

taking into consideration the whole surface of the globe, *the process of mountain-making diminishes with the increase of the time, and so also does the rate of continental evolution.*

(25) It cannot be said that the contraction theory on the hypothesis of solidity is entirely free from objections. Two very obvious ones have already been alluded to in the course of this paper, namely (1) The small calculated depth of the unstrained surface, especially in early geological periods; and (2) The small proportion of folded rock to stretched rock directly produced by secular cooling. But I do not think that these objections are by any means fatal to the theory. Assuming the Earth to be practically solid, and to have been originally at a high temperature throughout, I believe it may be concluded that the peculiar distribution of strain in the Earth's crust resulting from its secular cooling has contributed to the permanence of ocean-basins, and has been the main cause of the growth of continents and the formation of mountain chains.

VIII. *Note on Mr. DAVISON's Paper on the Straining of the Earth's Crust in Cooling.*

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MR. DAVISON's interesting paper was, he says, suggested by a letter of mine published in 'Nature' on February 6, 1879. In that letter it is pointed out that the stratum of the Earth where the rate of cooling is most rapid lies some miles below the Earth's surface. Commenting on this, I wrote:—

"The Rev. O. FISHER very justly remarks that the more rapid contraction of the internal than the external strata would cause a wrinkling of the surface, although he does not admit that this can be the sole cause of geological distortion. The fact that the region of maximum rate of cooling is so near to the surface recalls the interesting series of experiments recently made by M. FAVRE ('Nature,' vol. 19, p. 108), where all the phenomena of geological contortion were reproduced in a layer of clay placed on a stretched india-rubber membrane, which was afterwards allowed to contract. Does it not seem possible that Mr. FISHER may have under-estimated the contractibility of rock in cooling, and that this is the sole cause of geological contortion?"

Mr. DAVISON works out the suggestion, and gives precision to the general idea contained in the letter. He shows, however, that there is a layer of zero strain in the

Integrating, with regard to time, from the time t to zero, we shall get the total amount of stretching in the layer x from the epoch of consolidation down to time t . Thus

$$K = \epsilon \int_x^\infty \left(1 - \frac{3x}{c}\right) \frac{dv}{dx} dx. \quad (5)$$

Now with THOMSON'S notation

$$\int_x^\infty \frac{dv}{dx} dx = V - v.$$

Since

$$\begin{aligned} \frac{dv}{dx} &= \frac{V}{(\pi\kappa t)^{\frac{1}{2}}} e^{-x^2/4\kappa t}, \\ \int_x^\infty x \frac{dv}{dx} dx &= \frac{V}{(\pi\kappa t)^{\frac{1}{2}}} \int_x^\infty x e^{-x^2/4\kappa t} dx = \frac{2V\kappa t}{(\pi\kappa t)^{\frac{1}{2}}} e^{-x^2/4\kappa t} = 2\kappa t \frac{dv}{dx}. \end{aligned}$$

Hence (5) becomes

$$K = \epsilon \left[V - v - \frac{6\kappa t}{c^2} \cdot c \frac{dv}{dx} \right] \cdot * \quad (6)$$

This expression gives us the total amount of contraction since consolidation in terms of the temperature and the temperature gradient. I shall return to this expression later.

Differentiating (6) with respect to the time, we have

$$\frac{dK}{dt} = \epsilon \left[-\frac{dv}{dt} - \frac{6\kappa}{c} \frac{dv}{dx} - \frac{6\kappa t}{c} \frac{d^2v}{dx dt} \right].$$

But

$$\frac{dv}{dt} = -\frac{x}{2t} \frac{dv}{dx}, \quad \text{and} \quad \frac{d^2v}{dx dt} = \frac{1}{2t} \left(\frac{x^2}{2\kappa t} - 1 \right) \frac{dv}{dx}.$$

Hence

$$\frac{dK}{dt} = \frac{\epsilon}{2t} c \frac{dv}{dx} \left[\frac{x}{c} - \frac{12\kappa t}{c^2} - \frac{6\kappa t}{c^2} \left(\frac{x^2}{2\kappa t} - 1 \right) \right],$$

or

$$\frac{dK}{dt} = \frac{\epsilon}{2t} c \frac{dv}{dx} \left[\frac{x}{c} - \frac{3x^2}{c^2} - \frac{6\kappa t}{c^2} \right] \quad (7)$$

When x exceeds more than a small fraction of the Earth's radius dv/dx is very small; hence in (7) we may regard x/c as small, and write the equation

$$\frac{dK}{dt} = \frac{\epsilon}{2t} c \frac{dv}{dx} \left[\frac{x}{c} - \frac{6\kappa t}{c^2} \right] \quad (8)$$

* If x had not been treated as small, we should have got

$$\epsilon (V - v) \frac{c(c^2 + 6\kappa t)}{(c - x)^3} - \frac{2\kappa t}{(c - x)^3} \left[\frac{c^3 - (c - x)^3}{x} + 4\kappa t \right] \epsilon \frac{dv}{dx},$$

and it is easy to see that this leads to the same result as the above for all practical purposes.

When $x = 0$, dK/dt is negative, and hence at the surface there is contraction or crumpling. The crumpling continues for some depth downwards, and vanishes when

$$\frac{x}{c} = \frac{6\kappa t}{c^2}.$$

Taking, with THOMSON, the foot and year as units, κ appears to be about 400; if, therefore, t is τ million years, $\kappa t = 4 \times 10^8 \tau$. Now, c being 21×10^6 feet, $\kappa t/c^2 = \tau/10^6$ approximately, and

$$\begin{aligned} x &= \frac{24 \times 10^8}{21 \times 10^6} \tau \text{ feet,} \\ &= 114 \tau \text{ feet.} \end{aligned}$$

If τ be 100, $x = 2$ miles.

Thus, if the time since consolidation be 100 million years, the present depth of the stratum of no strain is 2 miles, and the depth is proportional to the time since consolidation. With a greater value of κ the depth is greater.

With regard to the value of dK/dt at greater depths, we observe that at a few miles below the stratum of zero strain $6\kappa t/c^2$ becomes negligible compared with x/c . Hence dK/dt is approximately proportional to $x dv/dx$. Now, if we take the figure drawn in THOMSON and TAIT'S 'Natural Philosophy,' Appendix D, and augment or diminish each ordinate NP' proportionately to the corresponding abscissa, it is clear that we shall get just such a curve as that drawn by Mr. DAVISON. His curve might thus have been drawn without any computations at all. The function $x dv/dx$ is proportional to dv/dt , and thus his dotted curve is of just the same kind as the other, excepting close to the surface.

Now let us return to the expression for the integral stretching, viz. :—

$$K = \epsilon \left[(V-v) - \frac{6\kappa t}{c^2} c \frac{dv}{dx} \right] \dots \dots \dots (9)$$

We have

$$\frac{dv}{dx} = \frac{V}{(\pi\kappa t)^{\frac{1}{2}}} e^{-x^2/4\kappa t}.$$

Hence, if x be small and t large (both of which conditions apply to the present time near the Earth's surface),

$$\frac{dv}{dx} = \frac{V}{(\pi\kappa t)^{\frac{1}{2}}}.$$

Hence

$$K = \epsilon \left[V - v - \frac{6(\kappa t)^{\frac{1}{2}}}{c\pi^{\frac{1}{2}}} V \right] \dots \dots \dots (10)$$

Now, as we have seen above, with such values as those with which we have to deal, $6\kappa t/c^2$ is a small fraction, notwithstanding that it increases with the time.

the total amount of crumpling at any depth, we require to find the integral effect taken from t' to t , which is greater than t' .

The integral stretching from consolidation to time t is

$$K = \epsilon \left[V - v - \frac{6\kappa t}{c} \frac{dv}{dx} \right].$$

Now near the surface v is nearly equal to $v_0 + x \frac{dv}{dx}$;

hence

$$K = \epsilon (V - v_0) - \epsilon \frac{dv}{dx} \left[x + \frac{6\kappa t}{c} \right].$$

But

$$\frac{dv}{dx} = \frac{V}{(\pi\kappa t)^{\frac{1}{2}}} e^{-x^2/4\kappa t} = \frac{V}{(\pi\kappa t)^{\frac{1}{2}}} \text{ near the surface.}$$

Hence

$$K = \epsilon (V - v_0) - \frac{\epsilon V}{(\pi\kappa)^{\frac{1}{2}}} \left[\frac{x}{t^{\frac{1}{2}}} + \frac{6\kappa t^{\frac{1}{2}}}{c} \right]. \quad (12)$$

At the time t' , given by $t' = cx/6\kappa$,

$$K = \epsilon (V - v_0) - \frac{\epsilon V}{(\pi\kappa)^{\frac{1}{2}}} \left[x^{\frac{1}{2}} \left(\frac{6\kappa}{c} \right)^{\frac{1}{2}} + x^{\frac{1}{2}} \left(\frac{6\kappa}{c} \right)^{\frac{1}{2}} \right] = \epsilon (V - v_0) - \frac{\epsilon V}{(\pi\kappa)^{\frac{1}{2}}} \cdot 2 \left(\frac{6\kappa}{c} \right)^{\frac{1}{2}} x^{\frac{1}{2}}. \quad (13)$$

This gives the total stretching from the time of consolidation until the surface of no strain has got down to x .

If, therefore, we subtract (13) from (12), we get the total stretching between the time t' and the time t , and the result is obviously—

$$K = - \frac{\epsilon V}{(\pi\kappa)^{\frac{1}{2}}} \left[x - 2 \left(\frac{6\kappa t}{c} \right)^{\frac{1}{2}} x^{\frac{1}{2}} + \frac{6\kappa t}{c} \right].$$

This is clearly

$$K = - \epsilon \frac{dv}{dx} \left[x^{\frac{1}{2}} - \left(\frac{6\kappa t}{c} \right)^{\frac{1}{2}} \right]^2. \quad (14)$$

This expression is essentially negative, and therefore the total effect from t' to t is a crumpling, as was foreseen.

This integral crumpling vanishes at the same place as does dK/dt , that is to say, when

$$x = \frac{6\kappa t}{c}.$$

This, we have shown above, will be at a depth of 2 or 3 miles. The amount of crumpling at the surface is given by putting $x = 0$, and

$$K = -\epsilon \frac{6\kappa t}{c} \frac{dv}{dx}.$$

Now we have seen that, if τ be the number of millions of years since consolidation,

$$\frac{6\kappa t}{c} = 114 \tau \text{ feet,}$$

and

$$\frac{dv}{dx} = 0.02 \text{ Fahr. per foot.}$$

Hence

$$K = -\tau\epsilon \times 2.28.$$

The total amount by which a great circle is contracted is 25,000 K miles; and, judging by the coefficient of expansion of metals, ϵ may be about 5×10^{-5} .

Thus, with these rough data, the amount of crumpling of a great circle of radius c is

$$2\pi cK = \tau \times \left(\frac{5}{10^5}\right) \times (2.5 \times 10^4) \times 2.28 = 2.85 \tau \text{ miles.}$$

Thus, in 10,000,000 years, $28\frac{1}{2}$ miles of rock would be crumpled up.

The area of rock crumpled is $4\pi c^2 \cdot 2K$, and with these numerical data

$$4\pi c^2 \cdot 2K = 4c(2\pi cK).$$

Now $c = 4000$ miles, and therefore

$$4\pi c^2 \cdot 2K = 22,800 \tau \text{ square miles.}$$

Thus, in 10,000,000 years, 228,000 square miles of rock will be crumpled up and piled on the top of the subjacent rocks.

The numerical data with which we have to deal are all of them subject to wide limits of uncertainty, but the result just found, although rather small in amount, is such as to appear of the same order of magnitude as the crumpling observed geologically.

The stretching and probable fracture of the strata at some miles below the surface will have allowed the injection of the lower rocks amongst the upper ones, and the phenomena which we should expect to find according to Mr. DAVISON'S theory are eminently in accordance with observation. It therefore appears to me that his view has a strong claim to acceptance.