length and proportions, with central batteries and armour-belts, and with a fall-back stem. The advantages of this bow in giving increased buoyancy are well illustrated by the fact that the foremost water-borne section $R^1 R^1$ in Plate XVI. fig. 3 is only 40 feet from the bow, and that the excess of weight over buoyancy on this length is only 45 tons. In the 'Minotaur,' as we have seen, the foremost water-borne section is 80 feet from the bow, and the excess of weight on this length is 420 tons. Part of this difference is undoubtedly due to the different arrangement of the armour of the two ships; but as the 'Bellerophon' has an armoured bow-battery about 20 feet long, in wake of which the armour-plating reaches up to the height of the upper deck, and as the 'Bellerophon's' draught of water is much less, forward, than that of the 'Minotaur,' it will be obvious that the very different form of the bows greatly influences the case. The aftermost water-borne section $R^4 R^4$ in the 'Bellerophon' is about 50 feet before the stern, the excess of weight on this length being about 220 tons; it will be remembered that for 70 feet of the after part of the 'Minotaur' the weight exceeds the buoyancy by 450 tons, this large excess being due, in great part, to the armour-plating on the stern*. In the 'Bellerophon' (as in the 'Victoria and Albert') there are four water-borne sections, the excess of weight over buoyancy on 70 feet of the length amidships between $R^2 R^2$ and $R^3 R^3$ amounting to 250 tons. This excess is caused by the concentration of the weights of the armour and armament of the central battery in a comparatively short length amidships;

* The 'Minotaur' is a much larger ship than the 'Bellerophon;' but the difference of size is almost entirely a difference of length, the breadths and heights of the two ships differing comparatively little.

while the heavy weights of machinery, coals, powder, water, &c. are also more concentrated than they would be in a long ship like the 'Minotaur.' On the length of 50 feet between the water-borne sections $R^1 R^1$ and $R^2 R^2$ the buoyancy exceeds the weight by about 105 tons; and between $R^3 R^3$ and $R^4 R^4$ there is an excess of buoyancy of 410 tons on a length of 90 feet. There is, it will be observed, a certain amount of resemblance between the distribution of the weight and buoyancy in the 'Victoria and Albert' and the 'Bellerophon,' and in both these vessels the conditions are very different from those of the 'Minotaur.'

A few words are needed to explain the peculiar form of the curve H H in fig. 3 of the 'Bellerophon's' weight of hull. In both the preceding cases this curve is very flat, and throughout the length is concave to the base-line A B; while in the 'Bellerophon' it is comparatively irregular, having cusps at the ordinates P P, P P. This difference is caused by the disposition of the armour in the 'Bellerophon.' Her central battery is shut in by transverse armoured bulk-heads, the weights of which are included in those of the divisions of the ship represensed by the ordinates P P, P P; hence it follows that there should be cusps in the curve H H at those ordinates. Between those ordinates the curve is nearly straight, as the armour on the ship's sides is of about uniform weight for the length of the battery, and the weight of hull per foot of length is also nearly uniform. Before and abaft the central battery the armour-belt only reaches to the main deck (except in wake of the armoured bow-battery), and its weight per foot of length of the ship is considerably less than it is in wake of the central battery; this reduction, in combination with the gradual *fineing* of the vessel, leads to the form of curve shown in the diagram.

It will be remembered that I have already intimated that the graphical method of representation adopted in this paper is not minutely accurate; and the case just considered being an extreme one, affords an excellent opportunity for illustrating this fact, and showing the extent of the error thus introduced. From the description of the method given above, it will be obvious that we have assumed spaces bounded by curves, which are practically continuous, to be graphic representations of the distribution of the weight and buoyancy. The assumption is fairly accurate for most of the divisions, especially in the case of the curve of buoyancy; but it is not nearly exact for the curve of weight in some parts of a ship, particularly where there are any great weights concentrated in a very small length. The transverse armoured bulk-heads enclosing the central battery of the 'Bellerophon' are each really concentrated in a space, measured fore and aft, of less than 2 feet; and it is obvious that, by spreading the weight over the 20-feet division in which such a bulk-head comes, we materially modify the distribution, and obtain an inaccurate curve of weight, even when we make some allowance for the bulk-heads, as is done in the cusps P, P in fig. 3. If we place the planes of division 18 inches apart instead of 20 feet, and calculate carefully the weights of the 18-inch lengths, we shall, of course, obtain a curve of weight much more accurately representing the actual distribution in the ship, and shall then be able to judge of the amount of error

in the curve W W obtained previously. This I have done in the present case for a length of about 40 feet in the neighbourhood of one of the armoured bulk-heads, and it has been found that the curve of weights between the stations *e* and *f* in fig. 3 (Plate XVI.) assumes the form W′ W′ W′. The concentrated weights (such as the armoured bulk-head, the armoured pilot-tower, the engines, and the mainmast) are in the calculations made for W′ W′ W′ put very nearly into the spaces they really occupy, instead of being spread over 20-feet spaces as in the curve W W; and a further approximation to accuracy might be made by taking the planes of division still closer together. In practice, however, this is quite unnecessary; and it will be seen from the diagram in fig. 3 that even in this exceptional case no very considerable error results from taking the curve W W as a fair representation of the distribution of the weight. This follows from the fact that the curve W W averages, so to speak, the inequalities of W′ W′ W′, and, on the length of 40 feet considered, includes a nearly equal area; that is, represents as nearly as possible the same weight. On this account, when considering the strength of a ship as a whole, we may avail ourselves safely of the method previously described, taking the planes of division, say, 20 feet apart in large ships; but excessively concentrated weights necessarily cause severe *local* bending- and shearing-strains, which must be specially provided against. It will suffice here to say that shipbuilders depend chiefly upon longitudinal bearers or keelsons to distribute such concentrated loads over a considerable length, and prevent their exercising a prejudicial effect on the structure.

My fourth example of the distribution of weight and buoyancy in a fully laden ship floating in still water is taken from the class of broadside ironclads of which the ‘Audacious’ and ‘Invincible’ were the first examples. The calculations have been conducted in the same manner as those for the ‘Victoria and Albert,’ and the results are represented by the curves shown on fig. 4. It is necessary to remark that in these vessels there is an armoured upper-deck battery amidships above the central main-deck battery, and that there is neither a bow nor a stern battery protected by armour, as the fore-and-aft fire, so essential in ironclads, is obtained with the central battery guns on the upper deck. By this means nearly all the principal weights are brought towards the middle of the length, and the extremities are much less heavily burdened than in most other ironclads, not excluding even ships like the ‘Bellerophon.’ In short, the present case is one of a ship with weights very unusually concentrated at the centre. The curve H H of weight of hull in this case bears a certain resemblance to that of the ‘Bellerophon’ in fig. 3, only the cusps P, P are more pronounced on account of the armoured bulk-heads of the upper-deck battery. This fact also accounts in part for the difference between the curves of total weight W W for the two ships. It is only necessary to add that the length of the ‘Audacious’ is 280 feet, and that the same scales have been used for lengths and weights in fig. 4 as in the preceding diagrams. The distribution of weight and buoyancy for this ship may be summarized as follows:— On the 35 feet of length between the bow and the foremost balanced section (R^1 in fig. 4)

the weight exceeds the buoyancy by 115 tons*; between R^1 and R^2, a length of 65 feet, the excess of buoyancy amounts to 220 tons; from R^2 to R^3, 80 feet, the excess of weight is 275 tons; and between R^3 and R^4, on a length of 30 feet, the weight exceeds the buoyancy by 210 tons. These figures furnish another illustration of the advantages resulting from the adoption of moderate length and proportions in iron-clad ships.

In all the preceding examples the ships have been supposed to be fully laden; but there must, of course, occur alterations in the amount and stowage of the weights on board, and consequently in the relative distribution of the weight and buoyancy. Earlier writers on the subject have fully recognized this fact, and nearly all of them concur in stating that the greatest strains experienced by a ship floating in still water are those incidental to the state when all the weights of equipment are removed, but that the character of the strains remained the same as when the ships were laden. In this opinion they were probably correct in so far as the ships of their period were concerned, as the older classes of ships always hogged considerably; but in modern ships when light, the characters as well as the intensities of the strains are often very different from those when they are fully laden. For example, if a paddle-wheel steamer, like the 'Victoria and Albert,' had her heavy weights of engines, boilers, coals, &c. removed, the buoyancy amidships would become greater than the weight, and the ship would be brought into a condition similar to that of the 'Minotaur,' when fully laden, illustrated by fig. 2 (Plate XVI.). A few illustrations will show more clearly that the conclusions drawn by the earlier writers by no means hold for all modern ships, some of which are strained in a very different manner when light and when laden. As extreme cases I will take the 'Minotaur' and the 'Audacious.'

The 'Minotaur's' distribution of weight and buoyancy, when a mere shell, with engines, boilers, &c. all out, as they would be if the ship were undergoing a thorough repair, is shown by fig. 5 (Plate XVI.). The curve H H in this diagram agrees with the curve similarly lettered in fig. 2, and the inner of the two curves marked D D represents the displacement of the various divisions, or, in other words, is the curve of buoyancy for the ship when light. At the extremities the curve H H falls considerably outside the curve D D, the excess of weight before the water-borne section $R^1 R^1$, 90 feet from the bow, being 560 tons, and that abaft the water-borne section $R^2 R^2$ being 500 tons on a length of 80 feet of the stern. When the ship is fully laden, we have seen that the excess of weight forward on a length of 80 feet is 420 tons, and that at the stern there is an excess of 450 tons on a length of 70 feet; so that when this ship is quite light the difference between the distribution of the weight and buoyancy is even greater than it is when the ship is fully laden, and the resulting strains will obviously be more severe.

There is, however, another aspect of this question to which I must briefly refer. It is only when the ship is undergoing a thorough repair that she would be left a mere shell; and usually the engines and boilers are left in place when the other weights are

* The stem of the 'Audacious' is more nearly upright, and the lower water-lines are finer than the 'Bellerophon's.'

removed for the purpose of refit. Supposing this to be so, the engines and boilers must, of course, add greatly to the weight of the middle part of the ship, making the curve of weight in fig. 5 assume the form H E E H. Their effect on the ship's immersion is but slight, and the curve of buoyancy consequently is but little altered, it being represented by the outer one of the two curves marked D D in fig. 5*. In this case it will be observed we have an excess of weight amidships as well as excesses at the extremities. The latter remain nearly the same as in the preceding case; but the change of condition possessing most interest for us is amidships, where we have passed from an excess of buoyancy to an excess of weight amounting to about 80 tons on a length of 55 feet, or thereabouts. In consequence of this there are four water-borne sections (marked $S^1 S^1$, $S^2 S^2$, $S^3 S^3$, $S^4 S^4$) in fig. 5 instead of two only, as there are when the ship is either fully laden or quite light. It would be difficult to give a better illustration of the changes that may take place in the distribution of weight and buoyancy in consequence of changes in the weights on board.

Very few words will suffice respecting the condition of the 'Audacious' under circumstances similar to those of the 'Minotaur.' When quite light, with engines and boilers out, the distribution of weight and buoyancy are shown respectively by the curve H H and the inner of the two curves D D in Plate XVI. fig. 6. There are still four water-borne sections ($R^1 R^1$, $R^2 R^2$, $R^3 R^3$, $R^4 R^4$) in this case; but it will be observed that the excess of weight between $R^2 R^2$ and $R^3 R^3$ is very small, amounting to 65 or 70 tons on a length of 12 feet, so that practically the conditions of strain are almost identical with those of a ship having excesses of weight at the ends only. This is therefore a very interesting change from the condition of the ship when fully laden (as in fig. 4), especially as the excess of weight at the bow is thus increased to about 160 from 115 tons, while that at the stern is about the same as that previously given, 210 tons. A still more interesting change is, however, produced by supposing the engines and boilers to remain in the ship when all the other weights are removed. The curve of weight then assumes the form H E E H in fig. 6, and the outer of the two curves D D represents the buoyancy. By this means it will be seen that six water-borne sections are produced, marked $S^1 S^1$, $S^2 S^2$, &c. in the diagram. At the extremities the excess of weight remains about the same as before, but now between $S^2 S^2$ and $S^3 S^3$ the excess of weight amounts to 155 tons on a length of about 20 feet, and between $S^4 S^4$ and $S^5 S^5$ to 80 tons on a little shorter space; while between $S^1 S^1$ and $S^2 S^2$, a length of about 60 feet, there is an excess of buoyancy of 180 tons, and between $S^5 S^5$ and $S^6 S^6$ there is an excess of buoyancy of 225 tons on a length of about 65 feet. This is by far the most complex case we have yet considered, and I shall revert to it hereafter.

The preceding cases show clearly the effect produced by varying the amount of weight on board a ship; it is also obvious that changes in the stowage of the weights must

* I have preferred marking the two curves of buoyancy in this diagram with one set of letters, as they lie so close together, the weight of engines &c. only giving the ship a little deeper draught in the second case.

produce similar changes in the distribution of the weight and buoyancy, and consequently in the strains brought upon the ship.

Thus far I have dealt only with the actual distribution of weight and buoyancy of ships floating in still water; I have next to investigate the amount of the VERTICAL SHEARING (or racking) and BENDING strains that result from the unequal character of the distribution. By "shearing"-strains are here meant those vertical forces which tend to shear off the part of a ship or girder on one side; and by "bending"-strains are meant those moments resulting from the unequal distribution of weight and buoyancy that tend to alter a ship's longitudinal curvature. In order to give exactitude to my remarks, I will take the case of the 'Victoria and Albert.' The buoyancy and weight of this ship have already been calculated for every 20 feet of the length, and the results have been represented by the curves D D and W W in Plate XVI. fig. 1. It will be obvious that at any station in fig. 1 the length of the ordinate intercepted between those two curves represents the resultant vertical force (that is, the excess of weight over buoyancy, or *vice versâ*) acting on the 20-feet division to which the station corresponds. If the weight be in excess, the resultant force of course acts downward; and if the buoyancy be in excess, the resultant force acts upward. Knowing therefore the amounts, directions, and points of application of all such vertical forces, it is possible to calculate their shearing and bending effect at any transverse section by the simplest mechanical methods.

A graphical method may again with advantage be applied to represent these operations; and in Plate XVI. fig. 7 I have given an example of its application to the case of the 'Victoria and Albert.' The base-line A B represents, as in fig. 1, the length of the ship, and the ordinates dotted in fig. 7 correspond to those drawn in fig. 1, while those drawn in fig. 1 correspond to the imaginary transverse planes of division of which the positions are shown midway between the ordinates in fig. 7. On the dotted ordinates lengths are set off representing on a certain scale the distances between the curves D D and W W on the corresponding ordinates in fig. 1, and through the points thus obtained the curve L L in fig. 7 is drawn. This is known as the curve of "loads," or resultant vertical forces, its ordinates representing in direction and position the excesses of weight over buoyancy, and of buoyancy over weight. Where there is an excess of weight, the ordinate representing it is measured downwards below A B, and where there is an excess of buoyancy the ordinate is measured upwards. In this case, as in the curves of weight and buoyancy previously constructed, we may pass from ordinates to areas, and regard the latter as representing the excesses or defects of buoyancy on a certain scale. We must here, however, add the convention that defects of buoyancy shall be represented by areas lying below the line A B, and excesses of buoyancy by areas lying above A B; and in estimating the excess or defect of buoyancy on any part of the ship we must take the *algebraical* sum of (*i. e.* must integrate) the areas of the loops of the curve L L L corresponding to that part. The scale of areas for this curve is marked on the diagram. It will be obvious, therefore, that the curve of loads L L represents, in a manner more

readily understood, the results that would be arrived at by comparing the curves of weight and buoyancy previously constructed for this ship; and I shall show immediately how to pass from it to similar representations of shearing-strains. In Plate XVII. figs. 8, 9, 10, and 11 the same method has been applied to the cases of the 'Minotaur,' 'Bellerophon,' and 'Audacious' (to the last ship both laden and light), the curves of loads being marked L L in each diagram. In order that the hydrostatical conditions of equilibrium may be fulfilled, the joint area of the loops of the curve L L lying above the line A B must, of course, equal the joint area of the loops lying below A B; and the moments of the areas above and of those below A B, about any line perpendicular to it, must be equal. The points R^1, R^2, &c., where the curves of loads cross the axis A B, obviously correspond to what have been previously termed water-borne or balanced sections, where the weight equals the buoyancy.

Next, as to the construction of the curves of shearing-forces. From what has been previously said, it will appear that the shearing-force at any transverse section equals the resultant upward or downward force, measuring the excess or defect of buoyancy on either of the two parts into which the ship is divided by the transverse section. Hence it follows that, to construct a curve of shearing-forces, we have only to integrate the curve of loads (or to obtain the algebraical sums of the areas of the loops of that curve) up to certain stations, and to use the results of these integrations as measures, on a certain scale, of the lengths of ordinates to be set upwards or downwards at the stations according as the areas above or those below the axis are in excess. In performing the integrations we may start from either end. As an example I will take the case of the 'Bellerophon' in fig. 9 for that purpose, remarking the fact that in Plate XVI. fig. 7 and Plate XVII. figs. 8 and 9 the curves V V represent the result of these operations for the 'Victoria and Albert,' 'Minotaur,' and 'Bellerophon' respectively, and that the similarly marked curves in figs. 10 and 11 represent the shearing-forces experienced by the 'Audacious' when fully laden and when she has only her engines and boilers on board respectively.

Turning to fig. 9, it is necessary to state that A has been chosen as the starting-point for the integration of the curve of loads L L, and that the stations up to which the integrations have been carried, in order to determine the ordinates of the curve of shearing-forces V V, are those (drawn midway between the dotted ordinates of the curve of loads) corresponding to the imaginary planes of division, 20 feet apart, with which we started. The ordinate at any section, say, R^3. R^3, is determined as follows. The area of the loops of L L lying below A B and between A and $R^3 R^3$ is found, as is also the area of the loop lying above A B between the same limits; and the difference between the areas, in this case in favour of the downward forces, is set off on a certain scale of tons per inch (marked on the diagram) on the ordinate $R^3 R^3$. Through points thus determined the curve V V is then drawn. Had the point B been taken as the starting place, or origin, of the integrations, we should obviously have obtained an equal value for the ordinate $R^3 R^3$, only its direction would have been opposite to that we have found; and generally we may

say that, by starting from B instead of A, the same curves would be obtained, only the loops which now lie above the axis A B would lie below it, and *vice versâ*. This would not be of the least consequence, however, since any ordinate of the curve V V simply shows the *amount* of the shearing-force at that station; and the question of *direction* is immaterial, as the same effect is produced whether the part before the station moves upward or downward relatively to that abaft it.

There are one or two features of interest in the curve V V, Plate XVII. fig. 9, to which brief reference may be made before passing on. Its ordinates have continually increasing negative values between A and the foremost water-borne section R^1, while beyond that section they gradually decrease (on account of the fact that the curve of loads has crossed the axis A B, and has part of its area above as well as part below that line) until at the station $a\,a'$ they pass through a zero value, the area of the part of the curve of loads below the axis being there equal to the area of the part above the axis. The section $a\,a'$ therefore divides the ship in such a manner as to render each of the parts before and abaft it separately water-borne, and I shall in my future remarks term such sections "sections of water-borne division"*. At the sections b and $c\,c'$ there are two other zero values of the shearing-force; each of these is also a section of water-borne division, and in passing through them the shearing-force changes sign. At the extremities there is, of course, no shearing-force. The water-borne section R^1 has been shown to be a station of maximum shearing, and all the other points where the curve of loads crosses the axis (R^2, R^3, R^4) are also positions where the curve V V has maximum ordinates.

If the curves of weight and buoyancy previously laid down in Plate XVI. fig. 3 were minutely accurate, the curves of loads and of shearing-forces in Plate XVII. fig. 9 would also be accurate; but as this is not the case, it becomes necessary to examine how great an error is introduced into the curve V V by the errors existing in the curves of weight and buoyancy, and consequently in the curve of loads. We should naturally look for the maximum error in that part of a ship where weights which are really concentrated have been spread out over a considerable space in the construction of the curve of weights; and the 'Bellerophon,' in wake of the armoured bulk-heads, affords, as we have seen, a very exceptional and extreme illustration of the kind. This has been made use of in order to determine what may be fairly assumed to represent the limiting value of the error introduced into the curves of shearing-force. For this purpose the corrected curve of weights $W'\,W'$ in fig. 3 has been employed instead of W W in estimating the shearing-forces between the stations e and f, and the result has been graphically represented by $V'\,V'$ between the corresponding stations e and f in fig. 9. It has been assumed here that the curve V V gives the correct shearing-force at e; but this is not strictly true, and, from the difference between the moments about the station e of the areas included by W W and $W'\,W'$ between the ordinates e and f in fig. 3, it appears that the curve V V indicates too small a shearing-force at e. The connective curve $V'\,V'$ ought therefore to lie somewhat further out from the axis A B, and should cross the curve V V at some

* "Sections of water-borne division" must not be confused with "water-borne sections."

points and lie partly outside it. The maximum correction at the station g would therefore be less than that indicated by the space between V V and V′ V′. At the station g the ordinate of V V represents a shearing-force of 165 tons, that of V′ V′ represents a force of 100 tons, and the difference between the ordinates represents 65 tons. This last amount undoubtedly exceeds the real error in the curve V V for the reasons set forth; and although it would be necessary to construct the curve V V from A up to e in order to determine what its actual ordinate at e would be, we shall be within the mark in assuming that in this very exceptional case the error involved by using the fair curve V V falls below one third of the actual amount of the shearing-force thus obtained. Proportionately this correction is very considerable, but in no ordinary ship would such an extreme example of concentration be met with, and the error would in nearly all cases be much less. It may be added that in practice the exact valuation of the shearing-force is not required, and a sufficiently good approximation can be obtained by the graphical method. Where weights are very concentrated that method will be most in error, and the preceding illustration exemplifies the necessity for great care in its application in such cases in order to ensure thoroughly trustworthy results. In all, or nearly all, ships there is an ample reserve of strength to resist vertical shearing-forces of the most severe character, and on this account comparatively little interest attaches to the determination of the maximum values of the shearing-forces experienced by ships afloat in still water. Even in the case of the 'Minotaur,' which is certainly one that may be expected to give a limiting value, the shearing-force for still water nowhere exceeds one twenty-second part of the total weight; and in the 'Bellerophon' the maximum value does not exceed one thirty-third part, having been reduced so much by means of the better distribution of the weight and buoyancy at the extremities. Without going in detail through the investigation of the shearing-forces experienced by the typical ships previously chosen, it will suffice therefore to indicate a few general properties of these curves of shearing, and to illustrate them by reference to the different ships.

First, as to the determination of the sections of maximum shearing-strain in a ship. The reasoning on which the method of constructing the curves is based obviously leads to the conclusion that these sections must coincide with the balanced or water-borne sections at which the weight exactly equals the buoyancy. It may be summed up in the statements that in proceeding from one end of a ship towards the other the resultant vertical forces, or loads, between the end and the water-borne section all act in the same direction, and that their sum represents the shearing-force at that section; while between any two water-borne sections, such as R^1 and R^2 in fig. 9, the same law holds respecting the resultant vertical forces; and since their sum reaches a maximum at the bounding water-borne sections, the shearing-force, which equals their sum minus a constant quantity, is also, in most cases, a maximum at those sections*. The number of water-borne sections varies, of course, with the distribution of the weight and buoyancy. In the

* Minimum values of the shearing-force sometimes occur at balanced sections, as I shall show almost immediately.

'Minotaur' (as shown in Plate XVI. fig. 2 and Plate XVII. fig. 8) there are only two sections, and this is the simplest case, besides being that which is most frequently met with. In the 'Victoria and Albert' (as shown in figs. 1 & 7) and in the 'Bellerophon' (as shown in Plate XVI. fig. 3 and Plate XVII. fig. 9) there are four water-borne sections, and consequently four sections of maximum shearing. In the 'Audacious,' when fully laden, there are also four such sections (see Plate XVI. figs. 4 and Plate XVII. 10); and in this case it is evident that, by transposing 40 or 50 tons weight from amidships towards the extremities, keeping the ship's trim unaltered, the curve of weights might be made to cross the curve of buoyancy in two other points, thus bringing up the number of water-borne sections to six. When the 'Audacious' has only her engines and boilers on board, she is, in fact, in the conditions here referred to, the balanced or water-borne sections being marked $S^1 S^1$, $S^2 S^2$, &c. in Plate XVI. fig. 6, and $R^1 R^1$, $R^2 R^2$, &c. in Plate XVII. fig. 11. We can also imagine a ship to be so loaded that there would be even a greater number of water-borne sections than this; and it appears, at first sight, difficult to determine what law regulates the number of these sections. On an inspection of all the cases we have given, however, it will be seen that the number of water-borne sections is always *even*; and it appears probable that, at least in the greater number of ships having an excess of weight at the extremities, this law will be conformed to. The reason is that the excesses of weight and buoyancy for all parts of the ship must balance, and that at the extremities of such ships there are always two loops of the curve of loads lying below the axis.

It will be remarked that we have not fixed the position of the absolute or true maximum shearing-force as yet, and it will be necessary to determine it, either by calculation or by means of the curve of shearing-forces, the latter being the simpler plan. In different ships the section of absolute maximum shearing-force is found in various positions. For example, in the 'Minotaur' it is at the aftermost water-borne section $R^2 R^2$ (Plate XVII. fig. 8), while in the 'Victoria and Albert' it is at the water-borne section $R^2 R^2$ (Plate XVI. fig. 7). In screw steam-ships, as a rule, the greatest shearing-force is experienced at the aftermost water-borne section, on account of the, to some extent, unavoidable excess of weight at the stern. The 'Bellerophon' and 'Audacious' are cases in point (see Plate XVII. figs. 9 & 10). The case of the 'Minotaur' is exceptional in one respect, viz. that there is only a small difference between the shearing-forces at the two water-borne sections; in most cases the absolute maximum would probably be greater relatively to the other maximum shearing-forces.

Next, as to the sections where the shearing-forces have zero values. The extremities of a ship obviously have no shearing-force to resist, and we have seen, in the case of the 'Bellerophon' (fig. 9), that at the sections of water-borne division the shearing-force is also zero. The condition established in that ship holds for all the other ships I have taken, and in fact for all ships when floating in still water. In the 'Minotaur,' which is the simplest possible case, there is (neglecting the extremities) only one such section ($a\,a'$ in fig. 8); in the other three ships, when fully laden, there are three such sections,

where the curves of shearing-forces (V V, Plate XVI. fig. 7, Plate XVII. figs. 9 & 10) cross the axis A B; and in the 'Audacious,' when she has only her engines and boilers on board, there are no less than five such sections (see Plate XVII. fig. 11). All these cases, therefore, give an *odd* number of sections of water-borne division and of zero shearing-forces; and the number of these sections is one less than the number of water borne sections where the curves of loads cross the axis and the shearing-forces are maxima. It may be assumed that in most ships having an excess of weight at the extremities, when floating in still water, the number of sections of no shearing-forces will be odd.

The rules here laid down for the number of sections of maximum and zero shearing-force must not be regarded, however, as universally binding, nor necessarily true; they are rether indications of what may be expected to be conformed to in very many ships. To render this clear we will again take the case of the 'Audacious' when she has only her engines and boilers on board, her curve of loads corresponding to that condition being L L in Plate XVII. fig. 11. Between the water-borne sections R^4 and R^5 there is an excess of weight of 80 tons only, but within those limits there falls a section of water-borne division ($d\,d'$), and the ordinary rule is conformed to as regards the number of sections of water-borne division being one less than that of the balanced sections. Now suppose 40 tons out of the 80 tons were removed from this space, 25 tons being placed 90 feet abaft its present position, between the stern and the water-borne section R^6, and 15 tons being placed 150 feet before its present position on the fore side of the water-borne section R^1. The ship's trim would obviously remain unaltered, since the moments produced by the alteration of the stowage balance each other; and the only alteration in the curve of loads would be that the areas of the foremost and aftermost loops would be increased, while the area of the loop between R^4 and R^5 would be diminished. These changes are indicated in the curve L L, Plate XVII. fig. 12. Although the number of the water-borne sections and their positions remain unchanged, this transposition of weight has a remarkable effect upon the form and character of the curve of shearing-forces, as will be seen by comparing the curves marked V V in figs. 11 & 12. Starting from the point A, we find the shearing-force at $R^1\,R^1$ (fig. 11) to be 175 tons, while that at the corresponding station in fig. 12 is 160 tons. On account of this increase in the shearing-force, the foremost section of water-borne division ($a\,a'$) falls somewhat nearer to the water-borne or balanced section $R^2\,R^2$ in fig. 12 than it does in fig. 11, and the shearing-force at R^2 is only 5 tons instead of 20 tons. The next section of water-borne division ($b\,b'$ in fig. 12) also falls closer to R^2 than it did before; in fact the small loop of the curve V V between $a\,a'$ and $b\,b'$ in fig. 11 almost disappears in fig. 12. After crossing the axis at b the curve V V has continually increasing negative values up to the balanced section $R^3\,R^3$, where the shearing-force is 150 tons instead of 135 tons as in fig. 11; and this causes the next section of water-borne division ($c\,c'$) to lie nearer the balanced section $R^4\,R^4$ than it did before. Up to this point, therefore, the curves V V for the two cases have very similar characteristics, although they differ in some respects;

but between $R^4 R^4$ and the stern they are altogether different. At $R^4 R^4$ in fig. 12 the upward or positive shearing-force has a maximum value of 45 tons; and as the excess of weight between $R^4 R^4$ and $R^5 R^5$ has been reduced to 40 tons, the shearing-force at $R^5 R^5$ will still act upward, and have there a maximum value of 5 tons. After passing through this minimum value, the shearing-force gradually increases again until it passes through a maximum value at $R^6 R^6$ of 230 tons, and then decreases to its zero value at B. Hence it will be seen that the chief differences between the curves of shearing in Plate XVII. figs. 11 & 12 consist in the facts that the latter has only *three* instead of *five* sections of water-borne division, and that at $R^5 R^5$ a minimum positive shearing-force takes the place of a small maximum negative shearing-force.

It is interesting to remark, further, that by another slight alteration in the distribution of the weight we can convert the small positive maximum shearing-force at R^2 (fig. 12) into a small negative minimum shearing-force, doing away at the same time with the two sections of water-borne division $a\,a'$ and $b\,b'$. For example, if the excess of weight (40 tons) still remaining between $R^4 R^4$ and $R^5 R^5$ be diminished by one half, and 8 tons be placed before $R^1 R^1$ while 12 tons are placed abaft $R_6 R_6$ in such a manner as to produce equal moments and keep the trim unaltered, we shall have the following values for the shearing-forces at the balanced sections:—at $R^1 R^1$ a force of 183 tons acting downwards, being a maximum value, at R_2 a minimum shearing-force of 3 tons, also acting downwards, at $R^3 R^3$ another maximum value (158 tons acting downwards), at $R^4 R^4$ a maximum force acting upwards of 37 tons, at $R^5 R^5$ a minimum upward force of 17 tons, and at $R^6 R^6$ a maximum upward force of 242 tons. The only section of water-borne division, where the shearing-force is zero and the curve crosses the axis, will obviously be a little nearer to $R^4 R^4$ than $c\,c'$ is in fig. 12.

From these examples, then, it will be seen that by slight changes in the distribution of the weights in a ship we may, while keeping the same total weight and the same number of balanced sections, have different numbers of sections of water-borne division and of zero shearing-force; and at the same time it is evident that these changes may turn a maximum value of the shearing-force in one direction into a minimum value in the opposite direction. It becomes necessary, therefore, to add to our previous rules for the sections of maximum and zero shearing-forces. Before doing this it will be convenient to repeat what was said respecting the number of balanced sections in a ship floating in still water. The general laws deduced from various cases are as follows:—if there be, as there generally is, an excess of weight at both the extremities, the curve of loads will cross the axis in an *even* number of points, and consequently the number of balanced sections is *even*. Hence it follows that the number of sections of maximum or minimum shearing must also be *even*; and it is obvious that between two maximum ordinates of the curve of shearing, both of which lie upon the same side of the axis, there must occur either a minimum ordinate also lying upon the same side, or a maximum ordinate lying upon the other side of the axis. Since these conditions hold, and we know in addition that the curve of shearing must cross the axis in at least one point, on account of the fact

that there is an excess of weight at both ends of the ship, it follows that if it crosses the axis in any other points, these additional sections of zero shearing must be either two in number or some multiple of two. In other words, the whole number of sections of zero shearing, which we have shown to be also sections of water-borne division, must be *odd* in ships having excesses of weight at the bow and stern.

Before concluding these remarks on shearing-forces, it may be well to compare the condition of the 'Minotaur,' 'Bellerophon,' and 'Audacious' when fully laden and floating in still water, as they represent different classes of ironclads. At the foremost balanced section (R^1 in Plate XVII. fig. 8) of the 'Minotaur' the shearing-force is no less than 420 tons; at the corresponding station (R^1 in Plate XVII. fig. 9) in the 'Bellerophon' it is only 45 tons, and at that in the 'Audacious' (R^1, in Plate XVII. fig. 10) it is 120 tons. The great difference between the 'Minotaur' and the other two ships is partly due to the fact that she is completely protected with armour, while they have armoured central batteries and water-line belts; and partly to the very fine entrance and V-formed sections which her designers considered desirable. It has already been explained that the pronounced U-form of transverse sections, and the fall-back contour of stem in the 'Bellerophon' have much to do with her small excess of weight at the bow; and it may here be added that in the 'Audacious' the excess of weight over buoyancy, though somewhat greater than in the 'Bellerophon,' is still very moderate. At the aftermost balanced sections the shearing-forces are for the 'Minotaur' about 450 tons, for the 'Bellerophon' 210 tons, and for the 'Audacious' 202 tons. When speaking of the weight and buoyancy we gave the reasons for the difference existing between the 'Minotaur' and the other ships; they are, principally, the armour plating of the stern and the extreme length and fineness of the run. The 'Bellerophon' and the 'Audacious' each have, as we have seen, two other sections of maximum shearing (R^2, R^3 in figs. 9 & 10). At the foremost of these the shearing-force is, for the 'Bellerophon' 50 tons, and for the 'Audacious' 96 tons; while at the aftermost the shearing-force is 200 and 168 tons respectively. These figures show that the concentration of weights amidships, in ships with central batteries and armour belts, while it renders the conditions of strain more complicated, reduces the actual shearing-strains far below those experienced by completely protected ships. There is doubtless a great reserve of strength in all ships against shearing-strains, so that this fact has not much practical weight; but, as I shall show hereafter, the severer *bending*-strains are similarly reduced when the weights are concentrated, and this is a much more important feature.

The BENDING MOMENTS resulting from the action of the vertical forces on a ship floating in still water next claim attention. I have already indicated the method by which these moments may be estimated when the distribution of the weight and buoyancy are known, and will therefore proceed at once to the graphical method of recording them.

As before stated, it is necessary, in calculating the bending-moment at any transverse section of a ship, to consider the part of the ship on one side of that section as fixed,

and to take account of the moments of the resultant vertical forces representing the excess and defect of buoyancy of the part on the other side of that section. The algebraical sum of these moments represents the bending-moment required. We have in the curves of loads, previously constructed, graphical representations of these resultant vertical forces, both in magnitude and direction, as well as in position, and consequently can find with ease the bending-moments at various stations. The operation simply consists in finding the moments about the various stations of those parts of the curve of loads which lie on one side. I will again take the case of the 'Victoria and Albert,' for which the curve of loads is shown by L L in Plate XVI. fig. 7. It will appear on consideration that the most convenient stations at which to calculate the bending-moments are those (midway between the dotted ordinates of the curve of loads) which have already been used as ordinates of the curve of shearing. Let a (fig. 7) be the station at which the bending-moment is to be determined; then starting from A (which is nearer a than B is, and which is therefore more convenient) the vertical pressure represented by each loop of the curve of loads between A and a must be multiplied by the distance of the centre of gravity of that loop from a; and the difference between the sums of the moments of the upward forces and the sums of the moments of the downward forces will equal the bending-moment at a. At this station the moments of the downward forces are greater than those of the upward forces, and the bending-moment consequently tends to produce hogging, to represent which a length $a\,a'$ is set off *above* A B, showing, on a certain scale of foot-tons per inch, the hogging-moment at a. A similar method is followed at all the other stations, and where (as at b, fig. 7) the resultant moment tends to produce sagging, the ordinate representing its amount is set off below A B. As the result of this process, a series of ordinates is determined for the curve of bending-moments M M, the approximation to accuracy being sufficient for all practical purposes. We have previously seen that for curves of weight, buoyancy, loads, and shearing-forces the graphical method does not cause any important error even in extreme cases, such as the 'Bellerophon's;' and I may add here that for bending-moments the errors resulting from distributing weights that are really concentrated are very much less in proportion than they are in the cases previously considered.

There are one or two matters of practical interest connected with the construction of the curves of moments to which I may briefly refer. The first has already been mentioned, viz. that in calculating the moments it is always better to start from the end of the ship nearer to the section, although the same value would obviously be obtained by starting from the other end, this being one of the hydrostatical conditions of the ship's equilibrium. Another point is, that in working from one end of the ship to the other, the moment at the end towards which we are working should always be zero, since there can be no bending-moment at either end; this constitutes a check on the accuracy of the calculation.

This graphical method of representing bending-moments has also been applied to the 'Minotaur,' 'Bellerophon,' and 'Audacious' (the latter when laden and when light), the

curves for which ships are marked M M in Plate XVII. figs. 8, 9, 10, & 11 respectively. In all cases the same scales for lengths and for moments have been used, so that a fair idea of the relative conditions of strain can be obtained from a comparison of the diagrams, and the scales employed are marked on the various plates. Hereafter I shall refer at some length to the comparative strains of these ships, but will first refer to a few general considerations respecting vertical bending-strains, using the preceding cases as illustrations.

The simplest case is obviously that of a ship which, like the 'Minotaur,' is subject to hogging-moments at every transverse section, these moments gradually increasing in amount from the extremities towards the middle of the length, and attaining their maximum value near the midship section, as shown by the curve M M in Plate XVII. fig. 8. Earlier writers, as I have shown, regarded this as the only case which deserved attention; and recent works on the strains of ships devote greater attention to it than to any other case of strain, doubtless with good reason, since it is that which is most commonly met with. Modern writers, however, have also clearly pointed out the possibility of sagging- as well as hogging-strains being experienced by ships when floating in still water, and have laid down the conditions which must be fulfilled in such cases. But while this is true, it is no less true that a very general belief exists among shipbuilders and others that where an excess of weight over buoyancy exists at the middle of a ship's length there must necessarily be sagging-moments; and in some published works on the subject this is laid down as a general rule. No better illustration of the error of this belief could, I think, be given than that afforded by the 'Bellerophon,' which has an excess of weight over buoyancy amounting to 250 tons at the middle, and yet no sagging-strains at any portion of her length. The curve M M in Plate XVII. fig. 9 shows that this is so; and at the section of minimum hogging-moment, *b*, there is a strain of about 100 foot-tons. The case of the 'Audacious' in Plate XVII. fig. 10 furnishes another illustration of the same kind. In her the excess of weight amidships, when fully laden, amounts to 265 tons; but the bending-moment, so far from becoming a sagging-strain, never falls below a hogging-strain of 3400 foot-tons.

Cases do undoubtedly exist, however, in which great excesses of weight amidships produce sagging-moments in still water, and of these we have an example in the 'Victoria and Albert.' On reference to Plate XVI. fig. 7 it will be seen that in this ship the central part of the curve M M falls below the axis, thus indicating the fact that about 30 feet of the midships length of the ship is subjected to a very small sagging-moment, of which the maximum value does not exceed 170 foot-tons, although the excess of weight over buoyancy amounts to 210 tons. A very small deduction from this excess of weight would suffice also to convert this sagging-moment into a small hogging-moment. For example, suppose 10 tons only to be taken from the excess, and a weight of 4 tons to be placed 150 feet before the position from which the 10 tons were taken, while the remaining 6 tons are placed 100 feet abaft it. The trim of the ship would remain unaltered, and the distribution of buoyancy would therefore be unchanged; but the removal

of the weights would cause a hogging-moment of 600 foot-tons, which, combined with the sagging-moment previously existing of 170 foot-tons, would leave as a final result a hogging-moment of 430 foot-tons at the station where sagging-strains previously existed.

These examples will serve to show the error of supposing that an excess of weight amidships, or at any other part of a ship, necessarily causes sagging, as well as the necessity for taking into account the effect of all the forces on one side of the section for which the bending-moment is being calculated. Unless this is done it is almost impossible to determine whether sagging will take place or not; for in some cases the hogging-moment due to unsupported weights at the extremities will more than counterbalance the sagging-moment due to the excess of weight amidships, as is the case in the 'Bellerophon' and the 'Audacious;' or the reverse may be true, as in the 'Victoria and Albert.'

The effect which alterations in the weights carried by a ship have upon the bending-strains is well illustrated by the comparison of the curves M M in Plate XVII. figs. 10 & 11. By removing all the weights except the engines and boilers, and altering the curve of loads in the manner previously described, the curve of moments in fig. 11 is made to assume a very different form from that in fig. 10. Hereafter I shall have occasion to refer to the principal points of difference; for the present it will suffice to say that in the latter the greatest bending-moment falls nearly amidships, instead of in the after body as before, notwithstanding the fact that at some parts of the middle body there is an excess of weight. The amount of the greatest bending-moment when the ship is light is also somewhat greater than that when she is fully laden; and the curve M M in fig. 11 approximates more nearly in form to that of the similarly marked curve for the 'Minotaur' (fig. 8) than it does in fig. 10, although striking points of difference still exist. Another illustration of the variation in bending-moments which variation in the amount of the weights on board a ship may produce is afforded by comparing figs. 10 & 12 (Plate XVII.); and here we see also how, by reducing the excess of weight between the stations R^4 R^4 and R^5 R^5, we have rendered the curve M M in fig. 12 more nearly continuous in its concavity towards the axis than is the similar curve in fig. 11. The differences between these last two curves have a special interest, on account of the fact that they are entirely due to alterations in the *stowage* of the weights on board, the total amount of weight carried being the same in both cases. The greatest bending-moment in fig. 12 is found at the station $c\,c'$, and amounts to 13,800 foot-tons, or about one-eighth more than it is at the corresponding station in fig. 11, this increase being due to the transposition of weights to the extremities.

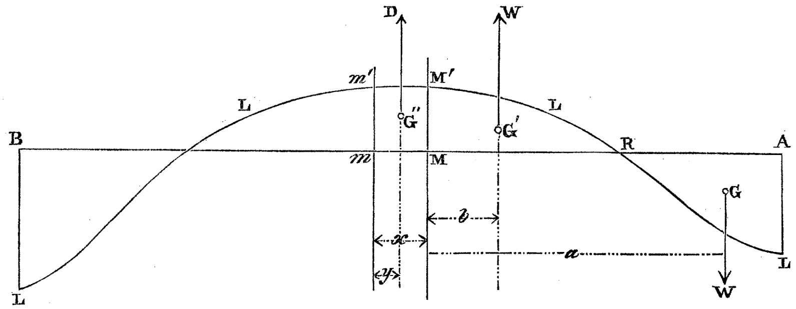
It may be well here to revert briefly to the fact that no ship having an excess of weight over buoyancy at the extremities (as all, or nearly all, ships have) can sag throughout her length when afloat in still water, although she may hog and not sag. The cases of the 'Minotaur,' 'Bellerophon,' and 'Audacious' prove the possibility of the latter condition, and no more need be said respecting it. That sagging alone cannot take place

follows from the fact, that in a ship which sags there must be an excess of weight amidships as well as at the extremities, and there must consequently be some intermediate portions at which there is an excess of buoyancy, and at which hogging-strains will result from the moment of the unsupported weights at the bow and stern. The curve of moments (M M in fig. 7) for the 'Victoria and Albert' illustrates these remarks; in the fore-and-after bodies we find considerable hogging-moments, while amidships, as I have shown, there are sagging-strains. While sagging alone cannot take place in still water, it may, however, occur at sea, or in exceptional positions ashore.

The various conditions of strain of ships floating in still water may, I think, be grouped under the following types:—First, the 'Minotaur' type, including the greater number of vessels, in which the weights are pretty evenly distributed, and the buoyancy is in defect at the extremities only. In some vessels which might be included in this class the weight and buoyancy are equal for a considerable length of middle body; but it will be obvious that in such cases the bending-moment is of uniform amount throughout the middle body, and that the length of middle body might be increased or diminished without affecting the vertical bending-strains. Second, the 'Bellerophon' or 'Audacious' type, in which there is a defect of buoyancy amidships as well as at the extremities, but the bending-moments throughout the length produce hogging-strains, having a minimum value amidships. Third, the 'Victoria and Albert' type, which has a greater proportionate defect of buoyancy amidships, and is there subject to sagging-strains, while in the fore-and-after bodies hogging-strains are experienced. Besides these there may be, and doubtless are, many special cases, wherein, to revert to the graphical method, the curve of loads would have a greater number of loops than in any of the ships we have considered; but the preceding classifications will, as I have said, probably include by far the greater number of ships.

The determination of the positions of those sections of a ship at which the maximum and minimum bending-strains are experienced has been satisfactorily performed by Dupin and later writers. Among these later writers I may particularly refer to Professor Rankine, who has done much to advance the application of scientific principles to the determination of the strength and strains of ships. At page 136 of his 'Shipbuilding, Theoretical and Practical,' are given a mathematical demonstration and a graphical representation of the theorem which Dupin first established, viz. that maximum and minimum bending-moments are experienced by, what I have termed previously, sections of water-borne division. Those who are desirous of following out these investigations will be repaid by a study of Professor Rankine's method; but that sections of water-borne division do possess this property may be shown by the following simple method.

Fig. 13.



Let L L in diagram fig. 13 represent the curve of loads of a ship that tends to hog simply, and let M M′ be the section of water-borne division. Then the area A L R must equal the area M′ M R, since the weight and buoyancy of each of the parts into which M M′ divides the ship must be balanced. Suppose W=weight in tons represented by each of the areas A L R, M′ M R. We may, as far as bending is concerned, consider the weight W to be acting through G, the centre of gravity of the area A L R, at a known distance a from the section M M′, and the buoyancy W to be acting through G′, the centre of gravity of M′ M R, at a known distance b from that section. Then we shall have, obviously,

$$\text{Hogging-moment at M M}'=\text{W}(a-b).$$

Next, take any section $m\,m'$ on the side of M M′ further from A, and let D tons be the buoyancy represented by the area M′ m' m M. Then if G″ be the centre of gravity of the area M′ m' m M, the force D may be considered to act through G″ at a known distance y from $m\,m'$. Hence if x=distance M m, we shall have

$$\begin{aligned}\text{Hogging-moment at } m\,m' &= \text{W}(a+x)-\text{W}(b+x)-\text{D}y\\ &= \text{W}(a-b)-\text{D}y\\ &= \text{Hogging-moment at M M}'-\text{D}y.\end{aligned}$$

That is to say, the bending-moment at $m\,m'$ is less than that at M M′. We shall obtain similar results if we suppose the section $m\,m'$ to be situated on the side of M M′ nearer to A; for in that case, to obtain the moment of the buoyancy of the area R $m\,m'$ about $m\,m'$, we must add the moment of M M′ $m\,m'$ about that station to the moment of the whole area R M M′ about it. Hence, as $m\,m'$ is now $-x$ distant from M M′, we shall have

$$\begin{aligned}\text{Hogging-moment} &= \text{W}(a-x)-\text{W}(b-x)-\text{D}y\\ &= \text{W}(a-b)-\text{D}y,\end{aligned}$$

as before. Hence it follows that for this distribution of weight and buoyancy the section of water-borne division is also the section of maximum bending-moment. For the more complex case, where there is an excess of weight amidships as well as at the extremities, it can be shown, by a method similar to the foregoing, that the central section of water-borne division is a section of minimum bending-moment, and that in the fore and after bodies there will be two sections of maximum bending-moment if there are two sections

of water-borne division. By a simple extension of the method, the cases similar to those illustrated by Plate XVII. figs. 10 & 11 can also be shown to come under this rule, and the theorem stated above may thus be established generally*.

It has already been shown that sections of water-borne division coincide with sections of zero shearing, and that the number of such sections must be *odd*; hence we have this rule, that the number of sections of maximum and minimum bending-moment must be *odd*. This rule will be seen to hold in all the preceding cases. For example, in the 'Minotaur' there is one section of maximum hogging-moment ($a\,a'$ in Plate XVII. fig. 8); in the 'Bellerophon,' and in the 'Audacious' when fully laden, there are three sections of maximum and minimum hogging-moment, the section of minimum moment being nearly amidships ($a\,a'$, $b\,b'$, $c\,c'$ in Plate XVII. figs. 9 & 10); in the 'Audacious,' when light, with her engines and boilers only on board, there are five sections of maximum and minimum hogging-moment ($a\,a'$, $b\,b'$, &c. in Plate XVII. fig. 11); and in the 'Victoria and Albert' there are two sections of maximum hogging-moment and one of maximum sagging-moment ($a\,a'$, $c\,c'$, and $b\,b'$ in Plate XVI. fig. 7). In the hypothetical case of Plate XVII. fig. 12, based upon the 'Audacious,' there are also three sections of maximum and minimum hogging-moment. If sagging be regarded as a negative phase of hogging, it appears from all these cases that sections of maximum and minimum bending-moment occur alternately, although it must not be forgotten that in its *absolute amount* the sagging-moment, which is termed a minimum, may exceed some of the so-called maximum hogging-moments. What is really meant may perhaps be better expressed as follows:—Between two sections of maximum hogging-moment there must fall either a section of minimum hogging-moment or a section of maximum sagging-moment. In the ships with which DUPIN was acquainted no sagging-moments were experienced, and hence we find him laying down the law, which I believe he was the first to observe, that sections of maximum and minimum bending-moment occur alternately. My statement is simply an extension of the same principle to more complex cases, such as are met with in modern ships. DUPIN also indicates a simple method of determining whether a section of water-borne division is one of maximum or minimum bending-moment. It is as follows:—If the resultant vertical force immediately adjacent to the section of water-borne division acts upward, that section is one of maximum hogging-moment; if downward, it is one of minimum hogging-moment. In order to extend this so as to embrace all the ships we have considered I must add, if the resultant vertical force acts downward, the adjacent section of water-borne division is either one of minimum hogging-moment or of maximum sagging-moment, and *vice versâ*.

All that has been said respecting the variations in the number of sections of zero shearing, or water-borne division, which may be produced by alterations in the stowage

* In the case previously considered, of ships with a long middle body of which the weight balances the buoyancy, this general law also holds; for any transverse section within the limits of the middle body will obviously fulfil the conditions of a section of water-borne division, and of one where the bending-moment is a maximum or a minimum.

of the weights such as keep the number of balanced sections constant, of course applies with equal force to the sections of maximum and minimum bending-moment. In both fig. 11 and fig. 12 (Plate XVII.), for example, we have six balanced sections ($R^1 R^1$, $R^2 R^2$, &c.), but only three sections of maximum and minimum bending-moment in the latter instead of five as before; while we have seen that another slight transposition of weight would leave only one section of water-borne division and maximum bending-moment. A comparison of the curves of bending and shearing in these two cases possesses further interest, on account of the fact that when the curve V V in fig. 12 has a minimum positive ordinate at $R^5 R^5$ instead of a maximum negative ordinate as in fig. 11, the maximum and minimum ordinates of the curve M M at $e\, e'$ and $d\, d'$ respectively in fig. 11 disappear, and we have instead of them a point of contrary flexure in the curve M M of fig. 12. Similarly, if the other transposition were made which does away with the two sections of water-borne division $a\, a'$ and $b\, b'$ in fig. 12, and turns the small positive maximum shearing-force at $R^2 R^2$ into a small negative minimum shearing-force, we should have a point of contrary flexure at $R^2 R^2$ instead of the maximum and minimum ordinates at $a\, a'$ and $b\, b'$ of the curve M M. In fact, from the relations which exist between the curves V V and M M, it is obvious that at the balanced sections, where the shearing-force has maximum or minimum values, the curve of moments has either points of contrary flexure (as in all the illustrations we have given) or singular points where there is a change of curvature. It may be added that we have by our construction supposed the points of contrary flexure in the curve of loads to lie at the balanced sections where the curve crosses the axis; and hence we may say that at the stations where the curves L L have maximum ordinates the curves V V have points of contrary flexure; in other words, the curve V V bears a relation to the curve L L similar to that which the curve M M bears to it*. The broad practical deduction to be drawn from the cases represented by figs. 11 & 12 is, however, simply this, that by transposing weights from the centre to the extremities we render the curve of moments M M less tortuous, which is a matter of no consequence; but we at the same time increase the maximum bending-moment which the ship has to resist, and which may thus become raised to a very undesirable amount. We are thus again reminded of the reduction of the strains of ironclads produced by adopting the belt-and-battery system instead of that of complete protection.

In all ships there must obviously be a section or sections of absolute maximum bending-

* Expressed in mathematical symbols, this relation stands as follows:—Suppose B to be the origin of coordinates in fig. 12, and let the distance of any station from B be called x. Then if y=resultant vertical force, or ordinate of the curve of loads at that station, S=shearing-force, or ordinate of the curve of shearing, and M=bending-moment, or ordinate of the curve of moments, we have by our method of construction, as previously explained,

$$S=\int_0^x y \, . \, dx,$$

$$M=\int_0^x y \, . \, x dx=\int_0^x S \, . \, dx.$$

That is to say, we obtain the ordinates of the curve of shearing by integrating the areas of the curve of loads up to various stations; and obtain the ordinates of the curve of moments, either by integrating for the moments of the curve of loads or for the areas of the curve of shearing.

strain, no matter what number of sections there may be at which that strain has a maximum value. For example, in the 'Victoria and Albert' there are two sections of maximum hogging-moment ($c\,c'$ and $a\,a'$ in Plate XVI. fig. 7), but the absolute maximum is found in the fore body; while in the 'Bellerophon' and 'Audacious' (see Plate XVII. figs. 9 & 10) the absolute maximum is found in the after body. In cases such as the 'Minotaur's,' where there is only one section of water-borne division, that is, of course the section of maximum bending. Attention has previously been drawn to the fact that in such cases the sections of maximum bending (such as $a\,a'$ in Plate XVII. fig. 8) usually lie very near the middle of the length; and many writers on the subject, considering the distribution of weight and buoyancy too exclusively, refer to the midship section as being most severely strained. For still water, however, I have shown above that this is by no means true in many ships; and it is only proper to add that some authors of standing have recently put forward views on this point which are not borne out by my calculations. For example, even Professor RANKINE, speaking of a ship that tends to hog in still water, says*:—"Let M be a transverse section which divides the ship into two parts, each separately water-borne (and which is seldom far from the midship section); then at the section M ..... the bending-moment is a maximum;" and further on he adds, "in the case of a ship that tends to sag amidships the greatest bending-moment is still at M ....., being a sagging-moment instead of a hogging-moment." Having proved above that the "greatest bending-moment" in ships which "tend to sag amidships" falls not at M but in the fore or after bodies, I need not dwell at length on the too hasty inference here made. It will suffice to say that the section of greatest sagging does in such cases sometimes come near the middle of the ship; but in Plate XVI. fig. 7 we have an illustration of the fact that it may come considerably abaft the middle; and there are doubtless cases in which it comes before the middle, its actual position being determined, as Professor RANKINE himself states, by the position of the central section of water-borne division. This sagging-moment, however, is by no means "the greatest bending-moment" experienced by the ship, the distribution of weight and buoyancy determining whether the true maximum bending-moment is experienced by some section in the fore or after body. I may add, that without accurate calculation it is generally possible to tell which body has to bear the greatest strain, by simply observing at which end the excess of weight is greater. For example, in the 'Victoria and Albert' the excess of weight over buoyancy is greater at the bow than at the stern, and the section of absolute maximum bending-strain falls in the fore body; had the reverse been true, the section would have fallen in the after body, as it does in the cases of the 'Bellerophon' and 'Audacious.' Professor RANKINE, while recognizing the existence of these sections of maximum hogging-moment, does not seem to have noticed the fact that the strains at them would generally be more severe than those at the section of maximum sagging-moment.

With respect to the actual amounts of the bending-moments experienced by the typical

* 'Shipbuilding, Theoretical and Practical,' p. 151.

ships I have chosen, when floating in still water, but few remarks are necessary, since these moments are much less severe than those experienced by the same ships when at sea, or when placed in exceptional positions ashore. The 'Minotaur,' on account of her great length, very fine form, and heavily burdened extremities, may be regarded as a limiting case in the amount of her hogging-moment, the maximum value of which (at $a\,a'$ in Plate XVII. fig. 8) is about 45,000 foot-tons, equalling the product of the displacement in tons by about one eighty-eighth of the ship's length. The 'Bellerophon' may be taken as an opposite limiting case, in so far as strain on the midship part is concerned, for the hogging-moment there does not exceed 100 foot-tons. Even at the station of absolute maximum hogging-moment ($c\,c'$ in Plate XVII. fig. 9) for this ship the strain only reaches 12,000 foot-tons, equalling the product of the displacement by about the hundred-and-seventy-sixth ($\frac{1}{176}$) part of the length. It will thus be seen that the changes made in the 'Bellerophon' from the 'Minotaur' have had the effect of rendering the maximum bending-moment about one fourth what it would have been if the long fine type had been conformed to. I may add that the concentration of weights amidships, due to the adoption of the central battery-and-belt system, has had much to do with this; while it has been shown that sagging-moments do not result from the excess of weights amidships. This fact adds one more to the numerous advantages previously shown to be possessed by this system, as compared with the system of complete protection exemplified in the 'Minotaur.' The case of the 'Audacious' (illustrated by Plate XVII. fig. 10) gives further support to this view. The hogging-moment amidships does not exceed 3400 foot-tons, and that at the section of absolute maximum strain in the after body is only 11,000 foot-tons,—that is, equals the product of the displacement by about the hundred-and-fiftieth ($\frac{1}{150}$) part of the length. With respect to the type represented by the 'Victoria and Albert,' it will be sufficient to state that the maximum hogging-moment (at $a\,a'$, Plate XVI. fig. 7) is about 5080 foot-tons, equalling the product of the displacement by the hundred-and-fortieth ($\frac{1}{140}$) part of the length of this ship. Hence it follows that in certain classes of unarmoured ships, with excess of weight amidships, moderate bending-strains may be anticipated, and that when sagging-strains exist their amount is comparatively small.

In this connexion it may also be proper to revert to the tendency which the over-weighted ends of a ship have to break off from the midship part, to which I have already alluded in general terms. The fore bodies of the 'Bellerophon' and 'Minotaur' afford excellent illustrations of the effect which changes in the form of the immersed portions of a ship have upon this tendency. At the foremost water-borne section of the 'Minotaur' (R^1 in fig. 8, Plate XVII.) the bending-moment of about 19,000 foot-tons is produced by the unsupported weight (420 tons) of the part before it; at the corresponding section of the 'Bellerophon' (R^1 in fig. 9) the bending-moment is less than 1500 foot-tons, the excess of weight producing it being only 45 tons. Some part of the difference between these bending-moments is, as I have said, due to the complete protection of the 'Minotaur;' but a considerable part is due to the difference between the length and

fineness of the entrances of the two ships, and the change from the V-form to the U-form of transverse section. At the sterns of these ships there is also a remarkable contrast between the bending-moments tending to break off the overweighted parts, although not so great as that at the bows, on account of the absolute necessity for fineness of form in the run of the 'Minotaur;' the bending-moment at the aftermost water-borne section (R^2 in Plate XVII. fig. 8) is upwards of 20,000 foot-tons, while at the corresponding station in the 'Bellerophon' it is only about 7000 foot-tons. These strains on the stern, unavoidable as they are, to some extent often develope weakness in screw steam-ships. With respect to the bending-moments experienced by the foremost and aftermost water-borne sections of the 'Victoria and Albert' and of the 'Invincible' nothing need be said; the curves of moments for these ships are constructed on the same scale as those for the other two, and the lengths of the ordinates afford the means of comparing the strains at various parts.

Before concluding my remarks on the still-water strains of ships, I must refer to another cause of bending to which brief allusion has already been made, viz. the horizontal longitudinal fluid pressure on the immersed part of a ship. The most recent writers on the subject have not taken account of this cause of bending, doubtless because they considered its effect out of all comparison with the effect produced by the vertical forces, an opinion which the greater number of earlier writers also entertained. EULER and Dr. YOUNG were exceptions, as I have shown; and the following results will prove that they were justified in attaching importance to the effect of the fluid pressure, although they did not correctly estimate it. Without criticizing their methods, however, I will proceed to indicate a simple plan for estimating the amount, and the bending-moment, of this pressure at any transverse section of a ship afloat in still water.

According to a well-established hydrostatical law, the resultant fluid pressure, in a horizontal direction, on a solid immersed in it equals the pressure of the fluid on the projection of the surface of the solid upon a plane at right angles to that direction; and this resultant acts through the centre of pressure of the plane area. Applying this principle to a ship, we see that the longitudinal pressure upon any part bounded by a transverse section equals the pressure upon the immersed area of that transverse section, and acts through the centre of pressure of the immersed area. By this means, therefore, we can determine the amount and the line of action of the resultant longitudinal pressure upon the parts of a ship, either towards the bow or towards the stern, cut off by a transverse section; and knowing these two features, we can determine the moment of the pressure about any horizontal line in the transverse section. For our present purpose it will suffice to say that the line about which moments must be taken in order to determine the bending effect produced by the pressure at the section coincides with the ecntre of gravity of the section*. Assuming that we know its position, and knowing

* It may be interesting to state here that EULER's mistake respecting the action of this longitudinal pressure arose from the fact that he considered the lower side of the keel as the "fulcrum," as it was then termed, about which moments should be taken.

the amount and the line of action of the resultant fluid pressure, the determination of the bending-moment due to it is very simple.

I will now consider the cases of some actual ships, in order to give a more definite idea of the amounts of the bending-moments due to the longitudinal pressure, and the proportions they may bear to the bending-moments resulting from the action of vertical forces. Taking first the 'Minotaur,' it has been found by calculation that the fluid pressure on the midship section amounts to 405 tons. The total depth of the ship being 41 feet, the centre of gravity of the section has been taken 20 feet above the keel, and the centre of pressure of the immersed midship section has been found to be about $9\frac{1}{3}$ feet below it; so that the bending-moment due to longitudinal pressure has in this ship a maximum value of 3780 foot-tons. The bending-moment due to vertical forces has been fixed at 45,000 foot-tons, and is therefore twelve times as great; in other words, the bending-moment due to the longitudinal pressure is only one twelfth of that due to the vertical forces. This seems a very small proportion, but it is obtained from a ship in which the overburdening at the ends and the length are both excessively great, so that the bending-moment due to vertical forces is very large. In ships of more moderate dimensions, having a less excess of weight over buoyancy at the extremities, and of buoyancy over weight amidships, the proportion of the horizontal force is much greater. Taking the case of the 'Bellerophon,' for example, the fluid pressure on her midship section is a little over 350 tons, and the distance between the centre of pressure and the centre of gravity of the section is about $8\frac{8}{10}$ feet, so that the bending-moment due to this pressure equals 3120 foot-tons. This is the maximum value occurring at the midship section, where we have seen the hogging-moment due to the vertical forces to be 100 foot-tons only, so that at that section the former bending-moment is that which virtually fixes the limit of strain. Greater interest attaches, however, to the comparison between the two maximum values of the bending-moment. The absolute maximum moment produced by the vertical forces has been found to be 12,000 foot-tons, and the moment produced by the longitudinal pressure is therefore a little more than one fourth of this amount. As compared with the 'Minotaur,' we find, then, that the bending-moment due to longitudinal pressure is very much less in actual amount for the 'Bellerophon;' but that, in proportion to the moment resulting from the unequal distribution of the weight and buoyancy, it is much larger in the shorter ship. This fact prepares us for the conclusion arrived at by Dr. YOUNG in his Report on the diagonal system,—that in the short full ships of war in use at the commencement of this century the moment due to longitudinal pressure sometimes amounted to more than one third of the maximum moment produced by vertical forces. Dr. YOUNG's method was incorrect, but his estimate of the relative magnitude of the bending-moments is probably not very far from the truth.

One other example must suffice for this branch of the subject. In the 'Audacious,' when afloat in still water, the pressure on the midship section has been found to be about 295 tons, and the distance between the centre of pressure and the centre of gravity

of the section is approximately a little more than $9\frac{1}{4}$ feet, the bending-moment due to the pressure being about 2740 foot-tons. This is, as nearly as possible, one fourth of the maximum bending-moment (11,000 foot-tons) produced by vertical forces in this ship; and in this respect the 'Audacious' resembles the 'Bellerophon,' although, being a smaller ship, the actual amount of the bending-moment due to the longitudinal pressure is less than that in the 'Bellerophon.'

On a review of these investigations, therefore, it appears that the limiting proportions of the bending-moments due to the pressure, and those due to the vertical forces, might be taken at one twelfth for very long fine ships, and one fourth for ships of moderate length and proportions, having their principal weights concentrated amidships. The cases which fall outside these limits may be considered exceptional amongst modern ships.

I have only determined the maximum values of the bending-moments due to the longitudinal pressure in the preceding examples, because they are the only values having much practical interest, and because my chief aim has been to show that it is not proper to omit all consideration of the bending effect of this pressure in all cases, as is usually done. In long fine ships, such as the 'Minotaur,' the error introduced by omitting it would be comparatively small for still-water strains; but in ships of more moderate proportions this is not the case. Were it at all desirable we might obviously determine the amount, and the moment, about the neutral axis, of the pressure on any transverse section, in a manner similar to that employed above for the midship section, and might represent the results graphically by curves of moments constructed similarly to those for the moments of vertical forces. In fact, by using the same scales of foot-tons, we might combine the curves of moments for longitudinal pressure and for vertical forces, and obtain a single curve which should represent their joint effect. This need not be done, however; for, as I have before remarked, the still-water strains experienced by a ship are not those which regulate the provision of strength; and we shall see hereafter that it is only for still water that the longitudinal has such an effect in proportion to the vertical forces as to require quantitative consideration.

The preceding considerations respecting the still-water strains resulting from the action of vertical and horizontal forces on a ship have been given at considerable length, because they furnish many extensions and corrections of existing knowledge, and also because methods similar to those here employed will hereafter be applied to the severer strains experienced by ships when at sea or when ashore. Besides this they have a special interest and importance themselves. They do not, it is true, exercise very much immediate influence on the distribution of material and the provision of strength in a ship, owing to the greater magnitude of the sea-strains; but they constitute what may be termed the *permanent* strains on the structure, and we have seen how very widely these permanent strains differ under different methods of distributing the weights and the buoyancy. In view of the illustrations chosen, there cannot fail to result a fuller appreciation of this fact than could result from any general statement; and the

importance of careful stowage must be fully realized, as well as the connexion which should subsist between the ship's form and the distribution of her weights. Some existing views have been shown to be erroneous,—notably those respecting the effect of an excess of weight amidships, and the position of the section of maximum strain. The classification I have adopted for ships is confessedly imperfect; but no general laws can possibly be laid down to include the very varied characters of the distribution of weight and buoyancy in all ships, and the types I have chosen have at least the merit of including a very large proportion of the cases met with, besides permitting a generality of investigation such as has not been previously attempted.

In attempting to approximate to the shearing- and bending-strains of *ships at sea*, we meet with a problem of great difficulty, and one which in the present state of our knowledge does not admit of complete or exact solution; in fact it may be doubted whether the very varied and rapidly changing conditions of strain in ships so situated will ever be completely expressed in mathematical language, and brought within the range of accurate calculation, in the same way as still-water strains have here been treated. It is, however, possible to distinguish the principal causes of straining in ships at sea, and in some cases to make approximations to what may be considered as their *limiting* values, as I shall show further on; but the dynamical aspect of the question, although the most important of all, is at present in some respects beyond our power, so far as its expression in *quantitative form* is concerned. That this is the case will be obvious on the most cursory glance at the condition of a ship in a sea-way. Neglecting, for the sake of simplicity, all consideration of rolling motion, and supposing the ship to lie directly bow on to the waves, the passage of each wave along her length establishes, or tends to establish, a vertical motion in the ship as a whole (except under certain special conditions), and a rotatory or pitching motion about some transverse axis, besides producing continual changes in the relative distribution of the weight and buoyancy all along the length. The ship's motion in pitching and ascending can be readily explained in general terms; but to express accurately the speed of that motion, and the corresponding accelerating forces, as well as the straining effect of the percussive shocks that are nearly certain to be caused by it, is an undertaking I shall not attempt. Even if this could be done, it would still be necessary to consider the heaving or vertical motion of the ship and the rapidly varying nature of the wave supports, both of which are causes of important straining-actions; and, in addition, to deal with the effect of the passage of a succession of waves (far from being of uniform dimensions and periods) as well as with the influence of the ship's onward motion. Altogether, therefore, we have before us a most complex question, which can only be touched, as it were, by some approximative method such as that I am about to describe.

From what has just been said, it will appear that there are three principal causes of increase in the longitudinal strains of ships at sea, as compared with their still-water strains,—the vertical or heaving motion of the ship as a whole, which is nearly sure to result from the wave motion, because ships share to some extent the motion of the waves;

the rapid changes in the positions occupied by ships relatively to the crests and hollows of the waves, by which most unequal distributions of the weight and buoyancy are, under some circumstances, produced and the straining forces are increased; and the pitching and ascending motions of ships, which further modify the strains, both by means of the accelerating forces thus developed, and by the percussive forces, shocks, and resistance of the water. These causes I propose to notice very briefly; it is to the quantitative investigation of the second, however, that I have given special attention, with detailed results, which I will presently record.

First, as to the modification of strain that may result from the heaving or vertical motion of a ship floating among waves. It is obvious that such motion must produce changes of strain; for when a vessel in moving downwards receives a check, the effect is to increase the straining-forces acting upon her; and when, in moving upwards, she reaches her highest position, and is for the moment partially abandoned by the water support, the strains upon the hull will be diminished. If the abandonment were total (that is, in the hypothetical case when the ship is left up in the air), all bending- and shearing-strains would, in fact, disappear; for then every particle in the ship would, for the moment, have impressed upon it the accelerating force of gravity, acting equally and in parallel directions throughout her.

Mr. W. Froude and Professor Rankine have both referred to this subject when dealing with the strains of ships at sea; and the latter gentleman has attempted to fix the *limiting* maximum increase of strain produced by vertical motion. This he considers to be about *one fourth* of the still-water strains (for both shearing and bending), this estimate being based upon two or three assumptions (see 'Shipbuilding, Theoretical and Practical,' pages 151, 152). The fundamental assumption made is that the ship may be considered so small in proportion to the waves as to closely accompany their motion, just as a float would do. This obviously differs from the condition likely to be fulfilled by any actual ship. A ship cannot be expected to closely accompany the wave-motion, and her heaving cannot be regarded as the result of the passage of one wave only, but of a succession of waves differing, in all probability, in sizes and forms. For these reasons I cannot adopt Professor Rankine's estimate (which doubtless has, however, a certain theoretic value) as a basis of practical calculation, nor am I prepared to substitute one of my own.

The practical deduction which should be kept in mind is the general one, that the heaving motion in ships at sea will, under some circumstances, produce increased strains. It seems probable that these strains are not so severe, in most cases, as those which result from variations in the wave supports and from pitching motions; but it must not be forgotten that all three causes may be operating simultaneously, and that their combined effect measures the actual strain on the structure.

Next, as to the additional strains resulting from the changing wave supports on a ship at sea. In dealing with this question I shall consider the two extreme positions of support illustrated by figs. 14 & 15. In the first of these a ship is supported on a single

Fig. 14.

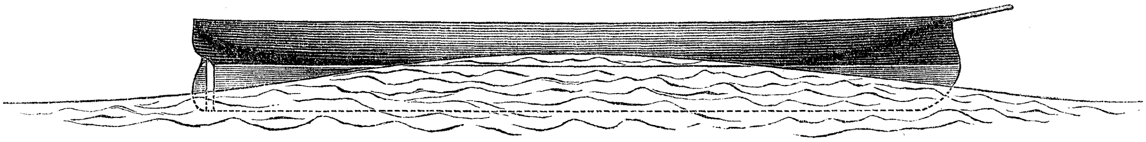
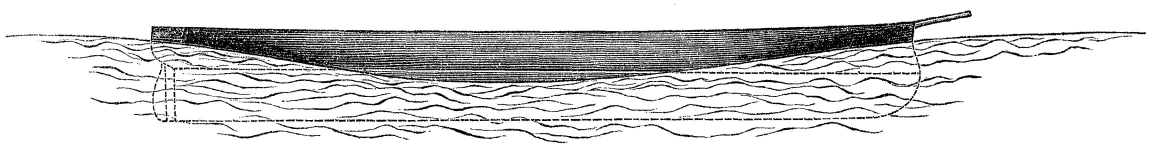


Fig. 15.



wave-crest, at or about the middle of her length; in the second she is situated in a wave-hollow, and has her extremities immersed in the adjacent wave-slopes. The first position obviously tends to develop hogging-strains of considerable amount, and the second to develop sagging-strains. It becomes necessary, therefore, to attempt some approximation to the amounts of these strains in various classes of ships, in order to compare them with the permanent still-water strains previously calculated; and for this purpose two or three assumptions must be made:—

(1) That for the moment the effect of the ship's vertical motion may be neglected.

(2) That for the moment the ship may be regarded as occupying a position of hydrostatical equilibrium.

(3) That the methods of calculating bending- and shearing-strains previously used for still water may be employed here also in order to approximate to the momentary strains.

Investigations based on these assumptions, although confessedly imperfect, will give us a better idea of the limits between which the strains produced by changing wave supports lie than we could otherwise obtain.

I have referred to the positions of support illustrated by figs. 14 & 15 as *extreme*, and it may be proper to state briefly my reasons for doing so. I will take the case of fig. 15, and suppose the ship to be moved forward some distance so that her bow comes nearer to the wave-crests, and her stern moves further down into the wave-hollow. The effect of this change will obviously be that a greater length of the vessel will be supported on one wave-slope than on the other, and that the ship as a whole will rest upon a *flatter* portion of the wave-form than it did before, as the curvature is much greater in the central part of the hollow than it is further up the slope. Hence, when the vessel has taken up her new position of equilibrium, it seems certain that the water-level will not sink so much below the height amidships corresponding to still water as it does in fig. 15, nor rise so much above that height at the bow and stern; consequently the bending-strains will be less. Similar considerations render it evident that the position shown in fig. 14 is the other extreme.

Without further preface I shall proceed to consider the strains brought upon our three

typical ships, the 'Minotaur,' 'Victoria and Albert,' and 'Bellerophon,' when supported on waves under the above-stated conditions. In still water the 'Minotaur,' as we have seen, has excesses of weight at the extremities only, and is subject to hogging-strains throughout her length; while the 'Victoria and Albert' has a large excess of weight amidships as well as at the extremities, and is subjected to sagging-strains in her middle body; and the 'Bellerophon,' although resembling somewhat the 'Victoria and Albert' in the distribution of the weight and buoyancy, has a maximum hogging-moment amidships instead of a sagging-moment.

When the 'Minotaur' floats on a wave-crest, it is clear she must be subjected to greater hogging-strains than are experienced by her in still water, because the excesses of buoyancy midships, and of weight forward and aft, will then be greater. On the other hand, when she floats in a wave-hollow, with her bow and stern deeply immersed in the wave-slopes, we naturally look for excesses of buoyancy at the extremities, and of weight amidships, because there the water-level has been lowered; hence it is reasonable to assume that in a wave-hollow such a ship will be subjected to severe sagging-moments *throughout her length*—a state of things that I previously showed could not exist in any ship, with excess of weight at the extremities, when floating in still water. These general considerations are confirmed by a closer examination and by actual calculation. Plate XVII. fig. 16 exhibits the relation between the weight and buoyancy of the 'Minotaur' when she is balanced on waves 600 feet long and 30 feet high, F F representing the buoyancy on the wave-crest (as in fig. 14), subject to the three conditions just laid down, and G G representing the buoyancy in the wave-hollow. The diagrams of the curves of loads, shearing-forces, and bending-moments are respectively shown for the wave-crest and wave-hollow in Plate XVIII. figs. 17 & 18; and the same scales have been employed in constructing them as were used for the corresponding curves representing still-water strains. It has been found by calculation that the excesses of weight at the bow and stern on the wave-crest become as nearly as possible *double* of those existing in still water, and that the shearing-strains become increased in about the same proportion, having a maximum value of 925 tons instead of 450 tons; while the maximum bending-moment amounts to more than 105,000 foot-tons instead of 45,000 foot-tons as in still water. The maximum shearing-force on the wave-crest is experienced by the station about 90 feet from the stern, and the maximum hogging-moment by a station not far from the middle of the length. In this case, therefore, the *character* of the strains remains practically unchanged as compared with those in still water, but their *intensity* is, roughly speaking, doubled. When the ship floats in the hollow of waves of the same dimensions (as in fig. 15), a very different state of things is met with, and one which presents some special points of interest. The deeper immersion of the stern in the wave-slope has the effect (figs. 16 & 18) of producing an excess of buoyancy of 410 tons on the first 110 feet of length, and the lowering of the water-level in the hollow causes a *defect* of buoyancy of no less than 880 tons in the middle portion of the ship, where in still water the *excess* of buoyancy is nearly as great. At the bow, however, even more striking results

are met with. Its transverse sections are so fine, and its weight is so great, that the increased immersion does not suffice to entirely do away with the excess of weight existing in still water; in fact on the first 20 feet of length there is a small excess of weight (about 50 tons) still remaining; and we thus meet for the first time with a case where the conditions of the two extremities of a ship differ, an excess of buoyancy existing at one end simultaneously with an excess of weight at the other. In actual ships at sea this must often be so, since the pitching and ascending motions are sure to produce a deep immersion at one extremity simultaneously with the emersion of the other extremity. Reverting to the 'Minotaur' in the wave-hollow, I need only state that the maximum shearing-force is found to be almost the same as that for still water (470 instead of 450 tons), and that the maximum bending-moment is also of nearly identical amount with that for still water (44,000 instead of 45,000 foot-tons), only it is a sagging-strain instead of a hogging-strain. In fact for five-sixths of her length the 'Minotaur' is subjected to sagging-strains when in the wave-hollow; and as the small excess of weight at her bow can only be expected to exist in armoured ships of her extreme length and fineness, under such circumstances we are quite warranted in assuming that, as a rule, ships of this type will sag *throughout the whole length* when floating in wave-hollows. In such ships also, so far as this example enables us to judge, the sagging-strain in a wave-hollow is likely to fall below one half of the hogging-strain on a wave-crest.

Within certain limits the strains of a ship of the 'Minotaur' type will be increased when the lengths of the waves become decreased and their steepness increased. For instance, I have previously supposed the 'Minotaur' to be balanced on waves 600 feet long, while her own length is only 400 feet; but if the waves were decreased in length, the strains would usually be increased so long as the decrease is not sufficient to cause the extremities of the ship to be immersed in the slopes of adjacent waves. The latter consideration roughly fixes the limit of decrease in the length of the wave by the condition that the wave and the ship shall be of equal lengths; and consequently to find the limiting values of the bending- and shearing-strains corresponding to the extreme positions of support, we will suppose our typical ships to be balanced on waves of their own length, and of such steepness as is likely to be met with in ocean-waves. The results of calculations made on these bases, and with the foregoing assumptions, I shall now briefly describe, as well as those for the corresponding wave-hollows.

First I will take the case of the 'Minotaur' on a wave 400 feet long and 25 feet high, instead of 600 feet long and 30 feet high. The results are graphically recorded in Plate XIX. figs. 19 & 20, and Plate XX. fig. 21. On the crest of such a wave the excesses of weight at the bow and stern respectively are found to be no less than 1275 tons and 1365 tons. The maximum shearing-force is 1365 tons, and the maximum hogging-moment is 140,300 foot-tons. By changing the dimensions of the wave, therefore, the maximum shearing-force has been increased by 435 tons, about one half, and the maximum hogging-moment by 35,000 foot-tons. The latter now equals $3\frac{1}{9}$ times the

maximum hogging-moment in still water. In the wave-hollow the deep immersion of the extremities and the less immersion of the middle leads to the following distribution of the weight and buoyancy. For the first 120 feet of the stern an excess of buoyancy of 695 tons, for the next 160 feet amidships a defect of buoyancy of 1380 tons, for the next 110 feet an excess of buoyancy of 695 tons, and right forward a very small excess of weight (10 tons on about 10 feet). The remarks previously made respecting the similar distribution on the 600-feet wave apply to this case also, and need not be repeated. Our chief interest centres in the determination of the maximum shearing-force and sagging-moment. The former amounts to 695 tons, and the latter to 74,800 foot-tons. As in the previous case, the section of maximum bending-moment falls nearly amidships, and the figures show that the sagging-strain (fig. 21) is a little more than one half the hogging-strain incidental to support on the wave-crest (fig. 20), while it is considerably greater than the still-water strain. These comparative results may be summarized approximately as follows:—

'Minotaur' type. Strains under various conditions.

	Still Water.	On Wave-crest.	In Wave-hollow.
Maximum shearing-force ÷ displacement.	$\frac{1}{22}$	$\frac{1}{7}$	$\frac{1}{14}$
Maximum bending-moment ÷ displacement × length.	$\frac{1}{88}$ (hogging)	$\frac{1}{28}$ (hogging)	$\frac{1}{53}$ (sagging)

By means of these proportions it is possible to approximate to the amounts of the bending-moments and shearing-forces in other ships of the type when their displacements and lengths are known; only it is necessary to remember that in many cases the results obtained must be regarded as strictly limiting values, on account of the fact that in the 'Minotaur' the excesses of weight at the extremities for still water and the fineness of form are both extremely great.

Passing on to the second typical ship, the 'Victoria and Albert,' we will suppose her to be balanced on the crest of a wave of her own length (300 feet) and 20 feet high from hollow to crest. This height has been taken because it bears nearly the same proportion to the length as that of the 400-feet wave on which we supposed the 'Minotaur' to be balanced, and fairness of comparison between the conditions of strain in the two types is thus ensured. Under these circumstances careful calculations, of which the results are recorded in Plate XIX. fig. 22, Plate XX. fig. 23, and Plate XXI. fig. 24, show that the additional immersion of the middle body does away altogether with the excess of weight existing there in still water, while the decreased immersion of the ends leads, of course, to an increase in their excesses of weight. In short, on the wave-crest the condition of this ship becomes similar to that of the 'Minotaur' in still water. At the bow the excess of weight amounts to 220 tons, and at the stern to 185 tons, while

between these parts, on a length of about 160 feet amidships, the buoyancy is 405 tons in excess. The resulting maximum strains are as follows:—Shearing-force 220 tons, bending-moment (hogging) 16,400 foot-tons. In still water the maximum shearing-force is 140 tons, and the maximum hogging-moment about 5080 foot-tons, this moment being experienced by a section considerably before the middle of the length, whereas on the wave-crest the section of maximum moment is comparatively near the middle.

This leads me to mention a point of some importance in the regulation of the longitudinal strength of ships. The ordinary assumption is that for both still-water strains and the strains of ships at sea, the midship section is that which has to bear the maximum moment, and that in moving from that section out towards the extremities we find the bending-moments continually decreasing in amount. I have already shown that this view is not correct so far as still-water strains are concerned, and that the position of the section of maximum moment may lie far away from the middle of the length—as it actually does in the 'Victoria and Albert' and in the 'Bellerophon.' The question arises, therefore, is the section of maximum moment generally near the middle when a ship is supported on a wave-crest or in a wave-hollow? or may that section be ordinarily expected to lie away from the middle in ships not of the 'Minotaur' type? So far as these investigations go, the answer to the question is, that the midship section, or some section comparatively near it, has to sustain the greatest bending-moment in all three types and in both the extreme positions of support. Hence it follows that the popular view, although incorrect in some respects, leads on the whole to conclusions that very properly influence practice; and general experience confirms the soundness of the opinion that, after keeping up uniformity of strength throughout a considerable length of the middle body, it is advantageous, from this point of view, to reduce the ship's scantlings as we proceed towards the extremities. What laws should regulate the reductions is a question upon which I shall not now enter.

When the 'Victoria and Albert' floats in the hollow of waves of the dimensions just stated, her condition of strain is, of course, entirely different from that in still water and from that on the wave-crest. The strains brought upon her are very remarkable. The excess of weight amidships (210 tons) existing in still water becomes exaggerated to 785 tons, and instead of having excesses of weight at the extremities we have considerable excesses of buoyancy, no less than 390 tons forward and 395 tons aft. The result of these changes is to produce sagging-moments throughout the ship's length, gradually increasing in amount as we proceed from the extremities towards the middle, and reaching their maximum at a station near the midship section. The maximum moment under these circumstances amounts to 31,000 foot-tons, and the maximum shearing-force to 395 tons.

From these figures we obtain the following approximate summary:—

'Victoria and Albert' type. Strains under various conditions.

	Still Water.	On Wave-crest.	In Wave-hollow.
Maximum shearing-force ÷ displacement.	$\frac{1}{16}$	$\frac{1}{11}$	$\frac{1}{6}$
Maximum bending-moment ÷ displacement × length.	$\frac{1}{139}$ (hogging)	$\frac{1}{43}$ (hogging)	$\frac{1}{23}$ (sagging)

The variation in the amount and character of the bending-moments is the most interesting feature in this summary. On the wave-crest the maximum hogging-moment is somewhat, more than three times as great as the maximum hogging-moment, for still water; and in this respect there is a clear resemblance to the similar proportions for the 'Minotaur,' although, in proportion to the products of the lengths and displacements, the 'Victoria and Albert' is much less severely strained than the 'Minotaur.' For the wave-hollow, however, the case is widely different. In the 'Minotaur' the bending- (sagging-) moment there has a maximum value less than *twice* as great as the still-water hogging-moment; but in the 'Victoria and Albert' the sagging-moment is about *six times* as great as the still-water hogging-moment. The 'Minotaur,' therefore, experiences her absolute maximum bending-moment when suspended on a wave-crest, while that for the 'Victoria and Albert' is incidental to flotation in a wave-hollow. This result would naturally be anticipated, when the conditions of strain previously shown to exist when the ships are floating in still water are taken in connexion with the distribution of the buoyancy in the extreme positions of support among waves. Hence follows the practical deduction that ships of the 'Minotaur' type are likely to be subjected to hogging-strains severer in their character than any sagging-strains that will be brought upon them; while vessels of the 'Victoria and Albert' type are likely to be most strained by sagging. Although, as I have said previously, the latter type only includes, at present, ships intended for special services, such as yachts, blockade-runners, &c., its investigation leads to a valuable extension of the ordinary theory respecting bending-strains, and appears to have been overlooked even by the most recent writers. For instance, Professor Rankine (at page 153 of 'Shipbuilding, Theoretical and Practical') says of the sagging-moment incidental to support in a wave-hollow, that "in all cases of ordinary occurrence in practice, the sagging-moment thus produced is less severe than the hogging-moment produced when the ship is balanced on the crest of a wave." This statement is undoubtedly true of many ships; but it would apparently lead to very erroneous results if it were supposed to hold universally, and I have therefore drawn attention to it. The method which in my opinion ought to be followed in approximating to the probable strains that a ship will experience, is first to determine, from her form and stowage, to what type she is likely to belong, and then to consider whether or not it is necessary to take the sagging- as well as the hogging-strains into account in calculating the strength required in her structure.

The third typical ship, the 'Bellerophon,' next claims attention; and I shall suppose her to occupy the two extreme positions of support amongst waves of the same dimensions as those taken in the 'Victoria and Albert,' 300 feet long and 20 feet high. The results of the calculations are graphically recorded in Plate XX. fig. 25 and Plate XXI. figs. 26 & 27. It has been previously shown that the still-water strains of the 'Bellerophon' bear much resemblance to those of the 'Victoria and Albert,' and the same thing is true of their strains on the wave-crests and in the wave-hollows. The fact that the excess of weight amidships in still water is proportionately less in the 'Bellerophon' than it is in the 'Victoria and Albert' prepares us, however, for the result which calculation develops, viz. that the hogging-moment of the 'Bellerophon' on the wave-crest bears a larger proportion to the sagging-moment in the wave-hollow than the corresponding hogging-moment of the 'Victoria and Albert' bears to the corresponding sagging-moment. On the wave-crest the excess of buoyancy amidships in the 'Bellerophon' amounts to 1000 tons (a striking change from the 250 tons *defect* existing in still water), while the excesses of weight at the bow and stern amount to 445 and 555 tons respectively; the resulting strains,—maximum shearing-force 555 tons, maximum hogging-moment 43,600 foot-tons. The latter is about $3\frac{2}{3}$ times the maximum hogging-moment for still water; and it is experienced by a section very near the middle of the length, although the section of maximum still-water strain is considerably abaft the midship section. In the wave-hollow the conditions of strain are exactly reversed. We then have an excess of weight of 1240 tons amidships, and excesses of buoyancy amounting to no less than 640 and 600 tons at the bow and stern respectively; the maximum shearing-force becomes 640 tons, and the maximum sagging-moment is 48,800 foot-tons, about four times the maximum hogging-moment in still water, and about 5200 foot-tons greater than the maximum hogging-moment on the wave-crest. Put into the form of a summary, these results stand as follows:—

'Bellerophon' type. Strains under various conditions.

	Still Water.	On Wave-crest.	In Wave-hollow.
Maximum shearing-force ÷ displacement.	$\frac{1}{33}$	$\frac{1}{13}$	$\frac{1}{11}$
Maximum bending-moment ÷ displacement × length.	$\frac{1}{176}$ (hogging)	$\frac{1}{48}$ (hogging)	$\frac{1}{43}$ (sagging)

Here, then, we have another case in which the maximum sagging-strain exceeds (although not largely) the maximum hogging-strain, and which confirms the opinion previously expressed respecting the impropriety of considering hogging as the only strain which need be considered when arranging the details of a ship's structure.

Some interest also attaches to a comparison between the strains of the two typical ironclads 'Minotaur' and 'Bellerophon,' viewed merely as types of form and distribution

of weight. In still water the bending-strain (in proportion to the products of the displacements and the lengths) is about one half as severe in the 'Bellerophon' as it is in the 'Minotaur,' on a wave-crest it is less than *two-thirds* as severe, and in a wave-hollow it is proportionately about one and one-fourth as great in the 'Bellerophon' type as it is in the 'Minotaur.' Another point of contrast is found in the fact that the maximum hogging- and sagging-strains are practically about equal in the 'Bellerophon' type, whereas in the 'Minotaur' the hogging-strain is very nearly double the sagging-strain. The longer ship must have, therefore, a very great reserve of strength against sagging if she is strong enough to resist hogging; and this fact prevents the material from being so well disposed as it might be if there were not so great a difference between the two extremes of strain. This follows from the consideration that an iron-built ship, such as the 'Minotaur,' is comparatively strong against sagging, because (if the longitudinal strength of the keel and keelson-work is kept up, and the bottom plates are properly butt-strapped) there is sure to be a reserve of tensile strength in the bottom, and because under compressive strains the wood upper deck becomes very effective; whereas against hogging the upper part of the ship is usually much weaker, both absolutely and as compared with the lower part. In the case of an armoured ship like the 'Minotaur,' the power to resist sagging is also added to very greatly by the resistance offered to compression by the upper strakes of armour, which contribute besides, in a considerable, although a much less degree, to her power to resist hogging.

The preceding comparison, as I have said, is limited to the consideration of the 'Minotaur' and 'Bellerophon' as *types*; but there remains to be considered the important question of the actual strains experienced by these two vessels when in their extreme positions of support among waves. These are as follows:—

On Wave-crest.

	Minotaur.	Bellerophon.
Excess of weight forward	1,275 tons.	445 tons.
Excess of weight aft	1,365 „	555 „
Excess of buoyancy amidships ...	2,640 „	1,000 „
Maximum shearing-strain	1,365 „	555 „
Maximum bending-moment	140,300 foot-tons.	43,600 foot-tons.

In Wave-hollow.

	Minotaur.	Bellerophon.
Excess of buoyancy forward	685 tons.	640 tons.
Excess of buoyancy aft	695 „	600 „
Excess of weight amidships	1,380 „	1,240 „
Maximum shearing-strain	695 „	640 „
Maximum bending-moment	74,800 foot-tons.	48,800 foot-tons.

The only matter to which attention requires to be specially drawn is the comparative amounts of the bending-strains, because against shearing-strains even the 'Minotaur' may be expected to have a large reserve of strength. On the wave-crest, then, we find the 'Minotaur's' bending-moment more than three times as great as the 'Bellerophon's;' and in the wave-hollow it is about half as much again as the 'Bellerophon's.' But we must not stop here. In the 'Bellerophon' the material at the midship section available against bending-strains is, to say the least, equal in efficiency to that of the 'Minotaur;' and consequently I shall be within the truth in saying that, in proportion to the strains previously calculated, the 'Bellerophon' is about three times as strong as the 'Minotaur' is against hogging, and about once and a half as strong against sagging.

From the preceding investigations it appears that the *statical* strains resulting from extreme positions of support of a ship floating among waves may, under the assumed conditions, reach amounts varying between *three* and *six* times the still-water bending-strains. (These results, of course, do not in any way represent the dynamical strains due to pitching and heaving motions.) This being so, it appears that some idea of the relative strengths of ships may be obtained by using the approximate values of the maximum bending-moments found above for the different types under different circumstances. Experience alone can enable us to judge whether the *absolute* strength of a ship is likely to prove sufficient against both dynamical and statical strains; and in order to do so we should have to take some ship which had answered and compare her construction with the proposed design. At the same time the investigations to which I have just drawn attention serve to indicate two features in which the strains of ships at sea differ from still-water strains,—the first, and most obvious, being their much greater severity, and the second their great and rapid variation in both intensity and character. On the latter feature a few additional remarks may be made.

Between the two extreme positions of support we have considered, others may be imagined, and undoubtedly occur, in which a ship is so circumstanced as to be subject to hogging-strains at some portions of her length and to sagging-strains at other portions. For instance, when the crest of a wave is intermediate between the bow and the

middle of a ship's length, and the after body is lying across the hollow, we should expect to find hogging-strains forward and sagging-strains aft; and as the wave-crest moved aft relatively to the ship, the character and intensity of the bending-strains must be continually varying in passing from one extreme to the other. Of the extent to which this variation reaches we already have some idea; but we must also introduce the idea of its *rapidity* before we can realize any thing like its full effect.

In order to make my remarks as definite as possible, I will again refer to the three typical ships on waves of the dimensions previously assigned. The time of transit of waves 400 feet long is rather less than nine seconds (really 8·83 seconds), and in half that time, or $4\frac{1}{2}$ seconds, we may suppose the 'Minotaur' to have passed from the wave-crest to the wave-hollow; while her maximum bending-strain has changed from 140,000 foot-tons of hogging- to 74,000 foot-tons of sagging-moment; and every transverse section has been subjected to similar changes in the character and amount of the strains brought upon it. The great rapidity in the variation of the strain is so apparent from this brief statement of facts that I need not dwell upon it.

The 'Victoria and Albert' and the 'Bellerophon' have been supposed to float on waves 300 feet long, for which the whole time of transit is less than 8 seconds (really 7·65 seconds). In half that time, therefore, or in less than 4 seconds, the ship may have passed from crest to hollow, and the bending-strains have passed through successive phases from one extreme to the other. On the crest the maximum hogging-moment of the 'Victoria and Albert' has been found to be about 16,400 foot-tons; in less than 4 seconds, in the wave-hollow, the maximum sagging-moment may have reached 31,000 foot-tons, at the same section of the ship which previously had to resist the hogging-strain. This is, proportionately, the greatest change which we have met with. In the 'Bellerophon' the change in the same brief interval, although not so great in proportion, is very striking; for on the crest the hogging-moment amidships is 43,600 foot-tons, and in the wave-hollow the sagging-moment is 48,800 foot-tons.

I give these figures merely as indications of what may be expected to happen in the changes of strain in ships at sea; and they probably fall much below the truth, since, as I have just said, no account has been taken of the effect of violent pitching-motions, which must lead to still more abrupt and violent changes. Enough has been said, however, to show how important this feature of the subject is; and I will simply add that a very convenient way of expressing the effect I have been attempting to describe is afforded by the supposition that the ship is fixed, and that what may be termed "waves of strain" roll through her structure. The introduction of this idea will help us to understand more clearly how *changes* in strain affect a structure; for a very small strain (considered statically), which would not affect a comparatively weak structure sensibly if it were constantly acting in one direction, will suffice to destroy a far stronger structure if its direction is continually and rapidly changed.

This subject has not escaped the attention of preceding writers; and Mr. FAIRBAIRN has made some interesting remarks upon it, at page 13 of his work on 'Iron Ship-

building,' where he refers to the results of a series of experiments on the endurance of iron-jointed beams when subjected to changes in the loads put upon them. He says "the joints of an iron-rivetted beam sustained upwards of three million changes of *one fourth* the weight that would break it, without any apparent injury to its ultimate powers of resistance. It broke, however, with 313,000 additional changes when loaded to *one third* the breaking-weight, evidently showing that the construction is not safe when tested with alternate changes of a load equivalent to *one third* the weight that would break it." In the case of ships, however, Mr. FAIRBAIRN thinks the strain brought upon the material should not exceed one fourth or one fifth its ultimate strength, on account of the fact that the changes of strain are not merely effective as regards its *amount*, but also as regards its *direction*. His final conclusion is that in iron ships "it seems highly probable that a strain of 5 tons per square inch on the material acting alternately in opposite directions would at least injure, if it did not ultimately fracture, the material after a great number of alterations." Professor RANKINE, I may add, also considers that the strain on the material in iron ships should not exceed one fifth of the ultimate strength, and thus provides for the changes of strain of which I have been speaking.

It has sometimes been taken for granted, by writers on this subject, that while the straining-actions in smooth water become greater as the figure of the vessel becomes finer and sharper, the additional straining-actions produced by waves become less, and that these two opposite changes in a rough way compensate for each other. This conclusion is based, however, upon hypothetical cases, and not upon actual ships. On turning to our typical ships we find that it is by no means always correct. Take, for instance, the cases of the 'Minotaur' and 'Bellerophon,' one a long fine ship, and the other a comparatively short ship with fuller water-lines. In still water we have seen that the maximum hogging-moment in the 'Minotaur' is about $\frac{1}{88}$ of the product of the displacement by the length, while the corresponding moment in the 'Bellerophon' is about $\frac{1}{176}$ of the corresponding product—that is to say, the hogging-moment in the short ship is about one half as great in proportion as that in the long ship. This is, so far, quite in accordance with the view that the bending-actions in smooth water become greater as the figure of the vessel becomes finer and sharper; but when we pass to the case of the wave-crests, we find that the long fine ship is still much more severely strained than the short one. The maximum hogging-moment in the 'Minotaur' then equals $\frac{1}{28}$ of the product of the displacement by the length; that in the 'Bellerophon' is less than $\frac{1}{48}$ of the corresponding product—that is to say, is about $\frac{7}{12}$ as great, in proportion, as the 'Minotaur's' maximum strain. We see, then, that in no sense is there such a compensation for the additional strains due to wave-support as has been supposed. In the two ships I have here taken the distributions of the weight are, of course, very different, as well as the forms of their immersed bodies, and this helps to make the strains less severe in the shorter ships; but even after allowing for this it appears that the adoption of finer lines does not produce the mitigating effect in waves which has been supposed.

I must next refer briefly to the subject of the bending-strains resulting from the

horizontal fluid pressure in a ship at sea. We have already seen that this pressure has an effect too considerable to be neglected in certain classes of ships floating in still water; and that in a ship like the 'Bellerophon' the effect might rise to one fourth of the bending-moment due to vertical forces. I have also stated that in ships at sea the conditions of strain are very different, and that no great error would be introduced by neglecting the effect of the horizontal fluid pressure, and considering only the vertical forces. This I shall now proceed to show, availing myself once more of the two extreme positions of support illustrated by figs. 14 & 15 (p. 446).

First, let us consider the case of a ship on a wave-crest. We have already seen that the vertical forces incidental to that position had the effect of making the maximum bending-moment from three to four times as great as it was in still water; but it will be evident that the effect of the horizontal fluid pressure does not increase in that ratio, if it increases at all. A glance at fig. 14 shows that the depression of the water-level at the bow and stern must have the effect of uncovering, so to speak, a large portion of the midship section at the middle, and so greatly reducing the pressure on that midship section below what it would be if the water-level at that section were preserved throughout the length*. In addition to this, at a wave-crest the tension and pressure of the fluid are reduced by means of the vertical motion; so that both these causes, acting conjointly, produce such a reduction of the bending effect of the horizontal pressure as to render it, in all probability, no greater than, even if so great as, that of the corresponding pressure in still water. Consequently in proportion to the bending-moment due to the vertical forces, that produced by the horizontal pressure may be safely neglected for this position.

The same thing is true for the wave-hollow, only for a different reason. In that position, as shown by fig. 15 (p. 446), the water-level at the extremities of the ship is higher, and that in the middle lower, than in still water. Hence it is obvious that, for some portion of her depth lying *above* the centre of gravity of the midship section, the ship will be subjected to compressive strains, as well as for that portion lying *below* the centre of gravity. The effect of this will evidently be to reduce the bending-moment of the horizontal pressure; and this reduction will in all probability more than counterbalance the effect of the increased tension of the water in the wave-hollow. On this account we may assume that the effect of the horizontal pressure may be neglected in comparison with that of the vertical forces, the latter being, as we have seen, from three to six times as great in the wave-hollow as it is in still water.

These general considerations are confirmed by roughly approximate calculations made for the 'Bellerophon' when on the crest and in the hollow of waves of her own length. From what has just been said, it will appear that in making such calculations we shall obtain results considerably exceeding the true ones if we assume that the pressure is always due to the maximum height of the water-level on the ship; so that if these

* It is true that an additional area of midship section is covered at the sides by the rise of the wave-crest, but, as the next sentence mentions, the tension and pressure of this wave-crest are small.

results may be neglected in comparison with the effect of the vertical forces, much more may the true bending effect of the fluid pressure be neglected. For the wave-crest I find that the water-level at the midship section is about $7\frac{1}{2}$ feet above that in still water; and that if the level were uniform throughout the ship's length (which it by no means is) this immersion would give a pressure of about 620 tons, the bending effect of which would be about 3700 foot-tons. This is certainly much greater than the true bending effect, yet it only reaches about *one twelfth* of the bending-moment due to vertical forces on the wave-crest. In the wave-hollow similar results are obtained. The water-level at the bow and stern then rises about 11 feet above that in still water; and supposing this level to be uniform throughout the length, the pressure on the midship section would be about 770 tons, producing a bending-moment of about 3450 foot-tons—that is to say, about one fourteenth of the bending-moment due to vertical forces in the wave-hollow. The true bending-moment of the pressure would be less than this, and might consequently be neglected. Hence we are confirmed in the opinion that for ships at sea it is only necessary to consider vertical forces.

I must now pass on to notice briefly the longitudinal strains of ships in exceptional positions—such as those produced by launching, grounding, and other causes. Although exceptional, these strains undoubtedly occur; and their effect may be, as I shall show, more severe than that of any of the statical strains to which approximations have been made. Preceding writers have recognized this fact to some extent, but no author has done so much as Mr. FAIRBAIRN towards giving quantitative expressions to these exceptional strains.

BOUGUER, ROMME, and other early writers had a very clear conception of the principal causes of launching-strains. They call attention to the fact that in all, or nearly all, launches the ship's bow or stern is water-borne before the other end has left the launching-ways, and that this cannot be prevented altogether even when the ways are extended further out into the water than is customary. Some of them go so far as to urge the policy of building ships in dock in order to avoid these strains, and support their opinion by statements of the large amount of "breakage" that takes place when a ship is set afloat for the first time from a slip-way. Others combat this opinion, and show by actual examples that ships built in dock also break when they are floated, although the breakage is not always so considerable as when ships are launched; so that the severity of the strains due to launching cannot be estimated simply by the amount of breakage. The latter opinion is undoubtedly the correct one; and the breakage recorded for wood ships when launched may be regarded as due in part to what is termed the ship's "settling" in her new position afloat—that is, to her reaching such a condition as to make her powers of resistance balance the bending-moments due to the unequal distribution of the weight and buoyancy. In part, however, the breakage is undoubtedly due to the dynamical strains connected with launching; and although we cannot separate the effects produced by these two causes, we may very properly regard the amount of the breakage as to some extent a measure of the relation between the strength of the structure and

the strains brought upon it. I will pass over the facts respecting the breakage in the older classes of wood ships, and confine attention to modern wood ships. Speaking of merchant vessels, Mr. Grantham says*, "It is the general custom with builders to leave the gangways of the bulwarks in modern ships unfinished, lest the hull should so much alter in form by settling in launching that the rails would not again fit their places; and no builder would willingly copper a vessel when new, but rather allow her first to find her own position in the water, as she would then be less liable to wrinkle the sheets." In wood-built ships of war there is also a considerable amount of breakage, as the following facts will show. Our finest screw line-of-battle ships, of which the length was about 260 feet, broke, on the average, about 2 inches, some ships proving weaker than others, and the breakage in one case (that of the 'Gibraltar') amounting to 4 inches on a length of 200 feet. Our finest screw frigates, which are 300 feet long, broke from 3 to 4 inches; the 'Galatea,' 280 feet long, broke 3 inches. Shorter ships, of course, usually broke less than these long fine ships. Iron ships, I need hardly say, display very little change of form or breakage when launched, the character of the materials and fastenings used in their construction being so much less yielding than those employed in wood ships, so that they resist more successfully strains of equal intensity.

The severest strains connected with the launching of ships are, however, those which occasionally result from partial launches. The well-known case of the early iron ship 'Prince of Wales' illustrates this statement; as, owing to an accident to the launching-gear, she was left for some time with her bow resting on the edge of a wharf and her stern supported by the water—in fact suspended by the extremities; but although so severely strained no breakage took place. Another case in point is found in the wood line-of-battle ship 'Cæsar,' which stopped on the launching-ways at Pembroke Yard in 1853, and remained for seventeen days with about 64 feet of the stern unsupported by ground-ways. The result was that the stern drooped about 2 feet in a length of 90 feet.

A similar but more recent case of stoppage in launching is that of the iron-clad frigate 'Northumberland,' which in March 1866 stopped with about 52 feet of the after part unsupported, and remained in that position for thirty days. The weight of the unsupported part may be roughly estimated at 440 tons, and the moment of this weight about the aftermost point of support at 11,700 foot-tons. At the corresponding station in the 'Minotaur,' when afloat in still water, the bending-moment is about 9200 foot-tons, and when supported on the crest of a wave of her own length, the bending-moment is about 12,000 foot-tons. Now the 'Northumberland,' although a sister ship to the 'Minotaur,' has had her disposition of armour altered to the central-battery-and-belt system, and by this means rather more than 100 tons weight of armour and backing have been removed on the 52 feet of length from the stern forwards. The effect of this may be fairly assumed to be a reduction of the still-water bending-moments at the station in question to about 7000 foot-tons, and of the bending-moments on the wave to 10,000 foot-tons; and hence it follows that the strains resulting from the stoppage in launching in her

* At page 92 of his work on 'Iron Shipbuilding.'

case were slightly severer than even the strains corresponding to the exceptional position afloat which we have considered. It is proper to add, however, that they were purely statical strains and acted only in one direction, whereas the strains of waves are, as we have seen, constantly changing in character and intensity, and are therefore very much more trying to the structure. The result of careful observations showed, however, that there was scarcely any change of form in this iron-built ship, the maximum amount of breakage being $\frac{7}{16}$ inch only in a length of 342 feet, and this becoming reduced to $\frac{5}{16}$ inch when the ship was floated.

The very various positions which ships occupy when they ground may all be supposed to lie between the position where the only support is found at the middle of the length, and that where there are supports only at the extremities. Mr. FAIRBAIRN has chosen these extreme positions as those by which the provision of longitudinal strength in a ship should be regulated; and while I cannot entirely agree with this choice, on account of the fact that such positions are never occupied except by a few ships, and by them only in consequence of accidents, I am prepared to admit that there are cases on record which show that such positions may be occupied. Ships have, for example, grounded on rocky bottoms and on causeways, and have been left by the tide with their ends unsupported; and others have grounded in such a manner as to be supported at the extremities only. Under these circumstances it may be well, therefore, to attempt an approximation to the limiting values of the strains incidental to the extreme positions of support ashore, and to compare them with the statical strains which have been calculated for ships at sea.

The most severe strains to which a ship aground can be subjected are those incidental to support at the bow and stern only. In this position the heavy-weighted amidship portion, of course, tends to make the ship sag; and as it is a very simple mechanical problem to determine the amounts of the upward pressures at the points of support, as well as the weights of the various portions of the ship between the points of support, it is possible to calculate the shearing-forces and the bending-moments at various stations by a method similar to that previously used for ships afloat. The graphical method of representing shearing-forces and bending-moments might also be applied to this case were it considered necessary; but this has not been done, on account of the fact that we are principally interested in determining an approximate value for the maximum bending-strain, and therefore care but little about the other values, which have no practical importance. Before giving quantitative examples, taken from actual ships, of the maximum sagging-strains that may occur in this position, it will be only proper to call attention also to the obviously great increase in the maximum shearing-forces which a ship then experiences. Roughly speaking, we may say that one half a ship's weight is taken at the bow, and the other half at the stern when the middle is unsupported; so that near the points of support the shearing-force is approximately one half the ship's weight. In preceding investigations it has been shown that for still water the shearing-force has an approximate maximum value of $\frac{1}{22}$ part of the total weight; and that for

support on a wave crest the shearing-force probably does not exceed $\frac{1}{7}$ part of the total weight; hence the above-mentioned fact, as to the increase of shearing-strains in a ship ashore, becomes obvious.

The quantitative examples which have been chosen to illustrate the maximum strains incidental to suspension by the ends only are based upon the three typical ships previously considered, viz. the 'Minotaur,' the 'Bellerophon,' and the 'Victoria and Albert.' As the result of calculations made for the 'Minotaur,' it has been found that the centre of gravity of the ship, when fully laden, is 210 feet from the bow, and that when she rests upon the extremities the upward pressure at the bow equals 4925 tons, while that at the stern equals 4975 tons. These quantities obviously give the limiting values for the shearing-forces experienced by the ship. Next, as to the maximum bending-moment which will be experienced by the transverse section of the ship that contains her centre of gravity. Other calculations have been made which show that the centres of gravity of the two parts into which the ship is divided by the transverse section of maximum bending-moment are respectively 85 feet before and 81 feet abaft this section. Hence, starting from the fore end, we have—

Maximum bending-moment = moment of upward pressure at the bow minus the moment of the weight of the fore part of the ship

$= 4925 \text{ tons} \times 210 \text{ ft.} - 4830 \text{ tons} \times 85 \text{ ft.}$

$= 579{,}300$ foot-tons (in round numbers); or about

$=$ displacement in tons $\times \frac{1}{7}$ of the length in feet.

In preceding investigations it has been shown that the approximate maximum bending- (hogging-) moments for this ship are for still water 45,000 foot-tons, and for support on a wave-crest 140,300 foot-tons, while in a wave-hollow the maximum sagging-moment amounts to 74,800 foot-tons. These figures speak for themselves, and illustrate the large increase in the bending-moments caused by the exceptional supports. The increase in the maximum shearing-force, from 450 tons in still water, 1365 tons on the wave-crest, and 695 tons in a wave-hollow, to 4975 tons when the ship is ashore, is no less striking.

The 'Bellerophon' furnishes our second example. By means of calculations similar to those made for the 'Minotaur,' it has been found that the transverse section passing through the centre of gravity of the whole ship is about 144 feet from the bow and 156 feet from the stern, and that the centres of gravity of the parts of the ship before and abaft this section are distant from it 54 feet and 64 feet respectively. The weights of these parts are respectively 3825 tons and 3225 tons, and the upward pressures at the bow and stern being respectively 3666 tons and 3384 tons, we have

Maximum bending-moment $= 3666 \text{ tons} \times 144 \text{ ft.} - 3825 \text{ tons} \times 54 \text{ ft.}$

$= 321{,}400$ foot-tons (in round numbers); or about

$=$ displacement in tons $\times \frac{1}{7}$ of the length in feet.

It has been previously shown that the approximate maximum bending- (hogging-)

moments experienced by this vessel are 12,000 foot-tons for still water, and 43,600 foot-tons for a wave-crest, while the maximum sagging-moment in a wave-hollow is 48,800 foot-tons. The shearing-forces have also been found to have the following approximate maximum values:—In still water 210 tons, on a wave-crest 555 tons, and in a wave-hollow 640 tons, while aground it equals 3666 tons. The comparative conditions of strain of the ship under these different circumstances can be fairly determined by comparing these figures. It is also interesting to remark the different manner in which changes of the attendant circumstances affect the strains of the two classes of ironclads represented by the 'Minotaur' and the 'Bellerophon.' The reader can trace these for himself; I would simply call attention to the fact that when aground and supported at the extremities the maximum bending-moments bear very nearly the same proportions to the products of the lengths and displacements, notwithstanding the very different distribution of the weight in the two ships. This similarity is mainly due to the fact that the centres of gravity of the fore and after bodies in the two ships are very nearly the same part of the length of these bodies distant from the bow and stern. For example, in the 'Minotaur' the centre of gravity of the fore body is 116 feet distant from the bow, and the fore body is 201 feet long, these quantities being very nearly in the ratio of 3 to 5; while in the 'Bellerophon' the centre of gravity is 90 feet from the bow, and the length of the fore body is 144 feet, the ratio between these quantities being nearly the same as before.

Our third example is drawn from the 'Victoria and Albert,' in which ship the centre of gravity is about 157 feet from the bow and 143 feet from the stern. When supported at the extremities only, the bow sustains a pressure of about 1120 tons, and the stern of 1230 tons; and the centres of gravity of the two parts into which the ship is divided by the transverse section containing her centre of gravity are respectively 60 feet before, and 46 feet abaft this section. Hence we obtain by the same method as before:—

$$\begin{aligned}\text{Maximum bending-moment} &= 1120 \text{ tons} \times 157 \text{ ft.} - 1020 \text{ tons} \times 60 \text{ ft.}\\ &= 114{,}700 \text{ foot-tons (in round numbers); or about}\\ &= \text{displacement in tons} \times \tfrac{1}{6} \text{ of the length in feet.}\end{aligned}$$

The approximate maximum bending-moments previously determined for this ship are, for still water 5080 foot-tons, on a wave-crest 16,400 foot-tons, in a wave-hollow 31,000 foot-tons; the approximate maximum shearing-forces under the same circumstances are respectively 140 tons, 220 tons, and 395 tons. When ashore the maximum shearing-force is, of course, 1230 tons. By comparing this ship with the 'Bellerophon' and 'Minotaur,' it will be seen that the maximum sagging-moment bears a larger ratio to the product of the displacement by the length than it does in the two ironclads. This is due to the fact, already illustrated, of the greater concentration of weights at the centre of the paddle-wheel steamship.

These three examples afford us the means of approximating to the limiting values of the shearing-forces and bending-moments of ships supported at the extremities only,

and from them we obtain the following limits. For shearing-forces the maximum lies between one half and three fifths of the displacement, and for bending-moments between one seventh and one sixth of the product of the displacement by the length. These are, of course, to be regarded simply as *limiting* values; in practice they can scarcely occur, because no ship is likely to rest at the extremities only without having a moderate base of support. The amount of the strains actually experienced would depend, obviously, upon the length of the base of support and its greater or less nearness to the bow and stern.

When a ship ashore rests upon a middle support and has her ends unsupported, which in the other extreme position she can occupy, she is, as I have said, less severely strained than when resting on the ends only, the reason for this fact being that the heavy weights carried in the amidship portion are comparatively close to the point of support in one case, whereas in the other they are much more distant. The greatest difference in strains should consequently be looked for in ships having very concentrated weights amidships; and it is interesting to remark that while such a concentration has been shown to be beneficial in reducing most of the principal strains experienced by ships afloat, it is the cause of the increase in sagging-strains in ships ashore. A ship having her weights uniformly distributed throughout the length would be subject, when supported at the middle only, to hogging-moments equalling in amount the sagging-moments incidental to support at the extremities. Actual ships, however, have not any thing like a uniform distribution of weights, and the greatest weights are usually found near the middle. In spite of the increase in the severest exceptional strains thus caused, however, it cannot be doubted that the ordinary distribution of the weight is beyond comparison better than uniform distribution would be; for its beneficial effect, in reducing strains in ships afloat, is continually called into play, while the other effect is seldom, if ever, produced.

In order to show the relative magnitude of the classes of exceptional strains in ships ashore, I will again take the three typical ships and give a few quantitative results. The only explanation required of the method I shall follow is that when a ship is supported on a single point, vertically below her centre of gravity, the maximum shearing-force will equal the weight of either the fore or the after body, and the maximum bending-moment will equal the product of the weight of one of these bodies by the distance of its centre of gravity from the point of support. The previous investigations therefore supply all the *data* required for the further calculations.

In the 'Minotaur' the after body is the heavier, and its weight, 5070 tons, constitutes the maximum shearing-force. The product of this weight by the distance of the centre of gravity of the after body from the transverse plane passing through the centre of gravity of the whole ship, 81 feet, gives us:

$$\begin{aligned} \text{Maximum hogging-moment} &= 5070 \text{ tons} \times 81 \text{ ft.} \\ &= 410{,}600 \text{ foot-tons (in round numbers); or about} \\ &= \text{displacement in tons} \times \tfrac{1}{10} \text{ of the length in feet.} \end{aligned}$$

The maximum shearing-force is a little greater in this case than in the case of support at the ends; and the maximum hogging-moment is considerably over two thirds the maximum sagging-moment. It may also be interesting to compare these exceptional hogging-strains with the strains incidental to support on a wave-crest. These are:—hogging-moment 140,300 foot-tons, shearing-force 1365 tons.

The 'Bellerophon' is divided into two parts by the transverse plane containing her centre of gravity, such that the foremost or heavier part weighs 3825 tons, and has its centre of gravity 54 feet before the plane of division. The maximum shearing-force for support at the middle is therefore 3825 tons, a little greater than in the previous position, and we have

$$\begin{aligned}\text{Maximum hogging-moment} &= 3825 \text{ tons} \times 54 \text{ ft.}\\ &= 206{,}500 \text{ foot-tons (in round numbers); or about}\\ &= \text{displacement} \times \tfrac{1}{10} \text{ of the length.}\end{aligned}$$

This moment is considerably under two thirds of the maximum sagging-moment previously found for this ship; and we have in this fact an illustration of the statement made above respecting the difference existing between the exceptional hogging- and sagging-strains in ships with concentrated weights amidships as compared with other ships. In the 'Victoria and Albert' we should expect to find a more striking illustration, and we really do so. Her after body is heavier than the fore body, and its weight, 1330 tons, constitutes the maximum shearing-force, while the centre of gravity is 46 feet abaft the centre of gravity of the whole ship. Hence we have

$$\begin{aligned}\text{Maximum hogging-moment} &= 1330 \text{ tons} \times 46 \text{ ft.}\\ &= 61{,}200 \text{ foot-tons (in round numbers); or about}\\ &= \text{displacement} \times \tfrac{1}{12} \text{ of the length.}\end{aligned}$$

The maximum sagging-moment in this ship is therefore not very much less than double the maximum hogging-moment experienced by the ship ashore; when compared with the 'Minotaur,' the case of the 'Victoria and Albert' appears still more striking.

These three examples lead to the conclusion that the limiting maximum values of the strains experienced by ships supported only at the middle may be fixed as follows:—for shearing-forces between one half and two thirds of the displacement; for bending-moments between one ninth and one eleventh of the product of the displacement by the length. As in the other extreme position of support, these limits can never be approached closely in actual ships, on account of the more or less extended base of support which is pretty certain to be found under ships ashore.

In order to facilitate a comparison between the various strains experienced by the three typical ships under different circumstances, the following tabular statement of the results arrived at in the preceding pages is given.

TABLE of Maximum Bending-moments and Shearing-forces determined for the 'Minotaur,' 'Bellerophon,' and 'Victoria and Albert.'

	Minotaur.		Bellerophon.		Victoria and Albert.	
	$\frac{\text{Shearing-force.}}{\text{Displacement.}}$	$\frac{\text{Bending-moment.}}{\text{Displacement} \times \text{length.}}$	$\frac{\text{Shearing-force.}}{\text{Displacement.}}$	$\frac{\text{Bending-moment.}}{\text{Displacement} \times \text{length.}}$	$\frac{\text{Shearing-force.}}{\text{Displacement.}}$	$\frac{\text{Bending-moment.}}{\text{Displacement} \times \text{length.}}$
In still water . .	$\frac{1}{22}$	$\frac{1}{88}$	$\frac{1}{33}$	$\frac{1}{176}$	$\frac{1}{16}$	$\frac{1}{139}$
On a wave-crest .	$\frac{1}{7}$	$\frac{1}{28}$	$\frac{1}{13}$	$\frac{2}{97}$	$\frac{1}{11}$	$\frac{1}{43}$
In a wave-hollow .	$\frac{1}{14}$	$\frac{1}{53}$	$\frac{1}{11}$	$\frac{1}{43}$	$\frac{1}{6}$	$\frac{1}{23}$
Supported at the extremities . .	$\frac{1}{2}$	$\frac{1}{7}$	$\frac{1}{2}$	$\frac{1}{7}$	$\frac{1}{2}$	$\frac{1}{6}$
Supported at the middle . . .	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{2}{3}$	$\frac{1}{12}$

The values of bending- and shearing-strains here given are not put forward as strictly accurate, although from the detailed calculations on which they rest they may be regarded as very close approximations to the strains experienced by various classes of ships under the assumed conditions. These conditions, as we have seen, do not include all the circumstances in which ships may be and are placed, but for still water the approximation made is doubtless very close; and for the exceptional positions of support ashore the limiting values of the strains are also very close to the truth, although in practice these limits can never be reached. For ships at sea, in the extreme positions of support assumed, the values given for the strains, of course, represent the statical aspect of the question; the attempt to put into figures the straining effects of pitching and ascending has not been made.

The voluminous calculations upon which the statements and tables comprised in this paper are based, together with the diagrams, were made with great care, under my direction, by Mr. W. H. WHITE, Jun., and Mr. JOHN, Fellows of the Royal School of Naval Architecture, and Draughtsmen at the Admiralty. Mr. WHITE also assisted me greatly in the detailed preparation of the Paper.

Scale used in diag.ms I – VI.
For areas of curves, 3 square inches = 8000 tons.
For lengths along line A B, 3 inches = 200 feet.

FIG. I.

"Victoria & Albert."

Curves of Weight and Buoyancy.

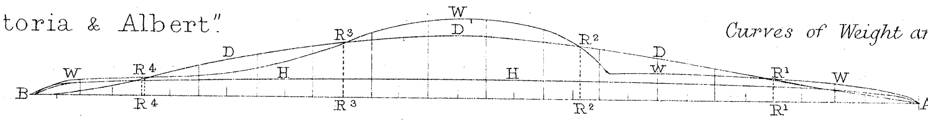


FIG. II.

"Minotaur."

Curves of Weight and Buoyancy.

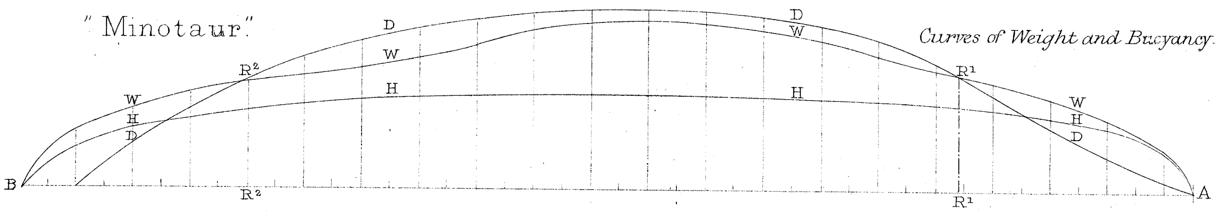


FIG. III.

"Bellerophon"

Curves of Weight and Buoyancy.

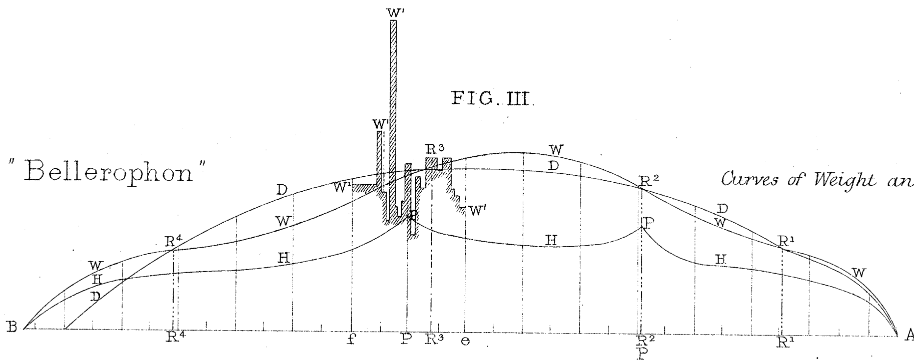


FIG. IV.

"Audacious"

Curves of Weight and Buoyancy.

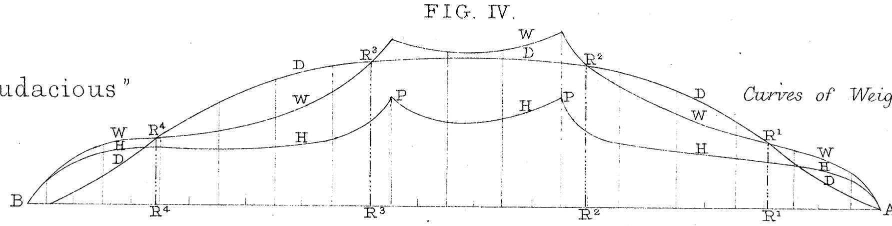


FIG. V.

"Minotaur."

Curves of Weight and Buoyancy when moveable weights are removed.

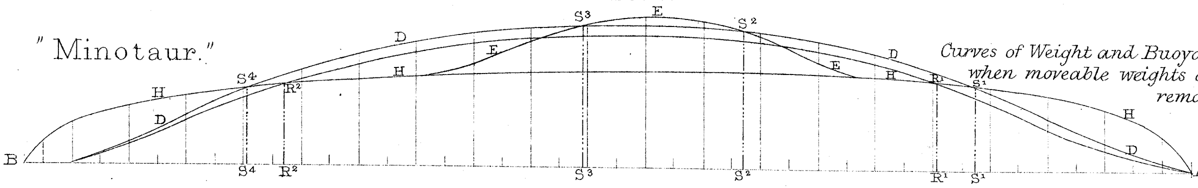


FIG. VI.

"Audacious."

Curves of Weight and Buoyancy when moveable weights are removed.

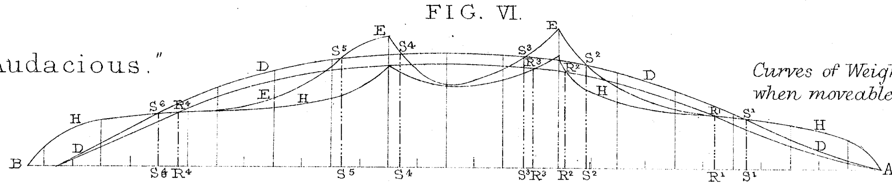
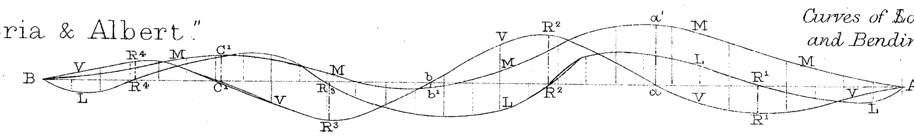


FIG. VII.

"Victoria & Albert."

Curves of Loads, Shearing Forces, and Bending Moments.



Scale used in diag.m VII.

For areas of curves L L L, *3 square inches* = *4000 tons.*
For ordinates of curves V V V, *1 inch* = *400 tons.*
For ordinates of curves M M M, *1 inch* = *16000 foot tons.*
For lengths along line A B, *3 inches* = *200 feet.*

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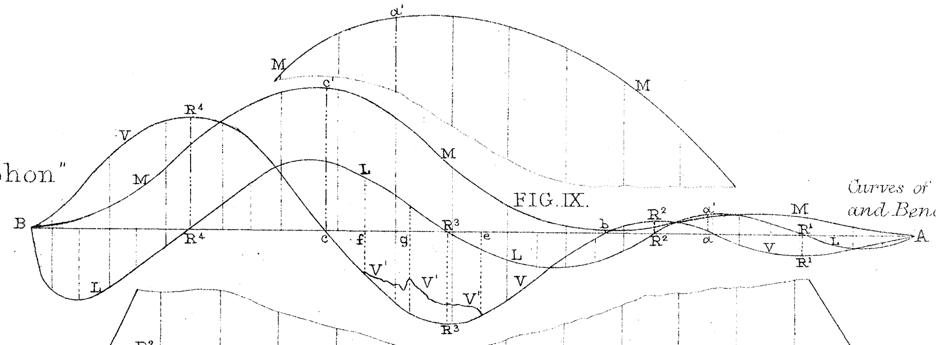
W. West & C.o imp.

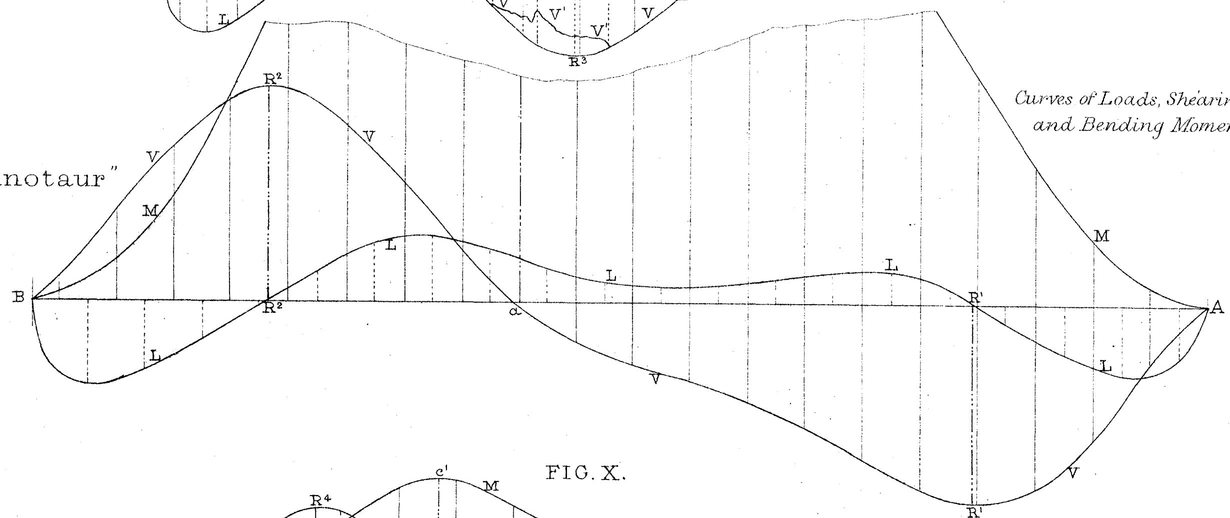
FIG. VIII.

"Bellerophon"



Curves of Loads, Shearing Forces, and Bending Moments.

"Minotaur"



Curves of Loads, Shearing Forces, and Bending Moments.

Scales used in diag^{ms} VIII – XII.

For areas of curves LLL, 3 square inches = 4000 tons

For ordinates of curves VVV, 1 inch = 400 tons

For ordinates of curves MMM, 1 inch = 16000 foot tons

For lengths along line AB, 3 inches = 200 feet.

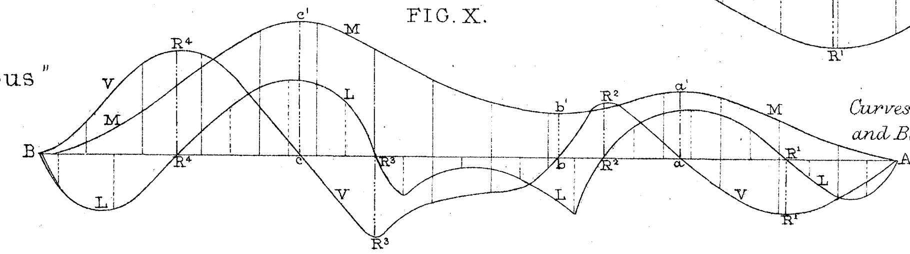
Scales in diag^{m} XVI.

For areas of curves, 3 square inches = 8000 tons.

For lengths along line AB, 3 inches = 200 feet.

FIG. X.

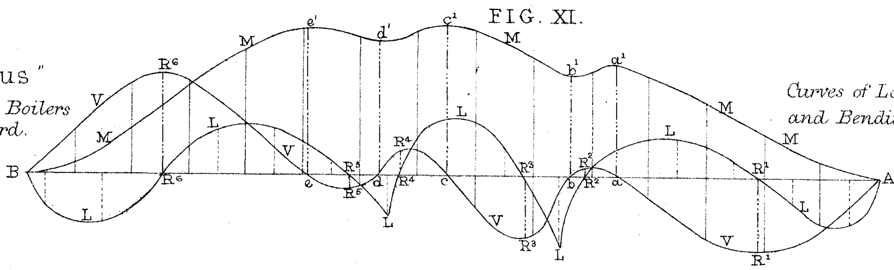
"Audacious"



Curves of Loads, Shearing Forces, and Bending Moments.

FIG. XI.

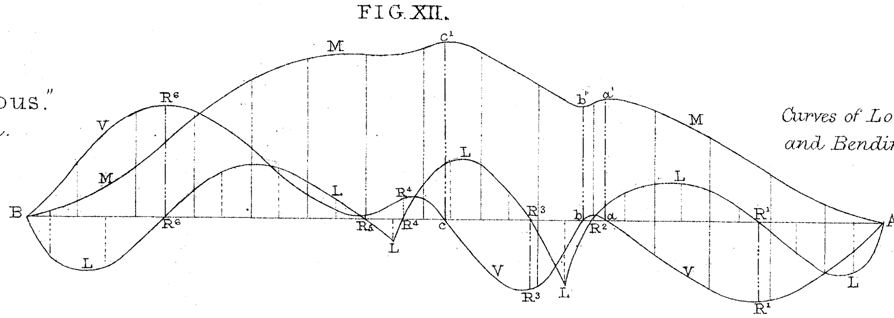
"Audacious" with Engines & Boilers only on board.



Curves of Loads, Shearing Forces, and Bending Moments.

FIG. XII.

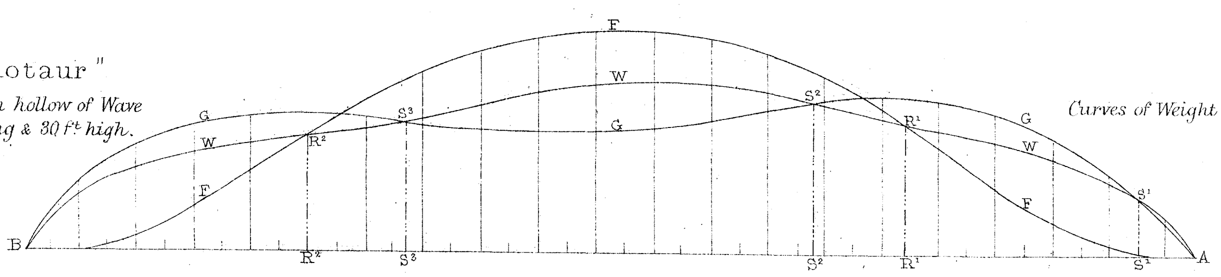
"Audacious." modified.



Curves of Loads, Shearing Forces, and Bending Moments.

FIG. XVI.

"Minotaur" on Crest & in hollow of Wave 600 f^{t} long & 30 f^{t} high.



Curves of Weight and Buoyancy

W.H. Wesley. sc.

W. West & C^{o} imp.

FIG. XVII.

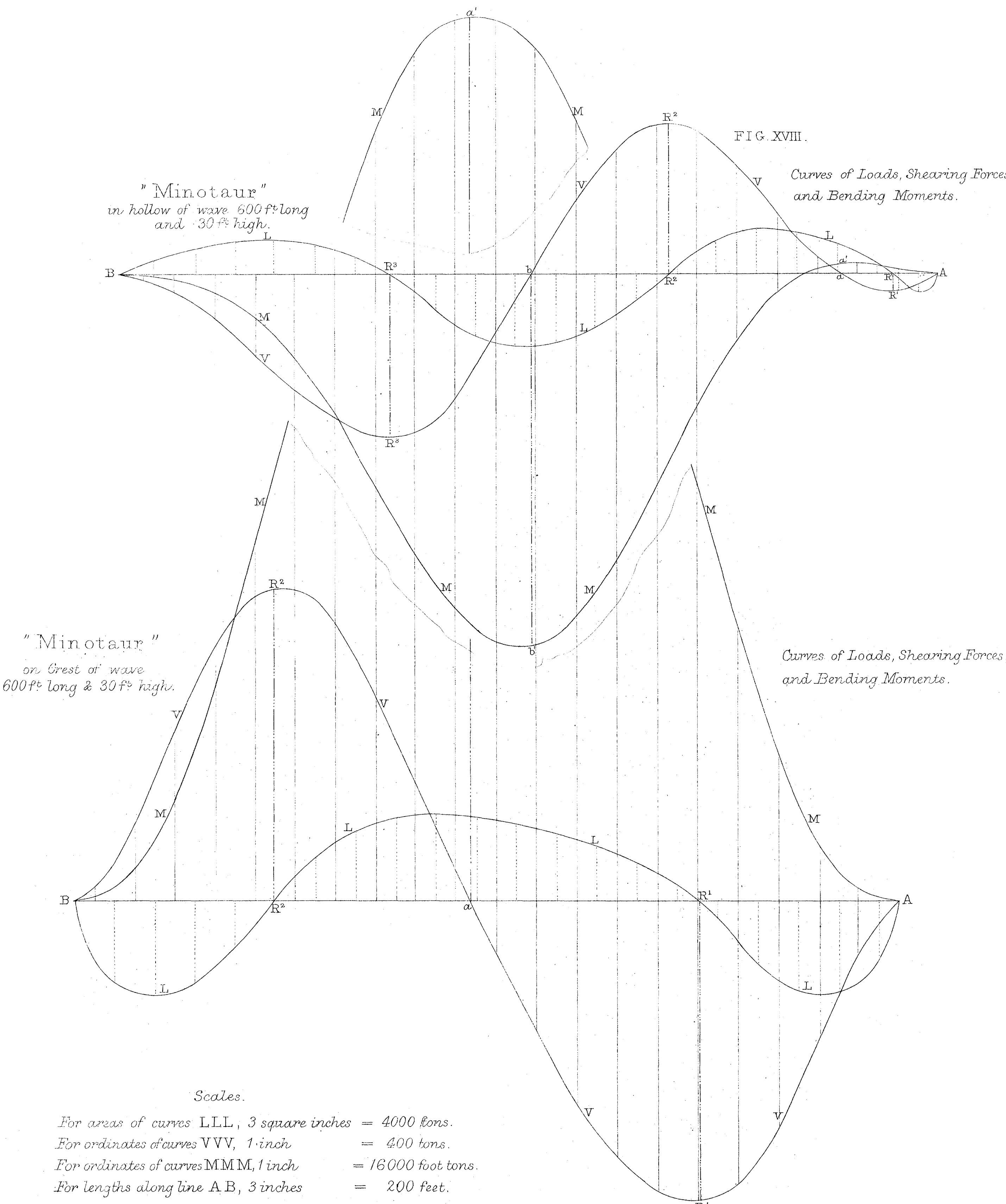


FIG. XVIII.
Curves of Loads, Shearing Forces and Bending Moments.
"Minotaur" in hollow of wave 600 ft. long and 30 ft. high.
"Minotaur" on Crest of wave 600 ft. long & 30 ft. high.
Curves of Loads, Shearing Forces, and Bending Moments.
Scales.
For areas of curves LLL, 3 square inches = 4000 tons.
For ordinates of curves VVV, 1 inch = 400 tons.
For ordinates of curves MMM, 1 inch = 16000 foot tons.
For lengths along line A B, 3 inches = 200 feet.

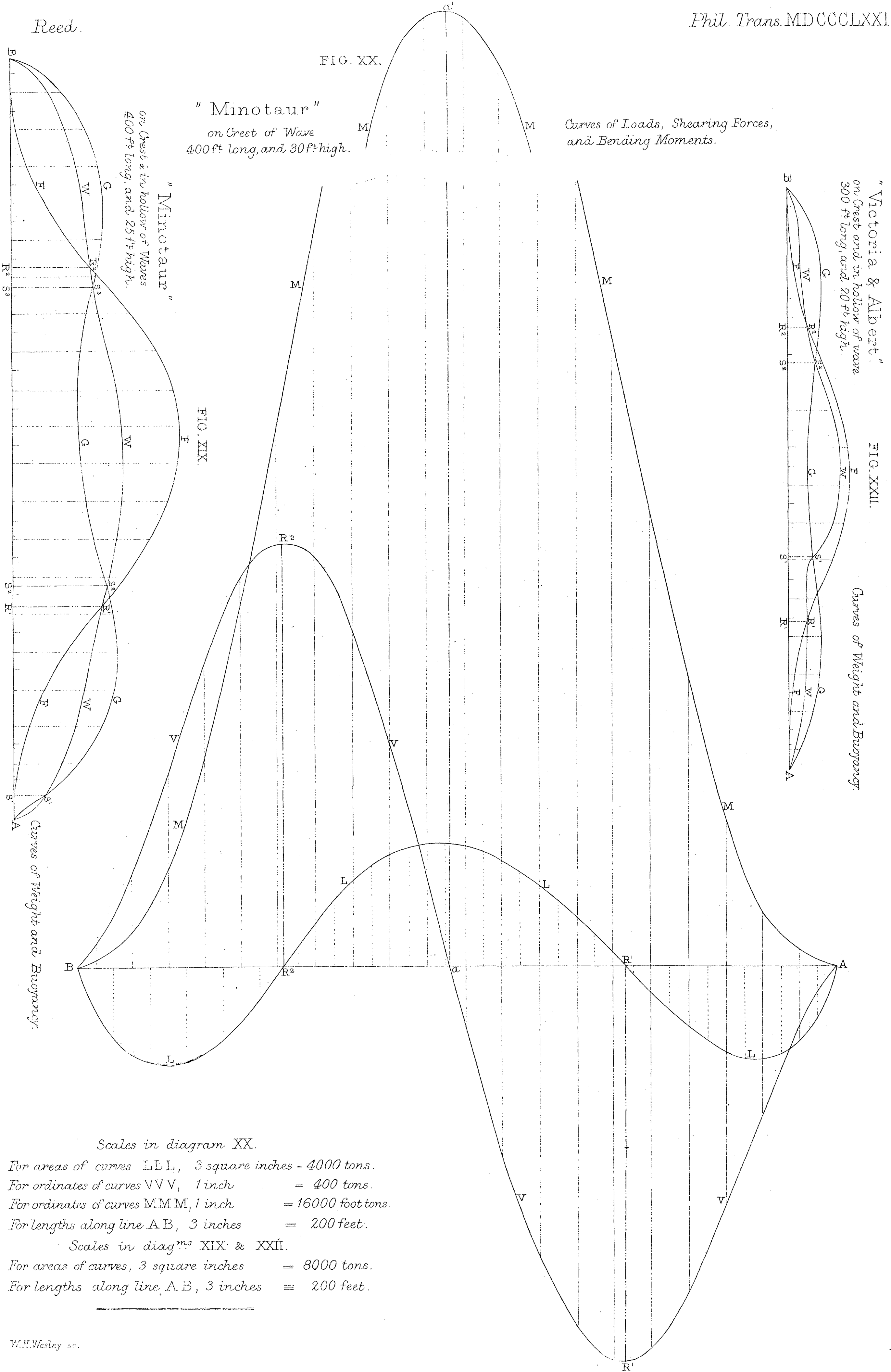


FIG. XX.

"Minotaur"

on Crest of Wave

400 ft long, and 30 ft high.

Curves of Loads, Shearing Forces, and Bending Moments.

Scales in diagram XX.

For areas of curves LLL, 3 square inches = 4000 tons.

For ordinates of curves VVV, 1 inch = 400 tons.

For ordinates of curves MMM, 1 inch = 16000 foot tons.

For lengths along line AB, 3 inches = 200 feet.

Scales in diagms XIX & XXII.

For areas of curves, 3 square inches = 8000 tons.

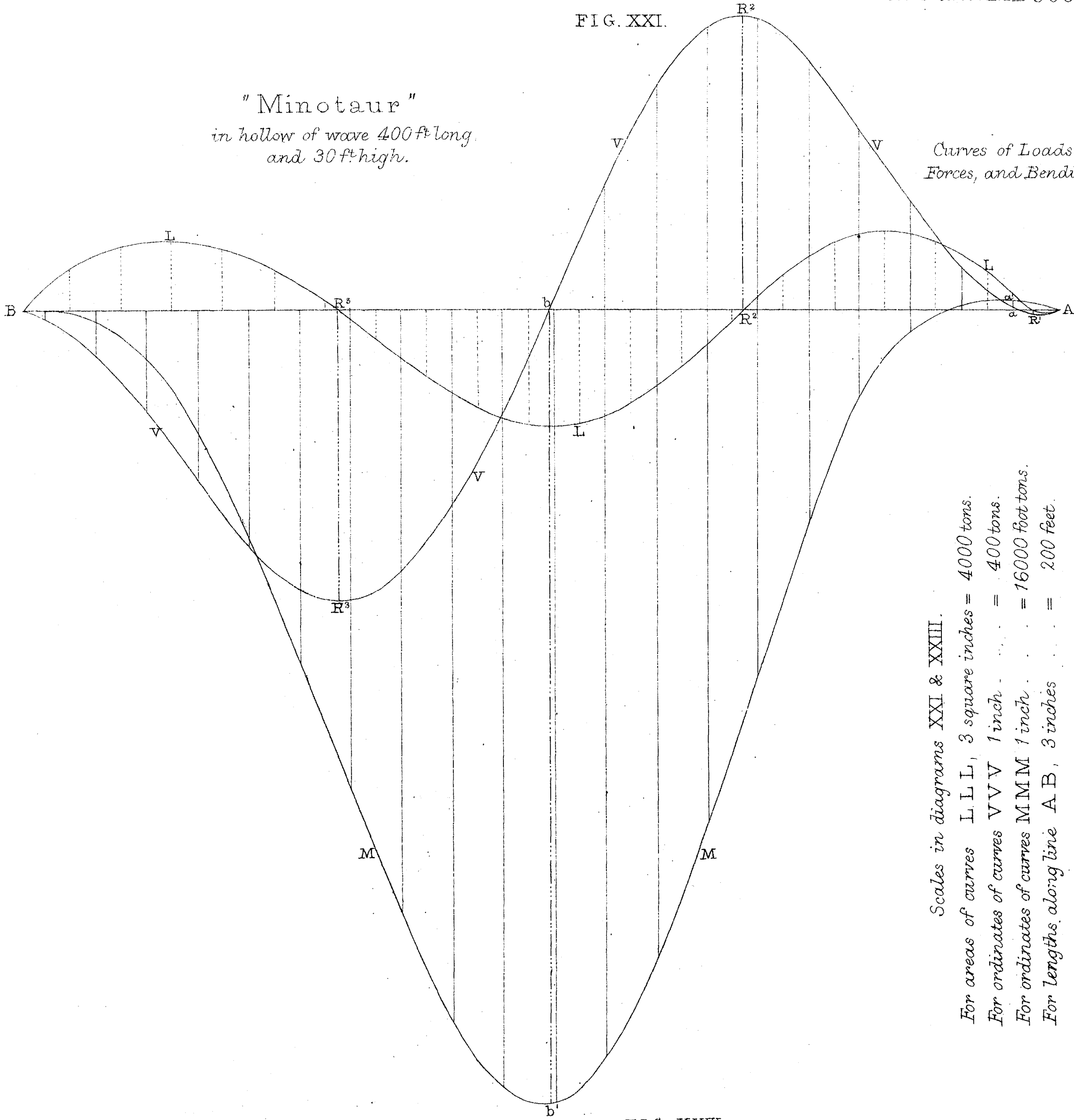
For lengths along line AB, 3 inches = 200 feet.

FIG. XXI.

"Minotaur"

in hollow of wave 400 ft. long and 30 ft. high.

Curves of Loads, Shearing Forces, and Bending Moments.



Scales in diagrams XXI & XXIII.

For areas of curves L L L, 3 square inches = 4000 tons.

For ordinates of curves V V V 1 inch . . . = 400 tons.

For ordinates of curves M M M 1 inch . . . = 16000 foot tons.

For lengths along line A B, 3 inches . . = 200 feet.

Scales in diagram XXV.

For areas of curves, 3 square inches . . . = 8000 tons.

For lengths along line AB 3 inches . . . = 200 feet.

FIG. XXIII.

"Victoria & Albert"

on Crest of wave 300 ft. long, and 20 ft. high.

Curves of Loads, Shearing Forces, and Bending Moments.

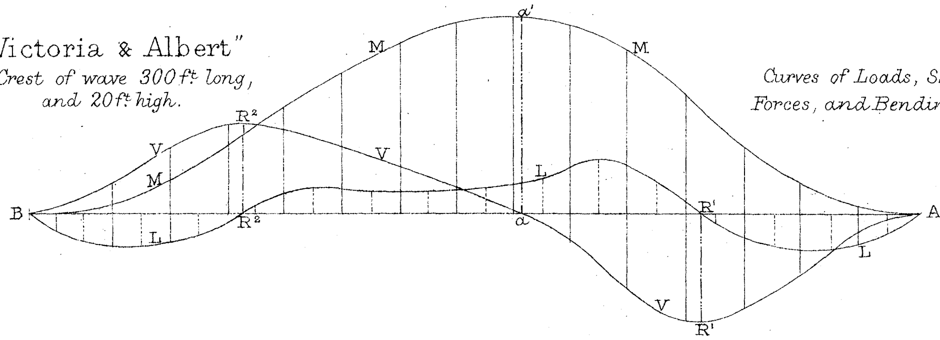
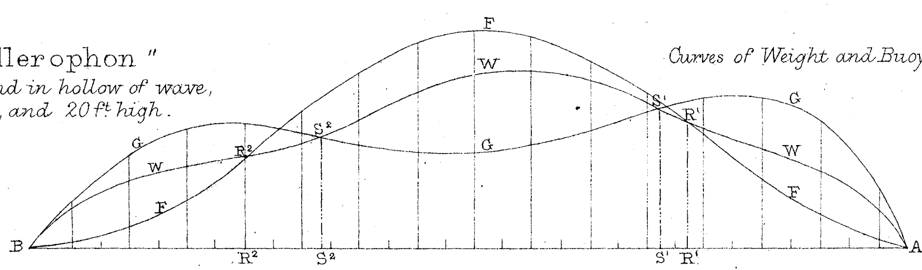


FIG. XXV.

"Bellerophon"

on Crest and in hollow of wave, 300 ft. long, and 20 ft. high.

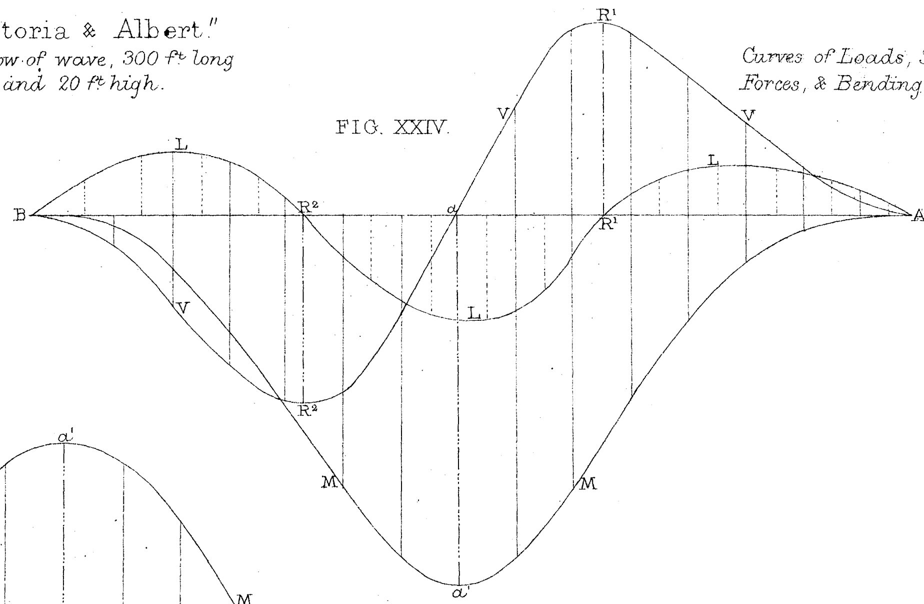
Curves of Weight and Buoyancy.



"Victoria & Albert."
in hollow of wave, 300 ft. long and 20 ft. high.

Curves of Loads, Shearing Forces, & Bending Moments.

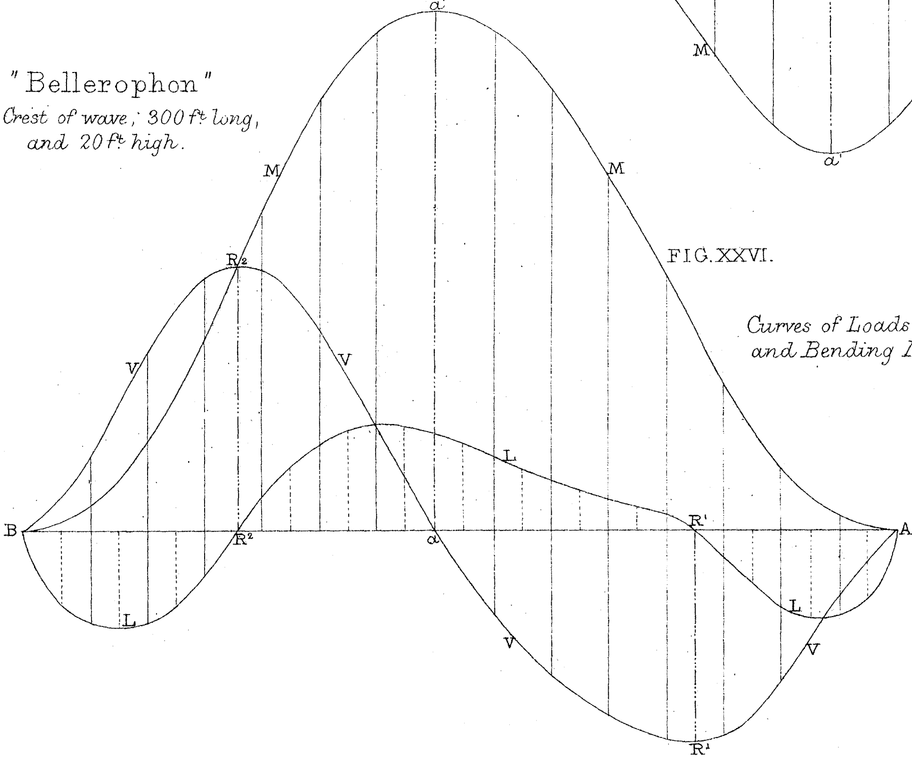
FIG. XXIV.



"Bellerophon"
on Crest of wave, 300 ft. long, and 20 ft. high.

FIG. XXVI.

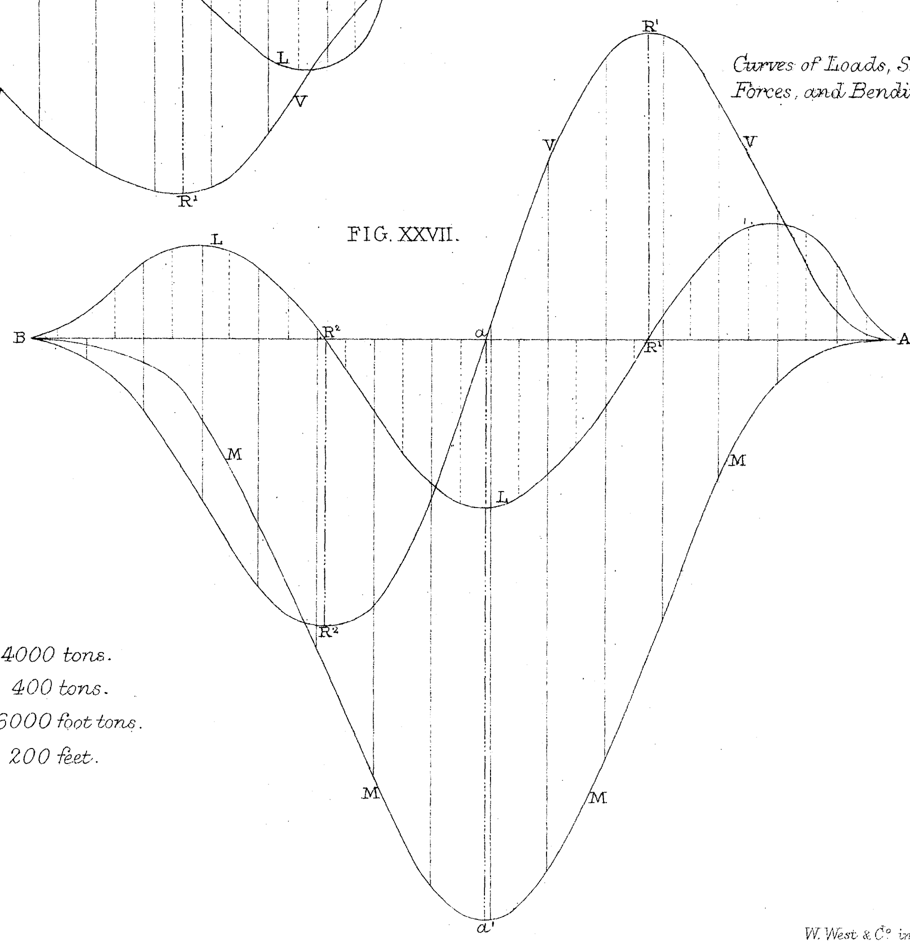
Curves of Loads, Shearing Forces, and Bending Moments.



"Bellerophon."
in hollow of wave 300 ft. long, and 20 ft. high.

FIG. XXVII.

Curves of Loads, Shearing Forces, and Bending Moments.



Scales in diag.ms XXIV, XXVI. & XXVII.

For areas of curves	L L L, *3 square inches*	=	*4000 tons.*
For ordinates of curves	V V V, *1 inch*	=	*400 tons.*
For ordinates of curves	M M M, *1 inch*	=	*16000 foot tons.*
For lengths along line	A.B, *3 inches*	=	*200 feet.*