

PHILOSOPHICAL TRANSACTIONS.

I. *On the Determination of the Rate of Vibration of Tuning-Forks.*

By HERBERT M'LEOD, *F.C.S.*, and GEORGE SYDENHAM CLARKE, *Lieut. R.E., Royal Indian Engineering College, Cooper's Hill.*

Communicated by LORD RAYLEIGH, *M.A., F.R.S.*

Received January 16,—Read January 30, 1879.

[PLATES 1-3.]

IN a paper in the Proceedings of the Royal Society for 1877, xxvi., 162, when describing a new method for determining the speed of machines, we pointed out that by the employment of a graduated drum rotating with a known velocity the true vibration number of a tuning-fork could be accurately determined. The following is an account of the apparatus employed, and subjoined are records of some experiments which have been made on tuning-forks with it, which, in view of the attention now being given to the subject, may prove of interest.

One essential instrument proved to be a good time-measurer. It will be seen in the sequel that an electric current is required once a minute, and in the earlier experiments a two-day marine chronometer was employed, a key being depressed by hand at the end of each minute. So many difficulties, however, arose from its use that it was soon rejected and a clock substituted. Near the pendulum two springs were placed which, being pressed together at the end of each vibration, made a contact at every alternate second. About the 59th second of each minute, a key was depressed by hand and held down until the clock completed the circuit. This, though a great improvement, was not entirely satisfactory, and endeavours had to be made to eliminate entirely all contacts made by hand. This was done by means of a compensated pendu-

MDCCCLXXX.

B

lum worked by electricity and connected with an electric clock. The method of compensation was copied from that of a pendulum exhibited in the Loan Collection of Scientific Apparatus, as a model of the one belonging to the standard clock of the Royal Observatory at Greenwich. Originally, the impulse was given by an electro-magnet placed under the bob, the contact being made by a small trailing piece of steel, which, catching in a notch in a piece of steel below, depressed a spring at every alternate second. This method, which is frequently employed in small clocks, is generally arranged to give a powerful impulse to the pendulum, and this so much increases the arc of vibration that the trailing piece passes over the notch without again engaging in it until the arc has sufficiently diminished. Although such clocks are accurate enough for ordinary purposes, it is obvious that the method would be inadmissible in cases where the period of all vibrations must be the same. To ensure uniformity in this respect the battery power was diminished until the contact was made at each vibration, but after many trials the method was abandoned, for it was found that the friction between the steel trailing piece and the notched piece below was not constant, and, besides, the pendulum was often found to be swinging in an elliptic arc.

An endeavour was next made to work the pendulum by a driver depending on the action of gravity alone, the electric current being employed merely to lift the driving lever, so that the impulse given to the pendulum was entirely independent of the power of the battery. It will be unnecessary to detail the various experiments that were made with more or less success. The following is a description of the form of driving arrangement ultimately adopted.

On each side of the supporting spring of the pendulum is pivoted a branched lever *a*, *b*, Plate 1, fig. 1. At the lower end of each lever is a platinum pin which comes in contact with the rod of the pendulum as in many forms of gravity escapement. The distance of fall of each lever is regulated by means of screws, *c c*. The lever arm, *a*, is so adjusted that the pendulum rod touches the pin at the end of its swing, thus making an electric contact with a platinum collar placed on the pendulum rod, and causing the current to flow from the battery through the electro-magnet *d*, attracting the armature and raising the lever *e*. [This arrangement is not shown in the figure, which represents a slight modification to be described presently.] A pin, *p*, projecting from the lever *e*, lifts the driving lever *b*. On the return swing of the pendulum to the right the contact with *a* is broken and the lever *e* falls, leaving *b* suspended by a small steel catch, *f*. When the pendulum rod touches the platinum pin on the end of *b*, a current traverses the releasing magnet *g*, withdrawing the catch, and allowing the driving lever *b* to fall with the pendulum and thus to give the requisite impulse. Many accurate determinations were made with the contacts as above described, but it was found that the roughening of the pin and platinum collar by the spark produced variable friction; this led to the adoption of the arrangement shown in the figure, which consists in attaching (by an ivory pin, *h h*) a thin piece of watch spring to each lever *a* and *b*, so that the electric contacts are made between the platinum pin

and a piece of gold soldered on to the spring. The pendulum rod thus presses against a piece of polished steel and all variable friction is prevented. The figure shows in the form of a diagram the arrangement of the connexions. The pendulum is driven by four cells of a gravity battery, but as the electro-magnet d has only a small resistance, the extra current at first occasionally fused together the contacts of lever a : this was prevented by connecting the wires on each side of the contact by a shunt consisting of a bobbin of fine wire of about 300 ohms resistance. This plan having, however, proved somewhat uncertain in action, the shunt has been replaced by a tube containing two platinum plates immersed in very dilute sulphuric acid.

At the back of the pendulum rod a short pin of chrysolite is fixed to collar k . This pin, passing under a small steel disk a , Plate 1, fig. 2, raises a thin brass spring fastened to the back of the pendulum case, and makes contact at each oscillation of the pendulum with a second spring carrying a platinum pin. The second spring is thereby raised sufficiently to make a contact with a third spring, and thus two different contacts are made at each second, one preceding the other by a small interval of time. The roughening and occasional sticking of the lower contact has been prevented by connecting a condenser, or two platinum plates in very dilute sulphuric acid, to the springs. The clock, placed at any distance from the pendulum, is actuated by the closing of the two lower springs, an electro-magnet raises a lever carrying a catch which turns a ratchet wheel with 60 teeth, the wheel being moved by the fall of the lever at the breaking of the pendulum contact. Plate 3, fig. 3, shows the arrangement of the minute wheel of the clock and the electric connexions. By an ingenious device of Mr. YEATES, who made the clock, the fall of the lever both turns and locks the wheel, the end of the catch falling on an inclined screw s which drives the point of the catch between the teeth of the wheel. On this minute wheel of the clock is a platinum pin a , which presses against a spring f once a minute. As this contact might possibly be irregular, since it is produced on the breaking of the pendulum contact and fall of the clock lever, whereby two elements of uncertainty are introduced, it is merely used as a switch, and the principal circuit is closed by the pendulum itself when the two upper springs are pressed together. The current then passes through the magnet of a relay and closes the circuit required in the tuning-fork apparatus. These minute contacts are made with great accuracy, for the upper contact is of very short duration, and it is made by the pendulum when moving with nearly its greatest velocity.

The rate of the clock was determined by observations with a small portable transit instrument. The arrangements were not as perfect as could be desired, but the time was determined from the mean obtained by the observations of at least four stars (generally more), and the rates of the clock and chronometer were thus both obtained; moreover the observations were made at such intervals that their inaccuracies did not much affect the determination of the rates. The clock was compared daily with the chronometer and the irregularities of the former corrected.

To prove that no great error can have crept into the calculations from any inaccuracy of the chronometer, the following table is given showing the dates of the observations and the chronometer rate in the interval.

	Gaining rate per day. seconds.		Gaining rate per day. seconds.
1878. Jan. 16	2.91	1878. May 4	2.463
Jan. 19		May 14	
Jan. 23	2.527	June 3	2.232
Feb. 25	2.741	June 11	2.282
Mar. 3	2.657	June 19	2.297
Mar. 10	2.323	June 22	2.673
Mar. 12	2.270	June 27	2.476
April 6	3.394	July 15	3.098
April 22	3.528	Aug. 7	2.504
May 4	3.546		

The error of the clock rate when checked was not more, and probably less, than one second per day.

The apparatus used for the measurement of the tuning-forks was made from a portion of an astronomical chronograph constructed by Messrs. R. and J. BECK. The brass drum, which is two feet long and one foot in diameter, rests on friction wheels and is driven by a weight and train of wheels. The original regulator, consisting of a conical pendulum, was removed and its place supplied by a fan kindly designed for us by our friend and colleague, Professor W. C. UNWIN, B.Sc. The fan and case are shown full size in Plate 2, fig. 4. The diameter of the fan is 3.8 inches, and it is attached to a vertical steel axis resting in an agate cup. The fan rotates in the lower part of a circular box 5 inches in diameter and 2 inches deep, the box being divided into two horizontal compartments by a fixed diaphragm $d d$, the upper compartment being .75 inch and the lower 1.2 inch deep; the diaphragm is perforated by four segmental apertures $a a$ which can be closed by a similarly perforated disk resting on the diaphragm and capable of being moved by an arm r on the top of the box. When the apertures in the diaphragm are opened, air is driven from the lower compartment into the upper, returning through the openings in the middle of the fan, and when the apertures are closed the air circulates with the fan in the lower half of the box. In the latter case the fan rotates about twice as fast as when the diaphragm is fully opened. The axis carries at the top a small pulley p by which it is connected to the drum of the chronograph by a piece of thread passing round the latter.

Round one end of the drum, strips of dark blue paper are wrapped, on which white equidistant lines parallel to the axis of the drum are ruled. The paper was graduated

before being attached to the drum; as the division of a long strip of paper by means of compasses was found tedious and somewhat inaccurate, it was effected by rolling a toothed wheel along the paper, thus obtaining fine equidistant marks through which lines were ruled in Chinese white by means of a drawing pen. For convenience of counting, every fifth and tenth line was made longer than the others, and after attaching the strip to the drum, figures were placed opposite every tenth line. The two strips which are most generally used for the determinations have 486 and 985 lines respectively.

Opposite the graduated paper on the drum is fixed a microscope placed in a horizontal position; the instrument is suspended from a shelf above the table on which the chronograph rests, and the shelf also supports the clock. In the sub-stage a 2-inch objective is placed which produces an image of the graduations at the focus of the object glass of the microscope, the portion of the scale used being illuminated by a lamp and condensing lens. At the common focus of the lenses is placed the tuning-fork, the stem of which is surrounded by a piece of sheet lead and fixed in a vice attached to the table, the upper part of one of the prongs being in such a position that about one-third of the field of view is cut off by the prong, Plate 3, fig. 6. The lower part of the fork is surrounded by a glass case made of four strips of glass glued to a square piece of wood through which the stem of the fork passes. A hole is bored in one of the plates of glass and through this passes a thermometer graduated to $^{\circ}2\text{ C.}$, the bulb being placed as close as possible to the fork without touching it. A plan of the arrangement is shown in Plate 3, fig. 8.

In the eyepiece of the microscope a horizontal hair and a vertical scale are fixed (Plate 3, fig. 6). A double-bass bow is suspended from the shelf by two parallel strings so that the operator may keep the fork in vibration while observing through the microscope. When fork and drum are at rest the white lines on the drum are seen across the field of the microscope but cut off on one side by the fork (Plate 3, fig. 6). When the drum rotates the lines can no longer be seen, but if at the same time the fork is in vibration wave-figures are produced extending from its edge. When the drum is rotating with such a velocity that a line passes over a distance equal to the interval between two adjacent lines during one vibration of the fork, a stationary wave (Plate 3, fig. 7) is seen of a length equal to the distance between two lines, and it only remains therefore to count the lines which pass in a known period of time to determine the number of vibrations of the fork. It was not found possible to obtain a perfectly uniform rotation of the drum by the regulator alone, as the slight irregularities of the clock-work and pulleys were sufficient to cause unequal rates of rotation. In order to obtain a final regulating power a piece of string connects the axis of the drum with a pulley, *g* (Plate 3, fig. 8), which can be turned by the operator's left hand. The motion of the wave upwards or downwards indicates that the drum is going too fast or too slow, and a gentle check, or assistance given by the hand to the axis of the pulley, is enough to keep the rotation uniform.

To count the number of lines passing across the horizontal hair of the eyepiece during an experiment it is necessary to know the number of complete revolutions, and also the additional fraction of a revolution which the drum has performed during the observation. The number of the revolutions is obtained by an electric counter attached to the axis of the drum, a pin on which, shown at *a* on the diagram, Plate 3, fig. 9, completes the circuit by pressing two springs together. To measure the fraction of the revolution, an electro-magnet, *b*, is made to produce a mark on the drum both at the commencement and conclusion of the experiment. Above the chronograph drum is a saddle supporting the electro-magnet; to the armature of the latter is attached a finely pointed tube, *t*, containing a solution of magenta mixed with an equal volume of glycerin to prevent it drying and so stopping up the jet. An electric current from the clock is made to pass through the electro-magnet, both at the beginning and end of the experiment, thus making two dots on the white paper covering the drum, the distance between the dots being the fraction of the revolution required. To find the number of lines in this fraction it is only necessary to place a straight-edge along the drum and to note the numbers of the lines corresponding to the two marks. The saddle carrying the electro-magnet is moved forwards or backwards parallel to the axis of the drum by a screw, which can be turned by means of a cord and pulley near the operator's hand, so that two consecutive dots may not be made on the same line on the drum, and the paper round the drum thus serves as a permanent record of the observations made. The electric current which makes the marks also starts and stops the counter. The circuit of the counter is broken at two mercury cups, *c*, which are connected by a copper wire attached to a small magnet resting between the poles of an electro-magnet, *d*. By means of the reversing switch, *e*, which is turned in one direction before the commencement of the experiment, the copper wire is dipped into the mercury cups, so that when the current is sent by the clock relay the counter is automatically started, and before the end of the experiment the reverser is turned over, so that the contact is broken and the counter stopped also automatically when the second mark is made on the drum.

In each determination of a fork, five experiments are usually made, lasting one, two, three, four, and five minutes respectively. The object of this is to diminish any error which may be due to variation in the duration of movement of the electro-magnets, and also to reduce to a minimum the errors of observation. The forks are placed in the vice many hours before the determinations are made so that they may attain a uniform temperature throughout, and that this temperature, as shown by the thermometer, may be as correct as possible. The glass case surrounding the fork prevents air currents from affecting the thermometer, and a large glass plate placed vertical on the table diminishes the rise of temperature due to radiation from the observer's body.

When a determination is to be made the counter is set at 0, and the velocity of rotation of the drum regulated by means of driving weights and by the fan until the

wave figure is nearly steady : when this is accomplished the observer keeps one wave in the field of view by judicious manipulation of the regulating string. He then begins to count seconds by the clock and turns the reverser so that the current sent at the 60th second will start the electric revolution counter. When the sound of the pen striking the drum is heard, he observes what part of a wave is cut by the hair in the eyepiece, and continuing to count the seconds he maintains as far as possible this part of the wave near the centre of the field. It is difficult to keep the rotation absolutely uniform, but it suffices not to allow the wave to escape from the field of view, or if it does, to bring it back as soon as possible. If the observation is to last for one minute the reverser is turned in the opposite direction immediately after the first dot has been made; but if for a longer period, the contact remains broken by the reverser until after the end of the first, second, third, or fourth minute, when the connexions are so made that at the end of the next minute the counter is stopped. As soon as the first dot is made the screw carrying the electro-magnet is slightly turned so as to shift the position of the pen. If at the sound of the marking of the second dot on the drum the particular part of the wave which was on the hair at the beginning of the experiment is in another part of the field of view, the alteration of position is estimated as accurately as possible in wave-lengths or portions of a wave-length. A correction corresponding to this alteration of position is subsequently applied. The only object of counting the seconds throughout is to direct the observer's attention to the approach of the minute intervals. The five experiments are made consecutively and without stopping the drum. After each experiment the alteration of position of the wave (if any), the temperature indicated by the thermometer, and the number shown by the revolution counter are noted. When the experiments are finished the dots on the drum are marked by consecutive numbers, two belonging to each measurement. A long straight-edge is then placed horizontally against the drum and the latter turned until the centre of the dot is at the edge of the straight-edge, the number belonging to the line of the graduated slip of paper opposite the ruler is then read off.

A determination made on May 8, 1878, with a 256 fork of KÖNIG'S, will best illustrate the method of recording and calculating the results, and will also serve to show the accuracy of which the method is capable.

		Number of line on circle.
No. of 1st mark	163	301
„ 2nd „	164	199·5
Duration of experiment, 5 minutes.		
Temperature by thermometer, 16°·1 C.		
Number shown by revolution counter, 158.		
Number of lines on circle, 486.		

Drum was going a little too fast, the part of the wave under observation had risen ·5 of a wave-length above the hair.

$158 \times 486 = 76,788$ lines passed in 158 revolutions.

$301 - 199.5 = 101.5$ lines between the two dots.

$101.5 - .5 = 101$ lines corrected for movement of the wave.

$76,788 + 101 = 76,889$ vibrations of fork in 5 minutes.

$$\frac{76889}{300} = 256.2966 \text{ vibrations per second.}$$

It frequently happens that the two numbers corresponding to the dots are on the opposite sides of the 0, in which case the counter indicates one more than the number of revolutions, and 486 must be added to the number corresponding to the first dot before subtracting the number corresponding to the second dot.

The remaining entries in the note-book serve to show the whole of these cases.

Mark 165	..	405.25	2 min.	
166	..	268	16°9	30618
		<u>137.25</u>		137.75
		+ .5	221 (counter)	
		<u>137.75</u>	158 (at last experiment)	120)30755.75
			$63 \times 486 = 30618$	256.298
Mark 167	407.25	407.25	3 min.	
168	444.5	486	17°0	45684
		<u>893.25</u>		448.75
		444.5	316	180)46132.75
		<u>448.75</u>	221	256.293
			$95 - 1 = 94 \times 486 = 45684$	
Mark 169	84	84	1 min.	
170	258	486	17°0	15066
		<u>570</u>		311.5
		258	348	60)15377.5
		<u>312</u>	316	256.292
		- .5	$32 - 1 = 31 \times 486 = 15066$	
		311.5		
Mark 171	252	252	4 min.	
172	464	486	17°2	61236
		<u>738</u>		274
		464	475	240)61510
		<u>274</u>	348	256.2916
			$127 - 1 = 126 \times 486 = 61236$	
		256.297	16.8	
		256.298	16.9	
		256.293	17.0	
		256.292	17.0	
		256.292	17.2	
		5) 22	5)34.9	
		Mean = 256.294	16.98	
			- 1.4 (correction for error of thermometer.)	
			<u>15.58</u>	

When this determination was made the rate of the clock was taken as $\cdot788$ second per day, losing; correcting for this rate, we have $256\cdot294 \times \frac{86399\cdot212}{86400} = 256\cdot292$ vibrations per second at $15^{\circ}\cdot58$ C.

The maximum difference between the above numbers is $\cdot006$ or $\cdot00234$ per cent.

An error of reading of one line, or mistaking one wave for the next, will produce, in a five-minute observation, a difference of $\cdot0033$ in the determination of a 256 fork, while the same error in an observation lasting one minute will produce a difference of $\cdot0166$; the value of an experiment lasting several minutes is therefore apparent. The method is of such a nature that no bias in the mind of the operator can have the slightest effect on the result, as the final numbers show no indication of their direction until the positions of the dots are read off on the drum; and, to prove that special training is not required for the observations, it may be mentioned that a sharp boy of 16 succeeded in keeping the figure sufficiently steady after two trials with the machine.

Three elements of possible error in the method above described suggested themselves—1st, the effect of the frequent bowing; 2nd, the variation of amplitude; and 3rd, the holding of the fork in a vice instead of fixing it, as usually employed, on a sounding box. These matters will now be dealt with *seriatim*.

1. *Effect of bowing.*

In order to dispense with the use of the bow, and to keep the amplitude constant, we had contemplated the employment of HELMHOLTZ'S method of maintaining the forks in vibration by means of an electro-magnet excited by an intermittent current from a contact maker worked by a large tuning-fork, the vibration number of which would bear a simple relation to the vibration number of the fork to be measured. The experiment, however, was not tried, as we feared that the current might to some extent control the fork, which would, of course, be inadmissible; the bow was therefore retained.

If the bowing altered the phase of the vibration, which seemed at first not improbable, there would be a sudden start of the wave figure, in one direction or the other, at each stroke of the bow; this has never been observed. Again, if the bowing altered the phase, even to an extent inappreciable by the motion of the wave, its frequent occurrence at each application of the bow would necessarily alter the number obtained in the determination. In order to put this to the test six experiments were made, in three of which the fork was bowed every 20 seconds, two strokes of the bow being generally made on the prong, and in the three others the fork was bowed every five seconds. The determinations were made alternately, and the first four lasted five minutes, and the others three minutes. The following numbers were obtained:—

Bowed every 20 seconds.	Temp.	Bowed every 5 seconds.	Temp.
512·470	16·1	512·460	16·2
512·460	16·25	512·452	16·25
512·460	16·3	512·456	16·4
Mean 512·463	16·22	512·456	16·28

(These numbers are not corrected for the rate of the clock)

The effect of frequent bowing, therefore, appears to be to make the fork vibrate a little more slowly, the diminution of rate being '00137 per cent. The fork was one of KÖNIG'S, and of stout metal ; lighter made forks may be affected to a greater extent. LISSAJOUS found that no change of form was exhibited by his figures when the forks were bowed, ('Comptes Rendus,' 1855, xli., 816) ; although a different result was obtained by POSKE (Pogg. Ann., 1874, clii., 449), who observed that the increase of amplitude produced by bowing seriously altered the rate of vibration. He, however, seems to have used a vibroscope, in which the ends of the prongs were weighted by a lens and counterpoise.* We have recently endeavoured to settle this question in another manner. The microscope being placed horizontally, a 256 fork was fixed vertically at the focus of the objective, and another fork was placed horizontally in such a position that an image of its edge was thrown by the lens in the substage on to the edge of the vertical fork. In this way the LISSAJOUS' figure was seen in the angle made by the two images. No change of phase was perceptible on bowing either of the forks. Where two 60 forks were tested in a somewhat similar manner, a change of phase was seen ; but one of these forks is thin in comparison with its length, and the prongs are weighted by a lens and counterpoise. It is evident, however, that the unskilful bowing of a light fork may alter the phase, by first damping and then re-starting the fork.

2. *Effect of the variation of amplitude.*

To determine the effect of different amplitudes, the scale in the eyepiece of the microscope was placed horizontally, and the amplitude kept as constant as possible by suitable bowing. Four experiments were made, each of five minutes' duration. In the first and third, the movement of the end of the prong was about '37 millim. ; in the second and fourth, about '15 millim. The results are as follows :—

Large amplitude.	Temp.	Small amplitude.	Temp.
256·277	16·2	256·275	16·25
256·273	16·4	256·274	16·45
Mean 256·275	16·3	256·2745	16·35

* KAYSER, also, in his determinations of the effects of temperature, employed weighted forks (Ann. der Physik u. Chemie, 1879, N. F. viii. 444). As far as our experiments go, such forks seem to behave, in many respects, differently from those of the ordinary form.—(Note added December 31, 1879).

Consequently, this difference of amplitude does not appreciably affect the vibration number.

3. *Effect of the mode of fixing the fork.*

To determine whether any difference was perceptible when the fork was attached to the sounding box, the latter was supported on a piece of wood and a heavy leaden weight placed within it, the top being left without any pressure upon it. The experiment was not easy, as the pressure of the bow slightly moved the fork in the field of view, and the amplitude was small, which made the waves somewhat difficult to see. Three five-minute measurements were made, giving—

	512·489	at	16·2
	512·482		16·25
	512·477		16·35
	<hr/>		
Mean	512·483		16·26

The results previously given of the effect of bowing were made with the same fork at the same temperature ; a comparison with these would seem to indicate that fixing in a vice slightly diminishes the vibration number, to an extent of about ·005 per cent. This might have been expected, as the fork communicates a slight motion to the body to which it is attached, and the heavier vice retards the fork.

Effect of temperature on the vibration number of forks.

In the previous paper some experiments were described by which the effect of a rise of temperature on a large fork, giving 60 vibrations per second, was found to be a diminution of the rate of vibration amounting to ·011 per cent. for each degree centigrade. It will be better to express this by the coefficient ·00011 for 1° C. This number has been confirmed by some determinations of a 256 fork at various temperatures. Since it is not easy to devise a thoroughly satisfactory means for altering the temperature of the fork whilst in the apparatus, the determinations were made when the temperatures of the surrounding air were very different. The following numbers were thus obtained :—

1.	256·310	at	15·01 C.
2.	256·273		16·31
3.	256·260		16·69
4.	256·175		19·75
5.	256·137		21·16
6.	256·052		24·29
7.	256·008		25·69

Calculating the coefficient for each degree centigrade from the determination made at the lowest temperature as the starting point, the following values were found :—

For 1° C.	From determinations at
·000116	15·01 and 16·31
·000111	„ 16·69
·000111	„ 19·75
·000110	„ 21·16
·000109	„ 24·29
·000110	„ 25·69
<hr/>	
Mean	·000111

Starting from the determinations made at the highest temperature, we have :—

For 1° C.	From determinations at
·000110	25·69 and 15·01
·000110	„ 16·31
·000109	„ 16·69
·000110	„ 19·75
·000111	„ 21·16
·000123	„ 24·29
<hr/>	
Mean	·000112

Rejecting the last number as being obviously too high, the mean of the other five is ·000110.

Determinations have been made with a new set of KÆNIG's forks, obtained for us by Mr. LADD last year, and stated to be correct at 15° C.* The following vibration numbers were found, and the corrections for temperature were made by the use of the coefficient ·00011 for each degree centigrade :—

* We afterwards heard that Mr. KÆNIG considered them to be correct between 19° and 20° C.

Date.	Mark on fork.	Number of measurements.	Percentage difference between highest and lowest numbers.	Temperature. ° C.	Vibration number.	Corrected to 15° C.	Mean.
1878. May 2	Sol ₃ 758 V.S. RK. .	7	·00755	17·6	384·330	384·440	} 384·437
" 3	" " . .	6	·00885	15·9	384·394	384·432	
" 3	" " . .	6	·00338	16·9	384·349	384·429	
" 4	" " . .	5	·00156	15·5	384·418	384·439	
May 4	Mi ₃ 640 V.S. RK. .	4	·00125	16·27	320·321	320·366	} 320·372
" 6	" " . .	5	·00219	15·12	320·365	320·369	
" 6	" " . .	4	·00156	16·27	320·327	320·372	
" 7	" " . .	5	·00312	18·05	320·273	320·380	
May 8	Ut ₃ 512 V.S. RK (I)	5	·00234	16·48	256·266	256·308	} 256·309
" 8	" " .	5	·00234	15·58	256·292	256·308	
" 9	" " .	5	·00195	17·39	256·217	[256·284]	
" 10	" " .	2	·00156	16·30	256·273	256·310	
" 10	" " .	2	·00039	16·35	256·272	256·310	
May 11	Ut ₃ 512 V.S. RK (II)	4	·00195	17·99	256·223	256·307	} 256·306
" 13	" " .	4	·00078	17·3	256·236	256·301	
" 14	" " .	4	·00234	16·14	256·278	256·310	
May 14	Ut ₄ 1024 V.S. RK .	4	·00078	16·82	512·446	512·548	} 512·551
June 6	" " . .	5	·00664	15·9	512·487	512·538	
" 7	" " . .	4	·00137	18·07	512·386	512·559	
" 8	" " . .	5	·00312	17·78	512·399	512·556	
" 10	" " . .	6	·00615	17·05	512·451	512·567	
" 11	" " . .	3	·00195	16·22	512·479	512·548	
" 11	" " . .	3	·00156	16·28	512·472	512·544	
June 17	Ut ₃ 512 V.S. RK (I)	4	·00117	15·01	256·310	256·310	} 256·310
" 17	" " .	4	·00078	16·69	256·260	256·308	
" 18	" " .	4	·00234	16·31	256·273	256·310	
" 21	" " .	4	·00312	19·75	256·175	256·309	
" 22	" " .	5	·00391	21·16	256·137	256·310	
" 25	" " .	5	·00195	24·29	256·052	256·314	
" 26	" " .	4	·00039	25·69	256·008	256·309	
Oct. 21	" " .	4	·00195	17·0	256·254	256·310	
" 22	" " .	5	·00117	15·07	256·302	256·304	
" 23	" " .	4	·00156	12·72	256·378	256·314	

A fork belonging to Sir WILLIAM THOMSON, and made by MARLOYE, gave the following numbers :—

Date.	Number of measurements.	Percentage difference between highest and lowest numbers.	Temperature. ° C.	Vibration number.	Corrected to 15° C.	Mean.
1878. June 13 . . .	5	·00391	17·48	255·938	255·945	} 255·953
" 15 . . .	5	·00430	16·12	255·962	255·962	

Method of comparing two forks.

Two forks, the vibration numbers of which differ very slightly from one another, may be conveniently compared by the method previously mentioned, namely, by placing them at right angles with one another, and in such positions that the images of their edges are seen to cross one another in the field of view of the microscope. When the forks are vibrating, a modification of LISSAJOUS' figure is seen in the angle made by the two images. If the forks are nearly of the same pitch, the corresponding LISSAJOUS' figure, obtained in the usual way, is, as is well known, a straight line changing to an ellipse which passes into a circle, then to another ellipse, and finally to another straight line inclined the opposite way to the first. When such a pair of forks, nearly in unison, is under observation, one of the lines is represented by a bright line bisecting the angle (due apparently to a diffraction phenomenon at the edge of each fork); this line then becomes double, and changes into an ellipse and finally to a circle. Directly the change from the line commences, the angle between the forks becomes filled by a dark shadow. The circle appears surrounded by a bright line, but the second ellipse is bounded on one side by a dark shadow, and on the other side by a bright line. At that part of the figure corresponding to the line lying across the angle, the angle itself appears to be filled up by a dark shadow with a well-defined edge. The figure then passes through the changes in the reverse order. The instant when the angle is free from shadow is very sharply defined, and, starting from this point, it is quite easy to count the number of optical beats which take place in a given time. The two forks may be kept in vibration by careful bowing, an assistant being required to bow the horizontal fork.

With two forks, one the octave of the other, the parabola seems to rest like a dark shadow alternately on the two forks. At these instants the angle is free from shadow; the figure of 8 is also perceptible surrounded by a bright line.

We found that the fifth could also be compared in a similar manner, but the observation was not quite so easy. In this case the sharpest indication was the filling up of the angle by shadow. With more complex relations between the two forks no sufficiently visible phenomena were perceptible.

Although, undoubtedly, great accuracy may be attained in counting beats by the ear, in some respects the optical method is more serviceable; it also permits of an extended observation, as the forks may be bowed without altering their phase.

As the question of absolute pitch has recently attracted much attention, and as tuning-forks are becoming more and more employed for measuring small intervals of time, we hope that the experiments above described may prove a useful contribution to the knowledge of the subject.

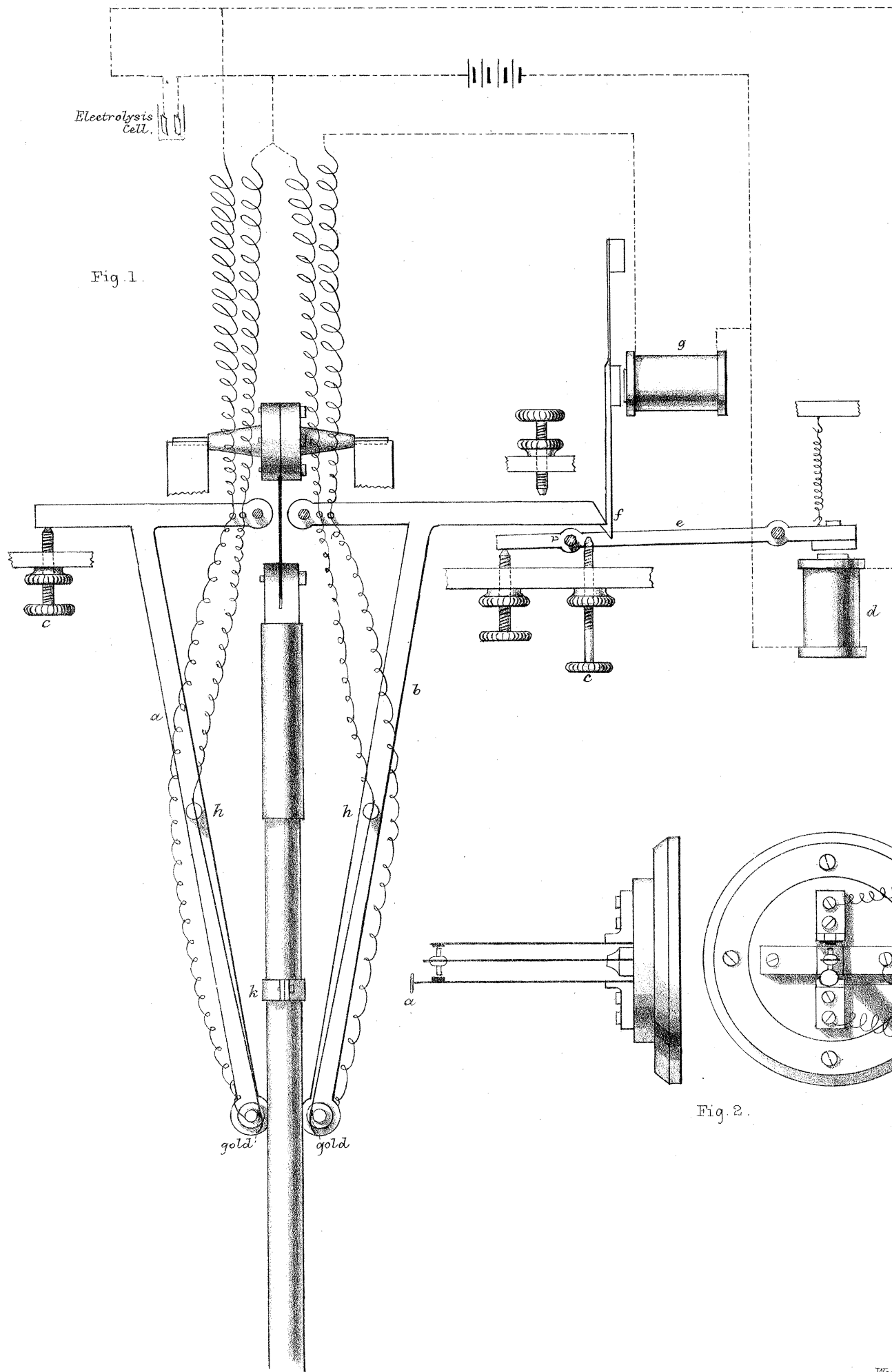


Fig. 1.

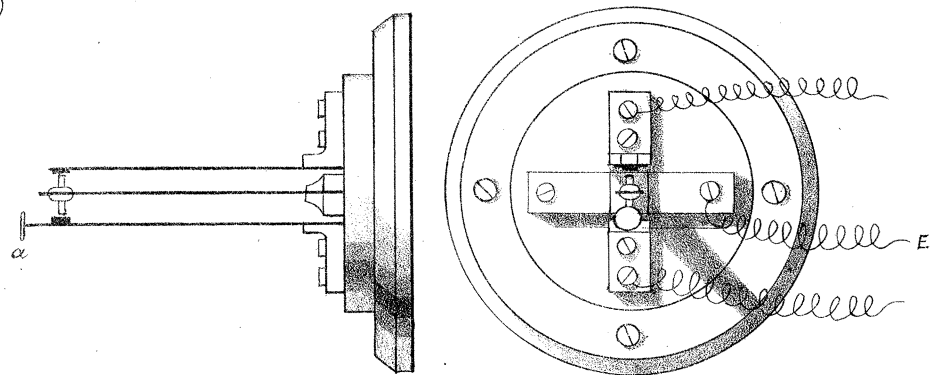
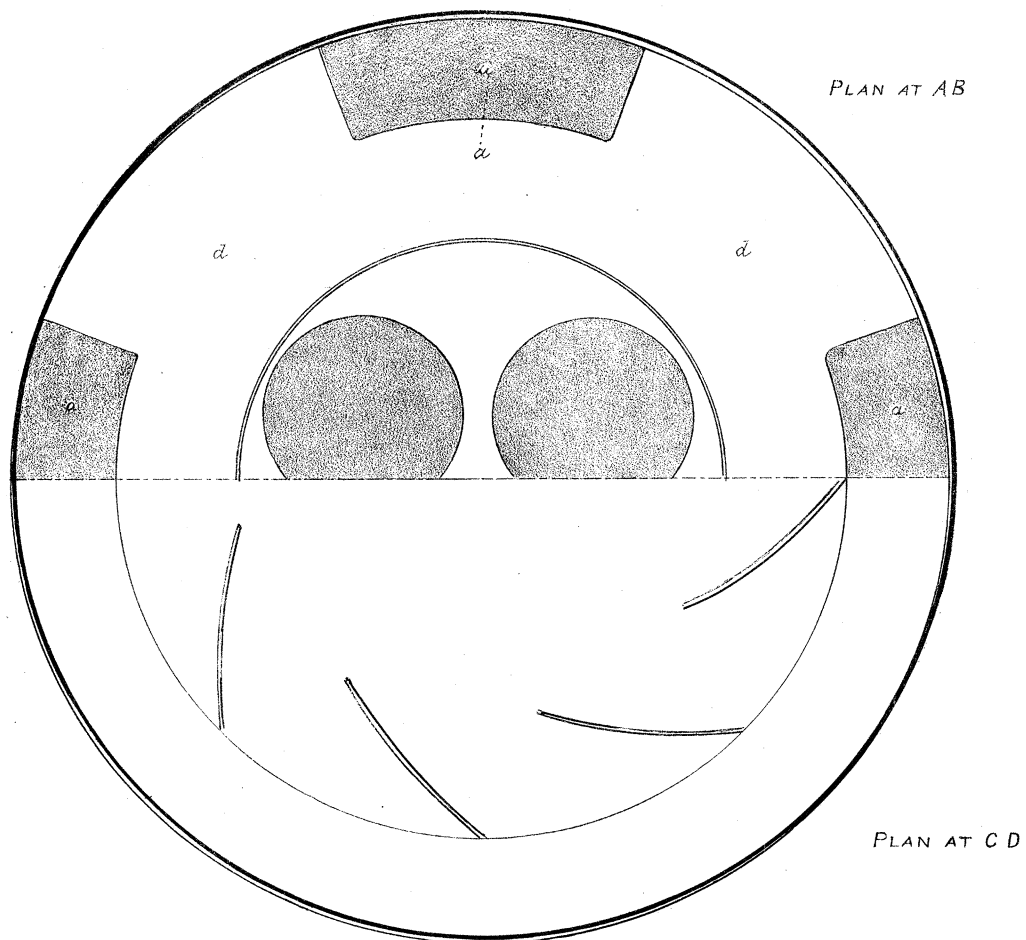
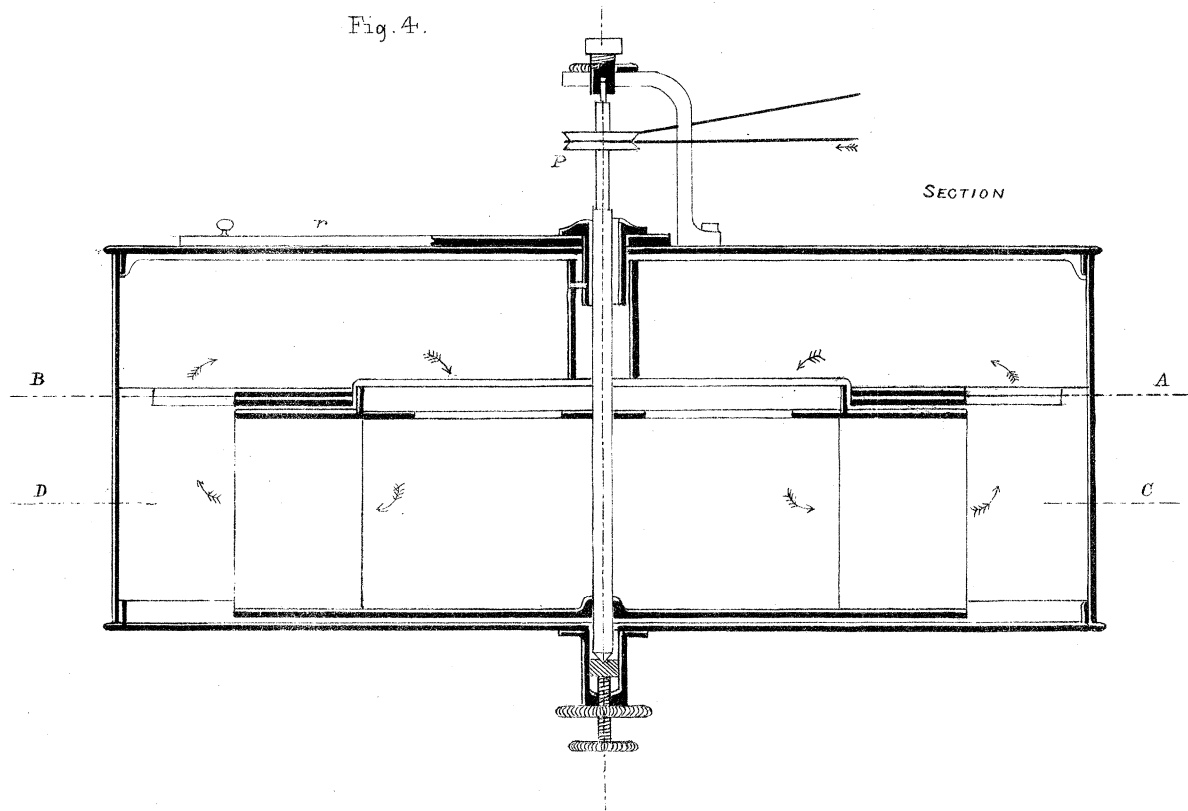


Fig. 2.

Fig. 4.



Chronograph Drum.

Lamp

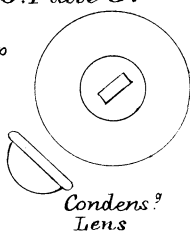


Fig. 9.

Clock Relay

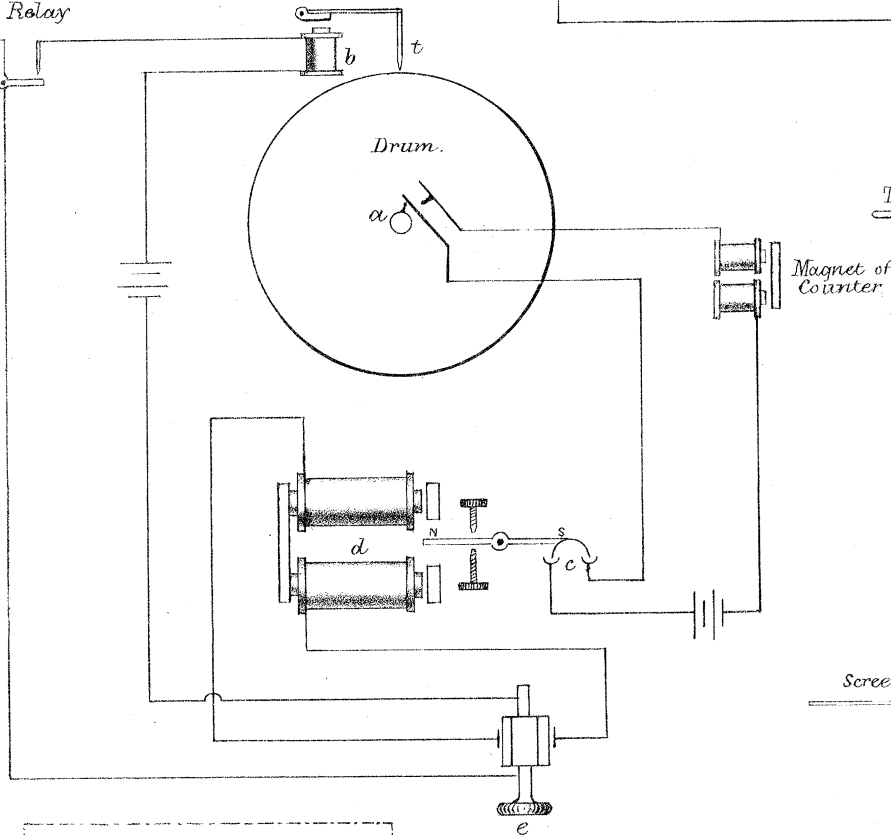


Fig. 8.

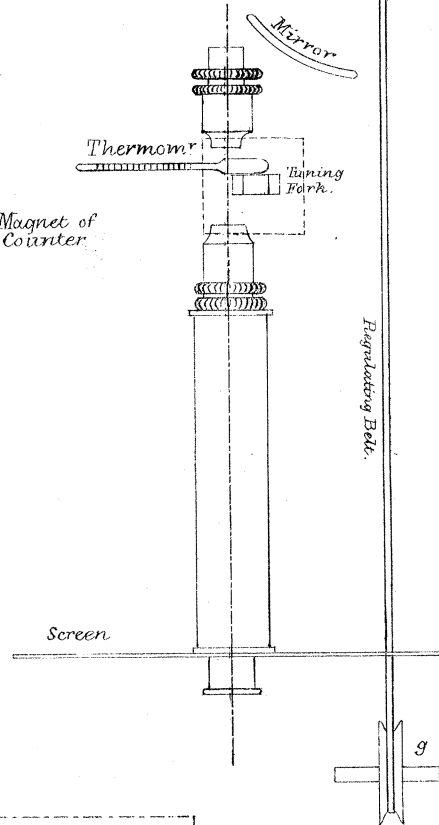


Fig. 3.

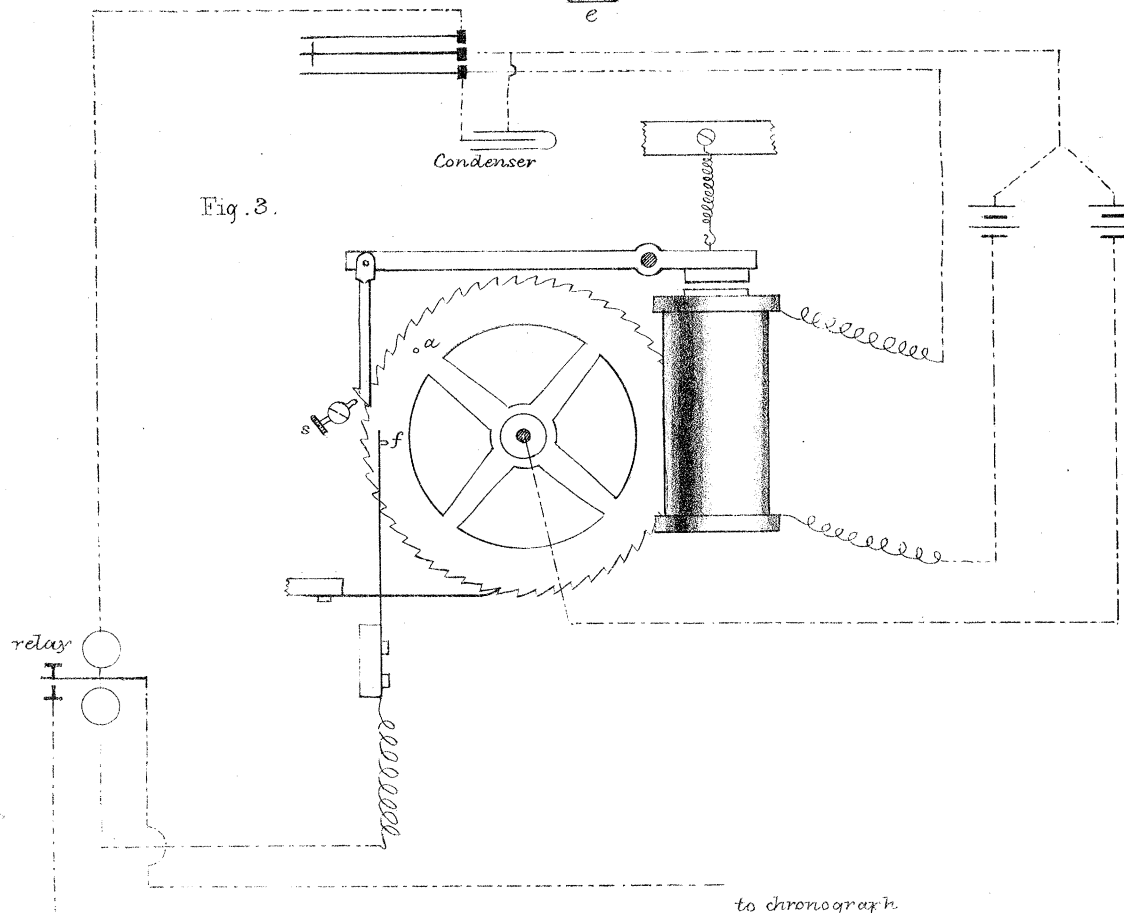


Fig. 6.

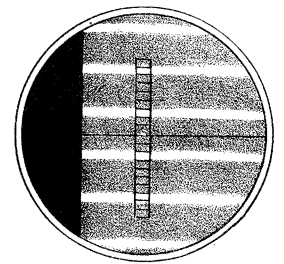


Fig. 7.

