XI. *On the Distribution of Frequency (Variation and Correlation) of the Barometric Height at Divers Stations.*

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[PLATES 9–17.]

I.—ON THE FREQUENCY AT DIVERS STATIONS OF THE SEVERAL BAROMETRIC HEIGHTS.

1. *Introduction.*

LET a curve be formed such, that if y be the ordinate falling between the abscissæ x and $x + \delta x$, the area $y\delta x$ represents the frequency of the barometer with height lying between x and $x + \delta x$ for any locality. This curve will be spoken of as the *barometer frequency curve* for the given locality. The curve in any series of actual observations will be represented by a polygonal line; in the present case the element δx has been throughout taken as $\frac{1}{10}$ inch of barometric height.

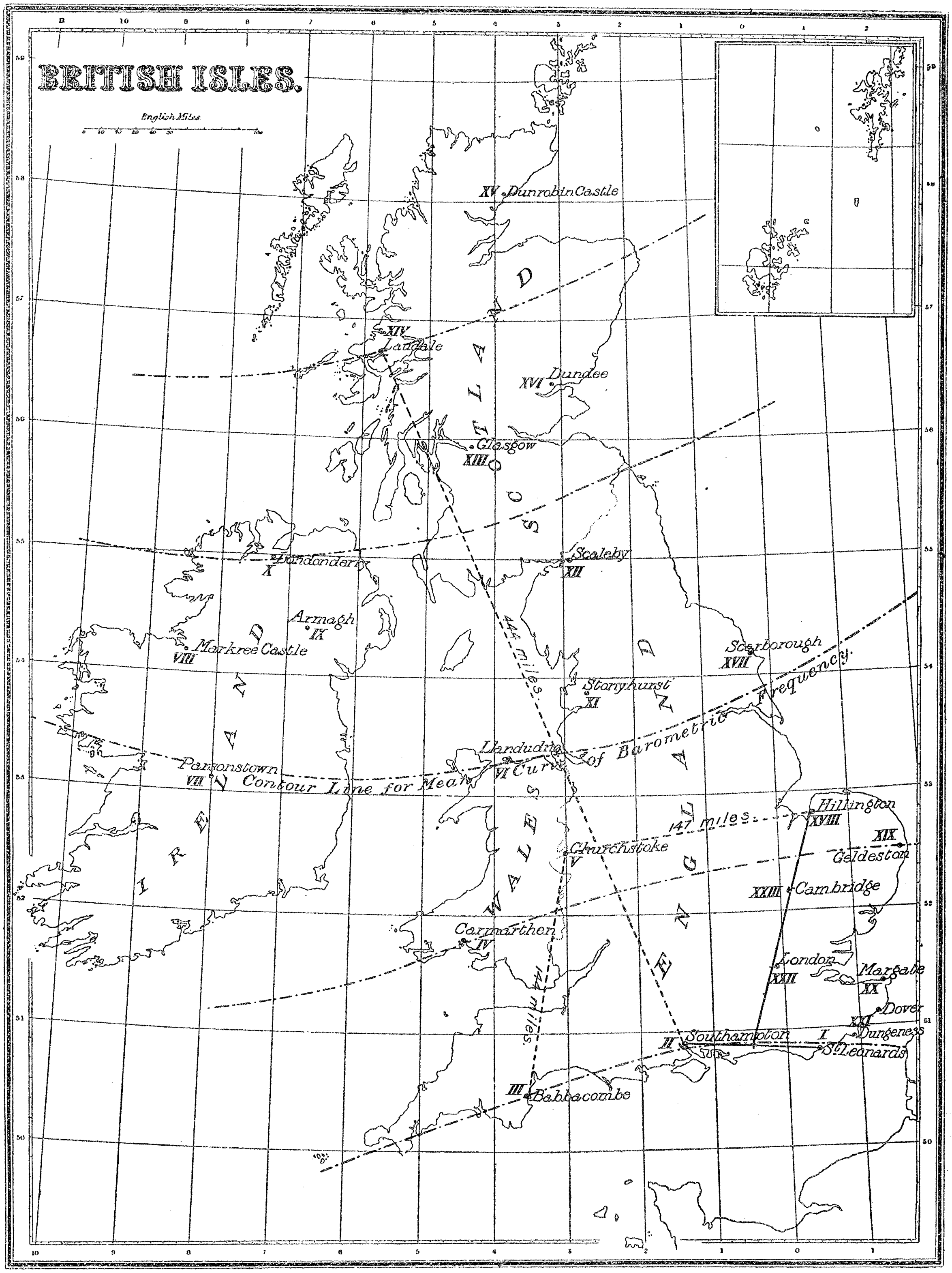
Such frequency curves occur in innumerable physical, anthropological and economic investigations, and can in many cases be fairly accurately represented by the normal curve of frequency, *i.e.*, LAPLACE's curve of errors. The barometric frequency curve is, however, a marked exception to this rule. The mean barometric height is very far from coinciding with the 'mode' or height of maximum frequency. While barometric frequency curves are remarkably smooth when a very large number of observations are dealt with, the distribution of frequency does not obey the normal law,* but some other law which up to the present has not been fully discussed.

In a memoir published in the 'Phil. Trans.,' A, vol. 186, pp. 343–414, a series of generalised frequency curves are introduced, and it is shown, pp. 351 and 382, that the asymmetry of the barometric frequency curve can probably be dealt with by one or other of these generalised curves.

The importance of this conclusion lies in the fact that the distribution of barometric frequency in any locality can then be fully described by the statement of the values of three or four well defined constants.

* This seems first to have been pointed out by DR. VENN, in a letter to 'Nature,' September 1, 1887.

7.3.98.


BRITISH ISLES.
English Miles
SCOTLAND
IRELAND
WALES
ENGLAND
XV Dunrobin Castle
XIV Laudale
XVI Dundee
XIII Glasgow
X Londonderry
XII Scaleby
IX Armagh
VIII Markree Castle
XVII Scarborough
XI Stonyhurst
VII Parsonstown
VI Llandudno
Contour Line for Mean Curve of Barometric Frequency.
444 miles.
147 miles.
XVIII Hillington
XIX Geldeston
V Churchstoke
XXIII Cambridge
IV Carmarthen
XXII London
XX Margate
XXI Dover
Dungeness
II Southampton
I St. Leonards
III Babbacombe
147 miles.

Accordingly, in order to test this theory of barometric frequency, series of observations have been reduced and the frequency distributions for divers localities fitted with generalised probability curves. On the basis of these curves the attempt has been made to answer the following questions :—

(*a.*) Is there any one type of curve especially characteristic of barometric frequency ?

(*b.*) If so, what are the constants by which the distribution of this frequency can best be described ?

(*c.*) Does there appear to be any numerical or geographical relation between these constants ? and

(*d.*) Does a knowledge of their values for a variety of localities enable us to make any statement with regard to the physics of atmospheric pressure ?

2. *Data of Observation.*

Our data were taken from the annual 'Meteorological Observations at Stations of the Second Order,' kindly placed at our disposal by Mr. R. H. Scott, F.R.S., Secretary to the Meteorological Council. The heights dealt with are 9 o'clock morning heights. Twenty stations were selected, sixteen in England and Scotland and four in Ireland. The stations selected are given on the accompanying map (page 424) of the British Isles. In their choice we were guided by (i.) the desire for a fairly uniform distribution and if possible a coast distribution, (ii.) the extent of data available, and (iii.) its continuity. The minimum number of years of continuous observation is five, the maximum thirteen. At one or two suitable places there were gaps in the record ; at others less than five years were available. In all cases the records were only dealt with for whole years, to remove any irregularity which might arise from an annual periodicity in frequency. It is hoped that the fraction of a monthly periodicity—if there be a lunar influence on the frequency—will not be very sensible, when the records deal with 65 to 170 lunar periods. Any long period, such as a 19-year period, would of course render less general a frequency distribution based on five to thirteen years of observation only. The possibility of such a periodicity would render it very desirable to recalculate the frequency distributions for the same localities after, say, another ten years of observations.

Observations at the selected stations, when unreduced, were reduced by formulæ kindly provided by the Meteorological Office.

The following is what we shall hereafter term the *geographical order* of selected stations, with the years of observation and dates :—

Place.		Years of Record.	Date.
South Coast	St. Leonards	6	Jan. 1, 1880–Dec. 31, 1885
	Southampton	13	„ 1878– „ 1890
	Babbacombe	13	„ 1878– „ 1890
Wales	Carmarthen	13	„ 1878– „ 1890
	Churchstoke	11	„ 1878– „ 1888
	Llandudno	8	„ 1878– „ 1885
Ireland	Parsonstown	13	„ 1878– „ 1890
	Markree Castle*	10	„ 1880– „ 1882 „ 1884– „ 1890
	Armagh	5	„ 1886– „ 1890
	Londonderry	7	„ 1879– „ 1885
North-west Coast	Stonyhurst	7	„ 1884– „ 1890
	Scaleby	6	„ 1880– „ 1885
	Glasgow	5	„ 1886– „ 1890
	Laudale	11	„ 1880– „ 1890
North-east Coast	Dunrobin Castle	8	„ 1883– „ 1890
	Dundee	11	„ 1880– „ 1890
	Scarborough	5	„ 1881– „ 1885
South-east Coast	Hillington	13	„ 1878– „ 1890
	Geldeston	6	„ 1880– „ 1885
	Margate	8	„ 1883– „ 1890

The frequency distribution† has been calculated for every $\frac{1}{10}''$. In reckoning the frequency, any height falling midway between two-tenths inches (*e.g.*, 28·75) has been split between those two-tenths (*e.g.*, half to 28·7 and half to 28·8).

Table I. of frequencies (p. 428) embraces the data from which the constants of the probability curves have been calculated. The lower or Roman figures in this table give the total number of days in which, during the years of observation, the barometer was at each particular height. The upper, or old-style figures, give the number of days per year at which the barometer (on the basis of these observations) may in the given locality be expected to be at each particular height.

We do not consider that the Stations of the Second Order give the best possible returns. Far from it; the irregularities which occur in the frequency distributions at several of these stations seem to us to indicate that either the methods of observation, the instruments, or the formulæ of reduction are not entirely satisfactory. It is probable that the Telegraph Stations would give far better results, and for a greater number of identical years. The ideal data would cover twenty or more well-distributed stations for identical periods of four "lustra," or twenty years. The observations of the Stations of the Second Order, however, were readily available in print, and our object has not been so much to make a contribution

* Omissions occur in the record for 1883, so that year is not included.

† Besides the above twenty stations, three telegraph stations, Cambridge, London (Brixton), and Dover-Dungeness will be found to have their constants recorded in our tables. The very great labour, however, of copying the manuscript records discouraged a larger selection of these stations. Our returns are for the thirteen years 1878–1890 inclusive.

to the meteorology of the British Isles as to illustrate what we believe to be a novel method of dealing with barometric frequency, in the hope that it may hereafter be taken up by professed meteorologists having at their disposal trained computators. We need not stay to lay stress on the large amount of arithmetical work involved in calculating the constants for even the short periods dealt with in the present memoir, it will be obvious to any one who has undertaken any statistical work of a like character. In order to save a portion of this arithmetical labour, a self-recording frequency barometer has been devised by Mr. G. A. YULE, some account of which will be found in an Appendix to this paper. We believe that with due precautions,* such an instrument would be a valuable addition to those already in use at the various meteorological stations, and serve for the ready calculation of the frequency constants.

Yet, admitting all the disadvantages of the non-contemporary character and short period of some of the data which we have used, we still believe that an examination of our diagrams will serve to convince the reader that our method is really capable of considerable service.

Putting aside one or two anomalies, we find a continuous and gradual change of the frequency constants as we pass from station to station round the coast, and it appears as easy to draw the contour lines of equal frequency as it has been hitherto supposed to be to draw isobars.

3. *Theory Applied.*

Let $y\,\delta x$ be the frequency of a deviation lying between x and $x+\delta x$ from the mean, then $\mu_n = \mathrm{S}\,(x^n y\,\delta x)\,\mathrm{S}\,(y\,\delta x)$ may be termed the n^{th} power of the mean n^{th} deviation. In the case of the Gaussian or normal distribution of the frequency of N observations

$$y = \frac{\mathrm{N}}{\sqrt{2\pi\mu_2}}\,e^{-x^2/(2\mu_2)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(i.)}$$

Here all the odd μ's vanish and we have the relation $\mu_{2\nu} = (2\nu - 1)\,(2\nu - 3) \ldots 5 \cdot 3 \cdot 1\mu_2^2$ for the even μ's. In particular $\mu_4 = 3\mu_2^2$.

When, therefore, the odd μ's are not nearly zero, and the relation $\mu_4 = 3\mu_2^2$ is not nearly satisfied, it is impossible on both counts to represent the frequency by a normal distribution.

Let $\beta_1 = \mu_3^2/\mu_2^3$, $\beta_2 = \mu_4/\mu_2^2$, then if β_1 be not zero and β_2 be not equal to three, a normal distribution is theoretically impossible.

Now a skew frequency distribution may be deduced from the skew-binomial $(p+q)^n$ as a limit, in the same way as it is possible to deduce the normal frequency distribution as a limit from the symmetrical binomial $(\frac{1}{2}+\frac{1}{2})^n$.

* Notably, frequent comparison with the standard mercurial barometer.

TABLE I.—Observed Annual Frequencies of the Several Barometric Heights

Old style figures, annual frequencies. *Roman figures, gross frequencies.*

	Twenty Stations in the British Isles.											
Station	St. Leonards.	Southampton.	Babbacombe.	Carmarthen.	Churchstoke.	Llandudno.	Parsonstown.	Markree Castle.	Armagh.	Londonderry.	Stonyhurst.	Scaleby.
Years of Record	6	13	13	13	11	8	13	10	5	7	7	6
Total Observations	2192	4748	4748	4748	4018	2922	4748	3653	1826	2557	2557	2192
Height, in inches.												
27·7	... ...	... ...	... ...	... ...	... ...	... ...	... ...	0·10 1	... ...	... ...	... ...	... ...
27·8	... ...	... ...	... ...	... ...	... ...	... ...	... ...	0 0	... ...	... ...	... ...	... ...
27·9	... ...	... ...	... ...	... ...	... ...	... ...	... ...	0 0	0·20 1	... ...	... ...	... ...
28·0	... ...	... ...	... ...	... ...	... ...	... ...	0·08 1	0 0	0 0	... ...	... ...	... ...
28·1	... ...	... ...	... ...	... ...	... ...	... ...	0 0	0·10 1	0 0	0·14 1	... ...	... ...
28·2	... ...	... ...	... ...	... ...	... ...	... ...	0 0	0 0	0 0	0 0	0·07 0·5	... ...
28·3	... ...	... ...	... ...	... ...	... ...	... ...	0 0	0 0	0 0	0 0	0·21 1·5	0·17 1
28·4	... ...	... ...	... ...	0·08 1	0·09 1	0·125 1	0·15 2	0·10 1	0 0	0 0	0 0	0 0
28·5	... ...	0·08 1	... ...	0·08 1	0·045 0·5	0·125 1	0·15 2	0·20 2	0 0	0 0	0 0	0·25 1·5
28·6	... ...	0·15 2	0·19 2·5	0·23 3	0·14 1·5	0·125 1	0·15 2	0·25 2·5	0·6 3	0·36 2·5	0·14 1	0·08 0·5
28·7	... ...	0·15 2	0·42 5·5	0·35 4·5	0·45 5	0·125 1	0·58 7·5	0·60 6	0·40 2	0·71 5	0·14 1	0·50 3·0
28·8	... ...	0·31 4	0·38 5·0	0·65 8·5	0·68 7·5	0·875 7	0·50 6·5	1·05 10·5	0·40 2	1·29 9	1·14 8	1·42 8·5
28·9	0·33 2	0·66 8·5	0·46 6	0·81 10·5	1·14 12·5	1·06 8·5	0·77 10	1·90 19	1·30 6·5	2·00 14	1·21 8·5	2·50 15
29·0	1·17 7	1·04 13·5	0·69 9	0·88 11·5	1·23 13·5	2·69 21·5	1·62 21	2·40 24	3·10 15·5	2·93 20·5	1·50 10·5	3·25 19·5
29·1	2·25 13·5	1·65 21·5	1·88 24·5	2·69 35	3·09 34	3·00 24	3·85 50	5·40 54	3·20 16	6·14 43	2·36 16·5	4·50 27
29·2	3·67 22	2·85 37	2·81 36·5	4·54 59	4·23 46·5	6·19 49·5	6·23 81	7·35 73·5	8·30 41·5	7·71 54	5·64 39·5	5·92 35·5
29·3	4·42 26·5	6·08 79	6·04 78·5	6·62 86	6·50 71·5	7·06 56·5	9·38 122	12·15 121·5	7·20 36	10·86 76	6·93 48·5	11·50 69
29·4	8·12 49	8·31 108	9·31 121	12·23 159	12·00 132	13·81 110·5	12·58 163·5	14·60 146	13·80 69	13·57 95	10·86 76	11·75 70·5
29·5	11 66	13·96 181·5	15·15 197	15·46 201	16·45 181	19·25 154	19·19 249·5	19·50 195	21·20 106	21·36 149·5	16·00 112	25·17 151
29·6	20·33 122	19·58 254·5	18·62 242	24·00 312	23·45 258	26·75 214	23·46 305	26·30 263	25·60 128	26·79 187·5	26·57 186	27·00 162
29·7	26·50 159	26·81 348·5	26·85 349	28·12 365·5	29·91 329	31·25 250	29·81 387·5	31·75 317·5	31·90 159·5	30·29 212	29·07 203·5	31·75 190·5
29·8	36·42 218·5	35·65 463·5	36·12 469·5	36·50 474·5	32·77 360·5	32·375 259	35·12 456·5	33·85 338·5	34·00 170	35·07 245·5	33·57 235	33·08 198·5
29·9	44 264	42·19 548·5	41·27 536·5	38·96 506·5	38·73 426	37·125 297	38·65 502·5	37·25 372·5	33·00 165	36·29 254	41·50 290·5	39·42 236·5
30·0	49·83 2·99	46·35 602·5	45·92 597	42·81 556·5	41·64 458	44·81 358·5	39·15 509	37·10 371	39·40 197	39·57 277	39·21 274·5	37·25 223·5
30·1	48 288	47·65 619·5	46·96 610·5	44·00 572	44·05 484·5	40·00 320	40·08 521	39·95 399·5	38·90 194·5	33·07 231·5	44·21 309·5	34·83 209
30·2	40·75 244·5	38·46 500	38·88 505·5	35·88 466·5	34·27 377	32·50 260	35·23 458	32·70 327	31·30 156·5	30·29 212	34·36 240·5	34·83 209
30·3	27·25 163·5	29·38 382	30·19 392·5	28·46 370	29·91 329	25·50 204	27·23 354	26·55 265·5	33·10 165·5	26·07 182·5	30·00 210	26·33 158
30·4	18·08 108·5	18·27 237·5	18·23 237	18·00 234	18·86 207·5	18·625 149	19·31 251	17·40 174	18·10 90·5	21·36 149·5	21·93 153·5	16·50 99
30·5	12·25 73·5	14·58 189·5	13·65 177·5	12·31 160	12·09 133	11·00 88	11·62 151	10·85 108·5	11·40 57	9·50 66·5	9·64 67·5	8·92 53·5
30·6	6·83 41	6·81 88·5	7·31 95	7·31 95	8·50 93·5	6·50 52	6·88 89·5	4·20 42	6·70 33·5	5·93 41·5	6·14 43	5·17 31
30·7	2·58 15·5	3·35 43·5	3·12 40·5	3·46 45	4·18 46	3·25 26	2·85 37	1·30 13	2·00 10	3·07 21·5	2·71 19	2·42 14·5
30·8	0·67 4	0·54 7	0·38 5	0·50 6·5	0·59 6·5	0·50 4	0·38 5	0·25 2·5	0·10 0·5	0·79 5·5	0·14 1	0·50 3
30·9	0·58 3·5	0·31 4	0·31 4	0·23 3	0·18 2	0·625 5	0·19 2·5	0·15 1·5	... ...	0·14 1	... ...	0·33 2
31·0	0·25 1·5	0·08 1	0·08 1	0·08 1	0·09 1	... ...	0·04 0·5	... ...	... ...	... ...	... ...	... ...

at Twenty Stations in the British Isles, and Three Supplementary Stations.

Old style figures, annual frequencies. *Roman figures, gross frequencies.*

Twenty Stations in the British Isles.								Supplementary Stations.			
Glasgow.	Laudale.	Dunrobin Castle.	Dundee.	Scarborough.	Hillington.	Geldeston.	Margate.	Dover-Dungeness.	London.	Cambridge.	Station.
5	11	8	11	5	13	6	8	13	13	13	Years of Record.
1826	4018	2922	4018	1826	4748	2192	2922	4748	4748	4748	Total Observations.
											Height, in inches.
... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	27·7
... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	27·8
... ...	0·09 1	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	27·9
0·20 1	0·09 1	0·125 1	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	28·0
0 0	0 0	0 0	0·18 2	... ...	... ...	... ...	... ...	... ...	... ...	... ...	28·1
0 0	0 0	0 0	0·18 2	... ...	... ...	... ...	... ...	... ...	... ...	... ...	28·2
0·20 1	0·045 0·5	0·25 2	0 0	... ...	0·12 1·5	... ...	... ...	... ...	... ...	0·08 1	28·3
0 0	0·23 2·5	0·125 1	0 0	... ...	0·04 0·5	... ...	0·125 1	0·08 1	... ...	0 0	28·4
0·20 1	0·18 2	0·19 1·5	0·27 3	0·50 2·5	0·08 1	... ...	0 0	0 0	... ...	0 0	28·5
0·20 1	0·41 4·5	0·06 0·5	0·18 2	0·50 2·5	0·08 1	... ...	0 0	0 0	0·115 1·5	0·08 1	28·6
0·60 3	0·77 8·5	0·625 5	0·68 7·5	0 0	0·15 2	0·33 2	0·25 2	0·08 1	0·04 0·5	0·15 2	28·7
0·80 4	1·09 12	1·50 12	1·36 15	0·50 2·5	0·54 7	0·26 1·5	0·375 3	0·15 2	0·115 1·5	0·50 6·5	28·8
1·50 7·5	2·95 32·5	2·25 18	2·64 29	1·90 9·5	0·96 12·5	0·12 7	0·69 5·5	0·35 4·5	0·77 10	0·81 10·5	28·9
3·80 19	4·32 47·5	3·625 29	3·27 36	2·50 12·5	1·77 23	1·58 9·5	1·50 12	1·885 24·5	1·42 18·5	1·77 23	29·0
5·60 28	6·55 72	7·06 56·5	6·32 69·5	4·50 22·5	2·62 34	2·42 14·5	1·44 11·5	2·15 28	2·04 26·5	1·85 24	29·1
6·40 32	7·95 87·5	8·81 70·5	7·64 84	4·40 22	4·96 64·5	4·67 28	3·06 24·5	3·31 43	0·385 44	4·885 63·5	29·2
11·70 58·5	11·41 125·5	13·31 106·5	10·91 120	8·50 42·5	6·15 80	6·42 38·5	5·875 47	5·73 74·5	6·46 84	6·23 81	29·3
14·70 73·5	17·05 187·5	15·75 126	15·95 175·5	12·70 63·5	10·58 137·5	10·25 61·5	9·44 17·5	10·35 134·5	9·23 120	9·77 127	29·4
22·30 111·5	23·23 255·5	23·375 187	21·00 231	20·10 100·5	17·54 228	14·42 86·5	12·125 97	13·115 170·5	14·615 190	16·385 213	29·5
22·90 114·5	27·64 304	27·94 223·5	27·05 297·5	26·40 132	22·54 293	21·67 130	18·56 148·5	22·00 286	20·58 267·5	22·23 289	29·6
34·40 172	31·50 346·5	34·375 275	34·32 377·5	35·80 179	29·81 387·5	31·92 191·5	26·81 214·5	28·92 376	28·85 375	29·85 388	29·7
35·80 179	35·82 394	34·50 276	36·14 397·5	32·40 162	37·08 482	37·00 222	36·125 289	38·50 500·5	36·50 474·5	36·885 479·5	29·8
38·00 190	34·59 380·5	37·56 300·5	36·64 403	40·20 201	41·92 545	40·75 244·5	44·56 356·5	43·85 570	42·23 549	41·35 537·5	29·9
37·10 185·5	36·36 400	39·44 315·5	32·77 360·5	40·50 202·5	43·85 570	47·42 284·5	44·94 359·5	49·58 644·5	46·77 608	45·08 586	30·0
30·60 153	32·82 361	33·44 267·5	37·00 407	41·60 207	42·69 555	44·75 268·5	49·625 397	46·50 604·5	45·92 597	42·31 550	30·1
31·80 169	34·18 376	29·19 233·5	32·14 353·5	32·40 162	35·96 467·5	36·75 220·5	40·875 327	36·08 469	37·615 489	37·54 488	30·2
29·30 146·5	24·23 266·5	22·81 182·5	24·55 270	25·10 125·5	26·58 345·5	27·67 166	29·44 235·5	26·08 339	27·46 357	26·96 350·5	30·3
16·70 83·5	16·55 182	16·00 128	17·77 195·5	16·40 82	18·77 244	16·25 97·5	18·875 151	17·00 221	18·81 244·5	18·92 246	30·4
9·90 49·5	8·55 94	7·31 58·5	8·50 93·5	9·40 47	10·81 140·5	9·83 59	12·625 101	11·00 143	12·19 158·5	11·54 150	30·5
6·00 30	4·59 50·5	4·00 32	5·23 57·5	4·90 24·5	6·12 79·5	6·58 39·5	4·31 34·5	5·96 77·5	6·54 85	6·58 85·5	30·6
2·50 12·5	1·91 21	1·625 13	1·95 21·5	3·40 17	2·69 35	2·17 13	3·25 26	1·92 25	2·81 36·5	2·69 35	30·7
... ...	0·18 2	... ...	0·64 7	0·40 2	0·62 8	0·42 2·5	0·125 1	0·385 5	0·50 6·5	0·58 7·5	30·8
... ...	... ...	... ...	... ...	0·40 2	0·23 3	0·67 4	... ...	0·27 3·5	0·19 2·5	0·19 2·5	30·9
... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	0·08 1	0·04 0·5	31·0

Referred to its modal (or maximum) ordinate this frequency distribution is given by

$$y = y_0 (1 + x/a)^{\gamma a} e^{-\gamma x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(ii.)}$$

where

$$\gamma = 2\mu_2/\mu_3, \qquad a = 2\mu_2^{\,2}/\mu_3,$$

and if* $p = \gamma a$,

$$y_0 = \frac{\mathrm{N}}{\sqrt{2\pi\mu_2}} \frac{\sqrt{2\pi}(p+1)\, e^{-p} p^p}{\Gamma(p+1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(iii.)}$$

In order that this curve should fit a skew frequency distribution it is theoretically necessary that

$$6 + 3\beta_1 - 2\beta_2$$

should be zero, or at least that it should in statistical practice be small.

The theory of this curve is discussed, 'Phil. Trans.,' A, vol. 186, p. 373. It is shown in the same memoir that if

$$6 + 3\beta_1 - 2\beta_2 > 0$$

the generalised probability curve

$$y = y_0 (1 + x/a_1)^{m_1} (1 - x/a_2)^{m_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(iv.)}$$

will be found suitable, whereas if

$$6 + 3\beta_1 - 2\beta_2 < 0$$

the generalised probability curve

$$y = y_0 \frac{1}{\{1 + (x/a)^2\}^m} e^{-\nu \tan^{-1}(x/a)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(v.)}$$

will, in a great variety of cases, represent the frequency. Methods for determining the constants of these curves are fully described, and these methods have been adopted in the present paper.

Of the curves (ii.), (iv.), and (v.), it may be noted that (ii.) has a range of frequency limited in one direction, (iv.) a range limited in both directions, while (v.), like the normal distribution (i.), has a range unlimited in both directions.

When a frequency distribution can be satisfactorily described by a curve of form (ii.), then it is generally possible to obtain a skew binomial representing, with a fair degree of accuracy, this distribution.†

In the 'Phil. Trans.' memoir above referred to, some Cambridge barometric data are dealt with, and it is shown for this case:

(i.) That $6 + 3\beta_1 - 2\beta_2$ is > 0, and this barometric frequency therefore fits a curve of limited range.

* The mean ordinate is given by $y_1 = y_0 \left(\frac{p+1}{p}\right)^p \frac{1}{e}$.

† The rule is not general, just as it is sometimes, but not always, possible to represent a normal distribution by a symmetrical binomial.

(ii.) That this expression not being large, a curve like (ii.) above describes with an equally small percentage error (p. 383) the frequency distribution.

(iii.) That a point-binomial could be found to fit the observations also with a close degree of accuracy.

The discussion of the thirteen years of Cambridge observations was not therefore conclusive as to the best means of describing barometric frequency, and more than one set of constants seemed capable of performing this function with an equal degree of accuracy. It was sufficient to indicate that the generalised probability curve in one type or another was fully capable of supplementing the obvious and admitted inadequacy of the normal curve.

The first stage of the present investigation was accordingly an inquiry as to the best type of generalised probability curve for barometric frequency.

4. *On the Value of the Criterion* $6 + 3\beta_1 - 2\beta_2$ *for Divers Localities.*

TABLE II.

Place.	Number of observations.	β_1.	β_2.	$6 + 3\beta_1 - 2\beta_2$.
St. Leonards	2192	0·07123	3·04212	+·12944
Southampton	4748	0·12260	3·36529	−·36277
Babbacombe	4748	0·13110	3·34150	−·28970
Carmarthen	4748	0·12575	3·26479	−·15231
Churchstoke	4018	0·12578	3·18891	−·00049
Llandudno	2922	0·08777	3·11777	+·02778
Parsonstown	4748	0·15202	3·23306	−·01006
Markree Castle	3653	0·16380	2·98237	+·52664
Armagh	1826	0·20204	3·31567	−·02522
Londonderry	2557	0·13185	3·06986	+·25581
Stonyhurst	2557	0·10401	3·42101	−·53001
Scaleby	2192	0·15122	3·12878	+·19610
Glasgow	1826	0·18256	3·21205	+·12359
Laudale	4018	0·21973	3·19784	+·26350
Dunrobin Castle	2922	0·16714	3·16067	+·18008
Dundee	4018	0·17885	3·23219	+·07217
Scarborough	1826	0·13441	3·30247	−·20172
Hillington	4748	0·13942	3·26642	−·11458
Geldeston	2192	0·11654	3·27888	−·20815
Margate	2922	0·21562	3·50141	−·35596

This table shows us at once that there is a comparatively small range of values for the constants β_1 and β_2 for very diverse localities. The mean values of the constants, weighted with the number of years over which the observations extend, are as follows :—

$$\beta_1 = \cdot 14621, \quad \beta_2 = 3\cdot 24179, \quad 6 + 3\beta_1 - 2\beta_2 = - \cdot 04495.$$

It will thus be seen that the mean value of the criterion is small. Actually, it

would mark a frequency distribution corresponding to a curve like (v.), but for practical statistical purposes a curve like (ii.), which fits closely when the criterion is even as large as 0·38421 (see Plate 10, fig. 6, of 'Phil. Trans.' memoir cited above), will suffice to describe the distribution of barometric frequency. The standard deviations of β_1, β_2, and $6 + 3\beta_1 - 2\beta_2$ are 0·03673, 0·12022, and 0·2068; these give for the probable errors of the means of the same three quantities (duly weighted), 0·00568, 0·01860, and 0·03200 respectively. Thus the probable errors are about 3·9, 0·5, and 71 per cent. of the observed means. Thus, while it is exceedingly improbable that the mean values of β_1 and β_2 differ much from their observed values, it is very possible that the true mean value of the criterion is zero and not $-$ 0·04495.* The comparatively large and opposite values of the criterion at Markree Castle and Stonyhurst must, of course, be duly regarded, and a longer series of observations at these two stations may some day serve to indicate how far their barometric conditions are peculiar to local conditions of climate or of observation.

As a matter of fact, the curves corresponding to equations (iv.) or (v.) above were originally calculated for all the stations, and drawn upon the diagrams as well as those corresponding to (ii.), but the mean percentage error in frequency, when tested by a planimeter, showed no very sensible improvement. An examination of Plates 9–17, giving the graphical representation† of the observed frequency by the theoretical distribution (the negative direction of x being towards high barometer)

$$y = y_0 (1 + x/a)^p e^{-\gamma x},$$

amply demonstrates that this limit to the skew binomial suffices to satisfactorily describe barometric frequency. The closeness of the approximation is one rarely met with in the usual representation of variation by the normal curve of errors.

To illustrate how closely curve (ii.) corresponds to curve (v.), even with a value ($-$ ·28970) of the criterion relatively large compared with the majority of those with which we are dealing, we give in the following table the frequencies *per annum* for Babbacombe as observed, and as calculated from (ii.) and (v.) respectively :—‡

* The probable error of the criterion for skew variation will be discussed in a forthcoming memoir by one of the present authors, and it will there be seen that this probable error is frequently large.

† We have to thank Mr. C. JAKEMAN, Demonstrator in University College, for much aid in the preparation of our diagrams.

‡ Another instance of the same closeness is exhibited in the 'Phil. Trans.' memoir above cited for a curve like (iv.) with (ii.). *Cf.* Curves I. and II. of fig. 6, Plate 10. The criterion is here still larger, *e.g.*, 38·421.

Height in inches.	Frequency *per annum.*			Height in inches.	Frequency *per annum.*		
	Observed.	Calculated			Observed.	Calculated	
		From (ii.).	From (v.).			From (ii.).	From (v.).
28·6	0·19	0 07	0·09	29·9	41·27	40·99	40·99
28·7	0·42	0·14	0·17	30·0	45·92	44·35	44·37
28·8	0·38	0·29	0·32	30·1	46·96	43·93	43·64
28·9	0·46	0·58	0·60	30·2	38·88	39·21	38·66
29·0	0·69	1·09	1·09	30·3	30·19	31·21	30·16
29·1	1·88	1·99	1·94	30·4	18·23	21·79	20·75
29·2	2·81	3·50	3·34	30·5	13·65	13·08	12·40
29·3	6·04	5·86	5·58	30·6	7·31	6·61	6·38
29·4	9·31	9·31	8·91	30·7	3·12	2·76	2·80
29·5	15·15	14·21	13·67	30·8	0·38	0·87	1·04
29 6	18·62	20·41	19·82	30·9	0·31	0·21	0·32
29·7	26·85	27·56	27·11	31·0	0·08	0·03	0·09
29·8	36·12	34·94	34·64				

In view of the above remarks we shall confine our attention to curves of the form (ii.). We can now discuss the relations between the constants of this curve and the physical quantities associated with barometry.

5. *Constants of a Local Distribution of Barometric Frequency.*

It is convenient to term the height of the barometer, corresponding to the maximum frequency, the *mode*. This mode never coincides with the mean, and we shall represent them by M_o and M_e respectively. The divergence of the mode from the mean marks the skewness of the frequency distribution, but it is fitting to measure this divergence in terms of some standard of variation of the local distribution. It is convenient to select as this standard, σ, the standard deviation of the distribution, or $\sqrt{\mu_2}$. We shall represent the skewness of the frequency by *Sk.* Another interesting physical constant of the distribution is indicated by the theoretical law of frequency—this is the *maximum possibile* of barometric height in the locality. It is directly obtained by adding to the mode or mean the possible range above either. This maximum height we will indicate by H_p, while H_o shall indicate the maximum observed height.

The following relations hold between the constants of the theoretical distribution and the above physical quantities

$$\sigma = \sqrt{\mu_2} = \sqrt{p+1}/\gamma \quad \text{(i.)},$$

$$M_o - M_e = 1/\gamma = \tfrac{1}{2}\mu_3/\mu_2 = \tfrac{1}{2}\sigma\sqrt{\beta_1} \quad \text{(ii.)},$$

$$H_p - M_e = a' = 2\mu_2^2/\mu_3 = 2\sigma/\sqrt{\beta_1} = (p+1)/\gamma \quad \text{(iii.)},$$

$$Sk. = (M_o - M_e)/\sigma = \tfrac{1}{2}\mu_3/\sqrt{\mu_2^3} = \tfrac{1}{2}\sqrt{\beta_1} \quad \text{(iv.)}.$$

Table III. contains the values of the mean and the moments calculated directly from the observed frequencies. In determining the moments, the observation polygons were regarded as trapezia ('Phil. Trans.,' A, vol. 153, p. 350). Table IV. then gives the constants of the theoretical distribution and the physical constants of the frequency deduced from them. The constants y_o, p, γ and the mean M_e suffice to determine the form of the frequency distribution. But any other three constants would serve equally well. Thus, we might take the three physical constants, y_o the frequency of the modal height, σ the variation, and *Sk.* the skewness of the distribution. Or, we might use y_1 the frequency of the mean instead of the frequency of the mode. In a memoir, which one of us hopes to shortly publish,* the probable errors are worked out for the constants of any frequency curve, and it is shown that these errors are not, as in the case of a normal frequency distribution, uncorrelated. An error, for example, in γ, marks a correlated error in p and in the mean, while in the case of the normal frequency curve there is no correlation between its position (mean) and shape (standard deviation). It becomes accordingly of considerable importance to determine which are the quantities in barometric frequency which have the least percentage probable errors, and then to adopt these as our standard constants of barometric frequency.

TABLE III. (The unit of μ_2, μ_3, μ_4 is $\frac{1}{10}''$.)

	Station.	M_e.	μ_2.	μ_3.	μ_4.
Selected stations of the second order.	St. Leonards	29″·9834	10·1350	8·6111	312·4840
	Southampton	29·9814	10·8126	12·4493	393·4473
	Babbacombe	29·9787	10·9012	13·0321	397·0938
	Carmarthen	29·9518	12·0724	14·8749	475·8197
	Churchstoke	29·9545	12·6642	15·9835	511·4453
	Llandudno	29·9229	13·0338	13·6743	529·1235
	Parsonstown	29·9271	13·1640	18·6224	560·2627
	Markree Castle	29·8861	15·4049	24·4703	707·7482
	Armagh	29·9134	13·7001	22·7935	622·3322
	Londonderry	29·8905	14·7902	20·6538	666·3570
	Stonyhurst	29·9406	12·2702	13·8613	515·0601
	Scaleby	29·8862	14·0539	20·4879	617·9683
	Glasgow	29·8859	14·3712	23·2779	663·3914
	Laudale	29·8570	15·0354	27·3287	722·9168
	Dunrobin Castle	29·8457	14·3639	22·2559	652·1105
	Dundee	29·8695	14·7373	23·9262	701·9974
	Scarborough	29·9028	12·9644	17·1137	555·0735
	Hillington	29·9429	11·7452	15·0299	450·6021
	Geldeston	29·9476	11·1784	12·7585	409·7187
	Margate	29·9745	10·3937	15·5596	378·2518
Telegraph stations.	Dover-Dungeness	29·9558	10·3496	10·7772	—
	London	29·9660	10·8159	11·1621	—
	Cambridge	29·9524	11·4491	13·3671	—

* See PEARSON and FILON, "On the Probable Errors of Frequency Constants." (Abstract) 'Roy. Soc. Proc.,' vol. 62, p. 173.

TABLE IV.—Constants of the Frequency Distribution.

	Station.	Frequency of mean (y_1).	Frequency of mode (y_0).	a'.	p.	γ.	σ.	*Sk.*	M_e.	M_o. Calc.	M_o. Observ.	H_p.	H_a.	$M_o - M_e$.	$H_p - M_o$.
				"			"		"	"	" "	"	"	"	"
Selected stations of the second order.	St. Leonards	45·596	45·887	2·3857	55·1592	23·5395	0·3184	0·1334	29·9834	30·0259	30·0→30·1	32·3692	30·974	0·0425	2·3433
	Southampton	44·184	44·876	1·8782	31·6258	17·3706	0·3288	0·1751	29·9814	30·0390	30·1→30·0	31·8596	30·974	0·0576	1·8206
	Babbacombe	43·997	44·732	1·8238	29·5097	16·7298	0·3302	0·1810	29·9787	30·0385	30·1→30·0	31·8024	30·976	0·0598	1·7639
	Carmarthen	41·825	42·688	1·9596	30·8075	16·2319	0·3474	0·1773	29·9518	30·0134	30·1→30·0	31·9114	30·969	0·0616	1·8980
	Churchstoke	40·845	41·522	2·0068	30·8016	15·8466	0·3559	0·1773	29·9545	30·0176	30·1→30·0	31·9613	30·974	0·0631	1·9437
	Llandudno	40·546	40·996	2·4215	44·5717	18·8194	0·3587	0·1481	29·9229	29·9760	30·0→30·1	32·3444	30·926	0·0531	2·3684
	Parsonstown	40·032	40·930	1·8611	25·3121	14·1378	0·3628	0·1949	29·9271	29·9978	30·1→30·0	31·7882	30·949	0·0707	1·7904
	Markree Castle	37·005	37·783	1·9396	23·4206	12·5907	0·3925	0·2024	29·8861	29·9655	30·1→30·0	31·8257	30·890	0·0794	1·8602
	Armagh	39·190	40·163	1·6469	18·7976	12·0211	0·3701	0·2247	29·9134	29·9966	30·0→30·1	31·5603	30·750	0·0832	1·5637
	Londonderry	37·790	38·424	2·1183	29·3381	14·3221	0·3846	0·1816	29·8905	29·9603	30·0→29·9	32·0088	30·885	0·0698	2·0485
	Stonyhurst	41·616	42·166	2·1723	37·4596	17·7042	0·3503	0·1612	29·9406	29·9971	30·1→30·0	32·1129	30·870	0·0565	2·1158
	Scaleby	38·672	39·452	1·9281	25·4517	13·7192	0·3732	0·1944	29·8862	29·9591	29·9→30·0	31·8143	30·891	0·0729	1·8552
	Glasgow	38·332	39·278	1·7745	20·9105	12·2348	0·3791	0·2130	29·8859	29·9669	29·9→30·0	31·6604	30·710	0·0810	1·6935
	Laudale	37·398	38·494	1·6544	17·2041	11·0034	0·3878	0·2344	29·8570	29·9479	30·0→29·9	31·5114	30·796	0·0909	1·5635
	Dunrobin Castle	38·316	39·140	1·8541	22·9323	12·9079	0·3790	0·2044	29·8457	29·9232	30·0→29·9	31·6998	30·730	0·0775	1·7766
	Dundee	37·825	38·693	1·8155	21·3651	12·3190	0·3839	0·2115	29·8695	29·9507	30·1→29·9	31·6850	30·824	0·0812	1·7343
	Scarborough	40·342	40·910	1·9642	28·7600	15·1509	0·3601	0·1833	29·9028	29·9688	30·1→30·0	31·8671	30·869	0·0660	1·8983
	Hillington	41·652	42·882	1·8569	27·6901	15·6291	0·3427	0·1867	29·9429	30·0069	30·0→30·1	31·7998	30·930	0·0640	1·7929
	Geldeston	43·487	44·128	1·9588	33·3244	17·5231	0·3343	0·1707	29·9476	30·0047	30·0→30·1	31·9064	30·924	0·0571	1·9017
	Margate	44·997	46·212	1·3886	17·5512	13·3598	0·3224	0·2322	29·9745	30·0493	30·1→30·0	31·3631	30·770	0·07485	1·3138
	Mean	40·682	41·4675	1·9204	28·5996	15·1580	0·3581	0·1894	29·9221	29·9902	..	31·8425	30·879	0·0681	1·8523
	S.D	2·606	2·583	0·2310	9·0044	2·8927	0·0226	0·0257	0·0417	0·0338	..	0·0761	0·085	0·0118	0·2396
	P.E. of mean	0·393	0·390	0·0348	1·3581	0·4363	0·0034	0·0039	0·0063	0·0051	..	0·0115	0·0128	0·0018	0·0361
	Coefficient of variation	6·45	6·22	12·03	31·48	19·08	6·31	13·57	0·14	0·11	..	0·24	0·275	17·33	12·94
Supplementary stations.	Dover-Dungeness	45·182	45·789	1·9883	37·1792	19·2066	0·3217	0·1554	29·9558	30·0079	30·0→30·1	31·9441	30·94	0·0521	1·9362
	London	44·216	44·772	2·0961	39·6221	19·3798	0·3289	0.1569	29·9660	30·0176	30·0→30·1	32·0621	30·95	0·0516	2·0445
	Cambridge	42·839	43·487	1·9613	32·5973	17·1303	0·3384	0·1721	29·9524	30·0108	30·0→30·1	31·9137	30·96	0·0584	1·9029

The following are some of the formulæ given in the memoir above referred to:

Probable error of mean . $= \cdot 67449\sigma/\sqrt{n}$,

Probable error of γ . . $= \dfrac{\cdot 67449\gamma}{\sqrt{2n}}\sqrt{\dfrac{\frac{1}{2}+\mathrm{S}}{\mathrm{S}}}$,

Probable error of p . . $= \cdot 67449p/\sqrt{n\mathrm{S}}$,

Probable error of y_0 the modal frequency $= \dfrac{\cdot 67449y_0}{\sqrt{2n}}\sqrt{1+\dfrac{2\mathrm{T}^2}{\mathrm{S}}}$,

Probable error of y_1 the mean frequency $= \dfrac{\cdot 67449y_1}{\sqrt{2n}}\sqrt{1+\dfrac{2}{\mathrm{S}}\left(p\log\dfrac{p+1}{p}-\dfrac{p}{p+1}+\mathrm{T}\right)^2}$,

Probable error of σ . . $= \dfrac{\cdot 67449\sigma}{\sqrt{2n}}\sqrt{1+\frac{1}{2}\dfrac{1}{(p+1)^2\mathrm{S}}}$,

Probable error of skewness $= \dfrac{\cdot 67449}{2\sqrt{n}}\dfrac{p}{p+1}\dfrac{1}{\sqrt{(p+1)\mathrm{S}}}$,

Correlation of errors in p and γ . $= \sqrt{\dfrac{1}{1+2\mathrm{S}}}$,

Correlation of errors in p and mean $= 0$,

Correlation of errors in γ and mean $= \sqrt{\dfrac{2}{p+1}\mathrm{S}\Big/(\frac{1}{2}+\mathrm{S})}$.

Here S is the series

$$\frac{\mathrm{B}_1}{p}-\frac{\mathrm{B}_3}{p^3}+\frac{\mathrm{B}_5}{p^5}-\ldots$$

and T

$$\frac{\mathrm{B}_1}{2p}-\frac{\mathrm{B}_3}{4p^3}+\frac{\mathrm{B}_5}{6p^5}-\ldots$$

the B's being the BERNOULLI numbers.

Now a consideration of these formulæ gives the following results—which will be found to be fully verified by the Table V. of calculated values—

(*a*.) The errors in the mean, standard deviation and skewness are very small. The errors in both mean and modal frequencies are small, but the error in the modal frequency is considerably smaller than that in the mean frequency. This follows from the fact that the frequency curve is horizontal at the mode, but slopes at the mean, and accordingly any bodily shift of the curve produces far more effect on the mean than on the modal frequency.

(*b*.) The errors in p and γ are large in some cases very large. The question then arises how is it possible to determine the frequency curve correctly. The answer lies in the fact that p and γ are so highly correlated, that given a large error in p, there

TABLE V.—Probable Errors of the Chief Constants of the Frequency Distributions for the British Isles.

Station.	P.E. of mean.	P.E. of skew-ness.	P.E. of standard deviation.		P.E. of modal frequency.		P.E. of mean frequency.		P.E. of constant p.		P.E. of constant γ.		Corre-lation of errors in p and γ.	Corre-lation of errors in mean and γ.
			Gross.	Per cent.	Gross.	Per cent.	Gross.	Per cent.	Gross.	Per cent.	Gross.	Per cent.		
St. Leonards . .	0·0046	0·0172	0·0033	1·0451	0·4678	1·0195	0·4807	1·0542	14·4568	26·2092	3·0941	13·1441	0·9970	0·0146
Southampton . .	0·0032	0·0114	0·0024	0·7224	0·3110	0·6931	0·3236	0·7324	4·2648	13·4853	1·1774	6·7781	0·9948	0·0253
Babbacombe . .	0·0032	0·0114	0·0024	0·7243	0·3108	0·6931	0·3234	0·7350	3·8441	13·0265	1·0958	6·5499	0·9944	0·0271
Carmarthen . .	0·0034	0·0114	0·0025	0·7231	0·2959	0·6931	0·3067	0·7367	4·1004	13·3097	1·0860	6·6908	0·9946	0·0259
Churchstoke . .	0·0038	0·0124	0·0028	0·7860	0·3128	0·7534	0·3256	0·7973	4·4561	14·4670	1·1524	7·2725	0·9946	0·0259
Llandudno . . .	0·0045	0·0148	0·0033	0·9103	0·3620	0·8831	0·3730	0·9199	9·0954	20·4062	1·9273	10·2412	0·9963	0·0180
Parsonstown . .	0·0036	0·0113	0·0026	0·7291	0·2838	0·6933	0·2967	0·7412	3·0539	12·0650	0·8585	6·0721	0·9935	0·0314
Markree Castle .	0·0044	0·0129	0·0033	0·8343	0·2987	0·7905	0·3142	0·8490	3·0989	13·2313	0·8389	6·6626	0·9930	0·0339
Armagh . . .	0·0058	0·0179	0·0044	1·1938	0·4493	1·1186	0·4774	1·2181	3·1519	16·7677	1·0167	8·4578	0·9912	0·0419
Londonderry . .	0·0051	0·0155	0·0038	0·9873	0·3629	0·9445	0·3786	1·0019	5·1926	17·6991	1·1856	8·8997	0·9944	0·0272
Stonyhurst. . .	0·0047	0·0157	0·0034	0·9784	0·3981	0·9442	0·4121	0·9903	7·4914	19·9985	1·7782	10·0436	0·9956	0·0214
Scaleby	0·0054	0·0167	0·0040	1·0728	0·4026	1·0203	0·4217	1·0906	4·5318	17·8056	1·2294	8·9609	0·9935	0·0313
Glasgow . . .	0·0060	0·0180	0·0045	1·1868	0·4393	1·1183	0·4636	1·2094	3·6978	17·6841	1·0904	8·9122	0·9921	0·0378
Laudale	0·0041	0·0120	0·0031	0·8089	0·2903	0·7542	0·3090	0·8263	1·8505	10·8146	0·6007	5·4594	0·9905	0·0457
Dunrobin Castle .	0·0047	0·0143	0·0035	0·9338	0·3460	0·8839	0·3642	0·9504	3·3571	14·6392	0·9516	7·3726	0·9928	0·0346
Dundee	0·0041	0·0122	0·0031	0·7992	0·2919	0·7543	0·3079	0·8141	2·5745	12·0502	0·7480	6·0719	0·9923	0·0371
Scarborough . .	0·0057	0·0184	0·0042	1·1692	0·4573	1·1177	0·4788	1·1868	5·9640	20·7371	1·5800	10·4284	0·9943	0·0277
Hillington . . .	0·0034	0·0114	0·0025	0·7263	0·2973	0·6932	0·3072	0·7375	3·4941	12·6187	0·9920	6·3472	0·9940	0·0288
Geldeston . . .	0·0048	0·0169	0·0035	1·0610	0·4501	1·0200	0·4676	1·0753	6·7891	20·3728	1·7939	10·2372	0·9950	0·0240
Margate . . .	0·0040	0·0141	0·0031	0·9474	0·4087	0·8844	0·4352	0·9676	2·2481	12·8087	0·8637	6·4649	0·9906	0·0448
Mean . .	0·0044	0·0143	0·0033	0·8670	0·3618	0·8736	0·3784	0·9317	4·8357	16·0098	1·2530	8·0534	0·9937	0·0302
Dover-Dungeness	0·0031	0·0115	0·0023	0·7182	0·3173	0·6929	0·3285	0·7270	5·4360	14·6210	1·4104	7·3432	0·9956	0·0216
London	0·0032	0·0115	0·0024	0·7167	0·3102	0·6929	0·3206	0·7250	5·9804	15·0936	1·4687	7·5785	0·9958	0·0203
Cambridge . . .	0·0033	0·0115	0·0024	0·7215	0·3014	0·6930	0·3133	0·7314	4·4628	13·6907	1·1786	6·8803	0·9949	0·0245

is little variation possible in γ; it also has a large *correlated* error, and these errors tend very closely to balance each other, *i.e.*, to retain the same shape for the frequency curve.*

Indeed it can be shown, both theoretically and empirically, that very considerable changes may be made in the constant p, and the frequency curve will not sensibly change its shape provided the correlated error be made in γ.

But these results show us at once that while the frequency curves as determined from p and γ may fit—as indeed they do—the observations very accurately, still p and γ are not good constants on which to base (when treated separately) any discussion of the relative distribution of barometric frequency with geographical position. The best constants are, as we might hope they would be, the physical constants, namely the mean or modal height, the mean or modal frequency, the standard deviation or variability, and the skewness. It is accordingly to these constants, as the fundamental constants, that we have specially directed our attention. All the other quantities involved can be expressed in terms of them. If q be written for the skewness *Sk.* we have:

$$p = \frac{1}{q^2} - 1, \quad \mathrm{M}_o - \mathrm{M}_e = \sigma q,$$

$$\gamma = \frac{1}{\sigma q}, \qquad \mathrm{H}_p - \mathrm{M}_e = \sigma/q.$$

In Table V. the whole system of probable errors and of correlation between errors is given for the 23 stations, and an inspection of them will show at a glance the degree of accuracy which may be expected from any number of years of barometric observation. Curves will be found drawn on Plate 9, giving the probable percentage errors of the modal frequency, and the standard-deviation together with the probable errors of the mean and the skewness for any number of years up to 20, for a station having the mean values of σ and p. They will, we believe, be found of service by the practical meteorologist for estimating how closely any series of observations will suffice to determine the theoretical frequency of the station. It is true that these errors are not merely functions of the number of years of observation; they depend also upon the local values of p and σ, as an inspection of Table V. will illustrate. Compare, for example, Southampton with Babbacombe, or Armagh with Glasgow. The deviations of the probable errors from the graphical values owing to the variation in p and σ may, however, be looked upon as a second order deviation, and a reasonable, if rough, appreciation of the magnitude of the errors made may be obtained from Plate 9.

It will be seen from the Table V. as well as from Plate 9, that the value of the *physical* frequency constants are, for the years under consideration, given by the calculated values with a very close degree of accuracy.

* It should be noticed that the term involving p in the frequency, *i.e.*, $\left(1 + \frac{\gamma x}{p}\right)^p$, becomes more and more independent of p as p increases.

6. *On the Standard Barometric Frequency Curve for the British Isles.*

In order to reach a fair appreciation of the manner in which the distributions of frequency differ at any two local stations, it is desirable to have a standard frequency curve with which either of them may be compared. This curve was obtained in the following manner. The mean values of y_1, a', p and γ were found; they are recorded with their standard deviations, probable errors and coefficients of variation below the "selected stations" in Table IV. Naturally these values did not satisfy the relation $(p + 1) = \gamma a'$, which is necessary if the curve is to be of the required type. Accordingly small alterations δp, $\delta\gamma$, $\delta a'$ were made in their values, proportional in each case to the corresponding probable error of p, γ, a', so that the relation

$$(p + \delta p + 1) = (\gamma + \delta\gamma)(a' + \delta a')$$

was satisfied. The alterations in no case amount to 1 per cent. of the corresponding value, being 0·179 of the probable errors of those values. The curve thus determined has for its equation, the unit of y being a day, and of x one-tenth inch :—

$$y = 40{\cdot}682\left(1 + \frac{x}{1{\cdot}9267}\right)^{28{\cdot}3556} e^{-15{\cdot}2364x},$$

the origin being at the mean height 29″·9221. This curve we shall in future speak of as the Standard Frequency Curve for the British Isles. It may be at first looked upon as an arbitrary curve, artificially obtained with a view to having some standard of comparison, but when it has once been plotted on the diagrams of the barometric frequency, we can give the standard curve a physical meaning. On examining Plate 15, fig. xvii.; Plate 11, fig. vi. and Plate 12, fig. vii., it will be at once seen that the Standard Frequency Curve expresses the barometric frequency of places on a line running somewhat south of Scarborough, a trifle south of Llandudno, and slightly north of Parsonstown. Approximately we may state that the climate of Llandudno very nearly represents the standard of barometric frequency for the British Isles. Probably there would be hardly any sensible deviation at all between the standard frequency curve and the distribution of barometric frequency for places like Hull and Chester.

A comparison of the mean frequency contour as sketched in on the map, page 424, with Plate 13 of the 'Meteorological Atlas of the British Isles,' issued in 1883 by the Meteorological Office, shows that this contour is not very divergent from the isobar for 29″·90; it lies sensibly above the isobar given in that plate for 29″·92. The important suggestion now made is that a series of contour lines, generalised isobars, could be constructed, along which not only the mean barometric height would be the same, but practically all the constants which determine the distribution of barometric frequency. Our data are very far from sufficient to enable us to draw such a series satisfactorily, but an examination of Plate 10 will show that another such

generalised isobar passes very close to St. Leonards, Southampton and Babbacombe, where in each case the fitted distribution diverges almost to the same extent from the standard ; Plate 13, figs. x. and xii., and Plate 14, fig. xiii. shows Londonderry and some place between Glasgow and Scaleby marking a third, while Laudale, and a place about midway between Dunrobin and Dundee give a fourth ; lastly, Geldeston and Carmarthen mark a fairly satisfactory fifth generalised isobar. These isobars are roughly sketched in on the map (page 424) to indicate the general grouping of stations with approximately identical frequency distributions. The accurate determination of these isobars is a problem for the future ; it will require a knowledge of the frequency at a much larger number of stations and a delicate process of interpolation.

Turning now to the diagrams on Plates 10–17, the reader will find the standard frequency curve marked in strokes and dots, and the local frequency curve given by a broken line. Comparing these two curves, and neglecting, for the time, the irregular observation-polygon on which the latter is based, a continuous and regular change in the divergence of these two will be observed as the diagrams are taken in succession round, say, the coast line of England and Scotland. There are slight local deviations from uniformity, but on the whole there can be no doubt that the distribution of barometric frequency is a perfectly uniform and continuous phenomenon over the district treated. A fairly accurate distribution for any station not included among the twenty dealt with could be obtained by graphical interpolation from the generalised isobars we have roughly sketched.

7. *On the Modal Height of the Barometer.*

Attention has already been drawn by FECHNER, MAZELLE, H. MEYER in a series of papers to the importance of modal heights, which they term *Scheitelwerthe.** It seems, however, impossible to accurately determine the modal heights, even if the observations be grouped in very small ranges, until a theory of skew frequency for the barometer has been adopted. A glance at the diagrams for Churchstoke, Carmarthen, Dunrobin, &c., will sufficiently illustrate how delusive is the peak of the observation polygon. The true position of the mode depends, like that of the mean, on the *whole* series of observations, and not on the observational maximum only, which must always be largely the result of the grouping selected, and the elementary range taken as basis of the grouping. In Table IV., under the heading M_0, observed value, will be found nearly all our observation polygon by itself could tell us of the modal height. We should be able to determine the nearest tenth to

* See FECHNER: "Ueber den Ausgangswerth der kleinsten Abweichungssumme," &c. 'Abhandlungen der math.-phys. Classe der k. Sächsischen Gesells. der Wissenschaften,' vol. 11, No. 1, 1874; MAZELLE: 'Wiener Denkschriften,' vol. 60, p. 433, 1893, vol. 62, p. 57, 1895; dealing with air temperatures; H. MEYER: 'Anleitung zur Bearbeitung meteorologischer Beobachtungen,' Berlin, 1891, pp. 12–27; but compare JUL. HANN: 'Die Klimatologie,' 2te Ausgabe, Einleitung.

the mode and to say on which side of this tenth the mode lies. Thus in Table IV., for example, Glasgow $29''\cdot9 \twoheadrightarrow 30''$, means that the Glasgow modal height has $29''\cdot9$ for its nearest tenth, but lies on the $30''$ side of this.

A closer approximation to the position of the mode may be obtained by dealing with the three chief frequencies and finding the vertex (i.) of a parabola with vertical axis passing through their tops, or (ii.) of a normal curve of errors with vertical axis passing through the same three points, the latter, according to our experience, giving the better approximation to the true mode.

Let c be the unit of grouping, let y_2 be the maximum frequency, and y_1 and y_3 the frequencies on either side of it. Let z be the distance of the mode from y_2 towards y_3, then we have

(i.) For the parabola :

$$z = c\frac{\Delta_{21} + \Delta_{32}}{2(\Delta_{21} - \Delta_{32})}, \text{ where } \Delta_{rs} = y_r - y_s.$$

(ii.) For the normal curve :

$$z = c\frac{\delta_{21} + \delta_{32}}{2(\delta_{21} - \delta_{32})}, \text{ where } \delta_{rs} = \log y_r - \log y_s.$$

A third method, which is generally far more accurate, as it depends on all the observations, has been given in the memoir on skew variation in the 'Phil. Trans.' already cited (see pp. 375–6). This depends upon the principle that the distance of the mode from the mean is, with a close degree of approximation, thrice the distance of the median* from mean. It may be as well to illustrate these methods on an actual example.

Modal Height of the Barometer at Southampton.

(i.) By inspection of observation polygon	$30''\cdot1 \twoheadrightarrow 30''\cdot0$
(ii.) By using a parabola through three ordinates . . .	$30''\cdot0625$
(iii.) By using a normal curve through three ordinates .	$30''\cdot0615$
(iv.) By the principle of the modal third, as above . .	$30''\cdot0372$
(v.) By actual determination of the frequency curve. .	$30''\cdot0390$

By adding up the frequencies for Southampton in Table I., and then interpolating, it will be found that the median height of the barometer there is almost exactly $30''$. But by Table III., the mean height is $29''\cdot9814$, the third of the distance accordingly between mean and mode or the modal third $= 0\cdot0186$, whence we obtain $30''\cdot0372$ for the mode. It is clear that this method gives a close approximation to the true result, and is one which can be used by any ordinary observer. The value obtained will be,

* The median height of the barometer is the height given by that observation out of $2n + 1$ observations, which has the heights of n observations less and the heights of n observations greater than its own height, *i.e.*, it is the middle height of the series of observations arranged in order of magnitude.

as a rule, remarkably closer to the true mode than any application of methods such as (ii.) or (iii.).

If we invert the process and calculate from Table IV. the median values at our three southern stations, we find :—

Median,	St. Leonards.	29″·9976
,,	Southampton.	30″·0006
,,	Babbacombe	29″·9986

Thus we see that the median height of the barometer along the south coast of England approaches extremely closely to the 30″, so commonly adopted by physicists as a measure of the "standard atmosphere."

There is another convenient method of looking at this standard atmosphere of 30″. If we turn to the modal heights in Table IV., we notice that the mean modal height for all our stations is 29″·9902, with the very small probable error of 0·0051. Hence the mean modal height for the British Isles differs by less than twice its probable error from the customary standard atmosphere. On the other hand, the mean mean height differs by more than 12 times its probable error from the standard atmosphere. We may accordingly look upon the standard atmosphere of 30″, either as corresponding very closely to the mean modal height of the barometer for the British Isles, or as representing the median height of the barometer along the English southern coast.

Another interesting feature of the modal height is that it is less variable than the mean as we pass from station to station, and the probable error in the determination of the mean of the modes is accordingly less than that of the mean of the means. Whereas up and down the British Isles we find a coefficient of variation of 0·14 per cent. for the mean barometric height, the corresponding quantity is only 0·11 per cent. for the modes. On this account, and because the mode—as the most frequent barometric height—has a more direct physical interpretation than the mean, it seems to us that a record of local modes would be of greater significance than a record of local means.

Owing to the property we have already noticed, *i.e.*, that the distribution of barometric frequency is constant along contour lines, differing, at any rate, not very widely from the isobars in the narrower sense of the word, it follows that mean and mode oscillate, although not without deviations, in general accordance. Thus, both mean and mode are least at Laudale and Dunrobin Castle, the most northerly stations we have dealt with ; both means and modes are greatest at the four southernmost stations, Margate, St. Leonards, Southampton, and Babbacombe, although it is characteristic of some peculiarity of observation or climatological individuality, that while out of the four the means are greatest at St. Leonards and Southampton, the modes are greatest at Southampton and Margate. In particular we look upon the modal value at Margate (30″·0493) as standing geographically between Geldeston (30″·0047) and St. Leonards

(3″·0259) as not beyond suspicion, and accordingly open to revision when a wider range of data is available. The great value of the "skewness" in the Margate distribution is also unsatisfactory.

8. *On the Variability of the Barometric Height.*

The only method hitherto used by meteorologists to express briefly the variability of the atmospheric pressure is, so far as we are aware, the statement of the maximum and minimum heights reached during any given period. The fallacy of this method has been illustrated by one of us elsewhere.* It gives no real impression whatever of the manner in which the *bulk* of the variation is distributed, yet, for most climatological purposes, this is precisely what we require. Judged by such a test as this (namely, the range from maximum to minimum height observed) Hillington has a more variable climate than Scarborough, and Southampton than Babbacombe, but, as a matter of fact, Hillington is considerably less variable than Scarborough, and Southampton is slightly less variable than Babbacombe.

Another striking illustration of the defects of this method of measuring the variability has been mentioned to us by Mr. R. H. Scott, namely, that in 23 years of barometric observations at Valencia, the maximum was only reached in the *last* year.

Whatever be the form of the frequency distribution, the problem of determining how the bulk of the variability is distributed about either mean or mode, is exactly similar in character to the problem of determining how the inertia of a plate is distributed about any axis in its plane. One satisfactory and useful measure in both cases is the swing-radius, or radius of gyration. This is the quantity which, for distribution about the mean, appears under the heading σ the standard deviation in our Table IV.

Judged by this test the following is the order of variability in barometric pressure at our 20 stations :—

(1).	St. Leonards	[1].	(11).	Scarborough	[13].
(2).	Margate	[4].	(12).	Parsonstown	[11].
(3).	Southampton	[2].	(13).	Armagh	[12].
(4).	Babbacombe	[3].	(14).	Scaleby	[15].
(5).	Geldeston	[7].	(15).	Dunrobin Castle	[20].
(6).	Hillington	[8].	(16).	Glasgow	[16].
(7).	Carmarthen	[6].	(17).	Dundee	[18].
(8).	Stonyhurst	[9].	(18).	Londonderry	[14].
(9).	Churchstoke	[5].	(19).	Laudale	[19].
(10).	Llandudno	[10].	(20).	Markree Castle	[17].

* In an essay on "Variation in Man and Woman;" see 'The Chances of Death, and other Studies in Evolution,' vol. 1, p. 275.

In square brackets we have inserted the order of mean barometric pressures. It will be seen at once that the 10 stations of least variability are the 10 stations of highest pressure. Thus, there is a correlation between high pressure and small variability.* Some changes in the two orders may well be due to the doubt which attaches to the reduction to sea level, but taken as a whole the list illustrates the local character of the climate at the various stations, so far as it depends upon the height and variability in height of the barometer. This method of appreciating variability seems to us more satisfactory than a mere measurement of maximum to minimum ranges, which, with our data, while leaving St. Leonards first for steadiness of climate, would place Geldeston quite close to it, and make both that town and Scarborough superior to Southampton!

Instead of taking the variability about the mean, we might equally well have taken it about the mode, the only difference being that we should now have to calculate $\sqrt{p+2}/\gamma$ instead of $\sqrt{p+1}/\gamma$; see p. 433, Equations (i.) and (ii.). The comparatively large values of p, however, do not allow of any widely divergent differences in the results. The more interesting problem of the variabilities in excess and defect, which, owing to the skewness of barometric frequency curves, are not the same, will be dealt with in the next section.

9. *On the Skewness of Barometric Frequency.*

The comparative closeness of the mean to the mode enables us to easily find a formula for the probability that the barometric height in any locality shall be in excess or defect of the modal height. Using the property of the modal third we have to integrate

$$y = y_0\left(1+\frac{\gamma x}{p}\right)^p e^{-\gamma x} = y_0\, e^{-\frac{1}{2}p\left(\frac{\gamma x}{p}\right)^2 + \frac{1}{3}p\left(\frac{\gamma x}{p}\right)^3 - }$$

from 0 to $\dfrac{2}{3\gamma}$.

Hence the area as far as terms of the order $1/p^2$

$$= \frac{2}{3}\frac{y_0}{\gamma}\left(1 - \frac{2}{27}\frac{1}{p} + \frac{4}{135}\frac{1}{p^2} + \ldots\right).$$

Thus the total area on the mean side of the mode

$$= 0{\cdot}5\mathrm{N} + \frac{2}{3}\frac{y_0}{\gamma}\left(1 - \frac{2}{27}\frac{1}{p} + \frac{4}{135}\frac{1}{p^2}\right), \text{ nearly,}$$

$$= \mathrm{N}\left\{0{\cdot}5 + \frac{2}{3}\frac{p^p}{e^p\Gamma(p+1)}\left(1 - \frac{2}{27}\frac{1}{p} + \frac{4}{135}\frac{1}{p^2}\right)\right\},$$

[* It has been shown for this skew curve (PEARSON and FILON, 'Roy. Soc. Proc.,' vol. 62, p. 175) that the mean is *negatively* correlated with the standard deviation. Thus we have a theoretical indication that high pressure is correlated with small variability. The actual correlation for the mean value of p is ·25, approximately, this being for a random variation from the standard curve.]

$$= \mathrm{N}\left\{0\cdot5 + \frac{2}{3}\left(\frac{1}{2\pi p}\right)^{\frac{1}{2}}\left(1 + \frac{1}{12p} + \frac{1}{288p^2}\right)^{-1}\left(1 - \frac{2}{27}\frac{1}{p} + \frac{4}{135}\frac{1}{p^2}\right)\right\},$$

$$= \mathrm{N}\left\{0\cdot5 + \frac{2}{3}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{p+1}}\left(1 + \frac{37}{108\,(p+1)} + \frac{2309}{12960}\frac{1}{(p+1)^2}\right)\right\},$$

$$= \mathrm{N}\{0\cdot5 + \cdot26596\,\mathrm{S}_k(1 + \cdot3426\,\mathrm{S}_k^2 + 0\cdot1782\,\mathrm{S}_k^4)\},$$

where S_k is the skewness recorded in Table IV.

Since the skewness is in our case small, it is sufficiently close for barometry to take the chance of a reading of the barometer being less than the mode, as:

$$0\cdot5 + 0\cdot266\,\mathrm{S}_k,$$

and greater than the mode, as

$$0\cdot5 - 0\cdot266\,\mathrm{S}_k.$$

The corresponding probabilities for the barometric height less and greater than the mean will be found to be:

$0\cdot5 - 0\cdot133\,\mathrm{S}_k$ and $0\cdot5 + 0\cdot133\,\mathrm{S}_k$ respectively.

The following is the table of stations arranged according to their respective chances of the barometer exceeding its modal and mean heights:—

TABLE VI.

Station.	Skewness.	Chance of Height being in excess of	
		Mode.	Mean.
St. Leonards	0·1334	0·465	0·518
Llandudno	0·1481	0·463	0·519
Stonyhurst	0·1612	0·457	0·521
Geldeston	0·1707	0·455	0·523
Southampton	0·1751	0·453	0·523
Carmarthen	0·1773	0·453	0·524
Churchstoke	0·1773	0·453	0·524
Babbacombe	0·1810	0·452	0·524
Londonderry	0·1816	0·452	0·524
Scarborough	0·1833	0·451	0·524
Hillington	0·1867	0·450	0 525
Scaleby	0·1944	0·448	0·526
Parsonstown	0 1949	0·448	0·526
Markree Castle	0·2024	0·446	0·527
Dunrobin Castle	0·2044	0·446	0·527
Dundee	0·2115	0·444	0·528
Glasgow	0·2130	0·443	0.528
Armagh	0·2247	0·440	0·530
Margate	0·2322	0·438	0·531
Laudale	0·2344	0·438	0·531

This list would present in general the same features, as we have already noted—of a continuous change, distributed in contour lines running a little north-east to south-west of the parallels of latitude—were it not for the anomalous positions of Llandudno, Stonyhurst, and Margate. These stations appear to us, especially the last, to have anomalies in the values of their constants, which can hardly be entirely due to local peculiarities in climate.

It will be observed that the mode is here again more suitable than the mean as a method of recording high barometer. It might, if the point were only superficially considered, be deemed a climatological advantage to have the frequency of the barometer above its mean value as great as possible. But this is not really so, for the simple reason that climates which have an extreme range of low barometer have a low mean, and, other things being equal, places with low mean have most frequency above the mean. On the other hand, places with high means, as a rule, give the greatest frequency above the mode. Of course, this relationship is not invariable; it follows from the mode being more steady than the mean. Thus, there is a greater chance of the barometer standing above the mode in St. Leonards than Laudale, but a less chance of its standing above the mean.

The inequality of the frequencies above and below the mode is not the only point of interest connected with the skewness of the barometric frequency-curves. We have already noticed that the standard deviation about the mean is a good measure of the local variability of the barometer, and may well be used to replace the maximum to minimum range.* But a further question arises owing to the skewness, namely, what is the variability above, and what is the variability below, the modal height? These two will not be equal, and an appreciation of their value is of considerable importance.

Some idea of the range above the mode may be obtained by considering the columns marked H_p, H_a and $H_p - M_o$ in Table IV., which give the theoretical maximum height, the observed maximum height and the total theoretical range above the mode. The disadvantage of using H_a has already been referred to; it may make the range above the mode depend on a single observation in the last year of the whole series. H_p is calculated from the whole sweep of observations, and hence, although it may never be reached in actuality, it is a far better measure of range.

The order of the stations is given in the following diagram :—

* An arbitrary multiple of the standard deviation, σ, may be conveniently taken as the range, if required. Thus 6σ practically covers the whole range of barometric-frequency at any station.

Range above the mode according to

$H_p - M_o$	H_p	H_a
Llandudno	St. Leonards	Babbacombe
St. Leonards	Llandudno	St. Leonards
Stonyhurst	Stonyhurst	Southampton
Londonderry	Londonderry	Churchstoke
Churchstoke	Churchstoke	Carmarthen
Geldeston	Carmarthen	Parsonstown
Scarborough	Geldeston	Hillington
Carmarthen	Scarborough	Llandudno
Markree Castle	Southampton	Geldeston
Scaleby	Markree Castle	Scaleby
Southampton	Scaleby	Markree Castle
Hillington	Babbacombe	Londonderry
Parsonstown	Hillington	Stonyhurst
Dunrobin	Parsonstown	Scarborough
Babbacombe	Dunrobin	Dundee
Dundee	Dundee	Laudale
Glasgow	Glasgow	Margate
Armagh	Armagh	Armagh
Laudale	Laudale	Dunrobin
Margate	Margate	Glasgow

An examination of these lists seems to show that, even allowing for some disturbances in the Stonyhurst, Llandudno, and Margate returns, there is no geographical fact closely represented by range above the mode thus measured. There are but few and small changes introduced into the order of stations, whether we consider $H_p - M_o$ or H_p only. But H_a differs so widely from H_p that we must consider one or other of them as of little value in the determination of the nature of the frequency above the mode. In general the range above the mode dealt with in this manner appears to be more closely correlated with local conditions than with geographical position.

It would undoubtedly be most satisfactory in order to appreciate the skewness of the range of frequency to calculate the values of the standard deviation from the mode for the two portions of the frequency curve, which fall respectively above and below the mean. The extremely slow convergence, however, of the series which express incomplete Γ-functions, renders this calculation extremely tedious, and such mechanical methods as AMSLER's Integrator do not, in our experience, give very good results,* when the curves for which the second moment is to be found have, as is the case with nearly all frequency-curves, considerable "tails."

Fortunately, Mr. DE FORREST, in a paper published in the 'Analyst' (vol. 10, p. 69; Iowa, 1883), has found for a series of values of p the probable deviations in excess and defect of the mode, *i.e.*, the values of x on either side of the mode for which the corresponding verticals, y, cut off the half areas. Unfortunately, although he has interpolated for a considerable number of values of p, his values of the probable errors are only calculated for the small series $p = 4, 5, 7, 10, 20, 50$, and 200, doubtless on account of the great amount of arithmetic involved. Accordingly his table is only as strong as these seven values, which are as follows :—

DE FORREST's *Table of Probable Deviations.*

e_1 = probable deviation in excess of mode, *i.e.*, along negative x.
e_2 = ,, ,, defect ,, ,, positive ,,

p.	$-\gamma e_1$.	γe_2.
4	0·822	1·613
5	0·955	1·788
7	1·289	2·085
10	1·654	2·450
20	2·561	3·359
50	4·334	5·133
200	9·121	9·920

The completion of the Table would undoubtedly be a useful, if laborious, piece of work.

We have calculated the values of e_1 and e_2 for the twenty stations by interpolation

[* An AMSLER's Integrator, especially constructed for me, to determine μ_1, μ_2, and μ_3 is fairly satisfactory for "oval" sections; it gives a passable value of μ_1, but is not accurate enough to give working values of μ_2 and μ_3 for "tailed" areas.—K.P.]

from this Table. The results will, of course, only be correct to a corresponding degree of accuracy. The positive sign is given to e_1.

TABLE VII.—Skewness in Variability and Range. Unit, one inch.

Station.	e_1.	e_2.	$e_1 + e_2$.	σ.
St. Leonards	0·191	0·225	0·416	0·318
Southampton	0·187	0·233	0·420	0·329
Babbacombe	0·187	0·234	0·421	0·330
Carmarthen	0·197	0·246	0·443	0·347
Churchstoke	0·202	0·252	0·454	0·356
Llandudno	0·213	0·256	0·469	0·359
Parsonstown	0·203	0·260	0·463	0·363
Markree Castle	0·219	0·283	0·502	0·392
Armagh	0·204	0·270	0·474	0·370
Londonderry	0·217	0·273	0·490	0·384
Stonyhurst	0·207	0·248	0·455	0·350
Scaleby	0·210	0·268	0·478	0·373
Glasgow	0·214	0·279	0·493	0·379
Laudale	0·210	0·282	0·492	0·388
Dunrobin Castle	0·212	0·274	0·486	0·379
Dundee	0·214	0·279	0·493	0·384
Scarborough	0·203	0·256	0·459	0·360
Hillington	0·193	0·244	0·437	0·343
Geldeston	0·191	0·237	0·428	0·334
Margate	0·175	0·235	0·410	0·332
British Isles	0·200	0·253	0·453	0·356

It will be found that the following empirical formulæ give the values of e_1 and e_2 with an accuracy quite as great as that of their determination by interpolation from DE FORREST's table:—

$$e_1 = \sigma\,(0\cdot6520 - 0\cdot4728\ \mathrm{S}_k),$$
$$e_2 = \sigma\,(0\cdot6488 + 0\cdot3343\ \mathrm{S}_k).$$

These formulæ must, of course, only be applied with caution beyond the British Isles, and still less to other problems in skew frequency, when the skewness does not fall within the range of barometric skewness considered in this paper. Within this range, however, they give remarkably good results.

The above formulæ, or the table, show at once that e_1 and e_2 are quantities which follow the system of generalised isobars, and thus $e_1 + e_2$ and e_1, e_2, are good measures of the range and its skewness. It is, accordingly, these quantities which ought to be calculated for the purpose of obtaining an appreciation of the range above and below the mode at any station.

For example:—At St. Leonards half the frequency of the barometer falls into a range of ·416″, namely, from 29″·801 to 30″·217, *i.e.*, the mode being 30″·026, from 30″·026 − e_2 to 30″·026 + e_1. Further, the relative scattering of the frequency above and below the mode is given by the ratio of $e_1 : e_2$ or 0·191 : 0·225. At Markree Castle, on the other hand, it requires more than half-an-inch to cover half the frequency, *i.e.*, from 29″·6825 to 20″·1845, and the ratio of 0·219 : 0·283 gives the relative ranges of half the frequencies above and below the mode.

The comparatively small amount of labour necessary to determine Sk and σ,—

in fact, *Sk* may, by the rule of the modal third be determined from the median and the mean—and, thence, e_1 and e_2 by the above formulæ, make this a very convenient, as well as scientifically accurate, method of appreciating the range and the skewness of the variability at any station.

Our discussion has now led us to the following general conclusions :—

(1.) The mode, the standard deviation, and the skewness fully define barometric frequency. These three constants depend, in the first place, on geographical position, and appear to be constant along certain lines—the generalised isobars.

(2.) A knowledge of these three constants enables us, by means of very simple formulæ, to describe the chief physical features of the barometric frequency at any station.

(3.) By aid of the twenty stations dealt with in this paper, a fair appreciation can be obtained of the barometric frequency at any place whatever in the British Isles by means of interpolation.

For example :—A line from Hillington on the Wash to a point midway between St. Leonards and Southampton strikes the south coast between Littlehampton and Worthing, cuts the generalised isobars so that they make approximately equal angles with it, and passes very nearly at one-third and two-thirds distances through Cambridge and London. Thus, if the St. Leonards constants were based on a longer period and so somewhat more satisfactory,* we might fairly accurately predict the constants of the Cambridge and London frequencies from those of Hillington, Southampton, and St. Leonards. We find, as a matter of fact, by interpolation :

	Mean height.	Standard deviation.	Skewness.
	"	"	
London	29·969	0·330	0·165
Cambridge	29·956	0·336	0·176

Actual calculation for thirteen years at these stations corresponding to the years used for Southampton and Hillington gives :—

	Mean height.	Standard deviation.	Skewness.
	"	"	
London	29·966	0·329	0·157
Cambridge	29·952	0·338	0·172

* An attempt was made to replace the St. Leonards returns by the same thirteen years as have been dealt with for the other stations recorded at a suitable "telegraph station." Unfortunately, during these thirteen years the most suitable station, namely, that at Dover, was changed to Dungeness—a station some considerable distance off and subject to probably somewhat different conditions. A short missing period between the two sets of observations was most kindly supplied by Mr. R. H. SCOTT, by interpolation from the Meterological Office Records. The result of the calculations showed a considerable increase of variation, probably due to the combination of two stations, and it was very doubtful whether the result was of greater weight than the six years returns from St. Leonards.

These values are not widely divergent from the previous interpolated values, and would for many climatological purposes suffice to describe the barometric frequency; indeed, graphically it is hardly possible to show the difference between the frequencies curves corresponding to these two sets of constants on the scale of our diagrams. The chief difference is in the skewness of the London distribution, but as we have seen in the case of Llandudno, Stonyhurst and Margate, it is the skewness which is most influenced by local conditions. The constants and the observed and theoretical frequencies for the three telegraph stations London, Cambridge and Dover-Dungeness, are given in the form of supplements to our tables. They are of very considerable interest, but they have not been included in the general returns based on the selected distribution of twenty stations of the second order, as they would weight too much the eastern side of the British Isles unless an additional series of western stations had also been included.*

II.—On the Correlation of the Heights of the Barometer at Different Stations.

10. So far as we are aware, no tabulations have hitherto been made of the barometric heights at pairs of stations, and yet the degree of correlation between stations in different situations is one of extreme interest and importance. We have shown that the constants of barometric frequency vary continuously and gradually from one end to another of the British Isles. We should accordingly expect a close degree of correlation between the heights at different stations. This degree will probably be found to vary with the distance at a different rate along and perpendicular to the generalised isobars. It may also be greater when a certain interval is allowed between the observations at the two stations.† Our present object, however, being only to illustrate the general treatment of barometric correlation, we have dealt only with three pairs of stations and with contemporaneous observations.

The stations are the following:—

(1.) Babbacombe and Churchstoke for the eight years 1878 to 1885.

(2.) Southampton and Laudale for the eight years 1880 to 1887.

(3.) Hillington and Churchstoke for the eight years 1878 to 1885.

The results are exhibited in the following three tables:—

* We have to cordially thank Mr. R. H. Scott for allowing us to copy the manuscript records of the Meteorological Office for these three telegraph stations. We believe that the discussion of the frequency of twenty contemporaneous years of the whole system of telegraph stations would give most interesting results, but the labour of copying and reducing only thirteen years for but three stations has convinced us that it could hardly be undertaken by private individuals largely occupied with other work.

† A most interesting investigation would be the degree of correlation between suitable North-American and British stations, when the interval between the observations was varied from one up to six or seven days. Similar investigations for British and Continental stations might easily give important results bearing directly on the prediction of barometric changes.

TABLE VIII.

2922 days	*Babbacombe.* 31·0	30·9	30·8	30·7	30·6	30·5	30·4	30·3	30·2	30·1	30·0	29·9	29·8	29·7	29·6	29·5	29·4	29·3	29·2	29·1	29·0	28·9	28·8	28·7	28·6
Churchstoke. 31·0	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·9	..	2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·8	..	2	3	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·7	..	..	2	17	17	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·6	..	..	..	7	33·5	25·5	2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·5	..	..	..	..	17	53·5	27·5	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·4	..	..	..	..	2·5	25	61	54	10	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·3	..	..	..	..	..	6	27·5	112	62	6	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·2	..	..	..	..	..	..	5	55·5	126·5	83	9	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·1	..	..	..	..	..	..	..	11	86·5	161	78	14·5	2	..	..	..	..	..	..	..	..	..	..	..	..
30·0	..	..	..	..	..	..	..	..	13·5	92	157	58	7	2	..	..	..	..	..	..	..	..	..	..	..
29·9	..	..	..	..	..	..	..	..	..	26	101·5	113	54·5	11·5	1	..	..	..	..	..	..	..	..	..	..
29·8	..	..	..	..	..	..	..	..	..	3	23·5	112·5	105·5	22	5	1	..	..	..	1	..	..	..	..	..
29·7	..	..	..	..	..	..	..	..	..	..	3·5	27·5	100·5	79·5	31	9	2	..	..	..	..	..	..	..	..
29·6	..	..	..	..	..	..	..	..	..	..	1	1	33	70	52	23	1·5	..	..	..	..	..	..	..	..
29·5	..	..	..	..	..	..	..	..	..	..	..	..	7	26·5	41	46	15	2	..	..	..	..	..	..	..
29·4	..	..	..	..	..	..	..	..	..	..	..	..	..	3	19·5	33·5	21	8	2	1	..	..	1	..	..
29·3	..	..	..	..	..	..	..	..	..	..	..	1	..	..	1	14·5	21·5	11	5·5	1	..	..	..	..	..
29·2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2·5	8	12·5	7·5	2	0·5	..	..	..	..
29·1	..	..	..	..	..	..	..	..	1	..	..	..	..	..	..	1	3	8·5	6	4	0·5	..	..	..	..
29·0	..	..	..	..	..	..	..	..	..	..	..	..	..	0·5	..	..	..	2·5	3·5	3	1·5	..	..	..	..
28·9	..	..	..	..	..	..	..	..	..	..	..	..	..	0·5	..	..	..	1	..	2·5	2	0·5	1	..	..
28·8	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1·5	3	..	..
28·7	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	0·5
28·6	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	0·5

TABLE IX.

Southampton.

2922 days	31·0	30·9	30·8	30·7	30·6	30·5	30·4	30·3	30·2	30·1	30·0	29·9	29·8	29·7	29·6	29·5	29·4	29·3	29·2	29·1	29·0	28·9	28·8	28·7	28·6	28·5	28·4	28·3	28·2	28·1	28·0	27·9
Laudale. 31·0	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·9	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·8	1	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·7	..	1	..	4·5	4	3	1·5	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·6	..	1	3	7·75	9·75	8·5	3·5	2·25	0·25	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·5	..	1	0·5	6·25	9·25	17	15·75	6·5	5·5	1·75	0·5	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·4	..	..	0·5	5·75	11·75	22	24·75	39·75	19·75	12·25	4·5	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·3	..	..	..	5·25	11·5	15·5	24·75	55	50	25·5	6	5	1·25	0·25	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·2	..	..	..	1	4·25	21·75	24	38·5	55·5	66	32	12·5	4·75	2·75	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·1	..	..	..	..	2	10·75	21·75	23·75	55·75	55·5	46·75	26	15	3·25	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·0	..	..	..	..	..	5	11	26·25	48·25	63	58	27·5	24·5	10	1·5	1·5	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
29·9	..	..	..	..	..	4	4	19	31·75	54·25	63·25	57	27·5	11·5	8·75	1	0·5	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..
29·8	..	..	..	..	..	..	7	13	24·25	41·75	59·75	48	36·75	18	17·5	6·5	3·5	0·5	..	1	..	..	..	..	..	..	..	..	..	..	..	..
29·7	..	..	..	..	..	..	2	8	12	36·5	41·25	47	42·25	19·5	16	7·5	5	5	2·5	0·5	..	..	..	..	..	..	..	..	..	..	..	..
29·6	..	..	..	..	..	..	0·5	4	8·5	16	26·5	47	32·75	25·5	23·75	12·5	8	3	1	3	..	..	..	..	..	..	..	..	..	..	..	..
29·5	..	..	..	..	..	..	..	1	3	13	18	31	45	39·5	20	10·5	8	2·5	..	0·5	..	..	..	..	..	..	..	..	..	..	..	..
29·4	..	..	..	..	..	..	..	..	0·5	5	13·5	17·25	24·5	23·75	18	13·5	7·25	5·25	3	0·5	1	1	1	..	..	..	..	..	..	..	..	..
29·3	..	..	..	..	..	..	..	..	..	2	6	13·25	17·25	18·5	11·5	10·5	7·75	5·75	1	2	1	1	..	..	..	..	..	..	..	..	..	..
29·2	..	..	..	..	..	..	..	..	..	2	5·25	5·25	6	14	13	8	7	3	2	..	1	..	..	..	1	..	..	..	..	..	..	..
29·1	..	..	..	..	..	..	..	..	..	1	0·25	2·75	5·5	10·5	10	12·5	6	9·5	3	..	1	1	..	..	..	..	..	..	..	..	..	..
29·0	..	..	..	..	..	..	..	..	..	..	1	..	4	3	5·75	3·75	6·5	6	4·5	2·5	1·5	..	..	..	..	..	..	..	..	..	..	..
28·9	..	..	..	..	..	..	..	..	..	..	..	..	..	1	2·75	5·75	2·5	4·5	4	..	2	1	1	..	..	..	..	..	..	..	..	..
28·8	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	2·5	2	1·5	0·5	0·5	..	3	..	..	..	..	..	..	..	..	..	..
28·7	..	..	..	..	..	..	..	..	..	..	..	..	..	..	0·5	1·5	..	1·5	1	3	..	..	..	..	..	..	..	..	..	..	..	..
28·6	..	..	..	..	..	..	..	..	..	..	..	..	1	..	0·5	1	..	1	..	1	..	..	..	..	..	..	..	..	..	..	..	..
28·5	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
28·4	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	0·5	..	..	..	1	..	..	1	..	..	..	..	..	..
28·3	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	0·5	..	..	..	..	..	..	..	..	..	..	..	..	..
28·2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
28·1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
28·0	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..
27·9	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	..	..	..

TABLE X.

Hillington.

2922 days (*Churchstoke.*)	31·0	30·9	30·8	30·7	30·6	30·5	30·4	30·3	30·2	30·1	30·0	29·9	29·8	29·7	29·6	29·5	29·4	29·3	29·2	29·1	29·0	28·9	28·8	28·7	28·6
31·0	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·9	..	2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·8	..	..	4	2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·7	..	..	1	12	18·5	2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·6	..	..	..	5·5	32·75	25·75	3·5	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·5	..	..	..	..	3·25	39·75	45	8·5	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·4	..	..	..	..	..	9.5	72·25	62·75	8·5	0·5	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·3	..	..	..	..	..	..	20·75	107·75	73·75	11·75	4	..	..	..	..	..	..	..	..	..	..	..	..	..	..
30·2	..	..	..	..	..	..	1	17·25	142·75	99·25	13·25	1·5	..	..	..	..	..	..	..	..	..	..	..	..	..
30·1	..	..	..	..	..	..	..	2·75	49	172·5	108·75	18·5	2	..	..	..	..	..	..	..	..	..	..	..	..
30·0	..	..	..	..	..	..	..	1	3	50·5	164·75	88·25	17	3	..	..	..	..	..	..	..	..	..	..	..
29·9	..	..	..	..	..	..	..	..	0·5	6·5	54·75	130·5	88·25	22·5	3	..	..	..	..	..	..	..	..	..	..
29·8	..	..	..	..	..	..	..	..	..	..	16·5	64	113	69·5	9·75	2·25	1	..	..	..	..	..	..	..	..
29·7	..	..	..	..	..	..	..	..	..	..	1	14·25	66·75	98·25	58	11·75	2·5	..	..	..	..	..	..	..	..
29·6	..	..	..	..	..	..	..	..	..	..	..	1	13·5	43·25	69·5	43·75	11·5	1	1	..	..	..	..	..	..
29·5	..	..	..	..	..	..	..	..	..	..	..	..	1	11·75	39·5	52·25	20	6	4·5	..	..	..	..	..	..
29·4	..	..	..	..	..	..	..	..	..	..	..	..	..	1·25	10·25	29·5	30	15	6·5	2	..	..	..	..	..
29·3	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2·5	8·5	14·5	16	9	2·5	..	..	..	..	..
29·2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	0·5	0·5	3·5	9·5	7·5	11	2	0·5	0·5	..	..
29·1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2	1·5	10	5·5	4	1	..	..	..
29·0	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1·5	2	4	1·5	1	1	..
28·9	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1·5	0·5	3·5	..	2	..	..
28·8	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	0·5	2	1	1	..	..
28·7	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1·5	..	..	..
28·6	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	0·5	1	..	..

Now the distribution of frequency at each station being skew, the correlation surfaces as based upon these tables will also be skew, and the curves giving the mean height at one station for a given height at the other are no longer straight lines. We do not propose on the present occasion to discuss at length the properties of this type of skew correlation, but to refer the reader to a paper by Mr. G. U. YULE, published in the 'Roy. Soc. Proceedings,' vol. 60, pp. 477 *et seq.*, 1897. In that paper it is shown that the coefficient of regression is still significant in the case of skew correlation, it gives the slope of the line of closest fit to the curve of regression, or the locus of the mean heights of one station for successive heights at the other. Since the locus is not very far removed from a straight line in any of the cases dealt with, it follows that the line of closest fit will very approximately represent it. Calculating the coefficients of correlation and the regressions for the three pairs of stations by the usual formulæ (see "Mathematical Contributions to the Theory of Evolution, III.," 'Phil. Trans.,' A, vol. 189, pp. 265–6, 275–7), we have the following results :—

TABLE XI.—Barometric Correlation.

Pairs of stations.	Coefficient of Correlation.	Coefficient of Regression.	Probable deviation of array.
Babbacombe . . . Churchstoke . . .	0·9824	0·8901 1·0818	0″·0401 0·0441
Southampton . . . Laudale	0·7572	0·6260 0·9159	0″·1449 0·1752
Hillington . . . Churchstoke . . .	0·9576	0·9267 0·9895	0″·0663 0·0685

The application of this table to predict the height of the barometer at one station from a knowledge of the contemporaneous height at a second will be clear to readers familiar with the mathematical theory of correlation. For example, if the height of the barometer at Babbacombe be x'' above the mean Babbacombe height, then the height to be predicted at Churchstoke is 1·0818 x'' above the Churchstoke mean, with a probable error of 0″·0441. We may give a numerical illustration. The barometer at Churchstoke stands at 30″·0176 ; what are its probable heights at Babbacombe and Hillington ?

The mean height at Churchstoke = 29″·9545 (see Table IV.) ; hence the observed height at Churchstoke is 0″·0631 above the mean. The most probable heights at Babbacombe and Hillington will accordingly be 0·8901 × 0″·0631 and 0·9267 × 0″·0631, or 0″·0562 and 0″·0585 above the respective means of those places. Extracting these means from Table IV., we find: 30″·0349 and 30″·0014 for the probable heights

at Babbacombe and Hillington. The probable deviations in the two cases are $0''\cdot0401$ and $0''\cdot0663$. Thus, the prediction would probably be correct to within $\frac{1}{20}$ of an inch.

We should not propose, however, to base the prediction of the barometric height at one station on its correlation with the height at a single other station. We should think it desirable to apply the principles of multiple correlation, and endeavour by a suitable selection of stations to decrease the probable deviation of the array at the given station which corresponds to observed heights at the selected stations. We shall cite here the general formulæ for the prediction of the height of the barometer from a knowledge of its heights at correlated stations.

Let x_m be the probable height of the barometer at the m^{th} station above its mean value for that station. Let $r_{mm'}$ be the coefficient of correlation between the m^{th} and m'^{th} stations, where in finding $r_{mm'}$ the correlated heights may be taken at different times, or if it seems desirable on different days. Let $h_{m'}$ be the observed height at the m'^{th} station and $\sigma_{m'}$ the standard deviation, both measured from the mean of that station.

11. Case (i.).—*Prediction from one Correlated Station only.*

$$x_1 = r_{12}\frac{\sigma_1}{\sigma_2}h_2, \text{ with a probable deviation of } 0\cdot6745\sigma_1\sqrt{1-r^2_{12}}.$$

It is clear that unless r_{12} be very nearly unity, *i.e.*, the stations very close, the predicted height will be subject to a large probable deviation. For very close stations, such as London, Cambridge, Dover-Dungeness, where a rough investigation leads me to the conclusion that the correlation is as high as $0\cdot998$, we have the probable deviation about $0\cdot04487 \times \sigma_1$, or about $0\cdot015''$. In such cases the above formula will give fairly closely the height to be predicted at the first station. It may be used for purposes of interpolation.

The approximate linearity of the "regression" leads us to an interesting property of barometric correlation. There is a certain height of the barometer which, if it occurs at one station, will itself be the most probable height at the correlated station. This may be termed the *balance height*. This height may be easily found from the equation :

$$m_1 + x_1 = m_2 + h_2,$$

whence, if

$$\rho_{12} = r_{12}\sigma_1/\sigma_2, \qquad h_2 = (m_1 - m_2)/(1 - \rho_{12}),$$

and the balance height

$$= (m_1 - \rho_{12}m_2)/(1 - \rho_{12}).$$

Above and below the balance height the relative heights of the two stations are reversed. If above the balance height the first station has a probable height of its barometer invariably higher than the observed height at the second station, then below the balance height the probable height at the first station will be invariably lower than the observed height at the second station, and *vice versâ*.

The following are the balance heights of the three pairs of stations dealt with in Table XI. :—

Babbacombe . } Churchstoke . }	30″·2745	Above this value Churchstoke, below Babbacombe, has usually the higher barometer.
Southampton . } Laudale . . . }	28″·5022	Above this value Southampton, below Laudale, has most probably the higher barometer.
Hillington . . } Churchstoke . }	31″·0477	Above this value Churchstoke, below Hillington, has most probably the higher barometer.

In the case of close and highly correlated stations, the balance value, since the probable deviation is small, may be roughly found from a mere inspection of the barometer records for the two stations. Thus, we expect, it is about 30″·5 for London and Cambridge. Perhaps a better approximation might be obtained by estimating,

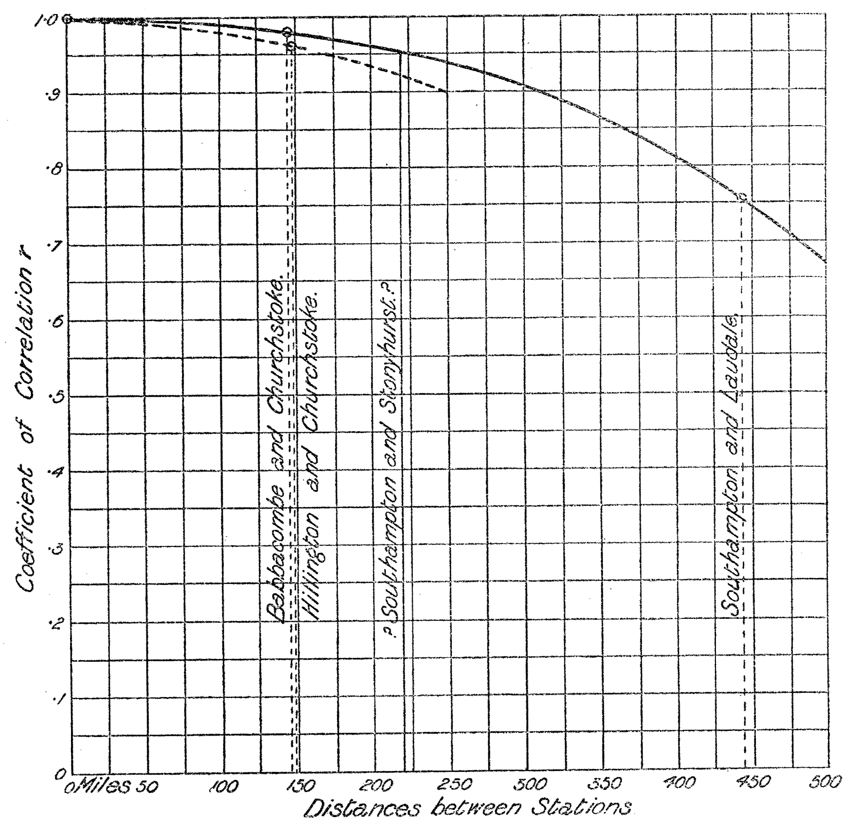


Diagram illustrating application of Theory of Correlation to predict Barometric Heights.

——— Stations approximately orthogonal to isobars.

- - - - - - - - - Stations approximately along isobars.

from their known distance, the coefficient of correlation for the two stations by means of the diagram above, and then calculating ρ_{12}, since σ_1 and σ_2 are given in Table IV., or can be found by interpolation from that table.

The diagram indicates graphically, what is fairly clear from Table XI., that correlation is not a single valued function of the distance between the two stations. It

suggests—we wish especially to note that it does not prove—that correlation differs in its variation with the distance according as the two stations have their distance practically along or orthogonal to the generalised isobars. Unfortunately, without crossing the Irish Channel, it was impossible to find two stations along a generalised isobar so far apart as Southampton and Laudale. There can be little doubt, we think, that such stations would, however, give a sensibly less correlation than the north and south stations. Although the diagram (owing to the very considerable labour of calculating the correlation for a pair of stations even for eight years) is based upon a very inadequate number of measurements, yet we should expect, with continuity in the correlation coefficient, which can hardly fail to be the case, that it would give fairly approximate values. For example, we should anticipate that the correlation between Southampton and Stonyhurst would be about 0·94 to 0·95. Since Stonyhurst is, roughly, about equally distant from Southampton and Laudale, the correlation between Stonyhurst and Laudale may with somewhat smaller probability be also put at 0·94 to 0·95. Clearly the general relationship between correlation, distance, and direction of distance will only be determined when a very great number of pairs of stations have been worked out, and these stations ought to be distributed, not over a small area like the British Isles, but over a large continental area.

12. Case (ii.).—*Prediction from two Correlated Stations.*

Here:

$$x_1 = \frac{r_{12} - r_{23}r_{13}}{1 - r_{23}^2} \frac{\sigma_1}{\sigma_2} h_2 + \frac{r_{13} - r_{23}r_{12}}{1 - r_{23}^2} \frac{\sigma_1}{\sigma_3} h_3$$

with a probable deviation

$$p_1 = 0{\cdot}6745\sigma_1 \sqrt{\frac{1 - r_{23}^2 - r_{13}^2 - r_{12}^2 + 2r_{23}r_{13}r_{12}}{1 - r_{23}^2}}.$$

Hence if we wish to predict the height of the barometer as closely as possible at one station from either an earlier or contemporary observation of the heights at two other stations, we ought to choose out intervals of time and the distribution of the three stations, so that p_1 may be as small as possible. It would thus seem, considering the great variety of times and places available, within our power to predict almost exactly the height at any selected station from a knowledge of the heights at two other selected stations at selected intervals of time. The importance of testing this principle seems to us very great; it might lead to quite novel methods of predicting barometric change. Unfortunately the needful knowledge of the correlation coefficients of widely separated pairs of stations for divers intervals of observation is still wholly wanting. The following example is merely illustrative. Suppose the second and third stations, so selected that they have equal correlation $r = r_{12} = r_{13}$ with the first, and a correlation $\rho = r_{23}$ with each other. Then p_1 would be absolutely zero if

$$\frac{1 - \rho^2 - 2r^2 + 2\rho r^2}{1 - \rho^2} = 0,$$

or if

$$1 = \frac{2r^2}{1+\rho},$$

that is, if

$$r = \sqrt{\tfrac{1}{2}(1+\rho)}.$$

For example, if the correlation between the second and third stations be 0·7572,

$$r = \sqrt{0{\cdot}8786} = 0{\cdot}9373.$$

In other words, if three stations could be found with coefficients $r_{12} = 0{\cdot}9373 = r_{13}$ and $r_{23} = 0{\cdot}7572$; then the barometric height at the first would be exactly a linear function of the contemporaneous heights at the other two stations. There seems no reason why stations should not be found with these correlations, or similarly related correlations. The correlation between Southampton and Laudale is 0·7572; the correlation between Stonyhurst and Laudale and between Stonyhurst and Southampton must be about 0·94 to 0·95. We increase the distance and make it less perpendicular to the generalised isobars by moving across towards the East coast. Probably somewhere near Whitby the required correlation of 0·9372 would be reached, and at such a station we should expect the barometric height to be very nearly a linear function of the heights at Southampton and Laudale.*

Supposing the relation $r = \sqrt{\frac{1}{2}(1+\rho)}$ to hold, then it is easy to deduce from the expression for x_1 above, that if H_1, H_2 and H_3 be the absolute heights of the barometer at the three stations,

$$H_1 = m_1 - \frac{\sigma_1}{2r}\left(\frac{m_2}{\sigma_2} + \frac{m_3}{\sigma_3}\right) + \frac{\sigma_1}{2r}\frac{H_2}{\sigma_2} + \frac{\sigma_1}{2r}\frac{H_3}{\sigma_3}.$$

This passage of correlation into causal relationship† is of such extreme importance, that it is worth while to see what approach we can find to it, even within the somewhat narrow limits of the British Isles. We have no details available for the barometer

* Let the reader imagine the heights at Southampton and Laudale replaced by the heights at two (or if necessary more) stations on the American continent, say, along the East coast, and the height at the Whitby station replaced by the height, after an interval of time, at a British station, say Valencia, and then he will grasp the sort of possibility—not the proven feasibility—of prediction, which the authors wish to lay stress upon.

† [This expression is used advisedly to draw attention to the importance of the limit when a correlation passes into a causal relationship. If the unit A be always preceded, accompanied or followed by B, and without A, B does not take place, then we are accustomed to speak of a *causal relationship* between A and B. On the other hand, if when A occurs amounts, $b_1, b_2, b_3 \ldots b_n$ of B are found with frequencies $p_1, p_2, p_3 \ldots p_n$ per cent. of the occurrences of A, we speak of a correlation between A and B. Now, if we approach the limiting case of $p_1 = p_2 = p_3 = \ldots = p_{r-1} = p_{r+1} = \ldots = p_n = 0$, and $p_r = 100$ per cent., it is clear that more and more nearly b_r of B will occur whenever A occurs, *i.e.*, a fixed amount of B on every occurrence of A. This is the transition of correlation into causal relationship. It is the construction of a q-dimensioned correlation surface, of which a particular $q-1$ dimensioned section approaches indefinitely close to a frequency curve of zero standard deviation.]

at Whitby, and if we had them and worked out the Whitby-Southampton and Whitby-Laudale correlations, it is unlikely that they would both be exactly equal, and equal to 0·9373. In fact the correlation for a number of Yorkshire stations, with both Laudale and Southampton, would have first to be worked out, and then the true station having an exact causal relationship with Laudale and Southampton would require to be found by interpolation.

But we should expect stations not so far removed from the right position to have their barometric height given in terms of those of Laudale and Southampton by an approximately linear relation. Hence, to indicate to the reader that our conclusion —namely, that a barometric correlation may pass into a causal relationship—is not so paradoxical as may appear at first sight, we have endeavoured to test how far the Stonyhurst height is a linear function of the heights at Laudale and Southampton. In order to do this we have neither assumed nor worked out the Laudale-Stonyhurst and the Southampton-Stonyhurst correlations. The former was too risky,* the latter too laborious for the end to be desired. We have simply assumed a linear relationship between the heights at the three stations, *i.e.*,

$$H_{St} = xH_{So} + yH_L + z,$$

where x and y are numerical constants, and z is a number of inches. To determine x, y, and z we chose twelve observations, taking the 15th day of each month for one year, and working by the method of least squares. Unfortunately the resulting equations for x, y, and z, throw back their determination on decimal figures, which are the limit of what is usually tabulated in barometric observations. The resulting equations were

$$30·159x + 30·020y + z - 30·130 = 0,$$
$$30·161x + 30·022y + z - 30·132 = 0,$$
$$30·161x + 30·024y + z - 30·133 = 0.$$

The solution of these equations is

$$x = 0·50, \qquad y = 0·50, \qquad z = 0·04'',$$

where the values of x and y are certainly not correct to the second place of decimals.

The resulting formula gives

$$H_{St} = 0·5H_{So} + 0·5H_L + 0·04''.$$

An attempt to approximate to the coefficients of correlation, gave the Southampton factor a somewhat higher value than the Laudale factor, and this is probably the case. But with the data available we shall hardly do better than the above formula. To test its degree of accuracy 50 values were taken out of the returns for Southampton, Laudale and Stonyhurst at fortnightly intervals, and the observed and calculated values at Stonyhurst are given in Table XII. The differences are distributed fairly evenly, positively and negatively, and their mean value is about 1/40″. We consider

* We have already noted that the Stonyhurst data are not, in our opinion, very satisfactory: see p. 446.

them sufficiently satisfactory to justify the view that a station could be found in Yorkshire for which correlation would pass into a causal relationship.

TABLE XII.—Illustrating Approach from Correlation to Causal Relationship.

Stonyhurst.		Difference.
Observation.	Calculation.	
30·58	30·61	+0·03
30·17	30·14	−0·03
30·03	29·98	−0·05
30·06	30·11	+0·05
29·92	29·94	+0·02
30·35	30·38	+0·03
29·85	29·84	−0·01
30·12	30·11	−0·01
30·13	30·13	0·00
30·19	30·20	+0·01
30·51	30·48	−0·03
30·65	30·62	−0·03
30·17	30·21	+0·04
29·52	29·55	+0·03
30·51	30·50	−0·01
29·91	29·90	−0·01
29·94	29·94	0·00
30·17	30·18	+0·01
30·13	30·10	−0·03
30·34	30·33	−0·01
30·74	30·73	−0·01
30·18	30·25	+0·07
30·16	30·18	+0·02
30·41	30·38	−0·03
30·01	30·00	−0·01
29·15	29·18	+0·03
30·28	30·26	−0·02
29·79	29·85	+0·06
29·93	29·90	−0·03
29·91	29·93	+0·02
30·11	30·11	0·00
29·99	29·92	−0·07
29·43	29·45	+0·02
30·15	30·15	0·00
30·22	30·22	0·00
30·16	30·13	−0·03
30·27	30·29	+0·02
29·56	29·61	+0·05
29·66	29·85	+0·19
30·23	30·25	+0·02
29·02	28·99	−0·03
30·35	30·30	−0·05
29·92	29·92	0·00
29·99	29·95	−0·04
29·95	29·96	+0·01
29·58	29·62	+0·04
30·11	30·11	+0·00
30·18	30·22	+0·06
29·92	29·89	−0·03
29·49	29·46	−0·03

As a rule there will exist for most triplets of stations a certain height, which may be termed the *balance height*, by which we are to understand that if the barometer stand at the balance height at two of the stations, its most probable value at the third correlated station is the balance height also. The balance height is at once found by putting $x_1 = H_b - m_1$, $h_2 = H_b - m_2$, $h_3 = H_b - m_3$ in the formula for regression and finding H_b. If we put $H_{So} = H_L = H_{St}$, in the formula on p. 460, we are led to $H_b = \infty$ for the balance height; this is probably very far from being the true balance height, and for the simple reason that the factors, 0·5 of H_{So} and H_L on p. 460, are only approximate.

If we found a Yorkshire station which had the correlation 0·9373 with both Laudale and Southampton, its balance height would be given by the formula, on p. 459; and supposing it to lie on, or nearly on, the same generalised isobar as Stonyhurst, *i.e.*, to have nearly the same mean and standard-deviation, then its balance height with Laudale and Southampton would be 29″·868. An error of between 0·001 and 0·002 in one of the factors, 0·5 of the Stonyhurst-Laudale-Southampton linear relationship, on p. 460, would thus have reduced the balance height of those stations from ∞ to about 29″·9.

The general expression for the balance height of a station 1, with regard to stations 2 and 3, is given by

$$H_1\,(\text{balance}) = \frac{(r_{12} - r_{23}r_{13})\dfrac{m_2}{\sigma_2} + (r_{13} - r_{23}r_{12})\dfrac{m_3}{\sigma_3} - (1 - r_{23}{}^2)\dfrac{m_1}{\sigma_1}}{(r_{12} - r_{23}r_{13})\dfrac{1}{\sigma_2} + (r_{13} - r_{23}r_{12})\dfrac{1}{\sigma_3} - (1 - r_{23}{}^2)\dfrac{1}{\sigma_1}}.$$

Hence if the three stations lie on the same generalised isobar, since m_1, m_2, m_3 and σ_1, σ_2, σ_3 have very approximately the same value, the balance height will be the mean height, and the same for every station with regard to the other pair.

One further general proposition may be noted before we leave the special case of prediction from two correlated stations. Suppose the barometer constant at one station, then the coefficient of correlation between the heights at the other two is given by*

$$\mathrm{r}_{23} = \frac{r_{23} - r_{13}r_{12}}{\sqrt{1 - r_{13}{}^2}\sqrt{1 - r_{12}{}^2}}, \text{ for a selected height at the first station.}$$

$$\mathrm{r}_{13} = \frac{r_{13} - r_{23}r_{12}}{\sqrt{1 - r_{12}{}^2}\sqrt{1 - r_{23}{}^2}}, \text{ for a selected height at the second station.}$$

$$\mathrm{r}_{12} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{1 - r_{13}{}^2}\sqrt{1 - r_{23}{}^2}}, \text{ for a selected height at the third station.}$$

* These values have been termed by Mr. G. U. YULE *nett* coefficients of correlation, to distinguish them from r_{23}, r_{13}, and r_{12}, which he terms *gross* coefficients. The difference would, perhaps, be best expressed mathematically by the use of such terms as *partial* correlation coefficient and *total* correlation coefficient, the former being the value of the coefficient when one variable is not allowed to vary, and the latter when it is. They are at once obtainable from the general expression on p. 287 of the Memoir, 'Phil. Trans.,' A, vol. 187, by putting, say x = const. and remembering that the coefficient of correlation for y and z in $P = P_0 e^{-\frac{1}{2}(a_2y^2 - 2a_{23}yz + a_3z^2)}$ is $a_{23}/\sqrt{a_2a_3}$.

The values of these expressions are peculiarly interesting, for they lead us to the general theorem that whenever the total correlation between two stations is less than the product of the total correlations at the other two pairs of stations, then the partial correlation between the latter two stations will be negative, *i.e.*, for a given value of the barometer at the first station, a rising barometer at the second station will on the average mark a falling barometer at the third station, and a falling barometer at the second station a rising barometer at the third.

Owing to the generally high values of barometric correlation it is comparatively easy for $r_{13}r_{12}$, for example, to be greater than r_{23}. Thus in the case where two of the stations have equal correlation with a third we have, if $r_{23} = \rho$, $r_{12} = r_{13} = r$,

$$\mathbf{r}_{23} = \frac{\rho - r^2}{1 - r^2}, \quad \mathbf{r}_{13} = \mathbf{r}_{12} = \frac{r}{\sqrt{1 - r^2}} \sqrt{\frac{1 - \rho}{1 + \rho}},$$

and $\mathbf{r}_{23}$ will always be negative if r^2 be $> \rho$. For example, we have Laudale and Southampton with a correlation 0·7572, while Stonyhurst and both these stations must have a correlation of about 0·94 to 0·95. It follows, therefore, that the partial correlation of Laudale and Southampton with regard to Stonyhurst is negative, or for a constant value of the barometer at Stonyhurst rising at Laudale would in general denote falling at Southampton, and *vice versâ*. This is best illustrated by noting that x_2 being the mean height at Southampton for heights h_1 and h_3 at Stonyhurst and Laudale (x_2, h_1 and h_3 being measured from the respective mean heights of the stations),

$$x_2 = \frac{r_{23} - r_{13}r_{12}}{1 - r_{13}^2} \frac{\sigma_2}{\sigma_3} h_3 + \frac{r_{12} - r_{23}r_{13}}{1 - r_{13}^2} \frac{\sigma_2}{\sigma_1} h_1,$$

or, in the special case approximately,

$$= \frac{\rho - r^2}{1 - r^2} \frac{\sigma_2}{\sigma_3} h^3 + \frac{r(1 - \rho)}{1 - r^2} \frac{\sigma_2}{\sigma_1} h_1.$$

Hence, if $r_{13}r_{12}$ be $> r_{23}$ or $r^2 > \rho$, an increase of h_3 for a constant h_1 means a decrease of x_2.

In the particular instance of correlation passing into causation referred to by us on p. 459, *i.e.*, when $1 + \rho = 2r^2$ we have

$$\mathbf{r}_{23} = -1, \qquad \mathbf{r}_{13} = \mathbf{r}_{12} = 1.$$

$$x_2 = -\frac{\sigma_2}{\sigma_3} h_3 + 2r \frac{\sigma_2}{\sigma_1} h_1.$$

Thus the partial correlation between the three stations is "perfect," but between the second and third it is negative. The ratio of the *fall* at the second to the *rise* at the third, for stationary barometer at the first, is σ_2/σ_3.

This principle, which at first sight appears rather paradoxical, namely, that at three stations, A, B, C, a rise at A will, on the average, be accompanied by a rise at C, but that a rise at A for a constant barometric height at B may, on the average

be accompanied by a fall at C, appears capable, when extended, developed, and illustrated by actual numerical examples, of throwing considerable light on the nature of barometric variation.*

To show the importance and truth of the principle, a table has been formed for the deviations from the means at Southampton and Laudale, when the barometer at Stonyhurst does not differ by more than $\frac{1}{100}$ inch from its mean value, 29″·94.

In the course of two years 31 values were found, *and with only one exception, possibly a misreading, the barometer at Laudale was always above or below the mean, according as it was below or above the mean at Southampton:* see Table XIII.

TABLE XIII.—Deviations of Barometer from the Means at Southampton (29″·98) and Laudale (29″·86) when the Barometer does not differ more than $\frac{1}{100}$ of an inch at Stonyhurst from its Mean.

Stonyhurst height.	Southampton deviation.	Laudale deviation.
29·95	+·15	−·20
29·94	+·11	−·27
29·94	−·12	+·07
29·93	−·07	+·12
29·94	+·03	−·11
29·94	−·07	+·00
29·94	+·09	−·13
29·95	+·07	−·05
29·93	−·03	+·03
29·94	+·04	−·07
29·93	−·00	+·04
29·95	+·07	−·06
29·94	+·12	−·13
29·94	+·09	−·07
29·95	+·25	−·18
29·94	−·04	+·01
29·93	−·19	+·30
29·95	+·04	−·08
29·93	−·06	+·06
29·94	+·03	−·08
29·94	−·04	+·17
29·95†	+·04	+·01
29·95	+·08	−·00
29·95	+·11	−·18
29·95	−·14	+·04
29·93	+·02	−·10
29·93	−·08	+·12
29·95	+·13	−·22
29·94	−·06	+·24
29·93	+·06	−·18
29·95	−·19	+·14

* The corresponding apparent paradox in the theory of heredity is referred to in 'Phil. Trans.,' A, vol. 187, p. 289.

† This is an exception to the general rule, that for the mean at Stonyhurst a high barometer at

13. Case (iii.).—*Prediction from three Correlated Stations.*

The general formulæ are given on p. 294 of the memoir previously cited. We do not reproduce them here, as we have no numerical data available at present by which to illustrate them. We will, however, consider a special application similar to that dealt with under Case (ii.).

Suppose it possible to select three stations so situated round a fourth that the three stations have equal correlation r with each other, and each a correlation ρ with the fourth. The latter being marked with the subscript 1 in the formulæ, the following are the proper relations which may be obtained after some algebraical reductions from the general case above referred to.

$$x_1 = \frac{\rho\sigma_1}{1+2r}\left(\frac{h_2}{\sigma_2} + \frac{h_3}{\sigma_3} + \frac{h_4}{\sigma_4}\right),$$

with a probable deviation $0{\cdot}6745\sqrt{1 - \frac{3\rho^2}{1+2r}}$,

$$x_2 = \frac{\rho\sigma_2}{1+2r}\frac{h_1}{\sigma_1} + \frac{(r-\rho^2)\,\sigma_2}{1+r-2\rho^2}\left(\frac{h_3}{\sigma_3} + \frac{h_4}{\sigma_4}\right),$$

with a probable deviation $0{\cdot}6745\sqrt{\frac{(1-r)(1+2r-3\rho^2)}{1+r-2\rho^2}}$.

Here the closest prediction will be obtained, if we select stations, if possible, such that $\rho = \sqrt{\tfrac{1}{3}(1+2r)}$, which is the point at which correlation passes into causation. In this case we find

$$x_1 = \frac{\sigma_1}{3\rho}\left(\frac{h_2}{\sigma_2} + \frac{h_3}{\sigma_3} + \frac{h_4}{\sigma_4}\right),$$

$$x_2 = \frac{\sigma_2}{3\rho}\frac{h_1}{\sigma_1} - \sigma_2\left(\frac{h_3}{\sigma_3} + \frac{h_4}{\sigma_4}\right),$$

with vanishing probable deviations. In the first case, if $\rho^2 > r$, which can easily be true for correlated stations within 300 miles of each other; in the second case always, a rise of the barometer at two of the "outer" stations for a steady barometer at the "inner" or first station marks a fall at the fourth station and *vice versâ*. Now, it is not contended that four stations can be found for which exactly $r_{12} = r_{13} = r_{14}$ and $r_{23} = r_{34} = r_{42}$, still less that it is possible to make $1 + 2r = 3\rho^2$. But it is suggested that with the values of the barometeric correlation coefficients such as we have found in the British Isles approximations to these relations can be found for selected stations, and that such stations are what we require for close prediction or interpolation. Further, such principles as we have noted with regard to the relation

Laudale is a low barometer at Southampton, and *vice versâ*. Or, the rule (by differentiation) may be stated for steady barometer at Stonyhurst a rise at Laudale indicates a fall at Southampton, and *vice versâ*.

of rise and fall at correlated stations are independent of the special relations between the coefficients, which we have selected to illustrate them, they are really a deduction from the sign of the regression coefficient, or of the coefficient of partial correlation, which has the same sign.

14. Case (iv.)—*Prediction from any Number of Correlated Stations.*

The general formulæ are given, p. 302 of the memoir above cited, namely :

$$x_1 = \left(\frac{R_{12}}{R}\frac{\sigma_1 h_2}{\sigma_2} + \frac{R_{13}}{R}\frac{\sigma_1 h_3}{\sigma_3} + \frac{R_{14}}{R}\frac{\sigma_1 h_4}{\sigma_4} + \dots\right)$$

with a probable deviation of $0·6745\ \sigma_1\sqrt{R/R_{11}}$ where R is the determinant below and R_{pq} is the minor formed by leaving out the p^{th} column and q^{th} row.

$$\begin{vmatrix} 1, & r_{12}, & r_{13}, & r_{14} & \cdot & \cdot & \cdot \\ r_{21}, & 1, & r_{23}, & r_{24} & \cdot & \cdot & \cdot \\ r_{31}, & r_{32}, & 1, & r_{34} & \cdot & \cdot & \cdot \\ r_{41}, & r_{42}, & r_{43}, & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}$$

In order to obtain close prediction, it might be supposed that all that is necessary is to take a sufficiency of correlated stations. This is very far from being the case. The true test of closeness of prediction is the smallness of $\sqrt{(R/R_{11})}$, and this can often be obtained by a few well-selected stations better than a great number. In order to roughly illustrate this, suppose the correlation coefficients of the stations to be all of the same order of magnitude, *i.e.*, about ϵ, then the order of $\sqrt{(R/R_{11})}$ for

1, 2, 3, 4 . . . n stations

is given by

$$\sqrt{(1+\epsilon)(1-\epsilon)},\ \sqrt{\left(1+\frac{\epsilon}{1+\epsilon}\right)(1-\epsilon)},\ \sqrt{\left(1+\frac{\epsilon}{1+2\epsilon}\right)(1-\epsilon)},$$
$$\sqrt{\left(1+\frac{\epsilon}{1+3\epsilon}\right)(1-\epsilon)} \dots \sqrt{\left(1+\frac{\epsilon}{1+(n-1)\epsilon}\right)(1-\epsilon)},$$

or the prediction is only increased in certitude in the ratio of $\frac{1}{\sqrt{1+\epsilon}}$ to 1 by taking an indefinitely great number of stations, or, since ϵ cannot be greater than unity, it can only be increased in the ratio of $\frac{1}{\sqrt{2}}$ to 1. Our object should accordingly be to make $\sqrt{(R/R_{11})}$ as small as possible* by a fit selection of comparatively few stations

* It is easy to illustrate its vanishing by taking one station equally correlated with $(n-1)$ others (ρ), which are equally correlated among themselves (r). In this case we have

ERRATUM.

Page 466, line 7 from top, the formula should be :—

$$x_1 = -\left(\frac{R_{12}}{R_{11}}\frac{\sigma_1 h_2}{\sigma_2} + \frac{R_{13}}{R_{11}}\frac{\sigma_1 h_3}{\sigma_3} + \frac{R_{14}}{R_{11}}\frac{\sigma_1 h_4}{\sigma_4} + \dots\right)$$

at different intervals of time. Nor should it be forgotten that theoretically we have much power of varying the magnitude of our correlation coefficients. They decrease from about 1 at the distance zero to zero as the distance increases. We have thus a zone round every station of zero-correlated stations; beyond this it is highly probable that the correlation becomes negative as we reach places which have cyclones corresponding to anticyclones at the given station. This zone is of course not reached in such a small area as the British Isles. Passing through an area of negative correlation, we should in all probability ultimately reach an extended area of zero correlation. In the next place the time interval is under our control, and the coefficient of correlation can be reduced by increasing this. Lastly, it varies also, although probably to a much less extent, by varying the direction in which the distance between two stations is taken. We can imagine no more useful piece of work than the determination of the correlation for a period of 10 or 20 years of a series of stations taken so far as possible round, say, a parallel of latitude. We believe that the future of barometric prediction, *i.e.*, the accurate foretelling of the arrival of depressions, &c., lies in an extended knowledge of the correlation between a system of barometric stations widely diffused over the surface of the earth; special attention being paid to the changes of the correlation with intervals of time.

The object of the present writers has not been to make an elaborate investigation of the numerical values of barometric variation or correlation, but rather to indicate to those more directly occupied with meteorological investigations how the mathematical theory of statistics may be applied to barometry with novel and, they believe, valuable results.

APPENDIX.

On a Frequency-registering Barometer by G. U. YULE.

In all ordinary forms of registering barometer the resulting diagram shows the height of the barometer at each instant of time. To construct a frequency curve from such a diagram, the heights must be read off for all the times desired, corrected if necessary, grouped, and replotted in the manner described in the preceding paper. This procedure is somewhat tedious, and it may be obviated by so constructing the barometer that it shall give the frequency record automatically.

$$x_1 = \frac{\rho\sigma_1}{1 + (n-1)r}\left(\frac{h_2}{\sigma_2} + \frac{h_3}{\sigma_3} + \frac{h_4}{\sigma_4} + \ldots\right),$$

with a probable deviation 0·6745 $\sqrt{\left(1 - \frac{n\rho^2}{1 + (n-1)r}\right)^{\frac{1}{2}}}$. Hence we should have absolute prediction if $\rho = \left(\frac{1}{n}\left(1 + \overline{n-1}\, r\right)\right)^{\frac{1}{2}}$. Similar propositions follow for partial correlation and for rise corresponding to fall. It seems doubtful, however, whether such a system of correlation could possibly be arranged for more than four stations.

The barometer devised for this purpose is illustrated in figs. A and B. Fig. A is a diagrammatic plan of the instrument; fig. B is from a photograph of a rough model

Fig. A.

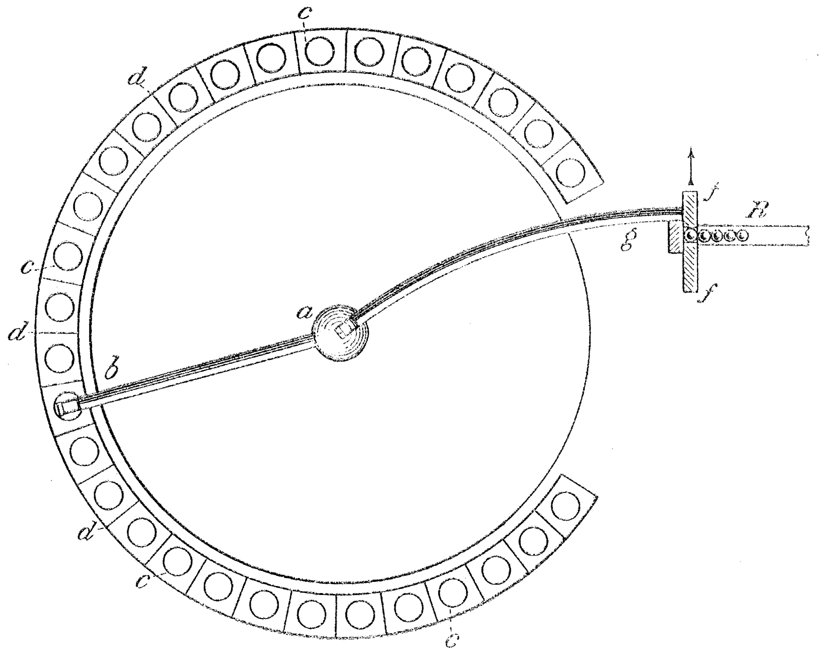
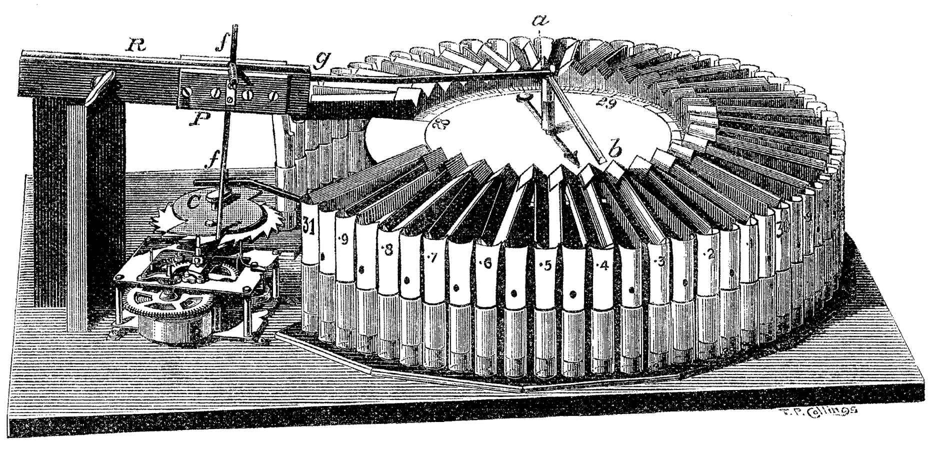


Fig. B.



made by the Cambridge Instrument Company. The barometer must be an aneroid, or some instrument working with a needle over a horizontal dial. This needle is removed, and replaced by a light V-shaped gutter of paper or metal-foil, sloping downwards from the centre to the periphery of the dial. This gutter needle is indicated by a, b in fig. A. Its outer extremity projects slightly beyond the dial over the tops of a series of small vertical tubes, c, c, c, which are equally spaced and separated by small wedge-shaped divisions, running along the lines d, d, d. If we suppose a ball to be dropped into the gutter at a, it will roll down to b, drop over,

and fall either straight into one of the tubes or on to one of the wedges, which will guide it into the corresponding tube. The shot will then be a record that the barometer needle stood once over the section corresponding to the tube. If balls be dropped in at regular intervals, a frequency record is obtained by simply counting up the number of balls in each tube. The record, of course, suffers from the disadvantage that the usual corrections cannot be made.*

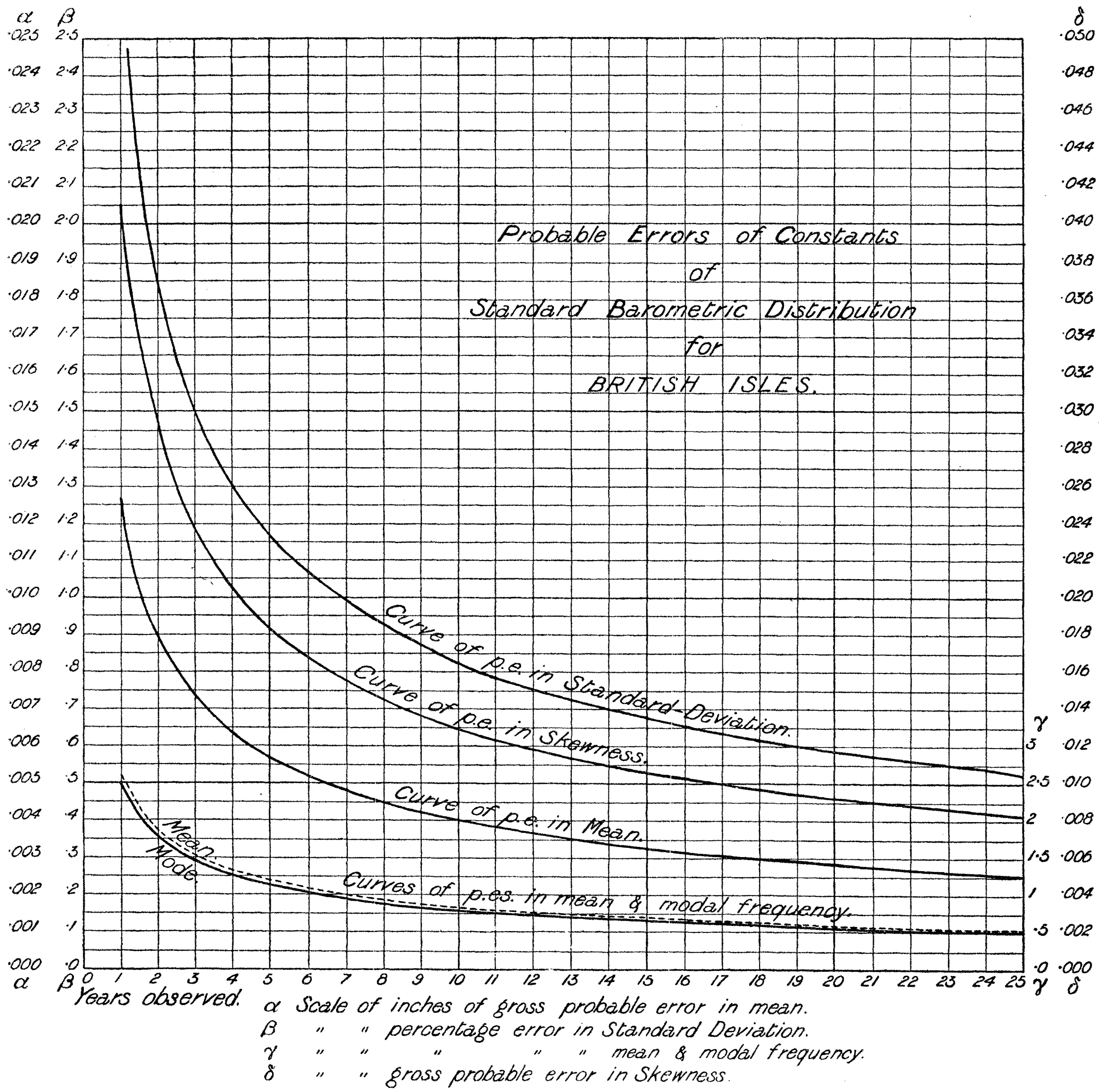
In practice, of course, the wedge edges *d, d, d* must be arranged so as to correspond exactly to the divisions between, say, the tenth inches. Any one tube will then record the number of times that the needle stood over a particular tenth when the record was taken. In the model itself a fixed gutter, *ag*, is brought over the centre of the needle gutter, just above its axis; it stands clear of it, and runs back to a reserve of balls, R. A trigger arrangement, *f, f,* is fired by a clock-driven cam as often as desired. The trigger gear can be seen in the second figure. The large cam, C, turns round once in the twenty-four hours, one of the saw teeth being pushed on each hour by the hand of the clock. The channel R, containing the reserve of balls† in single file, is normally closed by the forked brass lever, *ff,* which is pivoted near P, and bears at its lower end against the cam, being held forward by a spring. Only when the lever is fully cocked, *i.e.*, pushed over to its furthest reach by the cam, does the fork of the lever stand in line with the reserve channel. One ball then drops into the fork, which is just of the thickness of a ball. When the lever is freed again, the fork comes opposite the fixed gutter (*a, g* of fig. A), and the ball drops first into this, then falls into the needle gutter, and ultimately into a vertical tube.

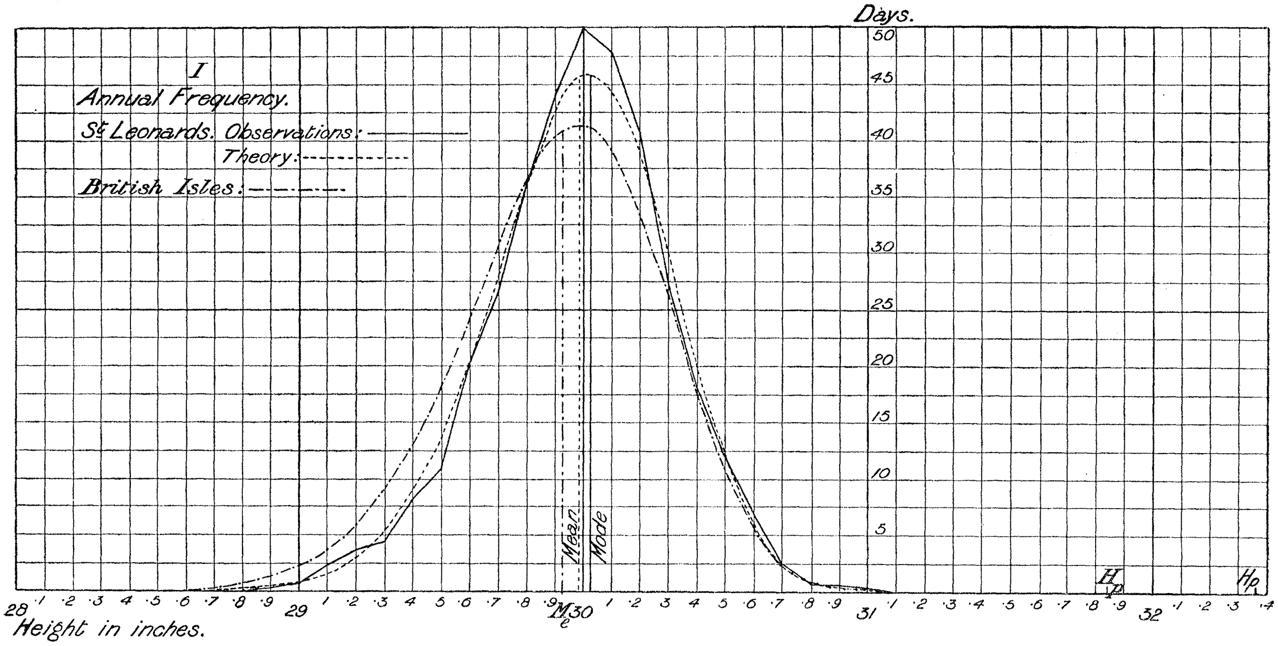
It may be desirable to take two or more frequency records for different hours of the day, and keep these records separate, so as to admit of the study of systematic differences. Mr. HORACE DARWIN, to whom the details of the working model are due, devised a very simple arrangement for doing this, which can be seen in the general view of fig. B. The balls do not drop straight from the needle-gutter into the collecting tubes, but run into radial gutters fixed to the dial-face of the barometer. At their outer terminals these fixed radial gutters stand so far apart that they only deliver balls into alternate tubes, *e.g.*, to the set marked on the figure with numbers not with black spots. Before the alternate records are made, the whole barometer, with the radial gutters attached, is given a slight turn by the same clock that serves to release the trigger. The radial gutters now stand opposite the tubes marked with black spots, and the record is made in them.

The working model was kept running for rather more than a month, and worked very satisfactorily, considering its somewhat rough construction. There were one or two "misfires," due to the balls hanging up, but such accidents could be easily remedied by a slight alteration.

* It must be remembered, however, that grouping in tenths of an inch gives quite sufficiently smooth and detailed returns for the purpose of calculating frequency curves.

† Bicycle bearing balls were actually used, as shot were found too irregular in shape and size.


Probable Errors of Constants
of
Standard Barometric Distribution
for
BRITISH ISLES.
Curve of p.e. in Standard-Deviation.
Curve of p.e. in Skewness.
Curve of p.e. in Mean.
Mean.
Mode.
Curves of p.es. in mean & modal frequency.
Years observed.
α Scale of inches of gross probable error in mean.
β " " percentage error in Standard Deviation.
γ " " " " " mean & modal frequency.
δ " " gross probable error in Skewness.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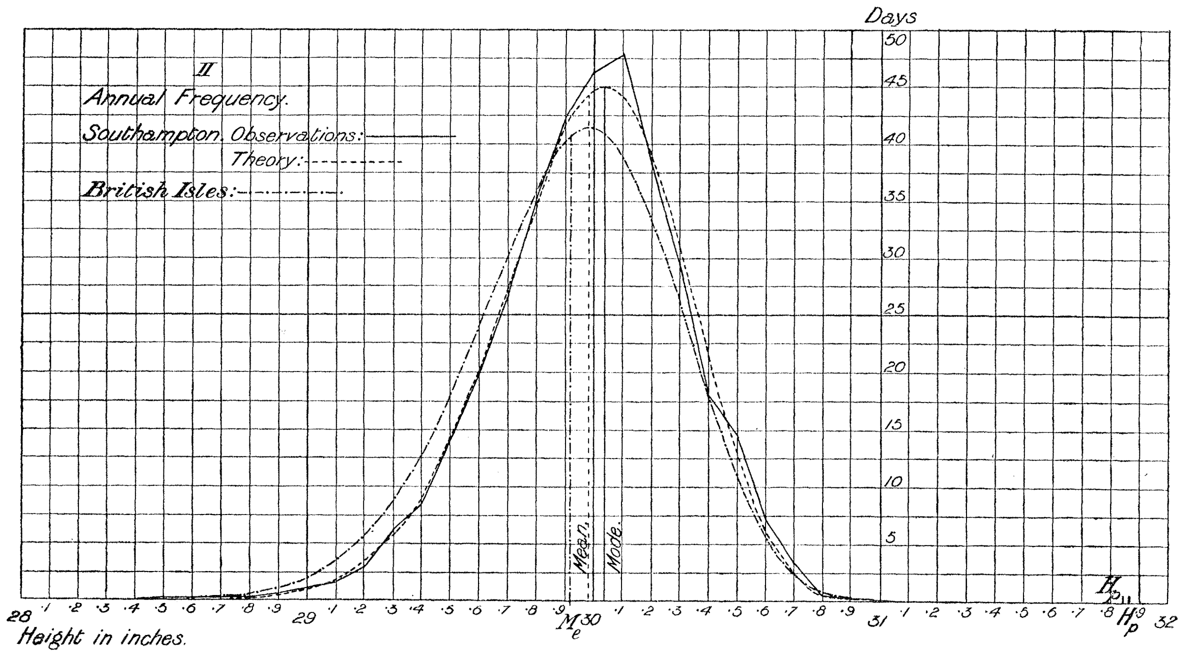
Annual Frequency.
St Leonards. Observations:
Theory:
British Isles:
Days.
Mean.
Mode
M_e
H_p
Height in inches.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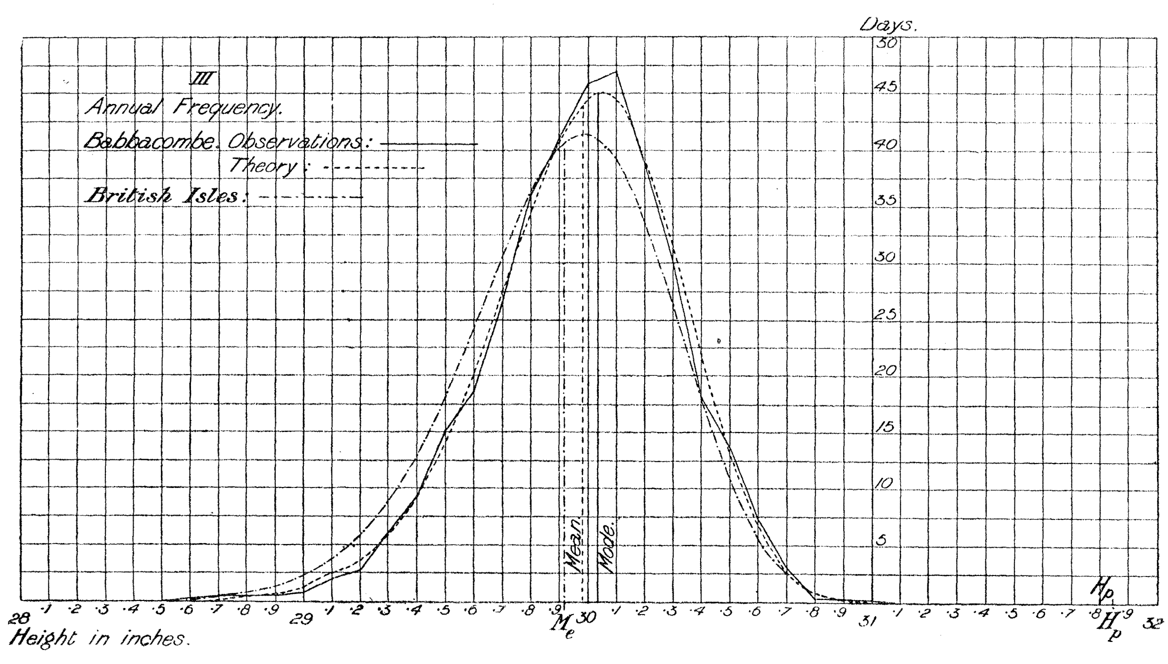
Southampton. Observations:
Theory:
British Isles:
Days
Mean.
Mode.
M_e
H_p
Height in inches.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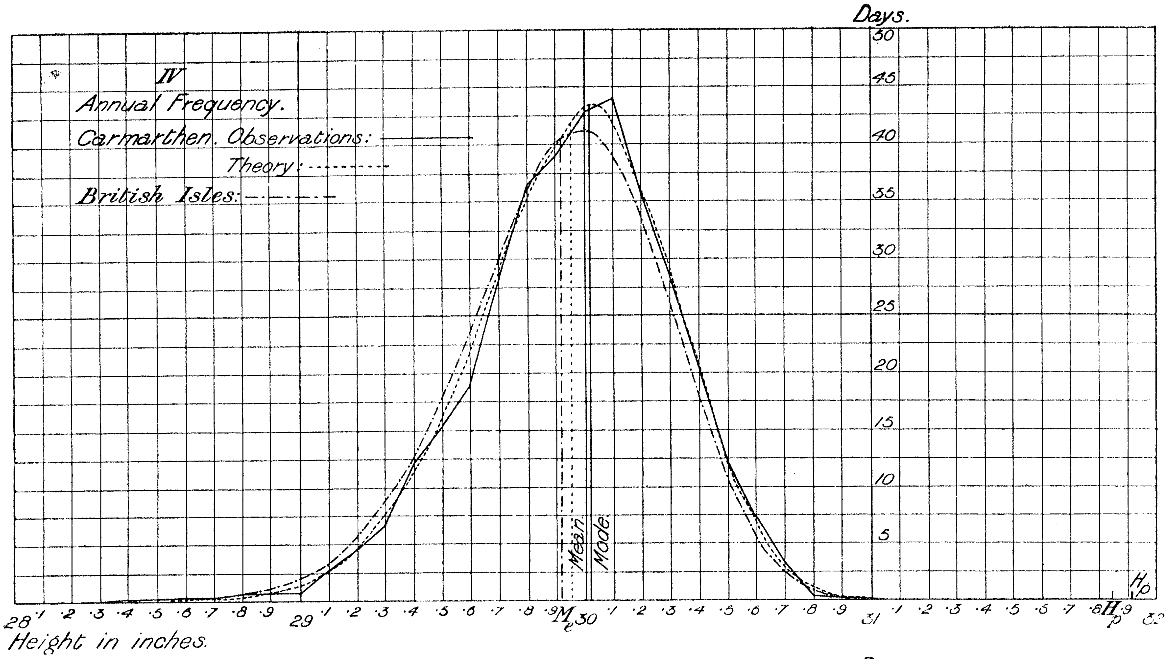
Theory:
British Isles:
Days.
Mean.
Mode.
M_e
H_p
Height in inches.


Days.
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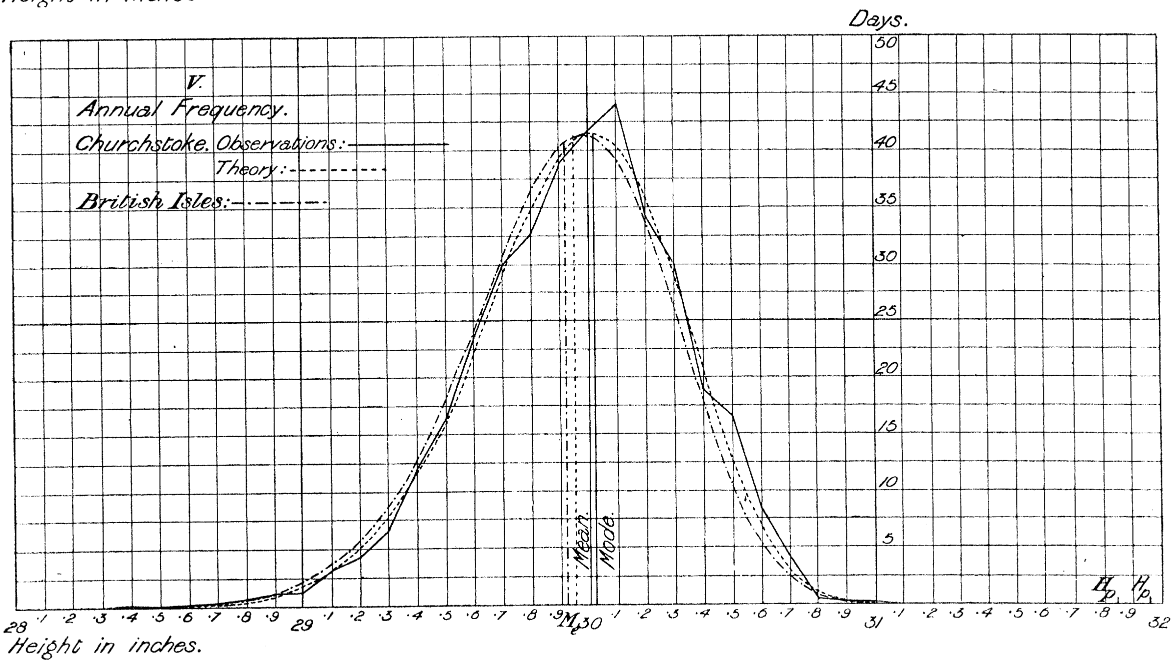
Theory:
British Isles:
Mean.
Mode.
M_e
H_p
Height in inches.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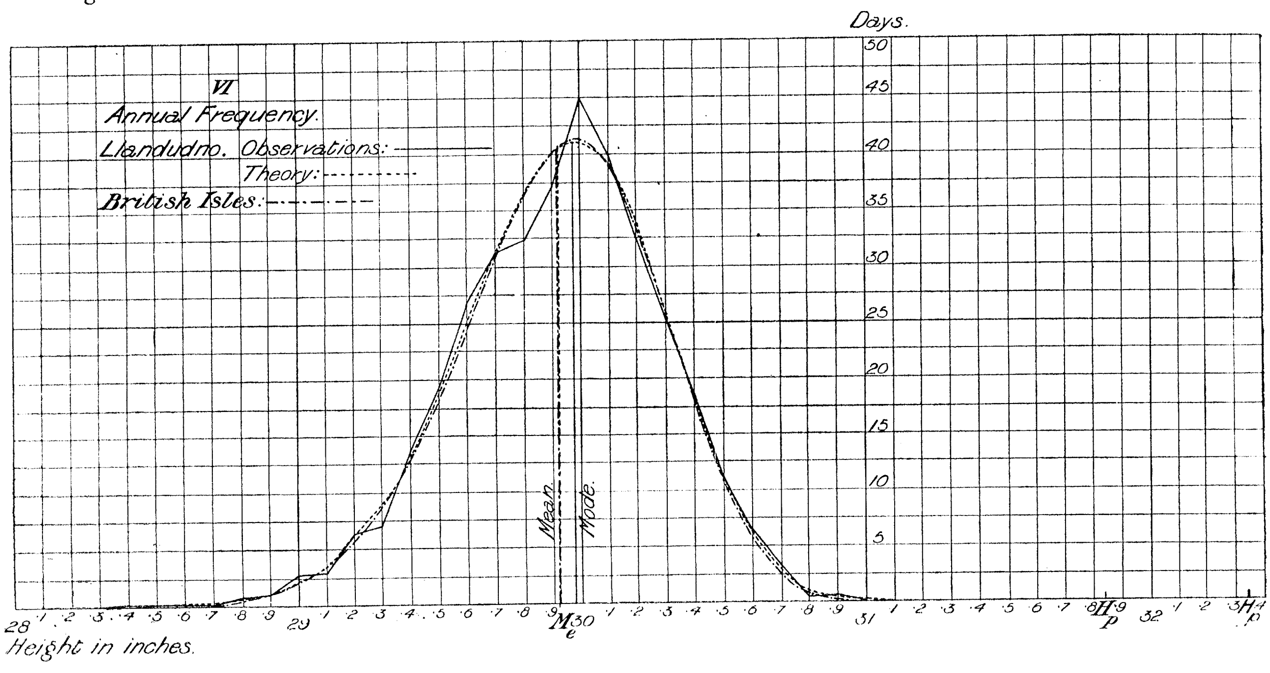
Annual Frequency.
Churchstoke. Observations:
Theory:
British Isles:
Mean.
Mode.
M_e
H_p
Height in inches.


Days.
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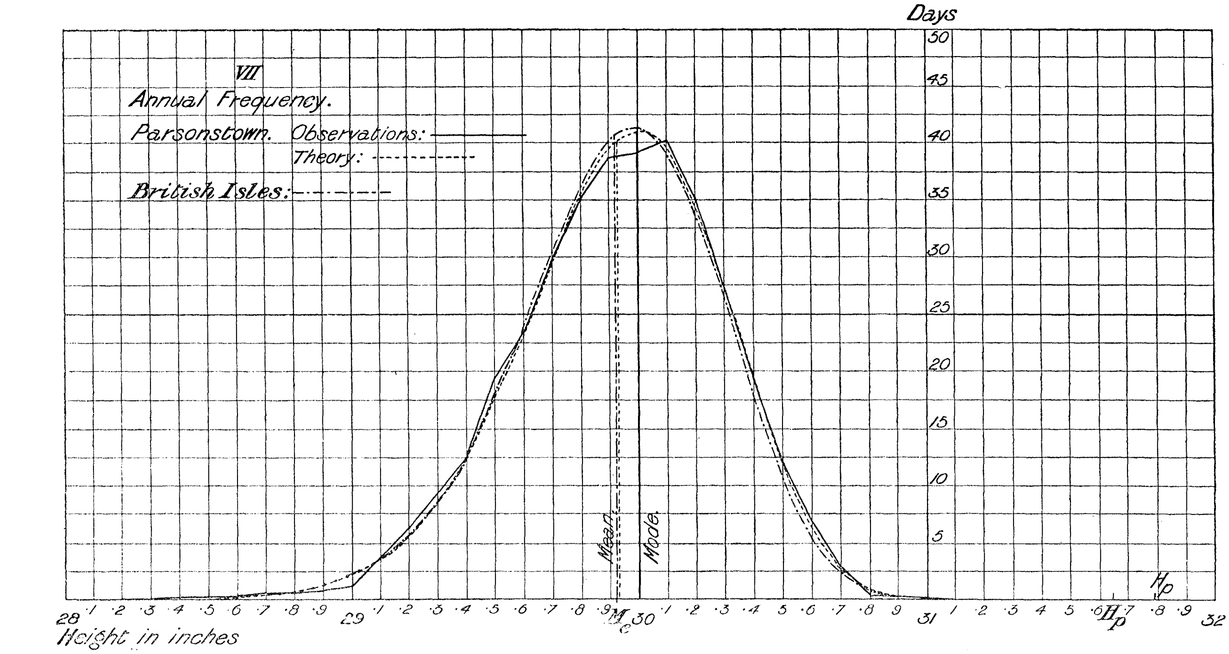
Annual Frequency.
Llandudno. Observations:
Theory:
British Isles:
Mean.
Mode.
M_e
H_p
Height in inches.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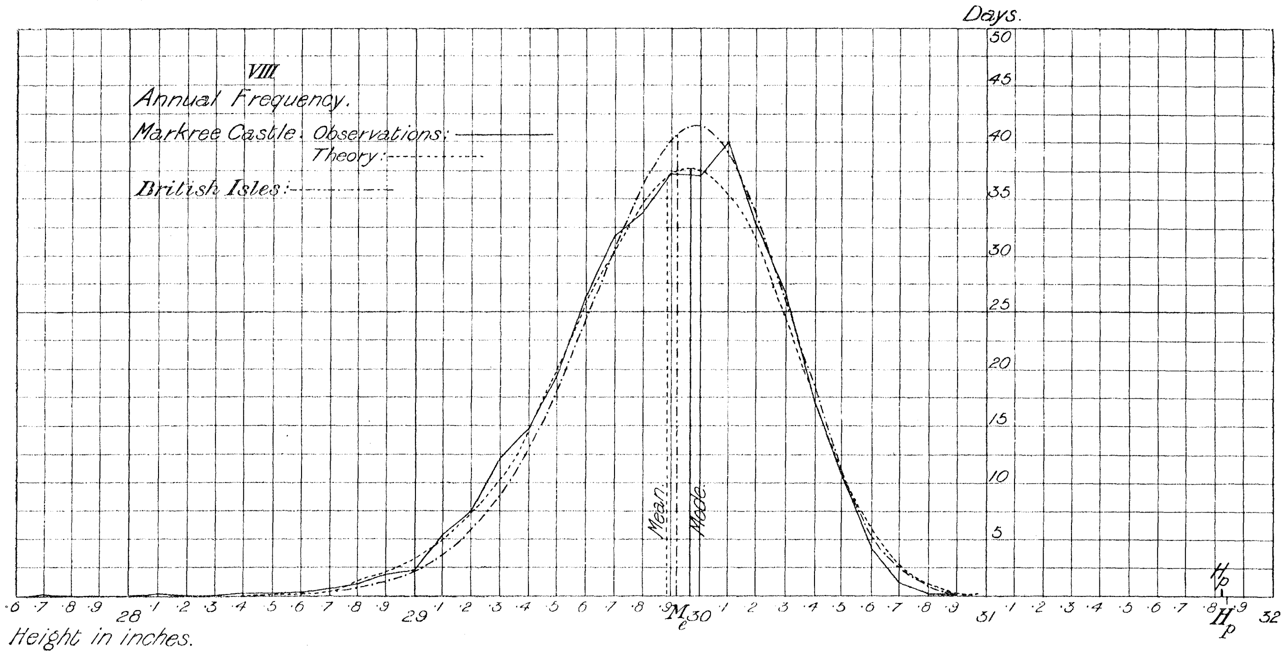
Annual Frequency.
Parsonstown. Observations:
Theory:
British Isles:
Days
Mean
Mode
M_e
H_p
Height in inches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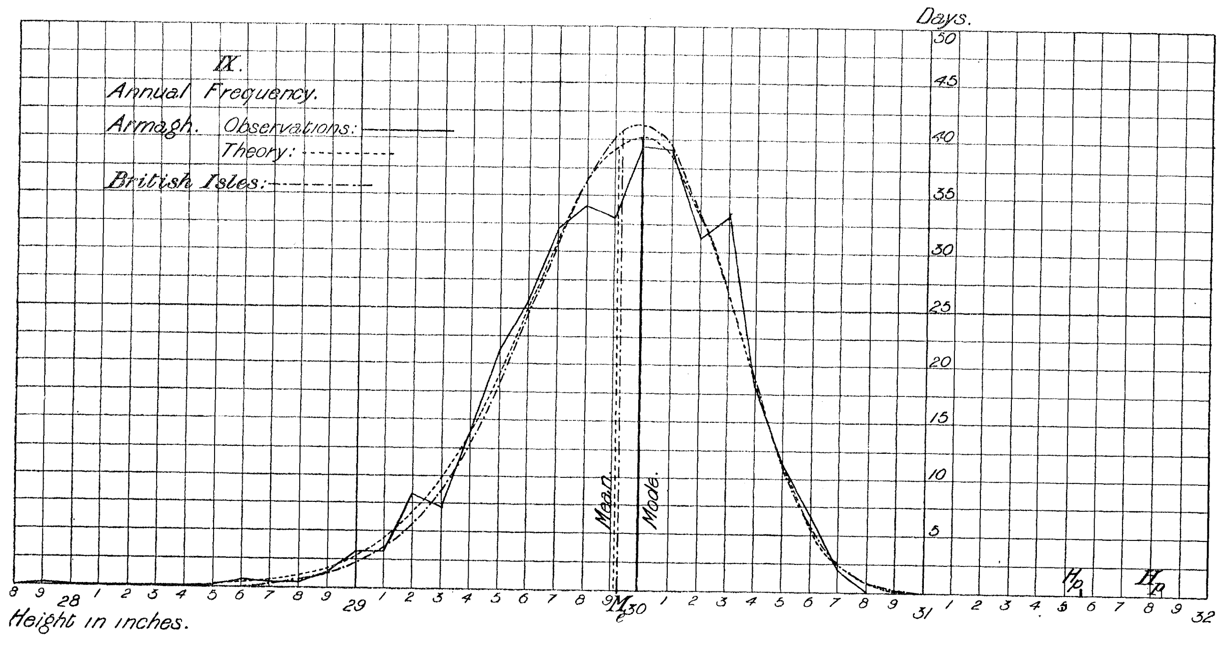
Markree Castle. Observations:
Theory:
British Isles:
Days.
Mean
Mode
M_e
H_p
Height in inches.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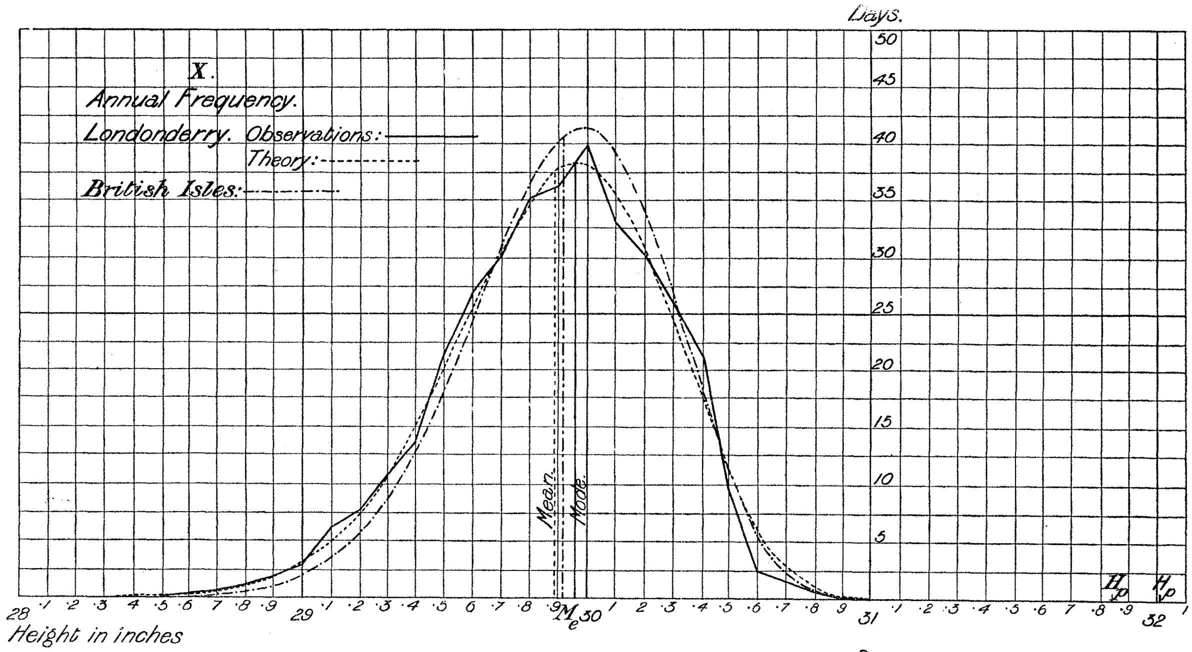
Armagh. Observations:
Theory:
British Isles:
Days.
Mean
Mode
M_e
H_p
Height in inches.


Days.
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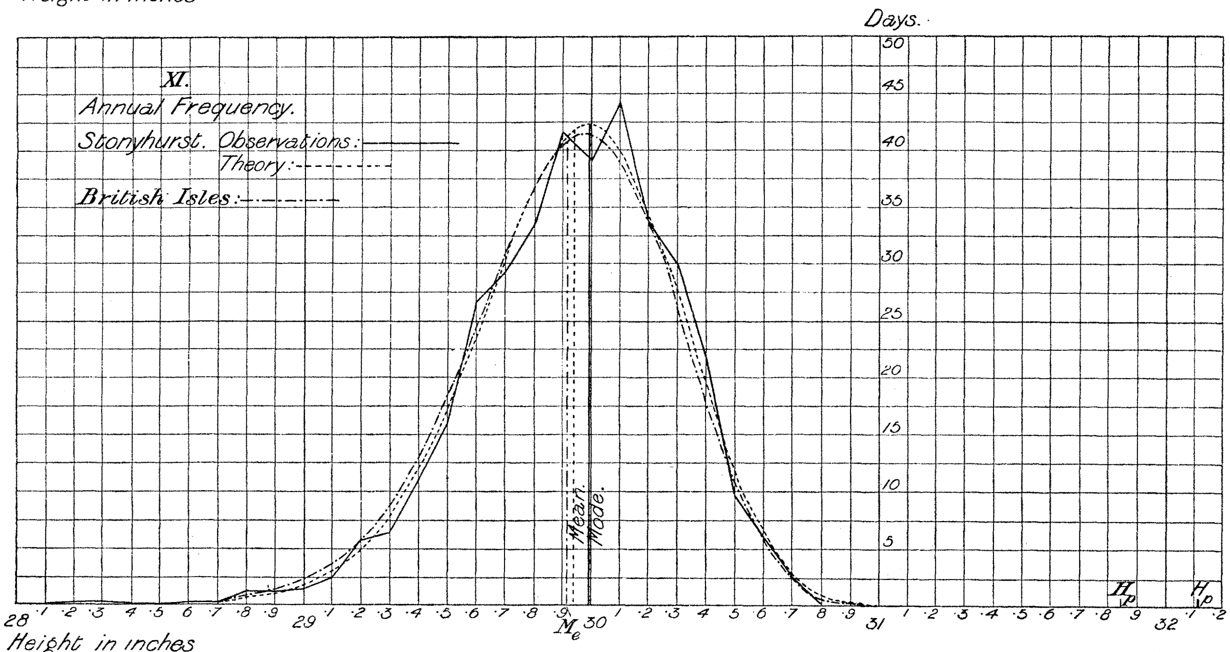
Annual Frequency.
Londonderry. Observations:
Theory:
British Isles:
Mean.
Mode.
M_e
H_p
Height in inches


Days.
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Annual Frequency.
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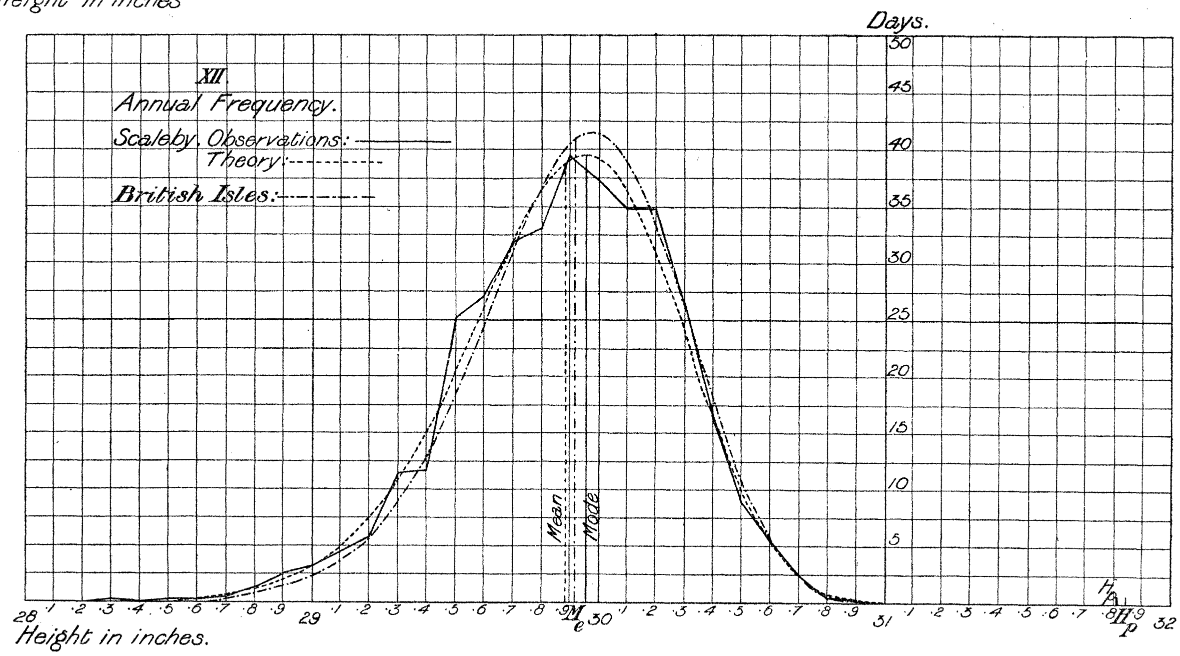
British Isles:
Mean.
Mode.
M_e
H_p
Height in inches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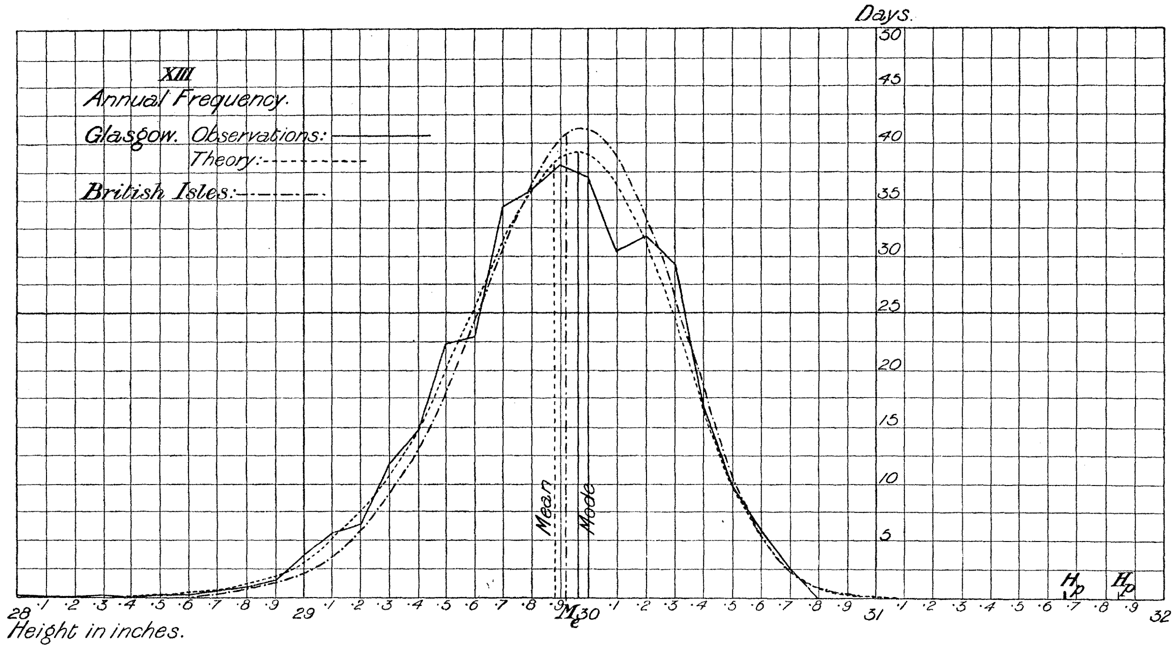
Scaleby. Observations:
Theory
British Isles:
Mean
Mode
M_e
H_p
Height in inches.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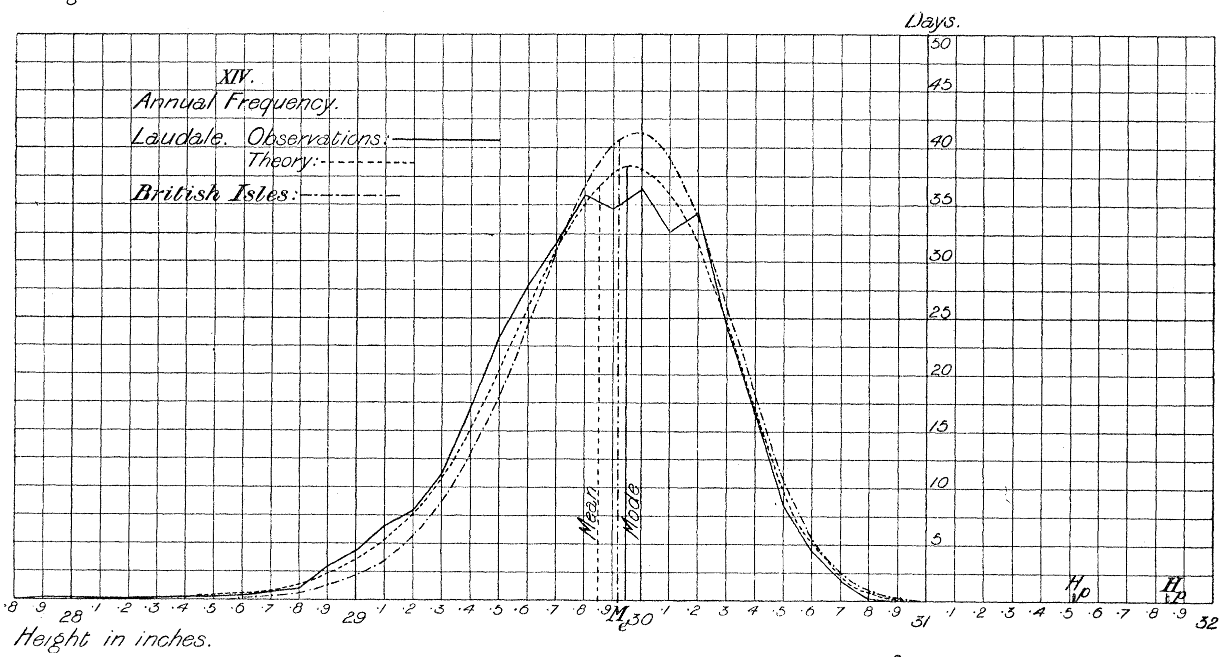
British Isles:
Days.
Mean
Mode
M_e
H_p
H_p
Height in inches.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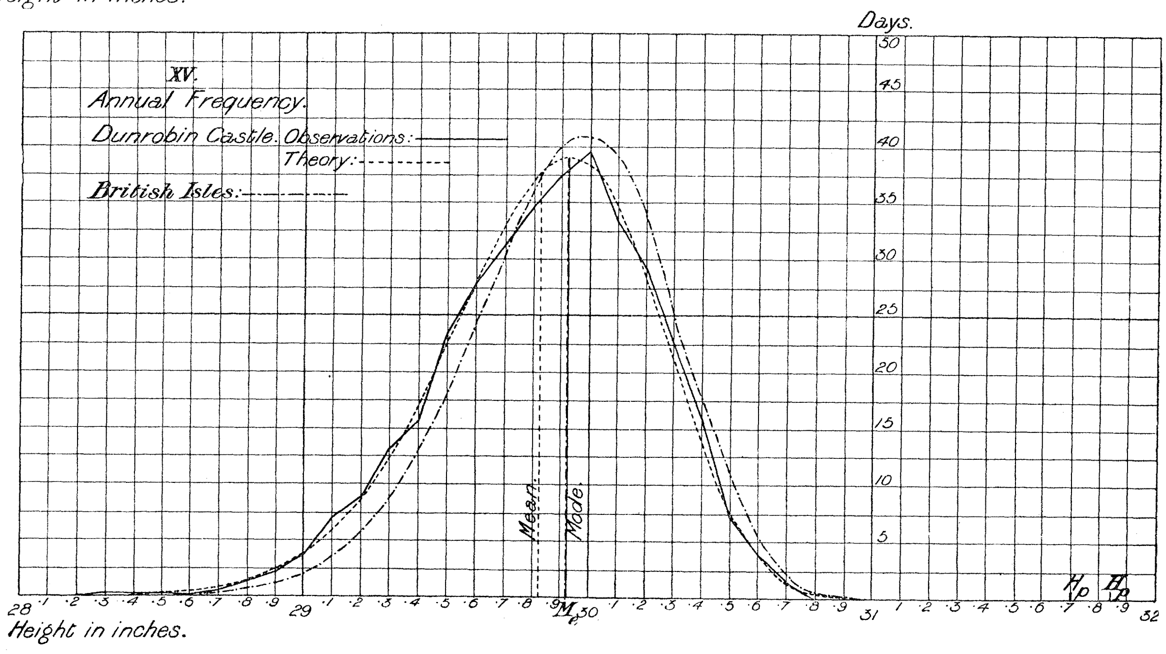
British Isles:
Days.
Mean
Mode
M_e
H_p
H_p
Height in inches.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Annual Frequency.
Dunrobin Castle. Observations:
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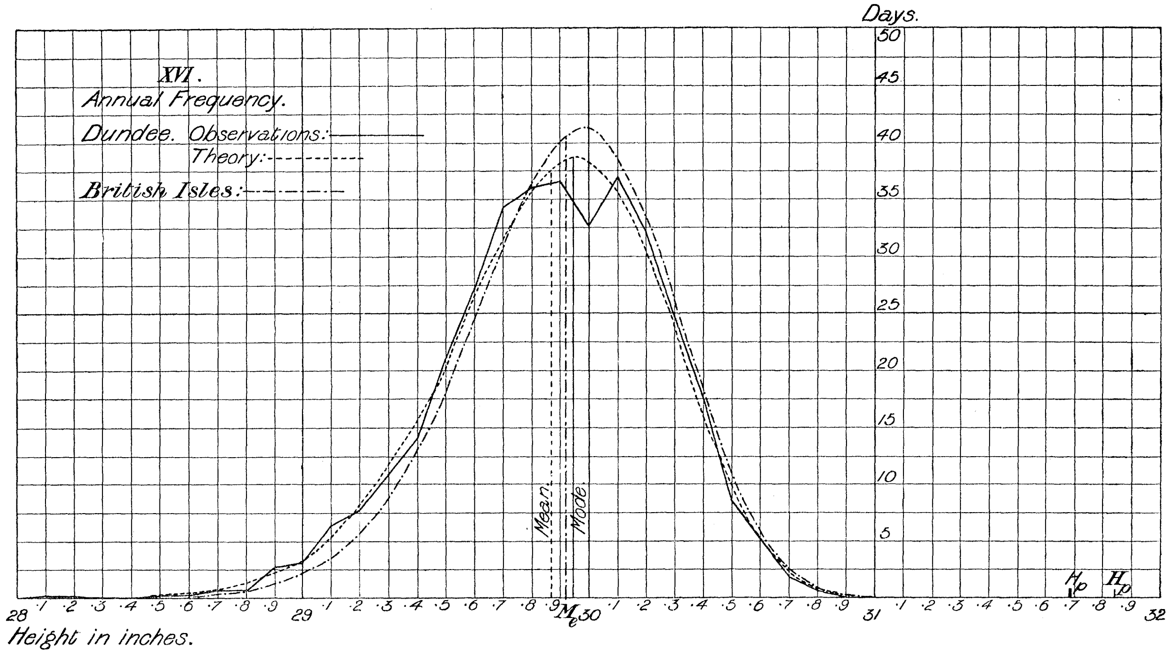
British Isles:
Days.
Mean
Mode.
M_e
H_p
H_p
Height in inches.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Days.
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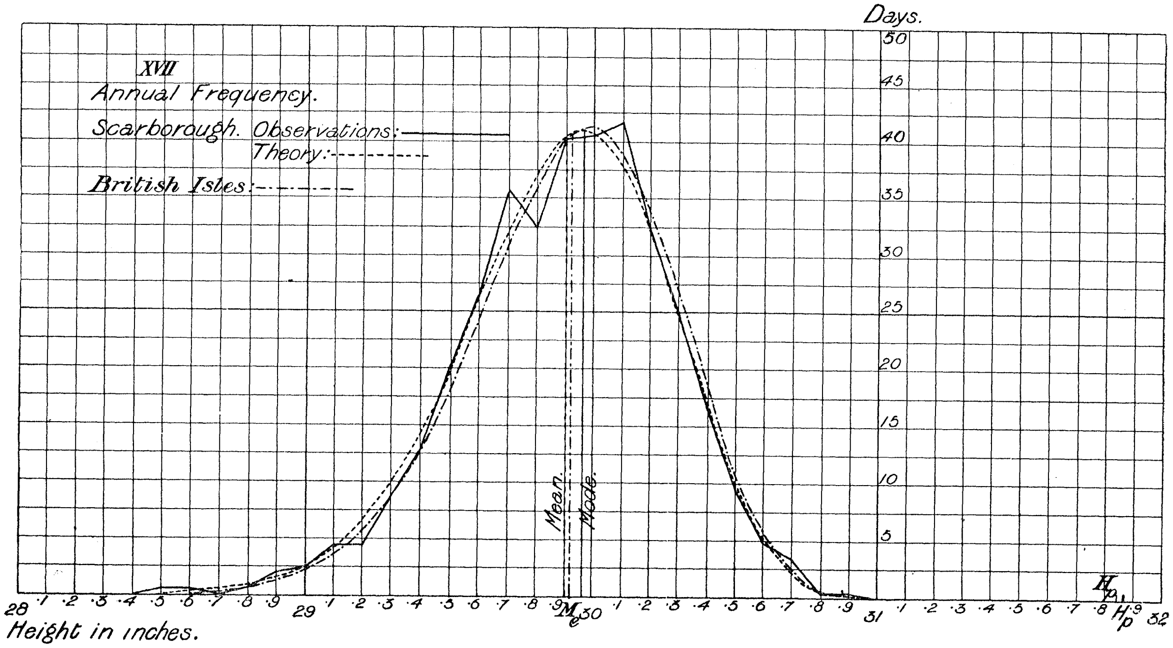
Mode.
M_e
H_p
Height in inches.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Scarborough. Observations:
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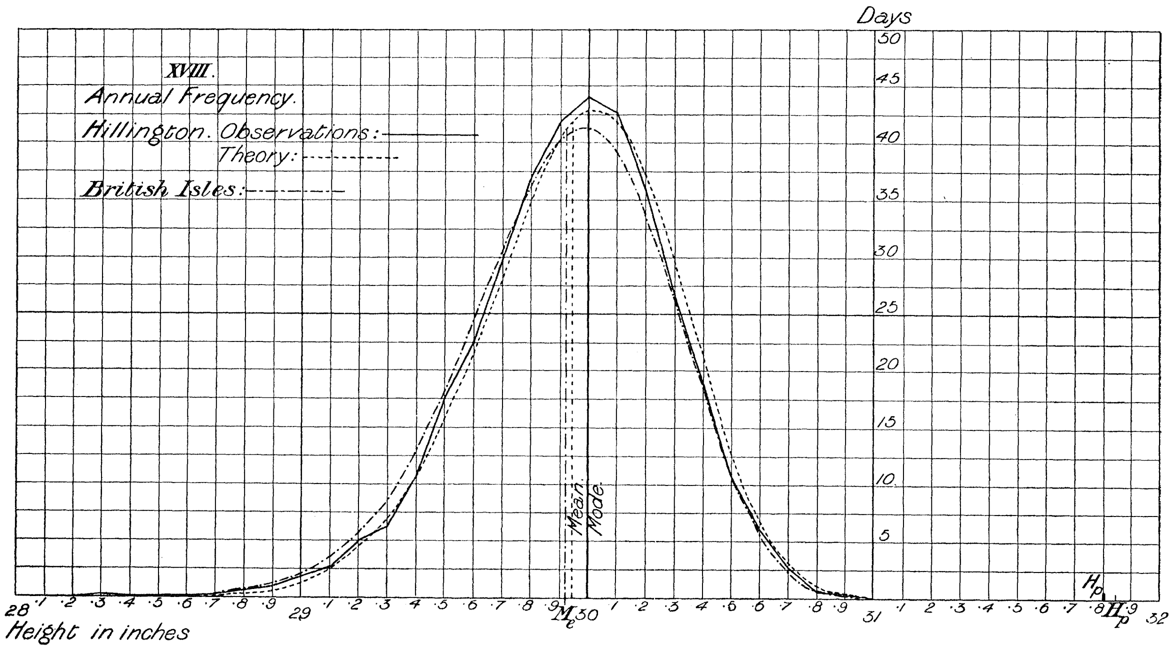
British Isles:
Days.
Mean.
Mode.
M_e
H_p
Height in inches.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Hillington. Observations:
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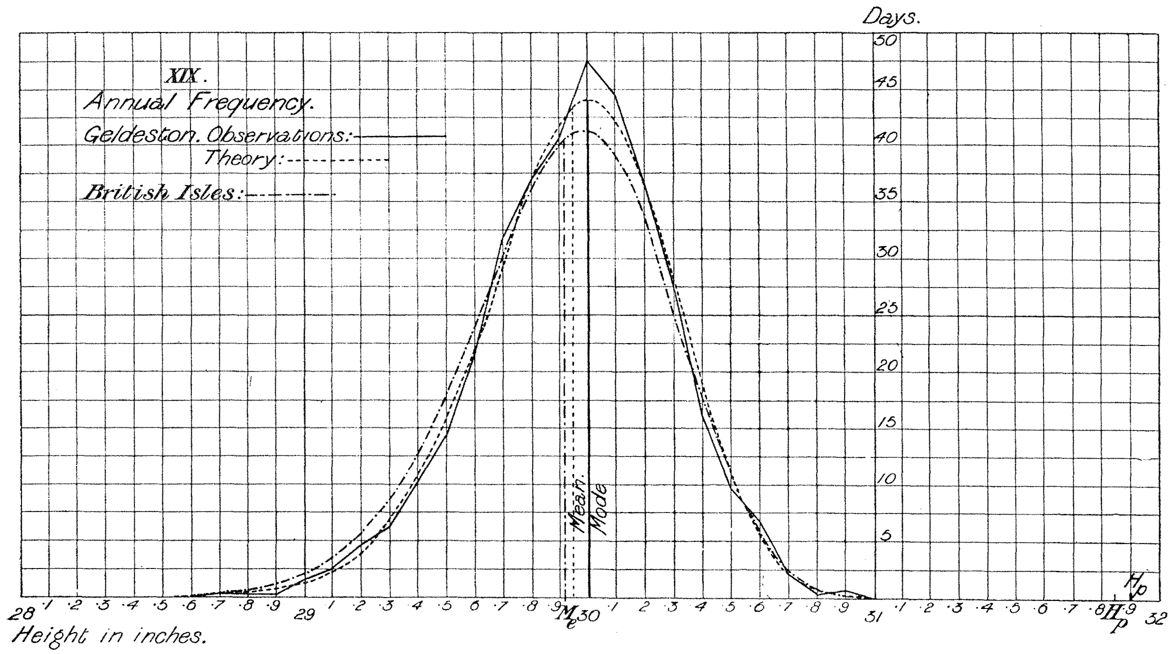
British Isles:
Days
Mean.
Mode.
M_e
H_p
Height in inches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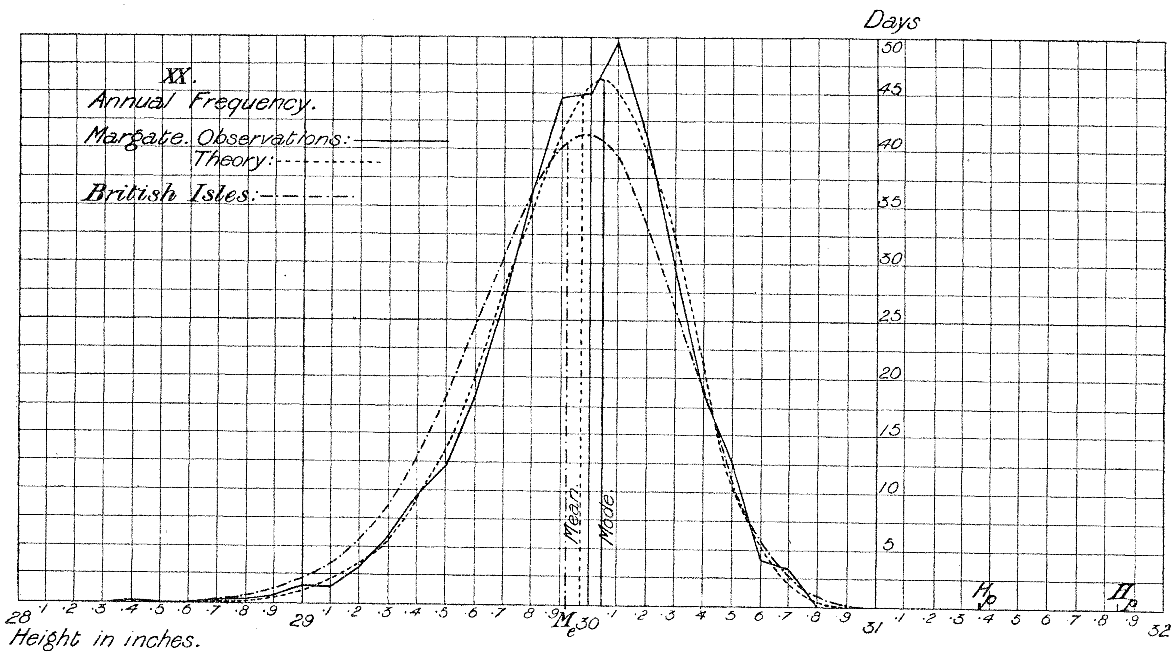
Theory:
British Isles:
Mean.
Mode
Height in inches.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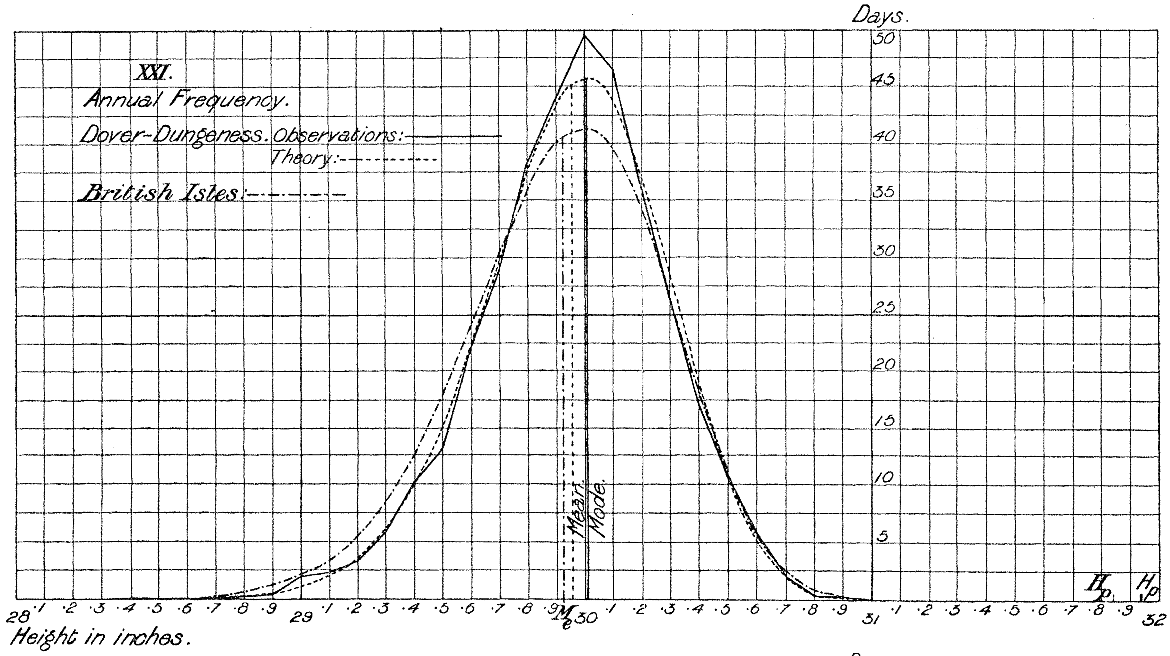
Margate. Observations:
Theory:
British Isles:
Mean.
Mode
Height in inches.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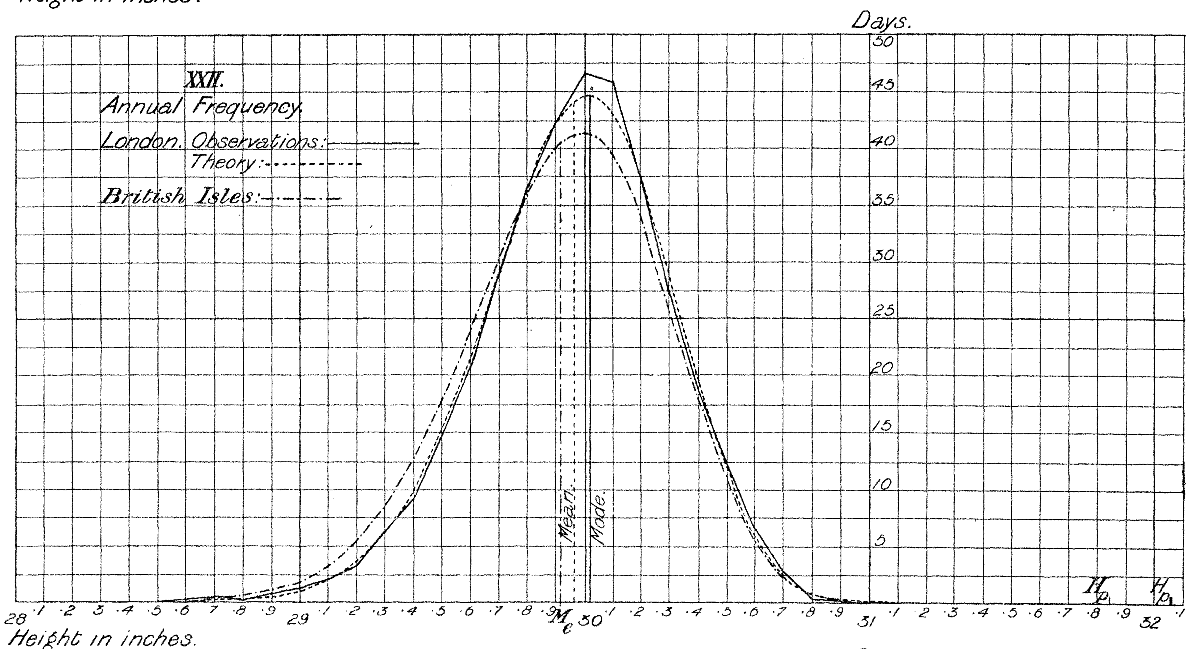
Theory:
British Isles:
Mean.
Mode.
M_e
H_{p_1} H_p
Height in inches.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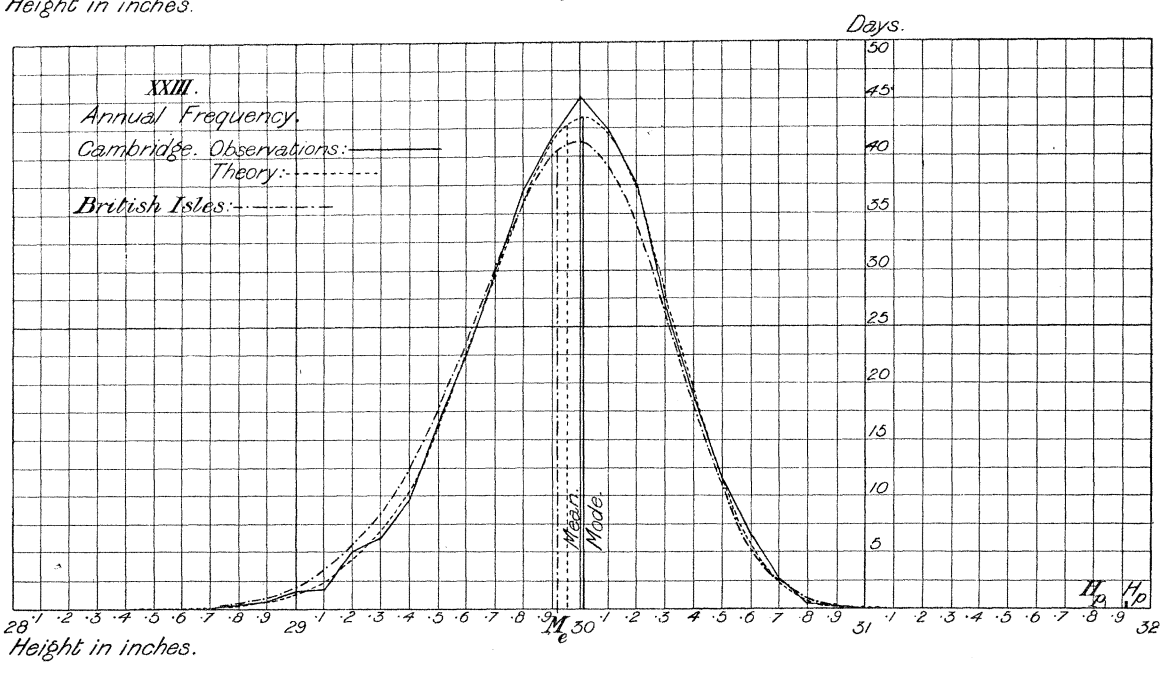
Mean.
Mode.
M_e
H_p H_{p_1}
Height in inches.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Theory:
British Isles:
Mean.
Mode.
M_e
H_{p_1} H_p
Height in inches.