VIII. *On the Measurement of the Magnetic Properties of Iron.*

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[Plates 5–16.]

This paper contains some of the results and a description of the methods employed in a series of experiments on the rate of change of an electric current, immediately after the application or reversal of a constant electromotive force, in a circuit containing the magnetizing coil of a large electromagnet or the primary coil of a transformer. The experiments have been carried out in the Electrical Laboratories of the Rose Polytechnic Institute, Terre Haute, Ind. The results seem to be of considerable interest, and the method of studying the magnetic properties of iron here proposed has, I believe, some important advantages in many practical cases where the magnetic circuit is closed and of large section.

In the experimental determination of the magnetic properties of iron it has been usual to determine by means of a series of successive experiments the value of the total magnetization produced by different magnetizing forces. From these results the magnetic permeability of the iron, the self-induction of the circuit, and so forth, can, of course be calculated. Several methods are well known, by means of which reliable results can be obtained in this way, but they are, in many cases, inconvenient. For closed magnetic circuits, for example, the method commonly employed has been to measure, by means of the current induced in a coil of wire surrounding the iron, and in circuit with a ballistic galvanometer, the changes of magnetization produced by different changes of the current in a magnetizing coil. By this method, the value of the integral $\int_{c_1}^{c_2} \mathrm{L}\, dc$, or its equivalent $\int_{t_1}^{t_2} e\, dt$, can be measured. In the first form of the integral, c_1 and c_2 are the initial and final values of the current in, and L the coefficient of induction of, the magnetizing coil. In the second form, $t_2 - t_1$ is the interval of time required for the current to change from the value c_1 to the value c_2, and e is the back electromotive force induced by the rate of change of the current at any instant between the times t_1 and t_2. When the masses of iron experimented on are large, the interval of time $t_2 - t_1$ becomes too great for the value of the integral

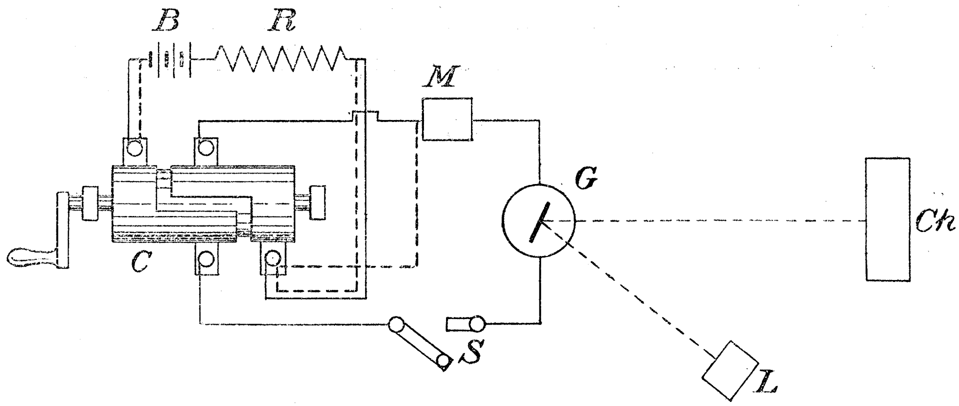
to be accurately measured by means of any ordinary form of ballistic galvanometer joined in circuit with a second, or induction, coil surrounding the iron.

It occurred to me, somewhat more than a year ago, that the cause of the difficulty in the ordinary method of measuring magnetic quality in large masses of iron might be made an advantage if the method of experiment were changed, that, in fact, the desired information might be more easily obtained by recording, not the total value of the induction for each of several particular values of the magnetizing force, but the curve showing the rise of current with time, immediately after the circuit is closed. Preliminary experiments showed that, for many of the cases most difficult by the ordinary method, this could be done both accurately and easily. In these preliminary experiments the coils of a large electromagnet, having laminated cores and pole pieces, were joined in circuit with a storage battery and a non-inductive resistance. An automatic arrangement was devised, by means of which the circuit could be closed and the instant of closing recorded by a chronograph. The ends of the non-inductive resistance were at the same time connected to the quadrants of an electrometer. At any desired interval after the closing of the circuit the electrometer could be disconnected from the resistance, and the instant of disconnecting recorded by the chronograph. The deflection of the electrometer needle then showed the difference of potential between the ends of the non-inductive resistance at the instant when it was disconnected. By this means a sufficient number of points on the curve of rise of current could be observed to enable it to be drawn. The results obtained in these early experiments agree with those given in this paper, and the method may be found useful in cases where the rise of current is too rapid to allow of the successful operation of the method since used. It is evident that a modification of this method with a ballistic galvanometer substituted for the electrometer could be easily arranged to give satisfactory results. The ordinary ballistic galvanometer and induction coil method can also be used with a break circuit arrangement, allowing the total time integral to be taken by steps. This method of experiment was not used after the preliminary results for which it was devised had been obtained. It was abandoned, partly because of its considerable complication, but mainly because a considerable number of separate experiments are required to obtain the complete curve, thus involving a corresponding number of independent magnetizations of the magnet.

The arrangement of the apparatus used in the experiments, the results of which are here illustrated, is shown in the annexed diagram (fig. 1). In this diagram M is the electromagnet experimented on; C, a commutator for reversing the battery, or, when the connection to it was as shown by the dotted lines, for simultaneously cutting out the battery and short-circuiting the magnet coils; B is the battery, and R a resistance of about one ohm in its circuit to prevent excessive current, when, by the operation of the commutator, it is for an instant short-circuited; G is a mirror galvanometer, nearly "dead beat" and of short period; L, the galvanometer lamp; Ch, a chronograph, and S, a key for closing the circuit. The drum of the chronograph is

14 inches long and 20 inches in circumference, and it can be run at a rate of one turn per minute or of six turns per minute, the speed being in each case almost perfectly uniform. The galvanometer lamp stand was provided with cross-wires, the galvanometer with a light plane mirror, and the cross-wires were clearly focussed on the chronograph sheet by means of a lens placed between the lamp and the mirror. The chronograph sheets were, for convenience of experiment and subsequent reduction, formed of "cross-section" paper, the cross-lines on which were one tenth of an inch apart.

Fig. 1.

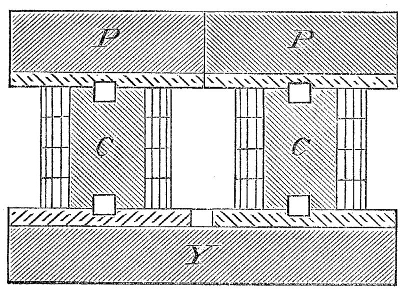


The method of experiment is simply to hold the finger on the switch *S* and observe the motion of the chronograph until the image of the cross-wires comes to a particular line on the paper sheet; at this instant close the switch, and then follow the image of the cross-wires with a soft sharp pencil, dotting the line occasionally, but taking care not to influence the motion of the chronograph. When the deflection has reached a maximum, the commutator is suddenly turned as the image of the wires passes a known position and the cycle of reversal marked out on the paper. The chronograph is now stopped, and the complete curve lightly marked. The experiment is then repeated, the image of the wires being simply observed as it passes over the curve, and any disagreement noted. Any disagreement being corrected and tested, the curve is ready for reduction. The time required to take a complete set of curves, check, and correct them, only requires a few minutes, after a little dexterity has been acquired in dotting out the curves. The curves obtained by this method have the advantage of being drawn to a large scale, the galvanometer sensibility being varied by means of shunts, so as to make the double deflection nearly cover the whole breadth of the chronograph sheet, while for many of the experiments made the length of the diagram extended several times round the drum. I am at present constructing an automatic recording apparatus on the principle of the Thomson recorder for the purpose of drawing diagrams for smaller masses of iron, where the interval of time is so short as to make the initial motion of the spot of light too rapid to be followed with certainty. Should this apparatus prove successful, I hope to apply it to investi-

gate the effect of cyclic speed on the dissipation of energy, using a varying impressed electromotive force.

The results here quoted were obtained from experiments on a large electromagnet belonging to the Electrical Laboratory of the Rose Polytechnic Institute. I have, however, found it easy to trace curves for an ordinary 6000-watt transformer, although the time is in this case considerably shorter than that required for the magnet. I hope to supplement the results here given by results for various transformers, as soon as they can be reduced. The electromagnet was constructed in the shops of the Rose Polytechnic Institute, and was designed by my colleague, Dr. C. L. Mees, and myself for convenient use in the instruction of students in magnetic measurements. It consists of two cores C C (fig. 2), each surrounded by four coils of

Fig. 2.



insulated wire, 2·5 millims. diameter. Each coil is divided into four concentric coils of five layers each, and there are 24 turns in each layer. The total number of turns in the magnetizing coils, when all are joined in series, is thus 3840, while the arrangement allows a considerable variation to be made, if desired, in the distribution, position, power, or resistance of the coil. For example, the inside set of concentric coils may be compared with the outside set to illustrate the law that the magnetizing effect is proportional to the number of turns, and for closed magnetic circuits practically independent of the diameter of the turns. Again, one set of alternate coils may be used for a primary, while the others, joined in any way, may be used for the secondary of a transformer. The magnet, when joined in this way, is capable of transforming from 30,000 to 40,000 watts. For transformer purposes this arrangement has the advantage for purposes of instruction and experiment that the action of an open and a closed circuit transformer can be directly compared. Various other modifications of the arrangement of the magnet coils and cores will readily be seen, with which we are not concerned in this paper.

The cores C C rest on a yoke Y, and on the top of the cores the pole pieces P P rest. The length of the iron circuit can be varied by moving the cores further apart and pushing forward the pole pieces. To facilitate this operation, a screwed rod is run from end to end of the yoke, sunk just beneath the surface; one half has a right, and the other half a left-handed screw, which work in half nuts fixed in the

cores. Thus, by turning the screw, the cores can be moved apart or brought closer together. A similar arrangement serves to move the pole pieces and to pull them apart after they have been strongly magnetized on closed circuit. When an air space is left between the pole faces, the magnetic attraction is resisted by distance pieces, placed between the poles. The necessity for the somewhat elaborate arrangement just described arises from the great weight of the pieces, the cross section of the iron circuit being about 50 square inches, and the length of the yoke 40 inches. The cores and pole pieces were constructed of soft homogeneous iron, laminated in sheets one-twentieth of an inch thick. Each part is held together by plates three-eighths of an inch thick, bolted together through the sheets. One plate of each piece forms a common nut for all the bolts, while the heads of the bolts are countersunk into the other plate. The insulation between the sheets and between the bolts and the holes is simply the oxidized surface of the rolled metal coated with shellac varnish.

Let the impressed electromotive force be E, the resistance of the circuit R and the current at any instant C. Then $\mathrm{E} - \mathrm{RC} = e$, the back electromotive force on the magnetizing coil due to induction. This is also equal to $\mathrm{L}\,(d\mathrm{C}/dt)$, and hence we have $e = \mathrm{E} - \mathrm{RC} = \mathrm{L}\,(d\mathrm{C}/dt)$ or $e\,dt = \mathrm{L}\,d\mathrm{C}$. Consider the curve *Obd* in the diagram,

Fig. 3.

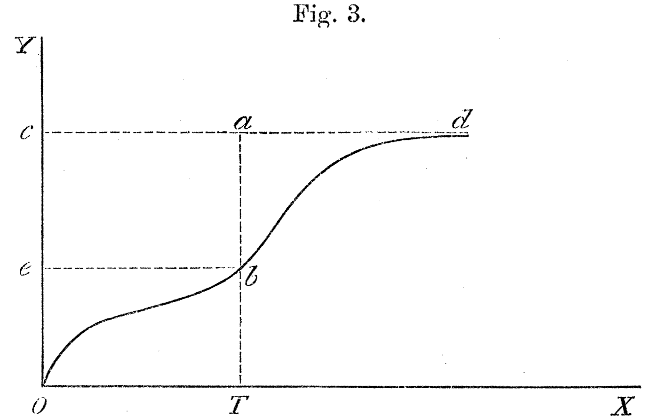


fig. 3, to be a curve showing the relation between current and time immediately after the magnetizing circuit is closed. Let the distance along *OX* to any point represent the interval of time $t_2 - t_1$ and the vertical ordinates the successive values of RC. Then the maximum vertical ordinate $Oc = \mathrm{E}$, and the line *ab* corresponding to any time T is the value of *e* at that time. Hence the area *Obac* is the value of the integral $\int_0^{\mathrm{T}} e\,dt$ or of $\int_0^{\mathrm{C_T}} \mathrm{L}\,d\mathrm{C}$, the total change of induction from the time of closing the circuit to the time T; the current being supposed zero when T is zero. By means of this curve the total induction produced by any current between zero and the maximum that the impressed E.M.F. is capable of producing can thus be studied. Again, by drawing tangents to the curve at points corresponding to different values of the currents flowing through the circuit, the value of $d\mathrm{C}/dt$ and, consequently, of

$e/(dC/dt) = L$, the coefficient of induction at any instant, can be obtained for different values of the magnetomotive force applied, or by comparison with the curve of total induction, for different degrees of magnetization. Similarly, if the curve of variation of current from maximum in one direction to maximum in the other direction, due to a reversal of the impressed E.M.F. be traced, the various changes of total induction, and of L through a complete cycle can be studied. The curve of total induction for the cycle forms generally, as is well known, a closed loop. The area of this loop is the difference of the values of $\int eC\,dt$ for the two halves of the cycle, and, consequently, is proportional to the energy dissipated in a cycle of magnetization. This area includes the dissipation due to magnetic retentiveness, and also that due to currents set up in the iron or in conducting currents surrounding the iron. It is interesting to study these curves, as is done for a few cases below, for different conditions of the magnetic circuit.

This method of experiment applied to electrical transformers has the advantage of giving directly the total dissipation of energy in the iron, and also of furnishing data from which the magnetic quality of the iron can be studied. The electromagnetic arrangement used for the experiments quoted in this paper was, as has already been stated, of such a form that it could be used either for a simple electromagnet or for an open or closed circuit transformer. Diagrams are given illustrating the action in these different cases. It should perhaps be stated that these curves are copies on a reduced scale of the original curves from which the calculations were made, and may not be sufficiently accurate to bear exact comparison. The values of the dissipation of energy, total induction, &c., calculated from the original curves, are therefore marked on the diagrams. The method usually adopted in reducing the curves was to integrate by means of the cross-section paper the successive values of $e\,dt$ for successive values of t, say, for one, two, three, &c., seconds, and plot a new curve with these areas as ordinates and the corresponding value of the current through the magnetizing coil as abscissæ. When the proper scale is adopted this gives the magnetization curve with magnetomotive force as abscissæ and density of magnetic induction as ordinates. From this curve, by a repetition of the integration of areas, the dissipation of energy was deduced. The values of L used in the curves for coefficients of induction were obtained by drawing tangents to the current curve at a sufficient number of points, and from these tangents reading the values of dC/dt. The corresponding value of e, for example, ab in fig. 3, was then divided by dC/dt, and the quotients used as ordinates in curves having current through the coil, that is, with proper change of scale, magnetomotive forces, as abscissæ. Curves of permeability can, of course, be also obtained by taking the ratio of the ordinates to the abscissæ of the induction or magnetization curves, that is $\left(\int_0^c L\,dC\right)\Big/4\pi nC$ as ordinates, and the corresponding values of $4\pi nC$ as abscissæ. These curves have not been drawn, as the curves on the diagrams seem to be already sufficiently numerous, and should the permeability be wanted at any time it can be readily obtained from those already given. The lines of impressed

E.M.F. have also been omitted from the diagrams as unnecessary. In the results, as shown by the derived curves, it should also be stated that no correction has been made for the induction through the coils which did not pass through the iron.

The curves in figs. 1–6 show the rise of current through the circuit of the electromagnet with different impressed E.M.F.'s. The full line curves were obtained after the magnet had been several times magnetized in the same direction as that produced by the current, while the dotted line curves show the rise of current when the magnet had been previously magnetized in the opposite direction by a current of the same maximum strength. The iron circuit was closed during these experiments, and had a mean length of about 265 centims. It will be observed from these figures that the time required to complete the magnetization is nearly inversely proportional to the E.M.F. applied to the poles of the magnet, and, also, that the shape of the curve changes considerably with change of impressed E.M.F. The time required for the current to reach different percentages of its maximum value is shown in figs. 7 and 8. The ordinates of these curves are proportional to the time required, and the abscissæ to the impressed E.M.F. It appears from these curves that for any particular percentage of the maximum current there is always a particular voltage which requires greatest time. The curves also indicate that above a certain voltage, higher than the percentage considered, the time required to complete the magnetization will become nearly constant. Figs. 9 and 10 show the values of the coefficient of induction for different values of the current flowing through the magnetizing coil, and the effect on this coefficient of the degree of magnetization ultimately reached. The most noticeable feature of these curves is the gradual change in the value and position of the maximum. This depends largely on the amount of residual magnetism previously existing in the magnet, and consequently, on the maximum value of the current which had previously flowed through the circuit. The exact value of the maximum coefficient is somewhat difficult to obtain by this method when the current produces a reversal of magnetization, because of the extreme smallness of $d\mathrm{C}/dt$ at the point of the curve where this maximum occurs. No particular importance is attached, therefore, to the apparent maximum of maxima shown in fig. 10. That there is a maximum of maxima, as shown in fig. 10, is, however, possible when we consider the cause of the change in position and magnitude of the maximum value of L from the point of view of the magnetization curve. It is well-known that this curve begins to rise somewhat slowly, and has a point of zero curvature corresponding to a comparatively small value of the magnetizing force. When the residual magnetism is great, and in the same direction as that which the current produces, the results show that the curve never attains to so great a steepness, and attains the greatest steepness later than if no residual magnetism existed. When the residual magnetism is of opposite kind from that produced by the current, the maximum steepness seems at first to increase with increase of residual magnetism, and afterwards to diminish; the position of the point of greatest steepness corresponding to greater and greater

magnetizing forces as the residual magnetism is increased. That there is a particular value of the residual magnetism, which gives greatest steepness of the reversal of the magnetization curve, is possible, but it is not here considered established. The interpretation of the results, with reference to this point, is somewhat complicated by the fact that Foucault currents no doubt produce considerable shifting of the position of the point of maximum, as indicated by the apparent magnetizing current. These currents may also modify the value of the maximum coefficient. This point will be more readily understood in connection with the effect of a secondary coil referred to below.

In fig. 11, a set of curves are shown which illustrate the rise of current when the previous residual magnetism is in the same and in the opposite direction, together with a continuation of the curve showing the rate of diminution of current when the battery is suddenly cut out, the magnet circuit being left closed. The dotted-line curves show the rise and fall of the magnetic induction for the two cases, the curve of fall of current being, of course, the same for both. It will be observed, from the total induction or magnetization curves here shown, that the residual magnetism has amounted to about 60 per cent. of the whole magnetization produced. These, and similar curves shown further on, also illustrate the points referred to in connection with figs. 9 and 10, as to the variation and great difference in the value of the coefficient of induction, as depending on previous magnetization. Fig. 12 shows the current and derived curves for a complete cycle of magnetization. The full line shows the variation of the current after reversal of the impressed E.M.F., while the dotted line shows the corresponding variation of the coefficient of induction. The dot-dash line shows the induction cycle, and the area enclosed by this curve gives the dissipation of energy due to the combined effects of magnetic retentiveness and Foucault currents.

The curves in figs. 13–16 are similar to those in figs. 11 and 12. The impressed E.M.F. was 23 volts for all three sets, but the iron circuit in the magnet contained half a centimetre of air-space for figs. 13 and 14, and two centimetres of air-space for figs. 15 and 16. Comparing the latter two sets of curves with those obtained when the iron circuit was closed, we find as one of the most noticeable features the evidence they afford of the great effect a small air-space has on the magnetic retentiveness of the magnet. Curves (1) and (2) of figs. 13 and 15 are now nearly the same, and the sharp rise in the coefficient of induction has disappeared. The maximum value of the coefficient of induction is now nearly the same, whether the current repeats or reverses previous magnetization, but, somewhat curiously, seems to be somewhat less for reversal than for repetition. The fall of current curves show slower fall at first, due, apparently, to the less firm hold of the residual magnetism and consequent larger initial value of the coefficient of induction just after reversal. This effect would probably be still more marked, were the total induction not considerably diminished by the air-space. Figs. 14 and 16 show the complete cycle, and thus correspond to

fig. 12. The sharp inflection in the current curve does not appear in these curves, and there is evidence that, for a given impressed E.M.F., there will be a particular air-space, which will give the maximum time for the current to pass through zero, and reverse after the reversal of the impressed E.M.F. The curves of coefficient of induction are now comparatively flat, and the curve of total induction is nowhere nearly so steep as it is for the closed magnetic circuit. There is an important difference also in the amount of energy dissipated in the cycle. Of course, a direct comparison of the amount of energy dissipated in the cycle cannot be made from these curves alone, but a separate set of experiments, illustrated in figs. 22–27, bring out the result that considerably less energy is dissipated on the open circuit for the same amount of total induction than is the case with the closed circuit. From the curve (fig. 26) it appears that the energy dissipated per cycle on closed magnetic circuit for total density of induction of 13,200 C.G.S. units is 92×10^7 ergs, while the actual experiment on open circuit only gives 74×10^7 ergs, the air-space being only half a centimetre. Again, from the same curve, it appears that, for closed magnetic circuit and a total density of induction of 8610 C.G.S. units, the energy dissipated per cycle is 4×10^8 ergs, while for the actual experiment, with two centimetres of air-space in the circuit, it is only 3×10^8 ergs per cycle. This is equivalent to a saving of about two and a half foot-pounds of energy per cycle. That the dissipation of energy due to magnetic retentiveness becomes comparatively small when the distance between the poles is as much as half a centimetre, was proved by static experiments made for the purpose. These experiments showed that, for a total induction of 14,000 C.G.S. units per square centimetre across the air-space, the dissipation, as measured by the variation of the field in the air-space, was only 590×10^6 ergs. The intensity of magnetization here quoted was obtained with about 130 volts on the terminals of the magnet, and approaches near to saturation of the cores. When the circuit is open, the dissipation of energy is largely due to Foucault currents in the iron, and, as will be seen from the values calculated and marked on the curves, it approaches towards being proportional to the square of the induction through the iron. This is a result which we should expect so long as the law of the variation of the induction is not very different in the cases compared. The practical bearing of the result here arrived at on the question of open *versus* closed circuit transformers is evident.

The group of figs. 17–21 illustrate results obtained with the magnet used as a transformer with equal numbers of turns on the primary and secondary coils. The coils were joined alternately so as to form two sets of four coils each, each pair of primaries containing between them one secondary and *vice versâ*. Fig. 17 is similar to fig. 12, except that the current is greater and the number of turns in the magnetizing coil less. The different curves in this set will be at once recognized. In fig. 18, the effect of closing the secondary coil through a total resistance, consisting of the coil and incandescent lamps, is shown. The curves of primary and secondary current are clearly marked in the figure, and can be better studied from it than from a description.

It may be remarked that the curve of secondary current is slightly affected by the heating of the lamps, which became a dull red towards the end of each reversal of current in the primary. The effect is slight, however, and the equality of the back E.M.F. on the primary, and the forward E.M.F. on the secondary, is as perfect as the experiment can show. The heavy dot-dash line encloses an area which represents the total dissipation of energy in the iron and in the secondary coil and circuit. We have here the means of illustrating the effect of Foucault currents on the L curve. It will be noticed, by comparing figs. 17 and 18, that the high values of the L curve are displaced to right and left by the existence of the secondary currents, which, by their demagnetizing action, allowed the primary current to become stronger before taking the inflection due to high permeability. The magnetizing force being now the difference of the magnetizing forces of the two coils, the curve of coefficient of induction should be plotted to this difference as abscissæ. This is done in figs. 20 and 21, which show that the maximum value of the coefficient comes earlier the stronger the current in the secondary. Fig. 19 is the same as fig. 18, except that the resistance of the secondary is now simply that of its own coil and the galvanometer circuit, namely, 5·62 ohms. There is now, of course, a greater dissipation area and a greater displacement of the L curve. There are one or two points of interest in connection with the L curves and the curves shown by fine dotted lines which may be noted. The L curve, when calculated from the primary current, besides being displaced, has a higher maximum value the greater the current in the secondary. The dissipation of energy in the magnet itself is less the greater the dissipation in the secondary circuit. When there was no current in the secondary, the energy dissipated per cycle was 1950×10^6 ergs; when the resistance in the secondary was 12·7 ohms, this fell to 1440×10^6 ergs, and when the secondary was short-circuited, it fell to 946×10^6, or less than half the first value. When the differences of the magnetizing forces are considered, the L curve has its maximum nearer to zero current, the greater current in the secondary, and has nearly a constant value.

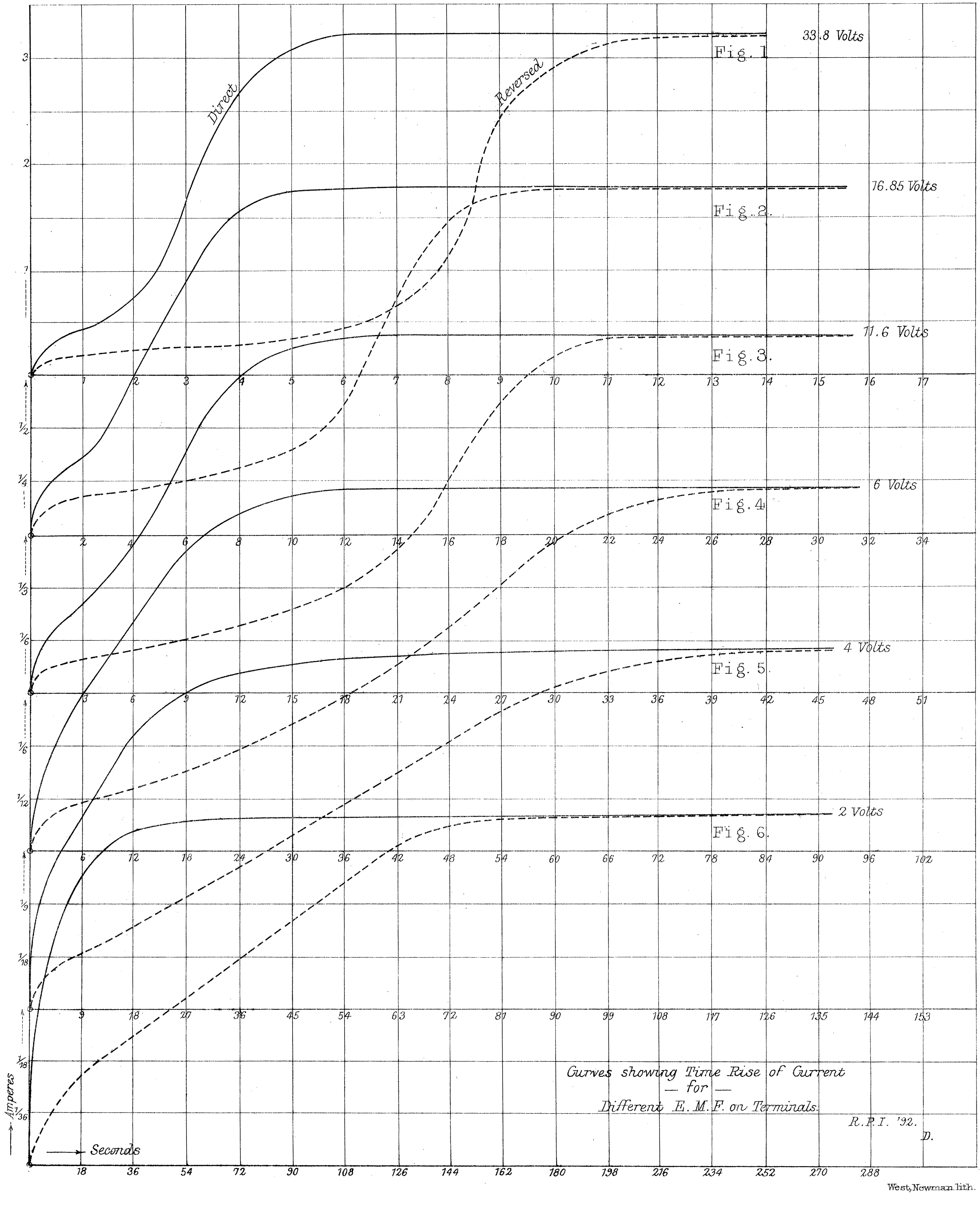
The effect of varying the impressed E.M.F. on the dissipation of energy is illustrated in figs. 22 to 25, the abscissæ of which are proportional to the impressed E.M.F., and the ordinates to the induction as calculated from the reversal of current curves which were similar to that shown in fig. 12. Although the number of experiments here shown are not sufficient to allow any law to be accurately deduced from them, the results are shown by the black dots in the curves figs. 26 and 27. Fig. 26 is drawn from an equation of the form $e = \text{AE} + \text{BI}$, where e is the energy dissipated, E the impressed E.M.F., I the total induction, A and B constants. For this particular curve $\text{A} = \frac{1}{2}$ and $\text{B} = 25300$. The curve shown in fig. 27 has impressed E.M.F. for abscissæ and dissipation for ordinates, and is drawn to show that for the range of experiment taken the dissipation was practically proportional to the impressed E.M.F. Independent experiments indicate that the amount of energy dissipated in the secondary circuit when the magnet is used as a transformer, in the

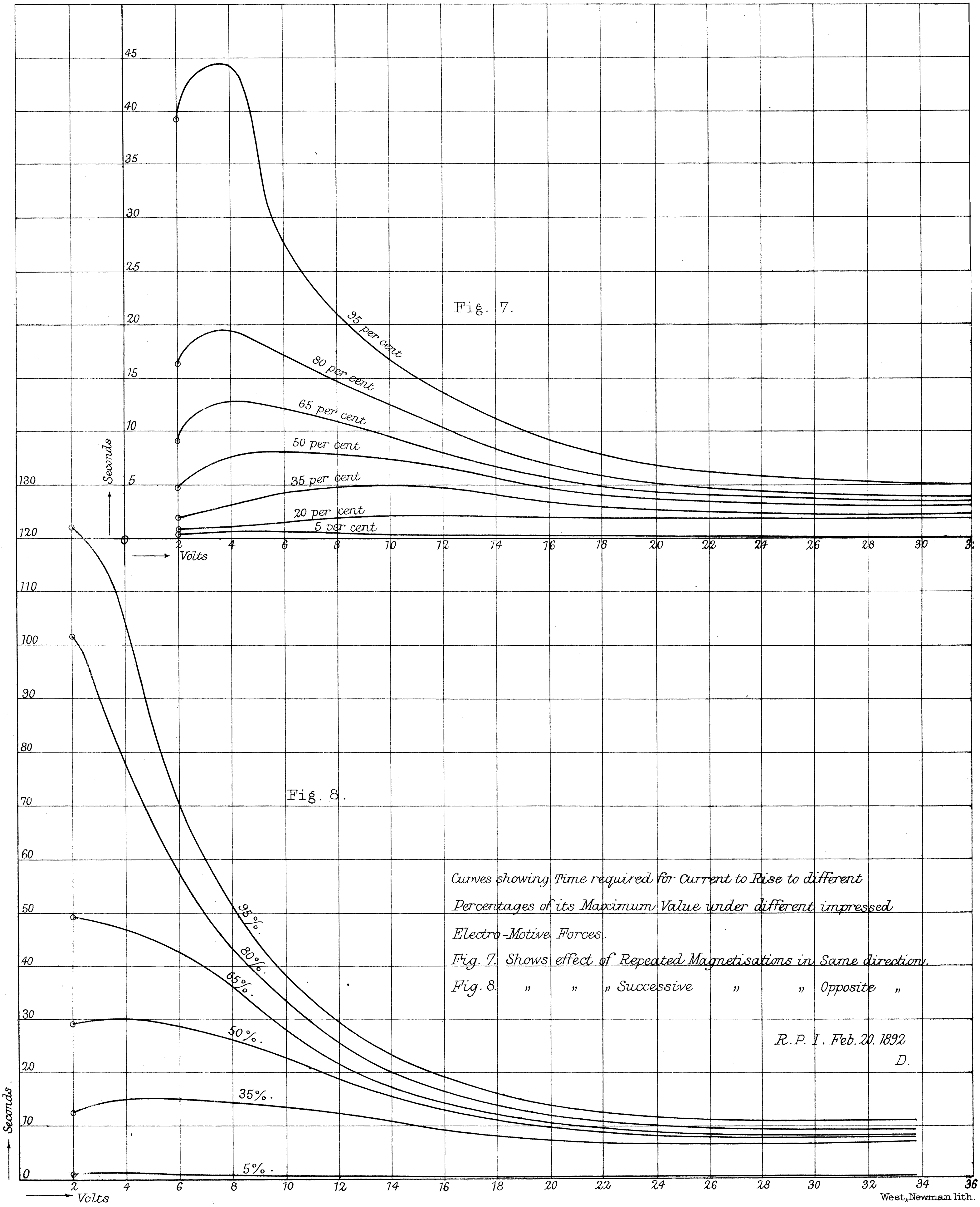
way above described, is nearly proportional to the impressed E.M.F., and hence this was assumed to be the case for the Foucault currents also. The other constants were then obtained by solving for them by means of simultaneous equations of the form $e = \mathrm{AE} + \mathrm{BI}^x$, the necessary data to form the equations being taken from the experimental results. It was found in this way that x was practically unity, not differing by more than one per cent. for the cases taken, and hence it is taken unity in the curve. The results of static experiments would indicate that the term BI^x, which represents the energy dissipated due to the retentiveness of the iron, should vary more rapidly than the induction, that is, that x should be greater than unity, about 1·6 for the experiments of EWING and others. This result does not seem to be reached by the kinetic method here adopted, and an examination of recent work on the subject seems to confirm the results here arrived at. In an elaborate series of experiments recently published by Mr. C. P. STEINMETZ an attempt is made to show that the exponent x is always 1·6. Unfortunately his results have most of them been reduced on this assumption, together with another based partly on calculation for Foucault currents. When the value of x given by the results is deduced from a series of equations by the methods indicated above the exponent unity seems to agree just as well, sometimes better, than 1·6. An excellent series of experiments for this purpose have recently been made by Mr. ALEXANDER SIEMENS and the results communicated to the Institute of Electrical Engineers.* In this paper also the losses from Foucault currents are calculated on an assumption of size of wire, specific resistance, &c. The estimate of the energy dissipated seems to be somewhat too small, and with a proper correction on it, about 25 per cent., the dissipation given under hysteresis becomes simply proportional to the induction. The results of these experiments treated by the simultaneous equation method for the determination of the law of variation of the different elements of the dissipation give for Foucault currents AI^2 and for hysteresis BI^1 with almost perfect exactness. The fact that a sufficient number of fairly accurate experiments furnishes data for the mathematical determination of the law of variation of the different elements entering into the dissipation of energy in cases like that here considered seems to have been very generally overlooked.

[Note added June 20, 1893.—Subsequent experiments, the results of which I hope soon to place before the Royal Society, show that the constant A in the above equation should have been zero, as there is practically no Foucault current loss. The exponent x is also shown by these experiments to agree closely with the number obtained by Mr. STEINMETZ from Professor EWING's experiments and, since this paper was written, more fully established by his own experiments.

* "Some Experimental Investigations on Alternate Currents." See 'Electrical Engineer,' February 19th and 26th, 1892.

The simultaneous equation method referred to above is more generally stated as follows:—Let the dissipation of energy e be expressed by an equation of the form $e = \mathrm{AI}^{\alpha} + \mathrm{BI}^{\beta} + \&c.$ Then, if, by proper experiments, the curve expressing e in terms of I be determined, the values of the constants A, B, &c., and α, β, &c., can be determined from the curve by taking a sufficient number of corresponding particular values of e and I from the curve to form the requisite number of equations. Experience shows, however, that it is difficult to obtain the experimental curve with sufficient accuracy. In the particular case given above, the results were neither sufficiently numerous nor sufficiently accurate for the purpose, and the conclusion arrived at is undoubtedly wrong.—T. G.]


33.8 Volts
Fig. 1
Direct
Reversed
16.85 Volts
Fig. 2.
11.6 Volts
Fig. 3.
6 Volts
Fig. 4
4 Volts
Fig. 5
2 Volts
Fig. 6.
Amperes
Seconds
Curves showing Time Rise of Current
— for —
Different E. M. F. on Terminals.
R.P.I. '92.
D.



Curves showing Time required for Current to Rise to different Percentages of its Maximum Value under different impressed Electro-Motive Forces.

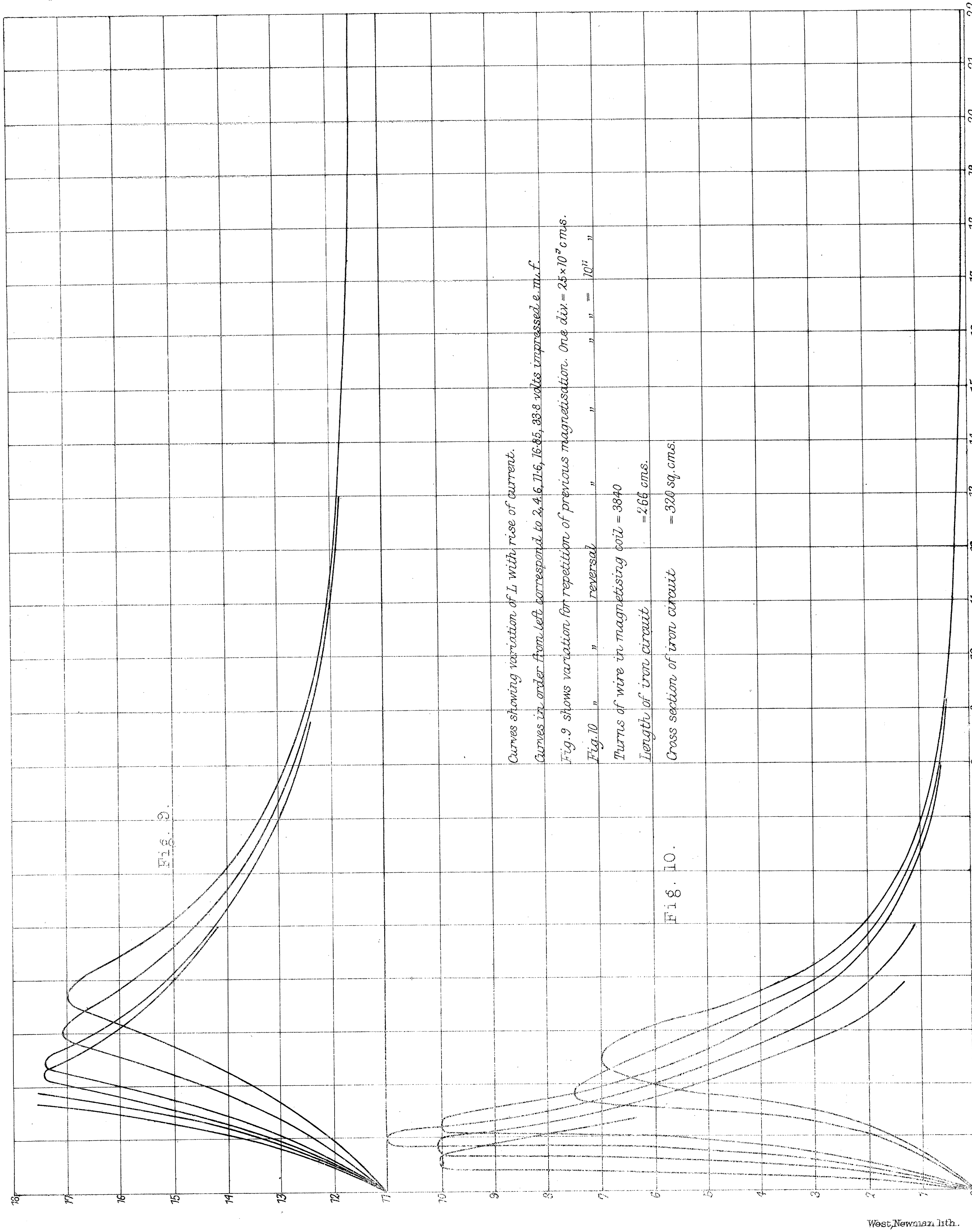
Fig. 7. Shows effect of Repeated Magnetisations in Same direction.

Fig. 8. „ „ „ Successive „ „ Opposite „

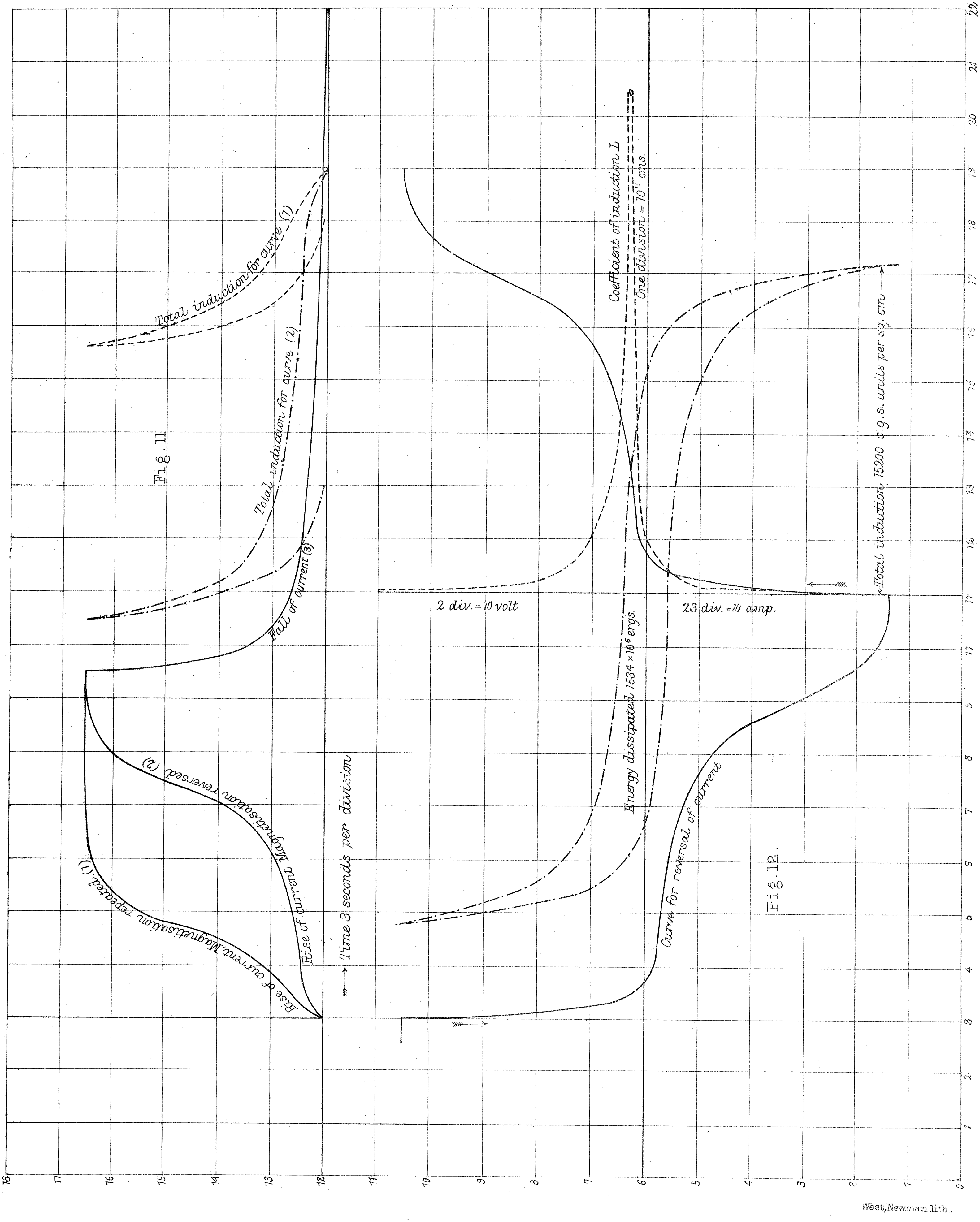
R. P. I. Feb. 20. 1892

D.

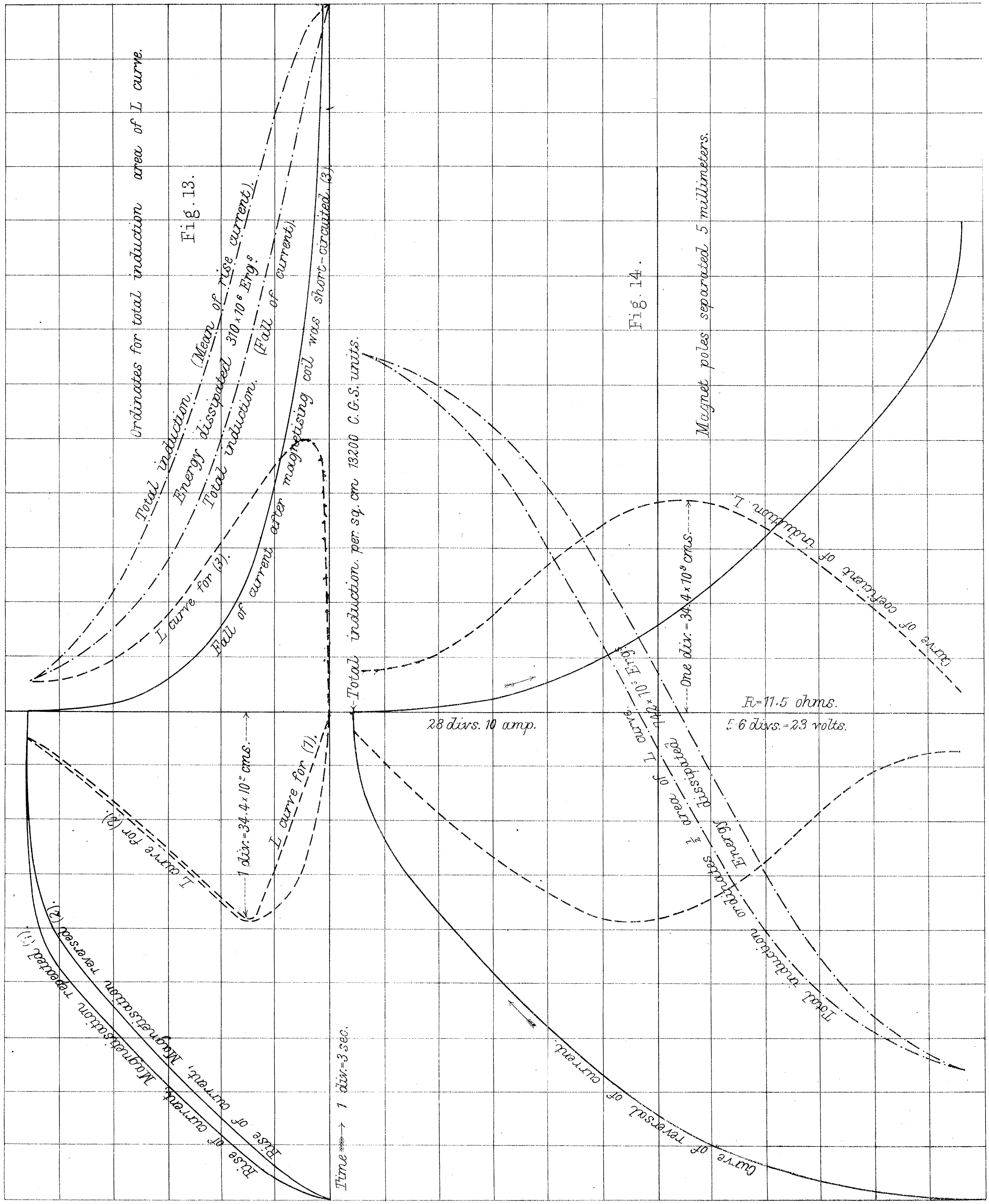
West, Newman lith.



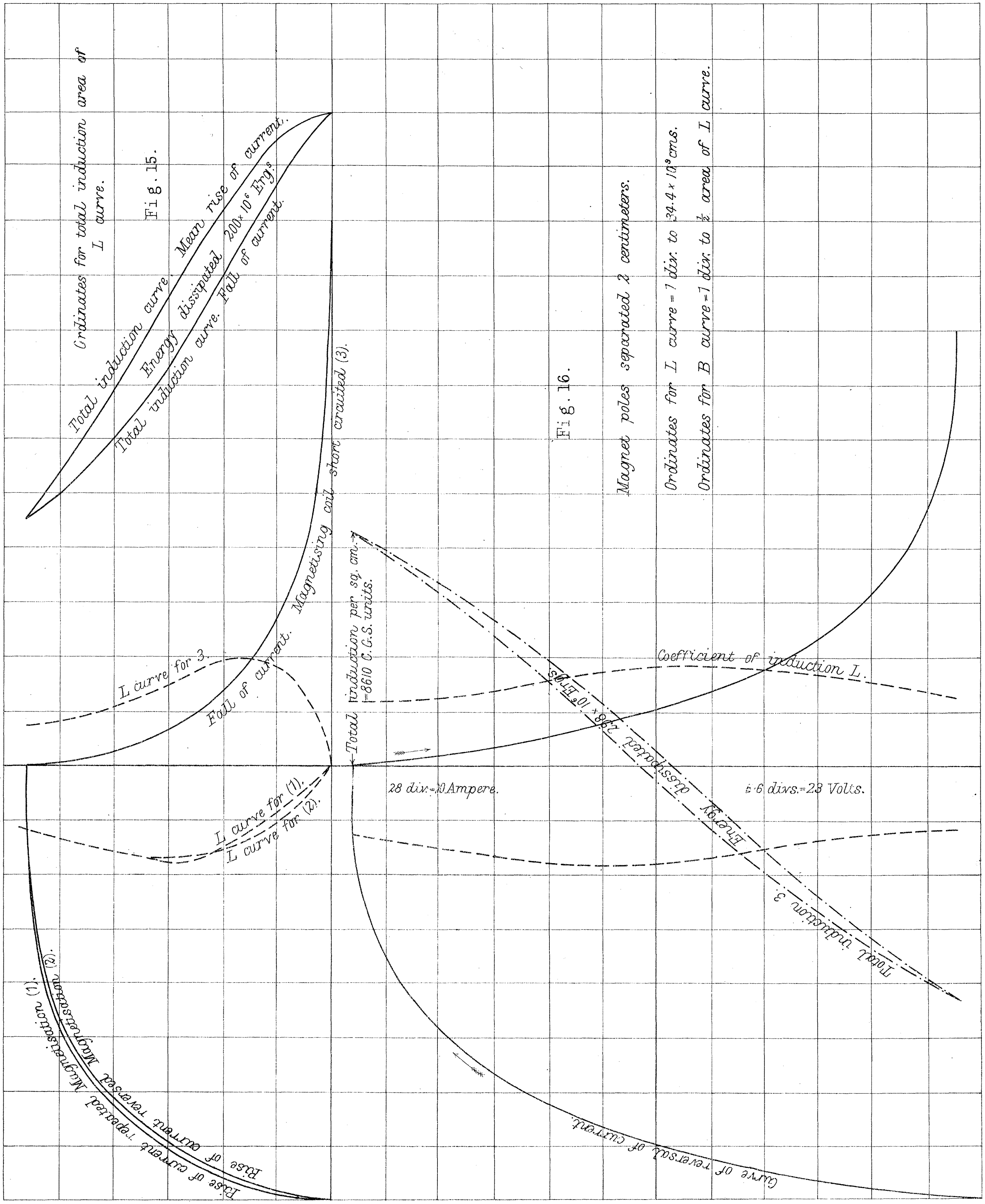
West, Newman lith.



West, Newman lith.



West, Newman, lith.



West, Newman, lith.

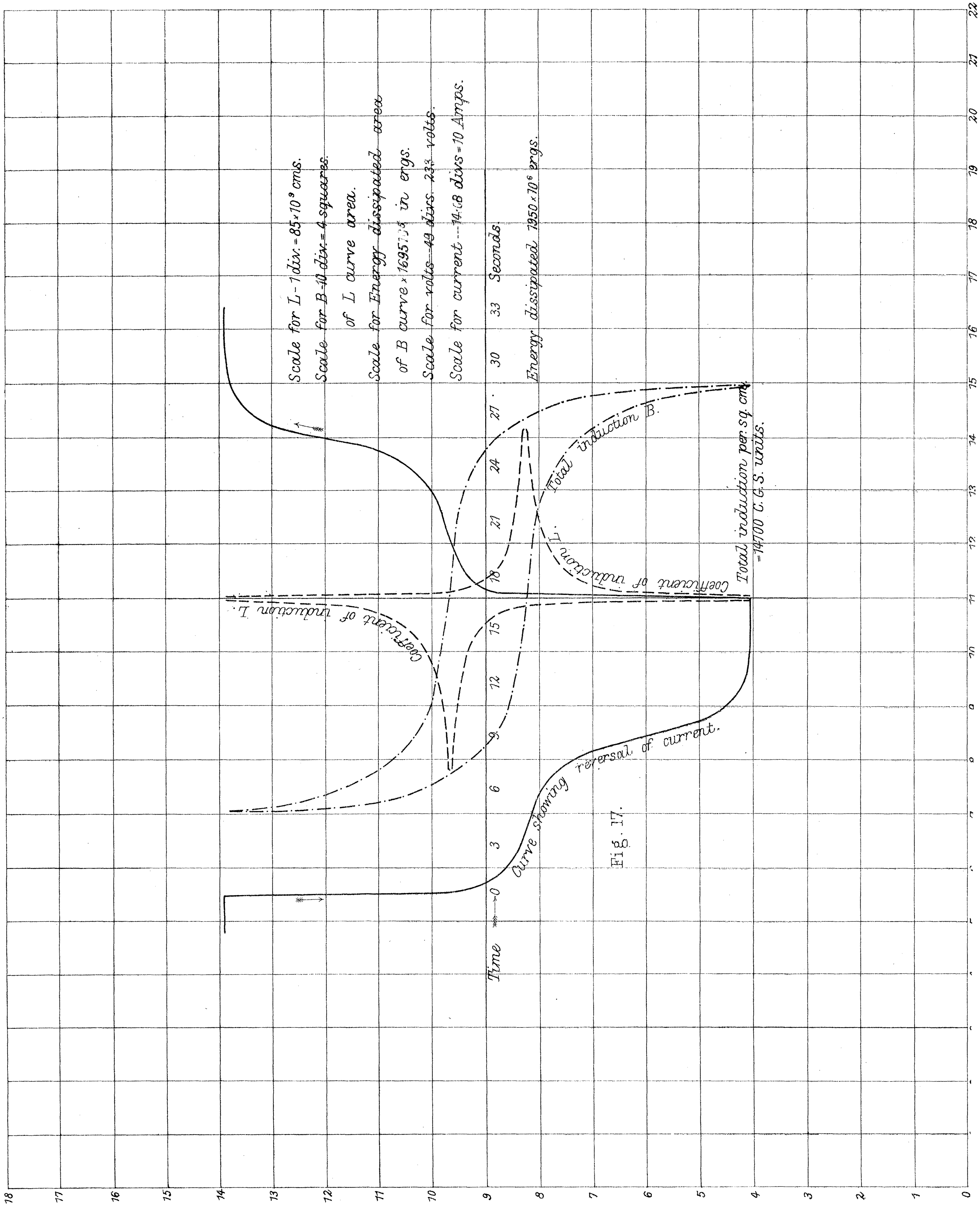
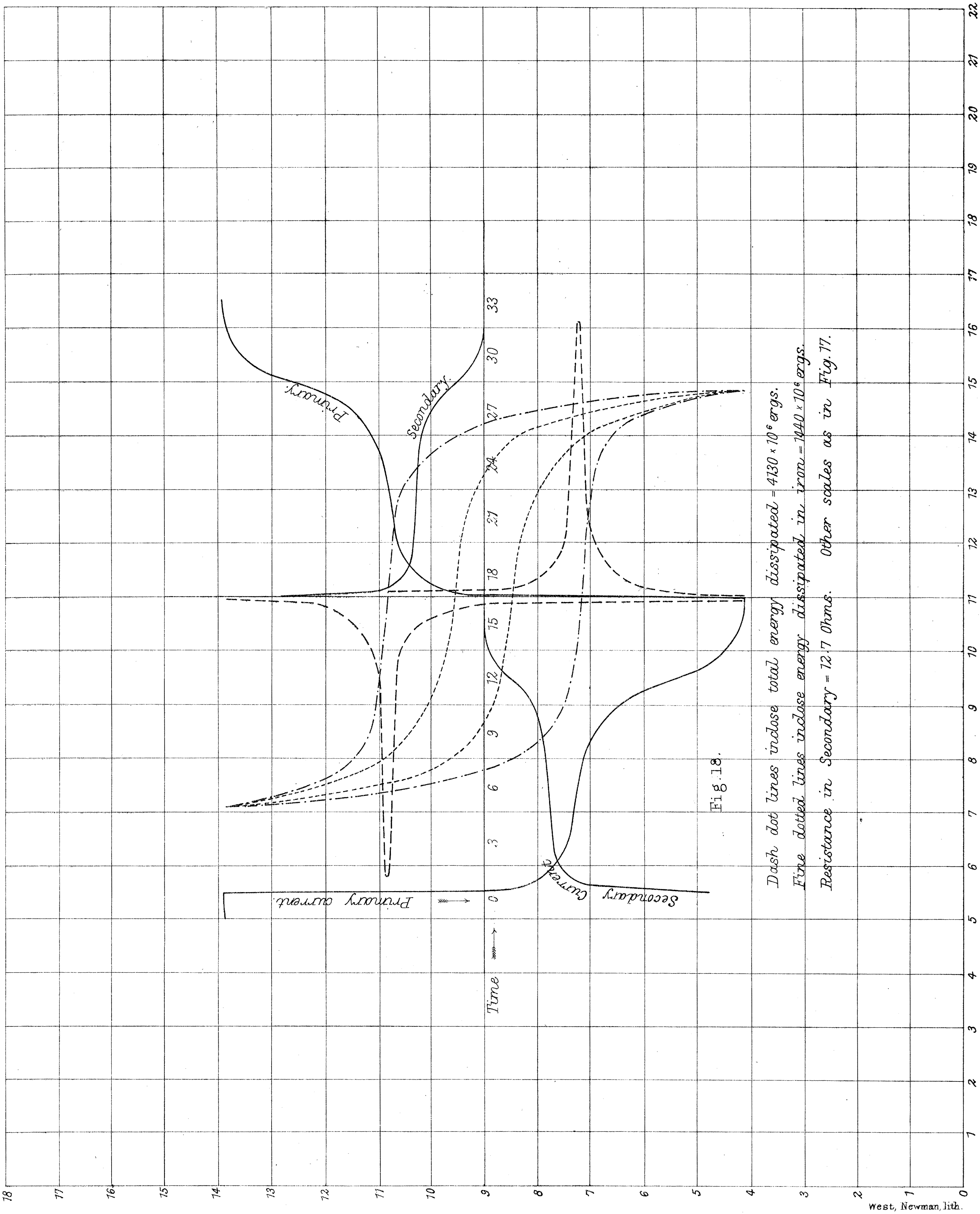


Fig. 17.

West, Newman lith.



West, Newman, lith.

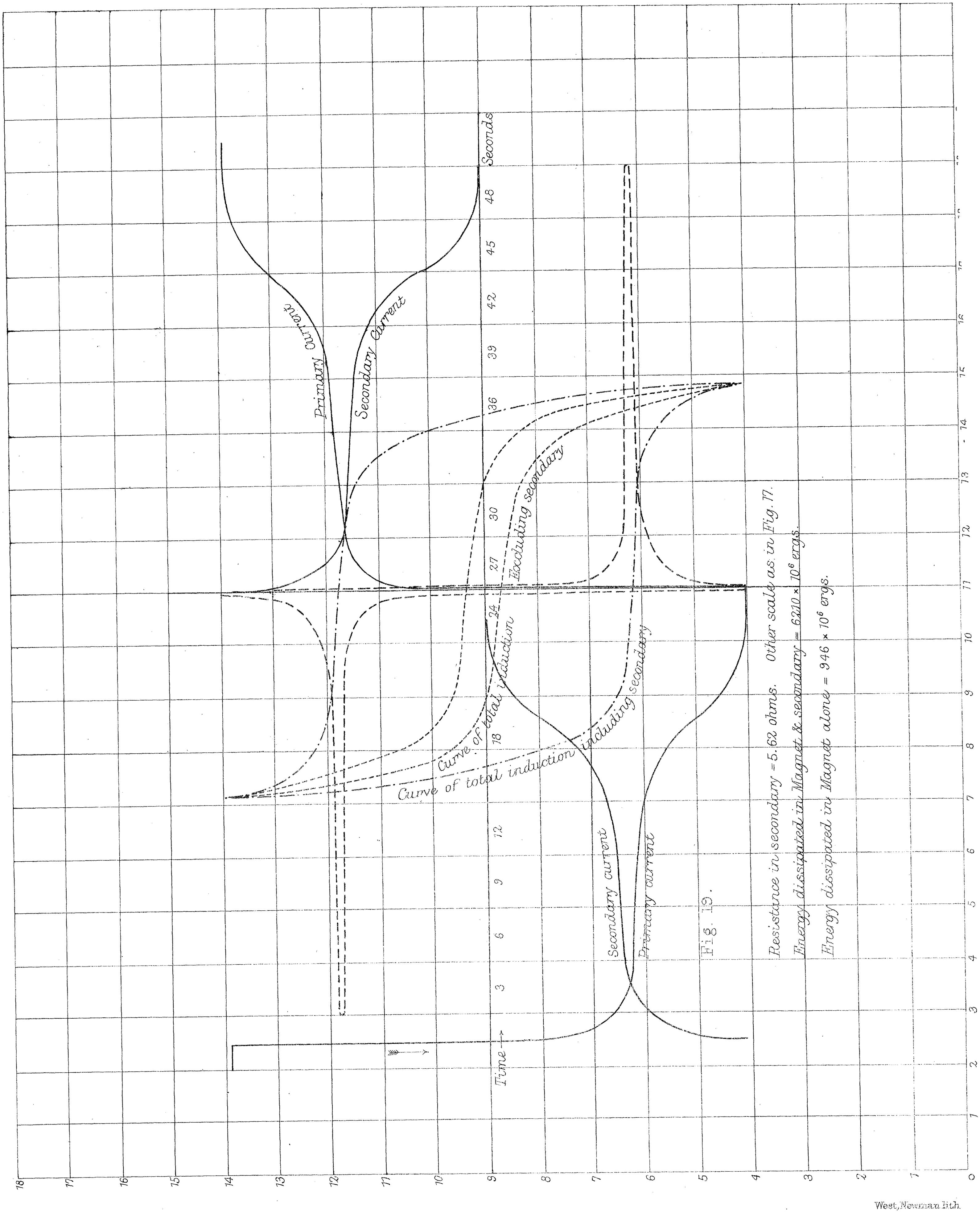


Fig. 19.

West, Newman lith.

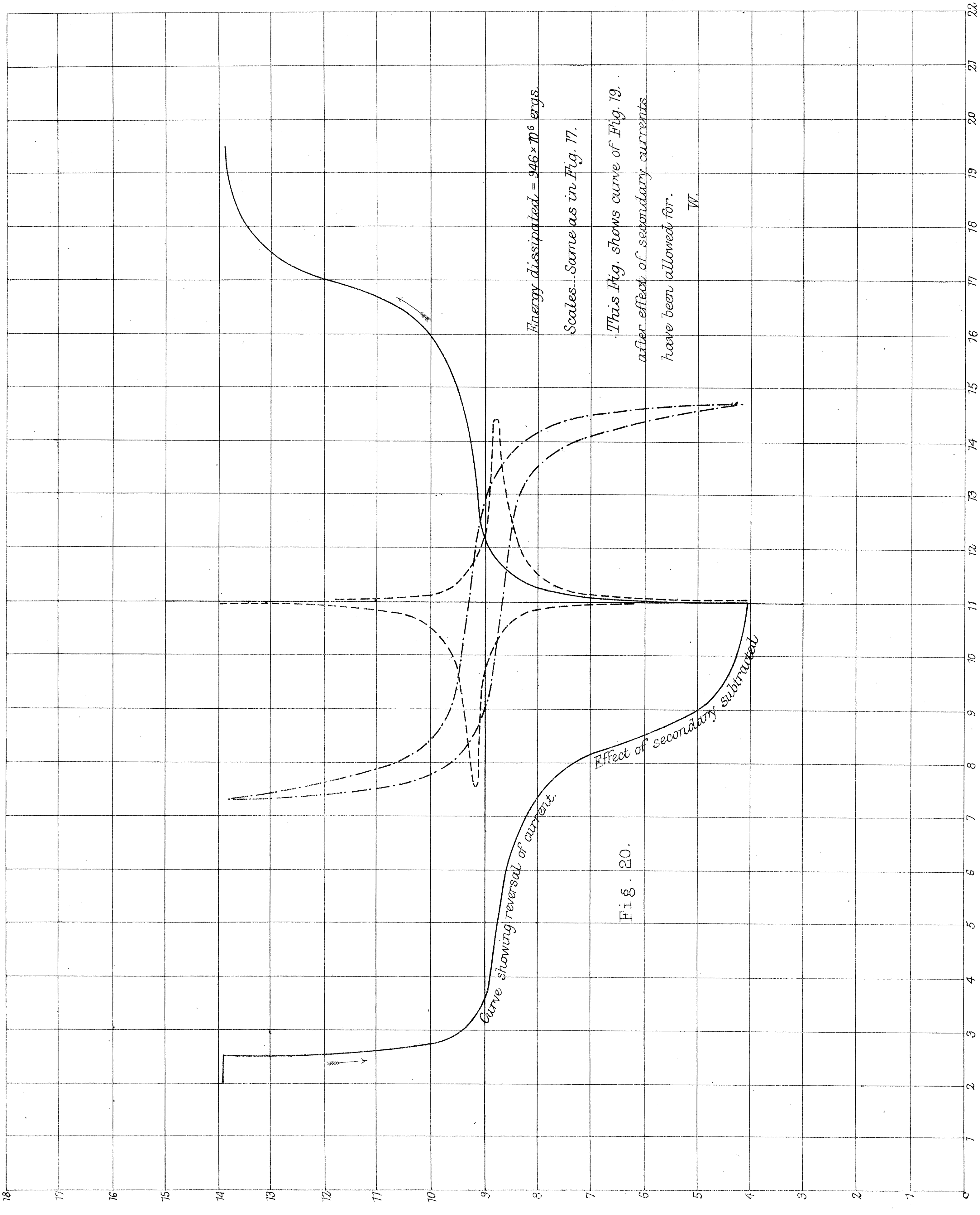


Fig. 20.

West, Newman lith.

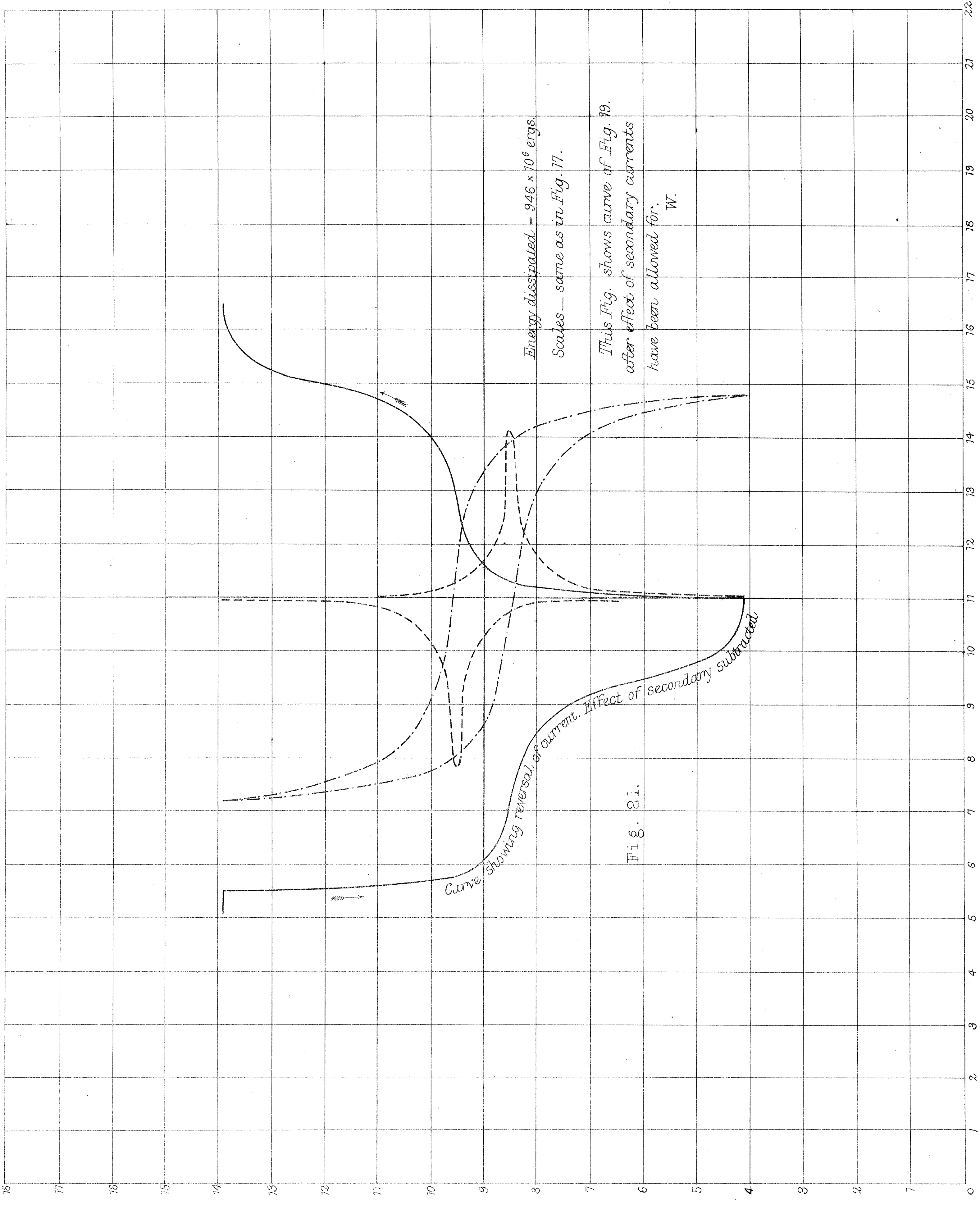
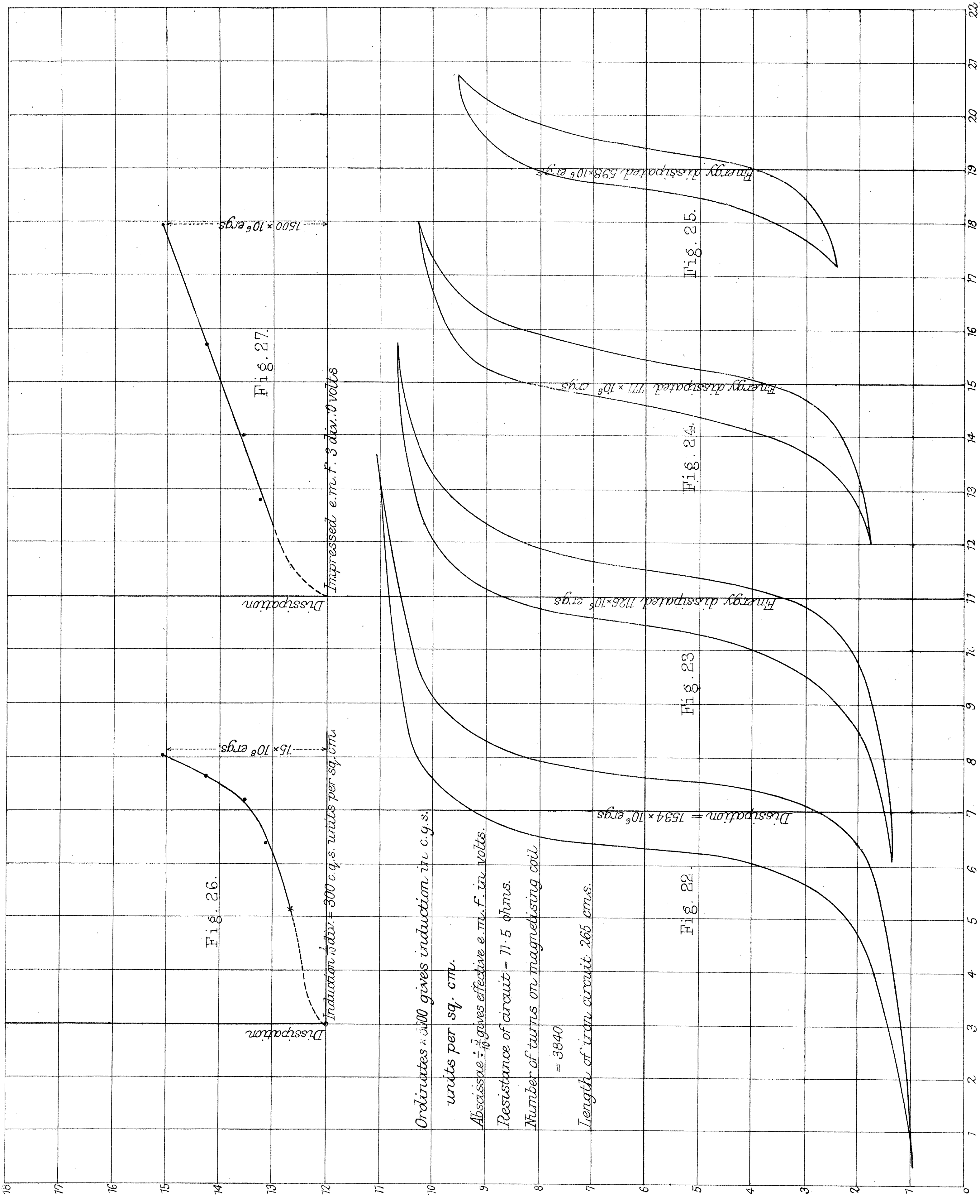


Fig. 21.

West, Newman lith.



West, Newman lith.