IX. *The Influence of Stress and Strain on the Physical Properties of Matter.*—Part III. *Magnetic Induction* (continued).—*The Internal Friction of Iron, Nickel, and Cobalt, studied by means of Magnetic Cycles of very minute range.**

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Received August 22,—Read December 5, 1889.
Revised May 19, 1891.

TABLE OF CONTENTS.

PURPOSE OF THE INVESTIGATION.

Three years ago the author had the honour of presenting to the Royal Society two memoirs relating to the internal friction of metals, as studied by the logarithmic

* An abstract of this paper has been published in 'Roy. Soc. Proc.,' vol. 47, p. 13.

In prosecuting the research, the author has received assistance from the "Elizabeth Thompson Science Fund," U.S.A., and from the English "Government Grant." For the aid rendered from these sources he begs to return his most grateful thanks.

31.7.91.

decrement of arc of torsionally oscillating wires;* these were followed by a third, dealing with the effect of magnetisation on the internal friction of iron.† The results recorded in these memoirs seemed to favour the view adopted by G. WIEDEMANN, that the main part of the dissipation of energy met with in a torsional cycle arises from *the permanent rotation of the molecules about their axes*, and served as an encouragement to attempt to ascertain how far the dissipation of energy which accompanies *cyclic changes of magnetisation* would be amenable to the laws which regulate the dissipation of energy in torsional cycles. The cyclic changes of magnetisation which attend on cyclic changes of magnetising force have been studied by EWING and others,‡ but mainly for comparatively great magnetising forces—for forces, in fact, far exceeding those for which there seemed any likelihood of any *simple* laws holding good.

Lord RAYLEIGH has shown, however,§ that when the magnetising force is sufficiently minute the ratio of induction to force (permeability) in iron, tends rapidly to a *finite constant value* as the force diminishes, and that the dissipation of energy in any cycle can be very simply expressed. The author proposed, therefore, to make use of very feeble magnetising forces, and to render their effects sufficiently evident by very sensitive arrangements. Since the same experiments would suffice to determine both the *constant* permeability mentioned above, and the dissipation of energy in any cycle, the two have been studied side by side. The metals which have been examined are iron, nickel, and cobalt; they have been tested not only at the ordinary temperature of the room but also at 100° C. Finally, the effects of mechanical stress on the *constant* permeability and on the dissipation of energy have received somewhat careful attention. The "ballistic" method of observation has been adopted in these researches.

DESCRIPTION OF APPARATUS.

Induction Coils.

Three induction coils were used at different times, the first of which was constructed as follows: A brass tube, slit throughout its entire length, was coated with paper stuck on with shellac varnish; the length of the tube was 360 cm., and the internal and external diameters were 5 and 5·6 cm. respectively. On the paper-covered tube, to within about 1 cm. of each end, was wound uniformly, and in a single layer, a silk-covered copper wire, there being 8·500 turns per cm. of length;‖ this served as

* 'Phil. Trans.,' vol. 177 (1886); 'Roy. Soc. Proc.,' No. 244, 1886 (vol. 40, p. 343).

† 'Phil. Trans.,' vol. 179 (1888), A.

‡ 'Phil. Trans.,' 1885.

§ 'Phil. Mag.,' vol. 23, March, 1887.

‖ The winding of the coils and the counting of the turns were in all cases carefully performed by the author himself.

the primary coil. The primary coil was then coated with paper, and on it, to within 20 cm. of each end, and, in a single layer, were wrapped no less than 2234 convolutions of silk-covered copper wire; this served as the secondary coil.

The second of the induction coils had its primary wire wrapped round the outer of two coaxial copper tubes, each 120 cm. in length, connected at the ends, and enclosing between them an annular air-space 0·6 cm. thick; there were 8·00 turns per cm. of length. Inside the inner of the two coaxial tubes was placed the secondary coil. This consisted of 2494 convolutions, wrapped in three layers, on a thin brass tube, slit from end to end, whose length was 100 cm., and inner diameter, 1·2 cm. This second induction coil was used both for experiments at the ordinary temperature of the room and at 100° C. In the latter case steam was made to pass through a tube soldered into one end of the annular air-space mentioned above, and out again through another tube at the other end, where it was condensed in cold water.

The third induction coil was constructed mainly with the view of testing the metal cobalt, which could not be procured in the same lengths as the iron and nickel. The primary coil was the same as in the last induction coil, but the secondary consisted of a thin brass tube slit throughout its length, round which were wrapped in six layers 5752 convolutions of silk-covered copper wire. The length of the coil was 48 cm., and its inner diameter, 0·9 cm.

The induction coils, when in use, were placed horizontally with their axes at right angles to the magnetic meridian, so that there might be no appreciable effect produced by the earth's magnetic force. The lengths of the primary coils were such that in all cases the metal cores may be regarded as endless.

The Galvanometer.

The magnetic induction was measured by the throw of the needles of a very sensitive Thomson's reflecting galvanometer, whose scale was placed 2 metres from the mirror. For reasons which will be mentioned presently, the time of vibration was made very short,* whilst the damping was rather considerable; but this, as will be seen, did not interfere with the accuracy of the observations.

The Battery.

For most of the experiments, a single Daniell's cell, charged with a saturated solution of sulphate of copper and a semi-saturated solution of sulphate of zinc, was used. The E.M.F. of the cell was compared from time to time with a Latimer Clark's standard cell.

* The needles took about $\frac{3}{4}$th second in swinging out from their position of equilibrium

The Earth Coil.

This was employed for determining in absolute measure the magnetic induction. It consisted of a wooden circular plate carefully turned, and capable of revolving about a horizontal axis. Round the rim of the plate were wound in a single layer seven complete turns of silk-covered copper wire, whose ends were soldered to amalgamated copper discs dipping in mercury cups, and revolving with the circular wooden plate. The plane of the coil was kept in a horizontal position, and at the end of each experiment was quickly turned through 180 degrees; at each turning the throw of the galvanometer was observed. This earth coil reading was the throw corresponding to

$$2 \times 7 \times 607 \times 0{\cdot}4371$$

lines of induction; where 7 is the number of turns, 607 the area of the earth coil in sq. cm., and 0·4371 the local value in C.G.S. units of the vertical component of the earth's magnetic force.*

Arrangement for Demagnetising by the Method of Reversals.

This was the same as that adopted by Mr. BIDWELL in his experiments described in the 'Philosophical Transactions,' vol. 179 (1888), A., p. 207.

MODE OF ANNEALING.

Those of the rods or wires which did not exceed 4 ft. in length were placed inside an iron tube of 1·5 cm. internal diameter; the tube was then put with its axis horizontal, and in a direction at right angles to the magnetic meridian in one of FLETCHER'S new tube gas furnaces. The furnace was 4 ft. in length, and was provided with 40 burners, which were capable of heating the tube and its contents to a temperature of about 1000° C. This high temperature was maintained for several hours, and afterwards the tube and its contents were allowed to cool slowly in the furnace. The longer rods were annealed by Mr. THOMAS ANDREWS, of the Wortley Iron Works, Sheffield, who was good enough to supply three rods of iron (Wortley best scrap iron) and three of steel (FIRTH'S soft cast steel), each 360 cm. in length and 1 cm. in diameter. The rods were enclosed in a wrought iron tube, and placed in a furnace at a temperature of 566° C., and allowed to cool gradually.

* In order to ascertain whether this value might be affected by masses of iron in and about the building, the earth coil readings inside the room were compared with those taken when the coil was turned in a garden some distance away from the house; the two sets of readings did not materially differ.

Relation between Change of Induction and Change of Magnetising Force in any Cycle.

Lord Rayleigh has shown* that provided the magnetising force does not exceed a certain limit, the relation between change of induction and change of force in any cycle is very simple.

If x be any *change* of force, and y the corresponding *change* of induction as the force varies from its maximum value to zero, if the force be always applied in the same direction, or from its maximum value in one direction to an equal maximum in the opposite, if the force be alternately applied in opposite directions, then

$$y = Ax + Bx^2$$

where A and B are constants. Lord Rayleigh experimented with *hard* iron, using the "*magnetometric*" method. When examined by this method, *annealed* iron was found to be a much less satisfactory subject, because the settling down of the iron into a new magnetic state proved to be far from instantaneous. As the main object of the enquiry was to compare the dissipations of energy in torsional and magnetic cycles, and as the laws of the former were deduced from experiments on wires freely vibrating with comparatively short vibration-periods,† it was decided to use the "*ballistic*" method with a galvanometer of *very short* vibration-period, so as to avoid as much as possible the effects of time-lag in the annealed metals. It has been already mentioned that the time taken by the galvanometer needles to swing out from their position of equilibrium was in most of the experiments only $\frac{3}{4}$th second, so that the complete vibration-period was only 3 seconds; but preliminary experiments *very carefully made with complete vibration-periods of* 2, 3, *and* 4 *seconds gave precisely the same results even with most thoroughly annealed iron.* It was accordingly assumed that with a sufficiently short vibration-period the effects of time-lag do not make themselves materially felt in the "ballistic" method.‡ Nevertheless it was thought advisable to make other preliminary trials, for, at first sight, the galvanometer from its very short vibration-period, and the rather considerable damping of its needles seemed unsuitable for observations by the ballistic method, in which it is usual to employ a galvanometer of long vibration-period, and with a small amount of damping. All that is required, however, in order to interpret the readings correctly in absolute measure, is to *compare* the throw of the needles produced by suddenly altering the magnetising force with that produced by suddenly turning the earth coil through 180 degrees; and provided the throw is proportional to the transient current, the reduction to absolute measure will be accurate.

* 'Phil. Mag.,' vol. 23, March, 1887, p. 236.

† Within the limits of the experiments the dissipation of energy was found to be independent of the vibration-period.

‡ It need hardly be said that as a consequence of this the present investigation is entirely concerned with the *instantaneous* changes which are produced by the application or removal of the magnetising force.

This point was closely examined in different ways, and it was found that the deflection of the galvanometer was strictly proportional to the transient current within the limits of the scale.*

The following experiments show that the simple relation between change of magnetic induction and change of force which Lord RAYLEIGH proved to hold good for *hard* iron when tested by the *magnetometric* method will equally hold for *well-annealed* iron, nickel, and cobalt when only the *instantaneous* changes of induction such as are registered in the ballistic method by a galvanometer of short vibration-period are taken into consideration.

Experiment I.

A well-annealed iron rod, 366 cm. in length and 1·116 cm. in diameter, was, as a preliminary,† put through several magnetic cycles whose maximum magnetising forces were $\pm$ 0·07441 C.G.S. units. The magnetising force was changed by steps, which could be accomplished by suddenly altering, by means of a single plug, the resistance in a resistance box connected with the primary circuit. The observations recorded in the next Table were then made.

TABLE I.

Sum of the throws obtained with the iron in the induction coil, as the magnetising force was changed by steps from its greatest value in one direction to its greatest value in the opposite direction. T.	Ditto with the iron out of the coil. C.	T − C.	Total *change* of magnetising force in C.G.S. units, reckoned from its highest value in one direction to an equal value in the opposite direction. x.	Total *change* of induction in C.G.S. units, reckoned from its highest value in one direction to an equal value in the opposite direction. y.	Ditto calculated from the formula $y = 196{\cdot}3x + 88{\cdot}8x^2$.
255·75	15·43	240·32	0·01750	3·454	3·461
528·06	31·60	496·46	0·03570	7·111	7·121
817·63	48·45	769·18	0·05467	10·998	10·996
1126·79	66·25	1060·54	0·07441	15·098	15·098
1438·53	84·05	1354·48	0·09415	19·240	19·286
1741·90	100·90	1641·00	0·11311	23·319	23·340
2036·25	117·08	1919·17	0·13132	27·284	27·307
2322·22	132·50	2189·72	0·14880	31·175	31·175

The numbers given in Table I. are the means of ten sets of observations. The observed changes of magnetic induction agree very closely with those calculated from the formula

$$y = Ax + Bx^2.$$

* Of course if a flat scale be used allowance must be made for the flatness of the scale. Even when the scale is 2 metres from the mirror the correction amounts to more than 1 per cent. when the deflection reaches 300 mm.

† Unless the contrary is stated, it must be understood that in all cases the metals were subjected to *preliminary* cycles of the same range of force as in the actual experiment.

The values of A and B, in the present instance, are 196·3 and 88·8 respectively; they were obtained by substituting in the formula the two values of x, 0·07441 and 0·14880, and the corresponding values of y. Of course, a still closer agreement between calculation and observation could have been obtained by adopting the method of least squares.

Experiment II.

Four pieces of well-annealed nickel wire, each 120 cm. in length and 0·1525 cm. in diameter, were tied together in a bundle and tested in the same manner as the iron with magnetising forces whose extreme ranges were ± 0·4043 C.G.S. units. The wire contained 97·5 per cent. of nickel, and 0·67 per cent. of iron.

TABLE II.—Nickel Wire.

Total change of force reckoned from its greatest value in one direction to an equal value in the opposite direction. x.	Total change of induction ditto. y.	Ditto calculated from the formula $y = 63{\cdot}31x + 4{\cdot}211x^2$.
0·1060	6·74	6·76
0·2085	13·39	13·38
0·3080	19·88	19·90
0·4043	25·60	25·60
0·5006	32·78	32·76
0·6001	39·56	39·51
0·7026	46·54	46·56
0·8086	53·96	53·96

Experiment III.

A strip of well-annealed cobalt, 56 cm. in length, 0·08547 cm. thick, and 0·7285 cm. wide. The strip contained 98 per cent. of cobalt, no trace of nickel, and barely a trace of iron.* The extreme ranges of magnetising force were ± 1·359 C.G.S. units.

* For the analyses of the metals nickel and cobalt, the author is indebted to Mr. J. S. JOHNSON and Professor J. M. THOMSON, of King's College, Strand.

TABLE III.—Cobalt Strip.

Total change of force reckoned from its greatest value in one direction to an equal value in the opposite direction. x.	Total change of induction ditto. y.	Ditto calculated from the formula $y = 65{\cdot}14x + 1{\cdot}26x^2$.
0·2620	17·23	17·16
0·5080	33·51	33·42
0·7395	48·88	48·85
0·9579	63·55	63·55
1·1640	77·54	77·53
1·3590	90·78	90·84
1·5540	104·28	104·28
1·7601	118·58	118·54
1·9785	133·91	133·78
2·2100	150·28	150·12
2·4560	167·69	167·58
2·7180	186·21	186·35

From Tables II. and III. it appears that the formula $y = Ax + Bx^2$ holds good for nickel and cobalt.

We may now regard it as established that not only for *hard iron*, but also for *well-annealed iron, nickel, and cobalt*, the *instantaneous* changes of induction follow the above-mentioned simple law. Lord RAYLEIGH has, moreover, proved that with *hard iron* A and B will have the *same* values for cycles of *different* ranges of force. The next experiments show that this is the case with *annealed iron, nickel, and cobalt* when tested as above.

The values of A and B the same for different ranges of force.

Experiment IV.

Ten pieces of well-annealed iron wire, each 120 cm. in length and 0·1 cm. in diameter, were tied in a bundle and tested with cycles of different ranges of force. As a preliminary the wires were subjected to a series of cycles whose ranges were gradually diminished from ± 0·2144 to zero.

TABLE IV.—Bundle of Iron Wires.

Maximum ± force used in the cycle. C.G.S. units.	A. C.G.S. units.	B. C.G.S. units.
0·01192	129·2	40·6
0·07300	129·3	41·9
0·13932	129·5	40·5
0·21440	129·4	42·0

Experiment V.

The same nickel wire as before, and treated like the iron to preliminary cycles of range gradually decreasing from the highest force to zero.

TABLE V.—Bundle of Nickel Wires.

Maximum ± force used in the cycle. C.G.S. units.	A. C.G.S. units.	B. C.G.S. units.
0·1043	57·58	4·758
0·1924	57·56	4·643
0·3902	57·66	4·835

Experiment VI.

The same cobalt strip as before, and treated like the iron and nickel to preliminary cycles.

TABLE VI.—Cobalt Strip.

Maximum ± force used in the cycle. C.G.S. units.	A. C.G.S. units.	B. C.G.S. units.
0·6195	65·090	1·261
0·8510	65·269	1·273
1·1097	65·086	1·263
1·3590	65·094	1·256

MATHEMATICAL FORMULÆ.

Having thus established that, within the limits of the forces used, the *change* of induction (y) can be expressed in terms of the *change* of force (x), as the force varies by any amount, in one direction going from $+$ X to $-$ X, or from $-$ X to $+$ X, where X represents the maximum force in any cycle by the equation

$$y = \mathrm{A}x + \mathrm{B}x^2 \quad \ldots \ldots \quad (1),$$

and having also shown that A and B are not only constant in any particular cycle, but have the same values for cycles of different ranges of force, it can be easily shown that

$$\mu_0 = \mathrm{A} \quad \ldots \ldots \quad (2),$$

where μ_0 is Lord RAYLEIGH'S *constant* permeability, *i.e.*, the value to which the permeability tends as the force becomes infinitesimally small.

Again, if ΔE denote the dissipation of energy per cubic cm. in any cycle of range of force $\pm X$

$$\Delta E = \frac{2B}{3\pi} X^3 \quad . \quad . \quad . \quad . \quad (3).$$

If the force be always applied in the *same* direction, and have a maximum value X the value of ΔE is, of course, $\frac{1}{8}$th of that in (3).*

Again, each time in a $\pm$ cycle the force is taken from 0 to $\pm X$, the change of induction exceeds the change of induction as the force is taken from $\pm X$ to 0 by a certain amount (P). If 2Y be the whole change of induction as the force varies from $\pm X$ to $\mp X$

$$A = \frac{Y - P}{X} \quad . \quad . \quad . \quad . \quad (4),$$

and

$$B = \frac{P}{2X^2} \quad . \quad . \quad . \quad . \quad (5).$$

These last are convenient equations for determining the values of A and B.

Also, we have

$$\Delta E = \frac{PX}{3\pi} \quad . \quad . \quad . \quad . \quad (6)$$

as a convenient expression for determining the dissipation of energy per cubic cm.

THE LIMIT OF MAGNETISING FORCE WITHIN WHICH A AND B ARE CONSTANT.

It has been shown that with annealed iron, nickel, and cobalt, the values of A and B are the same for cycles of different ranges of force *provided the force does not exceed a certain limit.* It is impossible to define this limit at all exactly for any metal, but the following table will give a rough notion of the value of the limit for *well-annealed* metals.

TABLE VII.

Metal.	Limit of force within which A and B have the same values in cycles of different ranges. C.G.S. units.
Iron	0·1 to 0·4†
Nickel	0·39
Cobalt	1·36

* This was actually proved by experiment.

† Depending upon the amount of "accommodation." See article on "Molecular Accommodation."

THE EFFECT OF ALTERING THE MAGNITUDE OF THE STEPS BY WHICH THE FORCE IS INCREASED OR DIMINISHED.

So long as the force is confined within the limits given in the last table, the *magnitude of the steps by which it is altered is of no moment,** but as soon as these limits are passed the magnitude of the steps becomes material.

Experiment VII.

A bundle of well-annealed iron wires was put through cycles of ranges of ± 0·6 C.G.S. units of force. At first this force was applied and removed in six steps† of equal magnitude; the number of steps was then diminished as in Table VIII.

TABLE VIII.

Number of steps by which the force was applied or removed.	Sum of the throws as the force was applied, starting from zero to + or − 0·6 C.G.S. units. U.	Ditto, as the force was removed. D.	U − D, or P.
6	812·5	655·0	157·5
4	825·0	653·0	172·0
3	837·0	654·0	183·0
2	884·5	657·0	227·5
1	908·0	656 0	252·0

The magnitude of the steps has very little influence on the sum of the throws on the *removal* of the force, but a very perceptible one on the sum of the throws on the *application* of the force, so that the dissipation of energy in the complete cycle, as is shown in the fourth‡ column of the table, is considerably *increased* by decreasing the number of steps.

MOLECULAR ACCOMMODATION.

When a recently annealed wire is vibrating torsionally the logarithmic decrement of arc is greatest at the very beginning of the vibrations, and gradually diminishes until it becomes sensibly constant. During this period the molecules, to use an expression adopted by WEBER, G. WIEDEMANN and others, are "*accommodating*" themselves to the motion

* This point will be further considered in connection with the relations between the dissipation of energy in torsional and magnetic cycles.

† Six for application and six for removal of both the + and − forces, *i.e.*, twenty-four steps in the complete cycle.

‡ See "Mathematical Formulæ," equation (6), where it is shown that the dissipation of energy is proportional to P.

The author has pointed out that the amount of "accommodation" depends, in the case of *iron*, upon the *length of time* during which the wire vibrates, and not merely upon the *number* of vibrations. Thus a well-annealed iron wire when tested within ten minutes after suspension was found to have a logarithmic decrement of 0·003011; after one hour, of 0·001195; and after one day, of 0·001078. After the last-mentioned period the logarithmic decrement became sensibly constant; the wire had apparently "accommodated" itself as far as possible. It was then, however, discovered that on heating the wire to 100° C., oscillating it at this temperature, and then allowing it to cool, next day the logarithmic decrement had considerably diminished. The process of heating and cooling was continued for six days, the wire on each day being maintained at 100° C. for eight hours, and then allowed to cool during the remaining sixteen hours. Ultimately the logarithmic decrement was reduced by this process from 0·001078 to 0·000412. It was by no means necessary to keep the wire oscillating during the whole, or even the greater part of the time, but evidently the reduction was facilitated by oscillations at intervals.

The following experiments were made for the purpose of ascertaining the effect of frequent repetition of a *magnetic cycle* on the dissipation of energy during the cycle, as also the effects of rest, and of the heating and cooling process mentioned above.

Experiment VIII.

A bundle of well-annealed iron wires was, shortly after cooling, put through magnetic cycles of ± 0·3 C.G.S. units of force range. The range of force was purposely taken *slightly* higher than the limit within which A and B have the same values in different cycles (see Table VII.), for in this case the "accommodation" is more striking than with lower ranges.

TABLE IX.—Molecular Accommodation Produced by Repetition of the Cycle.

Number of cycle.*	Sum of the throws as the force was applied starting from zero to ± 0·3 C.G.S. units. U.	Ditto on the removal of the force. D.	U − D, or P.
1	680·0	583·0	97·0
2	662·5	581·5	81·0
3	657·5	585·5	72·0
4	657·0	585·5	71·5

The fourth column shows that the dissipation of energy rapidly decreased with the first three cycles and then became nearly constant. A rest of eight hours, however, still further reduced the friction.

* In this column *one* preliminary cycle is not included.

TABLE X.—Accommodation of iron aided by rest.

Sum of the throws on the application of the force. U.	Ditto, on the removal of the force. D.	U−D, or P.	Remarks.
657·0	585·5	71·5	Before rest.
654·5	587·0	67·5	After a rest of eight hours.

The next experiment shows that when rest and repetition of the cycle have produced their full effect, the "accommodation" can be still further increased by heating and cooling.

Experiment IX.

A bundle of well-annealed iron was accommodated by rest and by frequent repetition of a cycle whose maximum range of force was ± 0·3 C.G.S. units. It was then heated to 100° C. for some time, tested at this temperature, suffered to cool for several hours, and then was again tested. The process was repeated for 25 days with a range of ± 0·3 C.G.S. units of force.

TABLE XI.—Accommodation produced by heating and cooling.

Number of times which the iron had been raised to 100° C.	Sum of the throws on the application of the force. U.	Ditto, on the removal of the force. D.	U−D, or P.
0	485·3	405·0	80·3
1	473·8	401·6	72·2
2	469·2	400·0	69·2
23	398·0	357·0	41·0
24	396·9	356·0	40·9
25	397·0	356·0	41·0

Evidently the effect of repeated heating and cooling on the dissipation of energy in a *magnetic* cycle is quite as striking as that in a *torsional* cycle. There is this difference, however, in the effect of heating and cooling in the two cases. In *torsional* cycles the process has very little effect on the *torsional elasticity*, whereas in *magnetic* cycles the *magnetic susceptibility* is very considerably diminished.

Several experiments in the same direction were now made with different specimens of iron and steel, with the results recorded in Table XII.

TABLE XII.—Accommodating effect of heating and cooling.

Metal.	Remarks.	A in C.G.S. units, before the application of the process.	Ditto, after the process.	B in C.G.S. units, before the process.	Ditto, after the process.
Iron	Ten pieces of wire, each 1 mm. in diameter, annealed by the makers .	126·0	104·8	52·3	26·9
Iron	Ten pieces of the same wire, annealed by passing slowly through a Bunsen's burner	213·0	107·9	76·0	24·9
Piano-steel .	Three pieces, each 1 mm. in diameter, well annealed	135·0	91·6	32·0	16·0
Firth's soft steel	One piece, 1 cm. in diameter, well annealed . .	187·0	181·2	13·0	10·8

There are several points to which it is advisable to call attention in this table. The first is the *very large* reduction of the *constant* permeability, as shown by the values of A (see "Mathematical Formulæ," equation (2)), and the still greater reduction of the dissipation of energy, as shown by the values of B (see "Formulæ," equation (3)), which occur in *bundles of wires* of iron and steel of *small* diameter.

A second point is the comparatively *small* effect of the process on the *rod* of soft steel.*

A third point to be noted is that, though in consequence of difference in the annealing, the values both of A and B for the same specimen may be very different *before* the application of the process, the process tends to bring these values to equality. Heating the wires to a high temperature restored the original values of A and B.

Similar experiments were made with *nickel* and *cobalt*. Both these metals were subjected to the heating and cooling process for several days, but neither of them exhibited any tendency to diminish either in permeability or dissipation of energy.

Nor again did *rest* assist in the "accommodation" which can be produced by repetition of the cycle. Iron, therefore, seems to be unique, both as regards the accommodating effects of rest, and of the heating and cooling process.†

The limit of magnetising force within which the values of A and B are constant (Table VII.) for different ranges of force, can be considerably raised by the heating and cooling process; thus the limit in the case of the iron wires last-mentioned was raised from 0·1 to 0·4 C.G.S. units.

* This did not arise from the fact that the length of the steel rod did not contain the diameter a sufficient number of times. A much longer piece of the same rod gave exactly the same values of A and B.

† This seems to be also true as regards the accommodating effects in *torsional* cycles.

FATIGUE OF ELASTICITY.

According to Sir WILLIAM THOMSON* a wire which has been kept vibrating torsionally for several hours or days through a certain range comes to rest much more quickly when left to itself than when set in vibration after it has been at rest for several days, and then immediately left to itself. This increase of dissipation of energy due to repeated vibrations has been shown by the author not to occur provided the vibrations are kept within sufficiently narrow limits;† but, on the contrary, the effect of the vibrations is in this case to *reduce* the dissipation of energy. "Fatigue of elasticity," in fact, *arises from upsetting molecular "accommodation,"* and occurs whenever the molecules are disturbed beyond a very small limit, *whatever be the cause of the disturbance.* Thus, as has been pointed out,‡ the logarithmic decrement of a torsionally oscillating iron wire is subpermanently increased by oscillating it beyond a certain limit, by a very slight mechanical shock, a small rise or fall in temperature, or a slight rotation of the molecules by magnetic stress. The following experiments show that in a *magnetic* cycle we have exactly the same kind of effect produced by the above-mentioned causes.

Experiment X.

A well-annealed iron rod which had previously been accommodated by repeated cycles, followed by rest, was given a slight tap with a hammer. The result is given in Table XIII.

TABLE XIII.—Upset of Accommodation caused by slight Mechanical Shock.

Sum of the throws obtained by changing the magnetising force from 0 to ± 0·07. U.	Ditto by changing the force from ± 0·07 to 0. D.	U − D = P.	Remarks.
1128·5	1067·5	61·0	Before the tapping
1152·0	1078·0	74·0	Immediately after the tapping and after two preliminary cycles
1145·0	1074·0	71·0	After 2 hours' rest
1138·5	1071·0	67·5	After 7 hours' rest
1124·0	1062·0	62·0	After 48 hours' rest

The upsetting of the "accommodation" by slight mechanical shock, and the restoration of the "accommodation" produced by rest, so well shown in this experiment,

* 'Encycl. Brit.,' 9th edit., Art. "Elasticity," § 30. The phenomenon is designated by Sir WILLIAM THOMSON the "*fatigue of elasticity.*"

† 'Phil. Trans.,' vol. 177 (1886).

‡ *Loc. cit.*

are not confined to *annealed* iron, but take place with unannealed iron and with the hardest steel.

Upset of Accommodation produced by Change of Temperature.

Experiment XI.

A bundle of well-annealed iron wires, which had been thoroughly "accommodated" by the heating and cooling process, was tested with the view of ascertaining whether change of temperature would subpermanently upset the accommodation. This was found to be the case. For whether the iron was raised from the temperature of the room to 100° C. or lowered from 100° C. to the temperature of the room, the dissipation of energy in a cycle was always greater *immediately* after the change than when the temperature was maintained constant for some time. Even a small change of temperature (say two or three degrees) had a sensible effect. This may, in all probability, account for the fact that if the accommodated metal be left for any length of time in the room the "accommodation" is almost always found to have been slightly disturbed, but it may be restored by merely putting the metal through one or two cycles.

Upset of Accommodation produced by Carrying the Magnetising Force beyond Certain Limits.

If a wire has been accommodated by repeated cycles within the limits of force given in Table VII., the accommodation is always disturbed when the force is taken beyond these limits.

Experiment XII.

A well-annealed iron rod was "accommodated" by repeated cycles of $\pm$ 0·074 C.G.S. units range of force. In the hope of increasing the accommodation, the metal was subjected to forces rapidly reversed, beginning with 1·3 C.G.S. units, and slowly diminishing to practically zero, as in the method of demagnetising by reversals. About a thousand reversals took place whilst the force thus varied, but in spite of this the process had just the opposite effect expected of it, as will be seen from the next table.

TABLE XIV.—Upset of Accommodation produced by using the Method of Demagnetising by Reversals.

Sum of the throws obtained by changing the magnetising force from 0 to ± ·074. U.	Ditto by changing the force from ± ·074 to 0. D.	U − D = P.	Remarks.
912·0	880·5	31·5	Before using the method of demagnetising by reversals.
935·0	896·0	39·0	Immediately after using the method and also after two preliminary cycles.
907·5	877·5	30·0	After 48 hours, rest with cycles at intervals.

Several other experiments of the same kind were made with other annealed specimens of iron and steel, and with similar result.

In the last experiment, the upset of "accommodation" produced by using the method of demagnetising by reversals was got rid of with comparative ease, but when the initial force used in the method is at all great, or if the temperature of the metal is at all high, the difficulty of restoring the accommodation may be very much greater.

Experiment XIII.

A bundle of well-annealed iron wires had been partly accommodated by the heating and cooling process. Afterwards, when at a temperature of 100° C., it was subjected to a series of cycles beginning with a maximum force of 1·4 C.G.S. units and ending with zero force as in the method of "demagnetising by reversals." When cool it was again tested with the results given in Table XV.

TABLE XV.—Upset of Accommodation produced by using the Method of Demagnetising by Reversals at the Temperature of 100° C.

Remarks.	Number of days during each of which the heating and cooling process had been applied.	Sum of the throws on changing the force from 0 to ± 0·3. U.	Ditto on removing the force. D.	U − D, or P.
Before demagnetising by reversals	2	469·2	400·0	69·2
After ditto	3	552·5	435·5	117·0
Method of demagnetising by reversals abandoned	4	518·5	420·0	98·5
	8	459·8	391·1	68·7

On consulting the last column in Table XV. we see that the method of demagnetising by reversals is responsible for increasing the dissipation of energy by nearly 70 per cent., and, further, that the upset of the accommodation thus produced requires five repetitions of the heating and cooling process for its removal.*

Providing the forces employed in the method of demagnetising by reversals do not exceed the limits of Table VII., the process of demagnetising by reversals acts beneficially as regards diminution of dissipation of energy in any cycle, as we have already seen, just as preliminary torsional oscillations of small amplitude bring down the logarithmic decrement of arc. Similarly, when the preliminary torsional oscillations exceed a certain limit of amplitude, we have Sir W. THOMSON'S "elastic fatigue," and more or less notable increase of logarithmic decrement. The analogy between "elastic fatigue" in a torsionally oscillating wire and in the same wire when put through magnetic cycles seems to be perfect.

Nickel and cobalt also showed the same disturbance of "accommodation" to be produced by the same causes as in iron.

EFFECT OF ANNEALING ON THE VALUES OF A AND B.

Annealing Iron whilst in the Magnetic Meridian.

It has been already mentioned that the metals annealed by the author were, during the process, lying in a direction at right angles to the magnetic meridian, whilst those annealed by Mr. ANDREWS (the long rods) had, by chance, been placed with their axes making an angle of about 45° with the meridian. Before, therefore, attempting any comparison of the values of A and B for different specimens of iron and steel, it seemed as well to ascertain how far a specimen of iron annealed in the magnetic meridian would differ from the same specimen annealed at right angles to the meridian. It is true that the horizontal component of the earth's magnetic force is not great (about 0·18 C.G.S. units), but it must be remembered that we are in many cases dealing with forces which are less; and, further, that at high temperatures the magnetic permeability of iron for feeble forces may be very much greater than its permeability at the temperature of the room. Very careful investigation, however, showed only a slight difference between the values of A and B for the same specimen when annealed in the two different positions.

The next table exhibits the effects of annealing at various temperatures.

* *Five whole days*, on each of which the metal was kept at 100° C. for eight hours, with cycles at intervals, followed by sixteen hours of cooling, with cycles at intervals, little more than sufficed for the purpose.

Table XVI.—ANNEALING at different temperatures.

Metal.	A in C.G.S. units.	B in C.G.S. units.	Remarks.
Soft iron wire . . .	129·5	42·2	Annealed by the makers (JOHNSON and NEPHEW)
,, ,, . . .	213·0	76·0	Annealed by passing several times slowly through the flame of a Bunsen's burner
,, ,, . . .	240·0	86·5	Annealed in Fletcher's furnace at 1000° C.
,, ,, . . .	77·7	1·8	Hardened by torsion
,, ,, . . .	185·4	18·61	Re-annealed by passing several times slowly through the flame of a Bunsen's burner
,, ,, . . .	96·1	8	Hardened by hammering
,, ,, . . .	241·7	64·4	Again annealed in Fletcher's furnace at 1000° C.
Firth's soft steel . .	170·9	8·1	Annealed by Mr. ANDREWS at 566° C.
,, ,, . .	187·0	13·0	Annealed in Fletcher's furnace at 1000° C.
Nickel wire . . .	6·24	0·04	Hard drawn
,, ,, . . .	63·3	4·21	Annealed by Bunsen's burner
,, ,, . . .	63·3	4·21	Annealed by Mr. ANDREWS at 580° C.
,, ,, . . .	111·95	6·90	Annealed in Fletcher's furnace at 1000° C.

The above table shows the importance when annealing of raising the temperature sufficiently high, and maintaining the high temperature for a sufficient time, also of cooling slowly.* That a high temperature is required is well exhibited by the behaviour of JOHNSON and NEPHEW's iron wire. This had been raised by the makers to a temperature above dull red, and the wire was rendered by their annealing exceedingly soft and, moreover, for high, or rather comparatively high, magnetising forces, of great magnetic permeability; but annealing in the Fletcher's furnace at the temperature of 1000° C. nearly doubled the permeability for very minute magnetising forces, and more than doubled the internal friction. The annealing, by passing the wire slowly several times through the flame of a Bunsen's burner, so as to heat all parts equally to a bright red, does not seem satisfactory, partly, it may be, because a sufficiently high temperature is not reached, and partly, doubtless, because the cooling is too rapid; in every instance, annealing by Fletcher's furnace at the temperature of 1000° C., produced higher values of both A and B than annealing by a Bunsen's burner. This last fact is especially noticeable in the nickel wire, where the annealing with the furnace nearly doubled the value of A in the wire which had previously been annealed with the burner, as well as by Mr. ANDREWS at the temperature of 580° C. Professor EWING has examined the magnetic permeability of nickel wire with very low magnetising forces, and he finds the value of A for *annealed* nickel to be 22·4.† This value is very much less than that obtained by the author for the thoroughly annealed

* The slow cooling is, at any rate, necessary for iron; for nickel and cobalt it may not be necessary.

† 'Phil. Trans.,' vol. 179 (1888), A, p. 332. Professor EWING finds the *susceptibility* to be 1·7; this would make the *permeability* equal to $1 + 4\pi \times 1{\cdot}7$ or 22·4.

nickel wire, and may, perhaps, be due to imperfect annealing* of Professor EWING'S nickel; the difference cannot certainly be ascribed to the presence of iron in the author's specimen.† The influence of permanent strain on the magnetic properties of nickel is evidently very great, the value of A for the drawn metal being only $\frac{1}{18}$th of the value for the thoroughly annealed metal, whilst the internal friction in the former is only about $\frac{1}{200}$th of the internal friction in the latter.

It would seem that the different specimens of wrought iron in ordinary use will, if *properly annealed*, be found to have not very different permeabilities for very low magnetising forces, as may be seen from the following table. The specimens mentioned in this table were all annealed by the author in the manner already described, whilst lying in a direction at right angles to the magnetic meridian, the temperature of annealing being about 1000° C.‡

TABLE XVII.

Substance.	A in C.G.S. units.	B in C.G.S. units.	Mean value of A.	Mean value of B.
Wrought iron rod, history unknown	240·0	64·7		
Soft iron rod furnished by Mr. ANDREWS	244·5	135·0		
JOHNSON and NEPHEW'S wrought iron wire	240·0	86·5	241·5	87·6
Soft steel rod furnished by Mr. ANDREWS	187·0	13·0		
Soft piano-steel wire	135·0	32·0	146·0	22·5
Nickel wire	111·95	6·90	111·95	6·90
Cobalt strip	65·14	1·26	65·14	1·26

The numbers given in the above table must be understood to be obtained 24 hours after annealing and after a large number of preliminary cycles. The magnetising forces used were all well within the limit up to which the values of A and B are the same in cycles of different ranges of magnetising force.

The values of B for the different specimens of wrought iron are not nearly so much alike as those of A; but it is not improbable that if the different specimens were subjected to the heating and cooling process already mentioned they would be found to become much more nearly alike.

It is evident that both for permeability for very minute magnetising forces and for dissipation of energy the descending order for well annealed metals would stand thus:—

* Professor EWING seems to have annealed his wire by passing it slowly through a Bunsen's burner.

† See the analysis of the nickel previously given.

‡ This temperature seems to be sufficiently high for iron, nickel, and cobalt; some experiments made at about 1200° C. did not sensibly increase the permeability.

Iron.
Steel.
Nickel.
Cobalt.

THE TEMPORARY EFFECT OF CHANGE OF TEMPERATURE ON THE VALUES OF A AND B.

The permanent diminution of the values of A and B, which can be produced by repeated heating to 100° C. and then cooling, have been already alluded to in Experiment IX. The temporary effects were studied at the same time as the permanent ones, with the results given in Table XVIII.

It will be noticed that the effect on the permeability is greatest in annealed iron wire which has not been "accommodated" by the heating and cooling process, and least in the same wire which has been thus accommodated.

In all cases, except for freshly annealed iron, the increase of the value of B produced by rise of temperature can be more than accounted for by increase of permeability, so that, if we subjected the metals to forces which produced the same magnetic *induction* at the two temperatures, the dissipation of energy in the cycle would be *less* for the hot than for the cold metals.

For magnetising forces below the limits given in Table VII. the temporary changes of both A and B are, in all cases, quite *independent of the value of the force.*

TABLE XVIII.

Substance.	A_t and A_0 represent the values of A at $t°$ C. and 0° C. respectively.	B_t and B_0 represent the values of B at $t°$ C. and 0° C. respectively.	Increase of A per cent. produced by rise of temperature from 0° C. to 100° C.	Ditto of B ditto.	A_0 in C.G.S. units.	B_0 ditto.	Remarks.
JOHNSON and NEPHEW's wrought iron wire	$A_t = A_0(1 + \cdot002890t)$	$B_t = B_0(1 + \cdot00739t)$	28·90	73·90	232	80	Freshly annealed
Ditto	$A_t = A_0(1 + \cdot000717t)$	$B_t = B_0(1 + \cdot001005t)$	7·17	10·05	107	22·5	After repeated heating and cooling
Steel rod of Mr. ANDREW'S . . .	$A_t = A_0(1 + \cdot001244t)$	$B_t = B_0(1 + \cdot002624t)$	12·44	26·24	185	12·7	Ditto
Piano-steel wire . .	$A_t = A_0(1 + \cdot001232t)$	$B_t = B_0(1 + \cdot000593t)$	12·32	5·93	89	15·9	Ditto
Nickel wire . . .	$A_t = A_0(1 + \cdot002789t)$	$B_t = B_0(1 + \cdot00535t)$	27·89	53·47	109	6·6	Ditto
Cobalt strip . . .	$A_t = A_0(1 + \cdot001450t)$	$B_t = B_0(1 + \cdot002979t)$	14·50	29·79	64	1·22	Ditto

The Effects of Sudden Cooling on the Values of A and B.

The effects of sudden cooling are exhibited in Table XIX. There are three points to be specially noted. The first is that the permeability of steel for *very minute* forces, though considerably diminished, is not nearly so much affected by sudden cooling as is the permeability for *moderate* or *great* forces. The second is that the diminution of A does not seem to depend so much on the *amount* of hardening* as does the permeability for greater forces or the other physical properties of steel. And the third is the large effect of the sudden cooling in the case of soft *iron*. The value of B, and consequently the dissipation of energy, for steel is enormously diminished by the sudden cooling, whilst that of soft iron is also considerably lessened.

Table XIX.

Substance.	A in C.G.S. units.	B in C.G.S. units.	Remarks.
†Soft steel rod .	187·0	13·0	Before chilling
,, ,, .	73·1	0·18	After sudden chilling from a temperature of 780° C.
,, ,, .	72·2	0·32	After sudden chilling from nearly a white heat
†Soft iron rod .	244·5	135·0	Before chilling
,, ,, .	159·6	43·1	After sudden chilling from nearly a white heat

The Effects of Permanent Extension, Hammering and Torsion on the Values of A and B.

The next three tables show the effects of permanent extension, hammering transversely, and torsion on well annealed iron wire of ·1 cm. diameter.

Table XX.—Permanent Extension.

Percentage of extension.	A.	B.
0	240·9	68·8
1	118·7	25·6
5	88·4	16·7
10‡	77·7	1·8

* The effect of cooling from a white heat (about 1200° C.) is sensibly the same as that in cooling from 780° C.

† These rods had been previously annealed by the author, at a temperature of 1000° C.

‡ On attempting to stretch further the wire broke.

TABLE XXI.—Transverse Hammering.

Percentage increase of length produced by the hammering.	A.	B.
0	240·9	68·8
5	96·1	8·7

TABLE XXII.—Permanent Torsion.

Number of turns of permanent torsion in a length of 160 cm.	A.	B.
0	240·9	68·8
10	125·5	36·4
100	98·0	14·7
300	90·2	9·8
350*	95·9	6·9
400†	97·0	16·3

The values of A and B given in these tables were all obtained when the metals had been allowed to rest for several hours after the strain had been imparted. When the metals were tested *immediately* after straining the permeability and the internal friction were always very sensibly greater.

Evidently permanent strain imparted by any of the processes mentioned above diminishes considerably the permeability for very feeble forces, and very much more considerably the dissipation of energy.

In all cases annealing in Fletcher's furnace at a temperature of 1000° C., restored the original values of A and B.

THE TEMPORARY EFFECT OF LOADING ON THE VALUES OF A AND B.

Experiment XIV.

A well annealed iron 1 mm. in diameter was loaded by small additions at a time, and with long rests between, with 20 kilos., a permanent elongation of 2 per cent. being thereby produced.‡ After the greatest load had been applied and removed a great many times until the values of A and B had become constant at the different

* The last 50 turns were applied in the opposite direction to the previous ones.

† The last 50 turns were applied, 10 at a time, alternately in opposite directions.

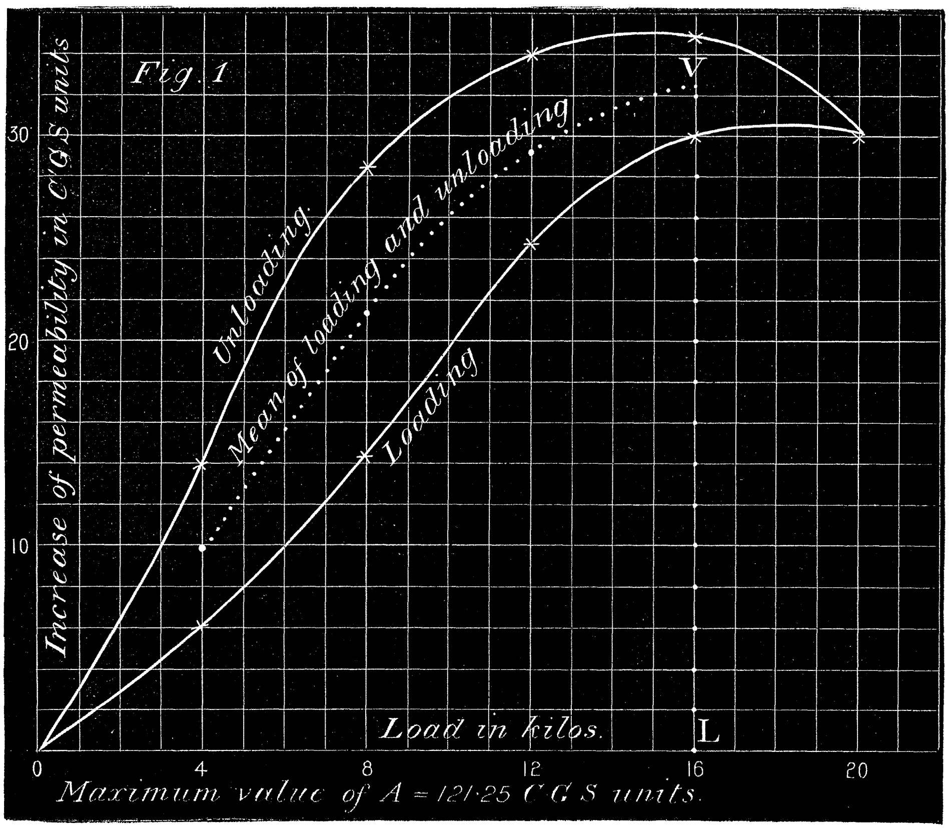
‡ A much greater permanent elongation would have been produced by the load had this been applied in the ordinary manner, *i e.*, without long intervals of rest between each addition

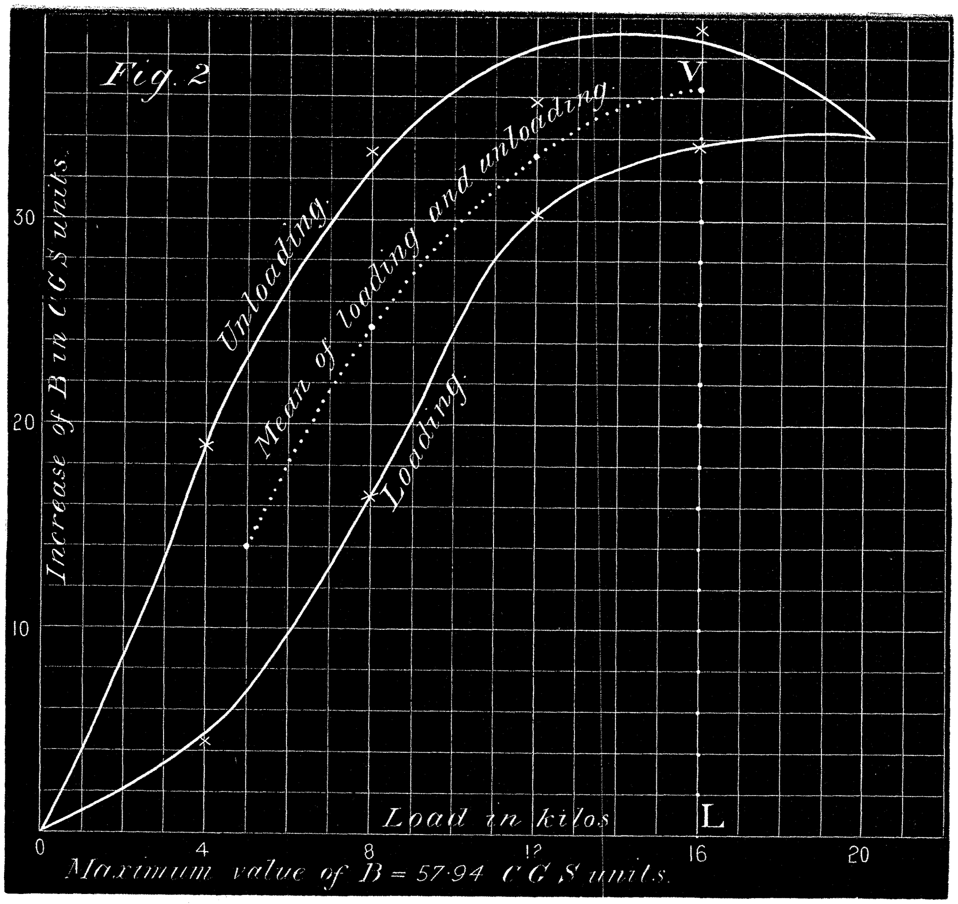
stages of loading and unloading the observations recorded in the next table were made. The loading and unloading were performed as carefully as possible so as to avoid jarring the wire, which was all the time lying in a direction at right angles to the magnetic meridian.

TABLE XXIII.

Load in kilos.	A in C.G.S. units. Loading.	Ditto. Unloading.	Mean value of A observed.	Ditto. A calculated.	B in C.G.S. units. Loading.	Ditto. Unloading.	Mean value of B observed.	Ditto. B calculated.	$\frac{B}{A^2}$
0	89·17	89·17	89·17	..	22·36	22·36	22·36	..	·0028
4	95·22	103·00	99·11	98·93	26·85	41·30	34·08	34·13	·0035
8	104·62	117·78	111·20	111·33	38·91	55·67	47·29	47·35	·0038
12	114·14	122·95	118·55	118·77	52·76	58·15	55·46	55·39	·0039
16	118·96	123·88	121·42	121·25	55·95	59·85	57·90	57·94	·0039
20	118·96	118·96	118·96	..	56·65	56·65	56·65	..	·0041

The results given in the table represent the means of several sets of observations made at each of the various stages of loading and unloading. There is an evident lagging of strain behind stress which is much more conspicuous in the values of B than in those of A. The means of the two values obtained at the same stages of





loading and unloading at each stage probably represent what would be the values of A and B if there were no such lagging. In the figs. 1 and 2, which supplement Table XXIII., the *dotted* curves show the relations which exist between the changes of these *mean* values and change of load. The values of both A and B increase with the load until the latter reaches 16 kilos.; further increase of load then causes decrease of A and B.*

The dotted curves of figs. 1 and 2 seem to be strictly parabolic, the vertex of the parabola being at V, and the axis a vertical drawn through V. Of course, the values of A and B corresponding to the loads 0 and 20 kilos. will not represent what would be the values of A and B if lag were absent, and they are therefore omitted in the comparison between observation and calculation given in columns 4 and 5, and 8 and 9 of the table. The calculated values of A and B are deduced respectively from the two formulæ

* A large number of independent observations were made which need not be recorded here, and in which the loads were altered by smaller amounts at a time; these observations seemed to fix 16 kilos. as *almost exactly* the load for which A and B were greatest.

$$A = 121{\cdot}25 - 0{\cdot}1550n^2$$
$$B = 57{\cdot}94 - 0{\cdot}1654n^2$$

where n is the number of kilos. in the actual load above or below 16 kilos.

The temporary increase of the value of B produced by the loading is greater than can be accounted for by mere increase of permeability, for if the increase of the value of B could be accounted for by increase of permeability, the ratio $B : A^2$ should be constant, whereas we see from the last column of Table XXIII. that this ratio increases with the load.

The above experiment serves but little more than to confirm the observations of VILLARI, Sir WILLIAM THOMSON, EWING, and others, that for small magnetising forces the magnetic permeability of iron is temporarily increased by moderate amounts of longitudinal traction. It does, however, go a little further, for it proves not only that the magnetic permeability of *iron* for *infinitesimal* magnetising forces is increased, but also that the *dissipation of energy* is increased. Now, Sir WILLIAM THOMSON has shown that the permeability of *nickel* is *diminished*, for moderate magnetising forces, by longitudinal traction; and it seems very probable, therefore, that the value of A for nickel will be diminished. But will the molecular friction—will the value of B—be also diminished? The author hopes to be able shortly to answer this question.

It had been rather expected that the "accommodation" by preliminary cycles at the various stages of loading and unloading might almost entirely do away with the lagging of strain behind stress. But though this lagging was considerably diminished, as subsequent experiments showed, by the "accommodating" process, yet, as we have seen, it was very sensible. In order to still further do away with the lag, the wire was now not only "accommodated" by preliminary cycles, but also tapped, or rather shaken, at each stage of the loading and unloading. The tapping still further diminished the lag, but by no means abolished it. Moreover, the dotted curves shown in figs. 1 and 2 remained sensibly unaltered, *i.e.*, the values which A and B would have if lag were absent remained the same.

RELATIONS BETWEEN THE DISSIPATIONS OF ENERGY IN TORSIONAL AND MAGNETIC CYCLES.

The following table shows in what respects the dissipations of energy in torsional cycles agree with those in magnetic cycles, and in what respects they differ:—

TABLE XXIV.

Torsional cycles.	Magnetic cycles.
1. The logarithmic decrement of arc is independent of the amplitude, and therefore the dissipation of energy in the cycle is proportional to the *square* of the maximum torsional force in the cycle.	The dissipation of energy is proportional to the *cube* of the maximum magnetising force in the cycle.
2. The logarithmic decrement of arc is independent of the vibration-period.	The dissipation of energy in the cycle does not depend upon the number of steps by which the maximum magnetising force in the cycle is reached.
3. The logarithmic decrement of arc is independent of the length and diameter of the wire.	The dissipation of energy per cubic unit of volume does not depend upon the dimensions.*
4. The logarithmic decrement of arc can be diminished by repeated oscillation within sufficiently small limits, by rest (in the case of iron), and (in the case of iron-wire) by repeated heating to 100° C. and then cooling.	The dissipation of energy can be diminished by repeatedly putting the metal through cycles of sufficiently small range of magnetising force, by rest (in the case of iron), and (in the case of iron-wire) by repeated heating to 100° C. and then cooling.
5. The logarithmic decrement of a very small arc is increased by recent oscillations, through arcs exceeding certain limits; it is also increased, either permanently or subpermanently, by mechanical shocks, by sudden changes of temperature, or by molecular disturbances due to magnetisation.	The dissipation of energy in a cycle of very small range of magnetising force is increased by previously putting the metal through cycles whose ranges of force exceed certain limits; it is also increased, either permanently or subpermanently, by mechanical shocks, by sudden changes of temperature, or by molecular disturbances due to torsional oscillation.†
6. The logarithmic decrement of arc is permanently *increased*, and the torsional elasticity is permanently *decreased* by excessive permanent extension and by permanent torsion.	The dissipation of energy is permanently *decreased*, and the magnetic elasticity is permanently *increased* by permanent extension, torsion, and hammering.
7. The logarithmic decrement of iron and nickel is *much less* at 100° C. than at 0° C.; the torsional elasticity is also *slightly less* at 100° C. than at 0° C.	The dissipation of energy of iron, nickel, and cobalt is *greater* at 100° C. than at 0° C.; the magnetic elasticity is *considerably less* at 100° C. than at 0° C.
8. The logarithmic decrement and the torsional elasticity of iron are *both independent* of the load.	The dissipation of energy of iron is *increased*, and the magnetic elasticity *decreased* by loading.

It will be seen from 1 that the dissipations of energy in the two kinds of cycles differ in the important particular that in torsional cycles the dissipation varies as the *square* of the maximum torsional force, whilst in magnetic cycles the dissipation varies as the *cube* of the maximum magnetising force. In both kinds of cycles the dissipation varies as PX, where P is the molecular set first on this side and then on that, as the wire vibrates torsionally in the one case, or as it is magnetised first in one direction and then in the opposite in the other case, and X is the maximum force, in the one case torsional and in the other magnetic; but for torsional cycles P varies as X

* Provided, of course, the length contains the diameter a sufficient number of times to enable us to neglect the effect of the ends.

† This last, though not mentioned in the present memoir, has nevertheless been determined by the author to be the case.

and in magnetic cycles as X^2. In attempting to account for this difference we might, perhaps, be inclined to ask ourselves whether, in spite of their minuteness, the magnetising forces employed have been *sufficiently* minute—whether, if we operated with forces which were, say, one-hundredth or one-thousandth of the least of those used in the present inquiry, we should not find P varying as X instead of as X^2. On the other hand, it is easy to show that the dissipations of energy in the torsional cycles employed by the author are *comparable in amount* with those encountered in true magnetising cycles; and if we are to reduce the range of the magnetising force to the extent mentioned above, the dissipations of energy in the two sets of cycles would quite cease to be comparable.

As regards 2 and 3, the dissipations in the two sets of cycles are analogous. We may notice, also (4), that in both torsional and magnetic cycles what have been called the "accommodating" influences are the same; and they are not only alike in kind, but comparable in amount. Further (5), the various causes which upset the "accommodation" are the same in the two kinds of cycles, and produce comparable effects.

When, however, we turn to both the permanent and temporary effects of mechanical stress (6 and 8), and to the temporary effect of change of temperature (7), we find *very essential* differences in the two kinds of cycles. How far these differences may be due to differences in the effects on the magnetic and torsional elasticities remains to be seen.

SUMMARY.

1. When a rod of iron, nickel, or cobalt is made to pass through a complete magnetic cycle of sufficiently minute range of magnetising force, the instantaneous changes of induction can be expressed by an equation of the form

$$y = Ax + Bx^2,$$

where x represents the total *change* of magnetising force as the force passes from its greatest value to zero, if the force be always applied in the same direction, or from its greatest value in one direction to its greatest value in the opposite, if the same force be alternately employed in opposite directions; y represents the corresponding *change* of induction; and A and B are constants throughout the cycle.

2. The values of A and B are the same for different ranges of magnetising force.

3. The constant A is a measure of the magnetic permeability for infinitesimal magnetising force; the constant B is a measure of the dissipation of energy in the cycle.

4. The dissipation of energy in any cycle may be decreased by repetition of the cycles; the molecules may be said to be "accommodated" by this process.

5. The molecular "accommodation" of freshly-annealed iron can be increased by repeatedly raising the metal to 100° C., and then allowing it to cool.

6. The "accommodation" of iron, nickel, and cobalt is upset by slight mechanical shock, by small changes of temperature, or by magnetisation beyond certain limits.

7. The values of A and B for iron, nickel, and cobalt may be very largely decreased by permanent mechanical strain. In the case of iron and steel they may also be largely decreased by sudden cooling.

8. The values of A and B for iron are decreased by temporary loading.

9. The values of A and B for iron, nickel, and cobalt are increased when the temperature is raised from 0° C. to 100° C.

10. In both torsional and magnetic cycles of sufficiently minute range the dissipation of energy is independent, apart from the effects of self-induction and time-lag, of the rate at which the cycle is performed.

11. The dissipation of energy in a magnetic cycle is proportional to the *cube* of the maximum magnetising force in the cycle, but in a torsional cycle the dissipation is proportional to the *square* of the maximum torsional force.

12. As regards "accommodating" influences, and also as regards those influences which upset "accommodation," the dissipations of energy in torsional and magnetic cycles are strictly analogous.

13. The temporary and permanent effects of mechanical stress on the dissipation of energy and the effects of change of temperature are very different in torsional and magnetic cycles.