VII. *An Enquiry into the Nature of the Relationship between Sun-spot Frequency and Terrestrial Magnetism.*

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Contents.

§ 1. In a recent paper,* termed here for brevity (A), I considered the relationship between certain phenomena of terrestrial magnetism on "quiet" days at Kew and Wolfer's sun-spot frequency. Some contemporaneous French and German magnetic data were also referred to, but in no great detail. I have now gone more fully into the question, in order to find out whether any of the phenomena conspicuous at Kew are peculiar to the place or to the period of time formerly dealt with.

* 'Phil. Trans.,' A, vol. 202, p. 335.

Assuming the relation between any magnetic quantity R—such as the daily range of declination—and sun-spot frequency S to be of the simplest type,

$$R = a + bS \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

I determined the values of the constants a and b in a number of cases. The chief results were as follows:—

Supposing each month of the year treated separately, it was found that both a and b were conspicuously lower in "winter" (the four months November to February) than at the "equinoxes" or in "summer" (May to August); but b/a was larger in "winter" than at the "equinoxes," and larger at the "equinoxes" than in "summer." The b constants were generally fully larger at the "equinoxes" than in "summer," but to this there were exceptions.

The values of b/a were distinctly larger in the case of inclination and horizontal force than in the case of declination and vertical force; generally they were larger for inclination than for horizontal force, and larger for declination than for vertical force.

In all the elements b/a was larger when R represented the sum of the 24 hourly differences from the mean value for the day than when it represented the range in the diurnal inequality.

In (A) the letters D, I, H, V were used for declination, inclination, horizontal force and vertical force respectively. This practice is continued here when it tends to brevity.

§ 2. The first question now to be considered is as to the dependence of a and b on the particular period to which the magnetic and sun-spot data refer.

This is not so simple as might appear at first sight. Very few observatories have magnetic records extending over any large number of years, and in the few cases where such long records exist their homogeneousness is seldom, if ever, beyond dispute. There has usually been change both in the apparatus and its environment, and it is difficult even for those in immediate charge of an old observatory to know what allowance ought to be made to put old and new records on a common footing. This is especially true of V and I. The element where least uncertainty should prevail is D, but even here there is usually cause for doubt.

Milan Declination Ranges.

§ 3. RAJNA* has recently considered a long series of data representing the mean value for each year of the diurnal range of D at Milan. Since 1871 the range seems to have been determined from regular daily observations at 8 A.M. and 2 P.M.; but previous to that date there seems to have been some lack of strict uniformity. RAJNA has calculated values for a and b in the formula (1) from the data for the 59 years 1836 to 1894, and independently for the 24 years 1871 to 1894, employing

* 'Rendiconti del R. Ist. Lomb.,' Series II., vol. 35, 1902.

the method of least squares. The formulæ he thus obtains, and one which he quotes as obtained by WOLFER, are as follows :—

RAJNA . . . . $\begin{cases} 1836 \text{ to } 1894, & R = 5{\cdot}31 + 0{\cdot}047\,S \quad \text{(I.)}, \\ 1871 \text{ ,, } 1894, & R = 5{\cdot}39 + 0{\cdot}047\,S \quad \text{(II.)}, \end{cases}$

WOLFER . . . . $R = 5{\cdot}67 + 0{\cdot}040\,S$ . . . . (III.).

WOLFER'S value 0·040 for b is based on data from Christiania, Prague, Greenwich and Vienna, as well as from Milan. RAJNA compares the values calculated from each of the three formulæ with the ranges observed at Milan from 1836 to 1901. Formula (I.) agrees rather better than (II.) with observation; formula (III.) seems distinctly inferior. The difference between the observed values and those calculated from (I.) varies from $-1'{\cdot}87$ in 1838 to $+1'{\cdot}87$ in 1866, the extreme values observed in R being $4'{\cdot}21$ and $12'{\cdot}03$. Since 1871 the agreement seems decidedly improved. Over the 24 years for which it was originally calculated (II.) gives a "probable error" of only $0'{\cdot}21$, the difference between observed and calculated values varying from $-0'{\cdot}49$ to $+0'{\cdot}53$.

RAJNA himself notices that there are several long runs of the same sign in the differences between observed and calculated values. Thus from 1837 to 1850 the observed value is in excess of the calculated (from either (I.) or (II.)) 12 out of 14 times; on the other hand, in the 14 years 1854 to 1867, the calculated value is 13 times in excess. In the 11 years 1890 to 1900 (to which the Kew data treated in (A) referred) RAJNA'S calculated value from either (I.) or (II.) has been in excess 8 times, including every year since 1893.

The two first specified predominances of one sign are certainly in excess of what one would expect from pure chance. To throw some further light on the question, I have calculated values for a and b for the above-mentioned series of years. Instead of least squares, I grouped the years, following the method explained in (A), § 52. The grouping of years and the corresponding mean values of R and S were as follows :—

	Mean R.	Mean S.
Period 1837 to 1850—	9·270	68·9
Years of sun-spot maximum, 1837, 1838, 1839, 1847, 1848, 1849 . .	10·950	107·7
,, ,, minimum, 1841 to 1845	7·552	25·4
Period 1854 to 1867—	6·586	41·4
Years of sun-spot maximum, 1858 to 1862	8·176	76·1
,, ,, minimum, 1854, 1855, 1856, 1865, 1866, 1867 . .	5·245	14·3
Period 1890 to 1900—	7·163	41·7
Years of sun-spot maximum, 1892 to 1895	8·753	75·0
,, ,, minimum, 1890, 1899, 1900	5·587	9·5

The values found in the several cases for a, b and b/a, with the corresponding values from RAJNA's formula (I.), appear in Table I.

TABLE I.—Milan Declination Ranges.

Period of years . . .	1837 to 1850.	1854 to 1867.	1890 to 1900.	1836 to 1894 (RAJNA).
a	6'·43	4'·62	5'·14	5'·31
b	·0413	·0474	·0484	·047
$10^4 \times (b/a)$	64	103	94	89

§ 4. It is certainly satisfactory that the values of a and b for the period 1890 to 1900 differ so little from RAJNA's values for the long period 1836 to 1894. The probable error, employing my values of a and b, is only some 4 or 5 per cent. less than that found when employing RAJNA's values for the long period.

In considering the results for the two earlier periods, we must remember the want of homogeneousness referred to above. The mere existence of RAJNA's formula (I.) seems, however, evidence that, in his opinion, the want of homogeneousness is not serious, and the similarity of formulæ (I.) and (II.) to a certain extent supports this view.

The period 1837 to 1850 gives a very high value for a and a distinctly low value for b. The outstanding features of this period were the high mean sun-spot frequency, and the largeness of the declination range in the years of sun-spot minimum. Unless the results are very sensibly affected by heterogeneousness in the data, we must conclude that values calculated for a and b from a period as long as 14 years *may* depart somewhat widely from those calculated from a different equal or longer period. The range of variability would seem least in b and (naturally) greatest in b/a.

The value calculated for b from the period 1854 to 1867 agrees well with RAJNA's, but the value found for a is distinctly lower than his. The sun-spot frequency during this period presented similar features to those occurring in the 11 years 1890 to 1900.

Greenwich Declination and Horizontal Force Ranges.

§ 5. A second long series of data is that employed by Mr. ELLIS in two papers,* in which he compares D and H ranges at Greenwich with sun-spot frequency. Mr. ELLIS gives the observed D and H ranges from the diurnal inequalities for each month of the period 1841 to 1896. These are based on *all* days, excluding those of

* 'Phil. Trans.,' vol. 171, for 1880, p. 541; 'Proc. Roy. Soc.,' vol. 63, 1898, p. 64.

large disturbance. Corresponding "quiet" day data appear in Mr. ELLIS' second paper for the 8 years 1889 to 1896. Mr. ELLIS specifies several sources which may have introduced some heterogeneousness into the earlier data as compared to the later. Prior to 1848 there were only eye readings at 2-hour intervals, whereas subsequently hourly data were available. Prior to 1864, when the magnetographs were transferred to a new building, some uncertainty seems to have prevailed as to the temperature correction for H, and the data for 1864 itself seem to be interpolated. The data subsequent to 1864 would seem to be strictly homogeneous.

§ 6. Mr. ELLIS employed no formula, and, whilst his graphical method appeals readily to the eye, it does not lend itself immediately to the present investigation.

I have accordingly calculated values for a and b for each month of the year in both D and H for the following periods: 1841 to 1896, 1865 to 1896, and 1889 to 1896 for both "all" and "quiet" days. In treating the first period, use was made of a group of sun-spot maximum years composed of 5 sub-groups each of 3 years, viz., 1847 to 1849, 1859 to 1861, 1870 to 1872, 1882 to 1884, and 1892 to 1894. The corresponding sun-spot minimum group consisted similarly of 15 years, made up of 5 sub-groups, viz., 1842 to 1844, 1854 to 1856, 1865 to 1867, 1877 to 1879 and 1888 to 1890.

For the period 1865 to 1896 the groups of sun-spot maximum and sun-spot minimum years were composed in either case of the last 3 sub-groups specified above.

For the period 1889 to 1896 the groups were: 1892 to 1895 for sun-spot maximum, and 1889 to 1890 for sun-spot minimum.

The values thus found for a, b and b/a appear in Tables II. and III. In addition to values for the individual months, the tables give values for winter, equinox and summer—each comprised of 4 months, as explained in § 1—and for the year. These seasonal and yearly values of a and b are simply arithmetic means of the individual monthly values; the seasonal and yearly values of b/a are derived from the seasonal and yearly values of a and b. The tables also supply corresponding data for Kew as given in (A), Table XL.

TABLE II.—Greenwich and Kew (unit 1′).

Ranges from Mean Monthly Diurnal Inequalities of Declination.

	a.					10^4b.					$10^4b/a$.				
	1841–96.	1865–96.	1889–96.		Kew.	1841–96.	1865–96.	1889–96.		Kew.	1841–96.	1865–96.	1889–96.		Kew.
			All.	Quiet.				All.	Quiet.				All.	Quiet.	
January	5·08	4·62	4·47	3·71	3·16	269	308	263	313	307	53	67	59	84	97
February	6·25	5·75	5·16	4·37	3·55	259	277	401	347	383	41	48	78	80	108
March	7·32	7·10	6·86	6·29	7·13	502	504	602	719	526	68	71	88	114	74
April	9·26	9·24	8·83	9·19	8·56	471	454	480	332	508	51	49	54	36	59
May	8·57	8·45	8·24	8·22	9·34	444	443	487	544	401	52	52	59	66	43
June	8·75	8·81	8·10	8·04	8·45	442	446	501	517	504	51	51	62	64	60
July	8·70	8·83	7·94	8·08	8·75	421	428	542	488	387	48	49	68	60	44
August	8·96	8·91	8·48	8·10	9·11	417	433	385	466	421	47	49	45	58	46
September	8·01	7·44	6·71	6·54	7·48	403	516	535	596	517	50	69	80	91	69
October	6·98	6·51	6·59	6·05	6·13	336	381	304	297	361	48	58	46	49	59
November	5·34	5·28	5·16	4·31	3·60	285	266	268	215	369	53	51	52	50	102
December	4·28	3·94	3·97	3·38	2·62	270	295	249	145	234	63	75	63	43	89
Winter	5·24	4·90	4·69	3·94	3·23	271	287	295	255	323	52	59	63	65	100
Equinox	7·89	7·57	7·25	7·02	7·32	428	464	480	486	478	54	61	66	69	65
Summer	8·75	8·75	8·19	8·11	8·91	431	438	479	504	428	49	50	58	62	48
Year	7·29	7·07	6·71	6·36	6·49	377	396	418	415	410	52	56	62	65	63

TABLE III.—Greenwich and Kew (unit 1γ).

Ranges from Mean Monthly Diurnal Inequalities of Horizontal Force.

	a.					10^3b.					$10^4b/a$.				
	1841–96.	1865–96.	1889–96.		Kew.	1841–96.	1865–96.	1889–96.		Kew.	1841–96.	1865–96.	1889–96.		Kew.
			All.	Quiet.				All.	Quiet.				All.	Quiet.	
January . .	14·2	10·7	11·4	11·2	11·6	157	147	116	181	118	110	137	102	162	101
February . .	13·8	11·0	13·0	15·1	10·5	203	187	143	219	172	147	170	110	145	164
March . . .	21·9	19·7	19·6	17·8	19·3	231	233	307	372	238	105	118	156	209	124
April . . .	33·3	32·2	29·7	31·5	25·7	265	246	310	262	284	79	76	104	83	111
May . . .	35·4	32·6	31·6	34·4	29·6	211	248	260	214	219	60	76	82	62	74
June . . .	36·7	33·4	34·1	37·2	32·6	223	301	281	194	142	61	90	82	52	44
July . . .	35·9	34·5	32·0	31·1	28·6	248	268	315	261	238	69	78	98	84	83
August. . .	35·8	32·9	33·3	38·3	31·6	169	234	238	154	160	47	71	72	40	51
September. .	32·8	30·3	31·7	30·1	27·0	155	205	178	223	192	47	68	56	74	71
October . .	27·1	24·5	24·1	29·0	22·2	142	182	201	130	171	52	74	83	45	77
November. .	15·9	13·7	13·8	15·3	13·0	168	180	177	237	225	106	132	128	155	173
December . .	13·6	8·2	10·6	9·1	6·7	108	145	91	114	131	80	176	86	125	193
Winter. . .	14·4	10·9	12·2	12·7	10·5	159	165	132	188	161	111	151	108	148	145
Equinox . .	28·8	26·6	26·3	27·1	23·5	198	216	249	246	221	69	81	95	91	94
Summer . .	35·9	33·4	32·7	35·2	30·6	213	263	274	206	190	59	79	83	58	62
Year . . .	26·4	23·6	23·7	25·0	21·5	190	215	218	213	191	72	91	92	85	89

§ 7. In the case of D, Table II., we see on the whole a close resemblance between the results from the two longest series of data. The later period shows, however, a slightly increased value of b and a slightly decreased value of a in winter, equinox, and the year as a whole. The last period, 1889–96, in the "all" day data shows a further increase in b and decrease in a; and the corresponding "quiet" day results give a still smaller value of a, especially in the winter months. The "all" and "quiet" day mean values of b for the year, from the 1889–96 period at Greenwich, are practically identical and very close to the corresponding Kew value. The values of b/a at Greenwich are in each season slightly larger for the "quiet" days than for the corresponding "all" days; and comparing the "all" day data amongst themselves we have an increase in b/a in passing from the longest to the mean period, and in passing from the mean period to the shortest period. This would imply that b/a has increased of late years.

At Greenwich, as at Kew, b is conspicuously lowest in "winter"; but no one of the four columns of Greenwich results gives so distinct an excess in the equinoctial over the summer value as appears at Kew, and, on the whole, we should infer that the equinoctial and summer values of b at Greenwich are practically equal.

At Greenwich, as at Kew, b/a is distinctly smaller in summer than in the other seasons; but the "winter" value of b/a at Greenwich, instead of markedly exceeding the values for the other seasons, as at Kew, would seem to be if anything slightly smaller than the equinoctial.

On the whole, the variation of b/a throughout the year at Greenwich is surprisingly small.

§ 8. The H ranges in Mr. ELLIS' tables are expressed in terms of the value of H at Greenwich. To make the results comparable with those for other stations, I have expressed the ranges in terms of 1γ as unit. In doing so, I treated H as constant for each period, and as possessing the following values:—

Period.	Value of H.
	C.G.S.
1841–96	·179
1865–96	·180
1889–96	·1829

Not knowing the exact procedure followed at Greenwich, I may not unlikely differ slightly from the exact values adopted there; but, for the purpose of the present enquiry, such small uncertainty as may exist is immaterial. The values of b/a are, of course, independent of the unit adopted.

The 1841–96 and 1865–96 data in Table III. present some conspicuous differences; a is larger in the former series than in the latter in every single month of the year, but b shows exactly the opposite phenomenon in 9 months out of the 12. This

implies a very conspicuous difference between the phenomena in years prior and subsequent to 1864. Not improbably the want of homogeneousness in the earlier data, already referred to, may be partly accountable for the apparent change. At all events, the results for the final period, 1889–96, show no distinct progressive diminution of a or increase of b as compared to the period 1865–96. In fact, the "all" day data for these two periods, and the "quiet" day data for the shorter period, give almost identical values for the mean b for the year.

As regards the seasonal phenomena, each set of Greenwich data makes b conspicuously least, but b/a conspicuously largest, in "winter." The December value of b is invariably the smallest. The "all" and "quiet" day data for 1889–96, as already stated, give very nearly the same mean value of b for the year, but the "quiet" days' value for b is much the larger of the two in winter, and the smaller in summer. The "quiet" day data at Greenwich present a remarkable similarity to the corresponding Kew data, as is best seen by comparing the seasonal values for b/a. The fact, however, that the absolute values of both a and b are some 10 per cent. higher at Greenwich than at Kew is rather suggestive of some misapprehension as to the scale values at one or both observatories.

§ 9. In cases such as the present, a comparison of calculated and observed values is useful. The Greenwich data do not lend themselves very readily to this, as Mr. Ellis does not give mean values for individual years, and the interest attaching to such a comparison for individual months of the year seems hardly sufficient to justify the necessary labour. In (A), § 75, some results were given for individual months of the year at Kew, but none for the year as a whole. This information is accordingly now supplied in Table IV., so far as concerns the ranges and the sum of the 24 hourly differences from the mean in the mean diurnal inequalities for the year in the several elements. The calculated values are derived from the values given for a and b in (A) Table XLIV. From the mathematical standpoint the nicety of agreement is best judged by considering the last line in Table IV., which shows what percentage the probable error in a calculated value is of the total variation exhibited by the element in the 11-year period. For practical purposes the mean difference between the calculated and observed values, and the percentage it forms of the absolute mean value of the element, are, however, fully as important.

It will be seen that the agreement is about equally good for the ranges and for the sum of the 24 differences. It is, on the whole, slightly better for D and H than for V and I.

TABLE IV.—Kew (Units 1′ for Angles, 1γ for Force Components).
Observed less Calculated Values.

Year.	Range from mean diurnal inequality for the year.				Sum of the 24 hourly differences in the mean diurnal inequality for the year.			
	D.	H.	V.	I.	D.	H.	V.	I.
1890	+0·49	+ 0·5	—	—	+ 2·12	+ 0·5	—	—
1891	+ ·40	+ 3·0	+ 1·4	+0·21	+ 2·55	+ 16·5	+ 6·3	+0·86
1892	+ ·24	− 1·4	+ 1·5	− ·12	+ 2·22	− 6·8	− 0·2	− ·54
1893	+ ·32	+ 0·9	− 1·1	+ ·05	− ·86	+ 2·9	− 0·2	+ ·12
1894	− ·16	+ 0·8	− 0·9	+ ·09	− ·49	+ 9·9	− 3·7	+1·12
1895	− ·28	− 0·4	+ 0·4	− ·01	− 1·05	− 6·4	+ 0·8	− ·61
1896	− ·14	− 1·4	+ 0·4	− ·08	·00	− 6·6	+ 7·7	− ·76
1897	− ·53	− 1·1	− 1·8	− ·05	− 1·68	− 6·4	−11·4	− ·24
1898	− ·41	− 1·9	− 0·2	− ·13	− 1·11	− 14·0	+ 0·3	− ·78
1899	+ ·07	+ 0·3	+ 1·2	− ·01	+ ·25	+ 8·3	+ 4·8	+ ·50
1900	+ ·01	+ 0·6	− 0·9	+ ·07	− 1·93	+ 2·3	− 4·2	+ ·33
Mean difference calculated ∽ observed . .	0·28	1·12	0·98	0·082	1·30	7·3	4·0	0·59
Probable error . . .	0·23	0·95	0·78	0·071	1·09	6·1	3·8	0·46
Mean value of element .	7·90	26·2	18·0	1·43	41·6	154·5	96·9	8·30
Range of element. . .	3·54	15·5	7·5	0·95	21·0	108·1	37·1	6·35
$\frac{\text{Mean difference} \times 100}{\text{mean value}}$.	4	4	5	6	3	5	4	7
$\frac{\text{Probable error} \times 100}{\text{range of element}}$.	6	6	10	7	5	6	10	7

Pawlowsk Data (59° 41′ N. lat., 30° 29′ E. long.).

§ 10. Magnetic observations at the three Russian magnetic observatories at Pawlowsk, Katharinenburg, and Irkutsk are published very nearly on parallel lines under the auspices of the Central Physical Observatory at St. Petersburg. The results are very complete, and are kept well up to date. For the present enquiry I have selected Pawlowsk and Katharinenburg. The former is, I believe, the furthest north magnetic observatory which has been in continued existence for any length of time. Its results include very full details of diurnal maxima and minima, and of the amplitudes of movements in magnetic storms. They are not confined to "all" days, but give in addition full particulars for selected "quiet" days, or "normal" days as they were called by WILD, to whom the idea of their separate treatment is due.

I have treated the Pawlowsk data for the 11 years 1890 to 1900 by the method of groups, taking the same combination of years as for Kew, viz., 1892 to 1895 for sun-spot maximum, and 1890, 1899, and 1900 for sun-spot minimum.

Table V. refers to the ranges from the mean diurnal inequalities for the several months of the year, both from "all" and "quiet" days. The seasonal and yearly values of a and b are arithmetic means from the included months, and these arithmetic means are employed in calculating the seasonal and yearly values of b/a.

TABLE V.—(Units 1′ for Angles, 1γ for Force Components.)

Ranges from Mean Monthly Diurnal Inequalities at Pawlowsk.

	Declination.						Horizontal force.						Vertical force.						Inclination.		
	a.		10^4b.		$10^4b/a$.		a.		10^3b.		$10^4b/a$.		a.		10^3b.		$10^4b/a$.		a.	10^4b.	$10^4b/a$.
	All.	Quiet.	All.	Quiet.	All.	Quiet.	All.	Quiet.	All.	Quiet.	All.	Quiet.	All.	Quiet.	All.	Quiet.	All.	Quiet.	All.	All.	All.
January	4·12	1·95	203	254	49	130	8·0	8·5	99	89	123	104	9·5	3·4	133	22	140	64	0·71	65	88
February	4·45	2·30	399	482	90	209	8·8	10·2	254	169	288	165	6·3	4·2	493	25	777	58	0·75	114	152
March	6·12	6·12	693	735	113	120	22·8	19·5	247	316	108	162	11·9	8·8	484	16	407	18	1·51	156	103
April	8·92	9·30	482	477	54	51	33·4	31·2	314	258	94	83	15·3	10·5	206	60	135	57	1·95	202	104
May	9·58	9·97	601	676	63	68	35·3	32·0	271	311	77	97	18·0	11·8	190	38	105	32	2·04	158	78
June	9·60	10·91	587	522	61	48	32·3	34·2	373	230	115	67	9·5	9·3	238	66	252	71	1·95	205	105
July	9·55	9·92	507	532	53	54	31·0	30·9	393	247	127	80	5·7	10·3	381	62	674	60	1·90	211	111
August	9·92	9·89	324	399	33	40	32·8	33·6	250	135	76	40	11·4	8·9	161	35	141	40	2·03	134	66
September	7·06	7·64	364	383	52	50	28·5	29·7	233	172	82	58	8·9	4·5	310	102	348	227	1·69	184	109
October	4·81	5·49	495	340	103	62	22·5	21·9	224	213	100	97	7·8	4·9	287	60	365	122	1·47	170	116
November	3·78	2·87	502	307	133	107	10·1	10·1	189	212	187	210	4·4	3·8	395	22	894	57	0·65	156	95
December	3·82	1·85	201	202	52	109	7·9	4·6	74	143	94	308	7·4	3·2	171	22	230	67	0·65	60	92
Winter	4·04	2·24	326	311	81	139	8·7	8·4	154	153	177	183	6·9	3·7	298	23	430	61	0·69	99	143
Equinox	6·73	7·14	509	484	76	68	26·8	25·6	254	240	95	94	11·0	7·2	322	59	293	83	1·66	178	107
Summer	9·66	10·17	505	532	52	52	32·9	32·7	322	231	98	71	11·1	10·1	242	50	218	50	1·98	177	90
Year	6·81	6·52	446	442	66	68	22·8	22·2	243	208	107	94	9·7	7·0	287	44	297	63	1·44	151	105

§ 11. Table V. presents several novel features. In the declination we see a conspicuous difference between the variation of a throughout the year on "all" and on "quiet" days. In December and January the "all" day value of a is more than double the "quiet" day value, and the excess of the "all" day value is also prominent in November and February. On the other hand, the "quiet" day value of a is, in general, distinctly the larger throughout the equinoctial and summer months. There is no such prominent difference between the "all" and "quiet" day values of b for declination. There are, of course, conspicuous differences in one or two individual months, but the seasonal and yearly values are closely alike. The rise of b/a in winter and its fall in summer are conspicuous, especially in the "quiet" days, where the phenomenon is even more prominent than at Kew. The mean values of b/a for the year from "all" and from "quiet" days are in close agreement with one another and with the corresponding Kew value.

In H the seasonal and yearly values of a in "all" and in "quiet" days are much alike. The winter and equinoctial values of b in the two cases are also nearly equal, but in summer the "quiet" day value is very decidedly the smaller. The mean "all" day value of b for the year is distinctly larger than the "quiet" day value, which is itself slightly in excess of the corresponding Kew value. The excess of b/a in winter is conspicuous in both "all" and "quiet" days; in the latter case the variation of b/a throughout the year is pretty similar to that at Kew.

In V the "all" and "quiet" day phenomena are vitally different. The fact that the mean diurnal range during the 11-year period for the "quiet" days was barely 40 per cent. of that for "all" days prepares one for a material difference between the phenomena in the two cases, but hardly for the "all" day mean yearly value of b being more than six times the corresponding "quiet" day value. The "quiet" day value of a for summer is not much less than the "all" day value, but in the equinoctial and winter months the latter greatly predominates. The value of b is greatest at the equinox in both "all" and "quiet" days; in fact, in "all" days the summer value of b falls short of the winter value, notwithstanding a marked depression in December and January. The variation of b/a throughout the year on "quiet" days is somewhat irregular. In "all" days we have the fall in summer and rise in winter seen at Kew, but there is a prominent depression in December and January. As regards the absolute magnitudes of b and b/a, the "quiet" day data are much closer than the "all" day to the Kew results.

Table V. gives only "all" day data for I, as no "quiet" day data for this element seem to be published.

Here again the phenomena resemble those observed at Kew, b being conspicuously small in winter and b/a small in summer. The values of a and b are, on the whole, distinctly larger than at Kew (where the diurnal range of I is less than at Pawlowsk), but the mean value of b/a for the year is 105×10^{-4} as compared with 111×10^{-4} at Kew.

§ 12. The data ascribed to the year in Table V. are arithmetic means from the 12 monthly values. Table VI. supplies values of a, b and b/a for the ranges from the mean diurnal inequality for the year as given in the Pawlowsk tables. They answer to the range data for Kew in Tables XLIII. and XLIV. of (A), and I give the results in the latter of these tables (calculated by least squares) for comparison.

TABLE VI.—(Units 1′ for angles, 1γ for Force Components.)

Mean Diurnal Inequality for the Year at Pawlowsk.

	Declination.			Horizontal force.			Vertical force.			Inclination.		
	a.	$10^4 \times b$.	$10^4 \times b/a$.	a.	$10^3 \times b$.	$10^4 \times b/a$.	a.	$10^3 \times b$.	$10^4 \times b/a$.	a.	$10^4 \times b$.	$10^4 \times b/a$.
All days . .	5·74	400	70	20·7	211	102	8·1	265	326	1·24	126	101
Quiet days .	6·17	424	69	20·6	195	95	5·9	27	46	—	—	—
Kew . . .	6·10	433	71	18·1	194	107	14·3	81	56	0·87	125	145

§ 13. The Pawlowsk tables give for each month the mean of the differences between the daily maxima and minima, irrespective of their time of occurrence. The range thus obtained is, of course, larger than that from the mean diurnal inequality for the month, and is a quantity considerably more influenced by magnetic disturbances. The mean of the 12 monthly means may be regarded as the mean for the year of the absolute ranges in individual days. This is the quantity to which the results in the first line of Table VII. apply. The figures in the second line refer to the mean of the 12 monthly ranges, a monthly range being defined as the difference between the highest and lowest values recorded during the month. The third line in the table refers to the annual range, *i.e.*, the difference between the highest and lowest values recorded during the year. Owing to occasional losses of trace, monthly and annual ranges are sometimes under-estimated, especially at times of large disturbance. Both quantities are mainly dependent on the amplitude of disturbances; the mean monthly range is the better measure of the generally disturbed character of the year.

TABLE VII.—Pawlowsk (Units 1′ for angles, 1γ for H and V).

	Declination.			Horizontal force.			Vertical force.		
	a.	$10^3 \times b$.	$10^4 \times b/a$.	a.	$10^2 \times b$.	$10^4 \times b/a$.	a.	$10^2 \times b$.	$10^4 \times b/a$.
Mean daily range . .	11·28	113	100	45·2	64	141	17·6	52	295
„ monthly range.	28·6	538	188	102	413	406	74	412	556
Annual range . . .	62·9	893	142	240	1226	510	338	698	207

§ 14. If we compare the results for the mean daily range in Table VII. with those for the range of the diurnal inequality in Table VI., we see that, though the values of a in Table VII. are about double those in Table VI. in the case of D and H, the increase in b is relatively so much greater that the values of b/a in Table VII. are some 40 per cent. in excess of those in the earlier table. This means that the amplitude of magnetic disturbance is in general more enhanced relatively in years of sun-spot maximum than is that of the regular diurnal inequality. The great rise in b/a as we pass from the mean daily to the mean monthly range in Table VII. is evidence of the same fact. That this rise in b/a is not continued, except in H, as we pass from the mean monthly to the annual range, is at least consistent with the view that the incidence of *exceptionally* large magnetic storms is determined by causes of which sun-spot frequency is no exact measure.

§ 15. Table VIII. supplies information as to the degree of accordance between the observed values of the ranges and those calculated from the values of a and b in Tables VI. and VII. Table VIII. is constructed on parallel lines to Table IV.

TABLE VIII.—Pawlowsk (Units 1′ for D and I, 1γ for H and V).

Observed less Calculated Values.

Year.	Ranges from mean diurnal inequalities.							Absolute mean daily ranges.		
	"Quiet" (normal) days.			"All" days.						
	D.	H.	V.	D.	H.	V.	I.	D.	H.	V.
1890	+0·2	+ 3	+ 1	+0·30	0	− 1	0·00	+ 0·06	− 1·1	− 0·2
1891	+0·3	+ 1	+ 2	+ ·15	+ 2	+ 2	+ ·10	+ 0·71	+ 2·5	+ 3·2
1892	+ ·6	− 4	+ 1	+ ·09	+ 1	+ 8	+ ·05	+ 1·51	+19·4	+17·5
1893	+ ·3	+ 2	+ 1	+ ·51	− 1	−11	− ·07	− 3·05	−20·5	−21·0
1894	+ ·3	+ 5	− 2	− ·28	+ 1	+ 1	− ·06	+ 0·33	+ 2·3	+ 3·4
1895	− ·8	− 1	− 1	− ·08	− 1	− 2	+ ·03	− 0·44	− 6·2	− 4·7
1896	− ·3	− 2	0	− ·02	− 1	+ 4	·00	+ 1·46	+ 2·0	+ 3·3
1897	− ·6	− 2	− 1	·00	0	0	+ ·02	+ 0·33	− 0·7	− 1·6
1898	− ·3	− 2	− 1	− ·55	0	+ 1	− ·02	+ 0·40	+ 5·0	+ 3·6
1899	− ·2	− 2	0	− ·20	+ 1	+ 1	+ ·04	+ 0·50	+ 4·9	+ 2·7
1900	+ ·3	+ 1	− 1	+ ·08	− 1	− 3	− ·08	− 1·81	− 7·6	− 6·2
Mean difference calculated ∼ observed	0·38	2·3	1·0	0·21	0·8	3·1	0·043	0·96	6·56	6·13
Probable error	0·30	1·8	0·8	0·19	0·7	3·2	·037	0·91	6·60	6·27
Mean value of element	7·94	28·7	7·0	7·41	29·5	19·2	1·77	16·0	71·8	39·3
Range of element	3·6	20	4	3·62	16	27	0·96	10·5	67·4	56·7
Mean difference × 100 / mean value	5	8	14	3	3	16	2	6	9	16
Probable error × 100 / range of element	8	9	20	5	4	12	4	9	10	11

The calculated values of H and V employed in Table VIII. for the inequality ranges were taken only to the nearest 1γ, because the Pawlowsk tables go no nearer than this; but in the "all" days' D and I, both tables and calculation go to $0'\cdot01$. The agreement between calculated and observed values is much closer in D, H, and I than in V; and in D and H it is considerably closer for the "all" day than the "quiet" day results. Probably this only means that the fewness of the "quiet" days (sometimes only two or three a month) introduces an element of uncertainty which more than neutralises the effect of the greater regularity in these days.

§ 16. If the range of magnetic elements were largely dependent on influences which did not proceed *pari passu* with sun-spot frequency, then what we should expect to see in Table VIII. would be a notable occurrence of large + values in *all* the elements in some years, and of large − values in other years. The same result would follow if, while an intimate connection subsisted, it were not of the linear type assumed in (1).

So far as the inequality ranges in D, H, and I are concerned, there is no indication of such a phenomenon. There is indeed an excess of + signs from 1890 to 1894 and of − signs from 1895 to 1899, but the differences themselves are small, and those for the "all" and the "quiet" days show no kind of regular relationship. In the case, however, of the "all" day V inequality, and of the absolute daily ranges for all the elements, especially H and V, the observed values are conspicuously in excess of the calculated in 1892, and as conspicuously below them in 1893. This phenomenon seems due beyond a doubt to the influence of the disturbance element.

§ 17. With a view to further elucidation of the phenomenon described in the last paragraph, I have placed side by side in Table IX. data as to the mean value for each year of a variety of quantities which are affected in different degrees by magnetic disturbance. The small figures in brackets attached to the annual figures show the position which the year in question would occupy on a list which followed the order of magnitude of the quantity in question. If two yearly items are equal, a common number is attached. In the case of the years themselves, the attached figures indicate the order when the arrangement follows sun-spot frequency. It should, however, be noticed that the excess of sun-spot frequency in 1898 over 1897 was very trifling, and that the differences between 1899, 1900, and 1890 were not large.

In the case of the diurnal inequalities in D and H, quantities but little affected by disturbance, 1893 heads the list, just as it does in sun-spot frequency. In the case of the mean daily range—a quantity more influenced by disturbance—1892 and 1894 come to the front, and 1893 falls to the fourth place. Coming to the mean of the monthly ranges, we see 1892 and 1894 still more in advance, while in the case of H and V 1893 stands lower than 1898, a year of less than one-third its sun-spot frequency.

In the case of the annual range, 1893 has fallen to the eighth place in D and ninth place in V, whilst 1898 mounts to the third or even the second place.

In the mean daily, mean monthly, and annual ranges, 1892 and 1894 are as conspicuously in excess of what one would expect from sun-spot frequency as 1893 and 1895 are below it. Thus when we treat these four years as a unit, and compare it with a similar unit made up of the three years 1890, 1899, and 1900, we may arrive at a conspicuous connection between sun-spot frequency and amplitude of disturbance; but at the same time there is a marked absence of the close and regular connection in individual years which characterises the inequality ranges in D, H, and I.

TABLE IX.—Pawlowsk (Units 1′ for D, 1γ for H and V).

Year.	Diurnal inequality range.		Mean daily range.		Mean monthly range.			Annual range.		
	D.	H.	D.	H.	D.	H.	V.	D.	H.	V.
1890 (11) . . .	6·32 (8)	22 (10)	12·14 (10)	49 (10)	28·2 (11)	118 (11)	80 (11)	42·1 (11)	169 (11)	179 (11)
1891 (6) . . .	7·31 (6)	30 (5)	16·01 (6)	70 (6)	46·3 (7)	218 (7)	233 (5)	92·3 (6)	550 (5)	614 (4)
1892 (3) . . .	8·75 (2)	37 (3)	21·04 (1)	111 (1)	93·6 (1)	698 (1)	575 (1)	194·0 (1)	2416 (1)	1385 (1)
1893 (1) . . .	9·64 (1)	38 (1)	17·82 (4)	79 (4)	48·3 (4)	241 (4)	210 (7)	87·1 (8)	514 (6)	457 (9)
1894 (2) . . .	8·58 (3)	38 (1)	20·42 (2)	97 (2)	84·1 (2)	493 (2)	493 (2)	145·6 (2)	1227 (2)	878 (3)
1895 (4) . . .	8·22 (4)	33 (4)	18·07 (3)	80 (3)	47·4 (5)	220 (6)	223 (6)	73·9 (9)	395 (9)	534 (6)
1896 (5) . . .	7·39 (5)	29 (6)	17·46 (5)	74 (5)	52·4 (3)	232 (5)	236 (4)	88·7 (7)	574 (4)	608 (5)
1897 (8) . . .	6·79 (7)	26 (7)	14·57 (8)	61 (8)	43·8 (8)	201 (8)	170 (8)	101·1 (4)	449 (8)	480 (8)
1898 (7) . . .	6·25 (9)	26 (7)	14·70 (7)	67 (7)	46·6 (6)	276 (3)	242 (3)	118·9 (3)	1136 (3)	888 (2)
1899 (9) . . .	6·02 (11)	24 (9)	13·14 (9)	58 (9)	38·3 (9)	178 (9)	150 (9)	63·8 (10)	382 (10)	527 (7)
1900 (10) . . .	6·20 (10)	22 (10)	10·54 (11)	44 (11)	32·8 (10)	134 (10)	89 (10)	94·2 (5)	457 (7)	365 (10)
Means . .	7·41	30	15·99	72	51·1	274	246	100·2	752	629

§ 18. It was pointed out in (A), §§ 74 and 75, that whilst an intimate general connection between sun-spot frequency and diurnal magnetic ranges is unmistakable, it is open to doubt whether the mean values of these quantities for so short a period as a single month can be regarded as directly interconnected.

If both phenomena proceed from a common cause whose intensity of action at a given instant varies throughout the solar system, then it might possibly be better to compare monthly magnetic ranges with sun-spot frequency for a longer overlapping period.

As shown in (A), Table I., the mean sun-spot frequency for individual months of the year from the period 1890 to 1900 varied from 35·0 in November and 35·5 in March to 45·4 in June and August. Clearly, if the connection between sun-spot frequency and magnetic range is of the more general kind indicated above, the values we have found for b and b/a at Kew and Pawlowsk will be too large in months such as November and March and too small in months such as June and August.

To obtain an outside estimate of the uncertainty thus existing, I have calculated values for a, b and b/a for the "quiet" day Pawlowsk data, employing WOLFER'S

smoothed sun-spot frequencies (Ausgeglichene Relativzahlen), each of which is a mean from observed values for 13 months, of which the individual month forms the central period. Table X. gives the mean seasonal and yearly values thus found; these answer precisely to the seasonal and yearly values based on observed sun-spot frequencies which appear in Table V.

TABLE X.—(Units 1′ for D, 1γ for H and V.)

Pawlowsk "Quiet" Days with WOLFER'S Smoothed Values (Ausgeglichene Relativzahlen).

	Declination.			Horizontal force.			Vertical force.		
	a.	$10^4 \times b$.	$10^4 \times b/a$.	a.	$10^3 \times b$.	$10^4 \times b/a$.	a.	$10^3 \times b$.	$10^4 \times b/a$.
Winter . . .	2·17	312	144	8·0	152	191	3·6	23	63
Equinox . .	7·29	428	59	26·2	216	82	7·4	55	74
Summer. . .	9·98	616	62	31·9	265	83	9·9	58	58
Year. . . .	6·48	452	70	22·0	211	96	7·0	45	65

So far as the mean yearly and winter values of a, b and b/a are concerned, Tables V. and X. are in practical agreement, but the equinoctial values of b in Table X. are decidedly lower, and the summer values decidedly higher, than the corresponding quantities in Table V. The fact that the equinoctial values of b/a for D and H in Table X. fall slightly below the summer ones seems hardly likely *à priori* to be a natural phenomenon, and it is not in accordance with the results obtained for Greenwich, in Tables II. and III., from the longer periods, where the variation of the mean sun-spot frequency from month to month is naturally less than in 1890 to 1900.

§ 19. The effect of the substitution of the smoothed sun-spot frequencies on the values of b and b/a from month to month is most easily followed by expressing the monthly values as percentages of their mean for the 12 months. Table XI. gives the mean of the results thus obtained for D and H, employing smoothed and observed sun-spot frequencies for the "quiet" days, and observed frequencies for the "all" days. The employment of smoothed frequencies for the "all" days would alter the results to about the same extent as it does in the "quiet" days.

The substitution of the smoothed for the observed sun-spot frequencies for "quiet" days removes an isolated prominence shown by the b variation in March, and removes slight depressions in June and August, but it produces a depression in September and adds materially to an already conspicuous prominence in May. Also the June depression and the March prominence are not apparent in the "all" day b variation using the observed sun-spots, and if we used smoothed frequencies for the "all" days we should have a marked depression in March and a largish prominence in June.

The b/a variation throughout the year proceeds most regularly when we use the observed frequencies.

TABLE XI.—Pawlowsk.

Variation of b and b/a throughout the Year.

	b.			b/a.		
	"Quiet" days.		"All" days.	"Quiet" days.		"All" days.
	Smoothed frequencies.	Observed frequencies.	Observed frequencies.	Smoothed frequencies.	Observed frequencies.	Observed frequencies.
January . . .	50	50	43	116	117	85
February. . .	96	95	97	188	187	180
March. . . .	116	159	128	89	135	124
April	123	116	119	66	63	76
May	171	151	123	90	78	75
June	126	114	142	59	55	90
July	124	120	137	63	63	89
August . . .	103	77	88	53	39	54
September . .	71	85	89	40	52	69
October . . .	84	90	101	67	75	113
November . .	72	86	95	121	147	170
December . .	64	57	38	248	188	75

Katharinenburg (56° 49′ N. lat., 60° 38′ E. long.).

§ 20. Table XII. gives values of a, b and b/a for the range of the diurnal inequality for each month of the year, and arithmetic means for the seasons and the year, at Katharinenburg, corresponding exactly to the "all" day results for Pawlowsk given in Table V.

In D, b appears decidedly less at Katharinenburg than at Pawlowsk, especially in winter. The dip in the December and January values of b in Table XII. is particularly striking. The summer and equinoctial values of b/a at Katharinenburg are very similar to those at Kew and Pawlowsk, but the winter value is much less, owing to the low values in December and January.

In H the mean b for the year is close to the Kew value, but the winter values of b and b/a are distinctly lower than at Kew and Pawlowsk.

In V, Katharinenburg occupies an intermediate position between the "all" and the "quiet" day results for Pawlowsk The mean value of b for the year in Table XII. is little over half the corresponding "all" day value at Pawlowsk, but it is more than thrice the "quiet" day value at Pawlowsk, and double that at Kew.

The winter values of b and b/a at Katharinenburg are exceptionally large, notwithstanding a marked depression in December and January.

TABLE XII.—Katharinenburg (Units 1′ for D and I, 1γ for H and V).

Ranges from Mean Monthly Diurnal Inequalities.

	Declination.			Horizontal force.			Vertical force.			Inclination.		
	a.	$b \times 10^4$.	$10^4 \times b/a$.	a.	$10^3 \times b$.	$10^4 \times b/a$.	a.	$10^3 \times b$.	$10^4 \times b/a$.	a.	$10^4 \times b$.	$10^4 \times b/a$.
January .	3·09	61	20	7·0	91	130	6·1	95	156	0·55	61	112
February .	3·07	293	95	9·8	115	117	4·8	249	514	0·69	60	88
March. .	5·48	605	110	20·6	231	112	9·5	224	235	1·21	138	114
April . .	9·18	483	53	24·5	308	126	15·8	121	76	1·32	184	143
May . .	9·62	509	53	32·0	185	58	16·5	121	73	1·77	111	63
June . .	9·11	545	60	27·6	264	95	11·8	159	135	1·54	144	94
July . .	9·12	438	48	26·1	296	113	9·9	235	238	1·39	177	128
August .	9·04	310	34	27·5	197	72	11·5	102	89	1·56	114	73
September	6·54	351	54	23·7	201	85	7·8	137	175	1·41	130	92
October .	4·33	305	71	16·7	246	147	5·9	169	285	1·02	164	161
November	2·85	251	88	6·9	164	239	4·9	169	345	0·48	112	232
December.	2·68	114	42	7·7	38	50	5·4	96	177	0·54	38	71
Winter .	2·92	180	61	7·86	102	130	5·32	152	286	0·565	68	120
Equinox .	6·38	436	68	21·38	246	115	9·77	163	167	1·239	154	124
Summer .	9·22	451	49	28·30	235	83	12·44	154	124	1·563	137	87
Year . .	6·18	355	58	19·18	195	101	9·17	156	170	1·122	120	106

In I, both a and b are distinctly smaller at Katharinenburg than at Pawlowsk; in summer and the equinox they are very similar to the corresponding values at Kew. The winter value of b at Katharinenburg is decidedly less than at Kew, there being specially low values in December, January, and February. The mean value of b/a for the year is very close to the corresponding values at Kew and Pawlowsk.

§ 21. Table XIII. gives results for the mean of the absolute diurnal ranges for individual months, with corresponding seasonal and yearly values. The last mentioned correspond nearly to the Pawlowsk data in the first line of Table VII. (see § 22 for nature of difference).

In D the values of a in Table XIII., whilst invariably larger than the corresponding values in Table XII., are not very conspicuously so, except in winter. The values of b, however, in Table XIII., are conspicuously larger than those in Table XII., the mean values for the year being roughly one double the other. The difference between the values of b/a in the two tables, though less prominent, is unmistakable.

TABLE XIII.—Katharinenburg (Units 1′ in D, 1γ in H and V).

Monthly Means of Absolute Daily Ranges.

	Declination.			Horizontal force.			Vertical force.		
	a.	$10^4 \times b$.	$10^4 \times b/a$.	a.	$10^3 \times b$.	$10^4 \times b/a$.	a.	$10^3 \times b$.	$10^4 \times b/a$.
January . .	6·34	247	39	25·2	200	79	11·9	139	116
February . .	5·62	920	164	21·7	502	231	10·2	384	377
March . . .	8·44	974	115	31·7	468	148	15·6	383	245
April . . .	10·38	604	58	35·4	392	111	20·7	225	109
May. . . .	10·97	615	56	45·7	242	53	22·8	159	70
June . . .	10·00	666	67	36·1	397	110	17·9	215	120
July. . . .	9·61	706	73	32·6	538	165	15·0	324	215
August. . .	9·92	503	51	37·9	304	80	16·9	203	120
September. .	7·61	803	105	32·8	365	111	15·3	219	143
October. . .	6·68	661	99	28·9	365	126	11·4	255	223
November . .	4·67	910	195	18·7	440	236	8·6	313	365
December . .	5·15	427	83	21·5	194	.90	10·1	155	152
Winter . . .	5·44	626	115	21·76	333	193	10·20	248	242
Equinox . .	8·28	761	92	32·21	398	123	15·76	270	172
Summer. . .	10·13	623	61	38·08	370	97	18·15	225	124
Year. . . .	7·95	670	84	30·68	367	120	14·70	248	168

In H the values of b in Table XIII. are on the average about double those in Table XII., but owing to the large values of a in Table XIII. the excess in its values for b/a is not striking, except in winter.

In V the values of a and b are again much larger in Table XIII. than in Table XII., but the seasonal and yearly values of b/a in the two are closely similar. In Table XIII. the December and January values of b/a are conspicuously low in all the elements as compared to the values for November and February.

§ 22. Table XIV. gives results for the range from the mean diurnal inequality for the year (corresponding to the Pawlowsk data in Table VI.), for the mean of the absolute daily ranges for the year (corresponding to the first line of Table VII.), for the mean of the 12 monthly ranges, and for the yearly range. The results for the second of these quantities, though practically accordant with those in the last line of Table XIII., are not absolutely identical. The figures in Table XIII. represent arithmetic means of a and b resulting from applications of formula (1) to individual months of the year. Table XIV. assumes the 12 monthly mean ranges for each year to be meaned, and these means dealt with by a single application of formula (1). The last two quantities dealt with in Table XIV. do not in reality accord very closely with the linear formula (1), but the figures at all events supply, as in the corre-

sponding case at Pawlowsk, a rough measure of the amplitude of the fluctuation throughout a sun-spot period.

In all the elements included in Table XIV., as we pass from the range of the diurnal inequality to the mean absolute daily range, and thence to the mean monthly range—quantities increasingly influenced by magnetic storms—we see that whilst a increases, b increases in a greater ratio, so that b/a notably rises.

The fall of b/a as we pass from the mean monthly to the annual ranges in D and V may not improbably possess no real significance, but a similar phenomenon, it should be remembered, presented itself in the corresponding Pawlowsk results in Table VII.

TABLE XIV.—Katharinenburg (Units 1′ in D and I, 1γ in H and V).

	Declination.			Horizontal Force.			Vertical Force.			Inclination.		
	a.	$10^4 b$.	$10^4\, b/a$.	a.	$10^3 \times b$.	$10^4 \times b/a$.	a.	$10^3 \times b$.	$10^4 \times b/a$.	a.	$10^4 \times b$.	$10^4 \times b/a$.
Mean diurnal inequality for the year	5·29	342	65	16·8	182	109	8·6	117	137	0·93	105	113
Mean of absolute daily ranges for the year	8·00	652	82	30·7	366	119	14·6	248	171	—	—	—
Mean of absolute monthly ranges for the year	18·56	2552	137	76·3	1680	220	46·3	1770	382	—	—	—
Absolute yearly range	41·85	4750	113	146·3	4590	314	178·9	3600	201	—	—	—

§ 23. Table XV. shows the excess of observed over calculated values at Katharinenburg; it answers to Table VIII. for Pawlowsk.

The results for the diurnal inequalities in D, H, and I in Table XV. are similar to the corresponding "all" day results in Table VIII., but on the whole show a slightly less close agreement between theory and observation. In V, however, the agreement is distinctly better at Katharinenburg than at Pawlowsk.

In the case of the absolute daily range the agreement between observed and calculated values is particularly good in D, and it is closer in all the elements than at Pawlowsk. This may be ascribed to the fact that magnetic disturbances are larger at Pawlowsk than at Katharinenburg.

The difference between the observed and calculated values in the monthly range is somewhat large, and there is now clear indication that sun-spot frequency is not by itself a sufficient guide. The observed values in 1893 are conspicuously below, and those in 1892 and 1894 conspicuously above, the calculated. The deficiency in the observed values in 1895 and the excess in 1898 are also marked. Even in the absolute daily range in Table XV. there is a distinct depression in the observed values in 1893 and corresponding enhancement in 1892, though not to the same extent as in the corresponding case at Pawlowsk (see Table VIII.).

TABLE XV.—Katharinenburg (Units 1′ in D and I, 1γ in H and V).

Observed less Calculated Values.

Year.	Ranges from mean diurnal inequalities for the year.				Mean of absolute daily ranges.			Mean of 12 monthly ranges.		
	D.	H.	V.	I.	D.	H.	V.	D.	H.	V.
1890	+0·30	0	0	+0·04	+ 0·03	0	0	− 2·4	− 7	− 11
1891	+ ·35	+ 2	+ 1	+ ·07	+ ·45	+ 3	+ 2	− 1·3	− 9	− 6
1892	− ·04	+ 2	+ 2	+ ·02	+ ·62	+ 6	+ 4	+ 6·5	+ 54	+ 56
1893	+ ·64	− 1	− 2	− ·03	− ·94	− 8	− 8	−10·2	− 69	− 90
1894	− ·15	− 1	− 1	− ·03	− ·07	0	+ 1	+ 7·0	+ 54	+ 94
1895	− ·18	0	− 1	+ ·03	− ·19	− 1	− 1	− 6·7	− 44	− 60
1896	− ·21	+ 1	+ 2	+ ·01	+ ·39	+ 2	+ 2	+ 0·2	− 3	− 18
1897	− ·17	− 1	0	− ·05	− ·03	0	+ 1	+ 1·9	+ 3	− 6
1898	− ·44	− 1	− 1	− ·02	+ ·23	+ 1	+ 2	+ 5·0	+ 18	+ 28
1899	− ·37	+ 1	+ 1	+ ·06	+ ·04	+ 2	0	− 0·1	+ 11	+ 15
1900	+ ·27	− 1	− 2	− ·10	− ·51	− 4	− 3	− 0·1	− 8	− 3
Mean difference calculated ∽ observed .	0·28	1·0	1·2	0·042	0·32	2·4	2·2	3·8	26	35
Probable error . . .	0·23	0·8	1·0	0·035	0·30	2·5	2·2	3·5	25	34
Mean value of element	6·71	24·4	13·5	1·373	10·72	46	25	29·2	146	120
Range of element . .	3·50	15	11	0·87	5·27	33	23	27·5	180	230
$\frac{\text{Mean difference}}{\text{mean value}} \times 100$	4	4	9	3	3	5	9	13	18	29
$\frac{\text{Probable error}}{\text{range of element}} \times 100$.	7	5	9	4	6	8	10	13	14	15

§ 24. Table XVI. supplies disturbance data at Katharinenburg, corresponding to those given in Table IX. for Pawlowsk. If we compare the 11-year means in the two tables we see convincing proof of the remark already made that Pawlowsk is more disturbed than Katharinenburg, the mean ranges in Table IX. being roughly double those in Table XVI.

The small bracketed figures in Table XVI. have the same significance as those in Table IX.

According to the mean monthly range—probably a better criterion than the annual range—1893 would seem to have been relatively less quiet at Katharinenburg than at Pawlowsk, but it stands much below 1892 and 1894. Whilst, however, all the Pawlowsk data made 1892 more disturbed than 1894, the monthly ranges at Katharinenburg give the first position to 1894. In all the columns 1890 appears as the least disturbed year. The monthly ranges—though not the annual ranges—assign to 1900 and 1899 the two next lowest places, the same positions as they occupy according to sun-spot frequency. But, as at Pawlowsk, 1895 is less disturbed, and 1898 much more disturbed than they should be if sun-spot frequency were the sole criterion.

TABLE XVI.—Katharinenburg (Units 1′ in D, 1γ in H and V).

Year.	Mean monthly range.			Annual range.		
	D.	H.	V.	D.	H.	V.
1890 . . .	18·0 $^{(11)}$	82 $^{(11)}$	48 $^{(11)}$	29·3 $^{(11)}$	163 $^{(11)}$	117 $^{(11)}$
1891 . . .	26·4 $^{(8)}$	127 $^{(7)}$	101 $^{(6)}$	47·1 $^{(8)}$	205 $^{(10)}$	257 $^{(5)}$
1892 . . .	43·8 $^{(2)}$	253 $^{(2)}$	232 $^{(2)}$	116·9 $^{(1)}$	837 $^{(1)}$	591 $^{(2)}$
1893 . . .	30·0 $^{(4)}$	150 $^{(3)}$	106 $^{(4)}$	45·5 $^{(9)}$	296 $^{(5)}$	172 $^{(10)}$
1894 . . .	45·5 $^{(1)}$	261 $^{(1)}$	278 $^{(1)}$	90·0 $^{(2)}$	674 $^{(2)}$	849 $^{(1)}$
1895 . . .	28·2 $^{(6)}$	140 $^{(5)}$	100 $^{(7)}$	47·4 $^{(7)}$	205 $^{(9)}$	184 $^{(9)}$
1896 . . .	29·4 $^{(5)}$	143 $^{(4)}$	102 $^{(5)}$	48·6 $^{(6)}$	320 $^{(4)}$	244 $^{(6)}$
1897 . . .	27·2 $^{(7)}$	124 $^{(8)}$	87 $^{(8)}$	67·1 $^{(4)}$	233 $^{(7)}$	210 $^{(7)}$
1898 . . .	30·4 $^{(3)}$	139 $^{(6)}$	122 $^{(3)}$	83·8 $^{(3)}$	338 $^{(3)}$	471 $^{(3)}$
1899 . . .	21·6 $^{(9)}$	108 $^{(9)}$	83 $^{(9)}$	40·8 $^{(10)}$	208 $^{(8)}$	333 $^{(4)}$
1900 . . .	20·9 $^{(10)}$	84 $^{(10)}$	60 $^{(10)}$	61·6 $^{(5)}$	237 $^{(6)}$	191 $^{(8)}$
Means . .	29·2	146	120	61·6	338	329

Batavia (6° 11′ S. lat., 106° 49′ E. long.).

§ 25. Prior to the introduction of electric tramways in 1899 the magnetic results at Batavia Observatory were treated with great completeness in the annual Batavia 'Magnetical and Meteorological Observations.'

Up to the end of 1900 the D and H results seem to have suffered comparatively little, but the V results even then were too disturbed for publication.

The Batavia magnetic records go back to 1882, but are incomplete until 1884. Inspection of the vertical-force data for 1884 and 1885 created some misgivings, which gathered force from an editorial statement that the scale value in that element was at first very variable and remained so until a new magnet was introduced.

After considering all the circumstances, I decided to confine myself to the results for the 12 years 1887 to 1898, coming down to the latest time at which all the elements were free from electric-tram effects. This period has the advantage of supplying a sun-spot minimum group of years 1887 to 1890 equal in length to the sun-spot maximum group 1892 to 1895.

The Batavia tables give not merely the hourly values, but also the sum of the 24 differences from the mean, in the monthly diurnal inequalities.

In (A) I found the sum of the 24 differences in D and H to show the sun-spot connection even more prominently than the ranges. Accordingly I decided to use the sum of the 24 differences at Batavia, in preference to the ranges, when dealing with the diurnal inequalities for the individual months, and to employ the sum of the 24 differences as well as the ranges when dealing with the mean diurnal inequalities for the year.

§ 26. Before giving the results, I would draw attention to a feature wherein Batavia

differs widely from European stations. At Kew, for instance, D, H, V, and I all show a large variation in the amplitude of the diurnal inequality throughout the year. The range is three or four times as large at midsummer as at midwinter, and the way in which the range, or the sum of the 24 differences, varies throughout the year is pretty similar for all the elements. Thus, assuming that the mean diurnal inequality for the year were derivable from a potential, one could obtain a fair first approximation to the mean diurnal inequalities for individual months by multiplying this potential by appropriate numerical factors.

How exceedingly far this is from being the case at Batavia will be seen on inspection of Table XVII. This gives the sum of the 24 differences in the diurnal inequalities for each month of the year, with their mean, and the sum of the 24 differences in the mean diurnal inequality for the year. Batavia being in the Southern hemisphere, May to August are the "winter" months.

The D data in Table XVII. proceed, on the whole, like European data. In V, too, the lowest value occurs in the winter months, but there is likewise a low value in December. While the average value for I from the four winter months is below the mean for the year, the lowest values of all occur in November and December. In H, three out of the four winter months show values above the average, while the four summer months are all below the average. Thus no two elements behave alike, and the phenomena exhibited by H are more nearly opposite than parallel to those observed in high latitudes.

TABLE XVII.—Batavia, 1887–1898 (Unit 1′ in D and I, 1γ in H and V).

Sum of the 24 Hourly Differences in the Mean Diurnal Inequality for the Month.

	Declination.	Horizontal force.	Vertical force.	Inclination.
January	22·04	313·8	276·7	31·77
February	22·55	310·4	289·7	32·36
March	16·87	357·0	282·1	33·72
April	11·96	373·0	231·3	30·87
May	12·34	346·3	189·5	26·85
June	10·26	322·5	189·8	26·02
July	12·14	347·1	202·6	27·84
August	15·25	373·7	187·4	27·87
September	18·56	396·2	214·7	30·08
October	20·57	348·5	249·8	30·51
November	22·06	281·9	215·8	25·82
December	22·01	263·4	196·7	23·82
Arithmetic mean from 12 months	17·22	336·2	227·2	28·96
From mean diurnal inequality for the year	13·73	334·5	216·6	28·36

§ 27. The results pointed out in the last paragraph help to explain some novel features in Table XVIII., which gives the values obtained for a, b and b/a by applying the method of groups to the sums of the 24 differences in the diurnal inequalities for the several months. As in similar tables, the seasonal and yearly values of a and b represent arithmetic means for the included months.

In D and V the lowest values of a are found in winter, but in H and I the lowest values occur in November and December, that is, at midsummer.

In D and V, again, b is distinctly below the average in winter, though not nearly to the same extent as in Europe, and the winter value for I is less than the summer value; but in H the winter value exceeds the summer.

In D, b/a is distinctly smaller in summer than in the other seasons; but in I, H and V the summer value is a trifle the highest. The winter and equinoctial values of b/a are almost identical in all the elements. As compared to northern stations, the variation in b/a throughout the year is exceedingly small.

TABLE XVIII.—Batavia. Sum of 24 Hourly Differences, 1887–98 (Units 1′ in D and I, 1γ in H and V).

	Declination.			Inclination.			Horizontal force.			Vertical force.		
	a.	$10^3 \times b$.	$10^4 \times b/a$.	a.	$10^3 \times b$.	$10^4 \times b/a$.	a.	$10^2 \times b$.	$10^4 \times b/a$.	a.	$10^2 \times b$.	$10^4 \times b/a$.
January .	18·09	109	60	25·37	178	70	244·7	192	78	225·4	142	63
February	19·11	87	45	25·70	168	66	232·4	197	85	238·8	128	54
March .	13·52	105	78	26·65	222	83	266·8	283	106	232·6	155	67
April . .	9·94	55	55	25·74	139	54	309·6	171	55	196·0	96	49
May . .	9·60	68	71	20·00	171	85	264·2	204	77	143·0	116	81
June . .	8·62	40	46	19·87	149	75	241·1	197	82	151·5	92	61
July . .	9·24	69	74	22·29	131	59	259·2	208	80	174·8	66	38
August .	12·16	69	57	22·27	125	56	281·3	207	74	156·4	69	44
September	14·04	108	77	22·51	181	80	303·9	221	73	161·4	127	79
October .	17·87	73	41	24·73	156	63	265·7	224	84	215·6	93	43
November	19·52	78	40	19·20	203	106	212·3	214	101	165·7	154	93
December	18·32	95	52	19·12	121	63	207·5	144	70	163·2	86	53
Winter .	9·90	61	62	21·11	144	68	261·5	204	78	156·4	86	55
Equinox .	13·84	85	61	24·91	174	70	286·5	225	78	201·4	118	58
Summer .	18·76	92	49	22·35	167	75	224·2	187	83	198·3	128	64
Year . .	14·17	80	56	22·79	162	71	257·4	205	80	185·4	110	60

§ 28. In applying the method of groups, it is evidently desirable that one group of years should fall near the middle of the period dealt with, and that part of the second group should precede, and part follow it. This arrangement helps to eliminate any long-period variation, or any gradual change in the conditions. The period 1887 to 1898 being by no means ideal in the above respect, in dealing with the diurnal

inequality for the year, I have employed both the method of least squares and the method of groups. If large differences had presented themselves between the results from the two methods, it would have become necessary to reconsider Table XVIII. As the question of the reliance to be placed on the method of groups is important, I give the results from both methods in Table XIX. The agreement, it will be seen, is least good in D, but it will, I think, be allowed that even there it leaves little to be desired.

TABLE XIX.—Batavia (Units 1′ in D and I, 1γ in Forces).

Mean Diurnal Inequalities for the Year.

		Declination.		Inclination.		Horizontal force.		Vertical force.		Total force.	
		Groups.	Least squares.	Groups.	Least squares.	Groups.	Least squares.	Groups.	Least squares.	Groups.	Least squares.
Ranges.	a	2·455	2·470	3·61	3·60	38·74	38·74	30·13	30·11	20·94	20·90
	$10^4 \times b$	183	179	215	218	2738	2739	1550	1559	1541	1552
	$10^5 \times b/a$	746	725	597	605	707	707	514	518	736	743
Sum of 24 differences	a	10·30	10·34	22·21	22·19	258·5	258·0	173·3	173·1	145·9	145·4
	$10^3 \times b$	88·3	87·6	159·1	159·8	1967	1980	1121	1127	1190	1204
	$10^5 \times b/a$	862	847	716	720	761	767	647	651	816	828

§ 29. In the case of I, H and V the values of a, b and b/a given in Table XIX. for the sum of the 24 differences in the mean diurnal inequality for the year do not differ much from the yearly mean in Table XVIII. In the case of D, however, the results in the two tables are widely different, the a in Table XVIII. being nearly 40 per cent. in excess of that in Table XIX. The closeness of the values of a in the two tables in I, H and V, and their divergence in the case of D, is what we might anticipate from the figures for the sums of the 24 differences in the last two lines of Table XVII. The real inference to be drawn is that in D the hours of maximum and minimum vary somewhat widely from month to month, though apparently to a smaller extent in years of sun-spot maximum than in years of sun-spot minimum.

The data in Table XIX. for Batavia correspond exactly to those given for Kew in (A), Table XLIV.

In D the Batavia value of b for the 24 differences is almost exactly the third of the Kew value; in the case of the ranges the Batavia value is relatively larger, but still less than half that at Kew, Greenwich or Pawlowsk.

The D values of b/a, however, at Batavia and Kew are nearly equal.

In I the Batavia value of b is 70 per cent. larger than the Kew in the case of the ranges, and almost exactly double in the case of the 24 differences. The Batavia

values of a, however, so much exceed the Kew that b/a is more than twice as big at Kew as it is at Batavia.

In H the Batavia values of b are both roughly 50 per cent. in excess of the Kew, but the Kew values of b/a are more than 50 per cent. in excess of those at Batavia.

In V the values of b are again very much larger at Batavia than at Kew; in the 24 differences the Batavia value of b/a is somewhat the higher, but in the ranges it is slightly the lower.

§ 30. The relation between the values found for b/a is probably the best measure of the relative importance of the sun-spot connection in any two cases. Applying this criterion to the 24 differences and range results obtained by least squares in Tables XIX., we obtain the following values for the ratio of

(b/a) from sum of 24 differences : (b/a) from ranges :—

Declination.	Inclination.	Horizontal force.	Vertical force.	Total force.
1·168	1·191	1·085	1·257	1·115

The mean of the first four of these ratios is 1·18 and the corresponding figure for Kew (as deducible from (A), Table XLIV.) is 1·19. Thus the greater variability of the sum of the 24 differences with sun-spot frequency observed at Kew is also seen at Batavia, and to approximately the same extent.

§ 31. The Batavia publications record the values of the constants in the 24-hour and 12-hour terms of the Fourier series

$$c_1 \sin (t + \alpha_1) + c_2 \sin (2t + \alpha_2) +$$

for the mean diurnal inequality for the year. Here c_1, c_2 replace the Batavia notation A_1, A_2.

Table XX. gives the values which I have found for a, b and b/a in this case from the method of groups. The results should be compared with those given for Kew and Wilhelmshaven in (A), Table XLII., though the slight difference in the method of obtaining the Kew results should be noted.

In I, H and V the values of b/a in Table XX. are nearly alike in c_1 and c_2, and they approach fairly closely to the corresponding values in Table XIX. applicable to the ranges of the mean diurnal inequalities. In declination and total force, however, the values of b/a in Table XX. are decidedly higher for c_1 than for c_2. This phenomenon was observed at both Kew and Wilhelmshaven in the case of the declination and the westerly component.

TABLE XX.—Batavia, 1887–98 (Units 1′ for Angles, 1γ for Forces).

Amplitudes of 24-hour and 12-hour Terms in Fourier Series for Mean Diurnal Inequality for the Year.

	Declination.		Inclination.		Horizontal force.		Vertical force.		Total force.	
	c_1.	c_2.	c_1.	c_2.	c_1.	c_2.	c_1.	c_2.	c_1.	c_2.
Mean value of amplitude (for 12 years) . .	0·748	0·778	1·793	0·837	20·89	9·07	13·62	7·27	11·71	5·45
a	0·548	0·614	1·427	·663	16·13	7·13	11·14	5·90	8·84	4·43
$10^4 \times b$ . . .	52	42	95	45	1233	502	641	353	744	263
$10^4 \times b/a$ . . .	94	69	66	68	76	70	58	60	84	60

§ 32. Disturbances have special attention paid them at Batavia. Following a practice, of which SABINE was an advocate, a reading at Batavia is regarded as disturbed when its difference from the mean reading at that hour during the month reaches or exceeds a certain limit. The limiting values adopted at Batavia are 1′·3 in D and 11γ in H and V.

The arbitrary nature of such criteria, and the difficulty of justifying one limiting value in preference to another, have been more than once pointed out. It is arguable that the limit should vary with the season of the year, and even with the sun-spot frequency. In a European station, for instance, the range of the regular diurnal inequality near sun-spot maximum at midsummer is very large compared to that near sun-spot minimum at midwinter, and a good deal might be said for a limiting value which bore a fixed ratio to the range from the mean diurnal inequality for the month.

The disturbed values which exceed the hourly mean, and those which fall below it, are termed respectively "positive" and "negative" disturbances; they are in the first instance treated separately at Batavia, tables being given of the sum of the values of the disturbances and of their number. A final summary gives the aggregates of the positive and negative totals treated numerically. Table XXI. gives these aggregate values and numbers as published in the annual Batavia 'Observations.'

The number of disturbances in D is less than half that in V, and little over a quarter that in H. We cannot, however, draw any safe inference as to one element being absolutely more or less disturbed than another. If we calculate the ratios borne by the disturbance limits accepted at Batavia to the mean ranges of the diurnal inequalities for the year in the respective elements, for the period 1887 to 1898, we find the following results :—

	D.	H.	V.
$\frac{\text{Disturbance limit}}{\text{range}}$ =	0·41	0·22	0·30

If instead of the ranges from the mean diurnal inequality for the year we had taken the arithmetic mean of the 12 monthly inequality ranges, we should have obtained a somewhat smaller fraction in the case of D. But the figures are at least suggestive that the explanation of the great difference in the number of disturbances in D, H, and V may be largely due to the disturbance limit being less exacting in one element than another.

TABLE XXI.—(Units for "Values" 1′ in D, 1γ in H and V.)

Aggregate Values and Numbers of Disturbances at Batavia.

Year.	Declination.		Horizontal force.		Vertical force.	
	Values.	Numbers.	Values.	Numbers.	Values.	Numbers.
1887	339·3	210	17,160	1023	9,612	671
1888	237·3	149	16,339	933	12,709	807
1889	237·5	153	11,686	700	11,581	783
1890	354·0	252	6,227	346	4,781	301
1891	425·4	262	22,605	1208	16,394	1016
1892	1020·8	571	40,582	1786	20,295	1095
1893	730·8	427	23,731	1286	11,021	715
1894	840·2	462	37,239	1666	13,418	751
1895	616·0	360	23,595	1380	11,441	757
1896	458·2	286	19,983	1139	5,790	409
1897	464·5	286	14,187	815	5,995	408
1898	434·2	262	18,605	1030	8,485	548
Means	513·2	307	20,995	1109	10,960	688

§ 33. On examining Table XXI. it will be seen that the number and aggregate value, though generally increasing or decreasing together, are far from being in a constant ratio in any of the elements. As to which is the better measure of disturbance, opinions may well differ; but the aggregate values constitute probably the nearest parallel to the Pawlowsk and Katharinenburg data in Tables IX. and XVI.

According to both numbers and aggregate values, 1893 was less disturbed than 1892 or 1894, but its relative quietness is not so conspicuous, especially in D and V, as it was at Pawlowsk or even Katharinenburg.

Table XXI. must, of course, receive contributions—at least, in the case of H

and V—from a number of days which are not days of large disturbance; but if this were the true explanation, we should expect the position of 1893 at Batavia and Pawlowsk to differ more in the case of H than in that of D, which is the reverse of what happens.

§ 34. Table XXII. gives values of a, b and b/a calculated for the data in Table XXI. The value of b/a answering to the "aggregate value" is in each case greater than that answering to the "number"; and, except in the case of V, both values of b/a are considerably higher than the corresponding yearly values in Tables XVIII. and XIX.

If we compare Table XXII. with Table XIV. for Katharinenburg, we see that in V the Batavia disturbance values of b/a are much less than the lowest value of b/a at Katharinenburg, viz., that for the diurnal inequality. In H the Batavia disturbance values of b/a are similar to the value of b/a in the absolutely monthly range at Katharinenburg. In D, however, the Batavia disturbance values of b/a are much in excess of any corresponding value at Katharinenburg.

The way in which disturbance influences the records at the two places are thus widely different.

TABLE XXII.—Batavia "Disturbances," 1887–98 (Units for "Values" 1′ for D, 1γ for H and V).

	Declination.		Horizontal force.		Vertical force.	
	Aggregate values.	Numbers.	Aggregate values.	Numbers.	Aggregate values.	Numbers.
a	217·7	153·7	10,312	657·9	8425	578·9
b	7·65	3·96	277	11·7	65·6	2·84
$10^4 \times b/a$ . .	351	258	268	178	78	49

§ 35. Table XXIII. compares observed and calculated values in the mean diurnal inequality for the year at Batavia, and in the aggregate value of the disturbances. The values employed for a and b in the case of the ranges are those calculated by least squares.

The nicety of agreement in the case of the ranges is very similar to what has been already observed at Kew, Pawlowsk and Katharinenburg; and, as at Kew, the agreement is practically the same for the 24 differences as for the ranges. As has been generally observed elsewhere, the agreement is least good in the case of V.

In the case of the aggregate value of the disturbances, the agreement is pretty similar to what was found for the mean of the absolute monthly ranges at Katharinenburg; and, as elsewhere, the failure of the formula to account satis-

factorily for the phenomena observed in 1892 and 1893 is conspicuous. Unlike Pawlowsk and Katharinenburg, Batavia shows, however, no abnormal excess of disturbances in 1898.

TABLE XXIII.—Batavia (Units 1′ for D, 1γ for H and V).

Observed less Calculated Values.

Year.	Mean diurnal inequality for the year.						Aggregate values of disturbances.		
	Ranges.			24 differences.					
	D.	H.	V.	D.	H.	V.	D.	H.	V.
1887	−0·25	+ 0·5	− 2·7	− 0·77	+ 2·2	− 18·6	+ 21·4	+ 3,235	+ 327
1888	− ·07	+ 0·9	+ 0·9	− ·28	+ 5·0	+ 0·2	− 32·4	+ 4,146	+ 3,838
1889	+ ·16	− 0·3	+ 3·0	+ ·36	− 1·1	+ 13·7	− 28·4	− 368	+ 2,743
1890	+ ·09	+ 0·4	+ 1·3	+ ·21	+ 8·4	+ 18·0	+ 82·0	− 6,049	− 4,110
1891	− ·22	+ 2·4	+ 5·3	− ·88	+ 15·2	+ 36·9	− 64·6	+ 2,448	+ 5,633
1892	+ ·16	+ 0·6	+ 2·8	+ ·40	+ 12·3	+ 17·9	+244·6	+10,082	+ 7,080
1893	− ·02	+ 1·1	+ 0·6	·00	+ 8·9	+ 3·9	−136·4	−10,061	− 2,975
1894	− ·04	− 1·6	− 1·8	− ·03	− 9·8	− 9·3	+ 25·7	+ 5,355	− 125
1895	− ·08	+ 1·4	+ 0·6	− ·53	− 0·3	− 0·7	− 91·4	− 4,418	− 1,184
1896	+ ·03	− 3·8	− 5·1	+ ·41	− 29·1	− 34·2	− 79·3	− 1,890	− 5,378
1897	+ ·19	− 0·6	− 3·4	+ ·66	− 6·3	− 23·0	+ 46·3	− 3,371	− 4,149
1898	+ ·04	− 1·0	− 1·6	+ ·44	− 5·5	− 5·1	+ 12·2	+ 909	− 1,692
Mean difference calculated ∽ observed	0·112	1·22	2·42	0·414	8·7	15·1	72·1	4,361	3,270
Probable error . .	0·096	1·09	2·02	0·344	8·1	13·4	61·2	3,734	2,730
Mean value of element	3·16	49·3	36·1	13·73	334·5	216·6	513	20,995	10,960
Range of element .	1·52	22·9	14·9	7·12	165·6	104·0	783	34,355	15,514
Mean difference × 100 / mean value	4	2	7	3	3	7	14	21	30
Probable error × 100 / range of element	6	5	14	5	5	13	8	11	18

Mauritius (Lat. 20° 6′ S., Long. 57° 33′ E.).

§ 36. Owing to the novelty in some of the features of the Batavia results, examination of the data from a second tropical station seemed desirable. I have accordingly made use of a number of tables of magnetic results* at Mauritius, published in a convenient form in 1899. In D, data are given for the period 1875 to 1890, in H for 1883 to 1890, and in V for 1884 to 1890. The shortness of the two latter periods, and the fact that the data are not contemporaneous with those for most of the other stations, are drawbacks, but there is small choice of magnetic data in low latitudes.

* 'Mauritius Magnetical Reductions,' edited by T. F. CLAXTON, F.R.A.S., Director Royal Alfred Observatory, Mauritius, 1899.

On examining the tables, I found that the mean D ranges in 1881 and 1882—years of fairly large sun-spot frequency—showed a remarkable depression, being only about half those in 1880 and 1883, and in the preface I found the following editorial reference to some readjustment of the declination magnetograph in December, 1882 : "In the latter part of the year 1882 the effect of torsion on the magnetograph is very pronounced." As the phenomenal smallness of the range seems to have ceased with the readjustment, and as the Milan and Greenwich records show no parallel to the reduction of the ranges in 1881 and 1882, I have omitted these years entirely from the calculations.

The Mauritius publication gives in special detail the mean value for each month and year of the absolute daily ranges. These ranges seem based entirely on hourly readings; and so are not absolutely equivalent to the Katharinenburg ranges dealt with in Table XIII.

As the variation in these daily ranges throughout the year has exceptional features, I give particulars in Table XXIV. There is a resemblance to phenomena at Batavia. In D the variation in the range, though much less conspicuous than in Northern Europe, is well marked; the values for the three midwinter months—May, June, and July—are well below the average. In H the variation is small, and somewhat irregular; on the whole, the range is smallest in winter, from May to August, but the next lowest values appear in February and December.

There is a distinct reduction in the V range near midwinter, but a very similar reduction occurs at midsummer.

TABLE XXIV.—Mauritius (Units 1′ for D, 1γ for H and V).

Monthly Means of Absolute Daily Ranges.

	Declination, 1875–80 and 1883–90.	Horizontal force, 1883 to 1890.	Vertical force, 1884 to 1890.
January	6·93	37·9	17·1
February	7·79	35·0	19·5
March	7·11	36·2	20·1
April	5·75	37·6	17·3
May	4·87	35·0	16·5
June	4·03	34·1	15·5
July	4·36	33·8	17·1
August	6·00	34·5	22·0
September	6·28	36·6	22·7
October	6·71	37·4	19·4
November	6·99	37·8	16·7
December	6·78	35·3	15·2
Mean	6·13	35·9	18·2

§ 37. Table XXV. gives the values of a, b, and b/a applicable to the monthly means of the absolute daily ranges at Mauritius. The method of groups was employed, the groups being as follows :—

Years of sun-spot.	For D.	For H.	For V.
Maximum	1880, 1883, 1884, 1885, 1886	1883 to 1886	1884 to 1886
Minimum.	1878, 1879, 1888, 1889, 1890	1887 „ 1890	1887 „ 1890

The seasonal and yearly values of a and b are arithmetic means from the included months, and these means are employed in calculating b/a.

The values of b in Table XXV. fluctuate somewhat erratically from month to month even for D, where 14 years' data are utilised. The exceptional features presented by the September and October data in V had better be regarded provisionally as accidental.

In D and V we have b largest in winter and least at the equinox, a very remarkable feature.

Comparing Tables XXV. and XIII., we see that whilst the mean values of a for the year are fairly similar, the values of b and of b/a at Mauritius are roughly only from a half to a third of the corresponding values at Katharinenburg.

TABLE XXV.—Mauritius (Units $1'$ for D, 1γ for H and V).

Monthly Means of Absolute Daily Ranges.

	Declination.			Horizontal force.			Vertical force.		
	a.	10^4b.	$10^4b/a$.	a.	10^3b.	$10^4b/a$.	a.	10^3b.	$10^4b/a$.
January . . .	6·02	381	63	35·1	89	26	14·0	110	78
February . . .	7·34	186	25	29·7	164	55	17·8	55	31
March. . . .	6·70	158	23	29·7	200	67	16·8	104	62
April	5·06	278	55	33·5	119	35	14·1	115	82
May	4·10	354	86	30·1	165	55	13·7	97	71
June	3·70	130	35	26·9	209	78	13·2	82	62
July	3·65	290	80	28·3	159	56	13·9	113	81
August . . .	5·20	362	69	28·6	214	75	18·1	157	87
September . .	5·93	152	26	30·8	214	69	20·6	9	4
October . . .	6·41	145	23	31·8	221	70	20·2	[− 47]	[−23]
November . .	6·35	331	52	29·2	376	129	16·2	33	21
December. . .	6·56	120	18	29·7	226	76	13·5	96	71
Winter . . .	4·16	284	68	28·5	187	66	14·7	112	76
Equinox . . .	6·03	183	30	31·5	188	60	17·9	45	25
Summer . . .	6·57	254	39	30·9	214	71	15·4	73	48
Year	5·58	241	43	30·3	196	65	16·0	77	48

§ 38. Table XXVI. deals with the mean of the absolute daily ranges for the year, and with the ranges and the sums of the 24 differences in the mean diurnal inequalities for the year. The grouping of the years is the same as for the previous table. The values of a, b, and b/a for the yearly mean of the absolute daily ranges do not agree quite so closely with the corresponding means of the 12 monthly values in Table XXV. as was the case at Katharinenburg (*cf.* Tables XIII. and XIV.).

In H the values of a and b for the absolute daily range are about double those for the range of the diurnal inequality; in D and V the differences between the two sets of values are smaller, but still considerable. In all three elements the values of b/a for the two species of ranges are fairly similar.

In the case of the mean diurnal inequality in D, the values of b are lower even than those given in Table XIX. for Batavia, and the values of b/a are the lowest we have yet met with. The values of b for the ranges of the mean diurnal inequalities in H and V are much lower than at Batavia, but the values of b/a at the two places are fairly similar.

The values of b/a for the 24 differences do not show that decided excess over the values for the ranges that was seen at Kew and Batavia.

TABLE XXVI.—Mauritius (Units $1'$ for D, 1γ for H and V).

	Ranges.						24 differences.		
	Mean of absolute daily values for the year.			From mean diurnal inequality for the year.			From mean diurnal inequality for the year.		
	a.	$b \times 10^4$.	$(b/a) \times 10^4$.	a.	$b \times 10^4$.	$(b/a) \times 10^4$.	a.	$b \times 10^3$.	$(b/a) \times 10^4$.
Declination . .	5·53	255	46	4·06	164	40	15·96	79	49
Horizontal force .	30·4	1859	61	15·0	956	64	116·0	695	60
Vertical force . .	16·2	840	52	11·9	685	58	66·6	292	44

§ 39. Table XXVII. gives the differences between observed and calculated mean yearly data at Mauritius. Comparing the figures in the last line of the table with the corresponding figures in Tables IV., VIII., XV., and XXIII., we conclude that the agreement is not quite so good at Mauritius as at the other stations. The agreement is closest for the mean of the daily declination ranges, where it is very fair; it is on the whole better for V than for H, which is exceptional.

TABLE XXVII.—Mauritius (Units 1′ for D, 1γ for H and V).

Observed Less Calculated Values.

Year.	Mean of absolute daily ranges.			Mean diurnal inequality.					
				Ranges.			24 differences.		
	D.	H.	V.	D.	H.	V.	D.	H.	V.
1875	+0·1	—	—	+0·4	—	—	+ 0·7	—	—
1876	+ ·1	—	—	·0	—	—	+ 0·2	—	—
1877	− ·2	—	—	− ·2	—	—	− 1·8	—	—
1878	+ ·3	—	—	+ ·6	—	—	+ 1·5	—	—
1879	− ·1	—	—	− ·2	—	—	− 0·3	—	—
1880	− ·3	—	—	− ·1	—	—	+ 0·1	—	—
1883	+ ·3	+ 0·3	—	+ ·5	− 0·9	—	+ 0·4	− 2·8	—
1884	+ ·3	− 4·1	+ 0·9	+ ·3	− 3·6	+ 1·1	+ 1·2	− 22·9	+ 6·9
1885	− ·2	− 0·4	− 2·2	+ ·1	+ 1·4	− 1·5	+ 0·7	+ 4·9	− 5·4
1886	− ·2	+ 4·2	+ 1·3	− ·6	+ 3·1	+ 0·5	− 1·4	+ 20·7	− 1·2
1887	− ·2	+ 1·8	− 1·7	− ·6	+ 0·8	− 2·3	− 1·2	+ 9·0	− 6·7
1888	− ·1	+ 2·1	− 0·1	− ·3	+ 1·4	− 0·4	− 0·6	+ 8·5	+ 0·6
1889	− ·1	− 1·4	+ 0·7	+ ·2	− 0·9	+ 1·4	+ 0·8	− 8·0	+ 4·3
1890	+ ·1	− 2·4	+ 1·0	− ·2	− 1·3	+ 1·2	− 0·2	− 9·6	+ 1·9
Mean difference calculated $\backsim$ observed .	0·19	2·09	1·13	0·31	1·67	1·20	0·79	10·8	3·9
Probable error . . .	0·14	1·80	·94	0·25	1·41	0·98	0·67	9·2	3·3
Mean value of element	6·11	35·9	18·2	4·44	17·8	13·6	17·8	137	74
Range of element. .	1·90	13·2	6·8	1·90	7·0	6·8	7·0	46	28
Mean difference × 100 / mean value	3	6	6	7	9	9	4	8	5
Probable error × 100 / range of element	7	14	14	13	20	14	10	20	12

Summary.

§ 40. A slight progressive decrease in a and increase in b in the sun-spot formula (1) is suggested by the Greenwich D and H data from 1841 to 1896, but this does not meet with support from Signor RAJNA's analysis of D ranges at Milan from 1836 to 1894. In both cases there is an element of uncertainty, arising from want of homogeneousness in the data.

According to RAJNA's earlier data, values of a, and to a lesser extent values of b, calculated from periods as long as 14 years may differ very sensibly from those calculated from longer periods, but differences of this kind seem to have diminished since observations became more homogeneous and are probably ascribable in part to observational uncertainties.

Results calculated for Milan from the period 1890 to 1900, which is the period chiefly utilised in the present paper, differ but little from those found by RAJNA for the periods 1836 to 1894 and 1871 to 1894.

The tendency for b to be small in winter and large in summer, described at Kew, is also, in general, conspicuous elsewhere; but there are exceptions, especially at tropical stations.

The tendency in b/a to be large in winter as compared to summer, so prominent at Kew, is also, in general, prominent at other northern stations, but the phenomenon is comparatively inconspicuous in the case of the declination range at Greenwich. At the tropical stations the seasonal change in b/a appears much reduced and is somewhat uncertain.

There is no conspicuous difference between the "all" and the "quiet" days' mean yearly values of b and b/a for the ranges of the D and H diurnal inequalities at either Greenwich or Pawlowsk; but at Pawlowsk there is a somewhat notable difference between "all" day and "quiet" day D results in winter, and the difference between "all" and "quiet" day V results is very large throughout the whole year.

If we exclude Mauritius, the values of $10^4 b/a$ for the ranges in the mean diurnal inequality of declination for the year at the several stations vary only from 65 to 73. The corresponding values of b show also a pretty close agreement at the northern stations, but the values for the tropical stations are much smaller.

In H there is no very conspicuous difference in the values of b or b/a for the ranges from the mean diurnal inequality for the year at the northern stations; but the values found for b/a at Batavia and Mauritius are considerably smaller, while the value found for b is smaller at Mauritius, but very materially larger at Batavia.

When the formula (1) is applied to any ordinary measure of magnetic disturbance, it gives much too high values for 1893—the year of sun-spot maximum—and much too low values for 1892. Thus the application of (1) to disturbances has not the same justification as its application to ordinary diurnal inequalities. It may, however, serve a useful purpose in giving a greater degree of definiteness to the comparison of contemporaneous disturbance phenomena at the same or at different stations.

In the case of results obtained by the application of (1) to individual months of the year a considerable latitude must be allowed to chance, especially in winter months when the diurnal range is small, unless an exceptionally long series of observations is available. Results obtained from arranging months in seasons are much less exposed to numerical uncertainties, but they are insufficient for the reason that there are conspicuous differences between months which have to be grouped under the same season. This remark applies more particularly to winter and equinoctial months in higher latitudes.

[*June* 8, 1904.—The following additional data—all obtained by the method of least squares—apply to the ranges of the mean diurnal inequalities for the year at Irkutsk ("all" days) and Colaba ("quiet" days), and to the mean difference between the

absolute daily maximum and minimum at Zi-ka-wei. The units are 1′ for angles, 1γ for force components.

Place . . .	Irkutsk (Siberia).			Zi-ka-wei (China).			Colaba (Bombay).		
Latitude . .	52° 16′ N.			31° 12′ N.			18° 54′ N.		
Longitude . .	104° 16′ E.			121° 26′ E.			72° 49′ E.		
Period of years	1890 to 1900.			1890 to 1900.			1894 to 1901.		
Element . .	a.	10^4b.	$10^4b/a$.	a.	10^4b.	$10^4b/a$.	a.	10^4b.	$10^4b/a$.
D	4·815	358	74	4·369	303	69	2·373	66	28
I	0·971	87	90	—	—	—	—	—	—
H	18·18	1896	104	—	—	—	31·65	2814	89
V	6·49	710	109	—	—	—	19·35	723	37

At Irkutsk the values of b for D and H are similar to those at Katharinenburg; the values of b/a for these elements are similar to those at Kew; in V the values of b and b/a are decidedly less than at Katharinenburg.

At Zi-ka-wei b/a is decidedly less, and b much less, than the corresponding values for Katharinenburg (second line of Table XIV.).

At Colaba the ("quiet" day) values of b and b/a for D are notably less than the corresponding ("all" day) values at Mauritius, the smallest occurring in the paper; but the value of b for H exceeds that at Batavia, the largest previously noted. The value of b/a for V is conspicuously small.]