IV. *A Determination of the Electromotive Force of the Weston Normal Cell in Semi-absolute Volts.*

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With a Preface by Prof. H. L. Callendar, *F.R.S.*

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PREFACE BY PROF. H. L. CALLENDAR.

THE measurement of the E.M.F. of the Weston cell affords the best means of comparing the performances of different methods and instruments for the absolute determination of the ampere. Great progress has been made in the last six years, but the most recent determinations by independent methods, giving equal promise of accuracy, still show discrepancies covering a range of 2 parts in 10,000, which must be debited for the most part to the difficulty of the absolute determination of current. Each method in itself appears to give an order of accuracy of repetition approaching, or even exceeding, 1 in 100,000. It is therefore of special interest and importance to compare the results of methods differing as widely as possible in experimental details in endeavouring to arrive at a value comparatively free from the constant errors which may beset any particular type of method.

The measurements described by Mr. SHAW in the following paper were made by the method of the Weber bifilar electrodynamometer, as modified by CLERK MAXWELL and LATIMER CLARK, which has not hitherto been employed for work of the highest accuracy, and which merits attention on account of its many fundamental points of difference from recent methods. The instrument originally supplied to McGill College for this purpose was a faithful copy of CLERK MAXWELL'S instrument at Cambridge, of which the theory is given together with a figure and description in his 'Electricity and Magnetism,' vol, 2, p. 367. The chief sources of error in this instrument were (1) the uncertainty of insulation of the coils, which proved to be of the

order of nearly one half of 1 per cent.; (2) the difficulty of determining the mean radii of the coils, which were wound with silk-covered wire; (3) the want of rigidity of the pulley arrangement for equalising the tensions of the suspending wires, and the imperfect elasticity of the control, which depended too much on torsion, and made it impossible to obtain readings consistent to 1 in 1000 for the deflections or the times of oscillation. These defects were so fatal to accurate work even of the order of 1 in 10,000, which was all that it was originally contemplated, that it was found necessary to reconstruct the instrument entirely until nothing remained of the original except the frame, and even that required stiffening to a material extent.

The arduous work of reconstructing the instrument was undertaken by Mr. R. O. KING as part of his work during the tenure of an 1851 Exhibition Scholarship during the years 1897–8. The coils were re-wound with a double winding to give a perfect check on the insulation. The large coils were wound with a carefully measured length of hard rolled copper tape, which gave a very high order of accuracy in the determination of the mean radius—this device possesses the great advantage of avoiding the excessive refinement necessary in measuring the linear dimensions of the coils in the majority of other methods. The coils were also made reversible and interchangeable to eliminate possible errors of symmetry, especially in the measurement of the distance between their planes.

The dimensions of the suspended coils were determined by an electrical method of comparison with the fixed coils, which proved to be one of the most difficult measurements owing to extraneous magnetic disturbances. Mr. KING'S results for this comparison showed extreme differences amounting to nearly 1 in 5000. This led me, in revising his work, to put the limit of accuracy at 1 in 10,000, although all the fundamental quantities could be measured to 1 in 100,000 or better. I have since observed that the largest discrepancy in this comparison was undoubtedly due to a slight failure of insulation duly noted by Mr. KING at the time. Another series was interfered with by an inopportune magnetic storm. Mr. SHAW employing the identical tube made by Mr. KING for supporting the small coils in a fixed position relative to the large coils, has succeeded after 15 years in recovering Mr. KING'S value for the ratio of the currents required for balance to 1 in 30,000. I am now of opinion that I had greatly under-estimated the probable accuracy of this comparison, which Mr. SHAW appears to have carried out to an order of accuracy little if at all inferior to 1 in 100,000.

A weak point in many of the methods of absolute measurement is the relative smallness of the electrical force to be measured. The arrangement of the coils in parallel, rendered possible by rewinding the coils, permitted the attainment of a forty times larger force for the same current in the small coils, *i.e.*, without introducing any unsteadiness or uncertainty of dimensions due to the heating effect of the current on the small coils. The relative importance of the gravity control in the bifilar suspension could thus be increased, with a corresponding improvement in the steadiness

and accuracy of the deflection observations. There still remained, however, an appreciable effect due to imperfect elasticity of the wires, and a good deal of time was spent in selecting a suitable suspension. With the round wires ultimately adopted, and subsequently employed by Mr. SHAW, the effect amounted, as nearly as I could estimate, to 3 parts in 10,000 under the conditions of the deflection experiments. It appeared to me at the time that the effect was largely due to the sharp bending of the wire at the four points where it was rigidly clamped. I accordingly prepared a rolled strip of equal carrying capacity to take the place of the round wire with the object of reducing the creep. This would probably have proved effective, but it was considered inadvisable to disturb the suspension at the time, when half the observations were completed, seeing that the correction was so small in relation to the order of accuracy then contemplated. The exact study of this suspension since carried out by Mr. SHAW, has made it possible to determine the correction to a much higher order of accuracy, so that this last objection to the deflection method has been removed.

Now that Mr. SHAW has repeated the observations and revised the whole theory of the experiment with such minute care and accuracy, I feel sure that the results should take rank among the best determinations of recent years, and should possess special interest as an independent determination on account of the many radical differences of procedure involved. At the same time I hope that Mr. KING'S share in the research as a student in reconstructing the apparatus and getting it into working order, and his generosity in providing the means for the ultimate completion of the observations will not be forgotten.

I. INTRODUCTION.

ACCOUNTS of several absolute determinations of current have been published in recent years, and it does not, therefore, appear necessary to present an extended introduction to the subject. The results of the most recent determinations of the electromotive force of the Mean Weston Normal Cell are as follows :—

1908, AYRTON, MATHER, and SMITH*	1·01818	semi-absolute	volts	at 20° C.
1908, GUILLET†	1·01812	,,	,,	,,
1908, PELLAT‡	1·01831	,,	,,	,,
1908–10, JANET, LAPORTE, and JOUANST§ . .	1·01836	,,	,,	,,
1910, HAGA and BOEREMA‖	1·01825	,,	,,	,,
1912, ROSA, DORSEY, and MILLER¶	1·01822	,,	,,	,,

* AYRTON, MATHER, and SMITH, 'Phil. Trans.,' A, vol. 207, pp. 463–544, 1908.

† GUILLET, 'Bull. de la Soc. Int. des Élect.,' 2, vol. 8, pp. 535–561, 1908.

‡ PELLAT, 'Bull. de la Soc. Int. des Élect.,' 2, vol. 8, pp. 573–633, 1908.

§ JANET, LAPORTE, and JOUANST, 'Bull. de la Soc. Int. des Élect.,' 2, vol. 8, pp. 459–522, 1908; also (II.), vol. 10, p. 482, 1910.

‖ HAGA and BOEREMA, 'Kon. Akad. Wetensch. Amsterdam, Proc.,' p. 587, 1910.

¶ ROSA, DORSEY, and MILLER, 'Bull. Bur. of Stan.,' vol. 8, p. 269, 1912.

where the semi-absolute volt is taken as the potential difference between the terminals of an international ohm (Britain, America, and Germany) when it is traversed by a current of one absolute ampere.

The writer desires as a result of the investigation recorded in this paper to add to the above list the value **1·01831** semi-absolute volts at 20° C., which has been determined with the aid of a Weber electrodynamometer in the possession of McGill University, Montreal.

This instrument was purchased by the University as a part of the general equipment of the Macdonald Physics Building about 1893. It was carefully made by NALDER, and was an exact copy of the dynamometer constructed by the Electrical Committee of the British Association, and described by MAXWELL.* The instrument was set up by Mr. R. O. KING in 1895 under the direction of Prof. H. L. CALLENDAR. The work by KING has been fully described by CALLENDAR† who made some important changes in the design of the instrument. Preliminary observations were obtained by KING for the electromotive force of the old Board of Trade form of crystal Clark cell, and it was intended that more accurate work should be obtained later. This work was not, however, carried out, and the dynamometer remained in disuse for over ten years. It was suggested by Prof. BARNES that the writer should complete this investigation, and through the generosity of Mr. R. O. KING, a Fellowship was provided, which enabled him to devote a year almost without interruption to the work.

Several sources of error which appeared in the early use of electrodynamometers have been eliminated, and it is considered that an absolute accuracy closer than 1 part in 40,000 has been obtained in this investigation. Measurements can be repeated with the instrument to a much higher order of accuracy, but it was estimated that there might possibly be a constant error of 2 or 3 parts in 100,000.

The deflection measurements, the determination of the ratio of the radii of the two sets of coils, and the evaluation of the controlling couple formed the main parts of the investigation. For the convenience of comparison all the observations on each factor are grouped together, and the final calculation for the mean electromotive force of the standard cell is calculated from the mean values for the various factors under known conditions. An illustration of the dynamometer is shown in Plate 1. Descriptive details of the various parts of the instrument appear in the respective sections of the paper.

II. THE THEORY OF THE WEBER ELECTRODYNAMOMETER.

(*a*) *The Expression for the Magnetic Couple Acting on the Suspended System when Currents are Flowing in the Coils.*

If W is the mutual potential energy of the two systems when currents i_1 and i_2 flow through the fixed and suspended coils respectively, and if ϕ is equal to $\frac{\pi}{2}-\theta$,

* MAXWELL, 'Electricity and Magnetism' (third edition), vol. 2, p. 367.

† CALLENDAR, 'Phil. Trans.,' A, vol. 199, p. 55, 1902.

where θ is the angle of deflection from the position in which the suspended coils are at right angles to the fixed coils, then the couple acting on the suspended coils and tending to increase θ is $\frac{dW}{d\phi}$.

The mutual potential energy, W, is equal to the sum of the four mutual potential energies of each fixed coil with each suspended coil. In fig. 1 a section of the coils is represented for $\phi = 0$. If we call the coils I, II, III, IV, as marked in the figure, then

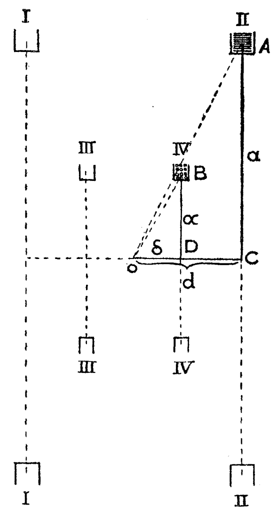


FIG. 1

$$W = W_{(I,\,III)} + W_{(I,\,IV)} + W_{(II,\,III)} + W_{(II,\,IV)}, \quad . \quad . \quad . \quad (1)$$

where the suffixes indicate the pairs of coils considered.

If we take 0 as the origin, a, α as the radii of the large and small coils, and d, δ their distances from the origin respectively, then it can be shown that

$$W_{(II,\,IV)} = i_1 i_2 [G_1 g_1 P_1(\cos\phi) + G_2 g_2 P_2(\cos\phi) + \ldots + G_n g_n P_n(\cos\phi) + \ldots],$$

where G_1, G_2, G_3, ... are constants depending on "a," "d," and the sectional dimensions of the winding, g_1, g_2, g_3, ... depending on α, δ (and the sectional dimensions of the winding), and $P_1(\cos\phi)$, $P_2(\cos\phi)$, $P_3(\cos\phi)$, ... are surface Zonal Harmonics of orders 1, 2, 3, ...

It is obvious from the symmetry of the dynamometer that

$$W_{(II,\,IV)} = W_{(I,\,III)}$$

and also

$$W_{(I,\,IV)} = W_{(II,\,III)},$$

but in the latter case we must have $\pi+\phi$ for ϕ, and the sign of either i_1 or i_2 must be reversed, hence

$$W_{(I,\,IV)} = W_{(II,\,III)} = i_1 i_2 [G_1 g_1 P_1(\cos\phi) - G_2 g_2 P_2(\cos\phi) + G_3 g_3 P_3(\cos\phi) - G_4 g_4 P_4(\cos\phi) + \ldots].$$

It follows, therefore, from (1), that

$$W = 4 i_1 i_2 [G_1 g_1 P_1(\cos\phi) + G_3 g_3 P_3(\cos\phi) + G_5 g_5 P_5(\cos\phi) + \ldots], \quad . \quad . \quad . \quad (2)$$

and hence the required couple $\frac{dW}{d\phi}$ is equal to

$$-4 i_1 i_2 \sin\phi\, [G_1 g_1 P'_1 + G_3 g_3 P'_3 + G_5 g_5 P'_5 + \ldots], \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where

$$P'_n = \frac{d\,[P_n(\cos\phi)]}{d(\cos\phi)}.$$

We shall require to evaluate this couple in terms of i_1, i_2, a, d, α, δ, and ϕ.

It can be shown that

$$P'_1 = 1\,; \quad P'_3 = \tfrac{3}{2}(5\cos^2\phi - 1)\,; \quad P'_5 = \tfrac{15}{8}(21\cos^4\phi - 14\cos^2\phi + 1)\,;$$
$$P'_7 = \tfrac{1}{16}(3003\cos^6\phi - 3465\cos^4\phi + 945\cos^2\phi - 35)\,; \quad \&c.$$

If we neglect temporarily the corrections due to the breadth and depth of the windings, it can be shown that the constants G_n and g_n are given by

$$G_n = \frac{2\pi N_1}{(a^2+d^2)^{\frac{n}{2}}}[\cos\overline{AOC}\,.\,P_n(\cos\overline{AOC}) - P_{n-1}(\cos\overline{AOC})]$$

and

$$g_n = 2\pi N_2(\alpha^2+\delta^2)^{\frac{n+1}{2}}[\cos\overline{BOD}\,.\,P_n(\cos\overline{BOD}) - P_{n+1}(\cos\overline{BOD})],$$

where N_1 and N_2 are the number of turns on each of the large and small coils respectively, and the angles are those shown in fig. 1. It will be seen that $\cos\overline{AOC} = \dfrac{d}{\sqrt{a^2+d^2}}$ and $\cos\overline{BOD} = \dfrac{\delta}{\sqrt{\alpha^2+\delta^2}}$.

We have, by substitution in the above expressions,

$$G_1 = -\frac{2\pi N_1 a^2}{(a^2+d^2)^{3/2}}, \qquad g_1 = \pi N_2\alpha^2,$$

$$G_3 = -\frac{4\pi N_1 a^2\left(d^2 - \dfrac{a^2}{4}\right)}{(a^2+d^2)^{7/2}}, \qquad g_3 = 3\pi N_2\alpha^2\left(\delta^2 - \frac{\alpha^2}{4}\right),$$

$$G_5 = -\frac{6\pi N_1 a^2\left(d^4 - \tfrac{3}{2}a^2d^2 + \dfrac{a^4}{8}\right)}{(a^2+d^2)^{11/2}}, \qquad g_5 = 5\pi N_2\alpha^2\left(\delta^4 - \tfrac{3}{2}\alpha^2\delta^2 + \frac{\alpha^4}{8}\right),$$

$$G_7 = -\frac{8\pi N_1 a^2\left(d^6 - \tfrac{15}{4}a^2d^4 + \tfrac{15}{8}a^4d^2 - \tfrac{5}{64}a^6\right)}{(a^2+d^2)^{15/2}}, \qquad g_7 = 7\pi N_2\alpha^2\left(\delta^6 - \tfrac{15}{4}\alpha^2\delta^4 + \tfrac{15}{8}\alpha^4\delta^2 - \tfrac{5}{64}\alpha^6\right),$$

&c., &c.

Substitutions of the values for a, d, α and δ for this instrument showed that $a = 2d$ and $\alpha = 2\delta$ to a degree of approximation such that the term $G_3g_3P'_3$ could be neglected as well as all the terms where $n > 7$.

It remains to find the corrections due to the breadth and depth of the windings. (A short method of calculation is given in MAXWELL's 'Electricity and Magnetism,' 3rd edition, § 700, Vol. II., p. 337). If ξ_1, η_1, ξ_2, η_2 represent the depths and breadths of the large and small coils respectively, it can be shown that we must multiply the value of G_1 recorded above by

$$1 + \tfrac{1}{24}\left(\frac{2}{a^2} - 15\frac{d^2}{(a^2+d^2)^2}\right)\xi_1^2 + \tfrac{1}{8}\frac{4d^2-a^2}{(a^2+d^2)^2}\eta_1^2 + \text{negligible terms},$$

but if $a = 2d$ approximately, this reduces to

$$1 - \tfrac{1}{60}\frac{\xi_1^2}{a^2}.$$

We may take, therefore,

$$G_1 = -\frac{2\pi a^2 N_1}{(a^2+d^2)^{3/2}}\left(1-\tfrac{1}{60}\frac{\xi_1^{\,2}}{a^2}\right)$$

and in a similar manner we have

$$g_1 = \pi N_2 a^2 + \tfrac{1}{12}\pi N_2 \xi_2^{\,2}.$$

In the case of G_5, G_7, g_5, g_7 we may neglect the corrections due to the depth and breadth of the windings.

(b) The Expression for the Current Flowing through the Suspended Coils.

Let the ratio of the current i_1 through the fixed coils, to the current i_2 through the suspended coils be r. If we assume that the magnetic azimuth of the planes of the fixed coils is zero, and that the angle which the axis of the suspended coils (in the undeflected position) makes with the planes of the fixed coils is also equal to zero,* then it can be shown that

$$i_2 = \sqrt{\frac{B \tan\theta}{r\,(-4G_1g_1 - 4G_5g_5P'_5 - 4G_7g_7P'_7)}} \quad \ldots \ldots \quad (4),$$

where B sin θ is the controlling couple due to the bifilar suspension, and tan θ is the mean of four values tan θ_1, tan θ_2, tan θ_3, tan θ_4 where

θ_1 is the deflection	when	i_1	is	+	and i_2	is	+
θ_2	,,	,,	,,	−	,,		−
θ_3	,,	,,	,,	+	,,		−
θ_4	,,	,,	,,	−	,,		+.

In this way the effect of the earth's magnetic field is eliminated to a sufficient degree of accuracy.

III. THE DIMENSIONS OF THE FIXED COILS.

(a) The Mean Radius of the Large Coils.

The radii of the fixed coils had been determined by Mr. R. O. KING. The following summary is taken from Prof. CALLENDAR'S account of the measurement:—

"The mean radius of the pair of large coils was determined from the length of the copper tape with which they were wound. This method is not satisfactory with soft

* If the axis of the suspended coils (in the undeflected position) makes an angle Ψ with the planes of the fixed coils, the above expression for i_2 is reduced to $\frac{i_2}{\sqrt{\cos\Psi}}$. The coils could, however, be adjusted by optical methods with sufficient accuracy for us to neglect this factor. The choice of θ_1, θ_2, θ_3, θ_4 eliminates the effect of small magnetic azimuth of the planes of the fixed coils.

annealed copper wire owing to stretching, but the hard rolled copper tape could be wound without any tension, and did not undergo any change of length. This was verified by graduating the tape itself on a 50-foot comparator, the errors of which were known; then winding the coil for trial, and unwinding and measuring the tape again, which was found not to have changed in length by more than a tenth of a millimetre in each 50 feet. The tape was supported horizontally on the polished surface of the comparator, and measured under a tension of 6 kgr. YOUNG's modulus was determined for each section. The tape was wound on each coil in two lengths of 19 turns each, starting at opposite ends of a diameter, with two thicknesses of paraffined paper between each turn, so that the insulation could be tested with absolute certainty at any time. There were 38 complete turns, and nearly 6000 cm. of tape, on each coil. The probable error of the measurement was less than 1 mm. on the whole length, *i.e.*, less than 1 part in 100,000. The coils could not be boiled in paraffin after winding, as this would subject the tape to uncertain strains owing to the contraction of the wax. It was found necessary to re-wind the coils two or three times with minor improvements before the insulation proved to be perfect. Finally, silk ribbon was adopted in place of paper."

The following is a quotation from the notes of Mr. R. O. KING, containing the summary of his measurements :—

" We have the following data :—

Coil I.

1st winding. Total length = 2970·17 cm.

Corrected for tension, it = 2969·90 cm.*

2nd winding. Total length = 2969·97 cm.

Corrected for tension, it = 2969·69 cm.

There are nineteen double turns so that the mean radius

$$=\frac{2969{\cdot}90+2969{\cdot}69}{2\times 19\times 2\pi}=24{\cdot}8767 \text{ cm.}$$

Coil II.

1st winding. Total length = 2972·88 cm.

Corrected for tension, it = 2972·60 cm.

2nd winding. Total length = 2973·20 cm.

Corrected for tension, it = 2972·92 cm.

* These tension corrections reduce the value of the length measured under a tension of 6 kilos. to its value when wound under no tension.

There are nineteen double turns so that the mean radius

$$= \frac{2972 \cdot 60 + 2972 \cdot 92}{2 \times 19 \times 2\pi} = 24 \cdot 9016 \text{ cm.}$$

The mean radius of both coils is therefore 24·8892 cm.

(A rough verification of these measurements was made by measuring the channel before and after with a steel tape.)"

The measurements were taken at an average temperature of about 17° C., hence correcting for the temperature change in the coil frames and tape we have the mean radius

$$a = \mathbf{24 \cdot 8905 \text{ cm. at } 20^\circ \text{ C.}}$$

The mean depth of the winding ξ_1 was approximately 1·72 cm.

It will be noted that the determination of this radius was made by Mr. KING some time before the present work. The constancy of the dimensions of the coils could, it was thought, be assumed without chance of error, as the coils had not been used at all in the interval and the insulation (silk ribbon in the case of the fixed coils) was not of such a nature that strains could be caused in the winding by temperature or humidity variations in the laboratory. A check on the validity of this assumption can, however, be found in § IV. (*c*), p. 167. It will be seen from the value of I_2/I_1 that Mr. KING's value for the ratio of the dimensional constants for the two systems of coils differed from the present one by less than 3 parts in 100,000. The construction and the arrangement of insulation in the two systems was so different that this constancy in ratio can be taken as evidence of their individual constancy.

The system of interchanges adopted in the series of observations recorded later made it possible to use the mean value of the radii without the introduction of any further correction on account of their slight difference.

(*b*) *The Determination of the Distance between the Large Coils.*

The distance between the planes of the large coils, $2d$, was determined by measuring the distance between the external edges at five evenly distributed positions round the coils, and subtracting the mean thickness of the channel-frames at these points from the mean of these measurements. The reversing of the coils during a series of observations eliminated errors due to any asymmetry in the position of the copper tape in its channel. An extension micrometer gauge of special size and shape for adjustment on the dynamometer was constructed by attaching a sensitive screw gauge to one end of a thick brass tube and a suitably shaped end-piece and stud at the other end. The brass part was wrapped in asbestos and a thermometer inserted in the tube. It was found that the variations due to the temperature changes while handling could be followed with sufficient closeness to avoid any possibility of error.

These calipers were calibrated by means of rectangular blocks with sharp edges

which were clamped on to the comparator (see § VI. (*c*)) at a distance approximately equal to the distance between the fixed coils. Comparisons of the comparator readings with the caliper measurements gave an accurate calibration. As it was found difficult to make the blocks absolutely rectangular the mean of the two measurements with the blocks placed on their opposite faces was taken in every case. The following is a summary of the calibration in which the values are reduced to those corresponding to a setting of the micrometer at 0·8000 inches and a temperature of 25°·0 C. Each value is the mean of three repetitions.

TABLE I.

	Blocks in one position.	Blocks in reversed position.	Mean.
	cm.	cm.	cm.
Using first pair of faces . . .	27·4106	27·4112	27·4109
	27·4103	27·4124	27·4114
	27·4085	27·4113	27·4099
	27·4100	27·4114	27·4107
Using second pair of faces . .	27·4088	27·4132	27·4110
	27·4093	27·4131	27·4112
	27·4087	27·4103	27·4095
	27·4083	27·4111	27·4097
			27·4105

A second calibration was made at the end of the investigation to ascertain whether the gauge had been disturbed in any way by usage. This was done by means of a standardized pair of steel calipers. The results are tabulated below. Each figure represents the mean of five settings. Although this method was not capable of the same accuracy as the first, it served as a very satisfactory check.

TABLE II.

Reading of the gauge in inches.	Temperature of the gauge.	Standard caliper reading for the same measurement.	Calculated value of gauge set at 0·8000 inch, at 25°·0 C.
	° C.	cm.	cm.
0·8146	29·0	27·450	$27{\cdot}411_0$
0·7805	29·0	27·363	$27{\cdot}410_5$
0·8042	29·1	27·424	$27{\cdot}411_6$
0·8191	29·0	27·460	$27{\cdot}409_6$
0·8097	29·1	27·437	$27{\cdot}410_4$
		Mean	$27{\cdot}410_6$

We may therefore take 27·4105 cm. as the equivalent of a reading 0·8000 at 25°·0 C.

A sample set of measurements for determining the distance between the fixed coils is given below. In every case where d is recorded in this paper the value is the mean of a set of observations taken in this way. The bracketed figures give the temperature of the extension calipers.

TABLE III.—Micrometer readings in inches. Dynamometer at 20°·0 C.

At 1st point.	At 2nd point.	At 3rd point.	At 4th point.	At 5th point.
0·8074 (25·1)	0·8255 (24·6)	0·8215 (24·2)	0·8125 (23·6)	0·8168 (23·0)
76 (25·0)	45 (25·3)	24 (25·4)	25 (25·4)	72 (25·4)
80 (25·0)	45 (25·0)	22 (24·0)	26 (24·4)	65 (24·8)
80 (24·3)	48 (24·8)	13 (25·3)	22 (25·2)	72 (25·0)
81 (24·3)	57 (24·8)	27 (25·3)	20 (25·2)	75 (25·0)
Mean 0·8078 (24·7)	Mean 0·8250 (24·9)	Mean 0·8220 (24·8)	Mean 0·8124 (24·8)	Mean 0·8170 (24·6)

The mean for the five points is **0·8168 at 24°·8 C.**

Now since we know that a reading of 0·8000 inches corresponds to 27·4105 cm. at 25°·0 C. it follows that 0·8168 at 24°·8 C. corresponds to a length of 27·4530 cm. and the mean thickness of a coil is 2·5556 cm., hence $2d = 24·8974$ cm. and we have $d = 12·4487$ cm. in this case.

Five measurements were taken with micrometer calipers at each of the five points on each of the fixed coils to determine the thickness of the coil frames, and a value of 2·5556 cm. was obtained for the average thickness of a coil frame at the points used for measurement.

All the deflection observations were reduced to the value for

$$d = \mathbf{12·4448\ cm.\ at\ 20^\circ\ C.}$$

IV. THE RATIO OF THE RADII OF THE FIXED AND SUSPENDED COILS.

(a) *The Method.*

The method employed for the determination of the ratio of the radii is similar to that outlined by CALLENDAR, and is a development of BOSSCHA'S method. A thick brass tube carrying a delicately suspended series of magnets and a mirror mounted at its centre was rigidly attached to the framework of the large coils. The small coils could be mounted co-axially on this tube against accurately turned shoulders at a distance of about 30 cm. on either side of the centre of the fixed coils (see Plate 2).

The large and the small coils were arranged in parallel, currents were passed through in opposite directions and adjusted until the balance point for equal magnetic forces at the centre could be determined by interpolation from the observations on the deflections of the small magnetic system. Readings were taken with the small coils in the four possible positions (viz., at either side of the dynamometer and then reversed in each case) for each of the four positions of the fixed coils (as enumerated in the account of the deflection measurements), making sixteen positions in all. In this way the effects of small asymmetries in the coils were eliminated and the mean equivalent value of the constants could be calculated. It is pointed out later that four positions are sufficient to determine the mean to a high degree of approximation. Further data concerning the arrangements, the observations, and the avoidance of temperature variations in the coils under the necessary experimental conditions, are given below. It is first necessary to determine the expression for calculating the constants of the small coils from such measurements.

(*b*) *The Theory of the Method.*

Fig. 2 represents the two sets of coils arranged co-axially with their centres at a distance x apart, a, d, α and δ have the same significance as before, and at o the

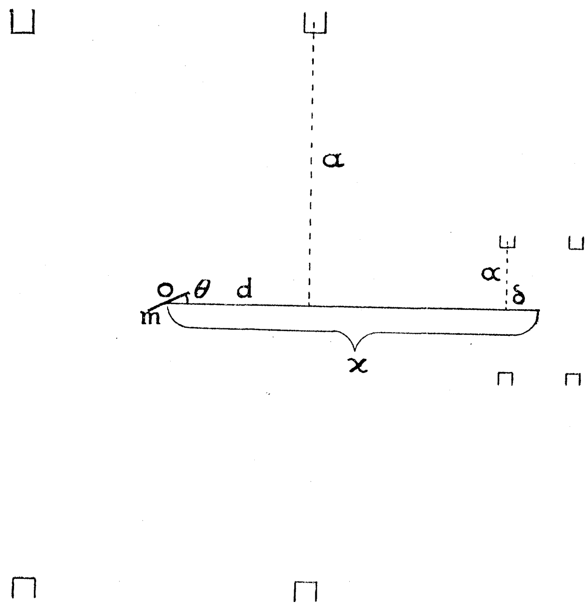


FIG. 2

magnetic system of strength m is represented as being deflected through an angle θ which should of course be in a plane at right angles to the paper. When no current is flowing, and also when a perfect balance is obtained, $\theta = \frac{\pi}{2}$.

Let us first find the couple on the magnets tending to decrease θ, due to a current I_1 in the large coils. If the magnetic system is small, it can be shown that this couple is given by

$$-2I_1 \sin\theta\,[\gamma_1 G_1 P'_1 + \gamma_3 G_3 P'_3 + \gamma_5 G_5 P'_5 + \ldots]$$

where G_1, G_2, G_3, &c., have the same significance as before;

$$P'_n = \frac{d\,[P_n(\cos\theta)]}{d(\cos\theta)}, \text{ and } \gamma_1 = M; \qquad \gamma_3 = ML^2; \qquad \gamma_5 = M\lambda^4 \ldots,$$

where M is the magnetic moment of the suspended magnets, L is their "equivalent length," and λ is a length differing slightly from L. Now we have $\theta = \frac{\pi}{2}$ and $G_3 = 0$, hence our required couple can be taken as

$$-2I_1\,[\gamma_1 G_1 + \tfrac{15}{8}\gamma_5 G_5].$$

In the same way it can be shown that the couple due to a current I_2 (in the opposite direction) in the small coils with their centres at a distance x from the centre of the magnetic system, is given by

$$-I_2\,[\gamma_1 \Gamma_1 + \tfrac{15}{8}\gamma_5 \Gamma_5],$$

where Γ_1, Γ_5, are constants for the two small coils added together. It should be noted that Γ_1 and Γ_5 are not derived in the same way as g_1 and g_5. In this case, the calculation of γ corresponds to that for g, and the method of calculating Γ is analogous to that for determining 2G for the large coils. It can be shown that

$$\Gamma_1 = -2\pi N_2 a^2 \left\{\frac{A}{\{a^2+(x-\delta)^2\}^{3/2}} + \frac{B}{\{a^2+(x+\delta)^2\}^{3/2}}\right\},$$

where

$$A = 1 + \tfrac{1}{24}\left\{\frac{2}{a^2} - 15\frac{(x-\delta)^2}{\{a^2+(x-\delta)^2\}^2}\right\}\xi_2^2 + \tfrac{1}{8}\left\{\frac{4(x-\delta)^2 - a^2}{\{a^2+(x-\delta)^2\}^2}\right\}\eta_2^2,$$

and

$$B = 1 + \tfrac{1}{24}\left\{\frac{2}{a^2} - 15\frac{(x+\delta)^2}{\{a^2+(x+\delta)^2\}^2}\right\}\xi_2^2 + \tfrac{1}{8}\left\{\frac{4(x+\delta)^2 - a^2}{\{a^2+(x+\delta)^2\}^2}\right\}\eta_2^2.$$

When the currents are such that there is no deflection in the magnetometer $\left(\text{that is, when } \theta = \frac{\pi}{2}\right)$ we have

$$2I_1\,[\gamma_1 G_1 + \tfrac{15}{8}\gamma_5 G_5] = I_2\,[\gamma_1 \Gamma_1 + \tfrac{15}{8}\gamma_5 \Gamma_5],$$

and therefore

$$\frac{I_2}{I_1} = \frac{2G_1}{\Gamma_1}\left\{1 + \tfrac{15}{8}\frac{\gamma_5}{\gamma_1}\left(\frac{G_5}{G_1} - \frac{\Gamma_5}{\Gamma_1}\right)\right\}. \qquad (5)$$

The magnetic system consisted of fourteen small magnets made from hair springs and each about 1 cm. in length, seven being placed on either side of a mica strip such that the system of magnets occupied a space about 1 cm. broad and 1·5 cm. high. It

will be seen that whatever the pole strength may be, $\frac{\gamma_5}{\gamma_1}$ for such dimensions is about unity in magnitude. It is not necessary to determine its value further, because substitution of our measurements show that $\left(\frac{G_5}{G_1}-\frac{\Gamma_5}{\Gamma_1}\right)$ is so small that we may assume that

$$\frac{I_2}{I_1}=\frac{2G_1}{\Gamma_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and therefore from (5) and (6) we see that

$$\frac{2G_1}{I_2/I_1} = -2\pi N_2\alpha^2\left\{\frac{A}{\{\alpha^2+(x-\delta)^2\}^{3/2}}+\frac{B}{\{\alpha^2+(x+\delta)^2\}^{3/2}}\right\} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

We can now solve for α in terms of G_1, I_2/I_1, x, δ, ξ_2, and η_2. G_1 is known, I_2/I_1 and x must be determined as accurately as possible; δ is less than a tenth of x in magnitude and need not therefore be known to a high degree of accuracy, while ξ_2 and η_2 are needed only to rough approximations. (In the case of our experiment $A = 1{\cdot}001463$ and $B = 1{\cdot}001421$, but the appearance of ξ_2 in the expression for g_1 lessens the apparent importance of ξ_2 and η_2. It can be shown that

$$2g_1 = \frac{2G_1\, I_1/I_2}{\left\{\frac{1}{\{\alpha^2+(x-\delta)^2\}^{3/2}}+\frac{1}{\{\alpha^2+(x+\delta)^2\}^{3/2}}\right\}(1+0{\cdot}001448)\left(1-\frac{1}{12}\frac{\pi N_2\xi_2^2}{g_1}\right)},$$

where $\left(1-\frac{1}{12}\frac{\pi N_2\xi_2^2}{g_1}\right)$ came to $(1-0{\cdot}001315)$, thus lessening the importance of ξ_2 and η_2 in the determination of g_1 by this method.)

(c) *The Determination of the Ratio of the Currents* $\frac{I_2}{I_1}$ *when the Magnets are Undeflected.*

The electrical connections are shown in fig. 3. S represents the suspended coils, and F the fixed coils. They are always connected in such a way that the current flows through them in opposite directions. r_1, r_2, are suitable resistances with shunts r_3 and r_4, capable of delicate adjustment. K represents a Kelvin-Varley slide which had been calibrated for refined comparisons according to the method outlined in the Appendix. G is a galvanometer, and BB are batteries. 1, 2, 3, 4, 5, 6, represent the terminals in a key which enables a sudden change to be made from the connection CD to the connections shown in dotted lines. The resistance of C (a thick copper rod) in the one case, and of the connections between the terminals 1, 2, and the Kelvin-Varley slide in the second case, are of such a value that they are quite negligible in our measurements.

The method of determination finally adopted consisted of a set of observations

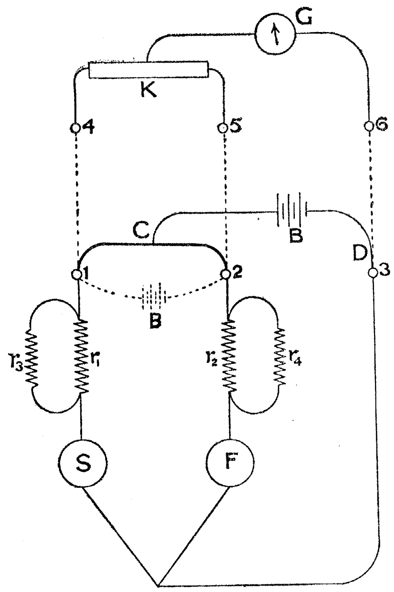


FIG. 3

taken in the following order. It is thought that by this system of measurement we have been able to eliminate or correct for all appreciable extraneous effects.

Order of Procedure.

(i) A current equal in magnitude to that used in the deflection measurements (approximately an eighth of an ampere in each of the four windings, in parallel as in use) is passed through the suspended coils.

(ii) r_2 and r_4 are adjusted until the magnetometer gives only a small deflection on the simultaneous reversal of both currents. This deflection is recorded.

(iii) The magnetometer scale is calibrated in terms of a small variation in the shunt r_4, and from the subsequent operation (xi), this can be expressed in terms of the Kelvin-Varley slide.

(iv) The connection C is replaced rapidly by the connections shown in dotted lines in the figure. Previous to the observations, the Kelvin-Varley slide has been set at the reading nearest to the correct ratio; hence if the galvanometer circuit be connected immediately after, it will show only a small deflection. The current now flowing through the coils is smaller than in the first connection, and hence there is a slow change corresponding to the cooling of the suspended coils. The change in the fixed coils by themselves was found to be quite negligible. Galvanometer readings are taken 15, 20, 25, and 30 seconds after making the second connection.

(v) The current is reversed and another galvanometer reading taken at 60 seconds; this will be in the opposite direction.

(vi) The current is reversed back to the original direction and the galvanometer reading at 90 seconds is noted.

(vii) The Kelvin-Varley slide is reversed (*i.e.*, the terminals are interchanged) and

FIG. 4

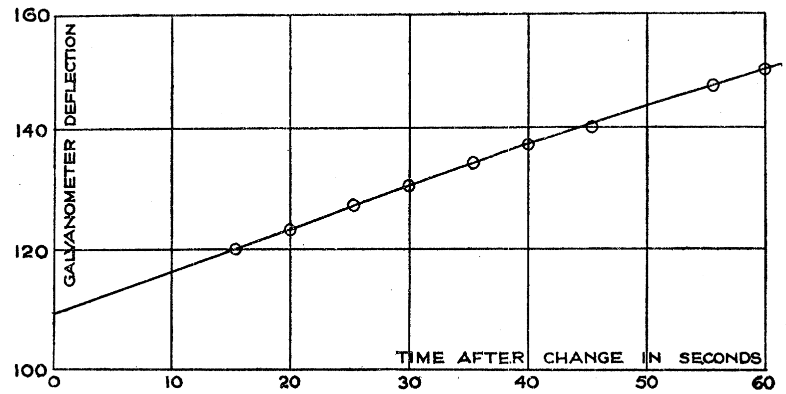
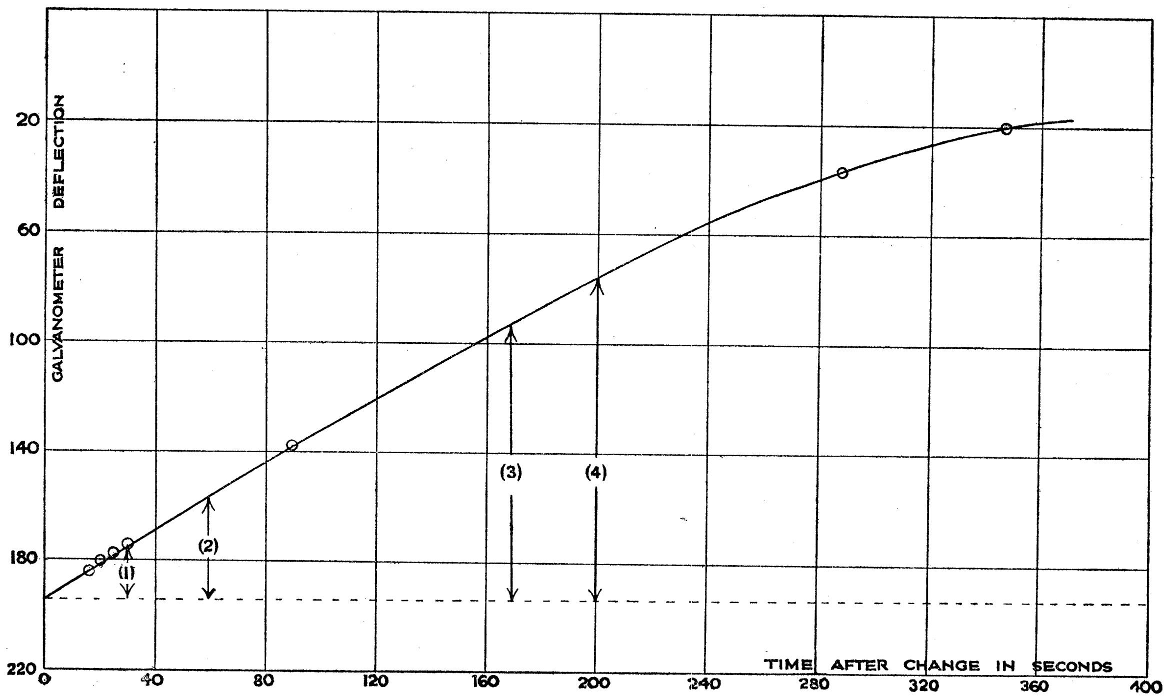


FIG. 5



the new reading and accompanying galvanometer deflection are noted at 210* seconds after the change.

(viii) The current is reversed, and the galvanometer read at 240* seconds.

* These intervals were decreased when proficiency in performing the operations rapidly had been attained.

(ix) The Kelvin-Varley is placed at the initial reading, and the galvanometer read at 330 seconds, and later at 390 seconds again.

(x) The value of a galvanometer scale division is determined in terms of the Kelvin-Varley slide.

(xi) The value of a small change in the shunt r_4 is determined in terms of the Kelvin-Varley slide.

We have thus obtained four measurements of the ratio for a given position of the coils chosen in such a way as to eliminate thermal and contact effects, and if the galvanometer deflections for 15, 20, 25, 30, 90, 330, and 390 seconds (which are all for the same arrangement of the circuit) are plotted, we can find by extrapolation the correction necessary to reduce the Kelvin-Varley readings taken 30, 60, 310, and 240 seconds after the change, to the value for the ratio under the conditions of the operation (ii).

In fig. 4 the type of behaviour at the commencement of the curve is shown, and in fig. 5 a curve is given which is typical of the curves taken with every set of operations. It will be seen how the corrections in column 8 of the table below can be determined with sufficient accuracy from such curves.

Table IV. (p. 165) gives a summary of all the observations on the determination of the ratio of these currents.

The first column gives the position of the coils. These are defined by A, B, C, D, E, F, G, H, I, J, K, L, M, N, O and P:—

In positions of—	Fixed coils are in position—	Suspended coils are placed—		Remarks.
		With marked face to the—	On— side of fixed coils—	
A, B, C, D	II.	—	—	As defined for the main deflection experiments (see § VII. (*e*)).
E, F, G, H	I.	—	—	
I, J, K, L	III.	—	—	
M, N, O, P	IV.	—	—	
A, E, I, M	—	West	East	
B, F, J, N	—	,,	West	
C, G, K, O	—	East	East	
D, H, L, P	—	,,	West	

The second column gives the arrangement of the circuit for the four readings on the Kelvin-Varley slide. (1) corresponds to the reading in operation (iv) above; (2) to that in (v); (3) to that in (vii); and (4) to that in (viii).

The slide reading is given in the third column; the calibration correction has already been applied.

The headings for columns (4) to (12) explain themselves.

TABLE IV.—Observations in the Ratio Determinations.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)	(9.)	(10.)	(11.)	(12.)	(13.)	(14.)	(15.)	(16.)
Position of the coils.	Arrangement of the circuit.	Kelvin-Varley slide reading (with calibration correction).	The change in the magnetometer reading for a reversal of the current.	The value of 1 magnetometer scale division in Kelvin-Varley ohms.	The magnetometer correction for the Kelvin-Varley reading.	The galvanometer deflections.	The deflection correction for the cooling of the small coils after the change of circuit.	The value 1 galvanometer scale division in Kelvin-Varley ohms.	The galvanometer correction for the Kelvin-Varley reading.	The corrected Kelvin-Varley reading.	The temperature of the dynamometer, t.	d at the temperature, t.	The correction reducing the Kelvin-Varley reading to that for $d = 12\cdot4448\,\{1+a'\,(t-20)\}$.	The equivalent reading for the true balance point.	
											° C.				
(A)	(1)	8311·2	+4·2	3·30	+13·8	+ 28	+ 20	0·128	+ 6·2	8331·2	26·8	12·4357	+4·0	8335·2	8334·9
	(2)	8311·2	+4·2		+13·8	+ 11	+ 37		+ 6·2	8331·2			+4·0	8335·2	
	(3)	91690·6	−4·2		−13·8	+ 55	−112		− 7·3	91669·5			−4·0	8334·5	
	(4)	91690·6	−4·2		−13·8	+ 67	−125		− 7·4	91669·4			−4·0	8334·6	
(A)	(1)	8311·2	+3·1	3·14	+ 9·7	+ 54	+ 22	0·124	+ 9·4	8330·3	26·9	12·4357	+4·0	8334·3	8334·3
	(2)	8311·2	+3·1		+ 9·7	+ 38	+ 42		+ 9·9	8330·8			+4·0	8334·8	
	(3)	91690·6	−3·1		− 9·7	+ 28	−117		−11·0	91669·9			−4·0	8334·1	
	(4)	91690·6	−3·1		− 9·7	+ 40	−129		−11·0	91669·9			−4·0	8334·1	
(A)	(1)	8311·2	+5·1	3·15	+16·0	+ 4	+ 20	0·125	+ 3·0	8330·2	26·5	12·4356	+4·0	8334·2	8334·4
	(2)	8311·2	+5·1		+16·0	− 9	+ 39		+ 3·7	8330·9			+4·0	8334·9	
	(3)	91690·6	−5·1		−16·0	+ 72	−110		− 4·7	91669·9			−4·0	8334·1	
	(4)	91690·6	−5·1		−16·0	+ 83	−112		− 4·9	91669·7			−4·0	8334·3	
(A)	(1)	8301·2	+0·9	3·15	+ 2·8	+169	+ 21	0·120	+22·8	8326·8	25·4	12·4223	+8·9	8335·7	8335·4
	(2)	8301·2	+0·9		+ 2·8	+151	+ 43		+23·2	8327·2	25·4		+8·9	8336·1	
	(3)	91700·6	−0·9		− 2·8	− 82	−118		−24·0	91673·8			−8·9	8335·1	
	(4)	91700·6	−0·9		− 2·8	− 67	−130		−23·6	91674·2			−8·9	8334·7	

The following values for the other positions were obtained in the same manner as those in Table IV. :—

(B) 8317·9	(F) 8306·8
(B) 8318·9	(G) 8299·3
(B) 8320·3	(G) 8298·7
(B) 8319·4	(H) 8329·0
(C) 8312·0	(I) 8334·6
(C) 8310·0	(J) 8318·6
(C) 8311·2	(K) 8310·1
(D) 8341·9	(L) 8341·8
(D) 8342·1	(M) 8325·6
(D) 8346·0	(N) 8307·8
(D) 8345·3	(O) 8298·8
(E) 8322·5	(P) 8334·1
(F) 8305·0	

In column (13) the values for d, half the mean distance between the planes of the large coils, are given. Each of these determinations was obtained from a full set of observations similar to those outlined in Section III. (b).

It is necessary to express the ratio of the currents in terms of d where d would equal 12·4448 cm. at 20° C., the figure used for the mean as explained in the section on the main deflection observations. The slide reading must therefore be reduced to its value for $d = 12{\cdot}4448\ \{1+a'(t-20)\}$ where "a'" is the co-efficient of expansion, taken as 0·000018. It can easily be shown that for small differences, an increase of x per cent. in d would mean a decrease of $0{\cdot}60x$ per cent. in the ratio of the currents, and hence an increase of $0{\cdot}56x$ per cent. in the Kelvin-Varley reading. This correction is given in column (14).

Columns (15) and (16) contain the corrected means. The readings at the upper end of the Kelvin-Varley scale are subtracted from 100,000. (The slide had been calibrated in terms of the exact value of 100,000 for the whole slide. See Appendix.)

The sixteen positions include two equally reliable groups of eight, namely, positions A, F, I, N, C, H, K, P and B, E, J, M, D, G, L, O. It will also be seen that there are a number of suitably chosen groups of four whose means should approximate very closely to the final mean.

TABLE V.—Summary of Mean Values.

The mean value for positions A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, is $8321{\cdot}0_4$.

Position	A	gives	8334·8	Position	B	gives	8319·1
,,	F	,,	8305·9	,,	E	,,	8322·5
,,	I	,,	8334·6	,,	J	,,	8318·6
,,	N	,,	8307·8	,,	M	,,	8325·6
,,	C	,,	8311·1	,,	D	,,	8343·8
,,	H	,,	8329·0	,,	G	,,	8299·0
,,	K	,,	8310·1	,,	L	,,	8341·8
,,	P	,,	8334·1	,,	O	,,	8298·8
	Mean	=	8320·9		Mean	=	8321·1

It will be seen from the table below that the means of the groups AFIN, CHKP, BEJM, DGLO, and also ABEF, IJMN, KLOP, CDGH have an average deviation from 8321·0 of less than 4 parts in 100,000.

TABLE VI.

AFIN	give	8320·8	ABEF	give	8320·6
CHKF	,,	8321·1	IJMN	,,	8321·6
BEJM	,,	8321·4	KLOP	,,	8321·2
DGLO	,,	8320·8	CDGH	,,	8320·8

We have for the mean value of our ratio with the four windings of the small coil in parallel

$$\frac{4I_2}{I_1} = \frac{100000-8321{\cdot}04}{8321{\cdot}04} = 11{\cdot}01773,$$

and hence $I_2/I_1 = \mathbf{2{\cdot}75443}$.

It is important to record that Mr. R. O. KING also made an accurate determination of this ratio, but unfortunately the notes containing his observations on "d" have been lost. However, assuming that the average d in his case was the same as that found for the dynamometer as it was set up when the writer first examined it in 1910, we calculated from his notes the mean value, $2I_2/I_1 = 5{\cdot}5087$ where one suspended coil is in parallel with the other, and hence

$$I_2/I_1 = 2{\cdot}75435.$$

This figure was the mean for the eight positions which we have called I, J, K, L, M, N, O, P.

The value of I_2/I_1 is independent of the temperature of the dynamometer.* Hence if the ratio is expressed for the value

$$d_t = d_{20}\{1+a'(t-20)\},$$

where d_t is the length of d at the temperature of observation t, and "a'" is the coefficient of expansion, we can use d_{20}, x_{20}, δ_{20}, and α_{20} in our general formula instead of d_t, x_t, δ_t, and α_t.

* The current through the fixed coils produces no perceptible heating (tested by resistance measurements), and that through the suspended coils during the magnetometer readings is the same as in the deflection experiments, hence any error due to the extra heating of the suspended coils is eliminated in the final results.

(*d*) *The Determination of x, the Mean Distance between the Centres of Symmetry of the Large and the Small Coils.*

The distance x can be most accurately determined by measuring the total distance between the shoulders at each end of the supporting tube, and also the thickness of the frame of the small coils where it fits against the shoulders. The reversing and replacing of the small coils during a complete series of observations for I_2/I_1 eliminated errors due to any asymmetry (in the direction of the axis) in the position of the frame or of the supporting tube with regard to the centres of the two systems of coils.

The following series of twenty readings taken at regular intervals round the shoulders was obtained with large calibrated vernier calipers* :—

TABLE VII.

Distance between shoulders.	Temperature of tube.
cm.	° C.
60·720	24·5
20	
22	
20	
20	
20	
22	
22	
20	
20	
22	
22	
20	
20	
22	
22	
22	
20	
20	
22	24·9
Mean . . . 60·7209	24·7

Hence at 20° C. the distance reduces to **60·7158 cm.**

This number was checked by two similar sets taken at 24°·0 C. and 22°·0 C.

The thickness of the coil frame where it fitted against the shoulders was determined by means of a Sweet Measuring Instrument and standard distance pieces. The following figures are each a mean value of a series of ten observations taken round the parts which fit against the shoulders :—

* The mean of the observations, rather than the maximum, is chosen because this variation depended on the setting of the calipers.

TABLE VIII.

Thickness.	Mean temperature of observation.	Value at 20° C.
cm.	° C.	cm.
4·9786	22·0	4·9784
4·9789	24·0	4·9785
4·9790	25·3	4·9785
	Mean	4·9785

We have therefore at 20° C.

$$x = \frac{60{\cdot}7158+4{\cdot}9785}{2} = \mathbf{32{\cdot}8472\ cm.}$$

(e) The Determination of "δ," the Mean Distance between the Planes of the Small Coils.

The distance between the planes of the small coils was determined by measuring the distance between the external edges at ten evenly distributed positions round the coils, and subtracting the mean thickness of the channel frames at these points from the mean of these measurements.

The measurement was taken with calibrated vernier calipers.

Mean of first set of measurements. . . .	7·4261 cm. at 20° C.	
,, second ,, ,, ...	7·4261 ,, ,,	

The mean thickness of the channel frames at the points used was measured with micrometer calipers. Twenty settings were made and the value for the mean thickness of a coil was found to be 1·4426 cm. Therefore, at 20° C.,

$$\delta = \frac{7{\cdot}4261-1{\cdot}4426}{2} = \mathbf{2{\cdot}9917\ cm.}$$

(f) The Calculation of the Mean Radius of the Small Coils.

We have from the above

$$x = 32{\cdot}8472 \text{ at } 20^\circ \text{ C.}$$

$$\delta = 2{\cdot}9917 \quad ,,$$

and

$$I_2/I_1 = 2{\cdot}75443 \quad ,,$$

for the mean complete balance. ξ_2 was 0·75 cm. and η_2 was 1·00 cm.

$2G_1 = -13{\cdot}7268$, see § V.

Substituting these values in expression (7) and solving for α, we get

$$\alpha = 5{\cdot}96076 \text{ cm. at } 20^\circ \text{ C.},$$

and hence

$$2g_1 = 2\left(\pi N_2 \alpha^2 + \tfrac{1}{12}\pi N_2 \xi_2^2\right) = \mathbf{88521{\cdot}8 \text{ at } 20^\circ \text{ C.}}$$

V. The Calculation of the Dynamometer Constants.

We have to evaluate

$$(-4G_1g_1 - 4G_5g_5P'_5 - 4G_7g_7P'_7).$$

The following data are given, elsewhere, in this paper:—

At 20° C.:—

The mean radius "a" of the large coils	$=$	24·8905 cm.
,, distance "d" of the large coils	$=$	12·4448 ,,
,, channel depth ξ_1 of the large coils	$=$	1·72 ,,
The number of turns N_1 on each large coil	$=$	38.

Hence we find by substitution that

$$2G_1 = -\frac{4\pi N_1 a^2}{(a^2+d^2)^{3/2}}\left(1 - \tfrac{1}{60}\frac{\xi_1^2}{a^2}\right) = \mathbf{-13{\cdot}7268} \text{ at } 20^\circ \text{ C.}$$

$$2G_5 = -\frac{12\pi N_1 a^2\left(d^4 - \frac{3}{2}a^2d^2 + \frac{a^4}{8}\right)}{(a^2+d^2)^{11/2}} = \mathbf{+\,0{\cdot}000008242} \text{ at } 20^\circ \text{ C.}$$

$$2G_7 = -\frac{16\pi N_1 a^2\left(d^6 - \frac{15}{4}a^2d^4 + \frac{15}{8}a^4d^2 - \frac{5}{64}a^6\right)}{(a^2+d^2)^{15/2}} = \mathbf{-\,0{\cdot}0000000104} \text{ at } 20^\circ \text{ C.}$$

We have also from § IV. (f)

$$2g_1 = 88521{\cdot}8 \text{ at } 20^\circ \text{ C.},$$

and since

$$\alpha = 5{\cdot}96076, \qquad \delta = 2{\cdot}9917 \qquad \text{and} \qquad N_2 = 396,$$

we have at 20° C.

$$2g_5 = 10\pi N_2 \alpha^2\left(\delta^4 - \tfrac{3}{2}\alpha^2\delta^2 + \frac{\alpha^4}{8}\right) = \mathbf{-106000000},$$

and

$$2g_7 = 14\pi N_2 \alpha^2\left(\delta^6 - \tfrac{15}{4}\alpha^2\delta^4 + \tfrac{15}{8}\alpha^4\delta^2 - \tfrac{5}{64}\alpha^6\right) = \mathbf{+\,4830000000.}$$

If $\theta = 7^\circ\ 55'\ 29{\cdot}1''$ (see § VII. (i)) then $\phi = 82^\circ\ 4'\ 30{\cdot}9''$ and therefore $P'_5 = +1{\cdot}3902$ and $P'_7 = -1{\cdot}14$.

Hence substituting these values we get

$$\begin{aligned} -4G_1g_1 &= +1215122 \\ -4G_5g_5P'_5 &= +\quad 1215 \\ -4G_7g_7P'_7 &= -\quad 57 \\ \text{and the total} \quad &= +1216280 \end{aligned}$$

VI. The Bifilar Suspensions and the Determination of their Directive Force.

(*a*) *The Suspensions and the Theory of the Method.*

The bifilar suspensions were made of hard drawn copper wire, diameter 0·0450 cm., length 80 cm., and were rigidly clamped 3·1 cm. apart. They had been hanging for over ten years under the tension of the suspended coils arranged as in actual use, and it was thought that the wires would be exceptionally satisfactory in their behaviour. An account of the influence of elastic changes in the part of the controlling couple, due to torsion, is presented in the section on the deflection measurements. It should be noted here that the examination and treatment of this problem is of fundamental importance to the whole investigation. A full discussion of the matter is given by the writer in a paper on "Increased Accuracy in the Use of Bifilar Suspensions" in the 'Philosophical Magazine' for September, 1911.

The suspended system was placed with its centre coincident with that of the fixed coils, and the wires were adjusted symmetrically for use according to the "method of tilting" described by Gray.* The position of the coils could be easily checked to within the desired limits of accuracy by simple optical methods. Each wire was finally clamped at both the top and the bottom.

If the controlling couple due to the bifilar suspensions is taken as B sin θ for a deflection θ then it can be shown that

$$B = \frac{Wa^2}{4(l-x-z)} + \frac{2\mu d^4}{l}\frac{\theta}{\sin\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

for symmetrical suspensions, where W is the weight of suspended system, "a" is the distance between the bifilars, "l" is their length, "x" is the constant correction for the effect of rigidity at the ends, "z" is the distance through which the suspended system is raised for a deflection θ, "d" is the diameter of the wire, and μ is Coulomb's torsional coefficient.

Under the conditions of our investigation it was found that an error of less than 5 parts in 1,000,000 in B would be introduced by taking $z = 0$ and $\frac{\theta}{\sin\theta} = 1$. If we

* Gray, 'Absolute Measurements in Electricity and Magnetism,' vol. 1, p. 242.

take B as the directive force *at the instant of deflection* we may, therefore, assume that B is a constant quantity for values of θ from zero up to the deflections considered in this investigation. As μ for our suspensions decreased rapidly with time after deflecting the system it was always necessary to apply a correction if the deflection was observed an appreciable time after it was produced. The calculation of this correction under the condition of continued reversals of θ is treated fully in the place quoted above.

B can be most accurately calculated from determinations of moments of inertia and times of oscillation of the suspended coils. The following method was adopted. An accurately constructed movable tube 45 cm. in length was placed coaxial with the coils, and into this a pair of similar cylindrical weights could be fitted either at the centre or at the extremities of the tube. It was thus possible to vary the moment of inertia of the suspended system without altering the total mass. A sliding support was inserted in the dynamometer to hold the suspended coils when it was necessary to handle the system.

If T_1 is the time of a complete infinitesimal oscillation (two transits), and K_1 is the moment of inertia with the cylinders at the outside position, and T_2 and K_2 the period and moment of inertia when they are at the internal position, then we have

$$T_1 = 2\pi\sqrt{\frac{K_1}{B}} \qquad \text{and} \qquad T_2 = 2\pi\sqrt{\frac{K_2}{B}},$$

and hence

$$B = \frac{4\pi^2}{T_1^2 - T_2^2}(K_1 - K_2),$$

and it can easily be shown that

$$K_1 - K_2 = M\{(\tfrac{1}{2}d_1)^2 - (\tfrac{1}{2}d_2)^2\}$$

where d_1 and d_2 are the distances between the centres of mass of the two cylindrical weights and M is the total mass of the two cylinders.

Thus

$$B = \frac{4\pi^2 M\{(\tfrac{1}{2}d_1)^2 - (\tfrac{1}{2}d_2)^2\}}{T_1^2 - T_2^2} \quad \ldots\ldots\ldots\ldots \quad (9)$$

It was important that the tube and especially the cylinders should be very carefully constructed in order that the above formula should hold to the desired limits of accuracy. After a trial of several pairs two very satisfactory solid brass cylinders were made, and the series of tests outlined below will indicate that the necessary refinement was obtained.

(*b*) *The Cylindricity of the Weights.*

Forty diameters of each cylinder were measured with micrometer calipers. Each column in the table below represents a set taken at regular intervals from end to end with the cylinder in each of four equally rotated positions.

TABLE IX.

Diameter of Cylinder I. in Centimetres.

2·0180	2·0180	2·0182	2·0183
81	80	82	81
80	79	80	81
80	81	81	81
85	83	81	84
85	82	82	85
82	83	82	83
82	81	80	83
84	81	80	85
81	81	80	82
Means . . $2 \cdot 0182_0$	$2 \cdot 0181_1$	$2 \cdot 0181_0$	$2 \cdot 0183_0$

Mean diameter of Cylinder I. = $2 \cdot 0181_8$ cm.

Diameter of Cylinder II. in Centimetres.

2·0180	2·0180	2·0181	2·0180
80	80	80	79
80	80	80	80
79	79	77	75
75	81	83	83
79	80	80	80
79	81	81	81
80	81	82	81
82	84	84	82
80	81	81	80
Means . . $2 \cdot 0179_4$	$2 \cdot 0180_7$	$2 \cdot 0180_9$	$2 \cdot 0180_1$

Mean diameter of Cylinder II. = $2 \cdot 0180_3$ cm.

(c) *The Linear Measurements.*

Before proceeding with the records of results involving linear and time determinations it is perhaps necessary to describe briefly the methods used. Linear measurements were made on a steel Queen Company comparator fitted with high power microscopes. Experiment showed that with care it was possible to transfer the tube from the dynamometer to the comparator without jarring the cylinders apart to any perceptible amount. A Geneva brass metre with inlaid silver scale was used as the standard for comparison. This involved careful attention to temperature, but a calibration of this scale showed such fine marking and accurate sub-division that it was used in preference to two Konstat metres which has been certified at the National Physical Laboratory, Teddington, for the full length, but which were not very accurately sub-divided. A comparison of the brass standard with the Konstat

standard at the certified marks gave, at different temperatures, the following results which are reduced in each case to their value at 19°·0 C. :—

TABLE X.

	cm.
Absolute length of 100 cm. on standard brass scale at 19°·0 C. . . .	100·0297
,, ,, ,, ,, ,, ,, . . .	100·0293
,, ,, ,, ,, ,, ,, . . .	100·0297
,, ,, ,, ,, ,, ,, . . .	100·0290
,, ,, ,, ,, ,, ,, . . .	100·0286
,, ,, ,, ,, ,, ,, . . .	100·0294
,, ,, ,, ,, ,, ,, . . .	100·0293
Mean	100·0293

It is necessary to test the assumption that the edges of the cylinders as seen in the comparator microscope lay quite in the plane of their ends. Comparisons were made by placing them end to end and pressing together tightly, then (1) measuring the distance between marks made on each cylinder, at this position; (2) placing the cylinders at the extreme position and measuring the distance between the marks, and (3) measuring the distance between the inner edges at the external position. The difference of the first two readings should equal the latter measurement if the sighted edge is quite at the extremity of the cylinder. Measurements of this kind were made with the weights in various rotated positions in the tube and a very good agreement was found. The variations between the results of the two methods were only of the same order as those obtained for different determinations of the distance between the marks at the inside position when the cylinders were placed end to end. The variation in this case was caused mainly by the difficulty of keeping them quite together without maintaining pressure. As the method with marks involves two measurements it will be seen from the figures below that it is more satisfactory to take the observations by sighting on the edges of the cylinders. The best and worst agreements of the two methods which were obtained were :—

TABLE XI.

Distance by method with marks.	Distance by sighting on edges.
cm.	cm.
(1) 30·2878	30·2877
(2) 30·2889	30·2878

Great care was taken to maintain constancy of temperature during all the linear measurements and in every case the lengths are reduced to their value at 20°·0 C. in standard centimetres.

The following small variations were obtained for the distances between the edges at the external position, with the cylinders in various rotated positions. The third column is the difference of the two measurements and represents the lengths of the two cylinders :—

TABLE XII.

Position of cylinders.	Distance between inner edges.	Distance between outer edges.	Sum of lengths.
	cm.	cm.	cm.
Initial position	30·2879	44·7850	14·4971
Rotated 45 degrees	79	47	68
,, 90 ,,	81	53	72
,, 135 ,,	77	59	82
,, 180 ,,	65	44	79
,, 225 ,,	59	42	83
,, 270 ,,	66	43	77
,, 315 ,,	76	43	67
Initial position	76	48	72
Means . . .	30·2874	44·7848	14·4974

Each cylinder was reversed in direction, and rotated haphazard into different positions, giving for the distances between the inner edges in the different positions—

TABLE XIII.

cm.	cm.	cm.	cm.
30·2875	30·2868	30·2886	30·2872
30·2875	30·2866	30·2881	30·2869
Mean . . . 30·2873 cm.			

The cylinders now placed end to end at the interior position gave the following values for the distance between the outer edges :—

TABLE XIV.

Position of the cylinders.	Distance between the outer edges.	Cylinders reversed in direction.
	cm.	cm.
Initial position	14·4978	14·4986
Rotated 90 degrees	81	82
,, 180 ,,	80	73
,, 270 ,,	77	78
Means . . .	14·4979	14·4980

By maintaining strong pressure it was possible to reduce these values to about 14·4974 as obtained at the outer position. This difference is due to the small distance apart at which the cylinders remained under ordinary circumstances. These observations show that this distance is approximately 0·0005 cm. Direct measurement on the distance between the edges, when no pressure was exerted to keep the cylinders together, gave from 0·0003 to 0·0006 cm.

It was advisable to check the apparent possibility of the influence of these small variations by some observations on T_1 and T_2 with the cylinders in the different positions.

TABLE XV.

Position of the cylinders.	T_1.
Cylinders in initial external position	11·6566
Reversed in direction	11·6563
Rotated 90 degrees	11·6564
,, 180 ,,	11·6564
,, 270 ,,	11·6565
	T_2.
Cylinders at the initial internal position	6·5621
Rotated through 90 degrees	6·5620
,, ,, 180 ,,	6·5622
,, ,, 270 ,,	6·5619
Reversed in direction	6·5620

(*d*) *The Time Measurements.*

The observations for T were obtained by recording the transits on an electric chronograph controlled from the McGill Observatory. It was necessary to take the time of a large number of swings in order to get the required accuracy. When the period T is determined in this way it is necessary to apply a correction to the calculated value in order to reduce it to the value T_0, for infinitely small oscillations. In the experiment we really obtain the average period T′. Now if the amplitude "α" is very small, we have, by a well-known calculation,

$$T = T_0\left(1+\frac{\alpha^2}{16}\right),$$

and hence

$$T' = T_0\left(1+\frac{\alpha_1^2+\alpha_2^2+\ \dots\ +\alpha_{2N}^2}{32N}\right),$$

where α_r is the amplitude $\left(i.e.,\text{ the arc described in }\frac{T}{4}\right)$ for the r^{th} transit, and 2N is the number of transits considered.

Now the amplitudes for successive transits are in geometrical progression such that $\alpha_1/\alpha_2 = e^{(n-1)\lambda}$ where λ is the logarithmic decrement; hence we have

$$T_0 = T' \left\{1 - \frac{\alpha_1^2}{32N}\left(\frac{1-e^{-4N\lambda}}{1-e^{-2\lambda}}\right)\right\},$$

and therefore approximately

$$T_0 = T' \left\{1 - \frac{\alpha_1^2 - \alpha_{2N}^2}{64N\lambda}\right\}.$$

If a_n is the distance between extreme scale readings for the n^{th} period or $2n^{th}$ transit and D is the distance from the axis of rotation to the scale we have $\tan 2\alpha = a/2D$ or approximately, $\alpha = a/4D$ and therefore

$$T_0 = T' \left\{1 - \frac{a_1^2 - a_N^2}{1024N\lambda D^2}\right\}. \quad \quad (10)$$

If the correction due to the resistance of the air had not been neglected there would have been a further factor, $(1-b)$. It can be shown that $b < (k\lambda)^2$ where k is a constant less than unity, and since λ^2 is quite negligible in our case we need not consider b.

Since the value of the correction in (10) was usually less than 2 parts in 100,000, the various figures recorded below for T_1 and T_2 have already been corrected when necessary.

In the case when the cylindrical weights were in the exterior position it was found experimentally that $\lambda = 0{\cdot}00090$. It was convenient to start observations with an oscillation of about 5 cm. on the scale. If the observations extended over 1500 periods the correction for T would be less than 1 part in 1,000,000. Starting at 5 cm. it took over 3000 periods before the oscillations were too small to be observed. When the weights were at the internal position $\lambda = 0{\cdot}0016$, and if desired it was again possible conveniently to choose α_1 and N so that the correction for T would be negligible.

The elastic fatigue produced by the continuous torsional oscillations was not of a sufficient magnitude appreciably to affect these observations in any way.

The transits were observed through a telescope and recorded electrically on the chronograph with the aid of a tapping key. With a little practice the time between transits could be repeated to within a tenth of a second, but the period as deduced from a full set of oscillations could be repeated to within 1 part in 100,000.

(e) *The Masses of the Cylinders.*

The weighings were made on a Troemner balance sensitive to 0·0001 gr. with the given mass of the cylinders, and using a set of standardized weights with corrections obtained at the Bureau of Standards in Washington. The readings were taken by the usual method of oscillations, and repeated with the weights and cylinders

interchanged. The following mean values for the two cylinders were obtained and repeated without a variation of more than three in the last figure:—

Mass of cylinder I.	196·1797 gr.
,, ,, II.	196·1711 ,,
	392·3508 ,,

(*f*) *Further Remarks on the Accuracy of the Method.*

An examination of the possible effects of small asymmetries appears to show conclusively that no error as large as 1 part in 100,000 could occur in these measurements for the determination of B. Let us consider—

(1) The effect produced by a small deviation in the position of the tube and cylinders as a whole, which might occur initially or vary during readjustment. (It must be noted, however, that this is quite a different matter from a variation of the cylinder with respect to the tube. In the final observations the tube was never readjusted between determinations for *a* "T_1" and *a* "T_2" which were to be taken together.)

(2) The effect produced by the deviation of the centre of mass of the two cylinders, in either the internal or external position, from the desired position on the axis of rotation of the suspended system. This effect might be produced by the effect mentioned in (1) and also by a variation of the position of the cylinder with respect to the tube.

(3) The effect produced by the slight difference in weight.

Suppose (*a*) that the centre of the tube is at a small distance x from the axis of the rotation; (*b*) that the centre of mass of the cylinders, at the external position, is at a small distance y_1 from *the centre of the tube*, and, at the internal position, at a small distance y_2; (*c*) that m_1-m_2 is not zero, where m_1 is the weight of one cylinder and m_2 that of the other.

Let

$$x+y_1 = z_1 \qquad \text{and} \qquad x+y_2 = z_2,$$

then it can be shown that

$$K_1-K_2 = M\left\{\left(\frac{d_1}{2}\right)^2-\left(\frac{d_2}{2}\right)^2\right\}+M\left(z_1^2-z_2^2\right)+(m_1-m_2)\left(z_1d_1-z_2d_2\right),$$

provided that we take z as lying in the direction of the axis of the tube. The possible displacements in other directions would be far less. Now d_1 and d_2, involving y_1 and y_2, could be reset to within a few thousandths of a millimetre; the centre of mass of the two cylinders could be reset to within a few hundredths of a millimetre; and m_1-m_2 was 0·0086 gr. Substitution in the above formula shows that K_1-K_2

would be changed by less than 5 parts in 1,000,000 through neglecting the last two terms, if we assumed, for example, that x was as much as half a millimetre, y_1 a twentieth of a millimetre, and y_2 a tenth of a millimetre.

It is thus quite apparent, considering this discussion in conjunction with the experimental tests on the variations of T for resettings of the cylinders (both in rotated and reversed positions), that B can be determined satisfactorily in this way.

During the deflection experiments the suspensions carried a current of half an ampere. The following observations on the period illustrate the constancy under these conditions.

The figures below marked with an asterisk represent the mean of the two values obtained when the current flowed through the suspended system, first in one direction and then in the other.

TABLE XVI.

	Period.
	seconds.
I. No current flowing	11·6672
Current of half an ampere	*11·6669
No current flowing	11·6669
Current of half an ampere	*11·6669
II. (Slight change in weight of system.)	
No current flowing	11·6571
Current of half an ampere	*11·6574
No current flowing	11·6575
Current of half an ampere	*11·6573
III. May 5	11·6566
„ 28	11·6564
June 3	11·6564
„ 13	11·6564
IV. (Moment of inertia changed.)	
August 2	6·5659
November 14	6·5652

The temperature of the dynamometer was about 20° C. during all recorded observations for T, but the clamping connections at the supports were of such a nature that B was nearly compensated for temperature changes.

(g) *The Value of* B *during the Deflection Observations.*

Observations for T_1, T_2, d_1, and d_2, were taken immediately before and after the set of deflection readings for the current measurement. As the repetition of the various sets of deflections took several weeks, it was necessary to test the constancy during these observations. This was done by several determinations of T_1 at a temperature of 20° C.

TABLE XVII.

	seconds
Mean value of T_1 before taking deflection set on May 28	11·6564
" " during " " " June 3	11·6564
" " " " " " June 13	11·6564
" " after " " " June 28	11·6564
Mean value of T_1	$11{\cdot}6564_0$
Mean value of T_2 before taking deflection set	6·5620
" " after " " "	6·5621
Mean value of T_2	$6{\cdot}5620_5$

The slight variation in T_2 is probably due to a variation in the very small distance which the cylinders remain apart, at the inside position. This distance was approximately 0·0004 cm. at the time of the final observations on d_2. The cylinders after careful polishing had been placed in positions giving values of d_1 and d_2 most closely corresponding to the mean of those recorded in Tables XII., XIII., and XIV.

The distance between the inner edges at the external position at 20° C. = 30·2874 cm.

The distance between the outer edges at the external position = 44·7848 cm.

Hence

$$d_1 = \tfrac{1}{2}(30{\cdot}2874 + 44{\cdot}7848) = 37{\cdot}5361 \text{ cm.}$$

The distance between the inner edges at the internal position = 0·0004 cm.

The distance between the outer edges at the internal position = 14·4978 cm.

Hence

$$d_2 = \tfrac{1}{2}(14{\cdot}4978 + 0{\cdot}0004) = 7{\cdot}2491 \text{ cm.}$$

Substituting for d_1, d_2, T_1, T_2, and M in formula (9) we have

$$B = \frac{4\pi^2 \left\{\left(\frac{37{\cdot}5361}{2}\right)^2 - \left(\frac{7{\cdot}2491}{2}\right)^2\right\} 392{\cdot}351}{11{\cdot}6564^2 - 6{\cdot}56205^2}$$

$$= \mathbf{56593{\cdot}2.}$$

This is the value of B during the deflection observations, and it is important to note again that it represents the directive force of the suspensions only at the instant of the initial deflection after a long period of rest.

VII. The Deflection Observations.

(a) *Method of Observation.*

The duplex system which had been installed by Prof. CALLENDAR was used for the observation of the deflections.* The suspended coils were fitted with a plane parallel mirror 0·3584 cm. thick, 5 cm. in diameter, and silvered on both sides. Two metre scales, accurately divided on plane milk-glass, were suitably mounted on a rigid frame of copper pipe at distances of approximately a metre and a half east and west of the mirrors respectively. The parts of the scales where the readings occurred were illuminated by strong reflectors, and the rest was wrapped in asbestos to preserve an even temperature. A pair of two-foot telescopes with two-inch apertures were mounted on the same frame immediately below and at right angles to the centres of the scales. Both the mirrors and the telescopes had been specially constructed by BRASHEAR for this work. The deflections were taken simultaneously through each of the telescopes which were provided with filar micrometers. It was found, however, that after some practice, readings could be estimated through the telescope directly to within three hundredths of a millimetre. (As the time for taking a reading was limited, this plan of estimation was adopted and proved to be of quite sufficient accuracy for the mean results.) A constant circulation of water was maintained in the supporting pipes, and the distances between the scales could be measured with great accuracy. The dynamometer was protected by non-magnetic covers which were provided with mica windows.

In these observations the value of $\tan\theta$ was deduced from readings taken first in one direction and then in the other. It is not necessary to introduce the zero positions; for if we let S equal the mean of the values obtained for the distances between the readings for opposite deflections, and if β is the average distance between the scales and m the thickness of the mirror, then

$$\tan 2\theta = \frac{S}{\beta - m},$$

and $\tan\theta$ can at once be deduced. This duplex method of reading eliminates small errors in the setting of the telescopes and scales.

Separate series of observations were taken with the fixed coils both *reversed* and *interchanged* in position. This eliminated errors introduced by any asymmetry in the position of the copper tape within its channel.

(b) *The Correction for Torsional Variations in the Controlling Couple.*

Preliminary observations showed slight variations which depended on the time elapsing between the reversing of the deflections and the taking of a reading. There

* CALLENDAR, *loc. cit.*

was a gradual increase of the deflection which reached a limiting value according to an expression of the type

$$y = a+b \log (1+ct),$$

where y represents the distance from an arbitrary origin at the time t after producing a given deflection and a, b, and c are constants. Deflections were observed only when the temperature of the whole circuit was quite constant, and it was found that the variation was due to a decrease in the controlling couple while the system was maintained in a deflected position. As the measurement of this couple depends on observing small oscillations of the suspended system, it is necessary to obtain the instantaneous value for a deflection reading. An investigation was undertaken by the writer for the purpose of examining this effect in bifilar suspensions, and the results, which have special application to the present work, have been published in the article already quoted. It is shown there how the behaviour of these suspensions may be subjected to accurate calculation. The direction of the deflection was reversed every five minutes throughout all the series of final observations and the readings were made in each case three and a half minutes after producing a reversal. All auxiliary observations were performed between each reversal and it was planned that any desired series of readings could be performed continuously in this way. A manipulation of the reversing commutators made it possible to bring the suspended system to rest in a few seconds. The object of this method was to maintain the wires in a known elastic state which could be referred to the initial one. The *problem* thus arose to find the equation representing the variation of the deflection after its direction had been reversed every T minutes for a comparatively long period, and to express this with reference to the instantaneous position of the first deflection. The general solution to this was shown to be

$$\begin{aligned} x = b \log (1+\overline{ct+n\mathrm{T}})-2b \log (1+\overline{ct+(n-1)\mathrm{T}}) \\ +2b \log (1+\overline{ct+(n-2)\mathrm{T}})- \ldots +(-1)^n 2b \log (1+ct), \end{aligned}$$

where x is the distance from the initial instantaneous deflection, t is the time after the last reversal, n is the number of previous reversals, and b and c are constants. The application of this formula is not practicable, but when $\mathrm{T} < 15$ and n very large, it was found that a much simpler formula could be deduced which was tested experimentally and found to be perfectly satisfactory. The formula obtained was

$$x = \mathrm{D}-\mathrm{D}\frac{2}{1+e^{-k\mathrm{T}}} e^{-kt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where D was shown to be the distance of the asymtotic limit of the creep from the position of the instantaneous deflection and k a constant depending on the rigidity of the suspensions and the dimensions of the system. D depends on the magnitude of the deflection. The formula was tested for different values of T, and curves were

given showing a splendid agreement between the calculated and observed results for x. It was thus possible to calculate accurately the correction for readings, but it was not always practicable to wait until the wires had made a complete recovery from any long period of distortion. Any further superimposed effects were eliminated, therefore, by (1) reversing several times at intervals of five minutes; (2) taking readings at the usual interval after the last reversal; (3) reversing and reading again at the right intervals; and (4) reversing and reading for the first position again. The mean of the first and third readings were taken with the second, and the mean of these for both scales gave a mean value of S, the double deflection on the scales, which could now be corrected according to the formula (11).

(c) *The Electrical Connections.*

Fig. 6 shows diagrammatically the arrangement of the circuit for the deflection experiments.

AB represents two ohm standards in series (2·00368 international ohms, see § VIII.) and AD represents two-tenth ohms standards in series (0·199625 international ohms, see § IX.); S and F represent the suspended and the fixed coils respectively; r_1, r_2, r_3, r_4, r_5, are suitable non-inductive resistances and shunts capable of very delicate

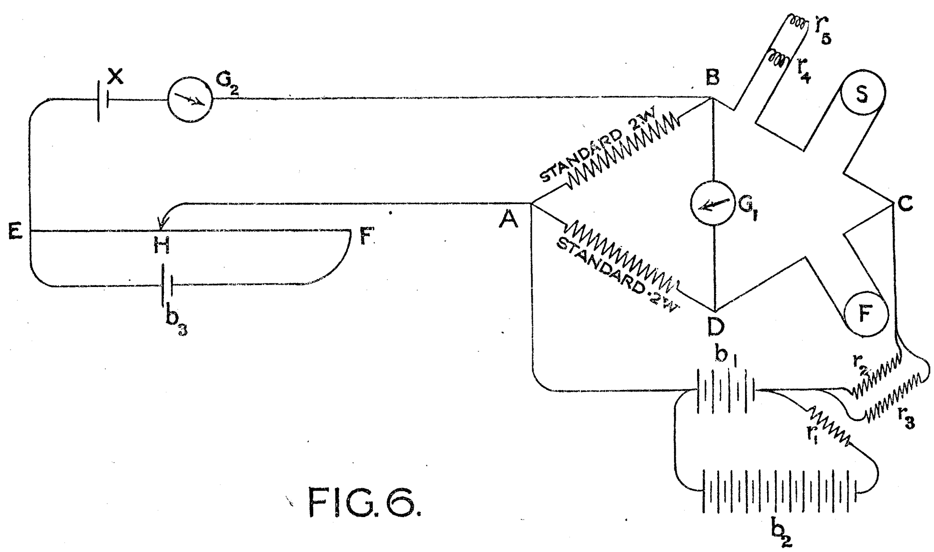


FIG. 6.

adjustment; G_1 is a sensitive low resistance galvanometer, and G_2 a sensitive high resistance galvanometer; EF is the potentiometer wire; b_1, b_2, b_3, are batteries; and X is the standard cell. Suitable commutators, not shown in the diagram, are arranged so that the current may be reversed (1) between A and C; (2) through the suspended coils; (3) through the fixed coils; (4) through G_1; (5) through G_2, (6) through EF.

The battery b_1 was composed of sixteen storage cells (each forty-ampere hour capacity) arranged in parallel four by four, to give an electromotive force of between

eight and nine volts. b_2 represents a large main battery of 100 volts. By proper adjustment of r_1, it was possible to supply a current of between five and six amperes for several hours with a constancy of better than 1 part in 100,000. It was arranged that b_2 supplied the larger part of the current, while b_1 served as a means of maintaining a constant voltage.

While taking deflection measurements r_4 and r_5 were kept adjusted so that there was only a small deflection in the galvanometer G_1. A change of 1 part in 100,000 was represented by seven scale divisions; r_2 and r_3 were kept adjusted so that the deflection in G_2 would be small, and this deflection could be eliminated and interpreted by the proper choice of the point H (see below).

It was arranged that there was a drop of a millionth of a volt per millimetre along EF; one scale division of G_2 corresponded to one millimetre of the wire.

(*d*) *The Potentiometer Correction.*

A complete record of the deflection measurements for this investigation is given in Table XVIII. The electrical connections are explained in detail above, and it will be seen that the current which passed through the suspended coils also passed through a standard resistance in an oil bath. It was arranged that the drop in potential across this differed from the electromotive force of a normal Weston cell at 25° C. by only a small number of microvolts. When opposing these two potential differences and measuring their difference on an ordinary potentiometer with a sensitive galvanometer in the circuit, it was possible to adjust the current to a constant value. It was not, however, necessary always to adjust the current in order to obtain a given reading, for if the difference on the potentiometer was read, the value of the current could be easily expressed in terms of that current which would produce a zero difference between the E.M.F. of the standard cell and the drop in potential across the resistance. Observations of the deflection were taken simultaneously with a potentiometer reading which never exceeded 700 microvolts and averaged about 250 microvolts. As this variation is so small and was taken to within five microvolts of the true balance, it was always possible to apply to the deflection a correction which could not be in error by as much as 0·001 cm. on the scale. (For example if the drop in potential across the standard resistance was 100 microvolts greater than the electromotive force of the Weston cell, then the current would have to be cut down by only 1 part in 10,180 to get the value which would produce the same drop in potential as the cell. As the circuits connecting the fixed and suspended coils of the dynamometer were in parallel and as the deflection varies with the product of the two currents which are kept in a fixed ratio we see that the deflection must be reduced by 2 parts in 10,180 or, in our case, where the readings lie between 42·3 and 42·4 cm. the correction would be $-0·0083$. We could thus determine this more closely than we could read the deflection and were able to save considerable time and much

trouble in regard to keeping the currents absolutely constant. It should be noted that the potentiometer corrections to the scale readings can be at once obtained from column four in the tables by dividing by 12·000.)

(*e*) *The Deflection Tables.*

In the first column an indication is given of the arrangement of the circuit. The different positions of the fixed coils are referred to as Positions I., II., III., and IV. where

In Position I. we have	Coil I.	on west	side	of dynamometer	with	marked face	out.
	,, II.	,, east	,,	,,	,,	,,	,,
,, II. ,,	,, I.	,, west	,,	,,	,,	,,	in.
	,, II.	,, east	,,	,,	,,	,,	,,
,, III. ,,	,, I.	,, ,,	,,	,,	,,	,,	,,
	,, II.	,, west	,,	,,	,,	,,	,,
,, IV. ,,	,, I.	,, east	,,	,,	,,	,,	out.
	,, II.	,, west	,,	,,	,,	,,	,,

A+ and A− corresponds to a reversal of the current through the whole circuit. (The A represents the point so marked in fig. 6.) Changes from (1) to (3) or from (2) to (4) correspond to a reversal of the current through the fixed coils only, while a change from (1) and (3) readings to (2) and (4) means a reversal of the current through the suspended coils only if the current has not been reversed at A also. Observations (1) and (4) are always at the zero end of the scales, and (2) and (3) are always at the "hundred" end. In the reading marked A+(1), A+(3), A−(2), A−(4), the earth's magnetic field acts to decrease S, while in A−(1), A−(3), A+(2), and A+(4) it acts to increase S. Wherever two consecutive readings (between groups) are not in opposite directions, it is due to the omission of a whole five-minute period in which it had not been possible to get all the auxiliary observations performed in time.

In the second and third columns, the uncorrected telescope readings are given, and the fourth contains the potentiometer readings, a + value meaning that the drop in potential across the standard resistance was greater than the E.M.F. of the cell by the amount given.

The fifth column contains the various temperatures t_1 of the standard two ohm coils at the time of the corresponding deflection; and t_2 represents the temperature of the two-tenths standard resistance in the circuit of the fixed coils.

The value of half the average distance between the centres of the fixed coils "*d*," is recorded in the next column. (See § III. (*b*) for details of measurement), and the distance between the scales, β in the eighth column (see § VII. (*h*)).

TABLE XVIII.—The Deflection Tables.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)	(9.)	(10.)	(11.)	(12.)	(13.)	(14.)	(15.)	(16.)
Arrangement of circuit.	East telescope reading.	West telescope reading.	Potentiometer reading.	t_1.	t_2.	d.	β.	Mean of east and west telescope readings reduced to values for zero potentiometer readings.	S.	Correction reducing S to value for $t_1 = 23^{\circ}\cdot 12$ C.	Correction reducing S to value for $t_2 = 28^{\circ}\cdot 0$ C.	Correction reducing S to value for $d = 12\cdot 4448$ cm.	Correction reducing S to value for $\beta = 298\cdot 450$ cm.	S reduced to standard conditions.	Mean S.
				° C.	° C.	cm.	cm.								
Position I. A + (1) (3) (1)	7·810 92·215 7·840	7·812 92·258 7·830	+110 + 60 − 10	23·62 23·67 23·72	27·8	12·4487	298·449	7·820 92·232 7·834	84·405	+0·012	+0·000	+0·016	+0·000	84·433	
Position I. A + (3) (1) (3)	92·215 7·860 92·195	92·250 7·855 92·238	− 40 −230 −370	23·77 23·77 23·77	27·8	12·4487	298·449	92·236 7·838 92·247	84·404	+0·014	+0·000	+0·016	+0·000	84·434	84·708
Position I. A + (2) (4) (2)	92·429 7·485 92·420	92·480 7·482 92·468	+110 +120 − 10	23·77 23·77 23·77	27·8	12·4487	298·449	92·445 7·494 92·445	84·951	+0·014	+0·000	+0·016	+0·000	84·981	
Position I. A − (1) (3) (1)	7·515 92·427 7·525	7·508 92·477 7·523	− 80 − 70 −180	23·82 23·82 23·82	27·8	12·4487	298·499	7·505 92·458 7·509	84·951	+0·015	+0·000	+0·016	+0·000	84·982	84·705
Position I. A − (2) (4) (2)	92·202 7·885 92·205	92·245 7·878 92·248	−250 −320 −410	23·82 23·87 23·87	27·8 28·3	12·4487	298·449	92·245 7·855 92·261	84·398	+0·016	+0·000	+0·016	+0·000	84·428	

The following values for columns 15 and 16 were obtained in the same manner as those in Table XVIII. :—

	(Column 15.) S.	(Column 16.) Mean S.
Position II.	84·844 84·327	84·586
	84·848 84·327	84·587
	84·329 84·844	84·586
Position III.	84·343 84·874	84·608
	84·863 84·340	84·602
	84·346 84·875	84·610
Position IV.	84·986 84·454	84·720
	84·456 84·997	84·726
	84·984 84·459	84·722
Position I.	84·963 84·441	84·702
	84·457 84·960	84·708
	84·981 84·452	84·716
	84·449 84·980	84·714
	84·970 84·436	84·703
Position II.	84·323 84·850	84·586
	84·869 84·320	84·594
	84·323 84·861	84·592

	(Column 15.) S.	(Column 16.) Mean S.
Position III.	84·866, 84·349	84·608
	84·356, 84·861	84·608
Position IV.	84·450, 84·988	84·720
	84·981, 84·448	84·714

The figures in the ninth column are obtained by applying the potentiometer correction to the mean of the two telescope readings; and the next column contains S which is obtained in the manner outlined above, by taking the difference of the middle reading from the mean of the first and last readings in each group.

To facilitate comparison and to illustrate the accuracy in repetition of readings, the next four columns contain corrections for reducing S to its corresponding value for the following standard conditions $t_1 = 23^\circ{\cdot}12$ C., $t_2 = 28^\circ{\cdot}0$ C., $d = 12{\cdot}4448$ cm., and $\beta = 298{\cdot}450$ cm., these numbers being chosen because they differed but slightly from the average values and agreed with those occurring in the various auxiliary calibrations or standardizations. The fifteenth column gives S reduced to the value corresponding to these standard conditions, and the last column contains the figures for the mean S, from which the effect of H, the earth's field, is eliminated.

The temperature coefficient of 0·00025 is taken for the standard two ohms in calculating the correction due to t_1, and 0·000018 is used for the two-tenths in correcting for t_2. It is sufficiently accurate to assume that S is changed by the same percentage as the resistance in each case, but in the opposite direction.

To obtain the correction due to d, it can easily be shown that for small differences a change of X per cent. in d means a change of 0·60 X per cent. in S in the opposite direction.

A change in β means a proportional change in S.

A summary of the value for the mean S for each of the different positions of the fixed coils is given below. The letters (a), (b), and (c), &c., represent a readjustment of the dynamometer and circuit. This involves a resetting of the fixed coils and general connections as well as the minor variations of condition recorded in the previous tables. As each position involves a resetting it will be seen that there are thus nine readjustments of the general conditions of the experiment, and it should be noted that the final mean involves 290 deflection readings. The controlling force of the suspensions and the position of the suspended coils were constant throughout the measurements.

TABLE XIX.

Position I.	Position II.	Position III.	Position IV.	Mean S.
(*a*) 84·708	(*b*) 84·586	(*c*) 84·608	(*d*) 84·720	**84·656**
·705	·587	·602	·726	**84·655**
(*b*) 84·702 ·708	·586	·610	·722	**84·656**
(*i*) 84·716	(*f*) 84·586	(*g*) 84·608	(*h*) 84·720	**84·657**
·714	·594	·608	·714	**84·657**
·703	·592	—	—	**84·656***
$84·708_0$	**$84·588_5$**	**$84·607_2$**	**$84·720_4$**	**$84·656_0$**

(*f*) *The Evaluation of Corrections.*

It is necessary to apply several further corrections to this value of S.

The following table contains data omitted above :—

TABLE XX.

The positions in the order given in Table XVIII.	Average temperature of the scales.	Average temperature of the dynamometer.
	° C.	° C.
Position I.	20·3	21·5
,, II..	19·5	20·9
,, III.	20·0	20·3
,, IV.	20·8	20·6
,, I.	20·7	20·7
,, I.	20·2	20·8
,, II..	20·5	20·0
,, III.	20·2	20·2
,, IV.	20·4	20·9
Means . . .	20·3	20·7

(1) *Temperature Correction.*—The average dynamometer temperature was 20°·7 C. If d were kept at 12·4448 cm. and the temperature reduced to 20°·0 C. the radius would be decreased 0·0013 per cent. and a single deflection would be increased, therefore, by 0·005 per cent. which would amount to a correction of +0·0008 cm. in S.

(2) *Calibration for the Scales.*—Each scale was compared with the standard scale described in § VI. (*c*) at the actual marks where deflection observations were taken.

* The mean values for III. and IV. are used in calculating this figure.

TABLE XXI.

Measurement between	7·5 and 92·0.	7·6 and 92·1.	7·7 and 92·2.	7·8 and 92·3.	7·9 and 92·4.	8·0 and 92·5.	Mean.	Temperature of scale.
								° C.
East scale . . .	84·5042	84·5036	84·5015	84·5034	84·5028	84·5039	84·5032	20·8
West scale. . .	84·5001	84·5002	84·5000	84·5007	84·5003	84·4999	84·5002	20·6
						Mean	84·5017	20·7

The average temperature of the scales during the deflection observations was 20°·3 C. (see Table XX.). Hence using the temperature coefficient 0·0000085 for milk-glass we see that the mean value for this distance at 20°·3 C. is 84·5014 cm. and a correction of +0·0014 cm. should therefore be added to our deflection mean in order to express it in true centimetres.

(3) *Correction for the Mica Windows.*—A very small displacement of the reflected scale reading is produced by the presence of the thin mica windows through which the suspended mirror is viewed. These windows are parallel to the scales and the telescope is at right angles to each, hence it can be shown that telescope readings for deflection measurements are each decreased by an amount x given by

$$x = z\,(\tan 2\theta - \tan\phi),$$

where z is the thickness of the mica, θ is the angle through which the suspended coils and mirror are deflected, and ϕ is the angle of refraction within the mica. In order to eliminate this deviation it is therefore necessary to add $2x$ to the mean value for S.

The mean thickness of the mica windows was 0·0061 cm.; 2θ was approximately 15° 50′ and ϕ can at once be deduced from $\sin\phi = \dfrac{\sin 2\theta}{\mu}$ where μ is the refractive index of mica and was taken equal to 1·58.

Substitution in the above equation gave $x = +0{\cdot}00066$ cm. and hence the desired correction $2x = +0{\cdot}0013$ cm.

(4) *Correction Due to the Change in the Controlling Couple.*—The last and most important correction is that required to reduce the deflection to its initial instantaneous value. It was pointed out above that this was equal to

$$\left(\mathrm{D} - \mathrm{D}\,\frac{2}{1+e^{-k\mathrm{T}}}\,e^{-kt}\right).$$

The experimental determination of D and k for these wires and for this magnitude of deflection is given in the paper on "Bifilar Suspensions," quoted before. We have

in this case $t = 3\frac{1}{2}$ minutes, $T = 5$ minutes, and it was found that $D = 0{\cdot}049$ cm. and $k = 0{\cdot}22$. Hence $x = 0{\cdot}0149$ cm. This is for a single deflection, and therefore for S we must subtract 0·0298 cm.

(*g*) *The Final Value of S.*

We have to apply the above corrections.

(1)	+0·0008	cm.
(2)	+0·0014	,,
(3)	+0·0013	,,
(4)	−0·0298	,,
Total	−0·0263	,,

The final value of S was therefore

84·629$_7$ cm.

(*h*) *The Determinations of* β, *the Distance between the Scales.*

This distance was measured by means of a rigid brass tube which was fitted with micrometer calipers at one end, and with an end stud at the other. The distance between the faces of the scales was determined directly by placing the end-stud against one scale, and the micrometer was so attached that it could be screwed against the face of the other scale. The temperature could be obtained from two thermometers which were inserted about 50 cm. from each end, respectively, and the tube was bound with thick asbestos cord to provide handling places.

The tube was calibrated on the standardized fifty-foot comparator at the Macdonald Engineering Building, McGill University. The method adopted was similar to that described in § III. (*b*), for the calibration of the smaller instrument used for determining the distance between the fixed coils. The following results were obtained for different calibrations. The figures given are reduced to the value of the length with the micrometer set at 0·300 cm. at a temperature of 20°·0 C.,

	117·503	inches,	
	117·505	,,	
	117·505	,,	
	117·504	,,	
Mean . . .	117·504	,,	*i.e.*, **298·460 cm.**

The scale distance was determined before and after each deflection series. It was found that the water circulation through the supporting pipes kept this quite constant.

Measurements were made at both the north and south ends of the scales *where the actual deflection readings occurred.* In order to insure delicacy of adjustment the tube could be supported from the ceiling in the correct position for measurement. A sample set is given to show the type of observations taken.

TABLE XXII.

Micrometer readings at north ends—	Micrometer readings at south ends—	Temperature of tube.
cm.	cm.	° C.
0·313	—	18·8
0·309	—	18·8
0·311	—	18·8
0·314	—	18·9
—	0·312	19·0
—	0·315	19·0
—	0·315	19·0
—	0·314	19·0
Means . . . 0·312	0·314	18·9

Mean micrometer reading = 0·313 cm.

At 20°·0 C. a reading of 0·300 corresponds to a total length of 298·460 cm., hence at 18°·9 C. this becomes 298·454 cm., and a value for a reading of 0·313 is, therefore, 298·441 cm.

Each determination of the mean distance between the scales which is recorded in the deflection tables was obtained in this manner.

It is necessary to subtract the correction m for the thickness of the mirror from 298·450, the standard value for β. A series of micrometer caliper measurements for m gave 0·3584 cm. We have therefore, $\beta - m = 298{\cdot}092$ cm.

(*i*) *Calculation of the Mean* tan θ.

Under the standard conditions enumerated, we have

$$\tan 2\theta = \frac{S}{\beta - m} = \frac{84{\cdot}6297}{298{\cdot}092} = 0{\cdot}2839046\,;$$

hence

$$2\theta = 15^\circ\ 50'\ 58{\cdot}14'',$$

and

$$\theta = 7^\circ\ 55'\ 29{\cdot}1'',$$

therefore

$$\mathbf{\tan\theta = 0{\cdot}139201_6}.$$

VIII. The Resistances.

The two ohms used as the standard in this investigation were made of platinum-silver wire wound on mica frames, annealed after winding, and immersed in an oil bath. Prof. H. T. Barnes, who constructed them in 1898, gives a detailed account of their special design on p. 173 of his paper "On the Capacity for Heat of Water between the Freezing and Boiling Points."* At that time their resistance in series was found to be 2·00218 true ohms at 20° C. with a temperature coefficient of +0·000248.

The present value of their resistance was obtained from a comparison with a manganin standard, No. 4086, which was sent to the Bureau of Standards for calibration. It was certified that its resistance could be expressed to within 0·005 per cent. between 15° C., and 35° C. by the formula

$$R_t = 1{\cdot}00032 + 0{\cdot}000018\,(t-20) - 0{\cdot}0000005\,(t-20)^2.$$

Each of the two platinum-silver ohms was compared with No. 4086 on a Carey-Foster bridge,† and the comparisons gave the following mean values (reduced to 20° C.) at different times during the investigation :—

The resistance of

(*a*)	Platinum-silver	Coil	No. I.	=	that of	No. 4086	+0·00138	ohms.
(*b*)	,,	,,	,,	=	,,	,,	+0·00140	,,
(*c*)	,,	,,	,,	=	,,	,,	+0·00137	,,
(*d*)	,,	,,	,,	=	,,	,,	+0·00138	,,
Mean	,,	,,	,,	=	,,	,,	+0·00138	,,

and the resistance of

(*a*)	Platinum-silver	Coil	No. II.	=	that of	No. 4086	+0·00011	ohms.
(*b*)	,,	,,	,,	=	,,	,,	+0·00007	,,
(*c*)	,,	,,	,,	=	,,	,,	+0·00014	,,
(*d*)	,,	,,	,,	=	,,	,,	+0·00013	,,
Mean	,,	,,	,,	=	,,	,,	+0·00011	,,

Hence

Coil I.	=	1·00170	international	ohms	at 20° C.
,, II.	=	1·00043	,,	,,	,,
Coil I. + Coil II.	=	2·00213	,,	,,	,,

* Barnes, 'Phil. Trans.,' A, vol. 199, p. 173, 1902.

† The same bridge is described and its accuracy indicated on p. 175 of the paper quoted above.

It will be seen that this differs from the earlier value of the resistance by only 5 parts in 200,000.

In this investigation the two ohms were used at 23°·12 C. At this temperature their resistance, obtained by using the temperature coefficient + 0·000248, is

2·00368 international ohms.

During the deflection observations a current of about half an ampere was flowing through these coils. If the oil was kept well stirred it was found that the temperature of the coils was given to a sufficient degree of accuracy by the temperature of the oil.

IX. The "Ratio of the Currents" in the Two Systems of Coils during the Deflection Experiments.

In § VII. (*c*) it will be seen that the ratio of the currents is determined by the ratio of the standard two ohms to a resistance composed of two standard manganin tenth ohms (Nalder, Nos. 4389 and 4388). This ratio was measured by means of the Kelvin-Varley slide (calibrated for refined measurements and used as described in the appendix). In each case the measurements were taken with a current of about half an ampere flowing through the coils to be compared; and a current of five amperes had been allowed to flow through the tenth ohms for some time preceding the measurement. In this way the actual conditions of the deflection experiment were closely approached, and any error due to the inaccuracy of the manganin temperature coefficient was eliminated.

Taking the temperature coefficient of 0·000248 for the two ohms and 0·000018 for the manganin resistances, and reducing the value of the ratio to that for the condition when the two ohms were at a temperature of 23°·12 C. and the tenths at a temperature of 28°·0 C. we got the following mean results for the ratio in two separate series of measurements:—

(1) 10·0357, (2) 10·0357.

Mean 10·0357.

This result is equivalent to a value of 0·199625 international ohms for the two-tenths at 20° C. Thermal or contact effects in the measurement were eliminated in the usual way, and each determination was checked with the Kelvin-Varley slide reversed.

The four windings of the suspended coil were arranged in parallel, and as errors due to any unsimilarity of the windings were eliminated by means of the series of observations involved in the deflection mean, and also by the method of determining the constant "g_1," we could assume that the effect was the same as that of a quarter of the current flowing through the four windings in series, and hence the actual ratio of the currents under the conditions of the deflection experiment was

40·1428.

X. The Standard Cells.

An account of the equipment of the laboratory with regard to standard cells has been given by Dr. H. L. Bronson and the present writer in a paper entitled : "Clark and Weston Standard Cells," which was published in 'The Electrician,' February 10, 1911.

During the deflection observations the electromotive force of the two cells P_1 and P_2 which were used in the determination, was repeatedly checked by comparison with a set of ten standards of especial constancy and reliability, viz., S 20, M 10, III., IV., A, B, N, O, Q_4, and Q_5, all of which are discussed in the paper quoted. S 20 and M 10 were constructed at the National Physical Laboratory, III. and IV. at the Bureau of Standards, and A, B, N, O, Q_4, and Q_5 at McGill.

Comparisons were made with the Bureau of Standards in 1908 and 1909, and with the National Physical Laboratory in 1909 and 1912. It has, therefore, been possible to express the mean of our cells in terms of the international mean for the Weston normal cell. The electromotive force of each of the cells P_1 and P_2 at the time of the deflection observations was 14 microvolts lower than the mean of the standards enumerated above, and the mean of these standards was found to be between 10 and 20 microvolts lower than the international mean. Calculations based on the exchanges with the National Physical Laboratory gave the larger figure, and those based on the exchanges with the Bureau of Standards gave the lower figure. We have assumed, therefore, that the two cells P_1 and P_2 were each 30 microvolts lower than the international mean value for the Weston normal cell.

XI. Insulation and Magnetic Tests.

Both systems of coils were wound with double winding so that it was always possible to perform a complete test on the insulation. In winter the dry Montreal atmosphere rendered insulation precautions unnecessary, but in summer it was found convenient to maintain the room at a higher temperature than the surrounding rooms and passages, in order to keep the connections and the instruments in the same dry condition as in winter. Tests were repeated throughout the investigation, and it was found that an insulation resistance of at least four times the necessary magnitude could always be maintained.

The framework of the dynamometer had been tested for magnetic qualities by Mr. R. O. King when the instrument was first set up, and found to be quite satisfactory. The influence of the covers, which were made for the sides and the ends of the dynamometer, was determined by comparing deflection observations taken *with* and then *without* the covers on the instrument. In the latter test air-currents were kept out by screens of paper at a distance of about two feet from the dynamometer. No difference in the deflections was observed. In the same way the presence of the

zinc reflectors (1·3 metres from the dynamometer) used for the illumination of the scales, was found to produce no magnetic effect on the deflection.

The "leading in" wires to the fixed coils were all wound non-inductively, and at the entrance to the coils the direction of the currents in the four sets of thicker wires, forming the small loops (visible in Plate 1) were such that their resultant magnetic effect at the centre was completely eliminated. A magnetometer test provided a simple proof of this matter.

The bifilar suspensions which constituted the leading in wires for the suspended coils formed, as will be seen from their dimensions given in § VI. (*a*), a long rectangular circuit of appreciable dimensions. It is, however, evident that this circuit extends only to the circumference of the suspended coils and is partly *outside* the fixed coils and partly *inside*. A calculation will show that the resultant effect from the suspensions is of a negligible order. The connections on the suspended coils made it possible to test this by reversing the direction of the current in the suspensions without changing its direction in the coils. No effect on our deflection could be detected.

XII. The Electromotive Force of the Mean Weston Normal Cell.

It has been explained that the electromotive force of the standard cell is given by the drop in potential across the two ohm standard when the current $4i_2$ passes through the resistance at a temperature of 23°·12 C. We now have all the factors. It has been shown that

$$B = 56593{\cdot}2, \qquad r = 40{\cdot}1428,$$

$$\tan\theta = 0{\cdot}139201_6, \qquad -4G_1g_1-4G_5g_5P'_5-4G_7g_7P'_7 = 1216280.$$

Hence by expression (4), [§ II. (*b*)], i_2 *in absolute amperes* is given by

$$i_2 = 10\sqrt{\frac{56593{\cdot}2\times 0{\cdot}139201_6}{40{\cdot}1428\times 1216280}} = 0{\cdot}127023_3 \text{ absolute amperes.}$$

Hence the drop in potential across the standard two ohms, 2·00368 international ohms, is given by

$$4\times 0{\cdot}127023_3\times 2{\cdot}00368 = 1{\cdot}01806 \text{ semi-absolute volts.}$$

This is equivalent to the electromotive force of our cell P_1 or P_2 at 25°·03 C. Using the temperature formula

$$E_t = E_{20}-0{\cdot}000040\,(t-20)-0{\cdot}0000009\,(t-20)^2,$$

we see that the electromotive force of our cell at 20° C. is 1·01828 semi-absolute volts. It has been pointed out in § X. that these cells are about 30 microvolts lower than the value of the international "Mean Weston Normal Cell," hence our final result for the electromotive force of the Mean Weston Normal Cell is

1·01831 semi-absolute volts at 20° C.

In conclusion the writer desires to express his gratitude to Mr. R. O. KING, M.Sc. for providing the fellowship which made this research possible.

The author is especially indebted to Prof. H. T. BARNES, F.R.S., Director of the Macdonald Physics Building, McGill University. The work was undertaken at his suggestion, and throughout the investigation he was a constant source of advice, personal help and kind encouragement.

Many thanks are due also to Dr. H. L. BRONSON and to Mr. F. H. DAY, M.Sc., for assistance in taking simultaneous observations.

APPENDIX.

NOTE ON THE CALIBRATION OF THE KELVIN-VARLEY SLIDE FOR ACCURATE ELECTRICAL MEASUREMENTS.

The Kelvin-Varley slide has been described in many electrical works,* and is well known as a valuable and accurate instrument. It has been found that an exceptional accuracy can be obtained if refined calibrations are occasionally performed. After making several trials of various methods, Prof. CALLENDAR, in connection with his research on "Continuous Electrical Calorimetry,"† briefly outlined the one which he considered to be the most convenient and accurate. This method with some slight modifications was adopted; and the table of corrections given on the next page is analogous to the tables on pp. 69 and 70 of the paper quoted. "n" represents the slide reading.

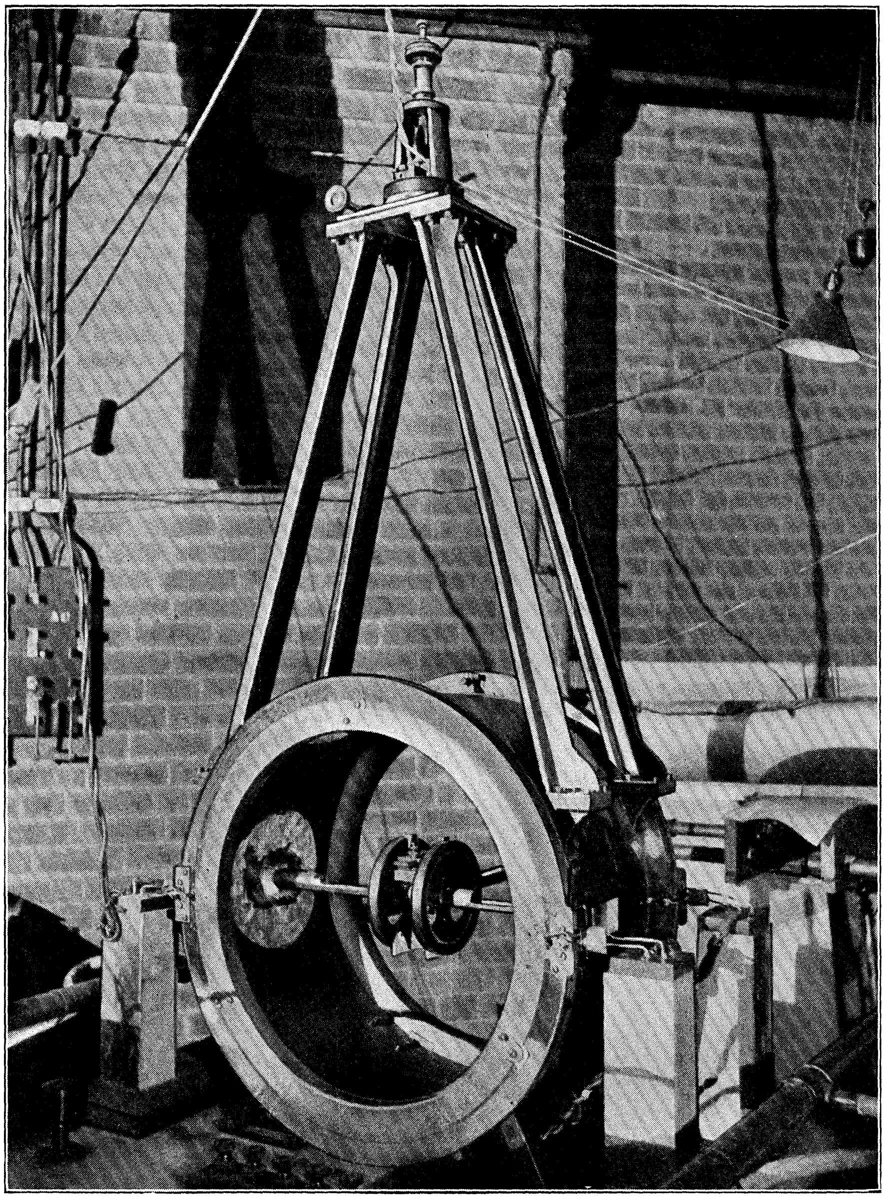
The validity of these corrections was tested both before and after the use of the instrument in the investigation. A comparison of the figures given in this table with those obtained by Prof. CALLENDAR (*loc. cit.*) for the same instrument in 1894 shows that a calibration of this kind must be repeated from time to time in order to maintain the same degree of accuracy in the use of the instrument. The slow alterations in the coils can attain considerable magnitude and ultimately render old calibrations quite useless. The changes are due, usually, to an average effect rather than to especial weakness in some particular coil or coils. It is thus possible to get a good indication of the condition of the whole box by taking and comparing readings for several ratios measured first from one end of the slide and then from the other. At the same time it is at once apparent in such a test, if the variation happens to be a purely local one. For measurements requiring an accuracy of more than 1 part in 10,000 we should not rely upon a calibration which is older than a year.

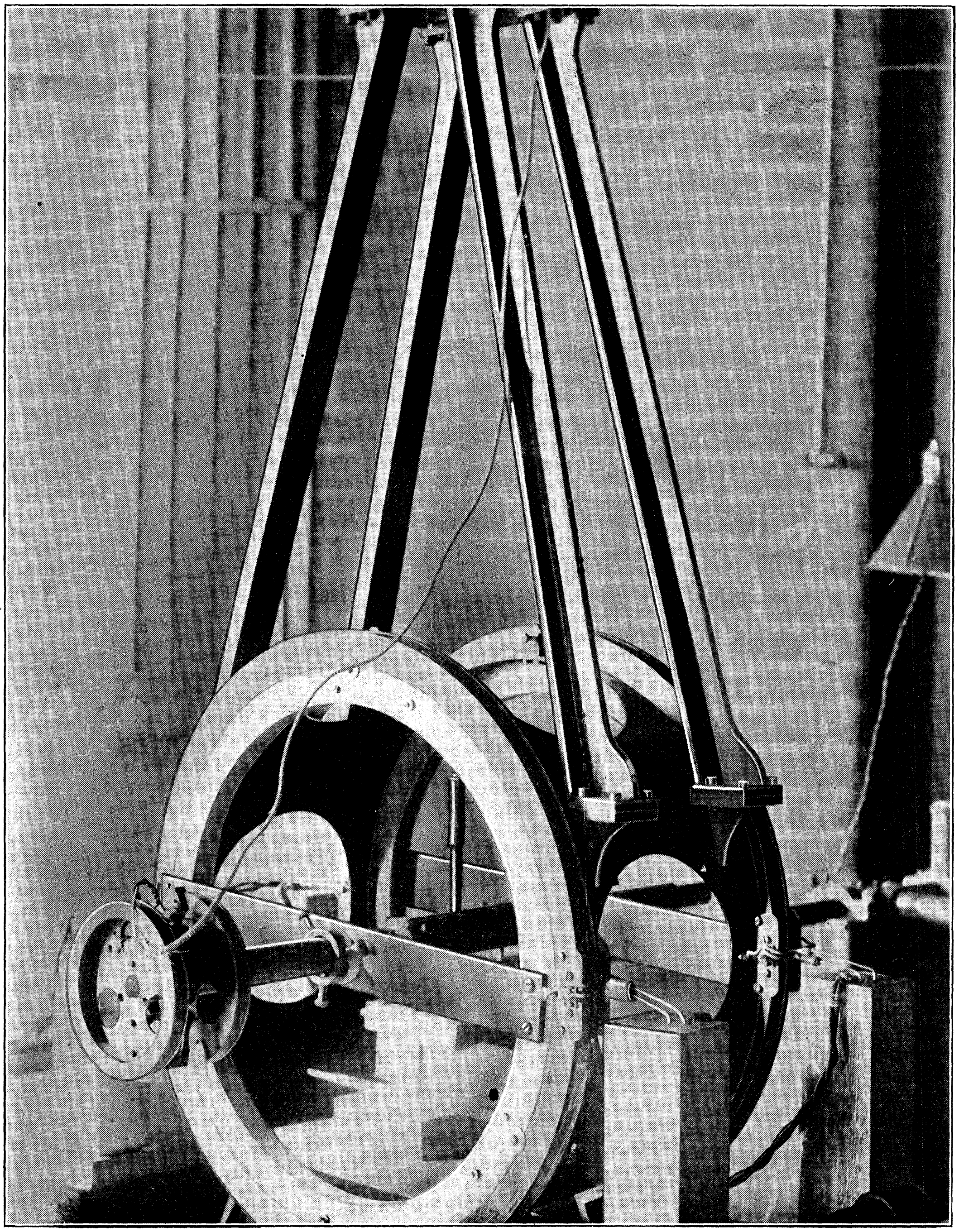
* PRICE, 'Measurement of Electrical Resistance,' p. 106; FLEMING, 'Handbook for the Electrical Laboratory and Testing Room,' vol. 1, p. 273; MUNRO and JAMIESON, 'Pocket Book of Electrical Rules and Tables,' p. 158; KEMPE, 'Handbook of Electrical Testing,' p. 219; ASPINALL PARR, 'Practical Electrical Testing,' p. 321, &c., &c.

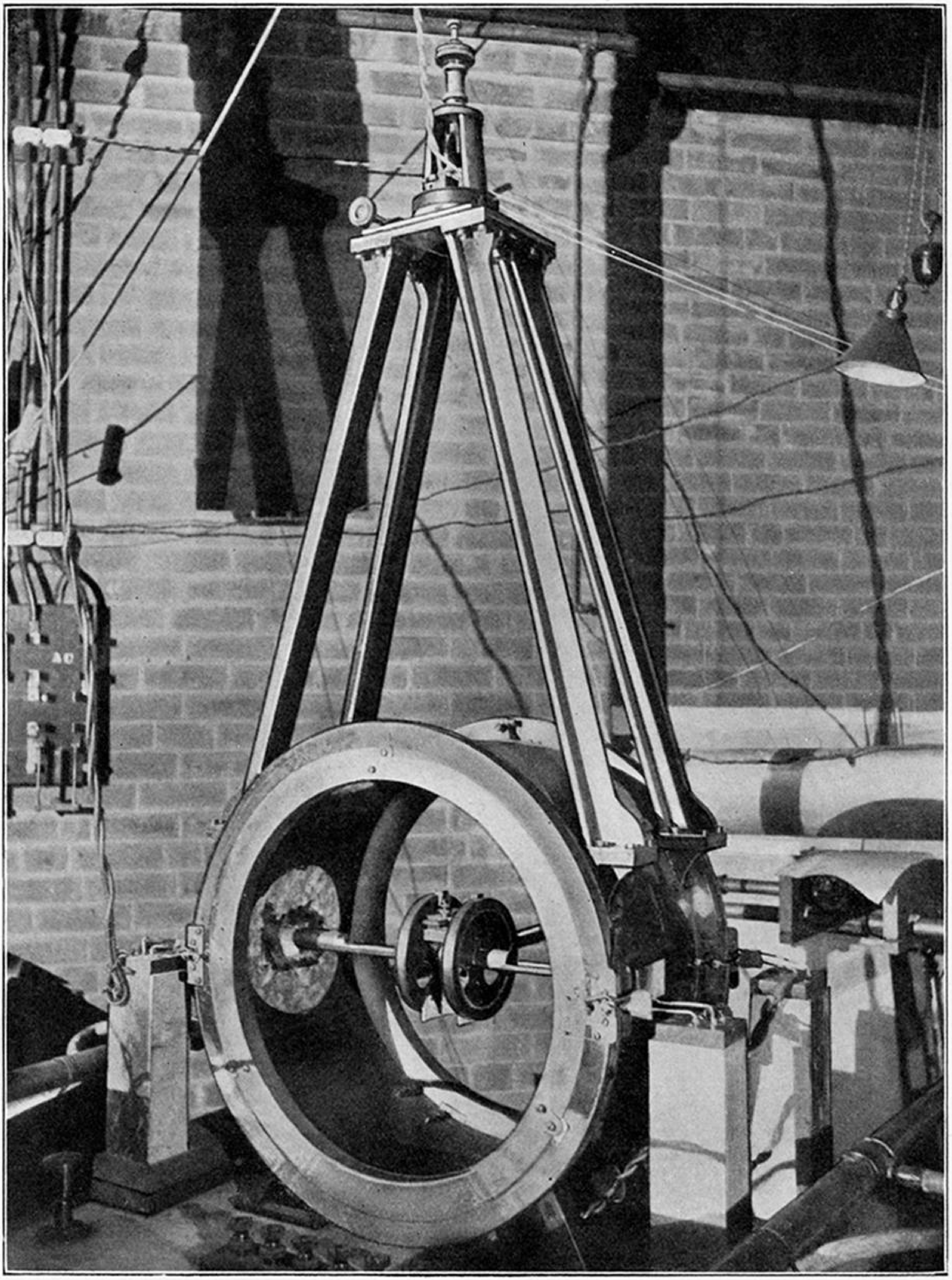
† 'Phil. Trans,' A, vol. 199, 1902, p. 65.

Corrections for the Kelvin-Varley Slide.

n.	Corrections.	*n*.	Corrections.	*n*.	Corrections.
0	0	34	−1·63	68	−2·57
1	+0·94	35	−2·12	69	−2·74
2	+0·74	36	−1·72	70	−2·14
3	+0·45	37	−2·33	71	−2·47
4	−0·07	38	−2·26	72	−2·16
5	+0·31	39	−2·66	73	−2·02
6	+0·43	40	−2·65	74	−1·78
7	+0·27	41	−2·42	75	−1·04
8	+1·19	42	−1·99	76	−0·39
9	+1·29	43	−2·15	77	−0·16
10	+1·16	44	−1·46	78	+0·46
11	+1·00	45	−2·16	79	+0·32
12	+1·20	46	−2·02	80	+0·75
13	+0·88	47	−2·64	81	+0·93
14	+2·33	48	−2·65	82	+1·00
15	+1·97	49	−2·98	83	+0·75
16	+1·87	50	−3·55	84	+0·54
17	+1·65	51	−4·02	85	−0·14
18	+1·44	52	−4·06	86	−0·23
19	+1·15	53	−4·69	87	−0·01
20	+1·19	54	−4·33	88	−0·18
21	+0·89	55	−4·74	89	+0·34
22	+1·33	56	−4·93	90	+0·75
23	+1·31	57	−4·84	91	+0·56
24	+1·43	58	−4·69	92	+0·32
25	+0·74	59	−4·70	93	+0·78
26	+0·51	60	−3·81	94	+1·02
27	−0·04	61	−4·15	95	+0·70
28	+0·07	62	−3·28	96	+0·54
29	−0·27	63	−3·34	97	+0·40
30	−0·11	64	−2·85	98	+0·14
31	−1·27	65	−2·70	99	− ·02
32	−1·14	66	−2·83		
33	−1·59	67	−2·92		



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