

XIV. *Magnetization of Liquids.*

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THE magnetization of liquids is a subject which has been examined by several methods, and although the relative values of the coefficient of magnetization for different bodies agree tolerably accurately, still there are large discrepancies in the absolute values given by different observers. An account of nearly all these methods is given in WIEDEMANN'S 'Electricität,' vol. 3.

The difficulty of finding the magnetizing force is no doubt a serious objection to the accurate determination of the coefficient k in absolute measure by magnetometer methods, which, on the other hand, are very convenient for finding the relative values of k for different bodies when magnetized by large forces.

The following null method of arranging an induction balance to determine k in absolute measure gives very accurate results, and is very sensitive even when the forces are as small as 1 C.G.S. unit.

The apparatus which was used consisted of a primary and a secondary circuit. A quantity of electricity $2Mi/R_2$ is induced in the secondary when the current in the primary is changed from $-i$ to $+i$, M being the mutual induction of the two circuits, and R_2 the resistance of the secondary. A commutator is used to commute the connection of the primary with the battery, and at the same time to alter the connections of the secondary with the galvanometer in such a way as to send the induced currents in the same direction through the galvanometer, so that it will give a deflection corresponding to a current $\frac{2Mi}{R_2} p$, p being the number of times per second the direction of i is changed.

By this means a very small value of M can be detected.

The mutual induction M consisted of three parts :—

1. The induction (α) of a long solenoid (wound on a glass tube) on a secondary wound round its centre.
2. The mutual induction (β) of two large circular coils.
3. An adjustable induction (γ) which could be altered by small known amounts.

$$M = \alpha + \beta + \gamma.$$

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The connections were so made that (α) and (β) almost cancelled one another, so that when the commutator revolved, and contact was made in the primary circuit with the battery, the *deflection* of the galvanometer in the secondary circuit could be reduced to zero by adjusting γ , thus making $M = 0$.

Let

A = internal area of the section of the solenoid tube.

N = number of turns in the primary coil.

N' = number of turns in the secondary coil.

$2l$ = length of primary.

The mutual induction $\frac{4\pi NN'A}{2l}$ is changed to $(1 + 4\pi k) \frac{4\pi MN'A}{2l}$ by introducing a liquid into the solenoid whose coefficient of magnetization is k . M is no longer zero, as the deflection of the galvanometer indicates, and if m is the amount by which γ must be diminished so as to reduce this deflection to zero we have

$$4\pi k \cdot \frac{4\pi MN'A}{2l} = m$$

The balance $M = 0$ being again restored.

We have thus a means of measuring k in terms of quantities which are easily found without involving in the formula the rate of rotation of the commutator or the value of the primary current.

The method is therefore very easily adapted to determine whether k varies with the force, since the latter can be changed by altering the resistance of the primary circuit.

The general plan of the arrangements is shown in fig. 1. The continuous line denoting the primary and the dotted line the secondary circuit. The solenoid A is connected with the other parts of the apparatus as shown. The two circular coils B are separated by an ebonite sheet; the upper one belonging to the secondary circuit could be moved about till the mutual induction of the pair cancelled the mutual induction of the two coils A . The adjustable induction γ is represented at C (and also in fig. 2), the primary circuit leading to the mercury cups at E by which the required changes in induction are made. D and G represent the commutator and galvanometer. The sensitiveness is not much diminished by increasing R the resistance of the primary, as it will be shown that in this case the rate at which the commutator revolves can also be increased without any danger of the Foucault current generated in the liquid having any effect on the secondary current, as far as its effect on the galvanometer is concerned.

It is convenient to have a key, K , in the primary circuit by means of which the current can be sent in either direction to the terminals, T , so as to reverse the

direction of the current in the secondary circuit. The *deflection* of the galvanometer could thus have been increased by the multiplication method, but in all the experiments which were *done* the apparatus was found sensitive enough to give k correct to the seventh decimal place by the ordinary deflection.

Fig. 1.

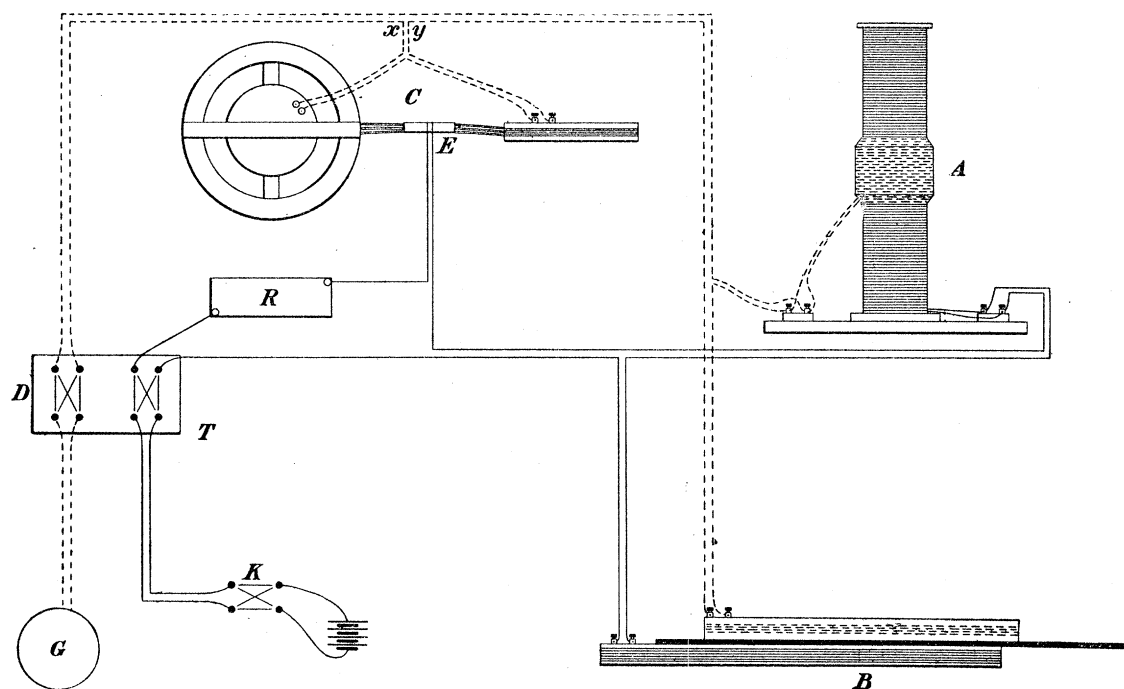
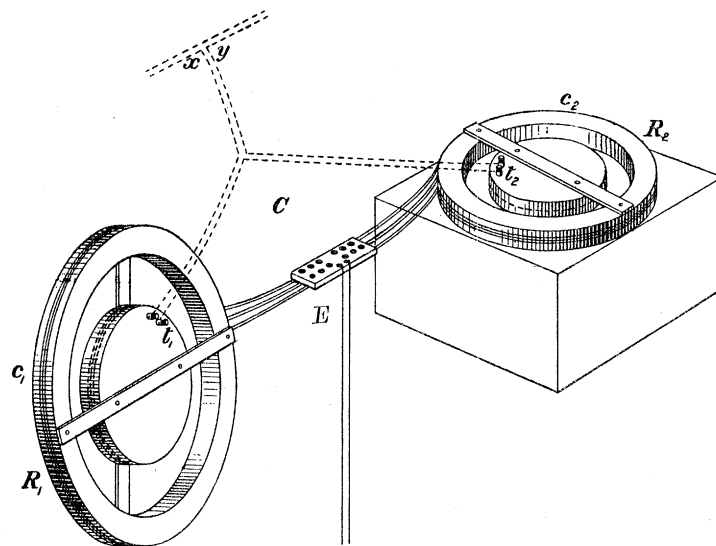


Fig. 2.



The great difficulty in the experiment is to ensure the perfect insulation of the secondary circuit, since the great electromotive force of the primary at break will

cause a current to leak to the secondary if good non-conductors are not used to separate the two circuits.

It is necessary that A, B, and C should be at long distances apart, so as to satisfy the two following conditions:—

1. The primary circuit of B must have no lines of force through the secondary of A.
2. Changing the number of coils in the primary of C should have no effect on the galvanometer when C is cut out of the secondary circuit (by joining x and y).

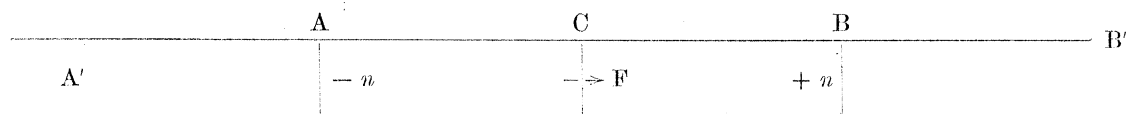
Details of Apparatus.

The primary circuit of the solenoid contained 1509 turns, wound in four layers on a cylindrical glass jar 47 centims. long and about 4 centims. internal radius. The secondary, of 841 turns, was carefully insulated from it by means of several layers of paraffin paper, and occupied a length 4 centims. each side of the centre.

The mean internal sectional area at the centre perpendicular to the axis of the tube was found by observing with a cathetometer the rise in level of water in the tube, when a known additional volume was added to it.

The area thus obtained was 50.6 sq. centims.

Let a = radius and l half the length of the solenoid AB. The force F due to unit current is less than $4\pi n$ by the amount contributed by the two productions AA' and BB' of the solenoid, A' and B' being theoretically at infinity.



The solenoids, A'A and B'B, can be replaced by uniform surface distributions of magnetism, $-n$ and $+n$, over the plane circular ends at A and B, as far as forces at places external to A'A' and BB' are concerned, n being the number of turns per centim.*

Hence the force F at the centre of the solenoid is less than $4\pi n$ by that due to a pole, $-\pi na^2$ at A and $+\pi na^2$ at B, so that

$$F = 4\pi n \left(1 - \frac{a^2}{2l^2} \right).$$

The second correction for the ends of the solenoid is found by considering the induction through the secondary circuit due to the magnetism induced on the surface of the liquid. In cases where k is small this correction is easily made since terms involving k^2 may be neglected.

* MAXWELL, 'Electricity and Magnetism,' vol. 2, sect. 676.

Let ϕ be the potential of the magnetic force; dv the element of the normal to the boundary of the liquid. There is a distribution σ of magnetism on the surface of the liquid, where $\sigma = -k d\phi/dv$, which gives rise to a small force f , at C, in the direction opposite to F. So that, if B is equal to the mean area of the secondary coil and N' its number of turns, there is a diminution in the total flux of induction through the secondary circuit equal to $AfN'\mu + (B - A)fN'$, due to the magnetism induced on the surface of the liquid.

Since $\mu = 1 + 4\pi k$, and f involves k as a factor, we may put $\mu = 1$, and the correction reduces to BfN' .

Now, since $\sigma = -k d\phi/dv$, we may, in determining the normal component $-d\phi/dv$ of the magnetic force, omit k and other small corrections and consider the case of a long solenoid of length $2l$ and find the force at the ends parallel to the axis, and the normal force at the cylindrical boundary, when the space inside is occupied by air.

It is easy to see that $\phi = -4\pi n z + \phi_1$, z being measured from A to B along the axis, and ϕ_1 the potential of the distribution $-n$ at A, and $+n$ at B, so that at the ends of the solenoid the component L of the magnetic force is

$$-d\phi/dz = 4\pi n - 2\pi n = 2\pi n \text{ at A or B.}$$

The following simple consideration also shows that the force parallel to the axis, at the ends is approximately half its value at the centre. The force at B due to the long solenoid AB' is $4\pi n$. This must be contributed to equally by the parts AB and BB', hence, when the latter is removed, the force parallel to the axis is $2\pi n$.

The double integral $\iint \frac{d\phi}{dv} ds$ vanishes over any closed surface within which $\Delta^2\phi = 0$ at every point, so that by considering the closed surface made up of the two planes perpendicular to the axis at C and B, and the intermediate part of a cylinder of any section whose generators are parallel to the axis, and integrate over it. We get

$$\begin{aligned} & - \iint \frac{d\phi}{dv} ds \text{ over the plane end at C} \\ & = \iint \frac{d\phi}{dv} ds \text{ over the length } l \text{ of the cylindrical surface and the plane end at B.} \end{aligned}$$

But $d\phi/dv = 4\pi n$ at C, and $-2\pi n$ at B.

Hence

$$\iint \frac{d\phi}{dv} ds \text{ over the length } l \text{ of the cylinder} = -4\pi n(A) + 2\pi n(A) = -2\pi n(A),$$

where (A) = area of the section of the cylinder.

Now let this cylinder be a body whose coefficient of magnetization is small, and let it extend from A to B, since the surface distribution is $-k d\phi/dv$, we see that :

When a current flows through a solenoid, and magnetizes a cylindrical body of equal length whose value for k is small, there are uniform distributions $+$, and $-$, $2\pi nk$ induced on the plane ends, and of the total quantities $+$, and $-$, $4\pi nk$ (A) induced, one-half resides on the ends and the other half on the cylindrical surface.

This latter distribution could not be got rid of by producing the ends of the cylinder beyond the magnetizing coil.

The actual value of the normal force N at a point distance y from the end of a solenoid of circular section of radius a is given by the equation

$$N = \left[\frac{2y^2 + 4a^2}{a\sqrt{y^2 + 4a^2}} K - \frac{2\sqrt{y^2 + 4a^2}}{a} E \right] n,$$

K and E being the complete elliptic integrals of the first and second kind, the modulus k being

$$\frac{2a}{\sqrt{y^2 + 4a^2}}$$

(N may also be found by differentiating the expression for V in THOMSON and TAIT's 'Natural Philosophy,' Ex. II., Section 546).

Making all the corrections, it is found that the change in induction m is

$$4\pi k \ 4\pi n N' \cdot A \left[1 - \frac{a^2}{2l^2} - \frac{B}{A} \left(\frac{0.530}{\pi} \right) \right].$$

Substituting the following values,

$$n = \frac{1509}{47}; \quad N' = 841; \quad A = 50.6; \quad a = 4.5; \quad 2l = 47; \quad B = 93;$$

we get

$$m = k \times 2155 \cdot 10^5 (1 - .049),$$

therefore

$$k = \frac{m}{205 \cdot 10^6}.$$

The adjustable inductance was made in two separate parts, c_1 and c_2 , shown in fig. 2. The larger changes in induction were made by c_1 , the secondary circuit of which consisted of seven turns wound round the inner bobbin, 18.61 centims. in diameter, and had its terminals at t_1 .

On the outer surface of the wooden ring R , 37.17 centims. in diameter, were wound 10 turns of wire of small diameter close beside one another. The ends of 1, 3, and 6 of these circles of wire were twisted together and terminated in 6 mercury cups. The ring R and the bobbin were placed in concentric and coplanar positions, and were then fixed rigidly together by means of ebonite bars.

The second part of c_2 was similar in principle, and by means of it smaller changes of induction could be made. It had only 1 turn in the secondary, the 6 ends of 1, 3, and 6 turns on the outer ring terminating in 6 mercury cups, beside those belonging to the larger inductance. The radius of the inner circle of wire was 12.4, and that of the outer circles 25 centims.

The planes of c_1 and c_2 were perpendicular to one another, each going through the centre of the other, so that when the number of coils of c_1 in series with the primary was altered there should be no change in the magnetic force perpendicular to the plane of c_2 , and *vice versa*. The induction of one circle of the primary on the corresponding secondary is, for

$$c_1, 707, \text{ and for } c_2, 65.*$$

Foucault Currents.

Objections have often been made to the induction method of finding k by using a commutator, as it was supposed that, in the case of conductors, the induced currents had an appreciable effect on the current through the secondary circuit.

This, however, is not the case when a galvanometer is used in the secondary circuit, but would introduce an error if a telephone were substituted.

In order to prove this, let us consider the case of currents generated in a circular cylinder by the current $i = \frac{E}{R} \left(1 - e^{-\frac{R}{L}t}\right)$ starting in the primary, E , R , and L being the E.M.F. resistance and self induction of the primary.

We see from symmetry that the induced currents are in circles round the axis, and, if r is the perpendicular distance of any point from the axis, and u the strength of the current per unit section,

$$u = f(rt).$$

Considering the circuit of radius r , and thickness dr , extending through unit length of the cylinder, the current flowing round the circuit is $u dr$, the resistance of the circuit is $2\pi\sigma r/dr$, where σ is the specific resistance of the liquid.

Therefore

$$2\pi r\sigma u = - \frac{dN}{dt},$$

where N is the total number of lines of induction going through the circuit.

$$N = 4\pi^2 \int_0^r r^2 u dr + 4\pi^2 r^2 \int_r^a u dr + 4\pi^2 r^2 \frac{nE}{R} \left(1 - e^{-\frac{R}{L}t}\right).$$

Hence we have

* MAXWELL, 'Electricity and Magnetism,' vol. 2, chap. iv., App.

$$(1.) \quad 4\pi^2 \int_0^r r^2 \frac{du}{dt} dr + 4\pi^2 r^2 \int_r^a \frac{du}{dt} dr + 2\pi r \sigma u = -4\pi^2 r^2 \frac{nE}{L} \epsilon^{-\frac{R}{L}t}.$$

Differentiating with respect to r , and dividing by r and differentiating a second time, we get

$$\frac{d^2u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{u}{r^2} - \frac{4\pi}{\sigma} \frac{du}{dt} = 0.$$

Let $u = v\epsilon^{-m\sigma t}$, where v is a function of r only and m as yet undetermined.

Hence we have :

$$r^2 \frac{d^2v}{dr^2} + r \frac{dv}{dr} + (4\pi m r^2 - 1) v = 0.$$

Hence

$$v = A_m J_1 \sqrt{4\pi m} r,$$

where J_1 is BESSEL'S internal function of 1st order.

Hence

$$u = \Sigma A_m \epsilon^{-m\sigma t} J_1 \sqrt{4\pi m} r,$$

substituting in (1) and using the equations

$$J_1(x) = -\frac{d}{dx} J_0(x), \quad \frac{d}{dx} (x^2 J_2) = x^2 J_1,$$

and

$$x^2 [J_2(x) + J_0(x)] = 2x J_1 x,*$$

we get

$$\Sigma A_m \sigma \epsilon^{-m\sigma t} \sqrt{4\pi m} r^2 \pi J_0(\sqrt{4\pi m} a) = -\frac{4\pi^2 r^2 nE}{L} \epsilon^{-\frac{R}{L}t}.$$

Hence $J_0 \sqrt{4\pi m} a = 0$, except when $m\sigma = R/L = p$, in which case

$$A_p J_0 \sqrt{\frac{4\pi p}{\sigma}} a = -\sqrt{\frac{4\pi p}{\sigma}} \frac{nE}{R}.$$

The other coefficients are determined by the condition that $u = 0$, when $t = 0$. From what follows, however, it will be seen that it is unnecessary to determine them, as they do not appear in the result.

The number of lines of force through the secondary due to the induced currents in the liquid is

$$4\pi^2 \int_0^a r^2 u dr = \Sigma B_m \epsilon^{-m\sigma t} + B_p \epsilon^{-pt}.$$

* FORSYTH'S 'Differential Equations.'

The equation determining the secondary current i_2 is

$$L_2 \frac{di_2}{dt} + R_2 i_2 = - \frac{dN}{dt},$$

where dN/dt consists of a number of terms of the form $-\lambda B \epsilon^{-\lambda t}$.

The solution of $L_2 \frac{di_2}{dt} + R_2 i_2 = \lambda B \epsilon^{-\lambda t}$ is

$$i_2 = \frac{\lambda B (\epsilon^{-\lambda t} - \epsilon^{-p't})}{R_2 - \lambda L_2}, \text{ where } p' = \frac{R_{(2)}}{L_{(2)}},$$

and the quantity q_2 is

$$q_2 = \frac{B \left[(1 - \epsilon^{-\lambda t}) - \frac{\lambda}{p'} (1 - \epsilon^{-p't}) \right]}{R_2 - \lambda L_2}.$$

Let us consider the combined effect of the increase in permeability, and the currents generated in the liquid, on the quantity q_2 .

The first gives rise to the term $4\pi k M i$, and the second to the series

$$\Sigma B_m \epsilon^{-m\sigma t} + B_p \epsilon^{-p't} \text{ in } N.$$

Hence

$$\frac{dN}{dt} = \frac{4\pi k M E}{R} p \epsilon^{-p't} - \Sigma m B_m \epsilon^{-m\sigma t} - p B_p \epsilon^{-p't}$$

Substituting and bearing in mind that $\Sigma B_m + B_p = 0$, we get

$$(2) \quad q_2 = \frac{4\pi k M E}{R} \left[\frac{(1 - \epsilon^{-p't}) - (1 - \epsilon^{-p't}) \frac{p}{p'}}{R_2 - p L_2} \right] + \Sigma \frac{B_m \left[\frac{m\sigma}{p'} \epsilon^{-p't} - \epsilon^{-m\sigma t} \right]}{R_2 - m\sigma L_2} + \frac{B_p \left(\frac{p}{p'} \epsilon^{-p't} - \epsilon^{-p't} \right)}{R_2 - p L_2}.$$

In order to select the important terms from this equation we must find the approximate values of m , p , and p' .

The smaller values of the roots of $J_0(x) = 0$ are*

$$\begin{aligned} &2.404, \\ &5.520, \\ &8.654, \\ &11.792. \end{aligned}$$

So that the smallest value of m is given by the equation

* Lord Rayleigh, 'Theory of Sound,' Vol. 1, section 206.

$$\sqrt{4\pi m\alpha} = 2.4,$$

therefore

$$m = .03,$$

and taking $\sigma = 10^9$, we get

$$m\sigma = 3.10^7, \quad R = 37.10^9, \quad L = 67.10^6,$$

so that $p = 500$, and p' about the same value.

Hence we see that in equation (2), if we seek the values of q_2 for times greater than $\frac{1}{50}$ th of a second after the primary current has been made, it is evident that terms having $\epsilon^{-m\sigma t}$ as a multiplier may be neglected in comparison with those multiplied by $\epsilon^{-p't}$ and $\epsilon^{-p't}$, so that the terms in the sum Σ , reduce to

$$\Sigma \frac{B_m m \sigma \epsilon^{-p't}}{R_2 (p' - m\sigma)} = - \frac{\epsilon^{-p't}}{R_2} \Sigma B_m = \frac{B_p \epsilon^{-p't}}{R_2}.$$

Hence :—

$$(3) \quad q_2 = - \frac{4\pi kME}{RR_2} - \frac{B_p \epsilon^{-p't} - \epsilon^{-p't}}{R_2 \left(\frac{p}{p'} - 1 \right)},$$

and

$$\begin{aligned} B_p &= 4\pi^2 \int_0^a r^2 A_p J_1 \sqrt{\frac{4\pi p}{\sigma}} r dr \\ &= -4\pi n N' \frac{\pi a^2 E}{R} \frac{J_2 \sqrt{\frac{4\pi p}{\sigma}} a}{J_0 \sqrt{\frac{4\pi p}{\sigma}} a} = - \frac{ME}{R_1 R_2} \frac{J_2 \sqrt{\frac{4\pi p}{\sigma}} a}{J_0 \sqrt{\frac{4\pi p}{\sigma}} a}. \end{aligned}$$

Hence the quantity induced in the secondary due to the presence in the solenoid of a liquid having conductivity σ is

$$\frac{ME}{R_1 R_2} \frac{J_2 \sqrt{\frac{4\pi p}{\sigma}} a}{J_0 \sqrt{\frac{4\pi p}{\sigma}} a} \cdot \frac{\epsilon^{-p't} - \epsilon^{-p't}}{\left(\frac{p}{p'} - 1 \right)},$$

and the ratio of this to the quantity induced owing to a value of $k = 10^{-7}$ is,

$$\frac{1}{4\pi 10^{-7}} \cdot \frac{J_2 \sqrt{\frac{4\pi p}{\sigma}} a}{J_0 \sqrt{\frac{4\pi p}{\sigma}} a} \cdot \frac{\epsilon^{-p't} - \epsilon^{-p't}}{\frac{p}{p'} - 1}.$$

p is increased when the resistance of the primary is increased, its smallest value being 500, which is also about the value of p' .

$$\sqrt{\frac{4\pi p}{\sigma}} \alpha = \frac{1}{100}, \quad J_0(x) = 1, \quad \text{and} \quad J_2(x) = \frac{x^2}{8}$$

when x is small, so that the above ratio reduces to :

$$\frac{1}{32\pi 10^{-3}} \cdot p't \epsilon^{-p't},$$

and letting $t = \frac{1}{30}$, which is about the time the commutator allowed the secondary to be connected in the positive direction, say, with the galvanometer after the process of changing the current in the primary from 0 to $i = \frac{E}{R} \left(1 - \epsilon^{-\frac{L}{R}t} \right)$ was completed, we get a value for this ratio far less than unity $\frac{5.00}{3} \epsilon^{-50/3}$; in fact, the commutator might have been going twice as fast without the currents induced in the liquid having any effect on the value of k in the eighth decimal place.

Results of the Experiments.

In carrying out an experiment for the determination of k , it is necessary in all cases to syphon the liquid out of the solenoid immediately after m has been found, and test the balance again to see whether it is restored by replacing m . When this condition is fulfilled, it is evident that the mutual induction of the system of coils has not been altered while the experiment has been going on.

It does not always happen that an exact balance can be got by the method of changing the number of turns in the primary of c , for if a certain adjustment gives a deflection of two divisions positive, a reduction of one turn in the primary of c_2 gives a deflection of three divisions negative, so that in this case the correct reduction can be taken as two-fifths of 65, which is sufficiently accurate, since all that is necessary in order to determine k in the 7th decimal place is to be able to detect an induction of 20 C.G.S. units.

In all the following experiments the magnetising force was varied from 1 to 9 C.G.S. units, and in no case was there any change in k .

Ferric Chloride.

The values of k were found for four solutions of ferric chloride, the quantities of the salt per cub. centim. being proportional to 1, 2, 3, and 4.

The values of $10^7 k$ were

$$69, \quad 148, \quad 220, \quad \text{and} \quad 298.$$

These numbers are not proportional to 1, 2, 3, and 4; but when we add 7.7 (the value of $-10^7 k$ for the solvent) to each, we get values for k which are proportional

to the weights of iron per cub. centim. This shows that the action of the solvent is to diminish the value of 10^7k by 7·7, which is the value of -10^7k for water.

The actual values found for distilled water in three different experiments were

$$10^7k = -8\cdot1, \quad -7\cdot5, \quad \text{and} \quad -7\cdot5,$$

the mean being $-7\cdot7$.

The values for 10^7k for water given by different observers are:—

HENRICHSSEN $-7\cdot51$, QUINCKE $-4\cdot278$, HOWARD $-4\cdot248$, and WÄBNER $-2\cdot758$.*

A solution of ferric chloride, whose value for 10^7k at 10° centigrade, was

$$230\cdot5 - 7\cdot7,$$

was found by gravimetric analysis to contain ·0865 gram of iron per cub. centim. Hence the general formula for k , for ferric chloride dissolved in water, is

$$10^7k = 2660 W - 7\cdot7$$

W being the weight of iron per cub. centim. of the solution.

Ferric Sulphate.

The values of k for three different solutions of ferric sulphate were found, the quantity of iron per cub. centim. in each being proportional to 2, 3, and 4, giving the values

$$52, \quad 79\cdot3, \quad \text{and} \quad 110, \text{ for } 10^7k.$$

If to these we add 7·7 we get the values of k for the salt alone which are in the same proportion as the quantity of salt per cub. centim.

A solution, whose value for 10^7k at 7° centigrade was $86\cdot2 - 7\cdot7$, contained ·0320 gram of iron per cub. centim., which gives for ferric sulphate the formula

$$10^7k = 2690 W - 7\cdot7.$$

It will be seen from the curves showing the variation of k with temperature that at 10° centigrade the value of k will be 1·5 per cent. less than at 7° , so that we get

$$10^7k = 2660 W - 7\cdot7, \text{ at } 10^\circ \text{ centigrade,}$$

exactly the same formula as for ferric chloride.

* WIEDEMANN, 'Electricitat,' vol. 3, pp. 1292 and 1322.

Ferric Nitrate.

An experiment was tried with ferric nitrate and estimated gravimetrically, like the chloride and the sulphate, and it also gave the formula

$$10^7k = 2660 W - 7.7.$$

thus showing that the coefficient k depends only on the quantity of iron present, and the nature of the solvent.

Ferrous Chloride.

A solution of FeCl_2 , which contained .0409 gram of iron per cub. centim., had a coefficient of magnetization k given by the formula

$$10^7k = 84.5 - 7.7 \text{ at } 12^\circ \text{ centigrade.}$$

The solution had been boiled for several hours with iron wire, so as to reduce any ferric salt to the ferrous state and then filtered. So that for ferrous chloride we have the formula

$$10^7k = 2060 W - 7.7.$$

Ferrous Sulphate.

A more complete examination was made of ferrous sulphate, the values for 10^7k for two solutions, one containing twice the amount of iron per cub. centim. as the other, were found to be

$$127.3, \text{ and } 60.5,$$

so that when they are corrected for the water in which they were dissolved they have the ratio 2 : 1.

A solution was examined similarly to the ferrous chloride and gave a formula $10^7k = 2090 W - 7.7$, the quantity of iron having been estimated gravimetrically by boiling a specimen with nitric acid and proceeding in the ordinary way.

As this formula did not coincide with that got for the ferrous chloride a more complete examination was made of a solution which contained a small amount of ferric sulphate. Having found the value of k , 25 cub. centims. of the solution were extracted and the quantity of ferrous salt found by a standard solution of permanganate of potash, another specimen having been treated with nitric acid gave the total quantity of iron present, thus providing a means of finding the amount of ferric iron in the solution.

The numbers thus obtained were :—

MDCCCXCVI.—A.

4 A

Weight of iron in ferrous state, per cub. centim., $\cdot 0689$,

Weight of iron in ferric state, per cub. centim., $\cdot 0064$,

$$10^7k = 158 - 7\cdot 7,$$

subtracting 17, which is the value of 10^7k for $\cdot 0064$ gram per cub. centim. in ferric sulphate, we get

$$10^7k = 141 - 7\cdot 7 \text{ for } \cdot 0689 \text{ of iron in the ferrous state.}$$

Hence

$$10^7k = 2050 W - 7\cdot 7,$$

differing from the formula obtained for ferrous chloride by only $\cdot 5$ per cent.

The same result was obtained for a solution containing $\cdot 01$ gram of ferric iron and $\cdot 0118$ of ferrous iron.

The presence of free sulphuric acid in the sulphates was found not to affect the value of k , by testing whether an addition of the acid altered it.

The liquid containing the two sulphates in nearly equal proportions was further examined by gradually neutralizing the free sulphuric acid with caustic potash till an almost neutral solution was obtained.

This did not alter the coefficient of magnetization of the salt, showing that no change in the acid radical had as yet taken place, but when the potash was added till the dark precipitate was formed, the value of k immediately rose.

This experiment shows that a salt corresponding to the magnetic oxide does not exist in solution, and that when the latter is dissolved in an acid it gives a mixture of ferrous and ferric salts.

Salts in the Dry State.

Estimations were also made in the dry state by filling a glass tube with the salt and putting it into the solenoid. These experiments could not with ease be performed with the same accuracy as those made on the solutions, since the density of the salt is variable along the tube, and the amount of moisture and water of crystallization in the body introduce errors when estimating the weight of iron.

The experiments showed roughly that the magnetic properties of the salts are the same as when they are in solution, the actual numbers got for ferrous sulphate were $10^7k = 841$, the amount of iron per cub. centim., found by weighing the body as $\text{FeSO}_4 + 7\text{H}_2\text{O}$, was $\cdot 241$. This value of k differs by only 4 per cent. from the value calculated from the formula $10^7k = 2050 W$.

Iron in the Acid Radical.

Solutions of potassium ferricyanide and ferrocyanide each containing $\cdot 029$ gram of

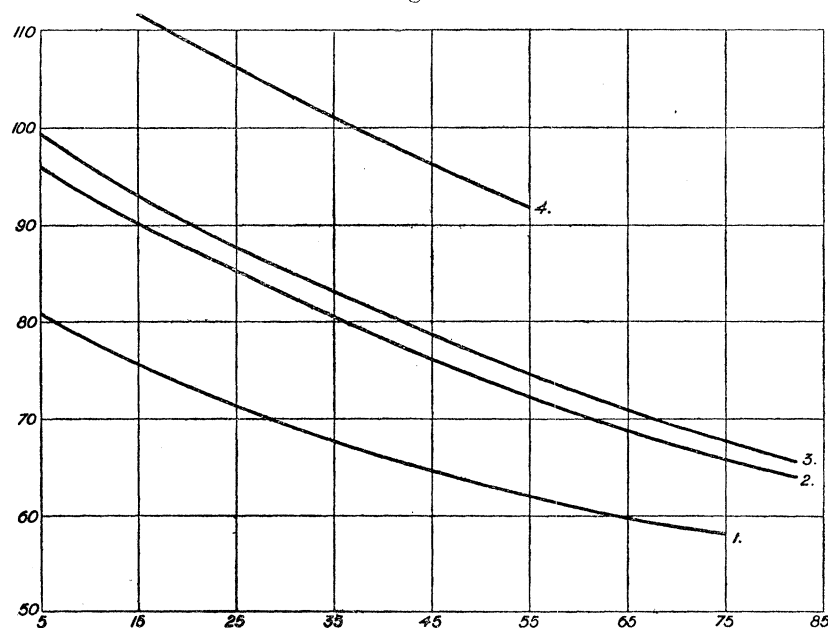
iron per cub. centim. gave values of k the same as if water alone were present, so that when iron appears in the acid radical it is at least 100 times less magnetic than when it takes the part of a metal in a salt.

Effect of Temperature.

In estimating the effect of temperature it is necessary to use a tube of about one-third the sectional area of that on which the solenoid is wound, so as to be able to prevent the latter from being heated by radiation and convection.

The time required to keep the hot tube inside the solenoid while adjusting is about half-a-minute, so that it is an easy matter to prevent the coils getting heated. The efficiency of the method employed was proved by the balance returning to its original zero when the liquid was taken out and the reduction in the turns in c restored. Uniformity of temperature along the column of the liquid is secured by inverting the tube containing it before inserting it into the solenoid.

Fig. 3.



The results of these experiments are best shown by drawing curves, fig. 3, the x ordinates denoting temperature centigrade, the y ordinates being proportional to k , a length corresponding to 50 being subtracted from the latter for convenience of representation.

The first curve was got from experiments done on a solution of ferric chloride containing .086 grm. of iron per cub. centim., the 2nd from ferrous chloride containing .148 grm. of iron per cub. centim., the 3rd from ferric sulphate containing

·105 grm. of iron per cub. centim., and the 4th from an alcoholic solution of ferric chloride containing ·127 grm. of iron per cub. centim.

The curves show that there is a great diminution in k as the temperature rises, amounting to more than ·5 per cent. per degree centigrade at the lower temperatures.

The curves show that $k = k_0(1 - \alpha t)$, where α is a function of the temperature, and is very approximately independent of the acid radical.

The diminution in density of the salt due to expansion contributes such a small amount to the rapid fall in k that it is not necessary to make any correction for it.

The solvent itself did not show any variation due to temperature which could be detected by this method, and as the alcoholic solution gives the same temperature coefficient as the solutions in water, it is probable that the change due to temperature is a property of the molecules of the salt itself.

The first conclusion that the above results leads us to is that the magnetization is entirely due to the iron, and is accurately the same for all acid radicals connected with it, so long as the iron remains either in the ferrous or the ferric state. The only other electromagnetic phenomenon we know of that also possesses this property is the atomic charge, so that possibly the polarity of the metallic atom may be due to its rapid rotation carrying with it its atomic charge. The ratio of the atomic charge in the ferric to that in the ferrous state is 3 : 2. There is a variety of suppositions as to the nature of the rotations of the iron atom in the molecule which will account for the ratio 266 : 206 of the two values of k .

A simple case presents itself by supposing the intensity of the polarity proportional to $q_1\omega_1$ and $q_2\omega_2$ in the two cases, q_1 and q_2 denoting the charges, and ω_1 and ω_2 the rotations, the axes of which coincide with the axis of polarity. In this case, if H is the applied force, we get the displacements proportional to $q_1\omega_1H$ and $q_2\omega_2H$, so that the ratio of the values of k is $q_1^2\omega_1^2 : q_2^2\omega_2^2 = 266 : 206$, so that $\omega_2 = 1\cdot32\omega_1$.

The next consideration is the controlling force, which acts in such a way that the induced magnetization is proportional to the applied force. It cannot arise from any action of the solvent, since the magnetic properties of the salts are the same in the dry state as when they are in solution. Also, since k is accurately proportional to the density, we see that each molecular magnet behaves in exactly the same way whether the neighbouring magnets are near it or not.

It is, therefore, highly improbable that the surrounding molecules contribute to the controlling force, as there is no variation in it when the mean distances undergo large changes.

We are, therefore, led to the conclusion that there is no controlling force due to external bodies, and that the magnetization is a phenomena due to the perturbations in the angle of inclination of the axis of polarity to the direction of force H .

Let θ denote this angle, which is a function of the time, $\theta = \theta_0 + \delta\theta$ where θ_0 is the value of θ when $H = 0$, and $\delta\theta$ the perturbation at any time due to H , and is propor-

tional to H , so that $\delta\theta = Hf(t)$ where f differs from the different atoms according to θ_0 and the initial circumstances.

The intensity I of magnetization is equal to

$$\Sigma \frac{1}{T} \int_0^T M \cos \theta dt,$$

where M is the magnetic moment of the molecular magnet, and T a time which is large compared with the periods.

Therefore,

$$I = \Sigma \frac{1}{T} \int_0^T M (\cos \theta_0 - \delta\theta \sin \theta_0) dt,$$

$$I = H \Sigma \frac{1}{T} \int_0^T M \sin \theta_0 f(t) dt.$$

This explanation accounts for diamagnetism, for if we consider the very simple case of a magnet rotating rapidly in the horizontal field of the earth, the effect of the applied force H will be to make the north pole move more slowly when it is south of the axis of suspension than when it is north, so that its mean position is south of the axis of suspension, thus causing the magnet to act like a diamagnetic body.

The following table shows the values of 10^7k for salts in solution, w being the weight of salt per cub. centim., the forces ranging from 1 to 9 C.G.S. units.

	$10^7k.$
Fe_2Cl_6	$916w - 7.7$
$\text{Fe}_2(\text{SO}_4)_3$	$745w - 7.7$
$\text{Fe}_2(\text{NO}_3)_6$	$615w - 7.7$
FeCl_3	$908w - 7.7$
Fe_2SO_4	$749w - 7.7$