XI. *Experiments on Aneroid Barometers at Kew Observatory, and their Discussion.*

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Preliminary.

§ 1. The ordinary aneroid barometer is an instrument whose chief recommendations are its portability and the ease with which it can be used by travellers. It is essentially a field instrument, and it would be largely waste of time to treat it as if intended for the laboratory.

The first object of the present investigation is to acquire knowledge likely to increase the usefulness of the aneroid under the conditions in which it is actually employed.

Aneroids have for many years been tested at Kew Observatory, and records exist of the performance of many hundreds of various sizes. The test has hitherto been applied as follows: The aneroid has been put in the receiver of an air pump, and,

pressure being reduced at the rate of 1 inch in 3 or 4 minutes, readings have been taken of the aneroid and a mercury gauge attached to the pump. The readings are taken with the pressure temporarily stationary, usually for each inch, but sometimes for each half-inch of pressure. After the reading has been taken at the lowest point for which verification is desired the pressure is reduced a very little further, and then maintained constant for some minutes. It is then allowed to rise by the readmission of air, stoppages being made and readings taken during the ascent of pressure precisely as during the descent.

From the observed differences between the aneroid and gauge—the readings of the latter being corrected of course for temperature—corrections are calculated separately for the descending and ascending readings of the aneroid, and these are given on the certificate issued.

If the aneroid possesses, as is most usual, a scale of altitudes in feet or metres, that is read as well as the pressure scale, and corrections are given to it based on Airy's table connecting altitude and barometric pressure. Whether the unrestricted application of Airy's table is the best means of interpreting observations taken during mountain ascents is an interesting question, but I do not propose to discuss it here, confining my attention to the aneroid as a measurer of pressure.

Aneroids intended for the Meteorological Office are usually constructed only for the narrow range, 31 to 26 inches. In their case there are exact regulations as to the size of error permissible and other points, and the issue of a certificate implies the attainment of a certain standard of excellence. With this exception, however, there exist no rules for rejection, and the certificate merely represents the report of an expert on an instrument submitted to him. The owner is supposed to draw his own conclusions from the figures submitted. This view of a certificate was apparently that originally dominant at Kew Observatory, but of late years there has been an increasing tendency on the part of the public to substitute the idea of a "certificate of excellence" for that of a "certificate of examination." What is sought is protection against inferior instruments rather than the means of applying corrections to observed readings. It has thus become desirable to ascertain how the excellence of an aneroid may be judged of, and one of the chief objects of the present investigation was to obtain data suitable for the purpose.

General Character of Phenomena.

§ 2. An aneroid is usually graduated by direct reference to a mercury gauge during a reduction of pressure. An examination, made under the same conditions as the graduation, tests the accuracy of the workman, but not necessarily the quality of the instrument. That something more is desirable becomes obvious, when, after reducing the pressure several inches, one keeps it constant and continues comparative readings of the gauge and aneroid for some time. A gradual fall in the readings of the latter

soon becomes apparent. Again, on allowing the pressure to rise to its original value, one finds the aneroid read lower than at the start, this depression gradually disappearing. The instrument, in fact, behaves as if an imperfectly elastic body. The strain, under a uniform stress, tends to increase, and as a natural concomitant, there is elastic after-effect (elastische Nachwirkung). That the whole of the difference between readings with pressure descending and ascending (or, as we shall call them for brevity, descending and ascending readings) represents, in all cases, a true after-effect, I am not prepared to say. If we regard a rise of temperature as analogous to a fall of pressure, a thermometer presents a somewhat parallel case; and we know that there the difference between ascending and descending readings is due to at least two causes, viz., change in the mercury meniscus, with consequent alteration of internal pressure, and temporary change of zero following exposure to the higher temperature. Of these two causes, the latter only is a true after-effect; but the former may also conceivably have its counterpart in the mechanism of the aneroid.

An aneroid showing large after-effect is not a suitable instrument for travellers. Readings taken with it, for instance, during a mountain descent, are largely influenced by the elevation of the summit and the time spent there. Thus, from the outset, I have regarded the after-effect phenomena as specially requiring investigation.

§ 3. As a first step, I examined the records of about 300 aneroids, tested over the ranges 30–15, 30–18, 30–21, 30–23, 30–24, and 30–26 inches. The differences between the descending and ascending readings were noted for the several ranges, and the general character of the phenomena ascertained. There existed, however, no data for connecting the results from the several ranges, it being customary to test each aneroid over one range only. Special experiments were thus requisite, in which the same aneroids should be taken over a variety of ranges. For such a purpose it was impossible to employ the ordinary working apparatus, with a due regard to the regular work of the Observatory. Application was accordingly made by the Kew Observatory Committee to the Government Grant Fund, and £30 was obtained for a new air-pump and receiver, and for a set of aneroids to be experimented on.

The only special feature of the apparatus* is a second or auxiliary receiver, between the pump and the main receiver which contains the aneroids. On the tube connecting the two receivers are two stop-cocks, a third cock being placed between the auxiliary receiver and the pump. One can thus exhaust the auxiliary receiver separately, and then by manipulating the cocks lower the pressure in the main receiver at any desired rate. This arrangement avoids the sudden changes of pressure to which aneroids are exposed when in a receiver connected directly to the pump. The pump, the receivers, and the mercury gauge—which shared the pressure of the main receiver—were rigidly attached to a stout board which rested on a table. The

* The apparatus was obtained from Mr. J. J. Hicks, 8, Hatton Garden, London. It was solidly constructed and proved very satisfactory. The experimental aneroids were likewise purchased from Mr. Hicks.

board was tapped before each reading, this being the invariable practice in the ordinary Observatory test. Though not free from objections, this seems to reproduce most closely natural conditions.

Differences of Descending and Ascending Readings, from Kew Verifications.

§ 4. It being most convenient to extract data from books not in current use at the Observatory, I have employed, in the following discussion, data from aneroids tested between 1885 and 1891. I have, however, examined a sufficient number of the more recent data to assure myself that the ordinary aneroid has since then undergone no important modification. On the whole, there is, perhaps, a slight reduction in the average size of the after-effect phenomena.

Table I. shows the mean excess, in inches, of the descending over the ascending reading at each inch of pressure for the several ranges specified, also the sum of the differences between the descending and ascending readings, and the mean difference for each range. It also mentions the number of instruments on which each set of results is based. The results, with the exception of those from the second group of 30 aneroids taken over the range 30–24 inches, are shown graphically in the several curves of fig. 1. Abscissæ represent air pressures, ordinates the excess of descending over ascending readings.

Fig. 1.

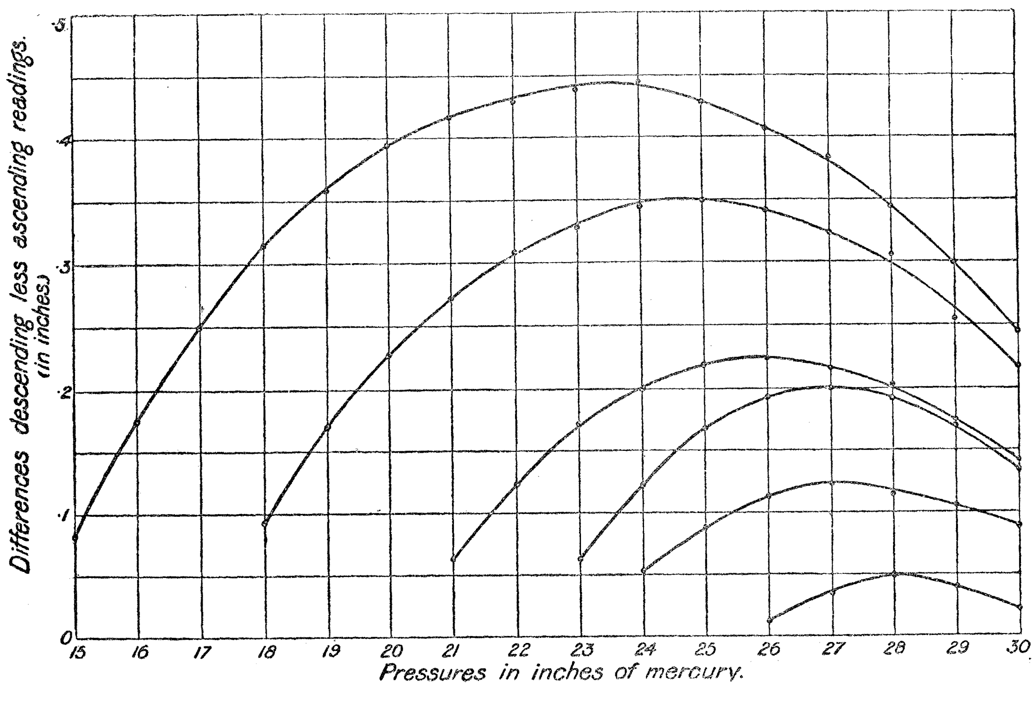


TABLE I.—Descending less Ascending Readings in inches (old records).

Range.	Number of instruments utilised.	Pressure in inches.																Sum of differences.	Mean difference.
		15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30		
inches. 30–26	101	..	..	..	..	..	..	..	..	..	..	..	·014	·035	·050	·037	·020	·156	·031
„ 24	13	..	..	..	..	..	..	..	..	..	·053	·086	·115	·124	·114	·105	·087	·684	·098
„ 24	30	..	..	..	..	..	..	..	..	..	·051	·086	·138	·135	·126	·084	·093	·102	·015
„ 23	33	..	..	..	..	..	..	..	..	·064	·120	·168	·193	·199	·191	·172	·133	1·240	·155
„ 21	44	..	..	..	..	..	..	·064	·120	·174	·199	·218	·223	·215	·204	·174	·141	1·732	·173
„ 18	35	..	..	..	·093	·171	·228	·271	·310	·328	·346	·350	·341	·323	·306	·254	·219	3·540	·272
„ 15	50	·083	·176	·249	·315	·357	·395	·417	·429	·438	·446	·428	·408	·385	·346	·299	·245	5·416	·338

The second group of aneroids tested over the range 30–24 inches were all large and of a special make, and so best treated apart. They showed such exceptionally small after-effect that I multiplied the actually-observed mean differences by seven, except in the last two columns, to make the figures similar in size to those given by the first group of 13 aneroids tested over the same range. In this way one sees more clearly that the general laws obeyed by the after-effect are to all appearance independent of its absolute amount.

In no case were the aneroids read to nearer than 0·01 of an inch, the third significant figure in Table I. being introduced merely in taking arithmetical means. Thus the law of variation throughout the range of the quantity tabulated is difficult to ascertain with great exactness over the shortest ranges, especially when the aneroids are of such exceptional quality as the second group tested between 30 and 24 inches.

When results are derived from a number of aneroids, such disturbing factors as local irregularities of graduation lose their influence; whereas with one or a small number of aneroids they may affect the smoothness of the results, however numerous be the observations.

§ 5. A glance at fig. 1 shows a similarity of type in the curves; but this is somewhat obscured by the variety in the pressure ranges and in the lengths of the maximum ordinates. The method of bringing this similarity into clearer relief will be most easily followed by taking a particular case, say, that of the range 30–21 inches. Here there are 10 differences of descending and ascending readings. Call these, starting from 21 inches and proceeding upwards, $d_1, d_2 \ldots d_{10}$. The mean difference is

$$\bar{d} \equiv (d_1 + d_2 + \ldots + d_{10})/10,$$

and $d_1/\bar{d}$, $d_2/\bar{d}$, &c., are the ratios of the several differences to the mean. Now draw a curve whose ordinates represent the size of these ratios on any convenient scale, while the abscissæ represent the corresponding fractions, 0, ·1, ·2, &c., of the range measured from the lowest pressure. The curves obtained in this way for the four longest ranges of Table I. are shown in fig. 2. In a short range the number of points on the curve determined by the data is so small as to leave the shape a little uncertain. Thus to have given together curves for all the data in Table I. would have sacrificed the clearness of detail without securing adequate compensation. It will be observed that the curves in fig. 2 are almost coincident near the central part of the range. At the lower end of the range there is a slight but appreciable difference, the ordinate tending to decrease as the range becomes longer. The closeness of the curves, striking as it is, would, I think, be improved by altering the cycle and the method of treating the results in the following three respects:—

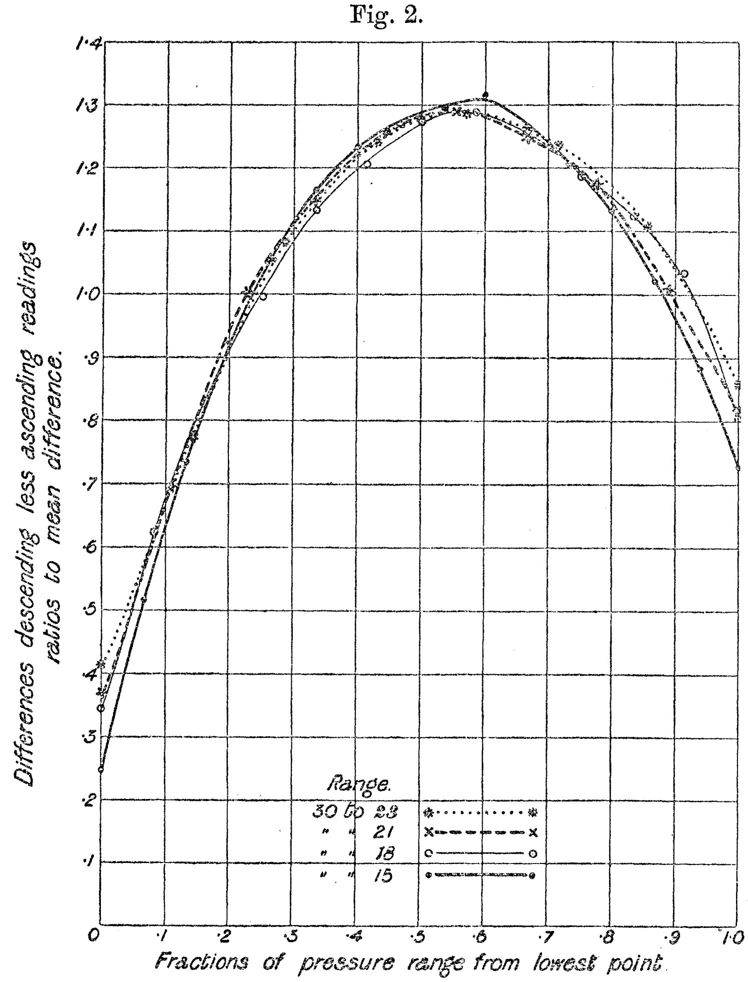
(i.) Making the duration of the stoppage at the lowest point proportional to the length of the range.

(ii.) Making the number of points of observation the same in all ranges by suitably varying the size of the pressure steps.

(iii.) Allowing only half-weight to the two terminal differences; for instance in the range 30–21 inches taking $\bar{d}$ as the mean of nine quantities, of which $(d_1 + d_{10})/2$ is one.

The last of these changes was, of course, within my power, but as its advantages after all could not be large, and might be nil, it did not seem worth while to undertake the large amount of arithmetic that would have been entailed by its substitution for the method originally adopted.

Fig. 2.



§ 6. In actual practice, the change (ii.) specified above would be inconvenient, entailing readings at fractions of inches difficult to determine exactly on an ordinary gauge. In ranges where the points of observation are sufficiently numerous the same object can be obtained pretty satisfactorily by interpolation. For instance, in the range 30–21 inches, the middle of the range is 25·5 inches, and $(d_5 + d_6)/2$ may be regarded as a tolerably good first approximation to the corresponding difference.

I have calculated in this way from Table I., in the case of all but the shortest range, the differences of the descending and ascending readings answering to the fractions 0, ·1, ·2, ·3, &c., of the range, measured from the lowest point. For the range 30–24 inches the first group of aneroids was used. The ratios borne by these calculated differences to the mean differences in Table I. are given in Table II.

It should be noticed that the mean of the 11 differences found for the fractions 0, ·1, &c., of the range is not generally identical with the difference $\bar{d}$ given in Table I.; consequently the sum of the 11 entries in each row of Table II. is not identically 11, though nearly so. In deciding to take the $\bar{d}$ of Table I. as the consequent of the ratios, I avoided the confusion entailed by having two slightly different *mean differences* for each range. There are, however, disadvantages, which seemed to me to prevail in the case of the mean results for the 5 ranges of Table II. In their case, accordingly, I made the small alterations required—an increment of less than $\frac{1}{7}$ per cent.—to make the sum of the eleven ratios exactly 11.

TABLE II. (Old Observations).—Ratios of Differences Descending less Ascending Readings to Mean Difference.

Range.	Fraction of range—										
	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0
30 inches to 24	·541	·743	·936	1·115	1·210	1·265	1·204	1·145	1·089	·998	·888
„ „ 23	·413	·666	·898	1·100	1·213	1·265	1·274	1·238	1·159	1·034	·858
„ „ 21	·370	·661	·943	1·106	1·215	1·274	1·270	1·223	1·143	·986	·814
„ „ 18	·342	·670	·900	1·081	1·192	1·271	1·278	1·226	1·149	·971	·804
„ „ 15	·245	·628	·931	1·111	1·232	1·281	1·318	1·235	1·137	·953	·724
Mean for 5 ranges	·382	·674	·923	1·104	1·214	1·273	1·270	1·215	1·137	·989	·819

Before treating further the results shown in Tables I. and II. it is convenient to notice some of the special experiments.

Differences of Descending and Ascending Readings. Special Experiments.

§ 7. The first 24 special experiments were intended to serve for comparison of the phenomena over different pressure ranges. They took place as follows, the numbers showing the chronological order :—

Range.	Order of experiments.	Number over range.
30 inches to 26	Nos. 1, 2, 12, 13, 22, 23, 24	7
„ „ 24	3, 4, 5, 20, 21	5
„ „ 21	6, 7, 18, 19	4
„ „ 18	8, 9, 16, 17	4
„ „ 15	10, 11, 14, 15	4

Two aneroids, No. 1 and No. 4, were subjected to all these experiments; two others, No. 2 and No. 3, divided only to 20 inches, were not taken below 21 inches of pressure.

The order of the experiments was adopted with a view to the possible permanent influence of the lower pressures on the aneroids.

The procedure in these experiments was uniform. The change of pressure during both the descent and the ascent was at the rate of 1 inch in 5 minutes. At the lowest point there was a stoppage of 10 minutes, preceded however by no slight lowering of pressure below the point at which the last descending reading was taken.

After each experiment the aneroids were left for some days at atmospheric pressure, so as to be fully rested at the beginning of each pressure cycle. The experiments were made in 1895, and extended over a period of five months. As will be seen later, there would not appear to have been any serious change in the after-effect phenomena during this interval.

§ 8 The first thing investigated was the law of variation throughout each range of the differences of the descending and ascending readings, it being advisable to make sure that the aneroids were fair specimens, and gave results sufficiently similar in type to those derived from the old verification books. I have thought it unnecessary to reproduce the results analogous to Table I., but pass at once to those analogous to Table II. The procedure followed was exactly the same as that already described, so that the results of Tables III. and II. are immediately comparable.

TABLE III. (Special Experiments).—Ratios of Differences Descending less Ascending Readings to Mean Difference.

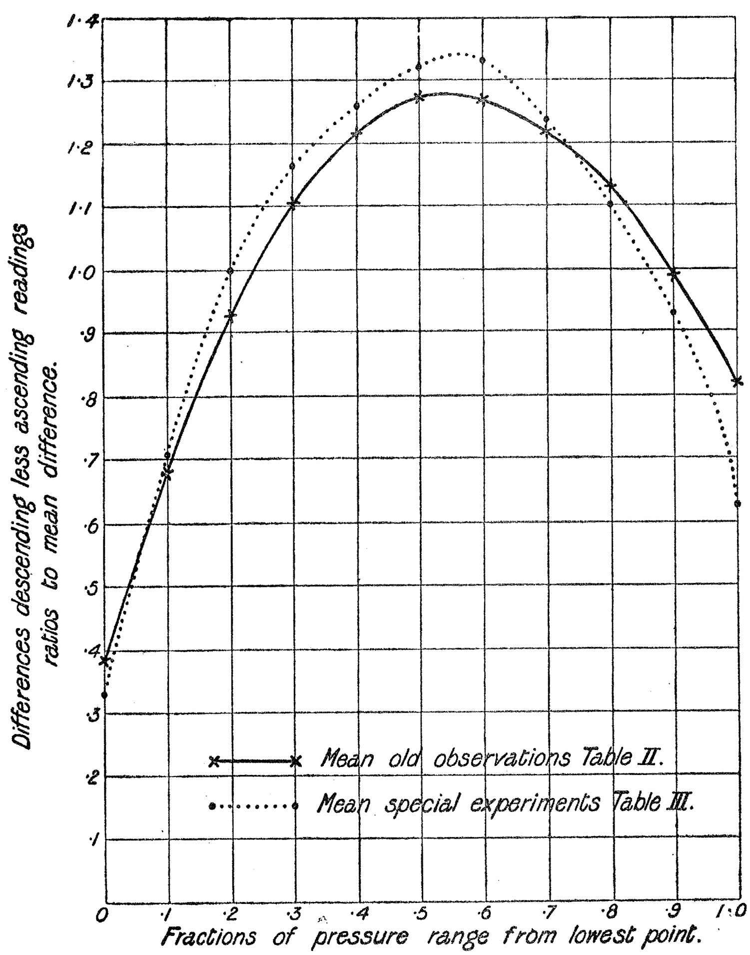
Range.	Fraction of range.										
	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0
30 inches to 26	·430	·707	0·984	1·168	1·260	1·352	1·292	1·231	1·140	1·017	·895
„ „ „ 24	·437	·771	1·029	1·137	1·195	1·228	1·293	1·309	1·230	0·974	·630
„ „ „ 21	·294	·708	0·992	1·163	1·306	1·334	1·250	1·169	1·116	0·975	·693
„ „ „ 18	·262	·715	0·940	1·142	1·276	1·416	1·425	1·227	1·049	0·887	·495
„ „ „ 15	·197	·650	1·084	1·228	1·255	1·323	1·442	1·274	0·989	0·823	·433
Mean for 5 ranges	·323	·707	1·002	1·163	1·252	1·325	1·334	1·237	1·100	0·931	·626

The alteration required to make the sum of the ratios in the last line exactly 11, was slightly greater than in the case of Table II., but still was less than one half per cent.

There is a tendency in the values at the extremities of all the ranges to be lower in Table III. than in Table II., entailing of course a slight difference in the opposite direction towards the centre of the range. This is probably due mainly to the difference between the procedure followed in the old and new observations. The

difference in the results, though worthy of notice, is for practical purposes of little consequence. As we shall see later in Table VI., unity in Table III. represents at the very utmost 0·25 of an inch, so that a difference of 4 or 5 per cent. in one of the ratios answers to a quantity which is in general considerably less than the probable error of an observation.

Fig. 3.



To facilitate comparison of the old and new observations, I show side by side in fig. 3, curves of the same type as in fig. 2, the one representing the mean results of Table II., the other the mean results of Table III. The former curve is exceptionally smooth, and the latter even is very fairly regular; but curves for the individual ranges of Table III.—which I do not reproduce—show very appreciable irregularities. This of course is only what one would expect from the limited number of observations. Whilst the data in Table III. thus possess defects which might have been much reduced by an increase in the number of experiments, they appeared quite sufficiently consistent for the purpose I had in view.

Formulæ for the Variation of the Differences of the Descending and Ascending Readings throughout the Range.

§ 9. The absolute size of the differences between the descending and ascending readings varies largely from aneroid to aneroid; thus formulæ reproducing the data of Table I. would not be immediately useful. On the other hand, the law of variation, as shown by the ratios of these differences, at different points of the range, to the mean difference, appears to be nearly the same in all ordinary aneroids. I have thus tried to represent the variations shown in Tables II. and III. by simple algebraic formulæ of the type

$$y = a_0 + a_1x + a_2x^2 + \ldots \qquad (1),$$

where y is the ratio of the difference of the descending and ascending readings to the mean difference, x the fraction of the range measured from the lowest point.

One can secure absolute agreement between the observed and calculated values in such a case by taking enough terms; but, for practical purposes, the question is whether with a comparatively small number of terms one can secure a sufficiently close agreement. In the present instance, four terms are enough for practical purposes. To get the best general agreement we ought, of course, to determine a_0, a_1, a_2, a_3 by the method of least squares, taking account of the whole eleven values in each range. In dealing, however, with material such as that here available, the slightly increased accuracy thus attainable would not, in my opinion, be an adequate return for the labour expended. I thus simply determined the constants so that the observed and calculated values were identical for the values ·1, ·3, ·7, and ·9 of x. The values thus found for the constants are given in Table IV., the first six sets of values relating to the old observations of Table II., the last set to the mean data of Table III.

TABLE IV.—Values of Constants in $y = a_0 + a_1x + a_2x^2 + a_3x^3$.

Range.	a_0.	a_1.	a_2.	a_3.
30 inches to 24	+·425	+3·680	−5·209	+2·031
„ „ „ 23	+·332	+3·677	−4·079	+0·958
„ „ „ 21	+·322	+3·808	−4·264	+0·948
„ „ „ 18	+·377	+3·219	−2·947	+0·115
„ „ „ 15	+·264	+4·067	−4·391	+0·802
Mean old observations	+·345	+3·699	−4·186	+0·969
„ new „	+·358	+3·922	−4·362	+0·792

The value of a_1 shows little variation. In the shortest range the values of a_2 and a_3 are undoubtedly unduly large numerically, and in the range 30–18 inches there is

a similar defect in the opposite direction. In these cases the method of least squares would no doubt have given values closer to the mean.

The closeness with which the data of Tables II. and III. are reproduced by the formulæ based on the above values of the constants is shown by Table V., which gives the algebraical excess of the observed over the calculated values. There is, it will be remembered, absolute agreement when $x = \cdot 1, \cdot 3, \cdot 7$, and $\cdot 9$.

TABLE V.—Observed less Calculated Values. (Ratios of Differences to Mean Difference.)

Range.	$x =$							Mean of differences.
	0	·2	·4	·5	·6	·8	1·0	
30 inches to 24	+·116	−·033	+·016	+·048	+·007	+·014	−·039	±·039
,, ,, ,, 23	+·081	−·014	+·002	−·006	−·003	+·006	−·020	±·019
,, ,, ,, 21	+·048	+·021	−·009	−·004	−·007	+·019	·000	±·015
,, ,, ,, 18	−·035	−·004	−·009	+·007	+·005	+·023	+·040	±·018
,, ,, ,, 15	−·019	+·024	−·006	−·018	+·022	+·018	−·018	±·018
Mean old observations	+·037	−·003	−·003	+·003	−·004	+·015	−·008	±·010
,, new ,,	−·035	+·028	−·024	−·002	+·022	−·010	−·083	±·029

There is not a single instance here where the difference between the observed and calculated values represents as much as 0·01 of an inch in absolute pressure.

For ordinary purposes—unless special importance is attached to the phenomena at the two ends of the range—the formula

$$y = \cdot 35 + 3\cdot 7x - 4\cdot 2x^2 + \cdot 97x^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

is quite good enough for experiments made on the old plan over any range not shorter than 6 nor longer than 15 inches.

Position of Maximum Difference, Descending less Ascending Reading.

§ 10. As illustrating the practical use of (2), let us find where it makes the difference between the descending and ascending readings largest. For this, putting

$$\frac{dy}{dx} = 0 \text{ in } (2),$$

we find

$$2\cdot 91x^2 - 8\cdot 4x + 3\cdot 7 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3),$$

whence

$$x = \cdot 54.$$

Here x, it will be remembered, represents the fraction of the range measured from the lowest point. To test the accuracy of this result, I have calculated the position of the maxima for the several ranges from Table I., which contains, it should be noted, the direct results of observation. The results are as follows :—

	Range 30 inches to—						Mean.
	26	24	23	21	18	15	
Position of maximum, $x =$	·56	·50	·54	·56	·58	·58	·55

Relations between Phenomena for different Ranges.

§ 11. Our previous work may be regarded as determining the differences between the descending and ascending readings at every point of any range in terms of the mean difference for that range. The next step is to express the mean difference for a range of n inches as a product of $f(n)$—a function whose sole variable is n—and a factor which is constant for a given aneroid.

The mean differences between the descending and ascending readings (defined as in § 5) given by the first 24 experiments with the 4 aneroids are recorded in Table VI.

TABLE VI.—Mean Differences (in inches) from the first 24 Special Experiments.

	Range 30 inches to—				
	26	24	21	18	15
Aneroid No. 1 . . .	·043	·084	·146	·217	·315
,, ,, 2 . . .	·033	·063	·123	..	..
,, ,, 3 . . .	·046	·089	·179	..	..
,, ,, 4 . . .	·027	·043	·077	·122	·190

The most convenient way of dealing with these results is to find the ratios borne by the several mean differences to one of their number for each aneroid separately. For this purpose, I have taken the differences in the range 30–21 inches as standards. The results are given in Table VII.

TABLE VII.—Ratios of Mean Differences to Mean Difference in Range 30–21.

	Range 30 inches to—				
	26.	24.	21.	18.	15.
Aneroid No. 1	·295	·575	1	1·486	2·158
" " 2	·268	·512	1		
" " 3	·257	·497	1		
" " 4	·351	·558	1	1·584	2·468
Mean for all 4 aneroids	·293	·535	1		
" " Nos. 1 and 4	·323	·566	1	1·535	2·313

Assume now that the ratio of the mean difference $\bar{d}_n$ for a range of n inches to the mean difference $\bar{d}_9$ for a range of 9 inches is given by

$$\bar{d}_n/\bar{d}_9 = \phi(n) = b_0 + b_1 n + b_2 n^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

Determining b_0, b_1, b_2 from the three ranges in Table VII. common to all four aneroids, we find

$$\phi(n) = -\cdot 028 + \cdot 053 n + \cdot 0068 n^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5),$$

with of course

$$\phi(9) = 1.$$

In arriving at this expression, we utilised only the data from the three shortest ranges in Table VII. Thus a check on the general suitability of the formula is supplied by the good agreement between the mean results for the two longest ranges in the table and the corresponding values

$$\phi(12) = 1\cdot 587, \qquad \phi(15) = 2\cdot 297$$

calculated from (5).

§ 12. As illustrating the practical application of (4) and (5), suppose we find that in a particular aneroid the mean difference between the descending and ascending readings for the range 30–15 inches is 0·338 inch. Then we should conclude that the mean difference for a range of m inches, m being any specified number, is given by

$$\bar{d}_m = \cdot 338 \times \phi(m) \div \phi(15).$$

For instance, we should find for this aneroid

$$\bar{d}_4 = \cdot 043, \quad \bar{d}_6 = \cdot 078, \quad \bar{d}_7 = \cdot 099, \quad \bar{d}_9 = \cdot 147, \quad \bar{d}_{12} = \cdot 234.$$

The value assumed in the above illustration is that found in Table I. for the range 30–15 inches. Hence the values just deduced for $\bar{d}_4$, &c., are what the mean differences for the other ranges would have been in the case of the average aneroid sent for trial over the range 30–15 inches. Comparing these calculated values with the mean differences actually recorded in Table I., we should draw the conclusion that the average aneroid sent for testing over the longest range is less subject to after-effect than the average aneroid sent for testing over any other range except the shortest. This is, I believe, really the case. The exception in favour of the shortest range is due to the exceptional size of a number of the aneroids intended to cover the range 30–26 inches.

§ 13. The sum of the differences of the descending and ascending readings is really the source of our knowledge of the mean differences, and for some purposes it may well be the more convenient quantity of the two. The formula for it is deducible at once from (5), for

$$\frac{\text{Sum of differences when range } n \text{ inches}}{\text{Sum of differences when range 9 inches}} = \frac{n+1}{10}\phi(n),$$
$$= -{\cdot}0028 + {\cdot}0025n + {\cdot}0060n^2 + {\cdot}00068n^3 \quad . \; . \; . \; (6),$$
$$= \psi(n), \text{ say.}$$

The following results are easily verified :—

$$\psi(4) = {\cdot}147, \quad \psi(6) = {\cdot}375, \quad \psi(9) = 1, \quad \psi(12) = 2{\cdot}066, \quad \psi(15) = 3{\cdot}680.$$

Using S_n to denote the sum of the differences for a range of n inches, then, according to the formula,

$$S_n \div \psi(n) = S_m \div \psi(m)$$

for all values of n and m.

To illustrate the degree of accuracy attained by the use of such formulæ, I have taken the case of the first 24 special experiments, and determined the arbitrary constant for each aneroid so as to make the total sum of the differences over all the ranges for which the instrument was tried the same for the calculated as the observed values. As it may be well to show how this was done, take the case of aneroid No. 2. Here the sums of the differences observed in the three shortest ranges were respectively ·163, ·443 and 1·228 inches, amounting in all to 1·834; while by the formula,

$$\psi(4) + \psi(6) + \psi(9) = 1{\cdot}522.$$

Thus, for a range of n inches with this aneroid, we may assume

$$\text{Sum of differences} = \psi(n) \times 1{\cdot}834 \div 1{\cdot}522.$$

In Table VIII. C denotes the values calculated in this way; O, those actually observed.

TABLE VIII.—Sums of Differences Descending less Ascending Readings.

	Range 30 inches to—									
	26		24		21		18		15	
	O.	C.	O.	C.	O.	C.	O.	C.	O.	C.
Aneroid No. 1	·214	·204	·589	·523	1·463	1·393	2·820	2·878	5·040	5·126
,, ,, 2	·163	·177	·443	·452	1·228	1·205	..	..	..	..
,, ,, 3	·231	·254	·622	·651	1·790	1·737	..	..	..	..
,, ,, 4	·134	·118	·304	·302	0·770	0·804	1·588	1·661	3·048	2·959

The differences between the observed and calculated values in Table VIII. are of the same order as those met with in different experiments over the same range under the most favourable conditions.

§ 14. The preceding results go a long way to secure one of the principal objects of the investigation—the more complete utilisation of the ordinary Kew test. Suppose, for instance, an aneroid has been taken over any convenient range in the usual way, and a summation made of the differences of the observed descending and ascending readings. This sum, being based on a considerable number of independent readings, is but little affected by errors of observation, if ordinary care is used. It is also but little affected by any ordinary irregularities in the sub-divisions, and thus affords a satisfactory basis for a general diagnosis of the quality of the instrument so far as after-effect is concerned.

By means of §§ 11, 12, and 13 we can deduce from the observed sum, or from the mean difference, the values of the corresponding quantities for any other range. In this way a common standard of excellence could be utilised for all aneroids of given type, irrespective of the range covered by their graduation.

§ 15. In the laboratory one can change the pressure at a uniform rate and keep the temperature within narrow limits. In the field, however, the conditions are very variable. It was thus obviously desirable to aim at a fuller knowledge of the properties of aneroids, which might be of service to a wider circle than that concerned with the mere testing of these instruments.

Some valuable work in this direction was done many years ago by Dr. BALFOUR STEWART, and more recently by Mr. EDWARD WHYMPER. Neither writer, however, made any very serious attempt to ascertain the exact connection between the several phenomena. I thus postpone consideration of their work to a later stage, when its significance will be more easily understood.

Whilst I regard the present experiments as throwing much fresh light on the subject, it will, I hope, be clearly understood that I do not profess to have established a complete physical theory, by the aid of which one can foretell the exact behaviour of an aneroid under any arbitrary set of conditions.

Fall of Reading at the Lowest Point.

§ 16. In the 24 experiments described in § 7, readings were taken at 2-minute intervals during the 10 minutes' stoppage at the lowest pressure. It was soon obvious that the fall in a given time was at least approximately proportional to the range, *i.e.*, to n, where $30-n$ represents the lowest pressure in inches.

To show this in a simple way, I give in Tables IX. and X. the mean results obtained by multiplying the observed falls by (18,000 ÷ range). The constant was chosen so as to avoid decimals. The actual fall in inches is of course obtainable by multiplying the figures in the table by (range ÷ 18,000).

TABLE IX.—Fall at Lowest Point × (18,000 ÷ range).

Range.	Aneroid No. 1.					Aneroid No. 4.				
	Time.					Time.				
	2	4	6	8	10	2	4	6	8	10
30 inches to 26	39	45	71	77	90	26	39	45	45	51
,, ,, 24	24	54	54	78	102	6	30	36	42	60
,, ,, 21	30	65	80	85	95	10	30	30	30	40
,, ,, 18	34	53	68	83	98	11	15	23	30	41
,, ,, 15	27	45	63	78	87	12	15	21	30	36
Means . .	31	52	67	80	94	13	26	31	35	46

TABLE X.—Same quantity as in Table IX.

Range.	Aneroid No. 2.					Aneroid No. 3.				
	Time.					Time.				
	2	4	6	8	10	2	4	6	8	10
30 inches to 26	32	39	58	64	71	32	45	64	71	77
,, ,, 24	18	54	54	66	90	24	54	66	84	108
,, ,, 21	20	55	60	65	80	30	60	65	80	95
Means . .	23	49	57	65	80	29	53	65	78	93

Considering the limited number of experiments, the irregularities in the tables are by no means surprising, when we reflect that even at 15 inches pressure the average fall of reading in 10 minutes was only about ·07 of an inch for aneroid No. 1, and ·03 of an inch for aneroid No. 4. In the case of the readings after 2 or 4 minutes' exposure to pressures such as 26 or 24 inches, the uncertainty is of course much greater. It will be noticed that the figures for aneroids Nos. 1 and 3 are very similar and are approximately double the figures for No. 4.

§ 17. Subsequently three experiments, Nos. 33, 34, and 35, were devoted to elucidating the law connecting the fall of reading with the time elapsed since the stationary pressure was reached. Pressure was reduced at the normal rate, 1 inch in 5 minutes, but kept for $2\frac{1}{2}$ hours at the lowest point of the range, readings being taken at intervals. The lowest pressures in these three experiments were respectively 24, 21, and 18 inches. It would occupy too much space to give full details, but Tables XI. and XII. show the falls observed at 30-minute intervals. To avoid decimals the observed falls are multiplied by 10^4/range; so, to deduce the falls in inches, the figures in the table must be multiplied by $n \times 10^{-4}$, where n is the number of inches in the range.

TABLE XI.—Fall at Lowest Pressure $\times$ (10^4/range).

Range.	Aneroid No. 1.					Aneroid No. 4.				
	Time, in minutes.					Time, in minutes.				
	30	60	90	120	150	30	60	90	120	150
30 inches to 24	83	133	150	155	172	50	78	83	83	100
,, ,, 21	89	111	133	156	156	44	44	66	78	89
,, ,, 18	92	108	142	158	175	50	67	83	83	92

TABLE XII.—Same quantity as in Table XI.

Range.	Aneroid No. 2.					Aneroid No. 3.				
	Time, in minutes.					Time, in minutes.				
	30	60	90	120	150	30	60	90	120	150
30 inches to 24	100	128	155	167	172	117	161	172	183	205
,, ,, 21	111	122	144	156	178	111	133	156	178	189

In these tables each entry depends on only one experiment, so the agreement with the law, "fall proportional to range," could hardly be better.

In the interval between these experiments and those dealt with in Tables IX. and X., aneroid No. 1 had broken down and been altered. The figures obtained from the other three aneroids stand to one another in almost exactly the same proportion as before.

§ 18. To show more clearly that the law of variation of the fall of reading with the time was nearly if not exactly the same for all four aneroids, I have in the following Table XIII. multiplied the figures actually found for Nos. 1, 2, and 4 by 6/5, 8/7 and 2 respectively. The results are the means of the several ranges included in Tables XI. and XII., but the observations at all the time intervals are included.

TABLE XIII.—Quantity for Aneroid No. 3, same as in Table XII.; for the others, multipliers are applied, as explained above.

Aneroid.	Time, in minutes.																	
	2	4	6	8	10	15	20	25	30	45	60	75	90	100	105	120	135	150
No. 1 . . .	7	34	68	86	83	101	89	96	106	128	140	158	170	173	174	187	196	202
„ 2 . . .	10	31	51	70	71	91	102	109	120	134	143	155	170	183	184	185	191	200
„ 3 . . .	9	33	47	69	75	89	98	103	114	133	147	156	164	173	175	180	189	197
„ 4 . . .	16	40	46	58	58	78	94	100	96	124	126	142	154	160	162	162	192	198

After trying some logarithmic functions, I found that the results in Table XIII. were more in accordance with a formula of the type

$$\text{fall, under stationary pressure} = Ct^{q} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7),$$

where t denotes time elapsed since the pressure became stationary, while C and q are constants *for a given previous rate of fall.* C varies, of course, from aneroid to aneroid, but to all appearance there was at least a close approach to equality in the values of q for the above four aneroids. Accordingly, I took the mean of the results from all the aneroids in Table XIII., and determined q by trial. Table XIV. compares the mean thus found for the observed values with results calculated from the formula

$$\text{constant} \times \text{fall} = (31{\cdot}3)\, t^{\cdot 369} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8).$$

The corresponding mean results from the first 24 experiments, as given in Tables IX. and X., are added.

TABLE XIV.—Law of Fall of Reading under Constant Pressure.

	Time, in minutes.																	
	2	4	6	8	10	15	20	25	30	45	60	75	90	100	105	120	135	150
Calculated. . . .	40	52	61	67	73	85	95	103	110	127	142	154	165	171	174	183	191	199
Observed from—																		
Expts. 33 to 35 .	11	34	53	71	72	90	96	10[illegible]	109	130	139	153	164	172	174	179	192	199
„ 1 to 24 .	22	42	51	60	73													

After the first 6 minutes the agreement between the calculated and observed values could hardly be better. The observed values at 2 and 4 minutes are conspicuously lower than the calculated, but this discrepancy is least in the case of the 24 experiments, which, on account of their much greater number, should give a more reliable mean than the 3 special experiments. Taking either set of figures, however, one would most naturally draw the conclusion that the formula (8) does not apply exactly when t is small. Another explanation is, however, possible. On an average, an aneroid's reading at two successive intervals would be taken as unchanged if the true alteration were less than ·005 of an inch, and this unquestionably was likely to happen frequently in the case of the readings at $t = 0$ and $t = 2$. That the falls observed, during at least this interval, suffered from this or some similar cause, can hardly be doubted, for it is most improbable, as would appear from the 3 special experiments, that the fall in the first 2 minutes should be less than the fall in the second 2. Again, as all the operations, such as the reading of the aneroids and gauge, occupied some time, it is possible that the method of procedure adopted really tended to curtail the first 2-minute interval. It is certainly a little suggestive that the values calculated for $t = 2, 4, 6$, are so nearly identical with those observed in the first 24 experiments at $t = 4, 6, 8$, respectively.

§ 19. In connection with §§ 16 to 18 several considerations should be borne in mind. The change of pressure prior to the stationary stage is gradual. The circumstances are not analogous to that of a wire suddenly loaded. Thus, the fact that the fall of reading in a given time is proportional to the pressure range can hardly be regarded as merely a special case of the ordinary after-effect law. As we shall see very clearly later, the phenomenon is largely dependent on the way in which the pressure is reduced. A preliminary stoppage on the way down, or a change in the rate of reduction of pressure, each exert an important influence; the one affecting the magnitude of the fall, the other its law of variation with the time.

Recovery after Pressure Cycle.

§ 20. As already stated, it is customary for an aneroid to read lower on completion of a pressure cycle than at its commencement. Let the departure from the original reading be called the *deficiency*, and let D_t represent the deficiency when t minutes have elapsed after return to the original pressure. Thus D_0—which we may call the *original deficiency*—is the difference between the descending and ascending readings, answering to the pressure of 30 inches.

According to the first 24 experiments, D_0 is, at least approximately, proportional to the pressure range. The evidence for this conclusion is summarised in Table XV.; the figures are deduced from the mean values calculated from the several experiments over the same range.

TABLE XV.—Value of $D_0 \times$ (1800/range) in inches.

Range.	Aneroid No.			
	1.	2.	3.	4.
30 inches to 26	15	13	16	13
,, ,, 24	19	12	17	9
,, ,, 21	19	15	29	11
,, ,, 18	17	..	..	9
,, ,, 15	16	..	..	10

The abnormal entry 29 for aneroid No. 3 is really due to the occurrence of appreciable permanent changes of zero in the course of 2 experiments over the range 30–21 inches. On one of these occasions, after a full day's rest, the aneroid read no less than ·08 inch lower than before the experiment.

The irregularity in the other results is due partly to the comparative fewness of the experiments, and partly to the fact that the pressure at the end of the experiment was in reality to some extent different from that at the start. It was usual, in fact, to treat the atmospheric pressure at the time being as 30 inches, unless it exceeded this value by several tenths of an inch.

§ 21. In the 24 experiments the aneroids were read at intervals of 5, 10, 15, 20, 60, 120, and 1440 minutes after the return to atmospheric pressure. The similarity of the phenomena for the several aneroids and the several ranges is brought out more clearly by tabulating D_t/D_0 than D_t itself. This is done in Table XVI. Initially, of course, *i.e.*, when $t = 0$, D_t/D_0 is unity.

TABLE XVI.—Recovery after Conclusion of Pressure Cycle, Values of D_t/D_0.

Range.	Aneroid No.	$t =$ 5.	10.	15.	20.	60.	120.	1440.
30 inches to 26	1	·74	·65	·52	·39	·22	·13	·17
	2	·65	·65	·50	·40	·25	·20	·25
	3	·72	·64	·48	·44	·32	·16	·20
	4	·62	·57	·57	·52	·43	·52	·19
	Mean	·68	·63	·52	·44	·31	·25	·20
" " 24	1	·84	·76	·60	·56	·44	·48	·16
	2	·81	·69	·62	·44	·25	·31	·25
	3	·83	·70	·57	·48	·39	·39	·13
	4	·92	·75	·67	·50	·58	·75	·17
	Mean	·85	·73	·62	·50	·42	·48	·18
" " 21	1	·74	·68	·66	·61	·42	·32	·03
	2	·77	·77	·68	·58	·45	·35	·16
	3	·86	·81	·74	·69	·53	·48	·29
	4	·81	·71	·67	·57	·43	·48	·14
	Mean	·80	·74	·69	·61	·46	·41	16
" " 18	1	·84	·73	·76	·67	·49	·40	·24
	4	·87	·74	·61	·57	·26	·22	−·09
	Mean	·86	·74	·69	·62	·38	·31	·08
" " 15	1	·79	·70	·66	·58	·32	·23	·04
	4	·82	·79	·70	·64	·39	·36	·33
	Mean	·81	·75	·68	·61	·36	·30	·19

In dealing with Table XVI. one must bear in mind the smallness of D_0 in the shorter ranges, especially in the case of aneroid No. 4. The law of variation of a quantity of the size ·03 or ·04 of an inch is difficult to ascertain with great exactitude unless one can read to much less than ·01. The figures, as they stand, certainly suggest that in its early stage the recovery was distinctly more rapid in the case of the shortest range than in any other. Since, however, there is no trace of a like phenomenon in the next shortest range, its reality is at least doubtful, and I should be disposed to attribute it to experimental errors.

Over the other ranges the law of recovery is unquestionably almost, if not exactly, identical; and if, instead of grouping the results under pressure ranges, one had grouped them under the several aneroids, one would have found that the mean results were almost identical for the four. Thus, in deducing the law of variation, I combined the results from all the aneroids, utilising all the ranges except the shortest. The mean values thus obtained are compared in Table XVII. with values calculated from the formula

$$D_t/D_0 = (1 + t \times \cdot 1357)^{-\cdot 369} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9),$$

where t represents as before the time elapsed in minutes since the return to atmospheric pressure.

TABLE XVII.—Law of Recovery, Observed and Calculated Values of D_t/D_0.

	t =							
	0.	5.	10.	15.	20.	60.	120.	1440.
D_t/D_0 observed . .	1	·83	·74	·67	·59	·41	·37	·15
,, calculated . .	1	·826	·729	·664	·616	·442	·349	·143

The agreement is certainly closer than I anticipated from the use of any formula with only two arbitrary constants. Of course I was led to try $-\cdot 369$ in the index of (9) from having already found $+\cdot 369$ in the index of (8); but this does not affect the significance of the fact that formulæ with the same numerical index should so closely reproduce the depression and recovery phenomena.

The expectation that (9) would apply to all aneroids treated as the 4 were, could, of course, only be justified by very wide experiment. As will be shown later, the law of recovery certainly depends on the rate and type of the pressure changes.

Effects of Temperature.

§ 22. Aneroids are usually "corrected for temperature," *i.e.*, there is a compensating arrangement to prevent the reading altering when the instrument is exposed to varying temperature at ordinary atmospheric pressure. The experimental aneroids, tried as usual at three temperatures at atmospheric pressure, appeared correctly compensated. It would appear, however, from the special experiments, that this is not incompatible with imperfect compensation at lower pressures.

Temperature is likely to influence several parts of the mechanism, and that possibly in more than one way. It presumably alters the elasticity of the vacuum box and iron spring, and causes slight changes in the dimensions of the apparatus. The compensation works by producing slight curvature in a lever, but as the position of the lever varies with the pressure, a curvature that suffices at one pressure may be insufficient at another. Again, temperature might influence the after-effect phenomena, and so modify the reading in a variety of ways.

I thus investigated the influence of temperature on the readings with pressure descending, on the fall at the lowest pressure, and on the differences of the descending and ascending readings. The experiments bearing most directly on the question consisted of a group, Nos. 51 to 55, made on five consecutive days, May 10 to 14, 1897. No. 51 was preliminary, and was not utilised in the calculations. Of the

others, Nos. 52 and 55 were made at a temperature of 50° F., Nos. 53 and 54 at a temperature of 81° F. Use was made of the room employed for testing chronometers, the temperature of which is readily controlled. Information as to temperature effects is also obtainable from the other special experiments, because, being taken at all seasons of the year in an ordinary room, they answered to considerably varied temperatures.

The investigation of the influence of temperature on the descending readings is complicated by the fact that the index error of an aneroid is a somewhat variable quantity. To get rid, so far as possible, of this uncertainty, I subtracted from all the readings a constant equal to the error observed at 30 inches. This is equivalent to making the instrument correct at the start of each experiment. The excesses of the readings so modified over the true pressures at lower points of the range I shall call the *corrected errors.*

Table XVIII. shows the algebraic excess of the corrected errors from the mean of experiments 52 and 55 over the corrected errors from the mean of experiments 53 and 54.

TABLE XVIII.—Corrected Errors temperature 50° F., less Corrected Errors temperature 81° F.

Aneroid.	Pressure in inches.								
	29	28	27	26	25	24	23	22	21
No. 2 . .	·005	·00	·04	·05	·055	·095	·09	·11	·13
" 3 . .	·01	·02	·055	·045	·065	·085	·085	·105	·12
" 4 . .	·015	·02	·03	·045	·065	·095	·085	·09	·11
" 8 . .	·02	·01	·035	·055	·055	·085	·075	·105	·115
Mean for 4 aneroids	·01	·01	·04	·05	·06	·09	·08	·10	·12
$\frac{\text{Mean corrected error}}{30 - \text{pressure}}$	·012	·006	·013	·012	·012	·015	·012	·013	·013

There is here a very decided fall of reading accompanying rise of temperature, and the fall appears to increase directly as the pressure interval measured from 30 inches.

§ 23. Similar conclusions follow from the experiments as a whole. For the two aneroids Nos. 2 and 3 there were 22 experiments over the range 30–21 inches at the normal rate. These I divided into two groups, the 11 colder in the one, the 11 hotter in the other, and found the mean corrected errors for the two groups separately. Aneroid No. 4, for some reason, changed its behaviour during a long rest between experiments Nos. 46 and 47. Prior to the change, however, there were 25 experiments in which the pressure was lowered to or below 21 inches. Twelve of these with temperatures from 77° to 71° formed the hotter group. The colder group

included the rest, but half-weight only was attached to each of two experiments in which the temperature was 70°. The treatment applied to these groups was the same as for aneroids Nos. 2 and 3. Aneroid No. 1 was somewhat erratic almost from the start, and the results from it are not reproduced: they showed clearly enough the same general phenomena as in the others. The results for aneroids Nos. 2, 3, and 4 are given in Table XIX. The temperatures quoted are the means for the colder and hotter groups.

TABLE XIX.—Corrected Errors, influence of Temperature.

Pressure in inches.	Aneroid No. 2.				Aneroid No. 3.				Aneroid No. 4.			
	56° F.	75° F.	Difference for 19° F.	Difference / 30 − pressure.	56° F.	75° F.	Difference for 19° F.	Difference / 30 − pressure.	61° F.	74° F.	Difference for 13° F.	Difference / 30 − pressure.
29	−·005	−·021	+·016	+·016	−·012	−·028	+·016	+·016	+·040	+·041	−·001	−·001
28	−·019	−·037	+·018	+·009	−·029	−·062	+·033	+·017	−·020	−·029	+·009	+·004
27	−·049	−·087	+·038	+·013	−·029	−·083	+·054	+·018	−·037	−·047	+·010	+·003
26	−·026	−·069	+·043	+·011	−·047	−·092	+·045	+·011	+·004	−·011	+·015	+·004
25	−·045	−·096	+·051	+·010	−·059	−·111	+·052	+·010	+·022	−·008	+·030	+·006
24	−·053	−·127	+·074	+·012	−·089	−·159	+·070	+·012	−·030	−·060	+·030	+·005
23	−·077	−·163	+·086	+·012	−·130	−·205	+·075	+·011	−·004	−·038	+·034	+·005
22	−·136	−·232	+·096	+·012	−·186	−·276	+·090	+·011	−·053	−·095	+·042	+·005
21	−·198	−·305	+·107	+·012	−·214	−·304	+·090	+·010	−·074	−·117	+·043	+·005

The experiments being numbered in chronological order, the arithmetic means of the experiment numbers in the several groups were, for aneroids Nos. 2 and 3, colder 38, hotter 37; for aneroid No. 4, colder 24, hotter 27. This alone would suffice to show that the phenomena cannot be ascribed to any gradual change in the aneroids.

§ 24. I next consider the possible influence of temperature on the fall of reading at the lowest pressure. Taking the 16 earliest experiments in which pressure was reduced at the normal rate to 21 inches, and then maintained steady for 10 minutes, I divided them into four groups, as follows:—

Group	I.,	Experiments	Nos.	6,	7,	18,	19.
,,	II.,	,,	,,	29,	30,	31,	32.
,,	III.,	,,	,,	34,	36,	39,	40.
,,	IV.,	,,	,,	41,	42,	43,	44.

Temperature did not vary much between individual experiments of the same group.

Table XX. gives the mean temperature for each group, and the mean falls of reading at the intervals 2, 4, 6, 8, and 10 minutes after the lowest pressure, 21 inches, was reached. To avoid decimals, the unit chosen is the $\frac{1}{400}$ of an inch.

TABLE XX.—Fall at Lowest Pressure (21 inches), unity = ·0025 inch.

Group.	Temperature, Fahrenheit.	Aneroid No. 1.					Aneroid No. 2.					Aneroid No. 3.					Aneroid No. 4.				
		Time.					Time.					Time.					Time.				
		2	4	6	8	10	2	4	6	8	10	2	4	6	8	10	2	4	6	8	10
I.	$66\frac{1}{2}^{\circ}$	6	13	16	17	19	4	11	12	13	16	6	12	13	16	19	2	6	6	6	8
II.	$50\frac{1}{2}$	3	7	11	18	18	4	7	10	16	17	4	9	10	15	18	0	2	4	6	6
III.	75	..	..	..	..	..	2	6	12	13	15	4	12	16	21	24	1	2	4	5	6
IV.	$72\frac{1}{2}$	..	..	..	..	..	3	8	12	16	18	4	8	11	16	20	1	3	7	11	12

The results in the last two groups for aneroid No. 1 are omitted as not comparable —owing to repair of the instrument—with the earlier ones.

In Table XX. there is no certain indication of any temperature effect.

In the special temperature experiments, Nos. 52 to 55, the observed mean falls in the colder and hotter groups were as follows :—

TABLE XXI.—Fall in 10 minutes at Lowest Pressure (21 inches) in inches (Experiments 52 to 55).

Experiments.	Temperature.	Aneroid.			
		No. 2.	No. 3.	No. 4.	No. 8.
52 and 55	50° F.	·035	·065	·03	·04
53 and 54	81° F.	·035	·05	·025	·025

There being only two experiments in each group, it would be unsafe to base on this any more definite conclusion than that if there is any temperature effect it is certainly not large.

§ 25. The influence of temperature on the differences of the descending and ascending readings next calls for consideration. The experiments bearing most directly on this point are the special ones, Nos. 52 to 55. The results derived from them are given in Table XXII., unity representing 1 inch.

TABLE XXII.—Mean Sums of Differences Descending less Ascending Readings (from Experiments 52, 53, 54, 55).

Experiments.	Temperature.	Aneroid.				Sum for 4 aneroids.
		No. 2.	No. 3.	No. 4.	No. 8.	
52 and 55	50° F.	0·96	1·35	1·19	0·98	4·48
53 and 54	81° F.	0·98	1·54	1·08	0·88	4·48

Here there is certainly no suggestion of any temperature influence.

The great majority of the later experiments had some peculiarity, such as a stoppage, which prevents their use for investigating the influence of temperature on the sum of the differences. I thus confine my attention to the earliest 24 experiments, subdividing those over each range into a colder and a hotter half.* The mean sums of the differences, in inches, are given in Table XXIII.

TABLE XXIII.—Mean Sums of Differences Descending less Ascending Readings (Experiments 1 to 24).

Aneroid.	Range 30 inches to 26.		Range 30 inches to 24.		Range 30 inches to 21.		Range 30 inches to 18.		Range 30 inches to 15.	
	Temperature.		Temperature.		Temperature.		Temperature.		Temperature.	
	57° F.	67° F.	66°½ F.	72° F.	62°½ F.	70°½ F.	68°½ F.	72° F.	69° F.	74°½ F.
No. 1	·207	·217	·530	·625	1·375	1·550	2·750	2·890		
„ 2	·170	·177	·400	·480	1·230	1·225				
„ 3	·217	·210	·500	·740	1·745	1·825				
„ 4	·143	·143	·290	·315	·785	·755	1·555	1·620	2·920	3·175

Some of the data, more especially those for the range 30 to 24 inches, suggest an increase in the sum of the differences as temperature rises. In the range 30 to 26 inches, however, where the temperature difference was greatest, there is no trace of temperature effect. In the two longest ranges the temperature difference was small and the experiments very few.

In the four experiments dealt with in Table XXII. the deficiency in the reading

* In the case of the range 30 to 26 inches the two groups consist of the three hotter and three colder experiments, the odd experiment being left out of account. The hotter were also the earlier experiments in this instance.

on return to atmospheric pressure was exceptionally small in the two high temperature experiments ; there was, however, no corresponding phenomenon in the case of the experiments dealt with in Table XXIII.

Secular Change in Aneroids.

§ 26. Variations in the reading of an aneroid at a standard pressure, say 30 inches, may arise from two causes—true secular change of zero, and elastic after-effect. In practice it is by no means easy to separate the two causes with absolute accuracy ; it is desirable, however, that the theoretical distinction should be clearly grasped. When a low pressure is maintained for a long time, the interval required for the after-effect to disappear on return to the standard pressure is, as Mr. WHYMPER has found, correspondingly great. This increases, of course, the difficulty of tracing the true secular change. In all but the latest experiments at Kew Observatory there was little trace of after-effect when 24 hours had elapsed since the return to atmospheric pressure. It is thus comparatively easy to trace the true secular change in the experimental aneroids, with at least a close approach to accuracy. This is done in Table XXIV. The quantity tabulated for each aneroid is the error or algebraic excess of its reading over the true pressure, as registered by the mercury gauge, at the commencement of experiments preceded by at least 40 hours' exposure to atmospheric pressure. Experiments during the same month were grouped together, and the mean of the errors found.

All the aneroids except No. 4 had a zero error to commence with.

TABLE XXIV.—True Secular Change of Zero, error in inches.

Date.	Aneroid.					
	No. 1.	No. 2.	No. 3.	No. 4.	No. 7.	No. 8.
1895.						
April	−·09	−·05	−·23			
May	−·05	+·13	−·23	·00		
June	−·02	+·16	−·39	+·05		
July	−·02	+·16	−·54	+·09		
August	−·11	+·17	−·54	+·10		
September . . .	−·19	+·19	−·51	+·11		
October	−·25	+·13	−·59	+·11		
November . . .	−·01	..	..	+·07		
December . . .	−·14	+·12	−·59	+·05		
1896.						
January	−·34	+·10	−·62	+·04		
February . . .	−·39	+·09	−·61	+·06		
March	−·34	+·09	−·62	+·06		
April	−·29	+·12	−·59	+·08		
May	−·42	+·13	−·57	+·08		
June	−·56	+·15	−·56	+·10		
July	−·67	+·16	−·57	+·13		
August	−·71	+·14	−·58	+·14		
December . . .	−·69	+·14	−·56	+·16		
1897.						
April	..	..	..	+·09	−·05	−·04
May	..	+·15	−·55	+·12	+·06	+·01
August	..	..	..	+·15	+·07	+·03
October	..	..	..	+·02	−·08	−·03
November . . .	..	..	..	·00	−·21	−·06
December . . .	..	..	..	−·04	−·23	−·08
1898.						
March	..	+·08	−·60	−·04	−·30	−·11

A horizontal line indicates the break down and repair of the instrument; readings above and below it are not comparable.

The changes were certainly in a good many instances discontinuous. Thus aneroid No. 1 experienced at least 9 sudden changes of reading of ·1 inch or more, 3 of these being rises and 6 falls. No. 2 had only one sudden change so large as this. It occurred in May, 1895, and amounted to + ·18 inch. No. 3 had two sudden falls of appreciable magnitude, both in June, 1895. No. 7 had two sudden changes exceeding ·1 inch, one a rise, the other a fall. In the case of Nos. 4 and 8, except on the occasion of the former breaking down, there was no sudden change as large as ·1 inch. The constancy of zero in the former aneroid was especially notable, because it experienced more variations of pressure than any of the others. The cause of the

discontinuities is not known. In some instances they may have been brought about by a somewhat too vigorous tapping preparatory to taking a reading. Sometimes they occurred during exposure of the aneroids to atmospheric pressure. Unless a discontinuous change were of a size considerably larger than ·01 of an inch it would be difficult to demonstrate its existence. It is quite possible, in fact, that all the changes in the experimental aneroids were discontinuous. There is certainly no clear indication of a general tendency in the zeros of these aneroids to alter in a definite direction with age. The fact that discontinuous changes may not unlikely arise from time to time in the zero of an aneroid, shows the absolute necessity of periodic comparisons with a mercury barometer.

§ 27. Apart from shift of zero, it is conceivable that secular change might occur in at least two ways. There might exist something equivalent to gradual alteration of elastic moduli, whereby changes of reading answering to definite pressure changes would either increase or diminish. Quite independently, there might be change in the qualities whereon depend the elastic after-effect, manifesting itself in the alteration of such quantities as the sum of the differences of the descending and ascending readings in a given pressure cycle.

To examine into the first possibility I took note of the size of the corrected errors (see § 22) in the descending readings of a number of the experiments. The earliest 24 experiments were dealt with first. Aneroids Nos. 2 and 3 were used in 16 of these experiments. In the earlier 8 the pressure 24 inches was reached 5 times, the pressure 21 inches only twice ; the later 8 experiments differed only in that the pressure 24 inches was reached no more than 4 times. Subtracting the mean corrected error at each inch of pressure in the earlier group from the corresponding mean error in the later group, we obtain the algebraical increase in the error (or rise in the reading) during the interval.

Aneroid No. 4 was exposed to the whole 24 experiments. In the earlier 12 the pressure 24 inches was reached 9 times, 21 inches 6 times, 18 inches 4 times, and 15 inches twice ; the later 12 experiments differed only in that the pressure 24 inches was reached but 8 times. The mean corrected errors in the two groups of experiments were dealt with exactly like the corresponding errors in aneroids Nos. 2 and 3. Aneroid No. 1, being so much more erratic than the others, is not discussed. The data obtained from the other three aneroids are recorded in Table XXV.

TABLE XXV.—Corrected Errors from later less corrected Errors from earlier of first 24 Experiments.

Aneroid.	Pressure in inches.														
	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15
No. 2	−·001	−·018	−·011	−·010	− 028	−·074	−·095	−·085	−·07						
„ 3	−·002	−·017	−·018	−·024	−·022	−·029	−·025	−·05	−·04						
„ 4	−·011	−·021	−·020	−·014	−·010	−·012	−·011	−·01	−·008	−·02	−·01	−·012	+·02	+·03	+·035

The table shows pretty decisive evidence of a slight fall in the readings of Nos. 2 and 3 as time advanced. In their case the differences in the table increase on the whole fairly regularly as pressure falls, a phenomenon at least consistent with a diminution of true elastic moduli. With No. 4 the figures suggest a slight fall in reading down to the pressure of 18 inches, but a rise at lower pressures. Both conclusions, however, are very doubtful. In only 4 of the experiments was the pressure carried below 18 inches, and in the two earlier temperature averaged 5° F. higher than in the two later. As we have seen in §§ 22 and 23, the difference in temperature would tend to relatively depress the readings in the earlier experiments. Again, while the entries in the last line of the table are persistently negative between 29 and 18 inches, they show no tendency to increase numerically as pressure falls. In aneroid No. 4, as might be conjectured from Table XIX., the scale was not very regular near 30 inches, so that the apparent index error, answering nominally to 30 inches, but in reality to a pressure which might be slightly greater, or slightly less, was exposed to a fictitious variation, liable to influence the corrected errors at lower points of the range.

§ 28. To throw more light on the general question, I calculated the mean corrected errors shown by aneroids Nos. 2 and 3 in the years 1895, 1896, and 1897 separately, confining my attention to the part of the scale most used, and including only those experiments in which pressure was lowered at the normal rate. Particulars are given in Table XXVI.

TABLE XXVI.—Corrected Errors, Yearly Means.

Year.	Number of Experiments.	Mean Temperature.	Aneroid No. 2.				Aneroid No. 3.			
			29 inches.	28 inches.	27 inches.	26 inches.	29 inches.	28 inches.	27 inches.	26 inches.
1895	18	64° F.	−·008	−·022	−·060	−·038	−·012	−·031	−·037	−·045
1896	13	68° F.	−·016	−·031	−·072	−·049	−·020	−·048	−·063	−·074
1897	7	64° F.	−·009	−·027	−·067	−·054	−·020	−·044	−·051	−·074

In accordance with § 22 a trifling + correction ought to be applied to the figures for 1896 to allow for its 4° excess of temperature over 1895 and 1897. On the whole, there appears to be satisfactory evidence of a slight secular lowering of reading in the two aneroids, such as would accompany alteration of a true elastic modulus; but it certainly did not increase to any sensible extent after the end of the first year.

In pushing the inquiry further in the case of aneroid No. 4, I took the 25 experiments already dealt with in Table XIX., but divided them into an earlier and a later half, treating experiments Nos. 16 and 17 as but one. The results appear in Table XXVII.

TABLE XXVII.—Mean Corrected Errors in Aneroid No. 4.

	Temperature.	Pressure in inches.								
		29.	28.	27.	26.	25.	24.	23.	22.	21.
Earlier half . .	68° F.	+·042	−·023	−·043	−·006	+·001	−·052	−·031	−·079	−·103
Later half . .	67° F.	+·039	−·025	−·041	−·001	+·012	−·038	−·012	−·068	−·088
Later less earlier	..	−·003	−·002	+·002	+·005	+·011	+·014	+·019	+·009	+·015

At the higher pressures the apparent difference between the earlier and later series is microscopic; at the lower pressures the figures would point to a secular *rise* in the corrected errors; but, as there is no visible tendency in this to increase at the very lowest pressures, its true character is somewhat doubtful. Probably the only safe conclusion to be drawn is that if any real change occurred in aneroid No. 4 it certainly was very small.

§ 29. For investigating possible secular change in the differences of the descending and ascending readings, the data are somewhat limited, because few of the later experiments were altogether of the normal type. The most suitable data are utilised in Table XXVIII., which gives the sums of the differences of the descending and ascending readings in experiments of the normal type separated by appreciable intervals of time. When two experiments are combined, the mean sum is taken to the nearest ·01 of an inch.

TABLE XXVIII.—Sums of Differences, Descending less Ascending Readings.

Range 30 inches to 26.					Range 30 inches to 24.					Range 30 inches to 21.					Range 30 inches to 18.		
Experi-ments.	Aneroid No.				Experi-ments.	Aneroid No.				Experi-ments.	Aneroid No.				Experi-ments.	Aneroid No.	
	1.	2.	3.	4.		1.	2.	3.	4.		1.	2.	3.	4.		1.	4.
1, 2	·22	·14	..	·13	3, 4	·63	·47	·68	·33	6, 7	1·48	1·18	1·83	·74	8, 9	2·85	1·52
12, 13	·21	·18	·24	·14	20, 21	·61	·44	·63	·30	18, 19	1·45	1·28	1·76	·81	16, 17	2·80	1·65
23, 24	·21	·16	·21	·14						36, 39	..	1·15	1·67	·72			
										56	..	1·09	1·67	·73			

The blanks are due to some uncertainty in the readings of No. 3 on one occasion, and to the later experiments with No. 1 not being comparable with the earlier. Between experiments No. 6 and No. 36 there intervened fully a year, and between No. 36 and No. 56 nearly a second year.

The table does not, I think, afford any clear proof of a change in the sum of the differences. If any change occurred, the evidence is in favour of its being a decrease.

In all the experiments utilised in Table XXVIII., the aneroids had been exposed to atmospheric pressure for at least two days previously. This introduces a difference between these experiments and five of the ordinary type, Nos. 52, 53, 54, 55, and 59, which were preceded by experiments over the same range, 30–21 inches, on the previous day. The difference in this respect is probably the chief reason for the relative smallness of the mean sums of the differences of the descending and ascending readings given by these five experiments, viz., ·96 for No. 2, 1·43 for No. 3 and ·66 for No. 4.

Influence of Rate of Change of Pressure.

§ 30. In ordinary mountain ascents the rate of change of pressure is much slower than 1 inch in 5 minutes; it is thus important to know how the various phenomena are modified when the rate of change of pressure is reduced. A variety of experiments were devoted to this investigation. In eight of these, Nos. 25, 26, 27, 28, 37, 38, 57, and 58, pressure was altered at the rate of 1 inch in 10 minutes, or half the normal rate, with a stoppage of the normal length, 10 minutes, at the lowest point. In experiments 27 and 28 the pressure range was 30–15 inches, in the other six instances it was 30–21 inches.

The two pairs of experiments, 37, 38 and 57, 58, were each preceded and followed by an experiment of the normal type over the same range. The experiments 36, 37, 38, 39 were taken on consecutive days, July 6–9, 1896; the experiments 56, 57,

58, 59 were taken on May 19, 21, 25, and 26, 1897. During either of these two groups of experiments the condition of the aneroids may reasonably be regarded as constant, except in so far as earlier experiments of a group may have affected the later. The temperature was practically the same for all the four experiments of either group; so that the corresponding observations are peculiarly well fitted for investigating the influence of the rate of reduction of pressure on the descending readings. The general nature of the phenomena will be found, I think, sufficiently illustrated by Tables XXIX. and XXX., which give the corrected errors at the lowest point of the range.

TABLE XXIX.—Corrected Error at Lowest Point.

Experiment.	Temperature, Fahrenheit.	Aneroid No. 1.		Aneroid No. 2.		Aneroid No. 3.		Aneroid No. 4.	
		Time to fall an inch.		Time to fall an inch.		Time to fall an inch.		Time to fall an inch.	
		5 mins.	10 mins.	5 mins.	10 mins.	5 mins.	10 mins.	5 mins.	10 mins.
36	73°	—·23	..	—·29	..	—·30	..	—·11	..
37	74	..	—·27	..	—·32	..	—·32	..	—·16
38	75	..	—·19	..	—·32	..	—·30	..	—·16
39	76	—·22	..	—·34	..	—·29	..	—·14	..
Means . . .		—·225	—·23	—·315	—·32	—·295	—·31	—·125	—·16

TABLE XXX.—Corrected Error at Lowest Point.

Experiment.	Temperature, Fahrenheit.	Aneroid No. 2.		Aneroid No. 3.		Aneroid No. 4.		Aneroid No. 7.		Aneroid No. 8.	
		Time to fall an inch.		Time to fall an inch.		Time to fall an inch.		Time to fall an inch.		Time to fall an inch.	
		5 mins.	10 mins.	5 mins.	10 mins.	5 mins.	10 mins.	5 mins.	10 mins.	5 mins.	10 mins.
56	66°	—·27	..	—·30	..	+·06	..	—·07	..	—·03	..
57	66	..	—·29	..	—·31	..	+·04	..	—·05	..	—·04
58	64	..	—·27	..	—·28	..	+·05	..	—·06	..	—·02
59	63	—·22	..	—·20	..	+·07	..	—·03	..	·00	..
Means. .		—·245	—·28	—·25	—·295	+·065	+·045	—·05	—·055	—·015	—·03

In both tables the mean corrected error is more negative, or the aneroid reads lower, when the rate of change of pressure is reduced. The difference between the

two rates appears larger in Table XXX. than in Table XXIX.; this is mainly due, however, to experiment No. 59, and very probably arises from the fact that the time interval between this and the previous experiment was shorter than the average.

Taking the figures as they stand, we should conclude that the mean reading at 21 inches was lowered by only ·02 of an inch when the time of reducing the pressure was increased from 45 to 90 minutes. At first sight, the smallness of this difference seems truly remarkable, because after lowering the pressure to 21 inches at the normal rate we should, in accordance with Tables IX. and X., have observed a mean fall of ·02 of an inch in some 4 minutes.

The influence of a still slower rate of pressure was tried in a very recent group of experiments, Nos. 71, 72, 73, 74, and 75, carried out on March 16, 18, 22–23, 29–30, and April 1, 1898. Of these, Nos. 71, 72, and 75 may be regarded as of the normal type, except that the stoppage at the lowest pressure, 21 inches, lasted 2 hours. Nos. 73 and 74 differed from these only in that all the pressure intervals were nine times longer. These last experiments lasted of course nearly two days, the length of stoppage at the lowest point being chosen so as to make this feasible without night work. In reality, it was found convenient to reduce the stoppage in experiment No. 74 by 45 minutes, and the corresponding curtailment, 5 minutes, was made in the case of No. 75. I have disregarded this trifling difference, treating Nos. 71, 72, and 75 as identical, and Nos. 73 and 74 as identical. From these experiments I have calculated, and give in Table XXXI., the differences between the mean corrected errors throughout the range for the slower rate (1 inch in 45 minutes) and the faster rate (1 inch in 5 minutes). The results are given to the nearest ·01 inch.

TABLE XXXI.—Mean Corrected Error Slower Rate, less Mean Corrected Error Faster Rate.

Aneroid.	Pressure in inches.								
	29.	28.	27.	26.	25.	24.	23.	22.	21.
No. 2 . .	+·01	·00	−·01	−·02	−·02	−·03	−·04	−·04	−·06
„ 3 . .	+·01	−·01	−·02	−·03	−·02	−·04	−·05	−·07	−·08
„ 4 . .	+·02	+·01	+·01	·00	+·01	·00	−·01	−·01	−·02
„ 7 . .	·00	−·01	·00	+·01	−·01	−·01	−·03	−·04	−·04
„ 8 . .	·00	−·01	·00	−·01	−·01	−·02	−·03	−·03	−·04
Means .	+·01	·00	·00	−·01	−·01	−·02	−·03	−·04	−·05

Here, as in Tables XXIX. and XXX., there is, at least at the lower parts of the range, a lowering of the reading accompanying reduction in the rate of fall of pressure. Also the mean lowering of the reading at the lowest point, ·05 of an inch, is notably

greater than in the two previous tables. As temperature was about 4° *lower* in the experiments at the slower rate, its influence must have tended if anything to reduce the apparent difference.

As we should have anticipated *à priori*, the differences in Table XXXI. are greatest in the case of the two aneroids Nos. 2 and 3, which showed the largest after-effect phenomena.

Taking both the Tables XIX. and XXXI. into consideration, we see that the fall of temperature naturally experienced in mountain ascents, and the reduction in the rate of change of pressure relative to that of the ordinary test, must occasionally neutralise one another's effects; but the consequences, however satisfactory, can hardly, I think, be justly ascribed to design. Though the differences exhibited in Table XXXI. are certainly quite appreciable, they are very much less than would naturally be anticipated from experiments in which pressure is reduced rapidly and then maintained constant for a lengthened period.

§ 31. The influence of the previous rate of reduction of the pressure on the fall of reading at the lowest point is hardly capable of being satisfactorily ascertained from the earlier groups of experiments, because with a stoppage of only 10 minutes we have to do with changes of reading so small relative to the probable error of observation that a large number of experiments are required to give results of much value. In the last group of experiments, Nos. 71 to 75, however, the stoppages were sufficiently long to produce considerable falls of reading. To avoid decimals I have applied a common large multiplier, and give the results in Table XXXII.

TABLE XXXII.—Fall of Reading at Lowest Pressure (21 inches).

Aneroid.	Rate 1 inch in 5 minutes.								Rate 1 inch in 45 minutes.			
	Time in minutes.								Time in minutes.			
	2.	5.	10.	15.	20.	30.	60.	120.	18.	45.	90.	1080.
No. 2 . . .	6	12	24	30	30	44	54	68	9	12	18	69
" 3 . . .	6	16	28	34	42	48	68	86	6	6	12	75
" 4 . . .	0	4	8	12	15	24	32	36	3	3	6	33
" 7 . . .	2	8	14	18	18	28	36	40	6	6	12	39
" 8 . . .	0	0	4	10	12	20	32	36	6	6	12	42
Sums . .	14	40	78	104	117	164	222	266	30	33	60	258

The agreement between the falls in 120 minutes in the one set of experiments and in 1080 minutes (120 × 9) in the other is remarkably close. A comparison of the falls at the three shorter intervals, 2, 5, and 10 minutes in the one case, and 18, 45,

90 minutes in the other, though naturally showing conspicuous irregularities, points to the same general conclusion. It is clear that the *creep*, or fall of reading, at the lowest point is largely dependent on the rate of the previous change of pressure. A slow rate of change affords opportunity for simultaneous creep, but at the same time it reduces the tendency to creep at lower pressures.

If we may judge from Table XXXII., supposing the time taken to produce a given fall of pressure to be increased in a given proportion, then the time required for the subsequent creep to attain a certain value is likewise increased in the same or nearly the same proportion. If the proportion were *exactly* the same in the two cases one would hardly, I think, expect to meet with the differences in the corrected errors shown by Tables XXIX., XXX., and XXXI.

§ 32. In treating of the influence of the rate of change of pressure on the differences between the descending and ascending readings, I shall consider first the final group of experiments Nos. 71 to 75. The following Table XXXIII. shows the ratio of the mean difference of the descending and ascending readings at each point of the range from the two experiments at the slower rate (1 inch in 45 minutes) to the corresponding mean from the three experiments at the faster rate (1 inch in 5 minutes).

TABLE XXXIII.—Ratios Differences Descending less Ascending Readings, slower : faster.

Aneroid.	Pressure in inches.									
	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
No. 2 .	1·02	1·05	1·03	1·08	1·14	1·08	1·05	1·08	1·19	·94
„ 3 .	·87	1·01	1·04	·97	1·11	·98	1·00	·99	1·04	·87
„ 4 .	·92	·99	·99	1·03	1·14	·99	1·07	1·11	1·24	·81
„ 7 .	·98	·90	1·01	1·15	1·09	1·18	1·03	·94	1·11	1·33
„ 8 .	1·17	·92	·94	·96	1·08	1·10	1·14	1·07	1·06	·98
Means . .	·99	·97	1·00	1·04	1·11	1·07	1·06	1·04	1·13	·99

The mean of the means is 1·04.

We can hardly avoid drawing the conclusion that, provided all the time intervals in the cycle be altered in the same proportion, the sum of the differences between the descending and ascending readings, and the law of variation of the differences throughout the range, are either absolutely unaffected or very nearly so. The departure of the mean ratio in Table XXXIII. from unity is mainly due to experiment No. 72. The interval allowed for recovery after the immediately preceding experiment was possibly insufficient. The agreement between experiments 71 and 73 and between experiments 74 and 75 could hardly have been improved. In the

latter pair, for instance, the sums of the differences of the descending and ascending readings were as follows :—

	Aneroid.				
	No. 2.	No. 3.	No. 4.	No. 7.	No. 8.
Experiment No. 74 (rate 1 inch in 45 minutes)	1·36	1·93	·84	1·14	1·15
,, ,, 75 ,, ,, 5 ,,	1·42	1·88	·85	1·12	1·24

In devising the earlier groups of experiments, for comparing the two rates 1 inch in 10 and 1 inch in 5 minutes, I did not at the time foresee the laws of the phenomena, and so allowed equal lengths to the stoppages at the lowest point in the two cases. The results are notwithstanding of considerable interest. Taking the experiments over the range 30–21 inches, in order to allow for alterations in the aneroids, I grouped them as follows :—

Group.		1 inch in 5 minutes.	1 inch in 10 minutes.
I.	Experiments Nos.	6, 7, 18, 19	25, 26
II.	,, ,,	36, 39	37, 38
III.	,, ,,	56, 59	57, 58

Taking Group III. for example, the differences of the descending and ascending readings at each inch were summed for the aneroids Nos. 2, 3, 4, 7, 8 in experiments 57, 58, and the ratios taken to the corresponding sums in experiments 56, 59. The preponderance thus allowed to aneroids Nos. 2 and 3, in which the after-effect was largest, is of little consequence, as all the aneroids showed the same general phenomena. The results appear in Table XXXIV.

TABLE XXXIV.—Ratios Differences Descending less Ascending Readings (1 inch in 10 : 1 inch in 5 minutes).

Group.	Pressure in inches.										Mean for group.
	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.	
I.	·94	·84	·86	·90	·84	·87	·90	·91	·84	·81	·87
II.	·62	·79	·87	·90	·83	·84	·88	1·04	·77	·75	·83
III.	1·00	·91	·81	·96	·98	·98	1·07	·86	1·01	·97	·95
Means .	·85	·85	·85	·92	·88	·90	·95	·94	·87	·84	·88

The ratios would seem to be somewhat above their average near the centre of the range, and below it at both extremities; but there are hardly sufficient experiments to justify any too positive conclusion. That the mean ratio is less than unity is absolutely clear; and there can be but little doubt that this is due to the fact that the stoppage at the lowest point in the case of the slower experiments was not doubled along with the other time intervals.

I likewise compared experiments Nos. 27 and 28 over the longest range, 30–15 inches, at the slower rate, 1 inch in 10 minutes, with earlier experiments, Nos. 10, 11, 14, 15, over the same range at the normal rate. Only one aneroid, No. 4, was available for calculation, owing to erratic behaviour in No. 1. It will thus suffice to mention that the results obtained were analogous to those in Table XXXIV., and that for the ratio of the sums of the differences of the descending and ascending readings at the two different rates I found

$$\frac{\text{case 1 inch in 10 minutes}}{\text{case 1 inch in 5 minutes}} = \cdot 93.$$

In both cases, as already stated, only 10 minutes stoppage was allowed at the lowest pressure.

§ 33. The influence of the rate of change of pressure on the rate of recovery at the end of a pressure cycle was less exactly ascertained. After experiments in which the pressure changes are slow, the variations observed in the reading during the first hour or two of the recovery are so small as to be largely affected by small errors of reading. After 12 or 24 hours recovery, readings are affected by various sources of uncertainty, *e.g.*, change of temperature or atmospheric pressure, and possible permanent shift of zero.

Table XXXV. compares the recovery following pressure cycles at the two rates 1 inch in 10 and 1 inch in 5 minutes.

The data for the normal rate are taken from Table XVII., and D_t/D_0 has the same meaning as there. The data for the slower rate were obtained from the experiments 25, 26, 37, 38 over the range 30–21 inches.

TABLE XXXV.—Recovery after Pressure Cycle, Values of D_t/D_0.

Rate in cycle.	Interval in minutes.					
	0.	5.	10.	15.	20.	60.
1 inch in 5 minutes	1	·83	·74	·67	·59	·41
,, 10 ,,	1	·87	·83	·72	·74*	·49

* If No. 4 omitted, only ·69. For some reason, reading recorded for No. 4 showed on two occasions apparent *fall* of ·01 inch in interval 15 to 20 minutes.

The number of experiments at the slower rate was insufficient to give very smooth results, and it would be waste of time to fit a formula to them. We should not be justified in regarding the coincidence between the data under 10 and 20 minutes in the second line with those under 5 and 10 minutes in the first as more than accidental. At the same time there seems conclusive evidence that the rate of recovery becomes slower when the rate of the previous pressure changes is reduced.

This is further illustrated in Table XXXVI., which compares the recovery in experiments Nos. 73 and 74 with that exhibited in experiments Nos. 71, 72, and 75.

TABLE XXXVI.—Recovery after Pressure Cycle, values of D_t/D_0.

Aneroid.	Pressure rate 1 inch in 5 minutes (stoppage at lowest point 2 hours).							Pressure rate 1 inch in 45 minutes (stoppage at lowest point 18 hours).					
	Time in minutes.							Time in minutes.					
	0.	5.	10.	15.	20.	120.	210.	0.	15.	30.	45.	1020.	2460.
No. 2 . .	1	·89	·85	·78	·70	·29	·20	1	·94	·88	·83	·65	·47
„ 3 . .	1	·91	·84	·81	·74	·38	·27	1	·96	·88	·80	·60	·44
„ 4 . .	1	1·00	·92	·85	·85	·67	·20	1	1·14	1·14	1·00	·14	·00
„ 7 . .	1	·89	·83	·89	·87	·67	·13	1	·87	·87	·86	·50	·38
„ 8 . .	1	·80	·65	·65	·60	·50	−·06	1	·85	·77	·80	·38	·31
Means .	1	·90	·82	·80	·75	·50	·15	1	·95	·91	·86	·45	·32

The uncertainties referred to above as unavoidable in a small number of experiments of this kind are conspicuous in Table XXXVI.; but as to the recovery being very much slower in the experiments with the slow pressure change there is no room for doubt. That it is exactly or almost exactly nine times as slow in the one case as in the other is a conclusion for which the evidence is insufficient; but there is at least nothing to demonstrate the contrary. The observations in the case of the experiments at the slower rate would naturally have been taken at the intervals 45, 90, 135, &c., minutes after return to atmospheric pressure, but daylight did not last long enough to admit of this. The extreme slowness of recovery after experiments such as Nos. 73 and 74 is a somewhat serious obstacle to research.

Effects of Stoppage.

§ 34. The influence of a stoppage on the readings at lower pressures is shown by three groups, each of three experiments, in all of which pressure was reduced at the rate of 1 inch in 5 minutes—in most cases with a subsidiary stoppage—to 15 inches, and maintained at that point for an hour.

In the first two groups there was in two out of the three experiments a subsidiary stoppage of 1 hour at a pressure above the lowest. In the third group there was a subsidiary stoppage of about 26 hours' duration in all three experiments. As the order of the experiments is of significance, I give particulars :—

Experiment.	Date.	Stoppage at—	Experiment.	Date.	Stoppage at—	Experiment.	Date.	Stoppage at—
		inches.			inches.			inches.
	1897.			1897.			1897.	
62	Oct. 11	18 and 15	65	Nov. 15	21 and 15	68	Dec. 21	16 and 15
							1898.	
63	„ 13	15	66	„ 17	15	69	Jan. 5	18 and 15
64	„ 15	21 and 15	67	„ 19	18 and 15	70	„ 20	21 and 15

As experiments Nos. 63 and 66 were of the normal type, so far as descending readings are concerned, I have used them as a standard to which to refer the others. To see, for instance, how an hour's stoppage at 18 inches influences the reading when the pressure 15 inches is reached, I compare the mean error at 15 less the mean error at 18, prior to the stoppage, in experiments 62 and 67, with the mean difference between the errors at 15 and 18 in experiments 63 and 66. For instance, in aneroid No. 4, I found :—

from experiments 62 and 67, mean error (before stoppage) at 18 = + ·01,
„ „ 62 and 67, „ .. .. at 15 = + ·015,
and so error at 15 less error at 18 = + ·005 ;

from experiments 63 and 66, mean error .. .. at 18 = + ·01,
„ „ 63 and 66, „ .. .. at 15 = + ·04,
and so error at 15 less error at 18 = + ·03.

Thus an hour's stoppage at 18 has lowered the reading at 15 by

·03 — ·005, or ·025 inch.

Again, to determine the influence of the above stoppage on the total creep which takes place as pressure is being lowered from 18 to 15, I compare the mean error at 15 less the mean error at 18, *subsequent* to the stoppage, in experiments 62 and 67 with the difference between the mean errors at 15 and 18 in experiments 63 and 66. For instance, in aneroid No. 4,

from experiments 62 and 67, mean error (after stoppage) at 18 = — ·06,
„ „ 62 and 67, „ .. .. at 15 = + ·015,
and so error at 15 less error at 18 = + ·075 ;

and this is greater than in experiments 63 and 66 by

$$\cdot075 - \cdot03, \quad \text{or} \quad \cdot045 \text{ inch.}$$

Thus an hour's stoppage at 18 diminished the total creep between 18 and 15 by ·045 inches.

These examples should suffice to explain Table XXXVII.

TABLE XXXVII.

Stoppage.		Consequent lowering of reading, in inches, at 15 inches.				Consequent diminution of total creep, in inches, between stoppage and 15 inches.			
At	For	No. 4.	No. 7.	No. 8.	Mean.	No. 4.	No. 7.	No. 8.	Mean.
inches.	hours.								
21	1	·015	·025	·005	·02	·04	·05	·05	·05
18	1	·025	·015	·05	·03	·045	·06	·04	·05
21	26	·04	·04	·07	·05	·08	·10	·06	·08
18	26	·12	·135	·14	·13	·06	·065	·06	·06
16	26	·185	·185	·19	·19	·035	·045	·04	·04

§ 35. In arguing from a limited number of experiments allowance must be made for accidental coincidences. The general conclusions to be drawn from Table XXXVII. seem however pretty obvious.* A stoppage has two effects, tending somewhat to neutralise one another. It lowers the reading, but while doing so it diminishes the tendency to further creep as pressure is further lowered. For instance, stoppage for 26 hours at 18 inches lowered the reading at 18 inches in aneroid No. 4 by ·18 inch, but lessened the subsequent creep between 18 and 15 inches by ·06 inch. Consequently on arrival at 15 inches the aneroid read only ·12 inch lower than if there had been no stoppage at 18 inches. When the stoppage occurred at 21 inches, and lasted only an hour, the diminution in the creep between 21 and 15 inches nearly obliterated all influence on the reading at 15 inches. With increased duration of stoppage the direct lowering of reading tended more distinctly to predominate.

The lower the pressure at which stoppage occurs, the greater influence does it exert in both directions. To realise this one should notice that stoppage at 21 inches influenced the creep over the 6 inches 21 to 15, whereas stoppage at 16 inches influenced the creep over a single inch only.

§ 36. The way in which the influence of a stoppage tends to disappear the further the pressure is lowered below the stationary point, is brought out most clearly by a consideration of the phenomena observed after the lowest pressure is attained.

In treating of this I have utilised, in addition to experiments Nos. 62 to 70, two

* At the same time, so far as concerns phenomena occurring during actual change of pressure, it would be difficult to distinguish between a temporary rise in true elastic moduli and a diminished tendency to creep.

earlier experiments, Nos. 60 and 61, made respectively on August 17 and 20, 1897. Of these 60 agreed in type with 63 and 66, while 61 was identical with 62 and 67. Taking the means of the observed falls in reading during stated intervals of exposure to 15 inches pressure in experiments 60, 63 and 66 as standards, I have found the ratios borne to these by the corresponding mean falls in the other groups of experiments during both the subsidiary stoppage and the stoppage at 15 inches. The results appear in Table XXXVIII. The other data in the table are of the following character :—

"Mean ratio" signifies the mean deduced from the values at the seven intervals 5, 10 . . . 60 minutes. "Theoretical ratio" is what the value should have been according to the law that the creep in a given interval of time is proportional to the pressure interval measured from 30 inches. For instance, in experiments 64 and 65 the reading during the subsidiary stoppage at 21 inches should according to the law have fallen (30–21) ÷ (30–15), or ·60, times as much in any given interval as it did in the same interval during exposure to the pressure of 15 inches in experiments 60, 63 and 66. In reality the fall in the former case was on the average only ·58 of that in the latter. Accordingly the presumption is that the tendency to creep in experiments 64 and 65 was less than in the three standard experiments in the ratio 58 : 60. This result is utilised in calculating the true influence of the stoppage at 21 inches on the fall of reading during the subsequent stoppage at 15 inches. According to the actual observation figures the mean ratio of the falls at 15 inches in experiments 64 and 65 to the falls in the standard experiments was ·93, but on the assumption that this suffered from the same cause as the creep at 21 inches, it should be multiplied by 60/58, in order to eliminate the difference between the aneroids' mean state during the two groups of experiments. The result of this multiplication is termed the "Corrected ratio."

TABLE XXXVIII.—Ratios to Falls of Reading (in same intervals) at 15 inches in Standard Normal Experiments.

Experiments.	Fall at—	Time in minutes.							Mean ratio.	Theoretical ratio.	Corrected ratio.
		5.	10.	15.	20.	30.	45.	60.			
	inches.										
64, 65	21	·57	·79	·55	·47	·55	·55	·60	·58	·60	. .
64, 65	15	1·07	·96	·88	·97	·87	·85	·91	·93	. .	·96
61, 62, 67	18	·62	·71	·65	·76	·70	·76	·77	·71	·80	. .
61, 62, 67	15	·71	·85	·77	·85	·81	·79	·83	·80	. .	·90
68	16	not ob	serve	d							
68	15	·29	·00	·20	·17	·08	·07	·10	·13	. .	. .
69	18	·86	·75	·85	·78	·76	·71	·65	·77	·80	. .
69	15	·43	·33	·33	·33	·42	·39	·39	·37	. .	·38
70	21	·57	·50	·65	·66	·55	·60	·65	·60	·60	. .
70	15	·71	·66	·85	·78	·72	·71	·72	·74	. .	·74

We should conclude from the table that an hour's stoppage at 21 inches lowered the creep during the subsequent stoppage at 15 inches by only 4 per cent. An hour's stoppage at 18 inches was distinctly more effective, but still not very serious. Stoppage for 26 hours had a much larger influence, reducing the creep at 15 inches from 100 to 74 when it occurred at 21 inches, from 100 to 38 when it occurred at 18 inches, and from 100 to 13 when it occurred at 16 inches.

According to the table:

$$\frac{\text{Influence of an hour's stoppage at 21}}{\text{Influence of an hour's stoppage at 18}} = \frac{4}{10} = \cdot 40,$$

$$\frac{\text{Influence of 26 hours' stoppage at 21}}{\text{Influence of 26 hours' stoppage at 18}} = \frac{26}{62} = \cdot 42.$$

This *suggests* that the influence of a stoppage on the creep at a lower stationary pressure may be expressible as a product of two factors, one a function of pressure only, the other having for sole variable the duration of the subsidiary pressure.

Again, taking the three experiments when the subsidiary stoppage lasted 26 hours, it will be noticed that the mean ratios, ·13, ·38, and ·74, are to one another nearly in the proportion 1 : 3 : 6, which corresponds to the pressure intervals 16 to 15, 18 to 15, and 21 to 15 inches.

Whether we have to do here with chance coincidences or with physical laws it would be impossible to say without further somewhat elaborate experiments.

§ 37. Before leaving Table XXXVIII. we may note its bearing on the laws established in §§ 16 to 19 for change of reading under steady pressure. The data deduced from the subsidiary stoppages at 21 and 18 inches, and the stoppages at 15 inches in the three standard experiments, are in good agreement with the laws that the fall of reading in a given time during a stoppage, preceded by a reduction of pressure at a uniform rate, is proportional to the pressure range, and that the mode of variation of the reading with the time is independent of the range. The first of these laws is supported by the closeness of the "mean" and "theoretical" ratios, the second by the absence of any clear tendency in the ratios of the falls at the specified intervals to either increase or decease as the intervals become longer.

Further support of the first law is supplied by the fact that the mean observed falls during 26 hours' exposure to the pressures of 21, 18, and 16 inches were respectively ·13, ·19$\dot{3}$, and ·22$\dot{6}$ inch, and we have

$$\cdot 13/9 = \cdot 01\dot{4}, \quad \cdot 19\dot{3}/12 = \cdot 0161, \quad \cdot 22\dot{6}/14 = \cdot 0162,$$

values nearly equal.

A very interesting point is that the ratios borne by the falls at 15 inches pressure, in the several experiments where a subsidiary stoppage existed, to the corresponding falls in the standard experiments are, to all appearance, constant throughout the hour's stoppage at 15. There are, of course, irregularities in the figures, but there

is no clear tendency in the ratios to increase or decrease as the time increases from 5 to 60 minutes. There would thus appear to be a very vital distinction between the influence of a stoppage and that of a reduction in the rate of fall of pressure. In both cases there is a reduction in the creep observed in a given time while the pressure is maintained steady at its lowest point, but the stoppage has apparently no appreciable influence on the law of variation of the creep with the time, while the reduction of the rate of fall of pressure has a notable influence in this direction.

§ 38. The influence of a stoppage is also brought out in an instructive way by comparing the differences of the descending and ascending readings at points below and at points above the stationary pressure with the corresponding data from experiments of the normal type. Taking as standards the normal experiments 63 and 66, I have instituted this comparison for experiments 62 and 67, 64 and 65, 68, 69, 70, and exhibit the results in Table XXXIX. The following example will show how the results were arrived at and what they signify.

The mean of the experiments 63 and 66 gave, in the case of aneroid No. 4, for the sum of the differences, descending less ascending readings, from 15 to 18 inches inclusive ·605 inch, and from 18 to 30 inches inclusive 2·635 inches. Now in experiment 69, with a prolonged subsidiary stoppage at 18 inches, the sum of the differences at 15, 16, 17 and 18 inches (after stoppage in descent) was ·26 inch, while the sum of the differences at 18 inches (before stoppage in descent) and 19 to 30 inches inclusive was 3·19 inches. Thus stoppage for 26 hours at 18 inches diminished the sum of the differences below this point by 100 (·605 − ·26) ÷ ·605, or 57 per cent., and raised the sum of the differences above this point by 100 (3·19 − 2·635) ÷ 2·635, or 21 per cent.

TABLE XXXIX.—Percentage Change in Sum of Differences, Descending less Ascending Readings, Below and Above Pressure of Stoppage (+ gain, − loss).

Aneroid.	Stoppage at 21 inches.				Stoppage at 18 inches.				Stoppage at 16 inches.	
	For 1 hour.		For 26 hours.		For 1 hour.		For 26 hours.		For 26 hours.	
	Below.	Above.	Below.	Above.	Below.	Above.	Below.	Above.	Below.	Above.
No. 4	−7	+3	−24	+12	−19	+10	−57	+21	−83	+57
„ 7	−6	+8	−22	+ 9	−23	+ 4	−48	+19	−68	+49
„ 8	−6	+4	−20	+13	−19	+11	−43	+17	−83	+51
Means .	−6	+5	−22	+11	−20	+ 8	−49	+19	−78	+52

The percentage loss in the sum of the differences below the stationary pressure is,

as a rule, considerably larger than the percentage gain throughout the higher part of the range. The latter, however, is a substantial quantity except in the case of the stoppage at 21 inches for an hour only.

§ 39. As pressure was raised above the point where it was stationary in the descent, the influence on the difference of the descending and ascending readings gradually diminished, and by the time atmospheric pressure was reached there was in every case a close approach to the normal deficiency in the reading. In some cases the deficiency was even below the normal. There is, however, ground for believing that even after return to the atmospheric pressure the stoppage exercised some influence. The evidence for this conclusion is contained in Table XL., which shows the rate of recovery in cases in which there was, and in other cases in which there was not, a subsidiary stoppage, in addition to the stoppage of an hour at the lowest point, 15 inches. The results are means for the three aneroids Nos. 4, 7, and 8.

TABLE XL.—Ratio of Deficiency to Original Deficiency, D_t/D_0.

Experiments Nos.	Additional stoppage.		Time interval in minutes—				
	At	For	0.	5.	10.	15.	20.
	inches.	hours.					
60 and 66	none	none	1	·88	·81	·61	·58
64 „ 65	21	1	1	·81	·75	·71	·67
62 „ 67	18	1	1	·92	·88	·86	·84
68	16	26	1	·89	·90	·87	·83

The experiments are, of course, not numerous enough to justify exact numerical deductions, but it seems quite clear that the recovery in the two last cases in the table was decidedly slower than in the first case.

§ 40. In addition to the experiments already referred to, there were two of earlier date, Nos. 45 and 46, in which there was a subsidiary stoppage, lasting 24 hours, during the descent. In the one the stoppage occurred at 26, in the other at 24 inches. In both, pressure was reduced to 21 inches, the stoppage there lasting only 10 minutes, and the ascent was of the normal type. At points below the stoppage the differences between the descending and ascending readings were very decidedly less than in the case of the normal experiments Nos. 36 and 39 treated as standards. The effect was considerably larger when the stoppage occurred at 24 than when it occurred at 26 inches. In neither case, however, was the recovery at the end of the experiment noticeably slow.

§ 41. After our discussion of the effects of a stoppage during the reduction of pressure, a brief reference will suffice to a variety of experiments, Nos. 29, 30, 31, 32, 40, 41, 42, 43, and 44, in which there was a prolonged subsidiary stoppage during the ascent of pressure. In all, the lowest pressure was 21 inches, and the stoppage there lasted 10 minutes.

Speaking generally, the reading of the aneroids tended to fall or to rise during the subsidiary stoppage according as the stationary pressure was below or above 26 inches; but unless it was below 25 or above 27 inches such change of reading was extremely small.

The influence of the stoppage in these experiments on the readings during the subsequent rise of pressure, can be traced with nicety, because the part of the cycle preceding the subsidiary stoppage was strictly of the normal type, and so could be utilised at once in conjunction with complete normal experiments to fix the standard. In this way I calculated the percentage increment produced by the subsidiary stoppage in the sum of the differences of the descending and ascending readings at points further up the scale. The results appearing in Table XLI. are the means for aneroids Nos. 1, 2, 3, and 4; + signifies an increase, — a diminution. The subsidiary stoppage lasted 24 hours when it occurred at 24 or 26 inches; in the other cases it lasted only one hour.

TABLE XLI.—Influence of Stoppage during Ascent of Pressure. Percentage Change of Sum of Differences (Descending less Ascending Readings) at Higher Pressures.

	Stoppage at						
	22.	23.	24.	26.	27.	28.	29.
Percentage change .	+10	+17	+27	+15	−11	−38	−40

A slight rise of reading during a prolonged stoppage is not incompatible with an increase in the sum of the differences of the descending and ascending readings at higher pressures. For, as we have already seen, there is reason to believe that a stoppage tends to make subsequent recovery slower.

Theoretical Deductions.

§ 42. Reasoning from the experimental data I have built up a theory, of a somewhat empirical kind it is true, which reproduces satisfactorily the phenomena presented by the normal type of test at Kew Observatory.

A lowering of pressure is supposed to produce two independent lowerings of an aneroid's reading. One is a perfectly reversible or wholly elastic phenomenon, the other is the source of the after-effect and is termed the *creep*. It is supposed that during the constant interval—called 5 minutes for brevity—occupied in the fall of pressure from $30 - n$ to $30 - \overline{n+1}$ inches the magnitude of the creep is $k(n + \overline{n+1})/2$. Here k is constant, for a particular aneroid, so long at least as the temperature is unchanged. During the fall of pressure to, say, $30 - p$ inches, the accumulated creep thus amounts to

$$\tfrac{1}{2}k(1 + 3 + \ldots + \overline{2p-1}) = \tfrac{1}{2}kp^2.$$

During the stoppage for a constant interval—termed 10 minutes for brevity—at the lowest pressure, the creep augments by kmp, where m is independent of the aneroid.

During the rise of pressure, matters are more complicated. As pressure rises from $30 - p + r - 1$ to $30 - p + r$ inches, it is assumed that in virtue of the rise of pressure above the lowest point the creep will diminish to the extent $kl(r - 1 + r)/2$, or $kl(2r - 1)/2$, where l is constant in the same sense that m is. On the other hand, the creep tends to increase in virtue of the pressure being still below 30 inches. During the 5 minutes ascribe to the pressure its mean value $30 - p + r - \frac{1}{2}$. The length of time elapsed from the beginning of this 5 minutes to the beginning of the corresponding 5 minutes during the descent is $5r + 10 + 5(r - 1)$, or $5(2r + 1)$ minutes. Then what I assume is that the creep downwards in this 5 minutes bears to the creep in the corresponding 5 minutes in the descent the same ratio that the creep in the interval from $5(2r + 1)$ to $5(2r + 2)$ minutes after pressure becomes steady bears to the creep in the first 5 minutes after pressure becomes steady. This does not absolutely assume the creep to be the same in *magnitude* when pressure is changing as when it is steady; but it is, of course, considerably speculative.

In the ordinary Kew test, the time occupied by the experiment is not in excess of the duration of the steady pressure during the special experiments Nos. 33, 34, 35, used in constructing Table XIV. In the following calculations I have employed, not the direct results of these experiments, but the values supplied by the formula based on them.

§ 43. An example of the calculations will serve, I hope, to illustrate the process. Take the cycle 30–24–30 inches, supposed of the normal type.

As pressure falls the creep augments by $\cdot 5k$ between 30 and 29 inches, by $(1 + 2)k/2$ or $1\cdot 5k$ between 29 and 28 inches, and so on. Thus the accumulated creep amounts to $\cdot 5k$ at 29, $2k$ at 28, $4\cdot 5k$ at 27, and so on.

During the stoppage at 24 inches, the creep augments by $6mk$. As pressure rises from 24 to 25 inches we regard it as 24·5, or 5·5 inches below 30. Also the 5 minutes occupied by the rise represents the interval 15 to 20 minutes elapsed since pressure in its descent entered the stage 25 to 24 inches.

Now, by Table XIV.,

$$\frac{\text{fall in interval 15–20 minutes}}{\text{fall in interval 0–5 minutes}} = \frac{10}{57} = \cdot 18.$$

Thus, for the increase of creep during this 5 minutes, we take $\cdot 18 \times 5\cdot 5k$, or $1\cdot 0k$, as against a decrease of $\cdot 5lk$.

The following scheme shows the magnitude of the creep as the pressure attains the values specified in the first column.

The common factor k is to be understood.

Pressure in inches.	Accumulated creep.		Descending less ascending reading.
	Descent.	Ascent.	
30	0	$6m + 20{\cdot}3 - 18{\cdot}0l$	$6m + 20{\cdot}3 - 18{\cdot}0l$
29	0·5	$6m + 20{\cdot}3 - 12{\cdot}5l$	$6m + 19{\cdot}8 - 12{\cdot}5l$
28	2·0	$6m + 20{\cdot}2 - 8{\cdot}0l$	$6m + 18{\cdot}2 - 8{\cdot}0l$
27	4·5	$6m + 20{\cdot}0 - 4{\cdot}5l$	$6m + 15{\cdot}5 - 4{\cdot}5l$
26	8·0	$6m + 19{\cdot}6 - 2{\cdot}0l$	$6m + 11{\cdot}6 - 2{\cdot}0l$
25	12·5	$6m + 19{\cdot}0 - 0{\cdot}5l$	$6m + 6{\cdot}5 - 0{\cdot}5l$
24	18·0	$6m + 18{\cdot}0$	$6m$

This gives sum of differences descending less ascending $= k(42m + 91{\cdot}9 - 45{\cdot}5l)$.

§ 44. As k is a common multiplier, all quantities such as ratios of differences of ascending and descending readings to the mean difference, or ratios of sums of such differences for different ranges, depend on only two constants, l and m. When these are known, k may be found for any given aneroid from the observed sum of the differences of its descending and ascending readings over any given range.

For the old Kew observations, summarised in Table I., I found by trial

$$l = 0{\cdot}83,$$
$$m = 1{\cdot}3,$$

employing these values for *all* the ranges.

To test the theory severely, it seemed expedient to compare with it the observed results for the law of variation of the ratios of the differences of the descending and ascending readings to the mean difference over the several ranges. For this comparison I have utilised the figures resulting directly from the observations at each inch of pressure, and not the data in Table II., as the latter arise from a combination of actual observations at adjacent points of the scale. This should obviate the suspicion that naturally arises when data compared with theory have been subjected to any kind of manipulation.

The observed, "O," and the calculated, "C," values appear side by side in Table XLII. To adequately appreciate the agreement, the reader should remember that unity in the table answers to the mean difference between the descending and ascending readings. It thus answers to only about ·34 inch even in the range 30–15 inches. In the range 30–26 inches it represents only a tenth of this.

TABLE XLII.—Ratios of Differences of Descending and Ascending Readings to the Mean Difference. "O," observed results (1885–91); "C," calculated from $\begin{cases} l = 0{\cdot}83, \\ m = 1{\cdot}3. \end{cases}$

Pressure in inches.	Range. 30–26 inches.		30–24 inches.		30–23 inches.		30–21 inches.		30–18 inches.		30–15 inches.	
	O.	C.	O.	C.	O.	C.	O.	C.	O.	C.	O.	C.
15	..	..	..	..	..	..	..	..	..	..	0·25	0·28
16	..	..	..	..	..	..	..	..	..	..	0·52	0·53
17	..	..	..	..	..	..	..	..	..	..	0·74	0·73
18	..	..	..	..	..	..	..	..	0·34	0·33	0·93	0·90
19	..	..	..	..	..	..	..	..	0·63	0·61	1·05	1·04
20	..	..	..	..	..	..	..	..	0·84	0·83	1·17	1·15
21	..	..	..	..	..	..	0·37	0·40	1·00	1·01	1·23	1·23
22	..	..	..	..	..	..	0·69	0·72	1·14	1·15	1·27	1·29
23	..	..	..	..	0·41	0·46	1·00	0·97	1·20	1·24	1·29	1·31
24	..	..	0·54	0·50	0·77	0·83	1·15	1·14	1·27	1·29	1·32	1·31
25	..	..	0·88	0·90	1·08	1·08	1·26	1·25	1·29	1·30	1·26	1·28
26	0·35	0·61	1·17	1·14	1·24	1·23	1·29	1·29	1·25	1·27	1·21	1·22
27	1·14	1·05	1·26	1·26	1·28	1·28	1·24	1·26	1·19	1·20	1·14	1·13
28	1·51	1·23	1·16	1·25	1·23	1·22	1·18	1·17	1·12	1·09	1·02	1·02
29	1·19	1·19	1·07	1·11	1·11	1·08	1·01	1·01	0·93	0·94	0·88	0·88
30	0·82	0·91	0·89	0·85	0·86	0·83	0·81	0·79	0·80	0·74	0·72	0·71
Mean difference observed ∽ calculated	± ·14		± ·037		± ·024		± ·016		± ·020		± ·013	
Representing in inches	± ·004		± ·004		± ·004		± ·003		± ·005		± ·004	

By varying l and m one could of course have slightly improved the agreement in individual ranges.

For instance, the assumption

$$l = 0{\cdot}78,$$
$$m = 1{\cdot}07,$$

would reduce the mean difference of the observed and calculated values in the range 30–23 to $\pm$ ·011, representing $\pm$ ·002 inch.

§ 45. As an example of the calculation of k, we have for the sum of the differences of the descending and ascending readings in the cycle 30–24–30 :—

from mean of aneroids in Table I. (first group) ·684 inch,
from formula . . . . $k\,(42m + 91{\cdot}9 - 45{\cdot}5l)$.

Putting $l = \cdot 83$, and $m = 1\cdot 3$, as in Table XLII., we find

$$k = \cdot 684 \div (54\cdot 6 + 91\cdot 9 - 37\cdot 8) = \cdot 0063.$$

This is a mean value for the 13 aneroids dealt with as a group in Table I.; individuals of the group differed, of course, amongst themselves.

§ 46. In the case of the experimental aneroids, it might appear that m and l are at once determined by the experiments.

For instance, it might be supposed that

$$m = \frac{\text{fall of reading in 10 minutes at lowest point}}{\text{fall of reading in 5 minutes at lowest point}},$$

$$l = \frac{\text{recovery in first 5 minutes at end of cycle}}{\text{fall in first 5 minutes at lowest pressure}}.$$

If this were so, we should have from the first 24 experiments for the mean of aneroids Nos. 1, 2, 3, and 4,

$$m = 73/47 = 1\cdot 55,$$
$$l = \cdot 65, \text{ roughly.}$$

In the case of m, if we preferred to use the formula (8), we should have

$$m = 73/57 = 1\cdot 28.$$

These deductions, however, assume the creep phenomena the same whether pressure is steady or changing, which may not be strictly true. Further, even if there were no theoretical objection, the method would not, in practice, be very satisfactory, owing to the large probable error involved in determining such small quantities as an increment or decrement of creep in 5 minutes.

Another method that naturally suggests itself is a comparison of the observed sums of the differences of the descending and ascending readings in three ranges. This supplies two simple equations involving l and m as unknowns. This method, however, is in practice no better than the other, as will be recognised on inspection of Table XLIII. The quantities tabulated there are the ratios borne by the sums of the differences of the descending and ascending readings over several ranges to the sum for the range 30–21 inches. The observed values are taken from the first 24 experiments.

TABLE XLIII.—Ratios of Sums of Differences to Sum for Range 30–21 inches.

	Range.				
	30–26 inches.	30–24 inches.	30–21 inches.	30–18 inches.	30–15 inches.
Observed ratios.	·146	·375	1	2·00	3·65
Ratios calculated from $l = 0·65$, $m = 1·5$ . . .	·145	·370	1	2·08	3·72
" " " 0·83, " 1·5 . . .	·148	·374	1	2·06	3·67
" " " 0·83, " 1·3 . . .	·144	·369	1	2·08	3·73
" " " 1·0, " 1·0 . . .	·138	·363	1	2·10	3·79

The difficulty here is that large variations in the values of l and m have but little influence on the calculated ratios. The differences between the observed and calculated values in Table XLIII. are in no case in excess of reasonable limits of experimental error. The agreement is certainly best for $l = ·83$, $m = 1·5$ and worst for $l = m = 1$.

The variation of the ratio of the difference of the descending and ascending readings to the mean difference over a range affords a much more sensitive method of determining l and m. The application of this method shows that, for the special experiments, $l = m = 1$ is in reality a better choice than $l = ·83$, $m = 1·3$. The evidence for this conclusion is supplied by Table XLIV.

TABLE XLIV.—Mean Differences between Observed and Calculated Values of Ratios of Differences of Descending and Ascending Readings to the Mean Difference (Experiments 1–24).

	Range.				
	30–26 inches.	30–24 inches.	30–21 inches.	30–18 inches.	30–15 inches.
Mean difference, using $l = 1$, $m = 1$ . .	±·066	±·070	±·057	±·043	±·048
" " " $= 0·83$, $= 1·3$ . .	±·080	±·094	±·063	±·086	±·095

Previous Work Bearing on the Subject.

§ 47. Of investigations into elastic after-effect, more especially in metal wires, there is an excellent summary in vol. 1 of WINKELMANN'S 'Handbuch der Physik.' This I studied, having previously read a number of the papers it refers to, notably some by KOHLRAUSCH. Of quite recent papers on the subject I have not seen many,

and with one or two exceptions—*e.g.*, a paper by Mr. L. AUSTIN, in 'Wied. Ann.,' vol. 50, 1893—the theories they dealt with seemed of a hopelessly involved character for practical application. I did indeed try one or two of the formulæ proposed, but conditions so complicated as those of the Kew aneroid test soon led to expressions of a formidable character. The numerical results I actually reached did not show a promising accord with experiment, and having regard to the many claims on my time, I did not push the calculations further.

§ 48. The following are the only earlier investigations on aneroid barometers, having any direct bearing on the results of the present paper, that have come under my notice :—

1. "On the Errors of Aneroids at Various Pressures" (Dr. BALFOUR STEWART), 'B.A. Report,' 1867, pp. 26, 27, of 'Transactions of the Sections.'
2. "An Account of Certain Experiments on Aneroid Barometers made at the Kew Observatory at the Expense of the Meteorological Committee" (Dr. BALFOUR STEWART), 'Roy. Soc. Proc.,' vol. 16, 1869, pp. 472–480 ; 'Phil. Mag.,' vol. 37, 1869, pp. 65–74.
3. "How to Use the Aneroid Barometer," by EDWARD WHYMPER ; London : JOHN MURRAY, 1891.

Of these, No. 1 is a preliminary report apparently of the work more fully dealt with in No. 2. It mentions the tendency in the reading to fall under continued exposure to low pressure, and also the general nature of the recovery after return to atmospheric pressure.

No. 2 commences with a slight reference to the secular variation of zero. It then describes some tests showing that at atmospheric pressure changes of temperature have little effect on "well-made compensated" instruments. The writer adds, "I am unable to say what effect a change of temperature would have at a diminished pressure."

The main part of the paper describes the phenomena observed in pressure cycles, 30–19–30 inches of the following kind. Pressure was reduced an inch and then kept steady for 10 minutes, when a reading was taken; it was then lowered a second inch, and so on. At the lowest pressure there was a stoppage of $1\frac{1}{2}$ hours, and thereafter pressure was raised at the same rate as it had been lowered. The aneroids were always tapped before reading. The general superiority of large to small aneroids, and the increase in the tendency to creep when the range is lengthened are duly noted. The effects of stoppage on the top of a mountain are also remarked on.

A considerable number of aneroids were examined. The object was apparently, however, rather to ascertain the conditions under which they behaved fairly consistently than to investigate whether the variation in the readings obeyed any ascertainable laws.

One experiment, at least, was, however, made to ascertain whether the rate of reduction of pressure was of any consequence. Two aneroids were tried, a comparison being made of their readings when pressure was reduced at the two different rates, 1 inch in 10 and 1 inch in 30 minutes. In one of the aneroids the readings were sensibly lower all through the range at the slower rate; in the other the change of rate had no certain influence.

§ 49. No. 3, which has been already referred to, is divided into parts. Part 1, treating of "Comparisons in the Field," enumerates changes of zero and of reading observed in a number of aneroids used at lofty stations, but hardly bears on the present paper. Part 2, "Experiments in the Workshop," gives numerous details as to the creep during several weeks' exposure to low pressures. It also records a considerable number of observations showing the recovery of aneroids on return to atmospheric pressure after a week's exposure to a lower pressure.

Mr. WHYMPER was apparently unaware of Dr. BALFOUR STEWART'S work, and regarded the creep at low pressures as a new phenomenon. He noticed that the rate of creep diminished as the time increased since the low pressure was reached, and that the recovery on return to atmospheric pressure was most rapid at first. He seems, however, to have made no serious attempt to study more exactly the laws of the phenomena. Judging from some remarks on his p. 25, he was discouraged from doing so by the belief that different aneroids varied very much in the laws they followed. The fall of reading during the first day's exposure to a low pressure might vary, he says, from about one-third* to more than three-fourths of the fall in the first week.

For evidence of this he refers to a table on his p. 26, giving particulars of the creep in 29 aneroids during the first day and the first week. The evidence seems to me somewhat inconclusive. In the first place, there seems no information as to whether the original lowering of pressure was carried out at an invariable rate. Thus it is not clear whether an exact comparison is possible between the aneroids which were exposed to different pressures. Those exposed to the same pressure were presumably exposed to the same conditions, and in their case there are no variations at all approaching those mentioned by Mr. WHYMPER. In fact, I notice only two instances in the whole table in which the creep in the first day was less than the half of that in the first week; and in both these cases there was no companion aneroid tried at the same pressure. In the first instance a 3-inch Watkin aneroid, exposed to a pressure of 24 inches, is credited with a creep of ·111 inch in a day and ·273 inch in a week; in the second instance, that of a $4\frac{1}{2}$-inch Watkin aneroid, the creeps were ·052 inch in a day and ·144 inch in a week. The presumption is that the aneroids were only read to ·01, or at most to ·005, of an inch, the third decimal coming from the reading of the mercury barometer; in the second instance mentioned above, a trifling error in the reading would make a large difference in the result.

* A footnote says "less than a fifth" in some exceptional cases not quoted.

In the other 27 aneroids the creep in the first day bore to that in the first week a ratio lying between ·6 and ·9. It is also not unreasonable to suppose that some at least of the extreme values were appreciably influenced by permanent changes of zero.

§ 50. Notwithstanding the absence of information as to the exact nature of the operations, Mr. WHYMPER's experiments are of much interest, inasmuch as they supply information of an almost unique kind relative to the effects of very prolonged exposure to low pressures. I have thus examined the results with considerable care, with a view to seeing what light they throw on the phenomena already described.

There are, in the first place, several tables whose contents afford the opportunity of determining how far it is possible to represent the creep over a series of weeks at a low pressure by a single algebraic term Ct^q, as in (7), § 18.

Thus, on his p. 17 Mr. WHYMPER gives the errors, relative to a mercury barometer, in six aneroids exposed together for six weeks to a pressure of $22\frac{1}{2}$ inches. Readings were taken at the end of each week. How exactly equal the "weeks" were is not stated; but a few hours' variability in such a case would hardly matter. At the end of four weeks the readings had become, if not absolutely stationary, so nearly so that the probable error of reading began to be too serious.

In Table XLV. I give the ratios I find borne to the first week's creep by the total creeps up to the end of the second, third, and fourth weeks; also the values found for q in the algebraic term by taking separately the mean values of the above three ratios.

TABLE XLV.—Creep Ratios, calculated from data on Mr. WHYMPER's p. 17.

	Aneroid						Mean.	Corresponding q.
	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.		
2 week's creep/1 week's	1·06	1·10	1·15	1·05	1·12	1·19	1·11	·151
3 " " "	1·13	1·20	1·26	1·21	1·15	1·33	1·20	·166
4 " " "	1·19	1·22	1·23	1·29	1·20	1·31	1·24	·155

The irregularities apparent in the data for some of the aneroids are not surprising, in view of the fact that the average creep recorded in the fourth week was only ·013 of an inch.

The mean results agree well with the single term formula (7).

The next set of data I have utilised are given on Mr. WHYMPER's p. 19. They refer to three aneroids—distinct from those of Table XLV.—exposed for eight weeks to the pressure of 16 inches. In these the increase of creep can be fairly traced up to the end of the sixth week. The individual aneroids show somewhat irregular behaviour. I give the mean results, obtained in the same way as the corresponding results in our last table. "$1w$" stands for "one week's creep,' and so on.

TABLE XLVI.—Creep Ratios, calculated from data on Mr. WHYMPER's p. 19.

	$2w/1w$.	$3w/1w$.	$4w/1w$.	$5w/1w$.	$6w/1w$.
Mean ratio	1·21	1·29	1·38	1·41	1·48
Corresponding q . . .	0·275	0·232	0·232	0·213	0·219

The mean value for q, viz., ·234, is considerably higher than that given in Table XLV. There is also at least a suggestion that the first two weeks' creep was, relatively speaking, somewhat greater than according to the single term law.

§ 51. The last and much the most varied set of data which I have utilised are taken from tables on Mr. WHYMPER's pp. 26, 27, and 28. The table on p. 26 has been already described. That on p. 27 gives with some blanks the errors at the end of the 1st, 2nd, 4th, and 7th days' exposure in 21 of the 29 aneroids included on p. 26. The table on p. 28 gives the errors at the end of the first hour, and the end of the first day, in all the 29 aneroids. Though not apparently mentioned, it is clear, from the size of the errors specified at the end of the first day, that the three tables refer to the same experiments. I have thus dealt with them together. The results of my calculations are given in Table XLVII. for all the aneroids for which complete sets of readings are given, with the exception of one, of which a reading is queried by Mr. WHYMPER. In the case of the 5 lowest pressures in the table, the results are means for the 2 or 3 aneroids exposed to the same pressure.

TABLE XLVII.—Creep Ratios, calculated from data on Mr. WHYMPER's pp. 26, 27, 28.

Constant pressure in inches.	Number of aneroids.	$\frac{\text{1 Hour's}}{\text{1 Day's}}$.	$\frac{\text{2 days'}}{\text{1 day's}}$.	$\frac{\text{4 days'}}{\text{1 day's}}$.	$\frac{\text{7 days'}}{\text{1 day's}}$.
14	2	·62	1·11	1·27	1·50
15	2	·58	1·17	1·27	1·42
21	2	·42	1·14	1·32	1·41
22	3	·47	1·17	1·35	1·50
23	2	·63	1·13	1·41	1·64
24	1	·44	1·50	1·92	2·46
26	1	·62	1·14	1·23	1·33
Mean from 13 aneroids		·53	1·17	1·36	1·55
Corresponding q . .		·200	0·227	0·222	0·225
Mean from 12 aneroids		·54	1·14	1·32	1·48
Corresponding q . .		·194	0·189	0·200	0·201

The aneroid exposed to the pressure of 24 inches is one of the two exceptional ones already referred to. The mean values in the last two lines of the table are obtained by omitting it.

The agreement between the mean results from the 12 aneroids and those deduced from the single term

$$Ct^{\cdot 196}$$

is extremely close.

There are naturally very considerable irregularities amongst the individual results in the table, but, with the solitary exception already alluded to, the phenomena observed at the different pressures are very similar. The aneroids were made mainly by CASELLA and HICKS, but they varied in diameter from 2 to $4\frac{1}{2}$ inches.

§ 52. Mr. WHYMPER's data as to recovery, though less varied, are also of interest. On his p. 30 he gives the original errors, the errors on return to the original pressure, and those observed 1 day and 1 week later for 36 aneroids, exposed for one week to pressures varying from 14 to 26 inches. In 12 instances the reading was higher after a week's recovery than it was prior to the experiment.

On his p. 31 Mr. WHYMPER repeats some of these data for 20 of the aneroids, adding also the errors observed after only 1 hour's recovery. I have omitted one of the 20 aneroids, No. 40, because after a week's recovery it read ·29 inch higher than prior to the experiment, the figures suggesting the occurrence of a large permanent change of zero before the end of the first day's recovery. According to the figures on p. 31 there was also a permanent rise of zero in three other aneroids, Nos. 42, 22, and 23. In the last-mentioned case, however, this seems due to a misprint of ·236 for ·206 in one of the data, the latter figure appearing on p. 30. In the case of Nos. 22 and 42 the final reading exceeded the original by so small an amount it seemed best to retain the aneroids. This retention accounts for the minus sign in the second entry of the last column of Table XLVIII. When more than one aneroid was exposed to a given pressure, I give only the mean values of the ratios for the group. In many cases the aneroids are the same as were dealt with in Table XLVII.

TABLE XLVIII.—Ratios of Deficiencies to Original Deficiency (data from Mr. WHYMPER's pp. 30, 31).

Constant pressure in inches.	Number of aneroids.	Time.			
		0.	1 hour.	1 day.	1 week.
14	2	1	·52	·17	·04
15	2	1	·70	·11	−·06
18	2	1	·62	·30	·22
20	6	1	·63	·37	·09
22	3	1	·63	·35	·19
23	2	1	·57	·28	·04
24	1	1	·61	·56	·35
26	1	1	·70	·50	·17
Mean from	19	1	·62	·32	·11

Evidently, little weight attaches to individual results after a week's recovery. A small permanent change or a slight initial fatigue in an aneroid would have a large effect on the final data. Even after a day's rest the results are somewhat erratic.

So far as one can judge from the recovery during the first hour, the lowness of the pressure to which the aneroids were exposed was immaterial. There is certainly no indication of the recovery becoming slower as the stationary pressure is more remote from 30 inches. It must be remembered, of course, that the aneroids exposed to the different pressures were different, so that individual peculiarities were not eliminated.

Comparing Table XLVIII. with Tables XVII. and XXXVI., it will be seen that the rate of recovery in Mr. WHYMPER's experiments was slower than in the case of the normal Kew experiments, but faster than in those experiments where pressure was reduced at the rate of an inch in 45 minutes, and where the lowest pressure was maintained stationary for 18 hours.

§ 53. There are other data on Mr. WHYMPER's p. 33 relating to the recovery of 22 aneroids, which had been exposed for a week to a pressure of 21·692 inches. These data I have not considered, for the reason that the reading during recovery proved higher than the reading prior to the experiment in

1 aneroid on return to atmospheric pressure,
5 aneroids 14 hours after return to atmospheric pressure,
9 ,, 12 days ,, ,, ,, ,,
16 ,, 30 ,, ,, ,, ,, ,,

Such a wholesale tendency to a rise of zero, whatever its cause, would have introduced great uncertainty into any conclusions drawn, even as to the recovery during the first 14 hours, the shortest interval for which results are given.

§ 54. Part 3 of Mr. WHYMPER's pamphlet, dealing with the "Determination of Altitudes," discusses the best method of utilising the readings of aneroid barometers. I hardly think that Mr. WHYMPER makes due allowance for the fact that in mountain ascents the reduction of pressure is very gradual, even at the lower stages. This allows accommodation to take place, so that on arrival at the summit the instrument will not behave as it would have done if rapidly transported there. Again the difference between descending and ascending readings is very different in aneroids whose readings as pressure falls at a given rate may be equally correct, and the difference increases *ceteris paribus* with the length of time spent on the mountain summit. Thus the advantage to be derived from taking the mean of ascending and descending readings—a course which Mr. WHYMPER seems to suggest in some instances—is very problematical.

On his p. 51 Mr. WHYMPER gives an example of a Kew aneroid certificate and then devotes several pages to a criticism of it. On his p. 54 he says, "In the absence of directions to the contrary, it may be assumed by persons into whose

hands similar certificates come that the corrections stated in them are good under all conditions and for all time." In view of the footnote to the certificate, calling attention to the probable recovery from the residual error, exhibited on return to the original pressure, this standpoint could hardly, I think, be justified; and judging by a footnote to his p. 52 Mr. WHYMPER would presumably allow this himself. At the same time, he apparently considers it a point of view likely to present itself to travellers, a class as to whose scientific knowledge and general intelligence he is better qualified to speak than I am. If he is right in his conclusion, a change in the certificate is certainly desirable.

Before concluding my reference to Mr. WHYMPER's interesting pamphlet, I would take the opportunity of explaining that the rate of change of pressure in the ordinary Kew test is not, as he states on several occasions, about 1 inch in 2 minutes; the actual rate is only about half this.

§ 55. The 75 special experiments—some of a very tedious and exacting character—on which this paper is mainly based, were carried out with great care and discretion by Mr. W. HUGO, Senior Assistant at Kew Observatory. In addition to the observational work, Mr. HUGO reduced all the barometer readings and carried out some of the subsequent arithmetical operations. The bulk of these and the checking of the reductions were undertaken by myself.

It is, I allow, anomalous, and from various points of view undesirable, that a scientific man should have himself performed none of the experiments which he discusses. When, however, as in the present case, observation is being pushed to the utmost capabilities of the instruments employed, the absence of preconceived ideas in the actual observer is a valuable compensation.