IX. *On the Alleged Slipping at the Boundary of a Liquid in Motion.*

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In treatises on hydrodynamics, the flow of a liquid through a straight tube is investigated on the supposition that there may be finite slipping between the walls of the tube and the outermost layer of liquid. This leads to the introduction of a "slipping coefficient" which vanishes when there is no relative motion between them.

Let r denote the radius of the tube,

p_1 the pressure at one end,

p_2 " " the other,

μ the coefficient of viscosity,

l the length of the tube,

ρ the density of the liquid.

Then it may be shown (Lamb's 'Hydrodynamics, p. 222) that when the motion is linear the flux is given by

$$\tfrac{1}{8}\frac{\pi r^4}{\mu\rho}\frac{p_1 - p_2}{l} + \tfrac{1}{2}\frac{\pi r^3}{\beta}\frac{p_1 - p_2}{l}$$

or

$$\tfrac{1}{8}\frac{\pi(p_1 - p_2)}{\mu\rho l}\left\{r^4 + 4\mu\rho\frac{1}{\beta}r^3\right\},$$

where $1/\beta$ may be defined as the slipping coefficient.

The experiments of Poiseuille* showed that the coefficient was certainly zero for glass tubes, but there was doubt whether this held for all materials.

Helmholtz and Piotrowski† attacked the problem in another way. They suspended bifilarly an accurately worked sphere, whose inner surface was gilded and polished, and by observing the time of swing and the logarithmic decrement when the sphere was filled with water and various other liquids, deduced a value for the

* 'Mémoires des Savants Étrangers,' 1846.

† 'Sitzungsber. der k. Akad. in Wien,' vol. 40, 1860.

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coefficient of viscosity and for the slipping coefficient, from the theory of spheres oscillating in a viscous medium, as worked out by STOKES and HELMHOLTZ.

For distilled water their value of the factor $\mu\rho\, 1/\beta$ in the above expression is $\lambda = 2{\cdot}3534$ mm.

If we apply this to the case of a tube we get a somewhat startling result. From equation (i.) it follows that the effect of slip varies inversely as the radius of the tube. The smallest tube practicable in the experiments to be presently described had a diameter of about a millimetre. It is easy to show that the result of the existence of a slipping coefficient of the magnitude given by HELMHOLTZ would be to produce an increase in the volume of liquid flowing through the tube in a given time, which could not only be detected, but would be of such importance that it could not easily be masked.

In HELMHOLTZ's notation equation (i.) is written

$$\frac{1}{8}\frac{\pi(p_1 - p_2)}{\mu l}\{r^4 + 4\lambda r^3\},$$

the density of water being taken as unity.

Putting $r = {\cdot}05$ and $\lambda = {\cdot}23534$, we get

$$\frac{1}{8}\frac{\pi(p_1 - p_2)}{\mu l} \times 117{\cdot}67 \times 10^{-6};$$

whereas if there is no slip, so that λ vanishes, the flux becomes

$$\frac{1}{8}\frac{\pi(p_1 - p_2)}{\mu l} \times 6{\cdot}25 \times 10^{-6}.$$

If we take HELMHOLTZ's coefficient to be correct, the flow through a polished gilt tube of a millimetre in diameter is nearly *twenty times* as fast as through a glass tube of the same size.

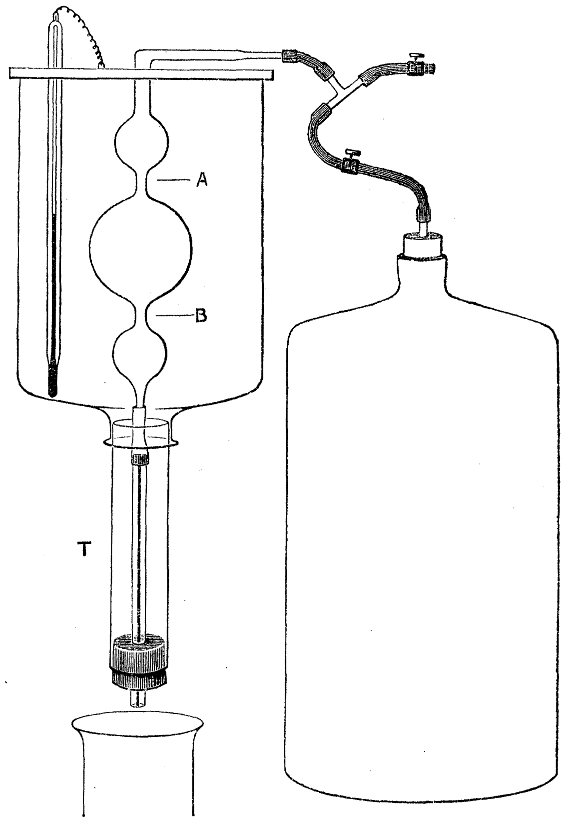
HELMHOLTZ refers to some experiments made by GIRARD with copper tubes* which make the flow some four times faster than do POISEUILLE's formulæ. I shall return to the consideration of these observations later. The value of λ which HELMHOLTZ deduces from them is $0{\cdot}3984$ mm.

The discrepancy between these results and the generally received opinion that no slip occurred with any material seemed worthy of further investigation.

It is evident that the alleged coefficient could be investigated with much greater advantage by observing the flow of liquid through a small tube than by any experiments on oscillating spheres. In order to avoid all absolute determinations while searching for the existence of such a slip, I decided to observe the time of flow of a

* 'Mémoires de l'Institut,' 1813–1815.

given volume of water through a glass tube, and then to deposit a coating of silver on the interior surface of the tube. If the time of flow was the same as before, allowing for the (usually very small) change in diameter, it would be conclusive evidence against the existence of the effect. Such evidence I have most satisfactorily obtained.



Since the experiments were to be merely comparative there was no object in attempting to keep the pressure constant throughout each observation, and the simplest apparatus could be used. The bulb tube AB was fixed in a large glass jar filled with water, and opening below into a wide tube T, also filled with water. The lower end of the bulb tube was attached to the capillary tube by an india-rubber joint, so that the ends of the two glass tubes should just meet. The top of the bulb tube could be put into connexion either with the air outside, or with an exhausted bottle which was used to fill the bulbs, as the apparatus was some distance from a pump. The temperature of the water in the glass jar and wide tube could be read off by a delicate thermometer.

The time taken by the upper surface of the water to fall from A to B was observed by a stop watch. Before silvering the tube the whole apparatus was repeatedly taken to pieces and set up again, and the time of flow shown to be unaltered.

The silvering solution was a modification of LIEBIG'S, and was made according to a

recipe kindly given me by Dr. A. S. LEA. Each tube was dried and weighed, and the silvering solution then run through till a bright metallic mirror was deposited. It was washed out with a current of water, then with air, and finally dried and weighed again. The increase gave the weight of silver. At the end of the experiments with the tube thus silvered, it was again dried and weighed, and the silver dissolved off with nitric acid. The mean of these two results, which usually agreed to one or two tenths of a milligram, was taken to represent the weight of silver adhering to the tube during the experiments. Assuming the deposit to be uniform, this at once gave the change in diameter. The correction for temperature was calculated by POISEUILLE'S formula.

The first tube had a length of 37·57 cms., and an average radius (determined by filling with mercury) of 0·0451 cm.

The following series of experiments were made :—

Temperature.	Time of flow.
18°·2	7′ 50″·8
18·3	7 50·4
18·3	7 50·6
18·2	7 50·4
18·3	7 51·0
18·3	7 51·0
Means 18·26	7 50·7

The apparatus was then taken completely to pieces as it would be for silvering, and again set up after some hours, with the following results :—

Temperature.	Time of flow.
17°·8	7′ 55″·4
17·9	7 55·2

If we correct this to 18°·26 by POISEUILLE'S empirical formula we get 7′ 51″·0, a value identical with the above. The apparatus could thus be taken to pieces with safety.

Weight of tube when dry . . . . . . = 16·3916 grams.
" with silver . . . . . . = 16·3932 "

The apparatus was then again set up.

Temperature.	Time of flow.
18°·4	7′ 52″·6
18·3	7 52·2
18·2	7 52·4
Means 18·30	7 52·4

The tube was then disconnected, dried, and weighed. Weight = 16·3931 grms., practically the same as before. The silver was then dissolved off, and the tube cleaned and dried. Weight = 16·3916.

The weight of silver deposited is, therefore, ·0015 grm., and its thickness ·000014 cm.

This gives a change in r^4 equivalent to 0·12 per cent. The time of flow for the unsilvered tube, corrected for change of radius, is 7′ 51″·4, while the observed time for the silvered tube is 7′ 52″·4. The difference of about 0·2 per cent. is probably due to slight irregularities in the thickness of the silver layer.

Two objections may be raised to this experiment. The first is that with such a thin film, the action between the water and the glass might still be effective, and prevent any slipping. When we remember, however, that the sphere of action of molecular forces is only about 10^{-8} cm., we see that no direct action can occur across a distance of 10^{-5} cm., and it is exceedingly unlikely that a layer of silver more than 1000 molecules thick, should be pervious to water, and thus allow of contact with the glass. In order, however, to entirely meet this objection, experiments were made with considerably thicker layers.

The second objection is that the silver might be deposited so irregularly that the choking effect might mask the quickening due to slip. It had been found, in a series of preliminary experiments, that if a tube was used whose diameter was much less than a millimetre, it was exceedingly difficult to get a uniform silver deposit, and the time of flow for the silvered tube was always much greater than for the unsilvered. To prevent this the tube had to be silvered in a vertical position, and various details, only to be learnt by experience, attended to. The time of flow for the silvered tube could, however, never be brought below that for the plain one, although as greater care was taken in the silvering it continually approximated to it, and this is conclusive against the existence of an effect at all comparable with that given by HELMHOLTZ for polished gold, or with that which he deduced from GIRARD's experiments on copper tubes. As the choking effect was naturally greater for small tubes, a series of experiments was next made on some of rather greater diameter.

Tube No. III.

Unsilvered.		Silvered.	
Temperature.	Time of flow.	Temperature.	Time of flow.
°	′ ″	°	′ ″
20·0	1 2·2	20·9	1 1·8
20·1	1 2·0	20·9	1 1·3
20·1	1 2·2	20·95	1 1·2
20·2	1 2·0	21·0	1 1·8
20·2	1 1·8	21·0	1 1·4
20 3	1 1·7	21·0	1 1·5
20·4	1 1·7	21·0	1 0·9
20·5	1 1·6	21·05	1 1·3
20·5	1 1·7	21·05	1 1·0
20·5	1 1·5	21·05	1 0·8
20·5	1 1·6		
20·6	1 1·5		
20·3	1 1·79	21·0	1 1·37

Thickness of film = ·0000237 cm.
Radius of tube = ·0776 cm.
Change in r^4 = 0·12 per cent.
Temperature correction = ·7 × 2·22 = 1·55 per cent.
Total correction 1·55 − ·12 = 1·43 per cent. = 0·87 second.
Time for glass tube, corrected = 1′ 0″·92
,, observed for tube, silvered = 1′ 1″·37
A change of + 0·7 per cent.

A rather smaller tube was then used, r = ·0642.

	Unsilvered.			Silvered.	
	Temperature.	Time.		Temperature.	Time.
	°	′ ″		°	′ ″
1	22·0	2 8·4	1	20·2	2 13·0
2	22·0	2 8·2	2	20·2	2 13·1
3	22·05	2 8·4	3	20·2	2 12·9
4	22·05	2 8·3	4	20·2	2 13·4
5	22·1	2 8·4	5	20·25	2 13·0
6	22·15	2 8·0	6	20·3	2 13·0
7	22·2	2 8·0	7	20·4	2 12·8
8	22·2	2 8·2	8	20·4	2 12·6
9	22·2	2 8·0	9	20·45	2 12·4
10	22·25	2 7·8	10	20·5	2 12·4
			11	20·5	2 12·8
			12	20·5	2 12·4
Means	22·12	2 8·17	Means	20·34	2 12·82

	′	″
Time for unsilvered tube, corrected for changes in temperature and radius	2	13·59
Time observed for silvered tube	2	12·82

A difference of $-$ 0·6 per cent.

Another series was made with the same tube which gave as the mean of ten observations for each state :—

	′	″
Unsilvered	2	18·78
,, corrected	2	15·92
Silvered, observed	2	16·51

A difference of $+$ 0·4 per cent. The silver was very thin, the change in r^4 being 0·10 per cent.

Thus, as the result of four series of observations with three different tubes, we have that the difference in the times of flow for the silvered and unsilvered tubes is never greater than 0·7 per cent. With both the first and second tubes, and in one series of observations with the third tube, the time of flow is slightly greater (by 0·2, 0·7, and 0·4 per cent.) for the silver surface, while in one case—the first series of observations with the third tube—the time is slightly less (by 0·6 per cent.).

These differences are all within the limits of experimental error, found by comparing the times of flow for the same tube in the same state on different occasions.

On the whole there is some evidence that the time is a little greater for the silvered surface, as would, of course, be the case if the deposit was not quite strictly uniform. In the one case, when the time was less, the temperature difference was largest, and errors likely to be most important.

These experiments may at any rate be considered conclusive against the existence of the large effect, for the existence of which I was searching.

A new series of observations was then undertaken to determine whether any slipping occurred in a silvered tube when the velocity of the water was greater than before, and the gradient of velocity was pushed near the limit beyond which the motion ceased to be linear.

This limit was calculated for each tube by means of a formula given by Professor Osborne Reynolds,* who found by experiment that in order to insure linear motion,

$$D v \rho / \mu \text{ must be } < 1400,$$

where D is the diameter of the tube, μ the coefficient of viscosity, ρ the density, and

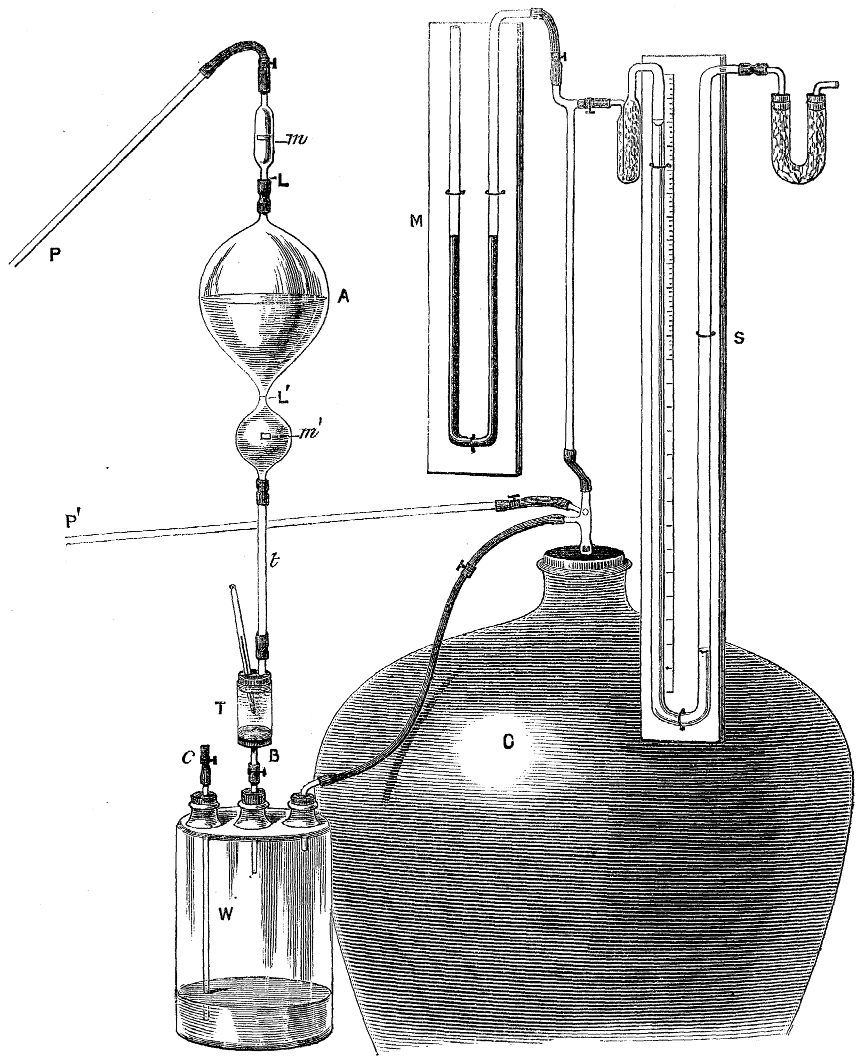
* 'Phil. Trans.,' 1886.

v the velocity of the water calculated from the quantity which flows through in a given time.

It was advantageous to use as small a tube as possible, for two reasons: firstly, because any slipping effect is inversely proportional to the radius, and secondly, because the gradient of velocity can be pushed farther without exceeding the limits of linear motion, the smaller the tube.

A tube, whose diameter was ·084 cm., and whose length was 27·30 cms., was therefore taken, as it was the smallest which could conveniently be silvered, and the greatest allowable pressure calculated.

From the relation given above, we find that the pressure must not exceed $5600\mu^2 l/r^3 g = 303{\cdot}5$ cms. of water column.



After many preliminary trials an apparatus was set up, which gave most excellent results.

The bulb had a capacity of about 850 c.c., and was fixed to a wooden frame to prevent breakage; this was screwed to the edge of a table. The water bath was eventually abolished, and the temperature of the water read off at intervals during each experiment as it passed through a broad tube, into which the capillary tube opened. The temperature of the water was thus read off immediately after it had passed the important place, and by taking readings at equal intervals while the bulb was emptying, a very accurate estimation of the average temperature could be obtained. Below the temperature tube was a three-necked WOLFF's bottle W; the second neck was connected to a large carboy C, and the third could be put into communication with the atmosphere. The carboy was connected to a three-way tube, the branches of which went, one to the WOLFF's bottle, one to an air pump P′, worked by the water supply, and one to the gauges S and M. The gauge S contained sulphuric acid, and M contained mercury. The latter was only used for the experiments in which higher pressures were employed.* The bulb was filled by disconnecting the joint B, and putting the lower orifice of the temperature tube in communication with c, the tube joining C and W being stopped. P was then put to the pump, and the water in W sucked up. When the bulb was filled, B was again connected, the carboy exhausted by putting P′ to the pump, and the tube from C to W opened. The pump was worked till the requisite pressure, as shown by the gauge, was reached.

The apparatus was then left till the temperature had become constant after the disturbances produced by exhausting, and the height of the gauge read off by a kathetometer. The pressure could be adjusted to within a millimetre or less by regulating the pumps, and small differences in corresponding experiments, were given by the readings of the kathetometer. The correction to be applied to the times of flow for a given small difference of pressure, was determined by observing the actual times of flow for pressures whose difference was considerably greater than that in the experiments to be compared, and keeping all other things unchanged. The small pressure correction could then be accurately estimated from the result of this auxiliary experiment. As before, the experiments were to be only comparative, and the pressure was allowed to fall during each observation. Any change introduced by this would affect the tube equally whether plain or silvered. In order to show the method of working and the degree of accuracy obtained, a complete account of an experiment is given in detail.

* All permanent joints, corks, &c., were thickly covered with marine glue, and were quite air-tight.

Tube No. 3.

9. *Time.*—Start 0′ 0″; finish 15′ 23″·0 . . . 15′ 23″·0.
Pressure.—Gauge scale, adjusted at start to 22·00, reading at finish 21·47.

$$\text{Kathetometer readings—Start}\left\{\begin{matrix}22\cdot680\\22\cdot583\end{matrix}\right\}22\cdot682\left.\begin{matrix}\\ \\ \\ \end{matrix}\right\} \quad . \quad . \quad 22\cdot956.$$
$$\text{Finish}\left\{\begin{matrix}23\cdot229\\23\cdot230\end{matrix}\right\}23\cdot230$$

Temperature readings at intervals of two minutes—

13·00 12·88 12·86 12·90 12·92 12·93 12·95 12·98 . . 12°·93.

In order to make the pressure readings strictly comparable, the level of the water before each experiment was adjusted to a mark, *m*, in the top bulb, and after each to a mark *m′* in the lower bulb, and all pressure readings were taken while the water stood at these levels. The pressure was adjusted by the pump till the reading on the gauge scale was as nearly as possible 22·00, and the exact height then read off by a telescope in terms of kathetometer scale.

Tube No. 1.—($r =$ ·042 cm.; $l =$ 27·30 cms.)

Total difference of pressure equivalent to about 250 cms. of water column. The limiting pressure is 305 cms.

	Pressure.	Temperature.	Time.
Unsilvered.	mm.	°	′ ″
1. Gauge reading	407·1	12·59	12 25·7
2. " "	407·0	12·71	12 24·1
Silvered.			
3. Gauge reading	407·1	12·79	12 23·5

The temperature correction is 2·22 per cent. for 1° C., and in order to correct (2) to 12°·79, we must subtract 1″·4.

The pressure correction is for 0·1 mm. of sulphuric acid in a total pressure equivalent to about 1400 mm., *i.e.*, 1 in 14,000, and is therefore negligible.

The weight of silver deposited is 0·0008 grm., and the thickness 0·00001 cm., a change in r^4 of 0·1 per cent.

The time for the unsilvered tube corrected = 12′ 23″·4 }
" " silvered " observed = 12 23·5 }

At slightly different pressures—

	Pressure.	Temperature.	Time.
	mm.	°	′ ″
Silvered	406·4	12·52	12 29·7
Unsilvered	406·5	12·62	12 24·3
Unsilvered, corrected for temperature and change in r^4 . . .			12 26·4
Silvered, observed time			12 29·7

Tube No. 3.—(r = ·036 cm.; l = 19·98 cms.)

Total pressure at beginning of each observation 84·15 cms. of water + 21·028 cms. of mercury; equivalent to 368·62 cms. of water. The critical pressure for this tube is 440 cms. of water. The tube was silvered with solutions of half strength very carefully.

	Mean reading of kathetometer.	Temperature.	Time.
		°	′ ″
Silvered	22·963	12·70	15 46·5
	22·956	12·93	15 23·0
	22·955	13·41	15 18·0
	22·966	12·76	15 34·0
	22·960	12·95	15 30·4
Unsilvered.	22·959	13·04	15 16·5
	23·004	13·69	15 7·3
	22·982	13·37	15 11·9
Silvered, corrected for change in radius − 0·16 per cent., for temperature − 0·93 per cent., and for pressure + 0·14 per cent.			15 21·5

In this very small tube there is thus a choking effect which increases the time by about 1 per cent.

The same tube was then re-coated with a very thin deposit which was just transparent to blue light.

Pressure.	Temperature.	Time.
mm.	°	′ ″
23·026	15·31	14 48·2
23·037	15·27	14 50·8
23·032	15·29	14 49·5

	Time.	
	′	″
Unsilvered.—Pressure 23·005 mm.; temperature 15°·64 . . .	14	43·2
,, Corrected for temperature and pressure	14	51·7
Silvered.—Observed time	14	49·5

A decrease of 0·25 per cent.

The change in radius was inappreciable.

Thus the result of four series of experiments at these large differences of pressure is that in three cases the time for the silvered tube comes out slightly greater [by 0·014, 0·44, and 1·0 per cent.], and in one case slightly less [by 0·25 per cent.].

This agreement may be considered a quite satisfactory proof of identity.

In support of his view that there is a finite slipping coefficient HELMHOLTZ refers to some experiments of GIRARD,* who examined the time of flow of water through copper tubes, found that the motion was linear within limits which agree fairly well with those given by REYNOLDS' formula, but got times of flow much less than those observed by POISEUILLE for glass tubes. Thus with a tube whose diameter was 1·83 mm. and length 1790 mm., a quarter of a litre of water flowed through in 624·5 secs. under a pressure of 100 mm. of water and at a temperature of 0°·5, while POISEUILLE's formula gives 2949 secs.

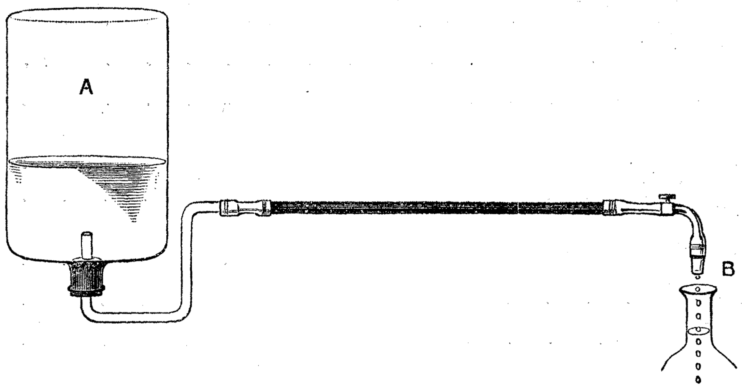
As I could detect no error in GIRARD's account of his experiments I determined to repeat them. Messrs. ELLIOTT, of Sellyoak, Birmingham, most kindly undertook to manufacture some solid drawn copper tubes of the dimensions required, and they were entirely successful. The apparatus employed was essentially the same as that used by GIRARD, and was of the simplest possible nature. The results depended on the value obtained for the diameters of the tubes, as determined by weighing when empty and when full of water, and as no allowance could be made for irregularities or non-uniformities in the bore, calibration being impossible, it was useless to determine the time of flow or the temperature to any great degree of accuracy.

A glass jar was arranged in the manner shown in the figure, and the difference of level between the surface of the water in A and the orifice B determined by a kathetometer to a tenth of a millimetre. Below B was placed a 100 c.c. flask, and the time taken to fill this was observed with a stop watch to an accuracy of about 1 sec. in 400 or 600.

The temperature was observed in the jar A and also in B, and the two seldom differed by more than a tenth of a degree. In GIRARD's investigations the level of the water in A was allowed to fall during each experiment, and its mean value assumed to represent the effective driving pressure throughout. In order to determine whether such an arrangement was allowable for these small pressures, where the change was a large fraction of the whole, a comparison was made with a series of

* 'Mémoires de l'Institut,' 1813–1815.

experiments, during which the level was kept constant by allowing water to flow into A at the same rate as it flowed out.



A glass tube was first used ($r = \cdot 0568$ cm., $l = 22{\cdot}53$ cms.).

Pressure constant.

Level of reservoir, 19·30 ; ,, orifice, 30·23	10·93 cms.
Temperature, 18°·1 ; 18°·1	18°·1
Time: start, 6′ 0″; finish, 17′ 28″	11′ 28″
Value of the coefficient of viscosity μ, deduced from this by POISEUILLE's formula	·01339

Pressure varying.

Level of reservoir {Start, 18·47; Finish, 20·22} 19·35 ; ,, orifice 30·24	10·89 cms.
Temperature, 18°·1 ; 17°·9	18°·0
Time: start, 8′ 0″; finish, 19′ 32″	11′ 32″
Value of μ	·01341

A copper tube was then substituted.

Tube No. 5.—($r = \cdot 0836$ cm., $l = 30{\cdot}86$ cms.)

Pressure constant.

Levels, 20·05 ; 26·57	6·52 cms.
Temperature, 16°·4 ; 16°·4	16°·4
Time: start, 0′ 0″; finish, 5′ 51″	5′ 51″
Value of μ	·01391

Pressure varying.

Levels $\left\{\begin{matrix}18·34\\20·06\end{matrix}\right\}$26·57	7·37 cms.
Temperature, 16°·4 ; 16°·6	16°·5
Time, 6′ 0″ ; 10′ 37″	4′ 37″
Value of μ	·01241
Levels $\left\{\begin{matrix}20·06\\21·78\end{matrix}\right\}$26·56	5·64 cms.
Temperature, 16°·5 ; 16°·5	16°·5
Time, 1′ 0″ ; 7′ 7″	6′ 7″
Value of μ	·01258

Pressure constant.

Levels, 23·42 ; 26·56	3·07 cms.
Temperature, 16°·5 ; 16°·6	16°·6
Time, 7′ 0″ ; 18′ 33″	11′ 33″
Value of μ	·01293
Levels, 21·56 ; 26·56	5·00 cms.
Temperature, 16°·6 ; 16°·6	16°·6
Time, 0′ 0″ ; 6′ 54″	6′ 54″
Value of μ	·01258

Pressure varying.

Levels $\left\{\begin{matrix}21·78\\23·48\end{matrix}\right\}$26·56	3·93 cms.
Temperature, 16°·6 ; 16°·6	16°·6
Time, 7′ 0″ ; 15′ 59″	8′ 59″
Value of μ	·01288

Thus the experiments, both with the glass and with the copper tube, show that the time of flow is the same if the pressure be allowed to fall, as it is if the pressure be kept constant at the mean value of the falling pressure.

The results also show that the copper tube gives a value for μ practically identical with that given by the glass tube, and a little greater than that given by POISEUILLE'S experiments, instead of about five times less.

The effect of modifying the interior surface was then investigated. Tubes were cleaned with acids and alkalies, polished with emery powder, coated with a film of oil, and amalgamated with mercury.

Tube No. 1.—($r =$ ·0803 cm. ; $l =$ 30·9 cms.).

Average pressure.	Temperature.	Time of flow.	Value of μ.
cms.	°	′ ″	
16·825	13·2	2 46·0	·01448
15·320	16·6	3 3·3	·01456

The tube was then cleaned with dilute nitric acid. The radius was re-determined, but was only changed by 0·08 per cent.

Average pressure.	Temperature.	Time of flow.	Value of μ.
cms.	°	′ ″	
17·215	14·0	2 37·5	·01406
16·37	14·6	2 42·8	·01382

Cleaned with hydrochloric acid and potash.

Average pressure.	Temperature.	Time of flow.	Value of μ.
cms.	°	′ ″	
17·86	13·7	2 32·5	·01412

Another tube was then taken and polished inside by working it along a stretched string, covered with fine emery powder.

Tube No. 2, polished with emery powder.—($r =$ ·08094 cm. ; $l =$ 23·30 cms.)

Average pressure.	Temperature.	Time of flow.	Value of μ.
cms.	°	′ ″	
16·43	13·0	2 9·5	·01510
14·69	13·0	2 23·6	·01497

The inside was then coated with a film of oil. The radius was re-determined, $r =$ ·08003 cm.

Average pressure.	Temperature.	Time of flow.	Value of μ.
cms.	°	′ ″	
16·56	13·6	2 13	·01494
15·07	13·6	2 27	·01502

Tube No. 5 was then cleaned with acid and amalgamated by leaving it for some time filled with mercury, and running a stream of mercury through several times.

The radius was re-determined, $r = \cdot 0833$ cm.

Average pressure.	Temperature.	Time of flow.	Value of μ.
cms.	°	′ ″	
3·36	15·4	10 56	·01324
3·35	15·5	10 54·5	·01317
4·94	15·7	7 12	·01282

In the experiments with the same tube described above, when the surface was copper, the following results were obtained at similar pressures :—

Average pressure.	Temperature.	Time of flow.	Value of μ.
cms.	°	′ ″	
3·07	16·6	11 33	·01293
3·93	16·6	8 59	·01288
5·00	16·6	6 54	·01258

Thus in none of these experiments does the value of μ differ much from that given by POISEUILLE for glass tubes, but, like his, agrees with the formula deduced from the supposition that no slip occurs. In all cases it is slightly greater, which is readily explained by irregularities in the tubes, owing to the difficulty of drawing them. According to GIRARD's results, the value of μ should have about a quarter of the value given by POISEUILLE, but in none of the experiments described in this paper did it fall below POISEUILLE's value, and more decisive still, no change in the nature of the surface changed the rate of flow; this is purely a comparative method, and seems much more reliable than the absolute method of GIRARD, which depends on accurate measurements of the radii of the tubes, differences in pressure, &c. GIRARD only used tubes of two sizes, and gives no account of the means he employed to estimate their radii. At the same time it should be noticed that his values for the two sizes agree fairly between themselves with the supposition that a slipping coefficient exists, whose value is about 0·4 mm. Any constant error in the estimation of the radius, would however be naturally of greater importance in the smaller tube, and may have led to the apparent agreement with the results of an effect, inversely proportional to the radius, and due to the existence of a finite slipping coefficient.

We must now return to the consideration of the experiments of HELMHOLTZ and PIOTROWSKI.

The discussion of the body of their paper I must leave to those with the requisite mathematical knowledge, merely observing in passing that it is remarkable that the value they deduce for the coefficient of viscosity of the liquid itself is considerably

greater (by about one-fourth) than that given by POISEUILLE's experiments. This seems to suggest that some slight modification in the application of the formulæ may be necessary, which will reduce the value deduced for the viscosity of the liquid, and increase that for its adhesion to the vessel to the value requisite for the condition of no slip.

By a preliminary series of experiments PIOTROWSKI claims to have shown that the friction on a body oscillating in contact with a liquid depends on the nature of the surface. He suspended a glass flask bifilarly, filled it with water, and observed the time of swing and the logarithmic decrement. He then silvered the inner surface and repeated his observations. The results are as follows :—

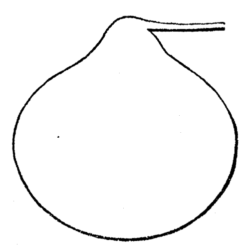
	Time of swing.	Logarithmic decrement.
Unsilvered	23·9333 23·9333	0·0622182 0·0628467
	23·9333	0·0625325
Silvered	24·0088 24·0076	0·0600305 0·0599622
	24·0082	0·0599964

As the result of these observations, PIOTROWSKI calculates that the ratio of the friction on glass to the friction on silver is as 1 : ·95645.

Independently of the fact that no account is given of any precautions to keep the temperature constant, or even to measure it, it is evident that the above determination is liable to errors due to changes in the suspension, which are very apt to occur, and that the agreement between the pairs of readings is not very close.

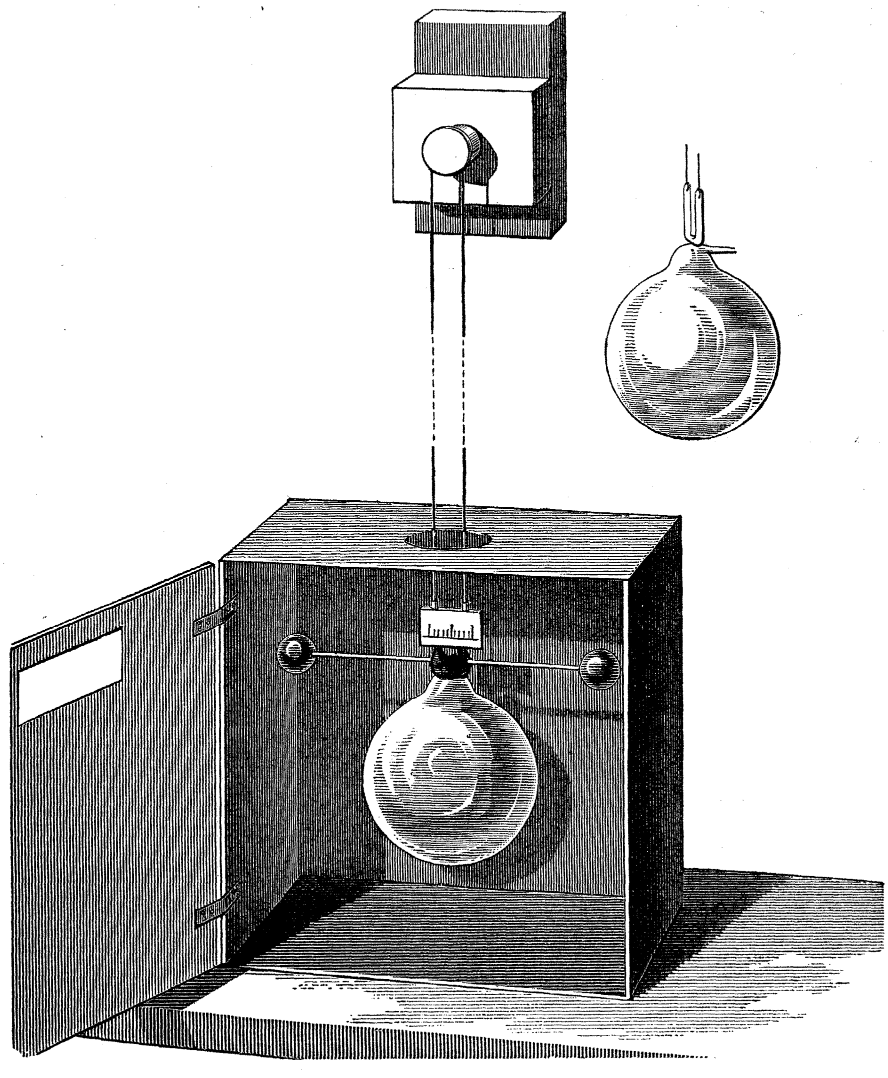
However, in order to test whether such an effect were appreciable, I undertook a series of experiments with an apparatus similar to that used by PIOTROWSKI.

A glass bulb was blown as nearly as possible spherical, and the neck drawn off



sideways into a fine tube. It was filled with water by means of an air pump, and always kept completely full; when left, a piece of india-rubber tubing filled with water was attached, so that if the temperature of the room sank, water, and not air was drawn in. During working the temperature was always slowly rising, and before

each observation the drop of water was removed by blotting paper. At any given temperature the apparatus was therefore in a definite state, and this was obtained at much less expenditure of time and patience than if the bulb had been always filled to a certain mark at a certain temperature, or the same mass of water always put in by adjusting the weight. A mirror was attached to the bulb by sealing wax, and the whole suspended bifilarly by a fine copper wire. The logarithmic decrement had to be reduced by suspending two brass balls at the end of a long bar magnet which was fixed to the bulb and by means of which the apparatus was set in oscillation.



A series of preliminary observations gave as the ratio of the frictions 1 : 1·0001, an accuracy in the proof of identity which was not justified by the roughness of the observations, but which, at any rate, showed that the difference could not be very great.

After a week spent in preliminary investigation, the apparatus was set up in the

manner which had been found to answer best, and a series of observations taken, one of which is given in detail :—

TIMES OF TRANSIT.

32′ 1″·5	10″·5	19	28	37	45·5	54·5	3·5	12	21	30	38·5
35′ 6″·5	15″·5	24	32·5	41·5	50	59	8	16·5	25·5	34	43

∴ Time of 1 vibration = 8″·794.

Logarithmic decrement.—Zero 1·0 ; at end 1·0. Readings at one end of swing :—

42·2	36·7	32·0	27·9	24·3	21·2	18·5	16·16
14·17	12·40	10·90	9·58	8·44	7·44	6·60	5·85
5·20	4·63	4·16	3·74	3·37	3·06	2·77	

∴ Logarithmic decrement = ·14273.

The bulb was kept in a beaker of water whose temperature could be easily observed, till just before each observation, when it was rapidly dried with blotting paper, allowed to come to rest, and set oscillating by means of a strong bar magnet. The limit of each swing was read by means of a telescope mounted at the centre of a curved scale at about two metres distance, and the times of transit over the centre of the scale taken by a chronometer.

The following determinations were made :—

	Temperature.	Time of swing.	Logarithmic decrement.
Unsilvered	13·5	8″·781	·14224
	13·6	8·783	·14063
	13·6	8·787	·14115
	12·8	8·801	·14269
	12·9	8·796	·14328
		8·806	·14276
	13·0		·14204
			·14278
	13·1	8·798	·14305
			·14307
			·14289
	13·2	8·794	·14273
			·14192
			·14125
	13·8	8·732	·14066
	14·0	8·736	·14052
	14·0	8·740	·14094
	14·2	8·732	·14106
	14·2	8·750	·14093
			·14090
Silvered	12·6	8·764	·14338
			·14310
	12·6	8·768	·14274
			·14380
	12·9	8·837	·14304
	13·1	8·810	·14270
	14·1	8·829	·14232
	13·9	8·828	·14125
	13·8		·14191
	14·0	8·806	·14143
	14·1	8·808	·14115
			·14121
Unsilvered	14·0	8·823	·14167
	14·3	8·830	·14126
	14·4		·14101

If we take the means of these observations we get :—

	Temperature.	Time.	Logarithmic decrement.
Unsilvered	13°·66	8″·779	·141801
Silvered	13·40	8·806	·142335

From the series of observations with the unsilvered flask we find that the alteration in the logarithmic decrement for a change in temperature of 1° C. is about ·00160. For 0°·26 the change will be ·000416, and the logarithmic decrement of the unsilvered bulb corrected to a temperature of 13°·40 is ·142217.

By HELMHOLTZ and PIOTROWSKI's paper we see that the ratio of the friction on silver to the friction on glass is as $1 : \frac{8 \cdot 806 \times \cdot 142217}{8 \cdot 779 \times \cdot 142335} = 1 : 1 \cdot 0022$.

The change, if it exists at all, is according to these experiments less than 0·3 per cent.

A modification of PIOTROWSKI's experiment was then tried. Instead of filling the oscillating flask with water, it was filled with sand, and oscillated as a rigid body in a large beaker of water. The temperature could then be accurately observed, and the ordinary investigation of the oscillations of a rigid body in a resisting medium, and acted on by a force proportional to the displacement, will hold.

Let k denote the frictional force proportional to the velocity,

M the moment of inertia,

λ the logarithmic decrement.

Then it is easily shown that

$$k = \frac{\lambda M}{\pi} p,$$

where

$$p = \sqrt{\left(\frac{\mu}{M} - \frac{k^2}{4M^2}\right)},$$

μ being the force of restitution for unit displacement.

$$\therefore k^2 = \frac{4\lambda^2 \mu M}{4\pi^2 + \lambda^2}.$$

Now in our case λ has a value of about 0·2, and λ^2 can therefore be neglected in comparison with $4\pi^2$,

$$\therefore k = \frac{\lambda}{\pi} \sqrt{(\mu M)} \quad \text{(approximately)}.$$

When the flask is silvered μ is unchanged, as the weight of silver is much too small to appreciably alter the bifilar couple, and, therefore, we get for the ratio of the frictions

$$\frac{k}{k'} = \frac{\lambda \sqrt{M}}{\lambda' \sqrt{M'}} = \frac{\lambda T}{\lambda' T'},$$

where T and T′ are the respective times of vibration.

A thick platinum wire was attached to the bulb, and the bifilar arrangement fixed to this above the surface of the water.

At the conclusion of the experiments, the bulb filled with sand was oscillated in air and the logarithmic decrement found to be a very small fraction of that observed when the bulb was in water. This meets the objection that the chief resistance

might be due to the suspension, and a small change in that part due to the water be inappreciable.

Logarithmic decrement in air . . = ·00284.
Time of swing . . . . = 9·952 seconds.

The logarithmic decrement due to the air, suspension, &c., is only about 2 per cent. of that due to the water.

	Temperature.	Time of swing.	Logarithmic decrement.
	°	s	
Unsilvered	12·6 12·6 12·7	9·766 9·769 9·772	·19745 ·19882 ·19830 ·19795 } (1)
	12·9 13·0	9·741 9·722	·19694 ·19729 ·19685 } (2)
	13·0 13·7	9·737 9·686	·19392 ·19331 ·19330 } (3)
	13·8 13·7 13·8	9·725 9·739 9·731	·19374 ·19333 ·19412 } (4)
	13·18	9·739	·19579
Silvered	13·6 13·5	9·884 9·891	·19588 ·19717 ·19622 } (5)
	13·55	9·887	·19642
Resilvered	12·5 12·6	9·892 9·892	·19945 ·19943 ·19943 } (6)
	12·9 13·0	9·882 9·907	·19811 ·19793 ·19775 } (7)
	14·5 14·6	9·894 9·885	·19406 ·19363 ·19331 } (8)
	9·75 9·75	9·891 9·904	·20688 ·20695 ·20772 } (9)
Unsilvered	10·0 10·0	9·939 9·936	·20687 ·20721 ·20636 } (10)
	12·7 12·8	9·917 9·920	·19881 ·19862 ·19860 } (11)

Between each of the series of observations marked (1), (2) . . . the suspending wire was re-adjusted, but an inspection of the results shows that they agree well among themselves, and that therefore the bulb might safely be moved for silvering. After

the observations marked (4) the bulb was suspended in the silvering solution for about an hour, and then removed. It was then found that the top hemisphere was covered with a black sediment from the solution on top of the silver, while the under hemisphere was silvered as usual. Nevertheless a series of three determinations of the logarithmic decrement was taken. The mean of these shows an increase in both the logarithmic decrement and in the time of swing, even though the temperature was actually higher. It was thought that this might be due to the black sediment, which could not be removed alone; so the whole deposit was dissolved off and the bulb resilvered, the solution being kept stirred and frequently changed. This time the deposit was bright and uniform. The observations were, however, identical with the last. The only explanation of this (unless we suppose that the friction is about 3 per cent. greater for silver, instead of 4 per cent. less as PIOTROWSKI deduced) is to suppose that a change had occurred in the suspension. To test this, a series of observations (6) to (9) were taken with the silver on to get the temperature correction, and immediately after (9) the beaker of water was removed from under the bulb, and one of nitric acid of the same temperature put in its place without disturbing the suspension. As soon as the silver was dissolved the bulb was washed, and the beaker of water replaced.

A series of observations (10) were at once taken with the glass surface. Thus by comparing (9) with (10) we get a comparison of the friction on glass with the friction on silver, free from all possible errors due to change of suspension, and at temperatures whose difference is only 0°·25. The means are—

	Temperature.	Time of swing.	Logarithmic decrement.
(9) Silvered	9°·75	9″·898	·20718
(10) Unsilvered	10·00	9·938	·20681

The temperature correction for the logarithmic decrement is by (8) and (9) ·002815 for 1° C. or ·00070 for 0°·25.

Therefore the logarithmic decrement for the glass surface corrected to 9°·75 is ·20751.

The ratio of the frictions is $\frac{9·938 \times ·20751}{9·898 \times ·20718} : 1 = 1·00564 : 1.$

Thus the effect is, if it exists at all, less than 0·6 per cent. instead of 4 per cent.

Another independent comparison can be taken between the series marked (6) and (7) and the series marked (11). The means are—

	Temperature.	Time of swing.	Logarithmic decrement.
(6) and (7)	12°·75	9·893	·19868
(11)	12·75	9·922	·19867

The ratio of the frictions is 1·00288 : 1, the change being less than 0·3 per cent. greater for glass. Thus within the limits of experimental error the friction on silver is the same as the friction on glass, and this part of PIOTROWSKI's paper is certainly misleading.

It is evident that the oscillation method is much inferior to that in which the flow of water through tubes is observed. The experiments described in the early part of this paper show that the difference in the time of flow for a glass and silver tube is less than *one-half per cent.*, in a case where the existence of a slipping coefficient of only one-half the magnitude of that deduced by HELMHOLTZ for gold, would make the time of flow for the silver tube about *twelve times* less than the time of flow for the glass tube.

The arguments sometimes used in favour of the contact theory of electromotive force, based on the differences in friction of a liquid on different solid surfaces, must now be admitted to be without value. It is certain that no slip occurs, at any rate in the case of substances which are wetted by the liquid.

In conclusion I must offer my most sincere thanks to Professor J. J. THOMSON and to Mr. GLAZEBROOK for the help they have given me, and the many valuable suggestions they have made.