X. *The Absolute Thermal Conductivities of Iron and Copper.*

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THE experiments described in this paper were undertaken at the suggestion of Professor A. GRAY, M.A., University College, Bangor, with the object of contributing something to the results still necessary to establish the experimental work bearing on the absolute thermal conductivity of metals on a more satisfactory basis. They were also intended to furnish a determination of the absolute conductivity of *pure* copper at different temperatures.

The data already accumulated* on the thermal conductivities of iron and copper are due chiefly to ÅNGSTRÖM, FORBES, NEUMANN, TAIT and MITCHELL, and more recently to KIRCHHOFF and HANSEMANN and LORENZ.

FORBES first published,† in 1861 and 1865, the details of his determination of the conductivity of iron, and showed that its value diminished with rise of temperature. Some years later TAIT confirmed‡ this result, and in addition gave determinations of the conductivities of copper, lead, &c. His results for copper showed that, unlike iron, its conductivity increased with increase of temperature. ÅNGSTRÖM, however, in 1861, published§ results for iron and copper, evidently obtained with great care and by a very reliable method, which showed that the thermal conductivities of both metals decreased with increase of temperature. As pointed out by TAIT, both ÅNGSTRÖM and FORBES omitted to correct their results for change of specific heat with temperature, but the application of this correction does not materially affect the nature of their results. NEUMANN‖ also determined the absolute conductivities of

* See table given below, p. 588.

† 'Trans. Roy. Soc. Edinb.,' vol. 23, pp. 133–146, and vol. 24, pp. 73–110. A preliminary communication had previously been made to the British Association at Belfast in 1852.

‡ 'Trans. Roy. Soc. Edinb.,' 1878.

§ Communicated to the Royal Swedish Academy in January, 1861. German and English translations of his paper are to be found in POGGENDORFF'S 'Annalen' for 1863 and in the 'Phil. Mag.,' 1863 (first half-year).

‖ 'Ann. de Chim.,' vol. 66, pp. 183–185.

several metals, but gave no results bearing on the change of conductivity with temperature. More recently KIRCHHOFF and HANSEMANN* and LORENZ† have published results for iron and copper, but their methods were not so reliable as their work, and the results for copper are exceptionally low.

Although there can be no doubt as to the substantial accuracy of most of the results obtained by the experimenters mentioned above, yet it is evident that those bearing on the change of conductivity with temperature are very unsatisfactory, and that a good deal of work has yet to be done on the subject.

The method of determination adopted in the work here described was that due to FORBES, with the exception that a single thermo-electric couple was employed for the determination of temperatures instead of a number of thermometers. In addition to the uniformity secured by measuring all temperatures on the same scale, there is the advantage that by this method the holes in the bar for the insertion of one junction of the couple may be very much smaller than those required for thermometers, and may therefore be drilled at shorter distances apart, thus admitting of a more accurate determination of the distribution of temperature along the bar. Further, as no mercury is put in the holes, no lining of iron is necessary in the copper bar. Another advantage in the use of a thermo-electric couple is, that in observing the cooling of the short bar used to determine the rate of cooling at different temperatures, the junction inserted in the bar cools at practically the same rate as the bar, whereas, in the case of a thermometer, the cooling of the thermometer always lags behind that of the bar; ‡ an experiment showed that at temperatures between 150° C. and 200° C. the temperature of the thermometer may be nearly a degree higher than that of the bar in which it is inserted even when this bar is sufficiently massive to cool rather slowly.

The bars used in the experiments were of iron and copper. The iron bar was a $\frac{3}{4}$-inch square bar, about $4\frac{1}{2}$ feet long, of ordinary commercial wrought-iron, having its surface filed up and very lightly polished with black-lead to secure uniformity of radiating power.

The copper bar was a round hard-drawn bar half-an-inch in diameter and about 7 feet long, of practically pure electrolytically deposited copper, prepared by Messrs. BOLTON and Co., Oakamoor Mill, Staffordshire. The sectional area of the bar was perfectly uniform and its surface was smooth and polished.

For the reception of the thermo-electric junction, holes about a millimetre in diameter were drilled in these bars at distances apart which preliminary experiment had shown to be suitable. The depth of the holes was about equal to three-quarters of the diameter of the bar, and care was taken in drilling that their axes were at right angles to the bar, and passed approximately through its central axis.

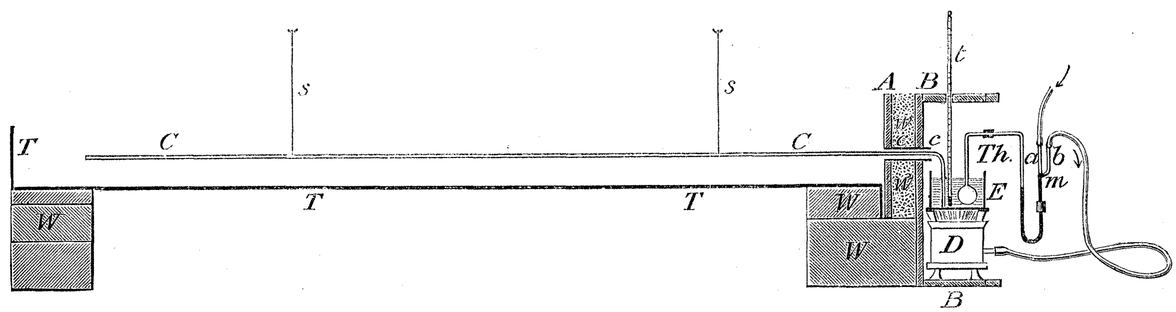
* WIEDEMANN'S 'Annalen,' 9, p. 1; 13, p. 406.

† WIEDEMANN'S 'Annalen,' 13, p. 422.

‡ See FOURIER'S 'Theory of Heat,' §§ 298–300.

The arrangement of apparatus finally employed was that shown in fig. 1. One end of the experimental bar was immersed in a vessel of melted lead maintained at a constant temperature, and the rest of its length was exposed to the air, but protected from draughts and external radiation by a wide trough of sheet zinc, *T T T*, arranged as shown in the figure. The molten lead was maintained at a constant temperature by a thermostat, *Th.*, which consisted of an air thermometer having its bulb immersed in the lead, and controlling, by means of a column of mercury in the U-shaped bend of its stem, the passage of the gas at *m*, the junction of the entrance and exit tubes *a* and *b*. By means of this arrangement the lead could be maintained at a temperature constant within a range of about half a degree for any length of time.

Fig. 1.



A. Wooden screen.
B. Wooden box lined with sheet iron.
C C. Metal bar.
D. Gas burner.
E. Pan containing melted lead.
T T T. Trough (of sheet zinc) protecting bar.
Th. Thermostat (air thermometer with mercury cut-off at *m*).
c c. Thin copper tube.
w w. Cotton wool.
s s. Strings supporting bar.
W W. Blocks of wood supporting *T*.

The bar was protected from the radiation from the heating apparatus by enclosing this apparatus in a box B, lined with sheet iron, and by screening off this box by a wooden screen A, and lightly packing the space between the box and screen with cotton wool. The tube, *c.c.*, shown in the figure was intended to shield the part of the bar within the box from irregular heating by the flame of the burner, and served also to prevent contact between the cotton wool and hot bar. The space between this tube and the bar was packed at one end with asbestos to prevent a current of hot air passing out of the box along the bar.

The thermo-electric couple employed was one of a good soft iron wire and German silver. The wires were about ·5 millim. in diameter, and the junctions were made by fusing the wires together under borax and filing down to the required size. Repeated calibration showed that with this couple the deflection on the galvanometer scale was,

between 0° C. and 200° C., strictly proportional to the difference between the temperatures of its junctions. The galvanometer in use with the couple was a low resistance reflecting galvanometer, adjusted so as to give a deflection of about four scale-divisions for a difference of 1° C. between the temperatures of the junctions. The scale deflections could be easily read to half a division, so that the difference between the temperatures of the junctions could be estimated to about one-tenth of a degree.

The distribution of temperature along the bars in the stationary state was determined by inserting one junction successively into the small holes drilled in the bar, while the other junction was maintained at a constant temperature* by inserting it in a hole in a short iron bar resting on the bottom of the zinc trough *T*. The deflections obtained were, in this way, proportional to the difference of temperature between the several points on the bar and the temperature of the surrounding air. It was found necessary to carefully insulate the ends of the wires of the couple from one another just above the junctions, and from the bars in which they were inserted; this was done by filing down the wires, inserting a thin slip of silk between them, and then whipping them round with a fine silk thread. The outer surface of the thread charred slightly at the higher temperatures, but not sufficiently to destroy its efficiency.

In taking a series of observations of the distribution of temperature along the bar, the heating was allowed to go on for five or six hours, and then the deflections corresponding to each hole along the bar were observed, up and down the bar, time after time, until a set of perfectly constant and concordant readings were obtained. It was not always possible to attain this end; a change in the temperature of the room, or any cause tending to produce draught in the room, destroyed the constancy of the results, but by repeated observations under favourable circumstances, and by shielding the bar from draughts by means of the trough *TT*, it was found possible to obtain perfectly constant data, except in the case of the copper bar, for the holes nearest the heated end. It was found that the temperatures of these holes when above 180° C. to 200° C. varied continually, through a range having a maximum value of about .1° for the hole nearest the end, and quickly diminishing to zero at the third or fourth hole from the end. This fluctuation in temperature was probably due, partly to the want of massiveness of the bar and partly to the formation of a film of oxide on the surface of this part of the bar; but by keeping the temperature from going much above 200° C. and by taking the mean of ten or twelve successive readings the mean temperature of these holes was determined with certainty to

* This temperature was indicated by a thermometer placed with its bulb in a hole drilled in the bar close to the small hole in which the junction was placed. In no case was the variation of temperature more than a fifth of a degree. The bar was a short massive iron bar, not the one used for cooling experiments.

within a quarter of a degree. A much greater error than this would have no effect on the results given below, which are for temperatures below 200° C.

The rates of cooling of the bars at different temperatures were determined by heating a piece of each bar, about a foot long, in a sand-bath to a temperature higher than required, and observing its rate of cooling under the same conditions as those to which the experimental bar was exposed. For this purpose it was suspended in the position originally occupied by the long bar, the thermo-electric junction inserted in a hole drilled at its middle point, and the deflection of the galvanometer observed at regular intervals during the cooling. It was found that the observations of the rate of cooling were not trustworthy for the first few minutes after it had commenced, so that it was necessary to heat the bar to a temperature considerably higher than the highest at which the rate of cooling was required. With this precaution it was found that the data obtained for rates of cooling under similar conditions agreed well together—for example, the data given in columns *a* and *b* of Table I. 2 were obtained on different days but under similar conditions. Experiments showed that the presence of a thin film of oxide on the surface of the copper bar had no perceptible effect on its rate of cooling, for it was found that the film did not form unless the heating was continued for a long time, so that by varying the time of heating in the sand bath it was possible to observe the cooling with the surface in the different initial stages of oxidation, and the rate of cooling was found to be practically the same in all cases.

In reducing the results, the graphical method was largely used—the distribution of temperature along the bars and the cooling observations were plotted in the usual way, and from the curves so obtained differential curves were plotted showing, in the one case, the gradient of temperature at any point on the bars, and, in the other, the rate of cooling at any temperature. In the case of the curves showing the stationary state of the bars, the scale adopted was such that abscissæ represented actual distances along the bar, and the lengths of the ordinates in millimetres gave the corresponding temperatures in scale divisions. In the differential curves, however, showing the gradient of temperature at any point on the bar, the ordinates were drawn on ten times this scale, so that one millimetre represented about $\frac{1}{40}$th of a degree Centigrade.

The cooling curves were drawn on a somewhat larger scale in overlapping sections. The scale varied, according to the data to be plotted, from one in which 4 centims. represented one minute and 5 millims. one scale division (1 millim. to $\frac{1}{20}$th of a degree Centigrade) to one in which 1 centim. represented one minute and 4 millims. one scale division. Owing to the size of the scale, it was necessary to draw the curves in sections, and experience showed that the sections must overlap considerably, in order to secure continuity. The differential curves showing the rate of cooling at any temperature were drawn with abscissæ on the same scale as the observational curves, but the ordinates were in each case on a scale ten times as great. As it is impossible to reproduce these curves on the scale to which they were drawn, two specimen curves are given on a reduced scale in figs. I and II (p. 590). Fig. I shows the upper portion

of the temperature curve for the copper bar, and fig. II gives the upper part of the cooling curve for the iron bar.

The experimental results obtained and the details of their reduction are set out in the Tables given below, which, taken in connection with the following general explanation of the notation adopted, indicate the successive steps of the reductions.

Let k denote the absolute thermal conductivity at any normal section of the bar at a distance x from the heated end, and where $d\theta/dx$ is the gradient of temperature. Then, if a denote the area of cross-section of the bar, $ka\,(d\theta/dx)$ denotes the heat which crosses the section considered in unit time. This heat is lost by radiation from the surface of the bar beyond this section, hence, if l denote the length of a short element in this part of the bar, θ its mean temperature, and $d\theta/dt$ its rate of cooling at this temperature, then the heat lost per unit of time from the surface of this element is $lasd\,(d\theta/dt)$, where s denotes the specific heat, and d the density of the material of the bar at the temperature θ. From this it follows that, if all the units involved be consistent, we have

$$ka\,(d\theta/dx) = \Sigma\, lasd\,(d\theta/dt) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

Introducing corrections for temperature this relation becomes

$$k\,(1 + 2\alpha\theta)\,(d\theta/dx) = d_0 \Sigma l_0 s\,(d\theta/dt) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

where l_0, a_0, d_0 are the values of l, a, d at 0° C., θ the temperature of the cross section of the bar, and α the coefficient of linear expansion of the metal. The value of k for any cross-section can evidently be found directly from (2) by calculating the corresponding value of $\Sigma l_0 s\,(d\theta/dt)$, where s has its proper value for each element of the bar;* the reduction can however be more conveniently made by taking $\Sigma l_0 s\,(d\theta/dt)$ as equal to $\bar{s}\,\Sigma l_0\,(d\theta/dt)$, where $\bar{s}$ is the specific heat at a mean temperature $\bar{\theta}$, which is given by

$$\bar{\theta} = [\Sigma l_0\,(d\theta/dt)\,\theta]/[\Sigma l_0\,(d\theta/dt)].$$

This gives us

$$k\,(1 + 2\alpha\theta)\,(d\theta/dx) = d_0\,\bar{s}\,\Sigma l_0\,(d\theta/dt),$$

or

$$k = \frac{d_0\,\bar{s}}{1 + 2\alpha\theta}\,\frac{\Sigma l_0\,(d\theta/dt)}{(d\theta/dx)} = \mathrm{M}\,\frac{\Sigma l_0\,(d\theta/dt)}{(d\theta/dx)},$$

where M denotes the factor involving $\bar{s}$.

In the reductions given below it will be seen that the absolute conductivity, k,

* [An error connected with this point, in the results first communicated to the Royal Society, w pointed out to me and has now been corrected.—R. W. S., 13.7.93.]

first calculated from the values of M, $d\theta/dx$, and $\Sigma\, l_0\, (d\theta/dt)$, and the diffusivity κ, afterwards calculated by dividing by the proper values of (sd), the thermal capacity of unit volume of the material of the bar.

To obtain the values of s and d the densities of the iron and copper were carefully determined in the usual way, and, as the iron was not pure its specific heat at different temperatures could not be obtained from previous results; it was therefore determined by BUNSEN'S calorimeter, as described below. For the specific heat of the copper the results given by BÈDE * were adopted.

It will be evident that if $d\theta/dx$ and $d\theta/dt$ are expressed in the same units, the absolute value of these units need not be known, hence, in what follows, temperatures are given in scale divisions (galvanometer deflections), and the absolute value of a scale division in degrees Centigrade has but little bearing on the results obtained.

IRON.

TABLE I. 1.—Table showing distribution of temperature along iron bar during steady state.

Number of hole.	Distance in centims. from fixed point at heated end. [x.]	Excess of temperature in scale divisions. [θ.]
1	1·00	539
2	3·00	458
3	5·55	375
4	8·55	283
5	12·30	221
6	20·00	130
7	27·65	81
8	35·35	50
9	45·60	26
10	55·90	14
11	66·20	7
12	81·55	4
13	96·95	2
(end of bar)	103·20	—

Temperature of air . . . . 83 scale divisions.
Between 0° C. and 100° C., 1° C. = 3·75 ,, ,,
,, 100° C. ,, 200° C., 1° C. = 3·74 ,, ,,

* 'Mémoires couronnés de l'Acad. de Bruxelles,' vol. 27, p. 1.

TABLE I. 2.—Table giving Cooling Data for Short Iron Bar.

[Temperature of the air 21° C.]

Time (t).	Temperature in scale divisions (θ).		
	a.	*b*.	Mean.
m. s.			
0 0	482	483	482·5
0 15	474	476	475
0 30	468	470	469
0 45	464	461	462·5
1 0	457	453	455
1 15	450	446	448
1 30	443	440	441·5
1 45	435	433	434
2 0	428	428	428
2 15	421	422	421·5
2 30	414	415	414·5
2 45	407	407	407
3 0	400	400	400
3 15	395	394	394·5
3 30	389	390	389·5
3 45	384	385	384·5
4 0	377	378	377·5
4 15	373	372	372·5
4 30	368	366	367
4 45	362	361	361·5
5 0	357	356	356·5
5 15	353	350	351·5
5 30	347	346	346·5
5 45	341	340	340·5
6 0	335	334	334·5
6 15	330	329	329·5
6 30	325	325	325
6 45	321	320	320·5
7 0	315	315	315
7 15	311	310	310·5
7 30	308	306	307
7 45	304	301	302·5
8 0	298	297	297·5
8 15	293	294	293·5
8 30	288	290	289
8 45	284	285	284·5
9 0	281	..	281
9 15	278	278	278
9 30	274	..	274
9 45	270	269	269·5
10 0	..	266	266
10 30	259	262	260·5
11 0	252	255	253·5
11 30	246	247	246·5
12 0	239	240	239·5
12 30	232	233	232·5
13 0	226	227	226·5
13 30	222	222	222

TABLE I. 2 (continued).

Time (t).	Temperature in scale divisions (θ).		
	a.	b.	Mean.
m. s.			
14 0	216	216	216
14 30	210	211	210·5
15 0	204	205	204·5
15 30	199	199	199
16 0	194	194	194
16 30	189	189	189
17 0	184	184	184
17 30	181	180	180·5
18 0	177	176	176·5
18 30	172	172	172
19 0	168	168	168
19 30	164		164
20 0	159		159
21 0	150		150
22 0	145		145
23 0	139		139
24 0	132		132
25 0	125		125
26 0	119		119
27 0	112	114	113
28 0	109		109
29 0	103	103	103
30 0	100		100
31 0	96		96
32 0	91		91
33 0	87		87
34 0	83		83
35 0	79		79
36 0	76		76
37 0	73		73
38 0	70		70
39 0	67		67
40 0	64		64
42 0	..	59	59
43 0	57	57	57
46 0	49		49
49 0	44		44
52 0	40		40
55 0	35		35
60 0	29		29
65 0	26		26
70 0	21		21
73 0	18		18
84 0	12		12

TABLE I. 3.—Table giving Loss of Heat from Surface of Iron Bar.

[For Notation, see p. 574.]

l_0.	θ (scale divisions).	$d\theta/dt$ (scale divisions per minute).	$l_0\,(d\theta/dt)$.	$\Sigma\,[l_0\,(d\theta/dt)]$.
centims.				
19·2	2·5	0·09	1·73	1·73
10	4	·15	1·50	3·23
10	6	·23	2·30	5·53
5	10	·39	1·95	7·48
5	14	·55	2·75	10·23
5	18	·63	3·15	13·38
5	25	·90	4·50	17·88
2	30	1·10	2·20	20·08
2	35	1·35	2·70	22·78
2	40	1·55	3·10	25·88
2	46	1·85	3·70	29·58
2	52	2·15	4·30	33·88
2	58	2·40	4·80	38·68
2	66	2·80	5·60	44·28
2	74	3·20	6·40	50·68
2	84	3·70	7·40	58·08
2	95	4·25	8·50	66·58
1	103	4·65	4·65	71·23
1	110	5·05	5·05	76·28
1	118	5·5	5·50	81·78
1	126	5·9	5·90	87·68
1	136	6·5	6·50	94·18
1	144	6·9	6·90	101·08
1	154	7·5	7·50	108·58
1	166	8·3	8·30	116·88
1	178	9·0	9·00	125·88
1	190	9·7	9·70	135·58
1	204	10·6	10·60	146·18
1	218	11·5	11·50	157·68
1	232	12·4	12·40	170·08
1	248	13·4	13·40	183·48
1	266	14·6	14·60	198·08
1	286	15·9	15·90	213·98
0·5	298	16·7	8·35	222·33
·5	310	17·5	8·75	231·08
·5	320	18·3	9·15	240·23
·5	332	19·1	9·55	249·78
·5	344	20·0	10·00	259·78
·5	356	20·9	10·45	270·23
·5	370	22·0	11·00	281·23
·5	385	23·0	11·50	292·73
·5	400	24·2	12·10	304·83
·5	415	25·4	12·70	317·53
·5	432	26·8	13·40	330·93
·5	450	28·2	14·10	345·03
·5	468	29·7	14·85	359·88
2·5	..	..	..	..

TABLE I. 4.

Table showing values of k, the absolute thermal conductivity of iron, at temperatures between 60° C. and 150° C.

T.	x.	$d\theta/dx$ (per centim.).	$\Sigma [l_0 (d\theta/dt)]$. (per minute).	$\bar{\theta}$.	M.	k [C.G.S. units].
°C.	centims.			°C.		
60	19·90	9·0	93·0	41	0·856	[·147]
80	13·70	14·2	146·7	51	·862	[·148]
100	9·20	20·1	206·6	63	·868	·148
110	7·50	23·4	235·3	68	·871	·146
120	5·90	26·9	267·0	73	·873	·144
130	4·60	30·6	297·6	79	·877	·141
140	3·45	34·5	325·7	84	·880	·138
150	2·50	38·5	352·7	88	·882	·135

From these values (between 100° C. and 150° C.), if we assume the conductivity at $t°$ C. to be given by

$$k_t = a - bt,$$

we have, on reducing by method of least squares,

$$k_t = \cdot 175 - \cdot 00026t,$$

or

$$k_t = \cdot 175 \, (1 - \cdot 0015t).$$

Determination of Specific Heat of Iron.

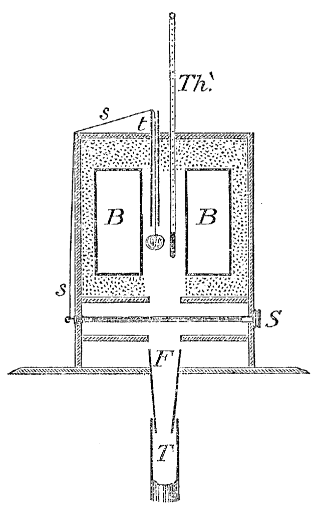
Advantage was taken of the snowfall in January to use BUNSEN'S calorimeter for this determination. The calorimeter was prepared for use by the method described by BUNSEN in his paper given in the 'Phil. Mag.,' 1871, p. 161, and all the precautions there detailed were observed. The tube used for determining the change of volume attendant on the melting of ice was of very fine bore, but was perfectly clean, so that there was no trouble with the sticking of the mercury thread. This tube was carefully calibrated, and a calibration curve drawn from which a length measured in any part of the tube could be converted into a length corresponding to the uniform bore taken as the basis of the calibration.

The heater used was made specially for the work. It consisted of a hollow cylindrical oil bath shown in section at BB (fig. 2). This bath was heated up to a temperature of about 250° C., and then quickly placed in the wooden case shown in the figure and packed round on all sides with cotton wool. Thus packed, the bath took nearly a day to cool, and could, therefore, be used for a complete series of determinations at temperatures below 200° C.

The piece of iron to be heated was suspended by a silk fibre in the cylindrical

space in the centre of the heater in close proximity to the bulb of the thermometer, *Th.*, which passes through a cork in the lid of the case. The silk thread passed through the glass tube, *t*, and was attached at the other end to a small hook in the end of the slide, *S*. The tube, *t*, was just long enough to be in contact with the piece of iron, so that when the slide, *S*, was withdrawn quickly the thread broke and the iron was free to fall through the holes in the double bottom of the case, between which the slide, previous to withdrawal, acted as a screen. When it was desired to make a determination the heater was lifted on to the lid of the calorimeter case and placed in a marked position, so that the holes in the bottom were directly over the mouth of a paper funnel, *F*, acting as a guide to the calorimeter tube, *T*. When placed in position, the thermometer was watched and the slide, *S*, withdrawn, when the required temperature was indicated. The iron at once fell straight into the calorimeter tube and the heater was then removed, the mouth of the funnel, F, closed

Fig. 2.



with cotton wool, and the observation of the mercury-thread in the indicating-tube proceeded with. In the meanwhile, a second piece of iron was suspended in the heater so as to be ready for another determination. The cooling of the heater was so slow that there was no doubt about the temperature of the iron when dropped into the calorimeter, and the time of fall into the tube, T, was too small to admit of any appreciable cooling. Experiment also showed that placing the heater on the lid of the calorimeter case, even for a long time, had no effect on the calorimeter.

In the reduction of the results obtained the latent heat of water and the specific heat of ice were assumed to be correct as given by BUNSEN.

The rapid melting of the snow and some accidents with the calorimeter limited the time of working and prevented any extension of the results given.

The principal data of two sets of experiments are given in the following Table.

I.

TABLE showing the Data for Determination of Bore of Tube.

Temperature.	Weight of mercury.	Length of thread. [Corrected by calibration curve.]	Volume of 1 centim. of bore.
° C.	grms.	centims.	
13	0·9092	99·80	·000672
13	0·8700	95·15	·000674
			·000673 (mean)

II.

TABLE giving Data for Specific Heat Determinations.

Mass of iron.	Temperature of iron (t).	Deflection of mercury thread.	Heat required to raise unit mass of iron from 0° C. to t° C. [h_t.]
grms.	° C.	centims.	
2·1025	41	16·0	4·5172
2·1025	55	21·7	6·1264
2·1025	98	39·3	11·0950
2·1025	102	40·6	11·4360
1·3440	150	39·7	17·5340
1·3440	170	44·7	19·7420

Assuming

$$h_t = at + bt^2,$$

and reducing by method of least squares, we have

$$h_t = \cdot 1095t + \cdot 0000401t^2,$$

and, therefore,

$$dh_t/dt = s_t = a + 2bt = \cdot 1095 + \cdot 00008t,$$

or

$$s_t = \cdot 1095\,(1 + \cdot 00073t)$$

where s_t denotes the true specific heat at t° C.

Determination of Density of Iron, and the Value of (sd) *the Thermal Capacity of Unit Volume.*

The density of the iron was found to be 7·556 at 0° C. For present purposes this may be taken as 7·56, and the density at t° C. will be given by

$$7\cdot 56\,(1 - \cdot 000038t).$$

Hence the value of (sd) at $t°$ C. is given by

$$7{\cdot}56\,(1 - {\cdot}000038t) \times {\cdot}1095\,(1 + {\cdot}00073t) = {\cdot}828\,(1 + {\cdot}000692t).$$

Value of κ, the Diffusivity of Iron, in Absolute Units.

From the results obtained above the value of κ at $t°$ C. will be given by

$$\kappa_t = (k/sd)_t = {\cdot}175\,(1 - {\cdot}0015t) \div {\cdot}828\,(1 + {\cdot}000692t).$$

That is,

$$\kappa_t = {\cdot}211\,(1 - {\cdot}0015t)\,(1 - {\cdot}000692t),$$

or,

$$\kappa_t = {\cdot}211\,(1 - {\cdot}00219t + {\cdot}00000105t^2).$$

This result is probably represented with sufficient accuracy by

$$\boldsymbol{\kappa_t = {\cdot}211\,(1 - {\cdot}0022t).}$$

Copper.

Table C. 1.—Table showing Distribution of Temperature along Copper Bar for Two Independent Determinations, I. and II.

Number of hole.	Distance from first hole.	Excess of temperature (θ) (in scale divisions).	
		I.	II.
	centims.		
1	·0	764	889
2	2·65	697	807
3	5·35	637	735
4	8·00	584	671
5	10·67	537	615
6	13·28	493	565
7	15·81	454	520
8	19·63	405	463
9	23·40	362	413
10	27·20	326	369
11	31·06	293	329
12	36·20	254	284
13	41·30	219	245
14	46·41	189	212
15	51·55	166	184
16	56·70	144	159
17	61·80	124	139
18	69·48	102	115
19	77·07	85	95
20	84·70	70	79
21	92·34	58	66
22	102·47	45	52
23	112·59	35·5	41
24	122·73	28	33
25	132·84	22	27
26	142·89	17·5	22
27	152·95	14	18
28	168·14	10	14
29	183·29	8	12
30	198·49	7	10·5

Temperature of room, I., 15° C. ; II., 17° C.

From calibration of couple before experiments,

1° C., between 0° C. and 100° C. = 4·24 scale divisions.

1° C., between 100° C. and 200° C. = 4·25 scale divisions.

From calibration of couple after experiments,

1° C., between 0° C. and 200° C. = 4·25 scale divisions.

In determinations I. the end of the copper bar was inserted in a socket in a

thick piece of copper standing in the melted lead, which was kept at a constant temperature of 345° C.

For II. the whole apparatus was taken down and the end of the copper bar bent round so that it could be directly inserted in the melted lead in the way shown in fig. 1. By a coincidence the constant temperature of the lead during this determination was 345° C. as in I.

TABLE C. 2.—Table giving Cooling Data for Short Copper Bar.

[Temperature of the room constant at 15° C.]

Time (t).			Temperature (θ) (in scale divisions).	Time (t).			Temperature (θ) (in scale divisions).	Time (t.)			Temperature (θ) (in scale divisions).
h.	m.	s.		h.	m.	s.		h.	m.	s.	
7	34	0	829	7	41	30	473	7	51	0	247
		15	814			45	464			30	239
		30	799	7	42	0	455	7	52	0	232
		45	785			15	446	7	53	0	219
7	35	0	771			30	439	7	54	0	205
		15	756			45	432	7	55	0	193
		30	742	7	43	0	424	7	56	0	181
		45	729			15	415	7	57	0	170
7	36	0	715			30	409	7	58	0	161
		15	702			45	403	7	59	0	152
		30	688	7	44	0	397	8	0	0	144
		45	675			15	389	8	1	0	137
7	37	0	663			30	382	8	2	0	129
		15	650			45	376	8	4	0	114
		30	638	7	45	0	369	8	6	0	102
		45	627			15	364	8	8	0	92
7	38	0	614			30	357	8	10	0	83
		15	602			45	351	8	12	0	75
		30	589	7	46	0	345	8	14	0	67
		45	579			15	339	8	17	0	57
7	39	0	568			30	333	8	20	0	49
		15	557			45	328	8	23	0	43
		30	547	7	47	0	323	8	26	0	38
		45	536			30	313	8	29	0	33
7	40	0	527	7	48	0	302	8	34	0	27
		15	515			30	292	8	39	0	22
		30	507	7	49	0	282	8	49	0	15
		45	497			30	273	9	9	0	7
7	41	0	489	7	50	0	265	9	29	0	3
		15	480			30	256				

TABLE C. 3.—Table showing Loss of Heat from Surface of Copper Bar for Determinations I. and II.

$[l_0]$.	$[\theta]$.		$[d\theta/dt]$.		$[l_0\,(d\theta/dt)]$.		$\Sigma\,[l_0\,(d\theta/dt)]$.	
Centims.	I.	II.	I.	II.	I.	II.	I.	II.
10	7	10·4	0·25	0·40	2·5	4·0	2·5	4·0
10	7·5	11·5	·28	·45	2·8	4·5	5·3	8·5
10	8·5	13·0	·30	·56	3·0	5·6	8·3	14·1
10	10·5	15·0	·40	·68	4·0	6·8	12·3	20·9
10	13·2	17·5	·50	·75	5·0	7·5	17·3	28·4
10	16·5	21·0	·70	·85	7·0	8·5	24·3	36·9
10	20·7	25·5	·85	1·00	8·5	10·0	32·8	46·9
10	26·2	31·0	1·05	1·30	10·5	13·0	43·3	59·9
10	33·6	39·0	1·41	1·75	14·1	17·5	57·4	77·4
10	42·5	49·0	1·94	2·35	19·4	23·5	76·8	100·9
5	50·5	58·5	2·37	2·88	11·85	14·4	88·65	115·3
5	57·3	65·5	2·78	3·30	13·90	16·5	102·55	131·8
5	65·0	73·8	3·21	3·82	16·05	19·1	118·60	150·9
5	73·5	83·0	3·75	4·35	18·75	21·75	137·35	172·65
5	83·5	94·0	4·38	5·05	21·90	25·25	159·25	197·90
5	94·5	105·5	5·05	5·85	25·25	29·25	184·50	227·15
5	107·3	119·2	5·88	6·85	29·40	34·25	213·90	261·40
5	122·0	136·0	6·90	7·85	34·50	39·25	248·40	300·65
5	140·0	155·3	8·00	9·25	40·00	46·25	288·40	346·90
5	160·0	178·5	9·38	10·75	46·90	53·75	335·30	400·65
2	176·5	197·0	10·62	12·20	21·24	24·4	356·54	425·05
2	187	208·0	11·45	13·00	22·90	26·0	379·44	451·05
2	197	220·5	12·10	13·90	24·20	27·8	403·64	478·85
2	209	233	13·00	14·90	26·00	29·8	429·64	508·65
2	221	247	13·95	16·00	27·90	32·0	457·54	540·65
2	234	261	14·90	17·10	29·80	34·2	487·34	574·85
2	248	277	16·00	18·2	32·00	36·4	519·34	611·25
2	262	293	17·04	19·4	34·08	38·8	553·42	650·05
2	277	311	18·20	20·7	36·4	41·4	589·82	691·45
2	293	329	19·40	22·1	38·8	44·2	628·62	735·65
2	309	349	20·70	23·6	41·4	47·2	670·02	782·85
2	328	370	22·10	25·4	44·2	50·8	714·22	833·65
2	347	393	23·50	27·3	47·0	54·6	761·22	888·25
2	367	418	25·05	29·5	50·1	59·0	811·32	947·25
2	389	444	26·9	31·9	53·8	63·8	865·12	1011·05
2	413	472	29·0	34·5	58·0	69·0	923·12	1080·05
2	439	502	31·3	37·1	62·6	74·2	985·72	1154·25
2	467	534	33·8	39·9	67·6	79·8	1053·32	1234·05
2	498	569	36·6	42·9	73·2	85·8	1126·52	1319·85
2	530	608	39·3	46·1	78·6	92·2	1205·12	1412·05
1	557	638	41·5	48·6	41·5	48·6	1246·62	1460·65
1	575	660	43·0	50·2	43·0	50·2	1289·62	1510·85
1	594	682	44·5	52·1	44·5	52·1	1334·12	1562·95
1	614	706	46·1	54·0	46·1	54·0	1380·22	1616·95
1	634	731	47·6	55·9	47·6	55·9	1427·82	1672·85
1	655	757	49·2	57·9	49·2	57·9	1477·02	1730·75
1	677	784	50·8	60·0	50·8	60·0	1527·82	1790·75
1	700	812	52·6	62·2	52·6	62·2	1580·42	1852·95
1	725	842	54·4	64·5	54·4	64·5	1634·82	1917·45
1	751	873	56·3	67·0	56·3	67·0	1691·12	1984·45

The values given in the last column of this Table are subject to a correction (a

reduction of about 2 per cent.), due to the fact that, in the case of the short bar used in determining the rates of cooling, the cooling surface includes the terminal faces of the bar.

TABLE C. 4.—Table showing the values of k, the absolute thermal conductivity of Copper, at temperatures between 40° C. and 200° C.

I.

T.	x.	$d\theta/dx$ (per centim.).	$\Sigma[l_0(d\theta/dt)]$ (per minute).	$\bar{\theta}$.	M.	k (C.G.S. units).
° C.	centims.			°C.		
40	68	2·70	192·50	28	0·816	[·956]
60	46·2	5·20	369·74	39	·821	[·973]
80	33·1	7·65	558·66	50	·828	1·007
100	23·5	10·55	758·50	59	·833	·999
120	16·5	13·60	950·70	71	·840	·979
140	11·0	16·80	1142·50	84	·846	·958
160	6·4	20·20	1334·50	93	·851	·937
180	2·5	23·90	1523·70	103	·856	·909

II.

T.	x.	$d\theta/dx$ (per centim.).	$\Sigma[l_0(d\theta/dt)]$ (per minute).	$\bar{\theta}$.	M.	k (C.G.S. units).
° C.	centims.			°C.		
40	75·50	2·40	191.5	29	0·816	1·085
60	51·65	5·00	375·2	40	·822	[1·027]
80	38·10	7·60	561·9	49	·826	[1·018]
100	28·60	10·30	753·6	60	·833	1·015
120	21·45	13·25	945·8	69	·838	·995
140	15·65	16·50	1145·7	80	·844	·976
160	11·00	19·93	1339·1	89	·850	·953
180	7·00	23·53	1532·3	100	·854	·927
200	3·70	27·20	1714·5	109	·860	·903

NOTE 1.—On comparing the results here tabulated, it will be noticed that the data of columns 3 and 4 are very similar in I. and II.; but reference to column 2 shows that the data of column 4 extend over a different length of the bar in the two cases.

NOTE 2.—In I. we find the same falling-off in the value of k at temperatures below 100° C. as occurs in the case of the iron bar.

Reducing the results given in these tables by the method of least squares, and assuming that the conductivity at t° C. is given by

$$k_t = a - bt,$$

we have

$$\text{I.}\quad k_t = 1{\cdot}113 - {\cdot}00112t = 1{\cdot}113\,(1 - {\cdot}001t)$$

$$\text{II.}\quad k_t = 1{\cdot}130 - {\cdot}00112t = 1{\cdot}130\,(1 - {\cdot}001t).$$

The mean value of these results gives

$$\mathbf{k_t = 1{\cdot}12\,(1 - {\cdot}001t).}$$

Determination of Density and Value of (sd), the Thermal Capacity of Unit Volume of Copper.

The density of the copper was found to be 8·907 at 0° C. Taking this as 8·9, we may take the density of copper at t° C., as given by 8·9 (1 − ·000056t).

The specific heat of copper at t° C. is given by BÈDE* as ·0892 + ·000065t. This result may be expressed as ·0892 (1 + ·00073t), hence the value of (sd) at t° C. is

$$8{\cdot}9\,(1 - {\cdot}000056t) \times {\cdot}0892\,(1 + {\cdot}00073t),$$

or

$${\cdot}794\,(1 + {\cdot}000674t).$$

Value of κ, the Diffusivity of Copper, in Absolute C.G.S. Units.

From the results obtained above the value of κ at t° C will be given by

$$\kappa_t = (\kappa/sd)_t = 1{\cdot}12\,(1 - {\cdot}001t) \div {\cdot}794\,(1 + {\cdot}000674t).$$

That is,

$$\kappa_t = 1{\cdot}41\,(1 - {\cdot}001t)\,(1 - {\cdot}000674t),$$

or,

$$\kappa_t = 1{\cdot}41\,(1 - {\cdot}0017t + {\cdot}0000007t^2).$$

This result is probably represented with sufficient accuracy by the formula

$$\mathbf{\kappa_t = 1{\cdot}41\,(1 - {\cdot}0017t).}$$

It will be seen that the results of the experiments here described go to show that for both iron and copper the conductivity *decreases* with rise of temperature.

* [Recent determinations of the specific heat of pure copper (probably identical with that used in these experiments) give the specific heat at 100° C. as ·0928. BÈDE's formula gives ·0957, a value about 3 per cent. too high. The determinations here referred to were made by J. JOLY, Esq, F.R.S., with his steam calorimeter, and were kindly communicated to me by Mr. JAMES H. GRAY, M.A., B.Sc., Glasgow University.—12.4.93.]

For convenience of comparison these results are given below, followed by a Table showing previous results.

Values obtained for Absolute Thermal Conductivities of Iron and Copper from Experiments detailed in this Paper.

[C.G.S. units.]

IRON . . $k_t = 0{\cdot}175\ (1 - {\cdot}0015t)$.

COPPER . $k_t = 1{\cdot}12\ (1 - {\cdot}001t)$.

TABLE showing the results of previous Determinations of the Absolute Thermal Conductivity of Iron and Copper.

[C.G.S. units.]

Iron.	Copper.	Authority.
$0{\cdot}209\ (1-{\cdot}00147t)$	..	FORBES
·164	1·108	NEUMANN
$0{\cdot}199\ (1-{\cdot}002874t)$	$\left\{\begin{matrix} 1{\cdot}027\ (1-{\cdot}00214t) \\ {\cdot}983\ (1-{\cdot}00152t) \end{matrix}\right\}$	ÅNGSTRÖM
$0{\cdot}197\ (1-{\cdot}00002t)$	$\left\{\begin{matrix} 1{\cdot}08\ (1+{\cdot}0013t) \\ {\cdot}71\ (1+{\cdot}0014t) \end{matrix}\right\}$	TAIT
$0{\cdot}17\ (1-{\cdot}00002t)$	$0{\cdot}72\ (1+{\cdot}00004t)$	LORENZ
$0{\cdot}17\ (1-{\cdot}002t)$	$0{\cdot}51\ (1+{\cdot}0057t)$	KIRCHHOFF and HANSEMANN

The Emissive Powers of the Surfaces of the Iron and Copper Bars.

From the data obtained in the cooling experiments involved in the conductivity research, it is evidently possible to calculate the emissivities of the surfaces of the bars at different temperatures.

As it may be of interest to place these on record, they are given in the following table.

The iron bar was a $\frac{3}{4}$-inch square bar, and its surface was blackened and lightly polished with blacklead.

The copper bar was a round bar of $\frac{1}{2}$-inch diameter, and its surface was smooth and polished.

TABLE showing the emissive powers of the surfaces of the Iron and Copper Bars.

The emissivities are given in gramme degrees of heat lost per square centim. of surface for each degree Centigrade difference of temperature between the surface and the surrounding air. The results are corrected for variation of specific heat with temperature.

Difference of Temperature.	Emissive Power.	
	Blackened surface of Iron Bar.	Polished surface of Copper Bar.
°C.		
20	0·000275	0·000220
40	313	253
60	348	280
80	378	297
100	405	315
120	429	333
140	454	346
160	476	360
180	..	374

On comparison of these results with those published by Mr. D. MACFARLANE* and by Professor TAIT,† for somewhat similar surfaces, it will be found that the agreement is fairly close.

The experience gathered in the conduct of the experiments described above goes to show that in order to obtain a thoroughly satisfactory determination of thermal conductivity and its variation with temperature, the bars employed should be somewhat massive—not less than an inch in diameter—and their surface should be plated (to prevent oxidation) and well polished. Further, to avoid the use of very long bars, and to diminish as far as possible the importance of the loss of heat from the surface of the bar as an element in the determination, ÅNGSTRÖM'S method should be adopted in preference to FORBES'.

It is noteworthy that with FORBES' method the influence of experimental errors seems to cause an apparent decrease of conductivity with fall of temperature. A trace of this effect is seen in the tables giving the values of κ for iron and copper, but the reduction of an unsatisfactory set of observations made with the copper bar before the use of the trough $T\,T$ (fig. 1) gave results which were not only much too low, but which decreased rapidly with fall of temperature.

In conclusion the author has to express his thanks to Professor GRAY for constant help and advice during the progress of the work.

* 'Proc. Roy. Soc.,' 1872, p. 93, or EVERETT'S 'Physical Constants,' p. 107.

† 'Proc. Roy. Soc. Edinb.,' 1869–70, p. 207.

Fig. I.

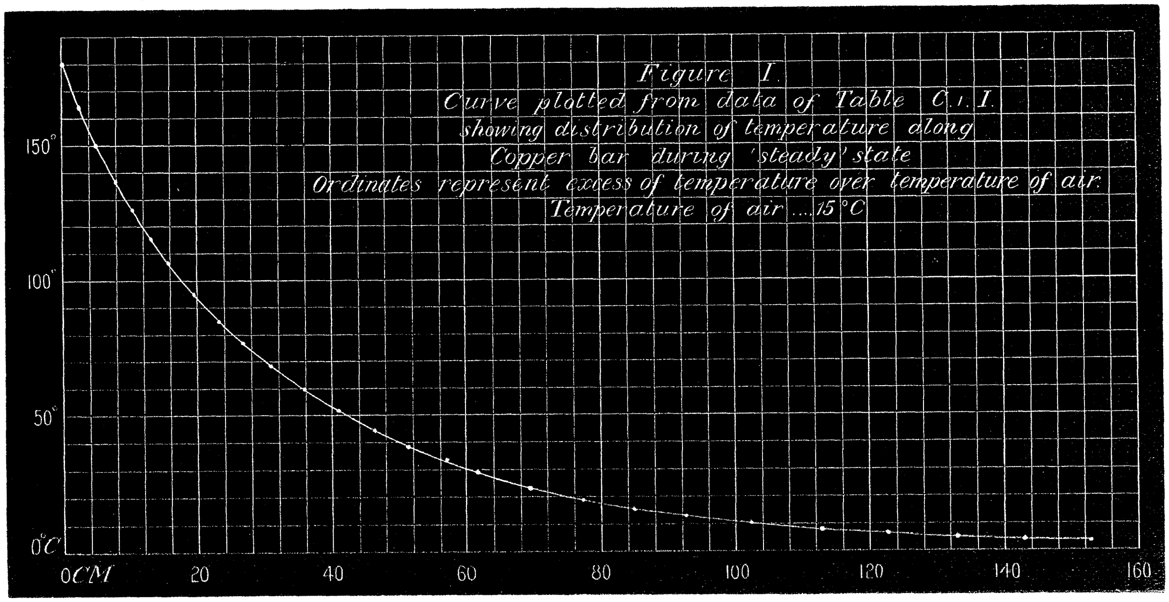


Figure I.
Curve plotted from data of Table C.1.I.
showing distribution of temperature along
Copper bar during 'steady' state
Ordinates represent excess of temperature over temperature of air.
Temperature of air ... 15°C
150°
100°
50°
0°C
0 CM
20
40
60
80
100
120
140
160

Fig. II.

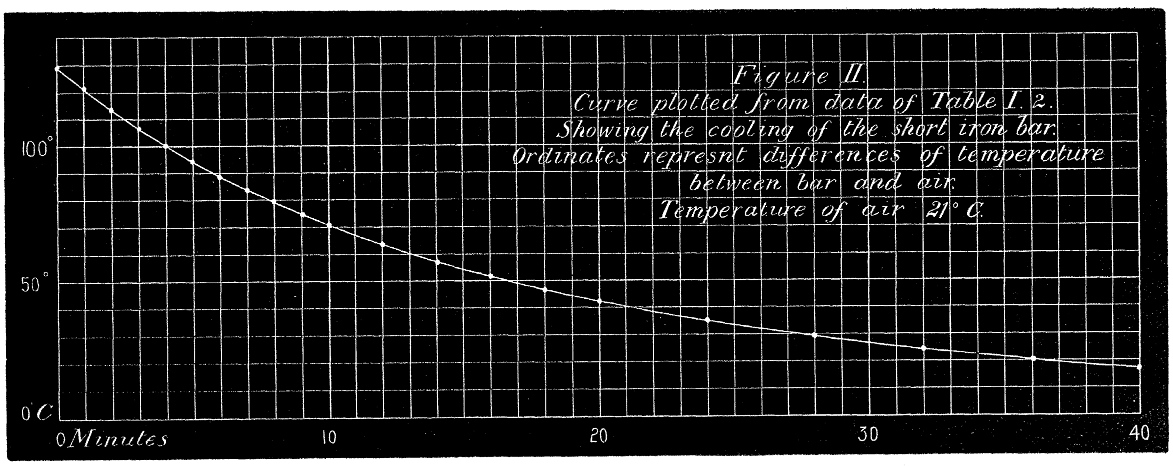


Figure II.
Curve plotted from data of Table I. 2.
Showing the cooling of the short iron bar.
Ordinates represnt differences of temperature
between bar and air.
Temperature of air 21° C.
100°
50°
0°C
0 Minutes
10
20
30
40