

XII. *On the Thermal Conductivities of Crystals and other Bad Conductors.*

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Introduction.

OUR knowledge of the Thermal Conductivities of Crystals is derived mainly from the experiments of DE SENARMONT, VON LANG, and JANNETAZ,* who, using the wax melting or analogous methods, have determined the ratios of what may be called the “principal conductivities” and the positions of the axes of conductivity within a number of crystals belonging to the simple systems. According to their experiments the isothermal surfaces about a heated point in a crystal are, in general, ellipsoids, having their axes parallel to the optical axes. In the case of a uniaxal crystal, this ellipsoid becomes a spheroid of revolution about the axis, and is, as a rule, oblate or prolate according as the wave-surface for the extraordinary ray is oblate or prolate. Although this rule has a number of exceptions, it is sufficiently general to render it probable that there may be some relation between the passage of light and of heat through a crystal. The recent determinations of the refractive indices of metals by KUNDT have shown that they stand in the same order as conductors of heat, and as to the velocity of propagation of light through them, and this fact brings again into prominence the old determinations with respect to crystals. That the comparison which KUNDT has made for the metals cannot be carried to other bodies is at once seen from the fact that the index of refraction of iron differs little from those of glass and several commoner crystals, the conductivities of which are shown to be very small compared to that of iron. A comparison may, however, be possible among transparent bodies themselves, and the following experiments were made with the object of furnishing data for this comparison, the results given by previous observers differing greatly from each other. They have, however, been extended to embrace non-transparent bodies commonly in use in a physical laboratory, and about the conductivity of which we have had a very meagre or absolutely no knowledge.

* SENARMONT, ‘Ann. de Chim. et de Phys.,’ (3), vols. 21, 22, and 23 (1848); VON LANG, ‘Pogg. Ann.,’ vol. 135 (1868); JANNETAZ, ‘Ann. de Chim. et de Phys.,’ (4), vol. 29 (1873).

Outline of Method.

The most important consideration in determining the method to be used is the fact that it is difficult to get large pieces of the crystals to be experimented on. This excludes methods requiring large plates, such as that of WEBER and TUSCHMID, or large spheres or cubes, such as that of KIRCHHOFF or THOMSON. A method which seemed to present several advantages was the one first suggested by LODGE,* and which may be called the "divided bar" method, and after some preliminary experiments had been made to determine its suitability, it was finally adopted. It consists in observing the temperature along a bar heated at one end and cooled at the other, and divided halfway between the two ends by a plane perpendicular to its axis, when (1) the divided ends are together, (2) a disc of the crystal or other body is between.

If temperature observations are taken at several points in each half of the bar, the corrections to be applied to the second set of observations for the distances of the points of observation from the contacts can be determined from the first set, and thus the temperature at each side of the crystal disc can be found. If, in addition, we know the thermal conductivity of the bar used, the amount of heat flowing through the disc can be found from the temperature slope; and thence we have the thermal conductivity of the crystal.

It may be noticed that there would be considerable uncertainty as to the nature of the contacts between bar and crystal unless some special precautions were taken. LODGE proposed to use pads of tinfoil to obviate this difficulty, but this method is not satisfactory. The difficulty has, however, been completely overcome by using bars of a material which would amalgamate, and making the contacts by means of mercury.

Description of Apparatus. (Fig. 1.)

The bar used in the experiments was one of brass, which presents several advantages.

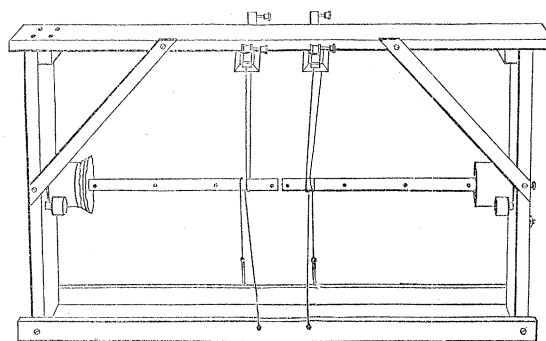
- (1) It readily amalgamates, and therefore enables good contacts to be made.
- (2) Its conductivity is not so high as to make comparison between it and that of crystals, &c., impossible.
- (3) Its conductivity, according to LORENZ, increases with the temperature, a fact which, as will be seen later, partly neutralises the deviation of the cooling from NEWTON's law.

The diameter of the bar was 1.93 cm., and its length 67 cm. To each end of the bar was soldered a can, through one of these cans steam was sent and through the other water. The whole was arranged in a wooden frame, so that the bar was horizontal and exposed to the air over a covered tank through which a current of water could be sent to keep the temperature constant. The apparatus was surrounded by paper screens to protect the bar as far as possible from air currents in the room.

* 'Phil. Mag.,' (5), vol. 5, p. 110 (1878).

The cans at the extremities of the bar were supported on slides attached to the framework in such a way that the bar could be moved parallel to itself by screws passing through the framework and bearing in conical indentations in the backs of the cans in the prolongation of the axis of the bar. These supports were sufficient while the bar was entire, and the experiments to determine its conductivity were being made. After the bar was cut for the insertion of the discs, it became necessary to support the cut ends in such a way as (1) to enable the motion of the bar parallel to its axis still to take place; (2) to enable the cut faces to be adjusted parallel to each other; (3) to cause as small as possible a disturbance of the isothermal surfaces. These objects were secured by supporting each free end in a loop of thin string,

Fig. 1.



hanging from the upper part of the framework. The ends of this loop were attached to screws for the purpose of raising or lowering it as required. The sides of the loop made an angle of 60° or 70° with each other at the bar. These loops were sufficient to support the bars properly if the arrangement was kept horizontal. It was, however, found that the mercury contacts could be made with much greater certainty if the bars were vertical and the mercury surfaces horizontal. On this account two other loops were attached to the lower part of the framework and passed over the bars. These were elastic, and served merely to keep the upper loops taut, and the bars in the same positions with respect to each other, whether the framework was horizontal, as in the experiments, or vertical, as in making the mercury contacts.

Measurement of Temperature.

The most direct method of measuring the temperature along the bar is to sink thermometers into holes in the bar, but this method, although it has been used by FORBES, TAIT, and MITCHELL for their large bars, is very objectionable, as it diminishes the available area of flow, and therefore makes the change of temperature along the bar more rapid than it would be for a continuous bar. The method first

used by WIEDEMANN and FRANZ of measuring the temperature by means of a thermo-junction brought into contact with the bar is free from this objection, and will produce no appreciable effect on the distribution of temperature in the bar if the mass of the junction is not large.

The bars used by WIEDEMANN and FRANZ had a silvered surface, and it was sufficient to bring the thermo-couple into contact with the surface in order to get the temperature. In the later of the following experiments the bars were painted, so that contact could not be made in this way. Small conical holes about .5 mm. deep were, however, made at opposite extremities of horizontal diameters of the bars, and at regular distances along. Eight such diameters were taken, the two nearest the centre of the uncut bar being 1 cm. apart, and the rest 10.5 cms. apart. Each of these small holes was amalgamated, and sufficient mercury left in them to make good contact with the ends of the two wires which were used as a thermo-couple. The thermo-circuit would thus consist at the bars of—first wire of couple; mercury of first hole; brass of bar; mercury of hole at opposite extremity of diameter; second wire of couple. The mercury in the holes was constantly cleaned by touching the surface with dilute nitric acid, washing and drying, and the ends of the thermo-couple wires were kept bright. Under these conditions the arrangement worked satisfactorily.

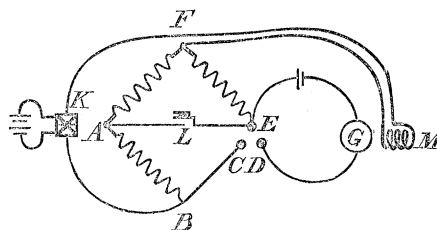
The wires used originally for the thermo-couple were of brass and iron, but these gave unsatisfactory readings, mainly on account of the poor contact between iron and mercury. As there is an advantage in using copper as one of the elements, a few combinations of copper with other metals and alloys were tried, and eventually the platinum-silver alloy used for resistance coils was selected as the other element. This couple gives an E.M.F. of about .00016 volt for 70° difference of temperature, and its constant only increases about 7 per cent. for 100° C. It is, therefore, well suited for thermo-electric measurement of temperature. The wires used for the thermo-couple were about 20 cms. long and .2 mm. diameter. The ends which served to make contact with the galvanometer circuit dipped into mercury cups in blocks of wood which could be moved along guides parallel to the bars in order that the whole of the temperature observations could be made with one couple. The cup into which the platinum-silver wire dipped was provided with a thermometer graduated in $\frac{1}{10}^{\circ}$ C.

The galvanometer was a low resistance one of the WIEDEMANN type, with SIEMENS' bell-magnet and copper damping sphere. The resistance of the whole circuit was only about 1.5 ohm, and as the resistances of the contacts at the holes in the bar might vary, it was necessary to take measurements of the resistance of the circuit when a galvanometer deflection was taken. For this purpose THOMSON'S modification of the bridge method was used. The diagram (fig. 2) shows the arrangement of the circuit.

G is the galvanometer in series with the thermo-element. C , D , E are three mercury cups, arranged so that when E and D are connected, the galvanometer and thermo-element are in circuit alone, and when C and D are connected the galvano-

meter and element form one side of the bridge. K is a reversing switch, and L a key. AB and AF are each 100 ohms from post-office box; FE is a multiple arc box capable of adjustment to $\frac{1}{500}$ ohm. The resistance R of the galvanometer circuit is found by adjusting FE so that there is no effect on the galvanometer on pressing down L , first with K in one direction and then in the other. The coil M is intro-

Fig. 2.



duced in the battery circuit to counteract the effect of the current through the galvanometer while making these observations. It will be noticed that the current in it is reversed by the operation which reverses the current in the galvanometer coils, and that the galvanometer needle is therefore kept close to the zero of the scale during the whole of the observations.

Let the values of the resistance EF determined as above be R_1 and R_2 , and let P be the resistance of each of the arms AF , AB , W that of the coil M , E the E.M.F. of the battery, e that of the thermo-element. Then writing down the condition that A and E have the same potential in each case, we have

$$(E + e) \frac{1 + e}{P + R + 2W} = \frac{E}{P + R_1 + 2W}$$

$$(E - e) \frac{E - e}{P + R + 2W} = \frac{E}{P + R_2 + 2W}.$$

Hence

$$\frac{2}{P + R + 2W} = \frac{1}{P + R_1 + 2W} + \frac{1}{P + R_2 + 2W},$$

which gives

$$R = \frac{R_1 + R_2}{2} - \frac{\{\frac{1}{2}(R_1 - R_2)\}^2}{P + \frac{1}{2}(R_1 + R_2) + 2W}.$$

The last term of the right member of this equation never exceeds $\frac{1}{500}$, and may therefore be neglected, and we have $R = \frac{1}{2}(R_1 + R_2)$.

The constants of the thermo-couple were determined by comparison with a thermometer reading to $\frac{1}{30}$ degree Centigrade. An auxiliary iron bar of 80 cms. length and 2.5 cms. square cross-section, was placed horizontal and heated at one end and cooled at the other by the waste steam and water respectively from the principal bar. The upper surface of this auxiliary bar was tinned and amalgamated. On this surface two copper mercury cups of 1 cm. diameter and 1.8 cm. height were placed. The under

surfaces of these cups were amalgamated so as to make good contact with the iron bar. The insides of the cups were amalgamated, and each cup contained sufficient mercury to fill it entirely when the thermometer was placed in it. At opposite extremities of a diameter, about half way down each cup, two small holes like those in the divided bar were punched, and into these holes, which were amalgamated, the wires of the thermo-couple could be inserted. The constants of the couple are in this way determined under exactly the same conditions as when they are in use. The iron bar was buried in sawdust, and the copper mercury cups placed in such positions on the upper surface that the thermometer read in them 90° C. and 55° C. respectively. After the insertion of the wires of the thermo-couple into the holes of the mercury cups, the cups were surrounded with sawdust, and 10 minutes allowed for the temperature of the cups to get as nearly as possible uniform before observations were taken.

In an experiment made in the early part of the work the following observations were taken :—

(a) Temperature of hot junction, $85^{\circ}90$ C.

Temperature of cold junction, $15^{\circ}67$.

Deflection of galvanometer in scale divisions, 296.

Resistance of circuit, mean of observations taken, as previously described
1.431 ohm.

E.M.F. of element in units used, 423.6.

(b) Temperature of hot junction, $55^{\circ}60$.

Temperature of cold junction, $15^{\circ}74$.

Deflection of galvanometer, 162.3.

Resistance of circuit, 1.448 ohms.

E.M.F. of element in units used, 235.0.

These observations are sufficient to determine the constants in the formula $e = a(t_2 - t_1)(1 + b\overline{t_2 + t_1})$ where t_2 and t_1 are the temperatures of the hot and cold junctions respectively. It was found more convenient for the reduction of observed E.M.F.'s to temperature, to draw up a small table of values of $a(1 + b\overline{t_2 + t_1})$ for different values of $t_2 + t_1$. Thus, for the above observations, we have :—

$t_2 + t_1$.	$a(1 + b\overline{t_2 + t_1})$.	Differences.	
101	6.030	$+ \overset{\circ}{2}$	$+ .009$
91	5.985	4	.018
81	5.940	6	.027
71	5.895	8	.036
61	5.850		
51	5.805		
41	5.760		
31	5.715		

By inspection, we have the divisor for converting E.M.F. observations to temperature difference, since in every case $t_1 + t_2$ is known approximately to within one or two degrees.

The value of $a(1 + b\overline{t_2 + t_1})$ is seen from the above table to change only about 7 per cent. for 100° C. of change of $\overline{t_2 + t_1}$; the platinum-silver-copper element is therefore well suited for thermo-electric observations of temperature.

The determination of the constants of the element was repeated every few days during the progress of the work. The value of a was found to diminish slowly, the total change during the three months during which the couple was used being about 1.5 per cent.

The temperature of the enclosure in which the divided bar was suspended was determined by means of a thermometer graduated in $\frac{1}{10}$ degrees, placed about 7 cms. above the top of the water-tank and about 15 cms. below the bar. The bulb of the thermometer was protected from radiation from the hot bar by a small sheet of paper, just large enough to screen the bulb.

Determination of the Thermal Conductivity of the Brass Bar.

The thermal conductivity of the bars was determined, following the method of FORBES, by two series of experiments, the first to determine the law connecting the rate of loss of heat from a heated surface, with the temperature of that surface, and the second to determine the law of steady distribution of temperature along a bar heated at one end, and exposed to the air along its entire length. These two observations furnish data for determining the thermal conductivity required.

Cooling Experiments.

It is customary in making these experiments to use a short length of the bar used in the steady experiments. I have, however, shown that the law of cooling is the same for bars of different material, but of the same cross-section and surface; and as a knowledge of the specific heat of the bar is required, it seemed better to make use of a bar of a material the specific heat of which is well known, rather than a brass bar in these experiments. As the values found by different observers for the specific heat of copper are practically identical, this metal was chosen. As it is a better conductor than brass, it also enables the assumption that the temperature throughout the bar is the same to be made without much chance of error. The bar was of the same diameter as the brass bar, 1.93 cm. and 26 cms. long. It had a polished nickel-plated surface. To make the cooling experiment correspond more closely to that of an infinitely long bar, the ends of the bar were covered with a layer of sulphur about 5 mms. thick. This was done by placing the bar vertical, wrapping a sheet of paper round the upper end so as to project about 1 cm. above the end,

and then pouring melted sulphur into the paper cylinder. Since sulphur is an extremely poor conductor of heat, it may be assumed that the loss of heat from the ends of the bar is very small.

The bar was suspended in the position occupied afterwards by the divided bar by means of threads. The temperature was determined by means of the platinum-silver-copper wires inserted into small holes amalgamated and filled with mercury, on opposite sides of a diameter at the middle of the bar. Observation of current, resistance, temperature of the cool junction, and of the enclosure, were made every two minutes during the experiments. From these observations the temperature of the bar at any time is determined, and curves drawn for each experiment showing the connection between time and temperature of bar and of enclosure. Of these curves that which corresponded most nearly with the mean of all was selected, and is referred to in what follows.

If m is the mass of the bar, s its surface, c the specific heat of the material of the bar, v the excess of the temperature of the bar over that of the enclosure, which will be supposed constant, we have, as the connection between v and time,

$$mc \frac{dv}{dt} + sh_v v = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

where h_v is a quantity which is generally assumed to be constant (NEWTON'S law), but which I have shown increases for a nickel-plated bar of the size used 70 or 80 per cent. for 100° C. rise of v .* In the paper referred to this rise is expressed by writing $h_v = hv^n$ where n is a small fraction $= \frac{1}{7}$ about. In that paper v was always positive. As in the following work v is sometimes negative, in which case v^n would require special interpretation, it is preferable to write $h_v = h(1 + bv)$ where b is a small positive quantity. The foregoing equation becomes, then,

$$mc \frac{dv}{dt} + sh(1 + bv)v = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

the solution of which is

$$\frac{1}{v} + b = Ae^{(sh/cm)t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3),$$

where A is an arbitrary constant.

The following table shows that this equation represents the cooling with a fair amount of accuracy for $b = .008$. The temperature of the enclosure has been assumed constant $= 15.2^\circ$ C., but in reality it varied from 15.3° at the commencement to 15.4° at $t = 10$ minutes, and then down to 15.2 at 40 minutes, 15.18 at 60 minutes, 15.1 at 70 minutes. On this account no attempt has been made to determine b with greater accuracy; the value given above is determined by trial.

* 'Phil. Mag.,' vol. 28, p. 429 (1889).

COOLING of Nickel-plated Bar.

t in minutes.	Temperature. — 15.2° C. = v .	$1/v$.	$\frac{1/v_t + b}{1/v_{t+2} + b} = e^{-(sh/cm)120}$.	
				Means.
0	75.4	.01326	..	} .9468
2	69.2	1445	.9469	
4	64.03	1562	504	
6	58.67	1704	434	
8	54.36	1840	485	
10	50.14	1994	448	} .9467
12	46.58	2147	482	
14	43.30	2309	479	
16	40.16	2490	450	
18	37.37	2676	466	
20	34.80	2874	460	} .9462
22	32.43	3084	459	
24	30.31	3299	475	
26	28.28	3536	454	
28	26.43	3784	459	
30	24.72	4045	463	} .9466
32	23.16	4318	467	
34	21.77	4593	490	
36	20.44	4892	474	
38	19.10	5236	430	
40	17.93	5576	467	} .9474
43	16.22	6165	428	
46	14.95	6689	528	
49	13.67	7315	479	
52	12.49	8006	470	
55	11.41	.08764	465	} .9454
60	9.88	.1013	} .9454	
65	8.51	1175		
70	7.21	1387		
75	6.26	1597		

The mean of the values of $e^{-(sh/cm)120}$ is .9466.

Hence we have

$$e^{120(sh/cm)} = \frac{1}{.9466} = 1.0564.$$

Therefore

$$\frac{sh}{cm} = \frac{.02383}{.4343} \times \frac{1}{120} = .0004572.$$

Now, $m = 676$ grms., $s = 157.6$ sq. cms., $c = .093$. Hence we have

$$h = .0001804.$$

The heat lost from a nickel-plated surface exposed to air is, therefore,

$$\cdot 0001804v(1 + \cdot 008v) \text{ gram degrees per sq. cm. per second,}$$

where v is the excess of temperature of the surface over that of the air.

Statical Experiments.

These experiments were made with the brass bar before it was cut. One end was heated by steam, and the other cooled by water. Eight observations of temperature were taken at different points of the bar in the way previously described.

Let v be the excess of temperature at point x of bar over that of surrounding air,

p = perimeter of cross-section of bar,

q = area of cross-section of bar,

k_v = thermal conductivity at temperature excess = v .

Then, at any point of the surface of the bar, we have

$$k_v \frac{dv}{dn} + hv(1 + bv) = 0,$$

where dv/dn is the rate of change of v along the normal to the surface, s , h , and b having the meanings assigned to them in last section.

At $v = 60^\circ \text{C.}$ this gives

$$\begin{aligned} k_v \frac{dv}{dn} &= - \cdot 00018 \times 60 \times 1\cdot 5 \\ &= - \cdot 0162. \end{aligned}$$

Now k_v will be seen later to have the value $\cdot 27$ approximately. Hence, at a point of surface, at the temperature of 60°C. excess, $dv/dn = - \cdot 06$.

At the same point on the surface $dv/dx = 2\cdot 5$ approximately. Hence, the inclination of the normal to the isothermal surface at this point to the axis of the bar is $\arctan \cdot 06/2\cdot 5 = \arctan \cdot 024 = 1^\circ 25'$. If the isothermal surface be assumed to be part of a sphere, the radius of curvature is about 42 cms. At 15°C. excess, a similar calculation gives the radius of curvature of the isothermal surface to be 60 cms.

Hence we may assume the isothermal surfaces to be planes perpendicular to the axis of the bar.

The equation for the motion of heat in the bar is under these conditions

$$q \frac{d}{dx} \left(k_v \frac{dv}{dx} \right) = phv(1 + bv). \quad \dots \dots \dots (4)$$

The conductivity k_v has generally been considered constant in treating this equation, but this is scarcely justifiable, as most experimenters find changes of

± 20 per cent. in conductivity for 100° C. change of temperature. Taking therefore k_v as equal to $k(1 + av)$ the above equation becomes

$$(1 + av) \frac{d^2v}{dx^2} + a \left(\frac{dv}{dx} \right)^2 = \frac{ph}{qk} v (1 + bv). \quad (5)$$

Multiplying through by $2(1 + av) \left(\frac{dv}{dx} \right)$, this becomes

$$2(1 + av)^2 \frac{d^2v}{dx^2} \frac{dv}{dx} + 2a(1 + av) \left(\frac{dv}{dx} \right)^2 \frac{dv}{dx} = 2 \frac{ph}{qk} v (1 + bv) (1 + av) \frac{dv}{dx},$$

or,

$$\frac{d}{dv} \left(\overline{1 + av} \frac{dv}{dx} \right)^2 = 2 \frac{ph}{qk} (v + \overline{a + b} v^2 + abv^3) \frac{dv}{dx}.$$

From which by integration we obtain

$$\left(\overline{1 + av} \frac{dv}{dx} \right)^2 = \frac{ph}{qk} \left(v^2 + 2 \frac{a + b}{3} v^3 + \frac{ab}{2} v^4 \right) + \text{constant}.$$

If the value of dv/dx when $v = 0$ be dv_0/dx , the above equation becomes

$$\left(\overline{1 + av} \frac{dv}{dx} \right)^2 - \left(\frac{dv_0}{dx} \right)^2 = \frac{ph}{qk} v^2 \left(1 + 2 \frac{a + b}{3} v + \frac{ab}{2} v^2 \right). \quad (6)$$

If $dv_0/dx = 0$, the integral of this equation can be expressed in terms of logarithmic and circular functions. If $dv_0/dx \neq 0$, the integration introduces circular functions and elliptic integrals of the 1st and 3rd kind. If the problem under discussion were—given the conductivities—to determine the distribution of temperature throughout the bar, this integration would be necessary, but as we are given the distribution and have to find the conductivity, the problem can be solved without further integration. Although this method is probably not as accurate as that depending on the integrated equation, its accuracy is sufficient for the present purpose, where the value of the conductivity is only required over a small range (25° – 37° C.) of temperature.

To determine dv/dx at the points of observation v is represented by an empirical function of x and the differential coefficient with respect to x taken. The known conditions which v satisfies, fix, to some extent, the function to be used; we see, *e.g.*, that it vanishes for some value of x , and that at that point the first differential coefficient is finite; that it increases in one direction with x in an approximately exponential manner.

These considerations lead at once to the function $A \sinh(\alpha x + \beta)$, and this function has been used. It evidently cannot express v accurately throughout the bar, since it is the solution of a linear differential equation, but it can be made by a proper choice of the constants to represent the main feature of the curve, the differences between its values and those observed being afterwards represented by an additional expression, which has generally only small values.

For the curve which corresponded best with the mean of the three sets of observations taken, the value of v is represented along the cooler part of the bar by $\cdot795 \cosh \cdot0378 x + 11\cdot86 \sinh \cdot0378 x$.

x .	v calculated.	v observed.	Difference.	$\frac{dv}{dx}$.
0	$\cdot8$	$\cdot8$	0	$\cdot448$
10.56	5.72	5.72	0	$\cdot497$
21.04	11.52	11.53	+ $\cdot01$	$\cdot624$
31.55	19.17	19.18	+ $\cdot01$	$\cdot852$
32.59	20.07	20.09	+ $\cdot02$	$\cdot881$
43.15	31.23	31.20	- $\cdot03$	1.262
53.59	47.24	47.88	+ $\cdot64$	1.971
64.03	70.64	73.47	+ 2.83	3.002

The differences, with the exception of the last two, are within the limits of error of observation, and may be neglected. The last two may be taken into account by adding to the above expression for v , between $x = 43$ and $x = 64$, an expression of the form

$$a \left(\frac{x - 43}{10.5} \right)^2 + b \left(\frac{x - 43}{10.5} \right)^3, \text{ where } a = \cdot572, b = \cdot067.$$

The values of the differential coefficient of this expression at $x = 43.15$, 53.59 , 64.03 are 0, $\cdot128$, $\cdot294$ respectively. Differentiating the hyperbolic expression for v we have $dv/dx = \cdot03005 \sinh \cdot0378 x + \cdot4483 \cosh \cdot0378 x$, which gives, on substituting for x the coordinates of the points of observation, the required values of dv/dx , to which, at $x = 53.59$ and 64.03 , we must add the quantities $\cdot128$ and $\cdot294$ to get the true temperature slope. The values thus obtained are given in the above table.

Having thus got the values of v and dv/dx at eight points on the bar, we can by trial determine a so that the equation (6) holds. A few trials show that a is positive and equal approximately to $\cdot002$. It is not necessary to determine a with any great accuracy as it affects very little the subsequent work. Assuming $a = \cdot002$ we have the following table of results:—

x .	v .	$v^2 \left(1 + 2 \frac{a+b}{3} v + \frac{ab}{2} v^2 \right)$	$\left(1 + av \frac{dv}{dx} \right)^2$	(5.)	(6.)
0	$\cdot8$	$\cdot643$	$\cdot201$	$\cdot001$	$\cdot0016$
10.56	5.72	33.96	$\cdot253$	$\cdot053$	$\cdot0015$
21.04	11.53	143.3	$\cdot406$	$\cdot206$	$\cdot00143$
31.55	19.18	415	$\cdot781$	$\cdot581$	$\cdot00140$
32.59	20.09	459	$\cdot839$	$\cdot639$	$\cdot00139$
43.15	31.20	1174	1.796	1.596	$\cdot00136$
53.59	47.80	3067	4.666	4.466	$\cdot00145$
64.03	73.47	8269	11.860	11.660	$\cdot00141$

Column (5) gives the values of $\left(1 + av \frac{dv}{dx}\right)^2 - \left(\frac{dv_0}{dx}\right)^2$, and column (6) gives the values of

$$\frac{\left(1 + av \frac{dv}{dx}\right)^2 - \left(\frac{dv_0}{dx}\right)^2}{v^2 \left(1 + 2 \frac{a+b}{3} v + v^2\right)},$$

which by equation (6) = ph/qk .

Neglecting the first two numbers in column (6), which are uncertain on account of the smallness of the quantity (5) at the cool end of the bar, we have, as the mean value of ph/qk , .00141.

Hence

$$k = \frac{ph}{.00141 q} = \frac{2.093 \times .0001804}{.00141} = \frac{.0003777}{.00141} = .268 \frac{\text{gram.}}{\text{cm. sec.}}.$$

The temperature of the air in this experiment was 17° C. hence we have for the thermal conductivity of the brass bar used*

$$k = .268 (1 + .002 v - 17^\circ) \frac{\text{gram.}}{\text{cm. sec.}}.$$

Experiments on Crystals, &c.

Before cutting the bar for the insertion of the discs the conductivities of which were to be measured, several observations were made of the distribution of temperature along the bar after it had been painted with Aspinall's enamel. It is necessary to have the radiating surfaces of bar and discs the same, and this is most easily secured by painting both. The loss of heat from a point of the surface at any temperature is about 40 per cent. greater than the loss at the same temperature for the nickel-plated surface, but it does not increase as rapidly with rise of temperature. On this account it is possible to express the distribution of temperature along the bar in the empirical form $v = A \sinh (\alpha x + \beta)$, and A , α , and β can be determined so that this equation holds with a closer approximation than in the case of the unpainted bar, throughout the whole length of the bar. It is assumed to hold in what follows for each half of the divided bar, α having the same, but A , β having different values for each half.

The bar was divided in the middle, between the contact holes 1 cm. apart, and the ends ground down to be as nearly as possible planes perpendicular to the axis of the bar. To secure this a vertical hole, a little larger than the bar, was drilled in a prism

* The following values of k for brass have been found by different experimenters:—NEUMANN, 'Ann. de Chim. et de Phys.' (III.), vol. 66 (1862), .302; WEBER, 'Monatsber. Berlin Akad.,' for 1880, p. 457, .150; LORENZ, 'Wied. Ann.,' vol. 13 (1881); 'red brass,' .252 ($1 + .0018 t$); 'yellow brass,' .212 ($1 + .0020 t$).

of wood, 30 cms. \times 20 cms. base, and 15 cms. height. This was done in a drilling machine the table of which was found to be perpendicular to the spindle. One of the bars was then fixed by wedges into this hole in such a way that a point, which when the bar was in position would be the highest point of the cut surface, came in contact with a mark on the block of wood. The block and bar were then moved about on a stone and afterwards on a slate slab, till the surface was plane and smooth. The other bar was then fixed so that the lowest point of the cut surface came into contact with the same mark, and the surface rubbed down. This process secured that the cut surfaces should be parallel to each other when the bars were coaxial. To secure and test that the surfaces were parallel when the bars were in position in the framework, two test gauges were made, one for testing the upper surface and the other the side of the bars. The bars were taken out of the frame and placed vertical, the upper bar resting with its cut surface accurately on the cut surface of the lower bar, and not being supported by any other means. Four screws on each gauge were then adjusted so that they would touch four points, two on each bar near its extremities. By applying these gauges to the bars when in position, either in contact with each other or with a disc between, it is therefore easy to make the required adjustment of the cut surfaces.

The surfaces were now amalgamated, the amalgamated surfaces brought together, and experiments made on the distribution of temperature throughout the bar, to test the efficiency of the mercury contact. When the bar was uncut, the difference of temperature observed between the points in the middle, 1.05 cm. apart, was, as a mean of three experiments, 76°C . With the cut bar and the mercury contact, the distribution of temperature in the other parts of the bars being the same, the difference of temperature observed was 64°C . The cutting and grinding of the bar had reduced the distance between the points of observation to .88 cm., hence if the mercury contact has no appreciable resistance the fall should be $\frac{.76 \times .88}{1.05} = 65^{\circ}$, which is, within the limits of error, what was observed. Hence, as far as the distribution of temperature along the bar is concerned, the mercury contact secures practical continuity. Occasional tests were made during the course of the work to see if this continued to hold. The resistance of the contact was found to increase slightly. Three tests made at the end of the work gave 78° as the difference of temperature. This is equivalent to an increase of distance between the points of observation from .88 cm. to 1.06 cm., and is probably caused by the gradual soaking of the mercury into the brass. Account is taken of this in the following calculations.

The conditions of the above experiments are not quite the same as in experiments with crystals, &c., for while in the above there is only one mercury film, in the crystal experiments there are two. To investigate the effect of this, a thin film of mica, about .0003 cm. thick, was inserted. Although there are then two films of mercury and a film of mica between the bars, no change could be detected in the

difference of temperature of the observing points. The same held on insertion of a piece of platinum foil .0025 cm. thick. The corrections for the short length of bar and mercury contact between the points at which the temperatures are observed and the surfaces of the discs, can, therefore, be determined from the observation with the amalgamated ends of the bars in contact.

In preparing the apparatus for an experiment, the bars and framework were placed so that the bars were vertical, with the cool bar at the top. The bars were taken out of the frame and the holes and contacts cleaned with dilute nitric acid, provided with mercury, washed, and then dried by filter paper. The hot bar was then inserted into lower part of the frame, and if a disc was to be experimented on, it was placed on the mercury surface and the excess of mercury forced from under it. In the case of a transparent disc it was possible to see that the mercury formed a perfect mirror. The cool bar was then put into the upper part of the frame, and held up, in contact with the centre screw at the top of the frame, by means of a spring. The amalgamated end would be thus brought about .5 cm. above the upper surface of the disc. It was then supplied with mercury, which hung down as a pendent drop. The upper screw was then turned till the end of the upper bar was about .5 mm. from the upper surface of the disc. In this position the gauges were applied to the bars, and the loops so adjusted by means of the screws at their ends, that the four points of each gauge touched the bars. The upper screw was then rotated till the upper bar came into contact with the disc, and the excess of mercury was forced out. The frame was then placed over the water tank with the bars horizontal and connections made to the steam and water supplies. The apparatus was allowed to stand about one and a half hours, in order that the distribution of temperature might become steady before observations were taken. Almost invariably on taking the bars apart at the conclusion of an experiment, the discs were found to adhere to the cool bar, and in the case of transparent discs this enabled it to be seen if the contact between the cool bar and the disc had been good. It was generally found as good a mirror as the contact between the hot bar and the disc. Some experiments were however made to try the effect of making contacts which were poor as far as could be judged by the appearance of the mirrors. No difference could be detected in the observations between an optically good and a bad mirror, so that a few experiments, in which the contacts were optically defective, have been included in the results given.

Reduction of Observations with Discs.

We have seen that the temperature throughout any short length of the bar may be represented by the empirical equation $v = A \cosh \alpha x + B \sinh \alpha x$. We assume that this equation holds for the first three observations on each side of the disc, and thus have, if x_1, x_2, x_3 are the coordinates of the points of observation, measured from the surface of the disc, and v_1, v_2, v_3 the observed temperatures,

$$\left. \begin{aligned} v_1 &= A \cosh \alpha x_1 + B \sinh \alpha x_1 \\ v_2 &= A \cosh \alpha x_2 + B \sinh \alpha x_2 \\ v_3 &= A \cosh \alpha x_3 + B \sinh \alpha x_3 \end{aligned} \right\}$$

These three equations are sufficient to fix the values of the constants A, B, α , but their solution is difficult unless $x_3 - x_2 = x_2 - x_1$. This relation is very nearly satisfied by the points of observation, in the hot bar, $x_2 - x_1 = 10.56$ cms. $x_3 - x_2 = 10.44$ cms.; in the cool bar, $x_2 - x_1 = 10.51$ cms. $x_3 - x_2 = 10.48$ cms.

The error introduced by assuming each of these intervals = 10.5 cms. is small enough to be neglected in the cool bar, but a small correction is necessary in the case of the hot bar.

If dv_2/dx be the value of dv/dx at the point x_2 we have for the temperature v_2' at the point $x_1 + 10.5$,

$$v_2' = v_2 - .06 \frac{dv_2}{dx}.$$

Now dv_2/dx is found to differ little from 1 for any of the experiments,* and we may, therefore, with sufficient accuracy, take $v_2' = v_2 - .06$.

Making use of v_2' we have from the three temperatures, v_1, v_2', v_3

$$\cosh \alpha l = \frac{v_1 + v_3}{2v_2'},$$

where $l = x_2 - x_1 = x_3 - x_2$, &c. This equation determines α from any three observations of temperature. The mean of the values thus determined for different points along the bars and for different experiments is used in the subsequent work.

We have then the equations

$$\left. \begin{aligned} v_1 &= A \cosh \alpha x_1 + B \sinh \alpha x_1 \\ v_2 &= A \cosh \alpha x_2 + B \sinh \alpha x_2 \end{aligned} \right\}$$

* The value of dv_2/dx may be determined as follows. By TAYLOR'S Theorem we have—

$$f(x+l) - f(x) = lf'(x) + \frac{l^2}{2}f''(x) + \frac{l^3}{3}f'''(x) + \&c.$$

$$f(x) - f(x-l) = lf'(x) - \frac{l^2}{2}f''(x) + \frac{l^3}{3}f'''(x) + \&c.$$

Therefore

$$\frac{f(x+l) - f(x-l)}{2l} = f'(x) + \frac{l^2}{3}f'''(x).$$

Putting $v = f(x)$ we have, since for any short length of the bar $\frac{d^2v}{dx^2} = \alpha^2v$,

$$f'''(x) = \alpha^2 f'(x).$$

Therefore

$$f'(x) = \frac{f(x+l) - f(x-l)}{2l \left(1 + \frac{l^2\alpha^2}{6}\right)}.$$

The value of $\frac{1}{6}(l^2\alpha^2)$ for this point of the bar is in all the experiments approximately .04.

where $\cosh \alpha x_1$, $\sinh \alpha x_1$, $\cosh \alpha x_2$, $\sinh \alpha x_2$ are known quantities, to determine the values of A and B.

Differentiating the equation $v = A \cosh \alpha x + B \sinh \alpha x$ and putting $\alpha = 0$, we have at the surface in contact with the disc, $v = A$, $dv/dx = \alpha B$.

The isothermal surfaces in the discs themselves may be assumed to be planes, for a small calculation, like that made (p. 490) for the bar itself, shows that the radius of curvature of these surfaces is about 40 cms. Writing α_1 , α_2 for the coordinates of the surfaces of the disc, A_1 , A_2 the temperatures determined as above, Q_1 , Q_2 the values of $k_1 \alpha B$ in the bars at the surfaces in contact with the disc, k_1 being the conductivity of bars, and k that of the disc, we have, for the temperature in the disc itself:

$$v = \frac{A_2 \sinh \sqrt{\frac{ph}{qk}} \cdot \overline{x - \alpha_1} + A_1 \sinh \sqrt{\frac{ph}{qk}} \cdot \overline{\alpha_2 - x}}{\sinh \sqrt{\frac{ph}{qk}} \overline{\alpha_2 - \alpha_1}}.$$

Differentiating and writing down the expressions for the flow of heat into and out of the disc we have the equations—

$$Q_1 q_1 = qk \sqrt{\frac{ph}{qk}} \cdot \frac{A_2 - A_1 \cosh \sqrt{\frac{ph}{qk}} \overline{\alpha_2 - \alpha_1}}{\sinh \sqrt{\frac{ph}{qk}} \overline{\alpha_2 - \alpha_1}},$$

and

$$Q_2 q_1 = qk \sqrt{\frac{ph}{qk}} \frac{A_2 \cosh \sqrt{\frac{ph}{qk}} \overline{\alpha_2 - \alpha_1} - A_1}{\sinh \sqrt{\frac{ph}{qk}} \cdot \overline{\alpha_2 - \alpha_1}}$$

where p , h , q , k refer to the disc and have the usual meanings, q being $= q_1$ in most cases, differing only slightly in others. Now $\alpha_2 - \alpha_1$, the thickness of the discs, is small enough to make $\sqrt{\frac{ph}{qk}} \overline{\alpha_2 - \alpha_1}$ small. Hence, writing $\alpha_2 - \alpha_1 = t$, and expanding the hyperbolic functions, we have, as a close approximation—

$$q_1 Q_1 = qk \sqrt{\frac{ph}{qk}} \frac{A_2 - A_1 \left(1 + \frac{ph}{qk} \cdot \frac{t^2}{2}\right)}{\sqrt{\frac{ph}{qk}} \cdot t \left(1 + \frac{ph}{qk} \cdot \frac{t^2}{6}\right)}.$$

Therefore

$$k = \left(\frac{q_1}{q}\right) \cdot \frac{Q_1 t \left(1 + \frac{ph}{qk} \cdot \frac{t^2}{6}\right)}{A_2 - A_1 \left(1 + \frac{ph}{qk} \cdot \frac{t^2}{2}\right)};$$

similarly

$$k = \left(\frac{q_1}{q}\right) \cdot \frac{Q_2 t \left(1 + \frac{ph}{qk} \cdot \frac{t^2}{6}\right)}{A_2 \left(1 + \frac{ph}{qk} \cdot \frac{t^2}{2}\right) - A_1};$$

either of which equations determines k , the conductivity of the disc, if its approximate value is substituted in the terms involving t^2 . The value of h used is that for a painted surface, at the mean temperature of the disc, and the value of $\frac{ph}{qk} t^2$ is given with sufficient accuracy for the above substitution by the equation

$$\frac{ph}{qk} t^2 = \cdot 00056 \frac{A_2 - A_1}{Q} t.$$

*Observations.**

I TRANSPARENT ISOTROPIC BODIES.—The temperatures given are the means of three experiments.

CROWN Glass Disc, 1.93 cm. diam., .169 cm. thick. Air, 16°·2 C.

Cool bar.	Hot bar.
v_1 8.58 A_1 8.83 v_2 4.20 Q_1 .1365 $\therefore k_1 = \frac{.1365 \times .169 \times 1.001}{18.25 - 8.83 \times 1.003} = \cdot 00246$	v_1 18.44 A_2 18.25 v_2 26.54 Q_2 .1336 $k_2 = \frac{.1336 \times .169 \times 1.001}{18.25 \times 1.003 - 8.83} = \cdot 00241$

k mean = .00244.

FLINT Glass Disc, 1.93 cm. diam., .177 cm. thick. Air, 16°·6 C.

Cool bar.	Hot bar.
v_1 7.60 A_1 7.83 v_2 3.54 Q_1 .1254 $\therefore k_1 = \frac{.1254 \times .177 \times 1.002}{18.84 - 7.83 \times 1.006} = \cdot 00203$	v_1 19.02 A_2 18.84 v_2 26.80 Q_2 .1246 $k_2 = \frac{.1246 \times .177 \times 1.002}{18.84 \times 1.006 - 7.83} = \cdot 00199$

k mean = .00201.

Rock Salt Disc, 1.95 cm. diam., .83 cm. thick. Air, 15°·0 C.

Cool bar.	Hot bar.
v_1 9.24 A_1 9.50 v_2 4.61 Q_1 .1457 $k_1 = (\cdot 98) \frac{.1457 \times .83 \times 1.005}{18.22 - 9.50 \times 1.014} = \cdot 0138$	v_1 18.47 A_2 18.22 v_2 27.31 Q_2 .1515 $k_2 = (\cdot 98) \frac{.1515 \times .83 \times 1.005}{18.22 \times 1.014 - 9.50} = \cdot 0138$

k mean = .0138.

* The results are collected in a table on page 506.

II. CRYSTALS.—Temperatures, mean of three experiments.

QUARTZ Disc, 1.91 cm. diam., 1.005 cm. thick. Thermal stream lines parallel to optical axis. Air, 15°·1 C.

Cool bar.	Hot bar.
v_1 10.09 A_1 10.50 v_2 4.64 Q_1 .1680 $k_1 = (1.02) \frac{.168 \times 1.005 \times 1.003}{16.40 - 10.5 \times 1.010} = .0298$	v_1 16.69 A_2 16.40 v_2 26.28 Q_2 .1763 $k_2 = (1.02) \frac{.1763 \times 1.005 \times 1.003}{16.40 \times 1.010 - 10.5} = .0299$

$$k \text{ mean} = .0299.$$

QUARTZ Disc, 1.93 cm. diam., .811 cm. thick. Thermal stream lines perpendicular to optical axis. Air, 16°·2 C.

Cool bar.	Hot bar.
v_1 9.09 A_1 9.39 v_2 4.07 Q_1 .1545 $k_1 = \frac{.1545 \times .811 \times 1.004}{17.45 - 9.36 \times 1.011} = .01574$	v_1 17.71 A_2 17.45 v_2 26.86 Q_2 .1616 $k_2 = \frac{.1616 \times .811 \times 1.004}{17.45 \times 1.011 - 9.36} = .01589$

$$k \text{ mean} = .0158.$$

ICELAND Spar Disc, 1.91 cm. diam., .70 cm. thick. Thermal stream lines parallel to optical axis. Air, 16°·4 C.

Cool bar.	Hot bar.
v_1 7.82 A_1 8.10 v_2 3.36 Q_1 .1436 $k_1 = (1.02) \frac{.1436 \times .7 \times 1.004}{18.49 - 8.10 \times 1.013} = .01001$	v_1 18.73 A_2 18.49 v_2 27.41 Q_2 .1478 $k_2 = (1.02) \frac{.1478 \times .7 \times 1.004}{18.49 \times 1.013 - 8.10} = .00998$

$$k \text{ mean} = .0100.$$

ICELAND Spar Disc, 1.93 cm. diam., .602 cm. thick. Thermal stream lines perpendicular to optical axis. Air, 16°·3 C.

Cool bar.	Hot bar.
v_1 7.95 A_1 8.12 v_2 3.33 Q_1 .1405 $k_1 = \frac{.1405 \times .602 \times 1.004}{18.37 - 8.12 \times 1.012} = .00837$	v_1 18.61 A_2 18.37 v_2 27.31 Q_2 .1475 $k_2 = \frac{.1475 \times .602 \times 1.004}{18.37 \times 1.012 - 8.12} = .00852$

$$k \text{ mean} = .00845.$$

MICA, large plate, .18 cm. thick. Thermal stream lines perpendicular to cleavage planes. Air, 15°·6 C.

Cool bar.	Hot bar.
v_1 6·36 A_1 6·58 v_2 2·77 Q_1 ·1093 $k_1 = \frac{·1093 \times .18 \times 1·002}{18·67 - 6·58 \times 1·05} = .00164$	v_1 18·90 A_2 18·67 v_2 27·44 Q_2 ·1436 $k_2 = \frac{·1436 \times .18 \times 1·002}{18·67 \times 1·05 - 6·58} = .00199$

$$k \text{ mean} = .00181.$$

As the mica was in the form of a large plate, the above method of finding k is not strictly correct, and this accounts for the difference between the calculated values k_1 and k_2 . It was found impossible to use a disc, as in the other experiments, on account of the mica splitting. The value of k given must be taken as approximate only.

III. ROCK SPECIMENS.

MARBLE Disc, 1·93 cm. diam., .26 cm. thick. The marble was the white variety used for statues. Air, 15°·5 C.

Cool bar.	Hot bar.
v_1 10·15 A_1 10·40 v_2 4·38 Q_1 ·1767 $k_1 = \frac{·1767 \times .26 \times 1·001}{17·01 - 10·4 \times 1·003} = .00699$	v_1 17·31 A_2 17·01 v_2 27·27 Q_2 ·1839 $k_2 = \frac{·1839 \times .26 \times 1·001}{17·01 \times 1·003 - 10·4} = .00718$

$$k \text{ mean} = .00709.$$

SLATE Disc, 1·93 cm. diam., .31 cm. thick, cut from an ordinary school slate. Air, 14°·9 C.

Cool bar.	Hot bar.
v_1 8·80 A_1 9·08 v_2 4·27 Q_1 ·1508 $k_1 = \frac{·1508 \times .31 \times 1·002}{19·15 - 9·08 \times 1·006} = .00468$	v_1 19·40 A_2 19·15 v_2 28·58 Q_2 ·1581 $k_2 = \frac{·1581 \times .31 \times 1·002}{19·15 \times 1·006 - 9·08} = .00482$

$$k \text{ mean} = .00475.$$

IV. VARIOUS BODIES CONSTANTLY USED IN PHYSICAL WORK.

Where no diameter is given the plate is of irregular shape, and rather larger than the section of the bars.

SHELLAC Plate, .068 cm. thick. Air, 14°·2 C.

This plate was made between two microscope slides having plane surfaces. One slide was placed horizontal, and the upper surface covered with a smooth sheet of tinfoil. On the foil was laid a sufficient number of the thin films of shellac supplied by varnishmen to make a plate of the requisite size and thickness. The slide was then warmed gently from underneath till the shellac was soft, and the cold slide brought down on to the soft mass. In order to get the requisite thickness, and the surfaces of the plate parallel to each other, the two slides were kept apart by two short lengths of wire .068 cm. diameter laid across the lower slide near its ends, the upper slide being pressed against these. The tinfoil which attaches itself to the plate is readily dissolved off by mercury. This method is very convenient and gives excellent plates.

Cool bar.	Hot bar.
v_1 7·16 A_1 7·40 v_2 3·32 Q_1 ·1183 $k_1 = \frac{·1183 \times .068 \times 1·001}{20·76 - 7·40 \times 1·002} = .000604$	v_1 20·95 A_2 20·76 v_2 28·74 Q_2 ·1159 $k_2 = \frac{·1159 \times .068 \times 1·001}{20·76 \times 1·002 - 7·40} = .000589.$

$$k \text{ mean} = .000596.$$

PARAFFIN Plate, .038 cm. thick. Air, 13°·4 C.

This plate was made by dropping a little melted paraffin on to the cold upper surface of the lower slide, and placing the upper slide on it as described above. The cold surfaces are sufficient to ensure the paraffin being detachable from the glass.

Cool bar.	Hot bar.
v_1 8·90 A_1 9·19 v_2 4·49 Q_1 ·1378 $k_1 = \frac{·1378 \times .038 \times 1·0002}{17·87 - 9·19 \times 1·0006} = .000604$	v_1 18·10 A_2 17·87 v_2 26·48 Q_2 ·1407 $k_2 = \frac{·1407 \times .038 \times 1·0002}{17·87 \times 1·0006 - 9·19} = .000615$

$$k \text{ mean} = .000610.$$

PARA RUBBER, pure. Sheet .0264 cm. thick. Air, 13°·8 C.

Cool bar.	Hot bar.
v_1 9·00 A_1 9·26 v_2 5·01 Q_1 ·1401 $k_1 = \frac{·1401 \times ·0264 \times 1·0002}{19·00 - 9·26 \times 1·0005} = ·000380$	v_1 19·23 A_1 19·00 v_2 27·87 Q_2 ·1433 $k_2 = \frac{·1433 \times ·0264 \times 1·0002}{19·00 \times 1·0005 - 9·26} = ·000388$

$$k \text{ mean} = ·000384.$$

SULPHUR Plate, .0584 cm. thick. Air, 14°·2 C.

This plate was made in the same way as the plate of shellac, except that it was found better to use a sheet of tinfoil on the surface of each slide.

Cool bar.	Hot bar.
v_1 6·60 A_1 6·83 v_2 3·08 Q_1 ·1082 $k_1 = \frac{·1082 \times ·0584 \times 1·0007}{20·74 - 6·83 \times 1·002} = ·000455$	v_1 20·92 A_2 20·74 v_2 28·42 Q_2 ·1085 $k_2 = \frac{·1085 \times ·0584 \times 1·0007}{20·74 \times 1·002 - 6·83} = ·000455$

$$k \text{ mean} = ·000455.$$

EBONITE Disc,* 1·93 cm. diam., .0414 cm. thick. Air, 14°·9 C.

Cool bar.	Hot bar.
v_1 7·35 A_1 7·61 v_2 3·36 Q_1 ·1227 $k_1 = \frac{·1227 \times ·0414 \times 1·0004}{20·10 - 7·61 \times 1·0013} = ·000407$	v_1 20·30 A_2 20·10 v_2 28·16 Q_2 ·1202 $k_2 = \frac{·1202 \times ·0414 \times 1·0004}{20·10 \times 1·0013 - 7·01} = ·000398$

$$k \text{ mean} = ·000403.$$

* Kindly supplied to me by Mr. C. H. GRAY, of the Silvertown Company.

GUTTA PERCHA Sheet, .0617 cm. thick. Air, 14°·4 C.

This sheet was made of the material used for insulating wire. A small quantity was cut from a wire, melted, and pressed into a sheet between the two cold microscope slides.

Cool bar.	Hot bar.
v_1 5·97 A_1 6·19 v_2 2·35 Q_1 ·1087 $k_1 = \frac{·1087 \times ·0617 \times 1·0006}{20·76 - 6·19 \times 1·002} = ·000461$	v_1 20·94 A_2 20·76 v_2 28·40 Q_2 ·1076 $k_2 = \frac{·1076 \times ·0617 \times 1·0006}{20·76 \times 1·002 - 6·19} = ·000456$

$$k \text{ mean} = ·000458.$$

PAPER Disc, 1·93 cm. diam., .019 cm. thick. Air, 15°·4 C.

This was cut from an ordinary visiting card.

Cool bar.	Hot bar.
v_1 9·42 A_1 9·73 v_2 4·56 Q_1 ·1515 $k_1 = \frac{·1515 \times ·019 \times 1·0001}{18·81 - 9·73 \times 1·0003} = ·000317$	v_1 19·05 A_2 18·81 v_2 27·91 Q_2 ·1496 $k_2 = \frac{·1496 \times ·019 \times 1·0001}{18·81 \times 1·0003 - 9·73} = ·000313$

$$k \text{ mean} = ·000315.$$

“ASBESTOS Paper” Disc, 1·93 cm. diam., .047 cm. thick. Air, 14°·9 C.

This was the ordinary asbestos millboard of commerce. It consists of paper to the pulp of which sufficient asbestos has been added to render it incombustible.

Cool bar.	Hot bar.
v_1 8·40 A_1 8·68 v_2 4·02 Q_1 ·1358 $k_1 = \frac{·1358 \times ·047 \times 1·0004}{19·93 - 8·68 \times 1·001} = ·000568.$	v_1 20·16 A_2 19·93 v_2 28·67 Q_2 ·1368 $k_2 = \frac{·1368 \times ·047 \times 1·0004}{19·93 \times 1·001 - 8·68} = ·000571$

$$k \text{ mean} = ·000570.$$

MAHOGANY Disc, 1.93 cm. diam., .056 cm. thick, cut from a sheet of "veneer"
used by cabinet makers. Air, 15°·9 C.

Cool bar.	Hot bar.
v_1 6.67 A_1 6.89 v_2 3.06 Q_1 .1107 $k_1 = \frac{.1107 \times .056 \times 1.0006}{20.57 - 6.89 \times 1.002} = .000454$	v_1 20.75 A_2 20.07 v_2 28.51 Q_2 .1167 $k_2 = \frac{.1167 \times .056 \times 1.0006}{20.57 \times 1.002 - 6.89} = .000477$

k mean = .000465.

WALNUT Disc, 1.93 cm. diam., .069 cm. thick, cut from a sheet of veneer.
Air, 16°·0 C.

Cool bar.	Hot bar.
v_1 5.11 A_1 5.29 v_2 2.14 Q_1 .0917 $k_1 = \frac{.0917 \times .069 \times 1.001}{22.46 - 5.29 \times 1.004} = .000363$	v_1 22.60 A_2 22.46 v_2 29.52 Q_2 .0888 $k_2 = \frac{.0888 \times .069 \times 1.001}{22.46 \times 1.004 - 5.29} = .000358$

k mean = .000360.

CORK Disc, 1.93 cm. diam., .05 cm. thick, cut by a section-cutting machine from a
sound cork. Air, 15°·1 C.

Cool bar.	Hot bar.
v_1 2.22 A_1 2.33 v_2 .34 Q_1 .0530 $k_1 = \frac{.0530 \times .05 \times 1.002}{23.30 - 2.33 \times 1.005} = .000127$	v_1 23.39 A_2 23.30 v_2 29.10 Q_2 .0547 $k_2 = \frac{.0547 \times .05 \times 1.002}{23.30 \times 1.005 - 2.33} = .000130$

k mean = .000129.

SILK Sheet, .0084 cm. thick. This was cut from a dress piece of plain brown silk.
Air, 16°·4 C.

Cool bar.	Hot bar.
v_1 10.06 A_1 10.39 v_2 4.95 Q_1 .1600 $k_1 = \frac{.16 \times .0084}{16.76 - 10.39} = .000210$	v_1 17.03 A_2 16.76 v_2 26.35 Q_2 .1690 $k_2 = \frac{.169 \times .0084}{16.76 - 10.39} = .000223$

k mean = .000216.

SILK Sheet, .017 cm. thick, cut from a piece of ribbed silk ribbon. The thickness given is the mean of that across ribs and across grooves. Air, 16°·6 C.

Cool bar.	Hot bar.
v_1 8·03 A_1 8·30 v_2 3·72 Q_1 .1333 $k_1 = \frac{.1333 \times .017}{18·65 - 8·30} = .00022$	v_1 18·88 A_2 18·65 v_2 27·37 Q_2 .1415 $k_2 = \frac{.1415 \times .017}{18·65 - 8·30} = .00023$

$$k \text{ mean} = .00023.$$

COTTON Sheet, .085 cm. thick. A piece of thick tape. Air, 16°·4 C.

Cool bar.	Hot bar.
v_1 5·23 A_1 5·43 v_2 1·95 Q_1 .100 $k_1 = \frac{.100 \times .085 \times 1·001}{21·24 - 5·43 \times 1·004} = .000539$	v_1 21·41 A_2 21·24 v_2 28·79 Q_2 .1044 $k_2 = \frac{.1044 \times .085 \times 1·001}{21·24 \times 1·004 - 5·43} = .000557$

$$k \text{ mean} = .00548.$$

FLANNEL Sheet, .1 cm. thick. Air, 15°·1 C.

Cool bar.	Hot bar.
v_1 2·34 A_1 2·44 v_2 .59 Q_1 .0504 $k_1 = \frac{.0504 \times .1 \times 1·006}{24·58 - 2·44 \times 1·018} = .000229$	v_1 24·67 A_2 24·58 v_2 30·39 Q_2 .0505 $k_2 = \frac{.0505 \times .1 \times 1·006}{24·58 \times 1·018 - 2·44} = .000225$

$$k \text{ mean} = .000227.$$

As the silk, cotton, and flannel yield somewhat, the measurements of thickness are not very accurate in these three cases. The error will be greatest in the cases of flannel and cotton, and probably small in the case of silk.

TABLE of Results.

Material and direction of stream lines.	Thermal conductivity between 25 and 35° C. C.G.S. units.		Refractive Index.
Copper	·7 to ·8 LORENZ, F. WEBER, &c.	
Brass	·25 to ·3 LORENZ, NEUMANN, &c.	
Brass bar used	·27		
Bismuth	·017 LORENZ	
Mercury	·018 ÅNGSTRÖM	
Crown glass	·00243	·0016 H. MEYER	1·53
Flint glass	·00201	·0014 „	1·64
Glass	{ ·0021 PECLET	
Rock salt	·0138	·0005 G. FORBES	1·55
		·016 TUSCHMIDT	
Quartz along axis	·0299	{ ·026 TUSCHMIDT	1·55
„ perpendicular to axis . . .	·0158	·001 G. FORBES	1·56
Iceland spar along axis	·0100	{ ·016 TUSCHMIDT	1·66
„ perpendicular to axis . . .	·0084	·004 G. FORBES	1·49
Mica perpendicular to cleavage .	·0018	·016 TUSCHMIDT	1·57
		·0086 „	
White marble	·0071	{ ·0075 PECLET	
Slate	·0047	·0073 YAMAGAWA	
		·0011 G. FORBES	
		·0008 „	
Water	·0015 WINKELMANN	
Glycerine	·0007 „	
Olive oil	·0004 F. WEBER	
Shellac	·00060		
Paraffin	·00061	·00014 G. FORBES	
Pure Pará rubber	·00038		
Rubber	·00046 PECLET	
Vulcanised rubber	·00009 G. FORBES	
Sulphur	·00045		
Ebonite	·00040	·00008 „	
Gutta percha	·00046		
Paper	·00031	·00045 G. FORBES	
Asbestos paper	·00057		
Mahogany across fibre	·00047		
Walnut across fibre	·00036		
Cork	·00013		
Silk	·00022		
Cotton	·00055		
Flannel	·00023		

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In the foregoing table the conductivities of a few metals and liquids are given in order to show the position of the bodies experimented on amongst other conductors.

Where experiments have been previously made by other experimenters their results are given for comparison.

Such comparison shows a fair degree of concordance between the present results and those obtained by totally different methods, with the exception of the results of G. FORBES, which were obtained by the following method. A can about 9 cms. diameter was filled with a freezing mixture, placed on a sheet of the material the conductivity of which was to be determined, and the lower surface of the sheet brought down on to a surface of water at 0° C. Ice is formed on the surface of the sheet, and FORBES works out an expression for the thickness of this ice in terms of the temperature of the freezing mixture (which was observed by a thermometer placed in it), the thickness of the material, the time, and the conductivities of ice and the material. In working out this expression, however, he assumes the conductivity of the materials between the thermometer in the freezing mixture and the upper surface of the sheet to be infinitely great, which, when one considers the layers of liquid, metal, and air present, is scarcely justifiable. On this account almost all FORBES'S results are low.

PECLÉ'S results for metals were all found to be low, and this led to the assumption that all his results were the same. It is, however, evident from the present experiments that his results for bad conductors are correct. This is probably owing to the fact that the layers of still water which, in his experiments, adhered to the surfaces of the sheet experimented on, are of much less importance when the sheet is a bad conductor.

TUSCHMIDT'S results are obtained by a method which WEBER used for liquids. The sheet of liquid is simply replaced by one of the crystal, the contacts being made by means of glycerine. As glycerine has a conductivity which is only about $\frac{1}{20}$ th of those of the crystal plates, the glycerine layers have a great effect on the flow of heat through the plates. This is a defect of the method, and, in addition, the plates themselves must be large, and are therefore expensive. TUSCHMIDT'S results and those of

the present paper agree fairly, the only great difference being in the case of Iceland spar along the axis.

MEYER'S determinations depend on the rate of rise of the temperature of a calorimeter into which a heated cube of the material is plunged. It is questionable whether the assumption which he makes, that the temperature of the surface of the cube at any instant is that given by the thermometer in the calorimeter, is justifiable on account of the difficulty of preventing a layer of water adhering to the surface. The same objection applies to the method which he used to confirm the results obtained by his first method.

Remarks.

From the Table of Results it is at once evident that, for transparent bodies, no such comparison between thermal conductivity and velocity of propagation of light can be made, as has been made by KUNDT for the metals. The thermal conductivities vary enormously for very small change of refractive index, and the variations are sometimes in the same direction as the variation of the index of refraction, sometimes in opposite. It is remarkable that the bodies quartz and rock salt, which are diathermanous bodies, should also be good thermal conductors, quartz being a better conductor than bismuth. To prove that the high value of the conductivity is not due to heat passing through these bodies by radiation from the hot bar to the cold, several experiments were made with the apparatus arranged as it was during the experiment in quartz, but the quartz disc was removed, thus allowing the hot bar to radiate heat through the intervening air space to the cool bar. Under these conditions, no change of temperature at the point of observation near the end of the cool bar, could be detected when the hot bar was suddenly cooled. As air is a more diathermanous body than quartz, it is thus evident that the amount of heat radiated through the quartz from one bar to the other, is too small to affect the above results.

The high conductivity of quartz would render the use of fused quartz (the conductivity of which does not probably differ much from that of quartz crystal) advantageous for vessels subject to sudden change of temperature, and for delicate thermometers.

The results for Iceland spar and marble seem to indicate that the irregular arrangement of the crystals in marble interferes with the passage of heat.

Water and salt solutions have conductivities about equal to that of glass.

The solid insulations used in electrical work rank with glycerine and the oils, so far as thermal conductivity is concerned.

Silk has about one-third the conductivity of shellac. Hence, if a silk covered wire has the covering saturated with shellac, it will have its heat conducted away to surrounding bodies much faster than previously. The practice of soaking galvanometer coils in shellac is, therefore, good from a thermal point of view, as it enables the heat generated in the wire to pass more rapidly to the exterior of the coils, and be radiated away.

On account of the low conductivity of paraffin, standard resistance coils with temperature coefficients ought not to be embedded in paraffin, as the B.A. standard is.

Cork is the worst conductor experimented on.

Almost all the substances experimented on are insulators electrically. Although tables of their electrical conductivities are available, the values given seem to depend more on the condition of the surface of the body experimented on than on its material, and, on this account, it is of little use to make any comparisons between the electrical and the thermal conductivities of these substances.

The above experiments were carried on in the Physical Laboratory of the Owens College.