VII. *On Phenomena Relating to the Spectra of Hydrogen and Helium.*

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[Plates 2 and 3.]

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Contents.

(I.) *Introductory.*

In recent years the spectra of Hydrogen and Helium have been perhaps more closely studied than those of any other element, owing partly to the importance of these substances in celestial bodies, and partly to the supposed simplicity of their atoms

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which should render their spectra especially suited to theoretical investigation. Researches have been mainly directed to the study of the series relations in the spectra, and production of "enhanced" or "spark" lines under conditions of powerful excitation. The present investigation is devoted to a study of the spectra of Hydrogen and Helium, with special reference to the relative intensities of the lines under different conditions, a quantitative knowledge of which must necessarily be of importance in any discussion of the relation of spectra to the constitution of the atom.

It has been the custom to record the intensities of spectrum lines on an arbitrary scale, ranging generally from 10 for the strongest lines to 1 or 0 for the weakest. The intensities have usually been assigned purely at the discretion of the observer, and without much regard to the conditions of observation, whether visual or photographic. In any case, the apparent intensities are affected by the optical system by means of which the spectrum is produced, and whereas in visual observations the sensitiveness of the eye to different wave-lengths should be taken into account, matters are even more complicated in the case of photographs of spectra, where the sensibility of the photographic plate varies very considerably with the wave-length in a manner which is not precisely defined, and which appears in fact to be to a great extent irregular. These remarks apply to the visible and ultra-violet regions of the spectrum. Measurements in the infra-red are usually carried out with the thermopile or bolometer, and in this case quantitative measurements of the intensities of lines are obtained. It is, however, the visible and the less refrangible part of the ultra-violet spectrum in which accurate measurements of intensity, on a precise and quantitative basis, are perhaps most urgently needed at the present time. For example, in the case of celestial spectra, the problem of the relative intensities of lines has become of considerable importance, and is probably, at present, the greatest obstacle in the direction of the further elucidation of the phenomena which occur in new stars. In these spectra, observations are of necessity limited to the visible and the less refrangible ultra-violet regions.

The great advances which have been made already in the study of the variations in spectra under different conditions of excitation must owe their success largely to the magnitude of the effects in question; for they have depended to a great extent on methods of observation which cannot take account of the more subtle and less conspicuous variations which would probably, if detected, provide the key to the elucidation of many fundamental problems of spectra, such as the distribution of intensity in spectral series and the relations of different series to one another. At the present time, when a radiating gas emits a series spectrum, we have no exact knowledge of the relative amounts of energy thrown into the individual members of the series, or even whether these amounts are definite functions of the "term number," or capable of variation according to circumstances, although a certain amount of evidence has hitherto favoured the latter alternative.

(II.) *Methods of Measurement.*

For a number of reasons, measurements of the intensities of spectrum lines in the visible region with the thermopile or bolometer appear to be impracticable. In the first place, the intensity of the source must be exceedingly great if galvanometer deflections are to be obtained of a magnitude suitable for accurate measurement, and it is doubtful whether any but the brightest lines could be dealt with in this manner.* Moreover, it is only possible by such methods to deal with one line at a time, and the difficulty of maintaining sources of light constant for the duration of a series of measurements would, in many cases, appear to be almost insuperable. It would be necessary to introduce corrections for the loss of light in the spectroscope, and the determination of the magnitudes of these corrections would necessarily be a matter of considerable difficulty. Measurements have been made of the relative intensities of spectrum lines with the spectrophotometer. With this instrument it is difficult to attain any high degree of accuracy, and the range of wave-length over which it can be used is small unless the intrinsic intensity of the radiations is very great. It is, moreover, necessary to encounter the difficulty referred to above of maintaining the light source constant during a series of measurements.

In a recent paper,† we have discussed the application of a method, involving the use of a neutral-tinted wedge, to the determination of the distribution of intensity in broadened spectrum lines. In this case, the problem is reduced to its simplest form owing to the fact that over the short range of wave-length covered in a single broadened line, we may consider that both the extinction-coefficient of the wedge and the sensibility of the photographic plate remain constant. Such a method cannot be applied directly to the present problem without important modifications and additions, for in the measurement of the relative intensities of lines of widely different wave-length neither of these essential conditions is even approximately fulfilled, and, in addition, the loss of light in the spectroscope must be considered.

The problem dealt with in the present communication is therefore of an essentially different nature, but it has been found possible to evade these difficulties without altering the character of the method employed.

The apparatus used in the investigation is identical with that which we have described previously‡ and consists of a spectrograph, in front of the slit of which is mounted a neutral-tinted glass wedge, cemented to a similar wedge of colourless glass so as to form a plane-parallel plate. The spectra are photographed through the neutral wedge, and the resulting photographs consist of lines which are dark along the edge corresponding to the thin end of the wedge, and which fade away towards

* *Cf.* JOLLY ('Phil. Mag.,' xxvi., p. 801, 1913). A bibliography of the subject is given by this author and by KONEN and JUNGJOHANN ('Astrophys. Journ.,' xxxii., p. 141, 1910).

† 'Phil. Trans.,' series A, vol. 216, p. 459.

‡ *Loc. cit.*

the region corresponding to the dense end. Thus the lengths of the lines on the plate correspond to their intensities, and it is necessary in the first place to obtain accurate measurements of the lengths of all the lines under consideration. This is accomplished most conveniently by preparing positives from the negatives and enlarging the positives on to bromide paper through a ruled "process" screen. This method provides an enlarged negative in which the lines are made up of minute dots, one hundredth of an inch apart, and the length of any line can be determined by pricking out the last visible dot, which is a perfectly definite and well-defined point, whereas in enlargements made without the "process" screen the determination of the "end of the line" is subject to a considerable amount of personal error. We believe that with the use of the "process" screen, personal error is almost entirely eliminated. We may now consider the method by which the lengths of the photographed lines may be used in the determination of an actual relative intensity scale. As a preliminary, a precise knowledge of the constants relating to the wedge must be obtained.

(III.) *Determination of the Photographic Intensities of Lines.*

The wedge employed was made of the so-called "neutral-glass" which shows no absorption of a selective character, but in which there is an increase of absorbing power with decreasing wave-length. If an incident intensity I_0 falls on such a wedge at any point, and if I is the intensity transmitted, the *density* at that point is defined as the value of $-\log_{10}(I/I_0)$. The theory of the wedge is briefly as follows, and indicates the necessary account to be taken of the enlargement of the photographs, and the fact that the density so defined is the most convenient form of specification for future calculations.

If ρ_λ is the "coefficient of extinction" of the glass for light of wave-length λ, it is such that light of intensity I_1, falling on a thickness y of glass, is reduced during transmission to I_2, where

$$I_2 = I_1 e^{-y\rho_\lambda}$$

this law being equivalent to an exponential one. If α is the angle of the wedge, and x the distance from the thin end, $y = x \tan \alpha$ and the complete length of wedge being l, the intensity ratio for the complete wedge is

$$I_2/I_1 = e^{-\rho_\lambda l \tan \alpha}$$

being the ratio of transmitted light at its ends, for the same incident intensity. The "density" is, therefore, $-\log_{10}(I_2/I_1)$ or $l \tan \alpha \,.\, \rho_\lambda$ and will be denoted by D_λ for this particular wave-length.

If l_λ is the visible length of a line before enlargement, the visible length after enlargement is h_λ or $l_\lambda H/l$, where H is the length of the wedge after enlargement. For the magnification is equally h_λ/l_λ and H/l.

Let I_c be the intensity at which a line is just photographically visible. Then if its photographic intensity in the original light was I_λ, and if it is reduced to I_c by the thickness corresponding to a length l_λ of wedge, we have the relation

$$I_c/I_\lambda = 10^{-\rho_\lambda l_\lambda \tan\alpha}$$

or

$$\log_{10}(I_\lambda/I_c) = \rho_\lambda l_\lambda \tan\alpha = \rho_\lambda l h_\lambda \tan\alpha/H = D_\lambda h_\lambda/H$$

by the preceding relations, where D_λ is the density for this wave-length. Thus

$$\frac{I_\lambda}{I_c} = \log_{10}^{-1}\left(\frac{D_\lambda h_\lambda}{H}\right)$$

gives the photographic intensity of the original line, where D_λ is the density of the wedge, H the height of the wedge on the enlarged photographs, and h_λ the height of the line on the same photograph. The actual degree of enlargement adopted is immaterial, and so also, in the estimation of relative photographic intensities of different lines, is a knowledge of I_c, the intensity just visible photographically. We may, therefore, define the photographic intensity of a wave-length λ as

$$\log_{10}^{-1}\left(\frac{D_\lambda h_\lambda}{H}\right)$$

where the notation signifies an anti-logarithm, or the photographic intensities may be arranged on any arbitrary scale in which the ratios of this quantity are preserved. The subsequent reduction of photographic to absolute intensities will be considered in a later section.

(IV.) *The Density of the Wedge as a Function of Wave-length.*

The density gradient of the wedge for different wave-lengths has been determined as follows:—As a primary standard of density we adopted two Nicol prisms, one of which could be rotated with respect to the other, the amount of rotation being read on a divided circle. These Nicol prisms were mounted in front of the slit of the spectrograph, the orientation of the fixed Nicol being such as to polarise the light in the same plane as the dispersing prism of the spectrograph. Light from a vacuum tube containing Helium passed through the two Nicol prisms and the neutral wedge, and an exposure was made for a definite time in the spectrograph, the movable Nicol being set in the same plane as the fixed Nicol. The gas in the vacuum tube was at a pressure estimated at about one millimetre of mercury, and was excited by means of an induction coil with a mercury break. For the comparatively short exposures which were involved, the light from this tube may be regarded as effectively constant. After the first exposure, a second exposure was made on an adjacent portion of the same plate and for the same time, the movable Nicol being now turned through the

appropriate angle to reduce the intensity of the light to one-tenth of its value in the previous exposure.

It is evident that the differences in the lengths of corresponding lines in the two exposures correspond to a reduction of intensity to one-tenth of its value, and are, therefore, the differences which would be produced by a step of density equal to 1·0 on the wedge for each of the various radiations measured. The calculation of the actual density of the wedge for each radiation then follows obvious lines. The following results were obtained for the wedge used in these experiments. Under λ are given the wave-lengths of the Helium lines which it was found convenient to use, and under D_λ the corresponding densities at the thick end of the wedge.

λ.	D_λ.	λ.	D_λ.
6678	4·1	4471	5·9
5876	4·5	3888	8·0
5015	5·0	—	—

These values were plotted on squared paper, and from the curve thus obtained (fig. 1) which is quite regular, the density of the wedge for any intermediate wave-length can be found. There is no very convenient interpolation formula over the

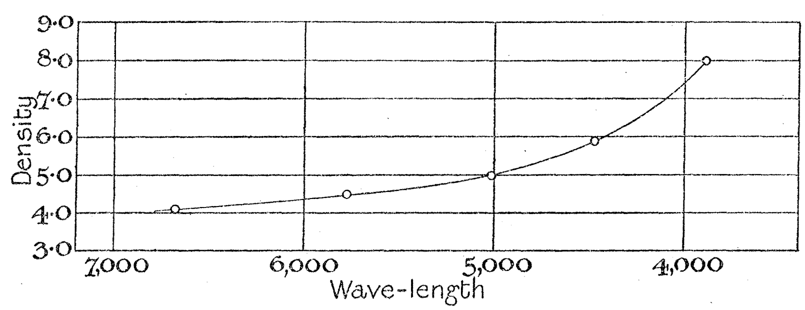


Fig. 1.

entire region, although the first four values of D_λ are nearly in the inverse ratio of the wave-lengths. But the graphical method is sufficiently accurate.

(V.) *Determination of the Intensities of Lines in Terms of the Intensity of the Radiation from the Positive Crater of the Carbon Arc.*

The reduction of photographic to absolute intensities implies the existence of a photographic record of some standard source of radiation extending over the whole region of wave-length examined. As a standard of radiation we have adopted the positive crater of the carbon arc burning in air at atmospheric pressure, and have

assumed that the distribution of energy in this source is that of a black body at the temperature of the boiling point of carbon. Assumptions involved in the use of this standard, and their general validity, will be discussed in a succeeding section.

The continuous radiation from the positive crater of the arc almost completely overpowers the band spectra which are superposed on it, except at certain regions in the spectrum where the intensities of the heads of bands cannot be neglected. But these regions, and points at which the lines due to impurities in the carbon are superposed, can easily be avoided in determining the photographic intensity curve of the arc. In Plate 2, A, is shown the photograph obtained of the spectrum of the positive crater of the carbon arc after the light had passed through the wedge. The Helium spectrum was superposed in order to provide lines of reference, and by measuring the height of the blackened area at a certain series of points, the photographic intensity curve of the positive crater has been determined. This curve refers, of course, only to the particular brand of plate employed, and must be re-determined for each batch of plates, or in the event of a particular batch being kept for a considerable time—for the sensibility curves of different batches may differ to some extent, and they are also subject to variation with time.

Plate 2, B, shows the Helium spectrum photographed through the wedge in the same manner, and, by measuring the heights of the lines, their relative intensities can be determined in terms of the intensity—for the same wave-length—of the carbon arc, by comparison with the photographic intensity curve of the positive crater. If, therefore, we can assume the distribution of absolute intensity in the continuous spectrum of the positive crater—corrected, of course, for the dispersion—we have the necessary data for determining the relative intensities of spectrum lines in absolute terms.

One of the main difficulties which at present somewhat restricts the accuracy which can be obtained by this method of comparison is our incomplete knowledge of the temperature of the crater. At the same time, very satisfactory results can be found from the existing data. The two temperatures usually quoted are T = 3700° C. absolute, found by WAIDNER and BURGESS in 1904, and T = 3620° C. absolute, found by HARKER in 1908, their mean being about 3650° C. absolute. These are black-body temperatures of the positive crater. In connection with this question we have referred to Dr. J. A. HARKER, F.R.S., of the National Physical Laboratory, who kindly informs us that the most probable temperature of the positive crater of the arc burning in air at atmospheric pressure should now be regarded as very close to 3750° C. absolute, or 100° C. higher than the mean. For purposes of comparison, we have, while accepting this estimate, computed according to both values in the following table.

PLANCK'S formula for complete radiation is not necessary for the present values of wave-length and temperature, and WIEN'S law may be used. A measure of the intensity for a given wave-length λ is therefore $\lambda^{-5}e^{-a\lambda/\tau}$ where $a = 1{\cdot}445$ cm.

degrees. Thus, for the wave-length 6678, or $6{\cdot}678 \times 10^{-5}$ cms., a/λ_T is equal to $(1{\cdot}445)10^5/(6{\cdot}678 \times 3750)$ and is of order about 6·0. With wave-lengths in centimetres, and

$$I = \lambda^{-5} e^{-a/\lambda_T}$$

the following results are obtained. The *ratios* exhibited are those for the given wave-length, with I for $\lambda = 3888$ taken arbitrarily as unity.

CARBON Arc with WIEN'S LAW.

λ.	I/I_{3888}. T = 3750.	I/I_{3888}. T = 3650.	Percentage differences.
7065	4·347	4·910	12·9
6678	4·200	4·703	12·0
5876	3·623	3·970	9·6
5047	2·639	2·808	6·4
5015	2·595	2·758	6·1
4922	2·464	2·609	5·8
4713	2·148	2·253	4·9
4472	1·810	1·8755	3·6
4438	1·761	1·821	3·4
4388	1·688	1·741	3·1
4144	1·339	1·362	1·7
4121	1·308	1·328	1·5
4026	1·178	1·189	0·9
3965	1·098	1·104	0·6
3889	1·000	1·000	0·0

These percentage differences correspond to 100° C. As there is no reason to believe that the possible error in the temperature is of this magnitude, we may in any case adopt the values for 3750° C. with sufficient accuracy for the present purpose. These are the absolute relative intensities in the carbon arc according to a certain scale.

Dispersion.—Uniformity of dispersion has been assumed in the calculation of these numbers. But the actual law of dispersion of the prism on the plates is approximately of the form

$$\lambda = \lambda_0 + \frac{B}{x + x_0},$$

where x is the distance measured along the plate from a certain standard position, λ, B and x_0 being constants. Thus

$$\delta\lambda = -B\,\delta x/(x + x_0)^2$$

and on the plate before enlargement an energy distribution of amount $f(\lambda)\,d\lambda$ between wave-lengths λ and $\lambda \times d\lambda$ is represented by the distribution of an amount

$f(\lambda)\,B\,dx/(x+x_0)^2$ within a distance dx centimetres, the apparent *intensity* at the point x being $Bf(\lambda)/(x+x_0)^2$. It is this law of intensity distribution on which the law of action of the wedge is actually superposed. The expression is equivalent to

$$f(\lambda)(\lambda-\lambda_0)^2/B$$

being the energy within a distance dx centimetres. After magnification m, this is spread over $m\,dx$ centimetres, but as the magnification is the same for all regions of the spectrum, the intensity *ratio* on the plate for two wave-lengths λ_1 and λ_2 is

$$f(\lambda_1)(\lambda_1-\lambda_0)^2/f(\lambda_2)(\lambda_2-\lambda_0)^2$$

where both m and B disappear. The true relative intensities of the carbon arc at λ_1 and λ_2 affecting the plate are, therefore, obtained by multiplying the entries in the preceding table for $\lambda=\lambda_1$ and $\lambda=\lambda_2$, $T=3750°$ C. absolute, respectively by $(\lambda_1-\lambda_0)^2$ and $(\lambda_2-\lambda_0)^2$.

The actual approximate law of dispersion for the plate used in the case of the carbon arc was

$$\lambda = 2257{\cdot}5+\frac{116802{\cdot}9}{x+x_0}$$

so that $\lambda_0 = 2257{\cdot}5$. But for λ close to λ_0, a better approximation is obtained by direct comparison of a prepared wave-length scale, which fits the lines, with an ordinary millimetre scale.

Again taking $\lambda 3888$ as the standard of reference, the following table has been constructed for the absolute relative intensities in the arc with which the individual lines in the Helium spectrum must be compared:—

TABLE of Absolute Relative Intensities (Energy Densities) of Carbon Arc on the Plate.

λ.	Ratio to $\lambda 3888$ (WIEN'S law).	Ratio corrected for dispersion.	λ.	Ratio to $\lambda 3888$ (WIEN'S law).	Ratio corrected for dispersion.
7065	4·347	28·98	4472	1·810	3·02
6678	4·200	28·00	4438	1·761	2·73
5876	3·623	16·10	4388	1·688	2·56
5047	2·639	7·04	4144	1·339	1·73
5015	2·595	6·92	4121	1·308	1·61
4922	2·464	6·08	4026	1·178	1·31
4713	2·148	4·475	3965	1·098	1·17

It is evident that no correction for dispersion is required in the case of individual lines in a series spectrum, the preceding investigation being only necessary in the case of continuous spectra. For the energy in the line spectrum is confined to narrow

regions of wave-length, and does not require expression as an energy density per unit length. The effect of unequal dispersion in any narrow region is negligible. If, however, the line were very broad, dispersion would become important within its extent, as in the case of H_α under the conditions of excitation employed in the work described in the preceding communication.*

When the wave-length scale is used, it is only necessary to multiply the theoretical ratio in the second column by $\frac{d\lambda}{dx}\Big/\left(\frac{d\lambda}{dx}\right)_{3888}$, which can be read directly.

(VI.) *The Energy in an Emission Line.*

In the case of emission lines, the intensities to which we have referred hitherto in this communication have been "central intensities," or the maxima of energy density in the lines. The central intensity determines the height of the line when it is photographed through the wedge, and "photographic intensity" has signified photographic central intensity. But every line, however narrow, has a definite breadth within which dispersion, for lines so narrow as those dealt with in the present experiments, may be regarded as uniform. If all lines were equally sharp, it is evident that this central intensity would be a satisfactory measure of the actual energy-content of the line, but the condition of equal and extreme sharpness cannot be assumed to occur, except under certain specified circumstances.

If I_c is the intensity just visible photographically, the height of the line, originally of intensity I_λ at its centre, on the plate after enlargement is, with the previous notation, h_λ, where

$$\frac{I_\lambda}{I_c} = \log_{10}^{-1}\left(\frac{D_\lambda h_\lambda}{H}\right) = 10^{\frac{D_\lambda h_\lambda}{H}}$$

and I_λ/I_c is the quantity defined already as the photographic intensity, representing actually the photographic central intensity. The critical intensity I_c affecting the plate is a function of λ, so that the use of the carbon arc is necessary as a standard of comparison, but in an individual line, I_c may be treated as constant, together with D_λ.

Taking account of both sides of the line, the whole energy in it is proportional to

$$2\int I_\lambda d\lambda = 2I_c\int 10^{\frac{D_\lambda h_\lambda}{H}} d\lambda$$

where the upper limit is effectively infinite, and the lower one is the wave-length of the central component. The law of energy distribution in the line determines the dependence of h_λ on the wave-length. There are two important cases :—

(1) The law of the simple exponential, found in the former paper* to be applicable in the case of the condensed discharge ; and

* *Loc. cit.*

(2) The probability law, valid for the ordinary discharge. If we take first the simple exponential, then before incidence on the wedge, if I is the intensity of the central component at $\lambda = \lambda_1$,

$$\mathrm{I}_\lambda/\mathrm{I} = e^{-k(\lambda-\lambda_1)},$$

and after incidence, the trace on the final plate is a straight line on either side of the central component of height h_{λ_1}. The height at wave-length λ is

$$h_\lambda = h_{\lambda_1}\left(1-\frac{\lambda-\lambda_1}{b}\right)$$

from the equation to the straight line, where B is the half breadth on the photograph. Substituting this value in the integral, the total energy becomes

$$2\mathrm{I}_c\,10^{(\mathrm{D}_\lambda h_{\lambda_1}/\mathrm{H})}\int_{\lambda_1}^{\infty} d\lambda\,.\,10^{-\mathrm{D}_\lambda h_{\lambda_1}(\lambda-\lambda_1)/bh} = \frac{2\,\mathrm{I}_c b\mathrm{H}}{\mathrm{D}_\lambda h_{\lambda_1}\log_e{}^{10}}\,.\,10^{(\mathrm{D}_\lambda h_{\lambda_1})/\mathrm{H}},$$

and is therefore proportional to

$$(\text{breadth})\ (\text{photo. intensity})/\log_{10}\ (\text{photo. intensity}),$$

where the breadth of the line on the base of the wedge photograph is implied, and the photographic intensity is that of the central component. It is not difficult to prove in a similar manner that when the law of distribution in a line of the original light is that of probability, the whole energy in a line is proportional to the quantity

$$(\text{breadth})\ (\text{photo. intensity})/\sqrt{\log\ (\text{photo. intensity})}$$

with the same definitions of breadth and intensity as determined from the wedge photograph. It is therefore necessary, in obtaining absolute measures of contained energy, for comparison along a given spectrum series, to consider the circumstances of production of the lines; but these two cases include those which have at present been explicitly recognised, and the effective one can at once be seen by inspection of the photographs.

The dispersion used in the present experiments has, in most cases, not been sufficient to overcome the effects of irradiation, which must be reduced to a relatively small magnitude in order to determine the true breadth of a line, which is small, at the base of the wedge photograph. The quantitative information contained in this portion of the paper is, therefore, limited to the maximum intensities of the lines, which provide a basis at present sufficient to elucidate the main phenomena, and from which the absolute ratio of the energy contents can be determined at any subsequent time by measurements of the relative breadths of the lines concerned, with a high dispersion, and without the necessity of repeating the present measurements. But it is important to notice that the use of the ruled "process" screen supplies a very

accurate method of determining the breadths, which is of much greater value than any method of direct measurement. For the measurement of such small distances cannot achieve a high degree of accuracy, even with the aid of a construction on the plate, giving a geometrical magnification.

According to the conditions of discharge, the bounding curves on the plate are either straight lines or parabolas, and in both cases the breadths at the base can be determined as follows :—The dots on the photographs produced with the screen are at the rate of 100 to the inch—in order of magnitude—in two perpendicular directions. They are so close that the *area* of any blackened region is very accurately proportional to the number of contained dots. Under a magnifying lens, these dots can be counted visually without difficulty, and the breadth of the line is proportional to the number of dots divided by the height. The preceding formulæ for the absolute contained energy could therefore be put into the form :—

$$(\text{Number of dots})\ (\text{photo. intensity})/(\text{height})\ (\log_{10}\text{ photo. intensity})^n$$

where $n = \frac{1}{2}$ for the ordinary discharges, and $n = 1$ for the condensed discharge. Even in the case of parabolas, the area is a definite numerical fraction of that of the containing rectangle.

The photographic intensity measures and the dot counts need not be done simultaneously on plates of the same magnification—the degree of magnification is not relevant to the formula—as is readily perceived, though, of course, the enlargements should be made from the same negatives.

The breadth of a line, of course, requires correction by the dispersion factor $(\lambda-\lambda_0)^2$, where $\lambda_0 = 2257{\cdot}5$ in these experiments, regarded as uniform across the line, and must be multiplied for each line by this factor. The effect of enlargement does not enter directly at this point into the question of integrating the whole energies in the lines, for it acts equally on all the lines. The energy is proportional to I_c which must be eliminated by comparison with the carbon arc.

We may, for purposes of clearness, summarise the results of this section and of the preceding sections in the following terms :—

In the quantitative analysis of a spectrum line by comparison with the carbon arc regarded as giving black-body radiation, there are two important magnitudes to be considered :—(1) the maximum energy-density at the centre of the line, I, such that the energy distribution in the line at its centre is $I_\lambda\, d\lambda$ for a small range $d\lambda$ of its width, and (2) the whole energy-content of the line, or the energy thrown into this wave-length when the gas is emitting its series spectrum. In the case of the carbon arc emitting a continuous spectrum, we only have energy-density to consider, and it can furnish a standard of comparison for both (1) and (2). The absolute relative intensities of the carbon arc for any wave-lengths can be calculated theoretically, and are exhibited in a previous table in the preceding section. Their photographic

effect as affected by the wedge can be thrown on the same plate with a series spectrum under the same conditions.

In order to determine from this plate the relative maxima of energy-density, or the relative central intensities of the lines in the series spectrum, as in (1), we apply the formula

$$\text{(photo. intensity of line) (absolute intensity of arc)/(photo. intensity of arc)}$$

for each wave-length. The dispersion factor enters into the absolute intensity of the arc, but into neither of the other quantities concerned. The relative central intensities are proportional to the results thus calculated.

But in order to determine the relative energy-contents of the lines, as in (2), we must use the formula, where $n = \frac{1}{2}$ or 1,

$$\frac{\text{(breadth of line) (photo. intensity of line) (absolute intensity of arc)}}{\text{(photo. intensity of arc) (log of photo. intensity of line)}^n}$$

which is the formula already deduced in this section, with the variable I_c removed by substitution of absolute intensity divided by photographic intensity in the arc, to which it is proportional. The dispersion ratio automatically cancels from the formula in this second case, as in fact it should in the present method whenever two magnitudes at the same wave-length, which involve a consideration of a range $d\lambda$ in the neighbourhood of this wave-length, are to be compared.

(VII.) *The Helium Spectrum under Different Conditions.*

One of the most interesting applications of the preceding theory does not involve the carbon arc, and consists of the examination of the relative behaviour of Helium lines under different conditions of excitation. In this case, we are not concerned with the intensity of one line in a spectrum relatively to the others, but with the changes of intensity of one line under different conditions relatively to those in other lines under the same conditions. Some interesting results in this connection have been obtained, and it is perhaps convenient to give some account of them before proceeding to the comparative intensities along the whole spectrum at any one time. The dispersion does not enter into the question in this case, when the lines are produced in the same manner on the same plate, and the ratio of the central intensities of any line from the two sources is the ratio of the photographic intensities which it shows, the photographic intensity being again $\log_{10}^{-1}\left(\frac{D_\lambda h_\lambda}{H}\right)$, where h_λ is the height of the line, D_λ the density of the wedge at the particular wave-length, and H the height of the wedge on the enlarged photograph.

In the following table are exhibited the results and intermediate steps in one important case—the ordinary Helium spectrum and the spectrum as given by the bulb. In each case, H was equal to 34·0 mm.

By "ordinary" Helium spectrum is meant the spectrum given by the capillary of a vacuum tube containing Helium at a pressure of about one millimetre, and excited by the discharge from an induction coil without capacity or a spark-gap. The bulb discharge was obtained by putting a small condenser in parallel with a very small spark-gap in the circuit. The intensity of the condensed discharge was not sufficiently great to give rise to any appreciable broadening of the lines. This observation was made by means of a Fabry and Perot interferometer. It must be pointed out that no attempt has been made to compare the intrinsic brightness of the lines under different conditions. The intensities given refer to an arbitrary unit, and the measurements refer to the relative intensities on the photograph. It is, therefore, the *ratios of the intensities under different conditions, for the various lines*, and not the intensities themselves from which conclusions may be drawn.

λ.	D_λ.	Helium (ordinary).			Helium (bulb).			Ratio.
		h_λ.	$h_\lambda D_\lambda/H$.	Photographic intensity.	λ.	$h_\lambda D_\lambda/H$.	Photographic intensity.	
7065	3·95	9·5	1·104	12·7	2·0	0·232	1·71	0·135
6678	4·1	17·2	2·074	118·6	12·5	1·507	32·1	0·271
5876	4·5	18·8	2·489	308	17·4	2·303	201	0·652
5047	5·0	4·0	0·589	3·88	—	0	—	—
5015	5·0	12·5	1·838	68·9	8·9	1·309	20·4	0·296
4922	5·15	9·2	1·394	24·8	8·9	1·348	22·3	0·900
4713	5·45	12·8	2·052	113	10·4	1·667	46·5	0·412
4472	5·9	14·8	2·568	370	17·8	3·089	1227	3·32
4438	6·0	3·0	0·530	3·39	6·4	1·130	13·5	3·98
4388	6·13	6·8	1·226	16·8	7·7	1·388	24·4	1·46
4144	6·9	2·2	0·447	2·80	3·4	0·690	4·90	1·75
4121	6·95	5·0	1·022	10·5	4·8	0·981	9·57	0·911
4026	7·4	6·0	1·306	20·2	7·9	1·720	52·5	2·60
3965	7·7	3·6	0·815	6·53	3·6	0·815	6·53	1·00
3888	8·0	8·7	2·047	111·4	6·0	1·412	25·8	0·232

The ratio in the last column is the intensity in the bulb divided by that with the ordinary discharge, and the series of values is very remarkable. Certain lines are, in the bulb, centrally enhanced to a great degree at the expense of others, and since experiments with the interferometer show that the widths of the lines are not sensibly different under these conditions, the results cannot be explained on the assumption that they are broader in one case and, therefore, for a given energy-content, less intense centrally. There is an actual preferential emission of energy which is essentially different in the two sources.

It is instructive to examine these ratios in the light of the known series relations between the lines. The Diffuse series of Helium contains the lines $\lambda\lambda$5876, 4471, 4026, whose "term numbers" are $m = 3, 4, 5$. In the bulb, the first is reduced to $\frac{2}{3}$ its former central intensity, the second is increased in the ratio $3\frac{1}{3}$, and the third

in the ratio $2\frac{1}{2}$. This appears to imply that in the bulb, a definite transfer of the energy of emission takes place towards the lines of higher term number—mainly in the present circumstances from the first to the second, but quite definitely also towards the third. If this be the correct interpretation, it has an obvious bearing on the anomalies met with in the spectra of nebulæ, where $\lambda\lambda$4472 and 4026, with λ4388 and, to a less extent, λ4143 are abnormally bright in many cases in comparison with such lines as D_3. Examination of the other lines only serves to indicate the generality of this interpretation. The Sharp series of Helium contains, as its leading members, $\lambda\lambda$7065, 4713, 4121, corresponding to $m = 3, 4, 5$. Instead of the actual enhancement found in the later members of the Diffuse series, these are all reduced in the bulb, so that there is relatively less radiation sent out in the form of the Sharp series. But the nature of the reduction is very significant. For λ7065 is reduced to about $\frac{1}{10}$, λ4713 only to $\frac{4}{10}$, and λ4121 only to $\frac{9}{10}$, so that the preferential transfer of the Sharp series energy is perhaps even more pronounced than that of the Diffuse series. The only line of the Principal series in the region we have examined is λ3888, so that no conclusions can be drawn regarding it.

The Diffuse series of Parhelium contains $\lambda\lambda$6678, 4388, 4144, corresponding to $m = 3, 4, 5, 6$ in the formula. This series exhibits the same phenomenon in a very pronounced manner. For in the bulb spectrum λ6678 is reduced to $\frac{1}{4}$, but λ4922 only to $\frac{9}{10}$, while the next member, λ4388, is actually enhanced to $1\frac{1}{2}$, and the next, λ4144, is even further enhanced to $1\frac{3}{4}$. These are the other two lines mentioned already in connection with their enhancement in nebulæ.

An even more striking fact in the same connection occurs with the Sharp series of Parhelium. Its first visible member appears in our ordinary Helium spectrum, but is quite invisible in the wedge photograph from the bulb. This is the line λ5047. But the next member, λ4437, is visible in both, and is enhanced in the ratio 4·0 in the bulb. The next member, λ4169, is visible in the ordinary spectrum of the bulb, though not in the wedge photograph, and is quite invisible with the ordinary discharge for the exposure given, even without the interposition of the wedge. This is a significant illustration of the effectiveness of the wedge method for determining variations incapable of detection by visual methods. The absence of this line in the ordinary spectrum, and its presence in the bulb under the same conditions, might have been noticed by visual or ordinary photographic methods. But without the present method of detecting comparatively small differences in order of intensity—as estimated by ordinary and very unsensitive methods—it would have been impossible to interpret this phenomenon as part of a general effect extending throughout the spectrum, and an effect which must be fundamental to any theory of the origin of spectra.

There remains the Principal series of Parhelium, whose members in the range examined are $\lambda\lambda$5015 and 3965. The same phenomenon is shown, for the former is reduced in the bulb to $\frac{3}{10}$, and the latter has the same intensity in both cases.

It is, therefore, a general fact that, without exception, the energy in the bulb spectrum has been transferred towards the members of higher term number in all the component series—provided, as later investigation shows, that the effect is not due to a mere change in the energy distribution within the lines.

The Helium Spectrum from the Capillary with a Condensed Discharge.—On Plate 3 is shown the wedge photograph of the Helium spectrum from the capillary produced by means of a powerful condensed discharge with a spark-gap in the circuit, and for comparison, the ordinary spectrum developed on the same plate. The quantitative analysis follows the same method as in the last section, and the results for one plate are exhibited in the table below. On this particular enlargement the full length of the wedge was 35 mm.

λ.	D_λ.	Helium ordinary).			Helium (condensed discharge).			Ratio cond./ord.
		h_λ.	$h_\lambda D_\lambda/H$.	Photo-graphic intensity.	h_λ.	$h_\lambda D_\lambda/H$.	Photo-graphic intensity.	
7065	3·95	3·7	0·418	2·62	—	—	—	—
6678	4·1	12·3	1·441	27·61	13·8	1·617	41·4	1·50
5876	4·5	18·2	2·340	219	18·2	2·340	219	1·00
5047	5·0	—	—	—	—	—	—	—
5015	5·0	11·5	1·643	44·0	10·0	1·428	26·8	0·596
4922	5·15	7·0	1·030	10·7	7·9	1·162	14·5	1·355
4713	5·45	10·2	1·588	38·7	10·7	1·666	46·3	1·20
4472	5·9	15·7	2·647	444	15·7	2·647	444	1·00
4438	6·0	—	—	—	1·8	0·309	2·04	—
4388	6·13	5·1	0·893	7·82	7·7	1·343	22·0	2·82
4144	6·9	—	—	—	1·5	0·296	1·98	—
4121	6·95	2·5	0·496	3·13	4·0	0·794	6·22	1·99
4026	7·4	6·0	1·269	18·6	6·0	1·269	18·6	1·00
3965	7·7	3·5	0·770	5·89	3·9	0·858	7·21	1·225
3888	8·0	9·1	2·080	120	8·5	1·943	87·7	0·729

The most significant feature of these results is the fact that the central intensities for the three lines of the Diffuse series of Helium are the same on both photographs. When their heights were found to be the same in both cases, they were re-examined, and no difference can be detected. The fact that their "absolute" heights were identical was of course accidental. At the same time the shapes of the lines are quite different (Plate 3), for $\lambda 4472$ is very much broader, and therefore contains, for the same height, very much more energy. The same applies to $\lambda 4026$, and to a less extent to $\lambda 5876$, but the increase in $\lambda 5876$ is not comparable with that in $\lambda 4472$ or even $\lambda 4026$. We are in the presence of the same phenomenon as before, but with the difference, that whereas in the bulb the transfer to higher members of the series takes the form of increased central intensity but not increase of breadth, with the condensed discharge the extra energy goes into the form of increased breadth.

The Sharp series of Helium, λλ7065, 4713, 4121 exhibits the transfer in a more normal manner. As regards central intensity alone, λ7065, while giving a definite trace on the wedge photograph in the ordinary spectrum, cannot be detected on the corresponding photograph with the condensed discharge. λ4713 is enhanced in the ratio 1·20, and λ4121 in the ratio 1·99, which is much greater. At the same time, each of these two lines is considerably broader, so that the enhancement of their energy-content is very great. The Principal line λ3888 is slightly broadened by the condensed discharge, and its central intensity is reduced to 0·7. Again no conclusion can be drawn without another line for comparison.

The Diffuse Parhelium series, λλ6678, 4922, 4388 is enhanced in the ratios 1·50, 1·355, 2·82. Both the Diffuse series are relatively enhanced with the condensed discharge. The lines become progressively broader towards the violet, so that the energy-enhancement of λ4388 is very great in comparison with that of λ6678, and that of λ4922 exceeds that of λ6678 when its breadth is taken into account.

In connection with the nebular spectrum, it is interesting to notice that λλ4472, 4388, 4026 are enhanced by the condensed discharge to an abnormal extent in comparison with any other lines, and their increase of breadth makes the effect more conspicuous.

The Sharp series of Parhelium is normal, for λ5047 is not visible through the wedge in either case, and λ4437 only with the condensed discharge. For the Principal series, λ5015 is reduced to $\frac{6}{10}$, whereas λ3965 is enhanced to $\frac{11}{9}$,—again a normal effect. The broadening with decreasing wave-length is, although definite, much less striking in these series, and its most pronounced effects occur in the two Diffuse series.

The transference of energy of emission to the higher members of the series is actually much greater with the condensed discharge than in the bulb, although it is not so strikingly apparent owing to the broadening of the Diffuse series, which are the most affected.

The fundamental importance of a phenomenon of this type is perhaps a sufficient justification for exhibiting the results derived from a third plate—in which all the three spectra are present together. The height of the wedge in the enlargement is 34 mm. for the condensed discharge and 33 for the others.

The first two columns of ratios are again in general agreement with the phenomenon already established, and more especially when the breadths of the lines on the plate are taken into account. There is, in fact, no case of exception. But other important facts are brought out in this table. In the first place, the *stage* of energy transfer in this experiment is not quite the same as before, and it therefore depends on the circumstances of the experiment—perhaps on the pressure of gas and so on, and further work is required in order to test the possibility of transferring the energy further down the series. For one very interesting question which is raised is the possibility of transferring all the energy into the final members, so that the series would ultimately behave like a band so far as it is visible. But if this be possible,

the experimental conditions necessary to produce the effect cannot be as yet predicted.

HELIUM under Three Conditions.

λ.	h_λ (ord.).	h_λ (bulb).	h_λ (cond.).	Photographic intensities.			Ratios of absolute central intensities.		
				Ord.	Bulb.	Cond.	Bulb/ord.	Cond./ord.	Cond./bulb.
7065	7·6	—	—	8·13	—	—	—	—	—
6678	14·4	10·2	10·9	61·5	18·5	20·6	0·301	0·335	1·11
5876	16·5	14·8	16·5	177·8	104·2	152·8	0·586	0·859	1·47
5047	2·9	—	—	2·75	—	—	—	—	—
5015	11·5	7·7	8·7	55·2	14·7	19·1	0·266	0·346	1·30
4922	7·9	9·0	7·8	17·1	25·4	15·2	1·485	0·889	0·598
4713	10·9	9·8	9·5	63·1	41·5	33·3	0·659	0·528	0·802
4472	14·8	16·0	15·5	442·6	726·2	489·8	1·64	1·107	0·675
4388	5·5	7·2	7·0	10·5	21·75	18·3	2·07	1·74	0·840
4144	1·0	2·2	1·5	1·62	2·88	2·01	1·78	1·24	0·687
4121	3·2	3·0	4·0	4·72	4·29	6·58	0·909	1·39	1·53
4026	5·8	6·1	6·5	20·0	23·3	26·0	1·165	1·30	1·11
3965	2·9	3·1	3·4	4·75	5·28	5·89	1·11	1·23	1·12
3888	6·6	5·2	7·4	39·8	18·2	55·1	0·457	1·38	3·03

Another interesting feature of this table is the evidence that the Principal series both of Helium (λ3888) and of Parhelium (λλ5015 and 3965), are relatively enhanced with the condensed discharge as compared with the bulb.

(VIII.) *Transfer of Energy in the Hydrogen Spectrum.*

The effects already described in connection with Helium are not peculiar to this gas. Experiments have been conducted in the same manner with Hydrogen, and with a mixture of Neon and Hydrogen. The results obtained from one of the typical experiments with this mixture may be given here, in so far as the Hydrogen spectrum is in question.

The lines H_α, H_β, H_γ, H_δ, are concerned, and the magnification of the wedge photographs is larger, being in this case such that the enlarged length of the wedge is 42 mm.

HYDROGEN (with Neon).

λ.	D_λ.	h_λ (ord.).	h_λ (bulb).	$D_\lambda h_\lambda/H$ (ord.).	$D_\lambda h_\lambda/H$ (bulb).	Photographic intensity (ord.).	Photographic intensity (bulb).	Central intensity ratio (bulb/ord.).
H_α	4·15	15·6	14·7	1·541	1·453	34·8	28·4	0·816
H_β	5·2	13·8	14·1	1·709	1·746	51·1	55·7	1·090
H_γ	6·25	11·5	12·6	1·711	1·875	51·4	75·0	1·459
H_δ	7·0	5·3	6·8	0·883	1·133	7·64	13·6	1·780

Hydrogen evidently shows the effect in a more pronounced manner than Helium, in that the fourth line, H_δ, shows it more than the third.

Our results are also in accordance with those of Sir J. J. THOMSON* who found that the intensity ratio $\frac{H_\beta}{H_\alpha}$ was greater in the negative glow than in the positive column.

The enhancement of the later members of the Balmer series in the presence of Neon is in accordance with the observations of LIVEING and DEWAR† who found that it was possible to observe ultra-violet members of the series in a mixture of Neon and Helium containing Hydrogen, although these lines could not be found in vacuum tubes containing pure Hydrogen.

(IX.) *The Spectrum of Neon.*

Except for certain series suggested by ROSSI‡ involving only a few lines, the greater part of the Neon spectrum is of unknown structure, although some constant differences of frequency have been detected by WATSON.§

One of the advantages of the mode of experiment described in this communication is that it may assist in elucidating the structure of very complicated spectra, and it may, in fact, be combined with investigations of the Zeeman effect, which has been used hitherto, as by DUFOUR|| in connection with the secondary spectrum of Hydrogen, in the detection of different classes of lines in such spectra. A preliminary examination of a portion of the Neon spectrum leads to some very interesting results which, however, at present cannot be brought within a general scheme of interpretation, owing to our lack of knowledge of series relations with which they are without doubt connected. The table contains a comparison of the spectra of Neon in the ordinary discharge and from the bulb, the gas containing a little Hydrogen. The enlarged length of the wedge was 42 mm. Only the stronger lines in the red and yellow have been subjected to examination, and if series exist in the spectrum, their leading members in the red and yellow should decrease considerably in intensity in the bulb, but on the whole, less considerably towards the yellow—provided that Neon behaves like Hydrogen and Helium.

There is a preponderance of small ratios in the case of the very strong lines, and the larger ratios are in general for the smaller wave-lengths, while the two smallest wave-lengths examined show enhancement, which is not found elsewhere in the list. These facts give some support to the view that a complicated system of series may be present. If, on the other hand, the spectrum is more in the nature of a band, as in

* 'Roy. Soc. Proc.,' vol. 58, p. 244, 1895.

† 'Roy. Soc. Proc.,' vol. 67, p. 467, 1900.

‡ 'Phil. Mag.,' XXVI., p. 981, 1913.

§ 'Astrophys. Journ.,' XXXIII., p. 399, 1911.

|| 'Ann. de Chim. et Phys.' (8) 9, p. 361, 1906.

NEON.

λ.	h_λ (ord.).	h_λ (bulb).	Photographic intensity (ord.).	Photographic intensity (bulb).	Ratio (bulb/ord.).	Tabular intensity (WATSON and BALY).
7032·6	6·7	4·2	4·35	2·51	0·578	3, —
6929·8	9·2	5·8	7·52	3·57	0·474	6, —
6717·2	17·6	12·2	47·4	14·5	0·306	7, 1
6678·5	20·5	12·5	100·3	16·6	0·166	9, —
6599·2	17·6	13·5	52·2	20·8	0·398	9, 4
6533·1	15·0	11·2	31·6	12·9	0·417	6, 4
6506·7	27·3	17·0	537·0	50·1	0·093	9, 6
6402·4	27·3	17·9	537·0	61·7	0·115	10, 10
6383·1	19·4	15·0	87·1	31·6	0·363	9, 8
6334·6	20·6	16·7	114·9	46·8	0·407	9, —
6305·0	15·0	12·1	31·6	16·2	0·513	6, 8
6266·7	20·0	14·4	111·5	29·8	0·267	6, 10
6217·4	13·8	10·6	25·9	12·2	0·470	6, 8
6163·7	15·5	11·0	38·6	13·4	0·346	6, 10
6143·3	21·6	19·0	162·6	88·1	0·542	7, 10
6096·4	19·4	15·7	96·8	40·5	0·418	6, 10
6074·5	16·0	14·8	47·4	35·5	0·749	6, 10
6030·2	11·0	9·0	14·2	8·77	0·618	5, 10
5975·*	9·5	9·0	9·89	8·77	0·887	5, 7
5945·0	16·3	13·7	51·0	27·2	0·534	6, 10
5902·6	8·0	5·7	7·04	4·02	0·571	5, 4
5882·0	13·7	10·8	28·3	13·9	0·493	5, 8
5852·6	24·5	16·0	421·7	51·8	0·123	10, 20
5820·3	5·8	6·0	4·18	4·40	1·050	5, 4
5764·5	8·0	10·0	7·19	11·78	1·638	7, 8

* A doublet, λλ5974·8 and 5975·8.

the corresponding region of the secondary spectrum of Hydrogen, the table contains the necessary information as to the variations of energy in the individual lines of such spectra. It is in any case noteworthy that the relative behaviour of the lines is very varied—and not comparatively uniform, as a mere visual inspection might suggest.

(X.) *Photographic Intensities in the Carbon Arc.*

An inspection of Plate 1A, which contains a wedge-photograph of the Carbon arc, indicates the irregular behaviour of the photographic plate through the visible region. We have calculated already the absolute relative intensities in the Carbon arc when corrected for the dispersion adopted. Below are given the measurements and subsequent determination of the photographic intensities in the arc, as registered on a plate which contains the ordinary Helium spectrum examined in an individual case, through the wedge. On the arc is also superposed the ordinary Helium spectrum without the wedge, in order to identify positions of given wave-lengths, and to facilitate the measurements of the heights to which the arc photograph extends at

the various wave-lengths characteristic of Helium. The total length of the wedge on the magnified photograph is 32 mm., and the values of D_λ, the density of the wedge, are obtained from the previous graph already described.

CARBON Arc, H = 32 mm.

λ.	h_λ.	D_λ.	$D_\lambda h_\lambda/H$.	Photographic intensity.	Ratios to value for 3888.	Absolute intensity ratio from previous table.
7065	17·3	3·95	2·135	136·6	13·7	28·98
6678	23·8	4·1	3·049	1120	112·0	28·00
5876	21·8	4·5	3·066	1163	116·3	16·10
5047	16·7	5·0	2·603	401	40·1	·7·04
5015	17·0	5·0	2·656	453	45·3	6·92
4929	18·5	5·15	2·977	949	94·9	6·08
4714	19·2	5·45	3·270	1863	186·3	4·475
4472	16·2	5·9	2·987	970	97·0	3·02
4438	16·0	6·0	3·000	1000	100·0	2·73
4388	14·4	6·13	2·758	573	57·3	2·56
4143	10·1	6·9	2·178	150·6	15·06	1·73
4121	9·9	6·95	2·026	106·3	10·63	1·61
4026	7·4	7·4	1·711	51·4	5·14	1·31
3965	5·7	7·7	1·372	23·54	2·35	1·17
3888	4·0	8·0	1·000	10·00	1·00	1·00

It is evident from the last two columns that the photographic effect is a very complicated function of wave-length, and that the present method of using the same plate for the standard spectrum and that to be measured is the only one which can give quantitative values for intensity, with a real physical significance.

Measurements of the ordinary spectrum of Helium on an adjacent portion of the same plate give the results shown in the next table.

HELIUM (Ordinary Discharge), H = 32 mm.

λ.	h_λ.	$h_\lambda D_\lambda/H$.	Photographic intensity.	λ.	h_λ.	$h_\lambda D_\lambda/H$.	Photographic intensity.
7065	4·5	0·555	3·594	4472	13·8	2·544	350·2
6678	11·8	1·512	32·5	4388	5·5	1·054	11·3
5876	14·4	2·025	105·9	4121	3·0	0·651	4·48
5044	2·9	0·453	2·839	4026	5·4	1·249	17·7
5015	10·3	1·609	40·7	3965	3·0	0·719	5·23
4932	8·3	1·336	21·7	3888	8·0	2·00	100
4714	10·5	1·788	61·4				

The lines $\lambda\lambda$4438, 4143 are omitted, for they were too weak to admit of accurate measurement.

These tables contain all the necessary data for the calculations of the relative central maxima of intensity of the Helium lines, according to the formula,

(photo. intensity of line) (absolute intensity of arc)/(photo. intensity of arc),

to which they are proportional. This calculation is exhibited in the next table, and it is convenient to take some definite line as the standard. For this purpose $\lambda 3888$ has been selected. The table contains the real intensity maxima of all the important lines relatively to that of $\lambda 3888$, measured on an absolute scale, and produced by the ordinary discharge.

(XI.) *Relative Intensities of Helium Lines on an Absolute Scale.** *Ordinary Discharge.*

λ.	Photographic intensity of line (α).	Photographic intensity of arc (β).	Absolute intensity of arc (γ).	Absolute intensity of helium ($\alpha\gamma/\beta$).	Tabular intensity (RUNGE and PASCHEN).
7065	3·594	13·7	28·98	7·61	5
6678	32·5	112·0	28·00	8·12	6
5876	105·9	116·3	16·10	14·7	10
5048	2·839	40·1	7·04	0·498	2
5015	40·7	45·3	6·92	6·22	6
4922	21·7	94·9	6·08	1·39	4
4714	61·4	186·3	4·475	1·47	3
4472	350·2	97·0	3·02	10·90	6
4388	11·3	57·3	2·56	0·505	3
4121	4·48	10·63	1·61	0·678	3
4026	17·7	5·14	1·31	4·51	5
3965	5·23	2·35	1·17	2·60	4
3888	100·0	1·00	1·10	100·0	10

The most remarkable feature of the table is the strength of $\lambda 3888$, the first visible member of the Principal series of Helium, which is much stronger than any of the lines of the two Subordinate series. Moreover, of the Parhelium lines, $\lambda\lambda 6678$, 4922, 4388, (Diffuse), $\lambda 5048$ (Sharp) and $\lambda\lambda 5015$, 3965 (Principal), the Sharp line is very weak, and the third member of the Principal series is stronger than that of the Diffuse—the first member of the Principal series is in the infra-red. It is evident, therefore, that under ordinary conditions, the two Principal series of Helium actually deserve their title. This extension to Helium of a result with which spectroscopists are familiar in some

* In a previous section we have given a table of the photographic intensities. We have done so, at the risk perhaps of some repetition, because we believe that the values given for photographic intensities are worthy of some confidence, and in the event of more accurate data being at some future time available for the temperature of the carbon arc, the accuracy of our "absolute intensities" can at once be correspondingly enhanced by a simple recalculation.

other elements is of importance in that it suggests that, under ordinary conditions, the Principal series of an element always contains the most important part of the radiation. This point has hitherto been obscured on account of the vagaries of the photographic plate and of the eye. It is interesting to compare the absolute intensities of members of any one series. For example, in the Diffuse series of Parhelium, $\lambda\lambda$6678, 4922, 4388, the intensities are 8·12, 1·39, 0·505, or roughly in the ratio 16 : 6 : 1, which fall much more rapidly than the usual tabular intensities.

In the Diffuse series of Helium, $\lambda\lambda$5876, 4472, 4026, the ratios are approximately 3·3, 2·4, 1, and in the Sharp series $\lambda\lambda$7065, 4714, 4121, they are 11·2, 2·2, 1, so that the Diffuse series of Helium falls off less rapidly than the others.

The discussion given already of the phenomenon of transfer of energy down the spectrum towards the violet is perhaps sufficiently complete. It is, however, interesting to append a table of the relative absolute intensities of the Helium lines, under various conditions, with that of λ3888 taken arbitrarily as 100. The results are deduced at once by a combination of the last table with those in previous sections, where the relative phenomena exhibited by the *same* line were examined. The following are the final results of the calculation :—

ABSOLUTE Intensities of Helium Lines.

λ.	Intensity (ordinary discharge).	Intensity (bulb).	Intensity (condensed discharge).
7065	7·61	1·03	very small
6678	8·12	2·20	3·30
5876	14·7	9·58	14·7
5044	0·498	very small	very small
5015	6·22	1·84	3·71
4922	1·39	1·25	1·88
4714	1·47	0·606	1·76
4472	10·90	36·2	10·90
4388	0·505	0·737	1·42
4121	0·678	0·618	1·356
4026	4·51	11·73	4·51
3965	2·60	2·60	3·18
3888	100·0	23·2	72·9

The extraordinary enhancement of λ4472 in the bulb, and to a smaller extent, of λ4026, is shown in a striking way in this table, together with the weakening of λ5876. For other points of interest, it is perhaps sufficient to refer back to the discussion of energy-transfer already given, but this table also indicates that the transfer takes place to a great extent also at the expense of the Principal series.

It is perhaps unnecessary to give the detailed working out of corresponding

problems in connection with the Balmer series, and only the final results are therefore exhibited. They are as follows, in the case of the ordinary discharge :—

HYDROGEN, Ordinary Discharge.

λ.	Photographic intensity.	Photographic intensity of arc.	Absolute intensity of arc.	Absolute intensity of hydrogen lines.
H_α	22·08	141·7	26·51	4·13
H_β	25·06	131·8	5·68	1·08
H_γ	17·18	54·25	2·39	0·757
H_δ	2·34	8·00	1·55	0·453

The fall of absolute intensity down the series proceeds fairly regularly. The measurements of the photographs of the Hydrogen spectrum taken in the presence of Neon give an estimate of the extent to which a large admixture of Neon affects the intensities of the lines. The particulars are contained in the following table :—

HYDROGEN mixed with Neon.

λ.	Intensity in hydrogen (ordinary discharge).	Intensity in hydrogen and neon (ordinary discharge).	Intensity in hydrogen and neon (bulb).
H_α	4·13	6·51	5·31
H_β	1·08	2·20	2·40
H_γ	0·767	2·26	3·30
H_δ	0·453	1·48	2·63

The enhancement of the later members of the series by Neon is very extraordinary, and is even more striking in the bulb than in the capillary.

(XII.) *The Nature of the Balmer Series.*

The investigation of the effect of Neon on the Hydrogen lines when produced in its presence, for which some quantitative data are given in the last section, cannot be interpreted in a satisfactory manner without a precise knowledge of the nature of the Balmer series. For in the case of Helium, the series arrangement into Principal and Subordinate series is known precisely, and considerable differences have been found in the behaviour of the three types ; more especially the Principal series behaves in a quite different manner from the other two. Until the type to which the Balmer series belongs is known, moreover, no discussion of the relative behaviour of

Hydrogen and Helium under similar conditions is possible. We are thus led directly to a problem which has many other important implications, some of which are stated below.

In the earlier paper* we suggested that the neutral-tinted wedge, as an accessory to the spectroscope, virtually increased the resolving power, and that in combination with the interferometer it should be able to solve problems which are beyond the power of pure spectroscopic analysis in its ordinary forms. Some experiments have been performed in connection with what is perhaps the most urgent of these problems at the present time—the elucidation of the nature of the Balmer series of Hydrogen. It has been generally accepted for some time that the series is one of doublets—we are not, at the present juncture, considering some very recent theoretical work by SOMMERFELD—but the exact nature of the series has always been an open question. For if it corresponds to a subordinate series (Diffuse or Sharp) in other elements, the separations in wave-number of the two components should be identical along the series. If, on the other hand, it corresponds to a Principal series, the separations should decrease rapidly towards the violet, after a manner which can be calculated from the separation in H_α, the first member of the series. There is at present no precise knowledge of the nature of these separations. MICHELSON† found H_α to be a doublet of separation 0·14, the intensities of the components being in the ratio 10 : 7, and the stronger one being on the red side. On the other hand, in H_β he found a separation only half as great, and the lines were so broad that no great degree of accuracy could be attached to this value. Others who have investigated the separation of H_α have obtained very discordant values. Thus HOUSTOUN‡ gives 0·065, FABRY and BUISSON§ 0·132, and PASCHEN and BACK|| 0·20, so that there is no certain knowledge even as regards the separation of H_α. FABRY and BUISSON have also stated that H_β is double, with a separation in accordance with the theoretical value. This statement is somewhat vague, especially as no numerical estimate was given, and the theoretical separation depends very much on the view we take as to the character of the series. For example, with MICHELSON'S value for H_α, $\delta\lambda = 0{\cdot}14$, the separation in H_β as a Diffuse or Sharp member would be

$$\delta\lambda' = 0{\cdot}14\,(4861)^2/(6563)^2 = 0{\cdot}075 \text{ Å.U.},$$

but as a member of a Principal series would only be

$$\delta\lambda'' = 3^3\delta\lambda'/4^3 = 0{\cdot}031 \text{ Å.U.}$$

An experimental arrangement, therefore, of more sensitiveness than any used

* *Loc. cit.*

† 'Phil. Mag.,' (5), vol. 34, p. 282, 1892.

‡ 'Phil. Mag.,' (6), vol. 7, p. 456, 1904.

§ 'Journ. de Phys.,' June, 1912.

|| 'Ann. der Phys.,' 39, p. 897, 1912.

hitherto, which could determine precisely an upper limit sufficiently low for the possible separation of H_β, would be decisive between these two alternatives, if the MICHELSON value were correct for H_α, but if HOUSTOUN'S value were correct, such an experiment for H_β would be much more difficult to perform. It is necessary, therefore, as an essential preliminary to an attack on this problem, to repeat the measurement of the separation in H_α in order to determine the possible magnitudes in the case of H_β.

Other possible interpretations of the Balmer series have been proposed, and these consist mainly in regarding it as a set of two or more practically *coincident* series. This has been suggested, for example, by FOWLER,* but it appears to involve a more complicated structure than doublets for the individual lines of the series. At the same time, it does not at present violate any definite experimental knowledge. Exact superposition of series cannot, however, be expected even in the case of Hydrogen. Hypotheses of this type are virtually a combination of the two alternatives already suggested, and an experiment which could decide between these two alternatives could, of necessity, also give a verdict for or against the present one. For it would apparently be necessary in this case that H_α should be a triplet.

From the point of view of series relations, another alternative, which has apparently not been noticed hitherto, may be put forward at this juncture. The Balmer series may primarily be a series of single lines, and the other components may be combination lines nearly coincident with these. If the normal series is given by CURTIS'S formula

$$n = \mathrm{N}\left\{\frac{1}{(2+p)^2} - \frac{1}{(m+\mu)^2}\right\}$$

where $\mathrm{N} = 109679{\cdot}2$, p is negligible, and $\mu = 0{\cdot}000007$, then the Principal series may be that of LYMAN in the Schumann region, following the law

$$n = \mathrm{N}\left\{\frac{1}{(1+p)^2} - \frac{1}{(m+\mu)^2}\right\}.$$

Combinations of an Arc series with itself are now well established if it is a Principal series, and on the present view we should expect a combination series of the form

$$n = \mathrm{N}\left\{\frac{1}{(2+\mu)^2} - \frac{1}{(m+\mu)^2}\right\}$$

with a constant difference of wave number from that of CURTIS, of amount $\mathrm{N}\,(2+p)^{-2} - \mathrm{N}\,(2+\mu)^{-2} = 0{\cdot}192$ on calculation. The corresponding separations of H_α and H_β then become 0·081, 0·048 Å.U. respectively, the first being nearer to HOUSTOUN'S estimate. There should also be a series

$$n = \mathrm{N}\left\{\frac{1}{(2+\mu)^2} - \frac{1}{(m+p)^2}\right\}$$

* 'Bakerian Lecture,' 1914.

very nearly coincident with the Balmer series, and this leads to separations in wave number which are not constant. The values for H_α and H_β are respectively $\delta n_1 = 0\cdot248$, $\delta n_2 = 0\cdot218$, and the corresponding separations in wave-length become

$$\delta\lambda_1 = 0\cdot105, \qquad \delta\lambda_2 = 0\cdot054 \text{ Å.U.}$$

These are nearer to MICHELSON'S estimate. It is just possible, therefore, that the presence of these two series with different intensities under different conditions may be the cause of the discrepancies which have been found in the separation of H_α. We do not at present pursue the subject beyond this conclusion, but it may be stated that such a view could give an account of the spectrum of Hydrogen, as at present known, which is in accord with the series relations known among other elements. The possibility of this new interpretation, in addition to such interpretations as that given by FOWLER, only serves to emphasise the need for an examination of the structure of the Balmer lines, which alone can be decisive. Another fundamental question with an intimate relation to the preceding remarks is the value of the Rydberg constant N for Hydrogen. The careful reduction of this value by CURTIS* depends ultimately on an accurate determination of the optical "centre of gravity" of the leading Balmer lines, and his formula is not applicable to the individual components. If the separations of H_α and H_β were very different, this formula and its contained value of N would cease to represent the series for any individual components, or even to give a series lying uniformly between two component series, and the value of N would need revision. In view of the theoretical importance which the exact value of N for Hydrogen and for Helium has recently assumed, it is necessary that the true value should not remain a matter of doubt, and only a precise determination of the separations can decide the question. If for example, the separation in H_β were found to be of the same order as that of a Principal series, a very important correction would be necessary in the observed wave-length of the optical centre of H_β which might change N very appreciably. This, in fact, as is shown by the experiments to be described later, actually occurs.

(XIII.) *Theoretical Considerations on the Basis of the Quantum Theory.*

SOMMERFELD* has recently published a remarkable investigation on the fine structure of the lines of the Balmer series. This is based on BOHR'S theory of the origin of this series, and it represents the only manner in which this theory at present can be applied in order to account for the existence of structure. On the view that the Hydrogen atom consists of one electron in orbital motion—its path being circular or elliptical—about a positively charged nucleus, with steady states characterised by discrete values of the angular momentum, the quantum theory requires the emission

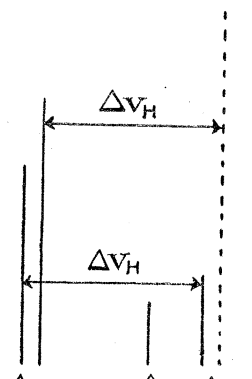
* *Loc. cit.*

† 'Sitz. der K. Bayr. Akad., München,' Jahrgang, 1916.

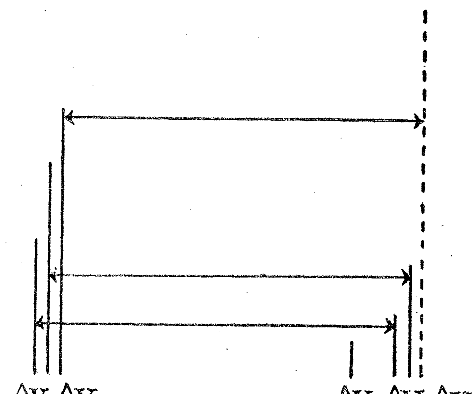
of a frequency ν where, if h is PLANCK's constant, $h\nu$ is the difference of energy between two states—when the electron passes from one state to the other. In the case of elliptic motion, SOMMERFELD finds it necessary for certain purposes to confine the eccentricities of the possible ellipses to definite discrete values, and thus obtains spectral lines which are not single, as in BOHR's theory in its ordinary form, but which have a definite structure. In fact, his theory involves the supposition that the Balmer series is effectively a supposition of Diffuse, Sharp and Principal series, as in some of the suggestions mentioned in the last section. These series are superposed in the case of Hydrogen, on account of the simplicity of the atom, and general considerations indicate—though not with great precision—that they would be widely separated in the case of other elements. This is not the appropriate place for any discussion or exposition of the theory, for we are concerned solely with the actual structure to which it leads.

The most important result is that the separation in H_β is of the order suitable for a Diffuse or Sharp series, in comparison with that of H_α. The latter is used—BUISSON and FABRY's value $\delta\nu_H = 0{\cdot}307$, corresponding to $\delta\lambda = 0{\cdot}132$ Å.U.—to determine a constant of the investigation, and the other separations are deduced in terms of this value. The results are as follows:—

In H_α the complete structure is that shown in fig. 2, where the separations are



Hα
Fig. 2.



Hβ
Fig. 3.

shown on a wave-number scale. The "main" separation is called $\delta\nu_H$, and the "satellite" separations are $\delta\nu_1$, $\delta\nu_2$, where

$$\delta\nu_1 = \tfrac{8}{81}\,\delta\nu_H, \qquad \delta\nu_2 = \tfrac{8}{27}\,\delta\nu_H = 3\,\delta\nu_1.$$

Since $\delta\nu_H = 0{\cdot}307$, we have

$$\delta\nu_1 = 0{\cdot}030, \qquad \delta\nu_2 = 0{\cdot}090,$$

and the corresponding wave-length differences are

$$\delta\lambda_1 = 0\cdot013 \text{ Å.U.}, \qquad \delta\lambda_2 = 0\cdot039 \text{ Å.U.}$$

The diffuse character of the components of H_α would render it practically impossible to detect so small a quantity as $\delta\lambda_1$ by any method, and since $\delta\lambda_2$ gives a weak component falling inside the main components, which themselves nearly overlap, the satellite corresponding to $\delta\lambda_2$ could not be expected either.

The structure for H_β is shown in fig. 3, and is more complex. The values of the separations indicated in the figure are :—

$$\delta\nu_1 = \tfrac{1}{48}\,\delta\nu_{\mathrm{H}}, \qquad \delta\nu_2 = \tfrac{1}{24}\,\delta\nu_{\mathrm{H}}, \qquad \delta\nu_3 = \tfrac{1}{8}\,\delta\nu_{\mathrm{H}},$$

and all the satellites are even more difficult to detect than those of H_α.

The most fundamental test of the theory, however, in a preliminary form, is to decide whether the main separation is again $\delta\nu_{\mathrm{H}}$. If it is smaller and of the magnitude required in a Principal series, the theory would apparently not hold. We confine ourselves for the present to a presentation of the results, and further discussion follows after the description and measurements of the plates.

(XIV.) *The Lummer Gehrcke Plate.*

The main theoretical features of this somewhat complex problem are now defined, and we may proceed to the experimental method. This consists merely in the use of a Lummer Gehrcke plate in addition to the former apparatus, the interference fringes being brought to a focus on the slit through the neutral wedge. The final pattern which is photographed—again with the use of the process screen—for any line such as $H\alpha$ consists of a set of fringes of the various orders. These consist of similar curves, with a general parabolic appearance, and approximately equidistant. From the shape of the contour of any one of these curves, we can deduce mathematically its analysis into components, from which by knowledge also of the density of the wedge the separations and relative intensities on an absolute scale of the components in the original light can be found,

The ordinary theory of the action of a Lummer Gehrcke plate on monochromatic light is, of course, familiar, but in its extension to a problem of this nature it becomes somewhat cumbrous. We have found it possible, however, to devise a more simple treatment of the theory of the plate which can be applied not only to the present case but apparently to all practical cases in which the use of the plate is necessary. We append, therefore, an account of this method, which is to some extent empirical, although it is not difficult to show that it is equally accurate. It is rendered the more necessary in that even the strict theory has apparently not been worked out completely for lines which are not monochromatic, but have a definite breadth as in the present experiments, where in fact the breadth is itself a matter of investigation.

The ordinary theory of the Lummer Gehrcke plate is, as stated above, well known so far as monochromatic radiation is concerned. If light falls on the plate, as in the figure (fig. 4), and if α and β are the rays respectively reflected once, and refracted twice and reflected once, their path difference is $D = 2\mu d \cos r$ where μ is the index of the plate for the particular wave-length, r is the angle of refraction, and d is the thickness of the plate. When the complete system of multiple reflections is considered,

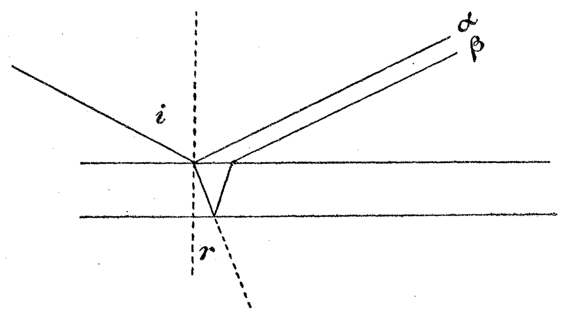


Fig. 4.

and the interfering rays are focussed on a screen S, the intensity corresponding to the angle of refraction r is

$$\frac{4\alpha^2\tau \sin^2(\pi D/\lambda)}{(1-\tau)^2+4\tau \sin^2(\pi D/\lambda)}$$

where λ is the wave-length, α^2 is the intensity of the incident light, and τ is the square of the internal reflection coefficient of amplitude, such that, for light polarized parallel to the plane of incidence, by FRESNEL's formula

$$\tau = \tan^2(\iota - r)/\tan^2(\iota + r),$$

but for light polarized perpendicular to this plane,

$$\tau = \sin^2(\iota - r)/\sin^2(\iota + r).$$

Maxima occur on the screen when $D = \frac{1}{2}n\lambda$ if n is an odd integer, and minima when n is even.

As regards the nature of these maxima and minima, a full discussion may be found, for example, in GEHRCKE's treatise.* It is not difficult to show that in order to obtain very sharp maxima nearly grazing incidence must be employed, so that τ is nearly unity, and this is adopted in the present experiments.

When we pass from this ordinary theory to the problem now in hand, difficulties begin to appear. In the first place, $D = 2\mu d \cos r$ is not actually a constant, although it may be nearly so for several successive maxima if the order is high. The successive separations of the maxima are therefore not identical, and there is also a variation in their intensities. This effect could be calculated, but owing to the

* 'Anwendung der Interferenzen,' Braunschweig, 1916.

difficulty of securing uniform illumination, such a calculation would be of no value. If measurements of the pattern to a high order of accuracy are required, it must in the first place be made "normal" in a manner subsequently explained. The variation in distance apart of the maxima is, in fact, in the present experiments, very small, as will appear from the measurements, and a high degree of accuracy could have been obtained by neglecting it.

The most important linear magnitude in the pattern, when it is obtained for two spectrum lines close together, whose separation is to be deduced, is, of course, the difference of wave-length corresponding to the distance between successive orders. Variations in D can be allowed for by making the pattern normal, and we may treat D as constant in the present theoretical investigation. Its value is

$$D = 2\mu\, d \cos r,$$

or, for grazing incidence, for which $\sin r = 1/\mu$,

$$D = 2\mu\, d \cos \sin^{-1}\left(\frac{1}{\mu}\right) = 2d\,(\mu^2-1)^{1/2}.$$

Between successive maxima, $\pi D/\lambda$ increases by π, and, therefore, if $\lambda'-\lambda$ is the wave-length separation corresponding to the distance between these maxima on the photograph,

$$\frac{\pi D}{\lambda} - \frac{\pi D}{\lambda'} = \pi, \qquad \text{or} \qquad \lambda'-\lambda = \lambda\lambda'/D.$$

To a first approximation, $\lambda'-\lambda = \lambda^2/D$. We may denote this quantity by ϵ, and

$$\epsilon = \lambda^2/2d\,(\mu^2-1)^{1/2}.$$

The accurate value is $\epsilon = \lambda^2/(D-\lambda)$, but its use would never be required. For a given plate, ϵ may be tabulated in a form suitable for interpolation. We have in this manner calibrated the plate used in these experiments. The plate was supplied by Messrs. Hilger, and its indices of refraction for the yellow sodium line and for the lines H_α, H_β, H_γ were known with great precision. With the old notation, C, D, F, G, for these lines, the values were :—

$$\mu_C = 1{\cdot}50746, \qquad \mu_D = 1{\cdot}50990, \qquad \mu_F = 1{\cdot}51560, \qquad \mu_G = 1{\cdot}52025,$$

and the thickness of the plate was $d = 4{\cdot}439$ mm.

The corresponding calculated values of ϵ are :—

$$\epsilon_C = 0{\cdot}43010, \qquad \epsilon_D = 0{\cdot}34615, \qquad \epsilon_F = 0{\cdot}23376, \qquad \epsilon_G = 0{\cdot}18533,$$

and they fit the interpolation formula

$$\epsilon_\lambda = 0{\cdot}1853 + 0{\cdot}8786\,(\lambda-4341) + 0{\cdot}0001\,(\lambda-4341)^2$$

with great accuracy. By means of this formula, the plate can be calibrated for all wave-lengths, but for the present purpose, only the values $\epsilon = 0{\cdot}4301$, $0{\cdot}23376$ for H_α and H_β are required.

Since the lines have a definite width, the theory outlined above is not strictly applicable in its entirety, but if the resolving power of the instrument is great in comparison with the observed (apparent) widths of the lines, it is well known that the maximum for any one wave-length is not altered except in magnitude, and that the plate gives reproductions of the lines in the successive orders with only a change in intensity—which is uniform along the narrow line—of each of the infinite number of components into which it can be resolved theoretically. Without the use of the plate, the wedge photograph should be parabolic for each line in the ordinary discharge as proved in a former paper*—according to the ordinary theory of broadening—and the combination of wedge and plate should, therefore, also give parabolic traces on the photograph under the condition that the lines are sufficiently narrow. This condition appears to be fulfilled in the case of the Hydrogen lines, and, in fact, it is proved later that the traces on the plate, when reduced to a normal pattern, are very accurately parabolic.

Reduction of the Fringes to a Normal Standard.—We have seen that in the case of a line broadened according to the probability law of intensity the fringes obtained from the wedge and plate should be a series of blackened patches with parabolic contours. Owing to the small variation of D, the path difference produced by two refractions in the plate, the parabolas are not uniform. They are not at equal intervals, and the distances between their axes must increase from the central member of the set of fringes. Their heights also vary continuously, and the appearance of the maximum heights of the individual fringes is as shown in the

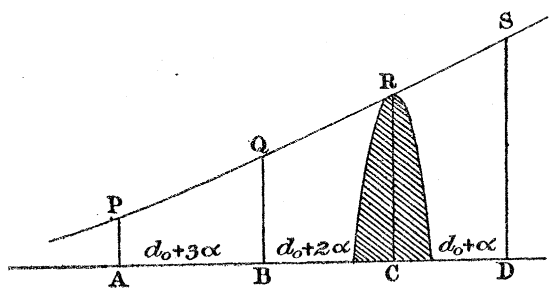


Fig. 5.

figure (fig. 5), where PA, QB, RC, SD, are four successive heights, and AB, BC, CD are in descending order of magnitude.

A theoretical consideration of the angle of refraction—nearly equal to $\sin^{-1}\left(\frac{1}{\mu}\right)$, corresponding to each fringe, shows that the distance, say AB, between the n^{th} and

* *Loc. cit.*

$n+1^{\text{th}}$ fringe should be very accurately given by the formula $\alpha+\beta n$ where α and β are constant. In other words, these distances are in arithmetical progression. Measurement of the plates shows that this law is almost exactly fulfilled in our photographs, the term in n^2, which would follow, having so small a coefficient as to be negligible for fringes not too far from the central one—signifying, in practice, the first five or six. This inequality of dispersion of the plate could be corrected graphically, but a simple interpolation formula deduced as follows is more convenient, for it elucidates the principal effects in a clear manner.

Let d_0 be the "normal" interval between two maxima, for a certain uniform dispersion, and $d_0+\alpha_3$, $d_0+2\alpha$, ..., the intervals actually found. Thus d_0 is the interval corresponding to the wave-length separation ϵ. The intervals $d_0+\alpha$, $d_0+2\alpha$ are DC, CB, in the figure, and may be called p and q. Then

$$d_0 = 2p-q$$

gives the normal scale to which the fringe of axis RC is being reduced. The whole visible fringe for this axis is shaded in the figure.

The law of the intervals between the fringes can be expressed by the formula

$$z' = z\left(1+\frac{\alpha}{2d_0}\right)+\frac{\alpha z^2}{2d_0^2}$$

where a "normal" interval z reckoned from any specified starting point becomes z' on the photograph. Thus if $z = d_0$, $z' = d_0+\alpha$, and if $z = 2d_0$, $z' = (d_0+\alpha)+(d_0+2\alpha)$. A line of breadth x on the normal scale, on one side of the point d_0, becomes situated at the point d'_0+x' on the photograph, where

$$d'_0+x' = (d_0+x)\left(1+\frac{\alpha}{2d_0}\right)+\frac{\alpha}{2d_0^2}(d_0+x)^2,$$

and as $d'_0 = d_0+\alpha$, we find

$$x' = x+\tfrac{3}{2}\frac{x\alpha}{d_0}+\frac{x^2\alpha}{2d_0^2}.$$

There is a difference between the right and left sides of the central line in the figure of the fringe. For on the left x' is positive, and on the right it is negative, while the last term in this formula is always positive. Thus the fringes are distorted—shortened on one side and expanded on the other. If x' is the breadth on the right, and x'' on the left, corresponding to a normal breadth x on either side

$$x' = x\left(1+\frac{3\alpha}{2d_0}\right)+\frac{\alpha x^2}{2d_0^2}, \qquad x'' = x\left(1+\frac{3\alpha}{2d_0}\right)-\frac{\alpha x^2}{2d_0^2},$$

whence

$$x'+x'' = 2x\left(1+\frac{3\alpha}{2d_0}\right),$$

and the total breadth at any point is *uniformly* expanded from the normal breadth $2x$. We may use a normal scale in which the interval $d_0\left(1+\frac{3\alpha}{2d_0}\right)$ corresponds to the wave-length separation $\delta\lambda = \epsilon$, and then the total breadth of the distorted parabolic fringe is the true normal breadth. In other words, if p and q are the observed intervals on the right and left of any fringe, the normal interval corresponding to ϵ has a length $d_0+\frac{3\alpha}{2}$, where $d_0 = 2p-q$, $\alpha = q-p$, and the central fringe is on the right. This length becomes $\frac{1}{2}(p+q)$, or the mean of the observed intervals. On this scale the apparent breadth of a distorted fringe is the true breadth of the undistorted one, if it is symmetrical about the central line when undistorted.

In this normal system, the apparent breadth on the right, of a real breadth x, is

$$x' = x - \frac{\alpha x^2}{2d_0^2}$$

to a sufficient order, and on the left,

$$x'' = x + \frac{\alpha x^2}{2d_0^2}.$$

Conversely, the real breadths on this normal scale corresponding to observed breadths x' and x'' are

$$x = x' + \frac{\alpha x'^2}{2d_0^2}, \qquad x = x'' - \frac{\alpha x''^2}{2d_0^2}$$

when insignificant terms are neglected. These results enable us in the next section to isolate the individual components of a broadened line, and to obtain their separation.

The second manner in which the fringes are not normal is in regard to their heights. This effect has a much smaller influence on the contours of the individual fringes, although it appears as a striking phenomenon in itself on the photographs. A theoretical investigation, which need not be reproduced here, indicates that the summits of the maxima—P, Q, R, S, in the preceding figure—should lie almost exactly on a parabola when the light falls on the plate at nearly grazing incidence. This parabolic locus of the summits is indicated on the photographs, but in actual fact the parabola is very nearly a straight line. Indeed, a straight line can be passed with some precision through any three consecutive summits, and it has been found sufficient, for the degree of accuracy we desired to obtain, to consider only three summits, so that the straight line locus can be used. For any given fringe, say CR in the figure, the "normal" heights of points on the right must be slightly smaller than the observed values, and those of points on the left must be slightly larger.

If axes are taken at C, the axis of y being along CR, and of x along CD towards the right, and if the equation to the "normal" parabola is

$$-kx^2 = y-h, \qquad h = \mathrm{CR},$$

then the equation to the parabola distorted by this effect should be

$$-kx^2 = y - h - \frac{h' - h}{d_0} x, \qquad h' = \mathrm{DS},$$

for the height increases from the normal value at a rate $(h'-h)/d_0$ very closely for a unit increase in x, when x is positive. It decreases at the same rate when x is negative, or to the left, so that both sides of the distorted curve are included. As stated above, the correction is, in fact, not very important in the subsequent calculations, its inclusion affecting only the third significant figure in the final separation of the components deduced in the case of H_α. In a calculation of separation, or distance between the axes of two overlapping parabolas, the effect along x is naturally the more significant.

(XV.) *The Law of Intensity Distribution in a Line Excited by the Ordinary Discharge.*

No *direct* proof of the validity of the usual law obtained for the intensity distribution around the maximum in a line broadened by the ordinary discharge—the law of probability $I = I_0 \exp(-k^2\lambda^2)$—has been given hitherto, although the work of Lord RAYLEIGH* and SCHÖNROCK has placed it on a secure theoretical basis, subject to certain well understood limitations in the conditions of excitement. It forms the basis of the calculations of atomic weights and emission temperatures of sources, made from the results of such experiments as those of BUISSON and FABRY†, and on this ground it is desirable that it should be confirmed experimentally in at least one definite case, for it is not possible to foretell readily whether the theoretical conditions are satisfied in any practical case. Moreover, we showed in the previous paper that it breaks down completely in the condensed discharge, and that all the components into which the line is then split by the electrical resolution which takes place under these circumstances follow the exponential law $I = I_0 \exp(-k\lambda)$. From the present photographs, it is possible to give a *direct* proof of the applicability of the theory to the ordinary discharge in Hydrogen. For all that is necessary is to show that the individual contours of the components making up, say H_α, are strictly parabolic.

Inspection of the photographs indicates that both H_α and H_β are double, and that each component is represented by a contour which appears very definitely parabolic—with, of course, the slight distortions dealt with in the preceding section. The parabolas overlap and represent lines of different intensities. We have not attempted to measure these intensities in the present experiments, but, as judged from the heights of the parabolas—one of these is calculated from the visible part of the contour—they are in at least rough accordance with MICHELSON's† estimate of 10 to 7. The

* 'Phil. Mag.,' XXIX., p. 274, 1915.

† *Loc. cit.*

contours intersect in a definite kink on one side of the main contour, and sufficiently above this kink the contour entirely represents the stronger component only and its exact form can be found. There is no evidence of any third component. Measurements have shown that the parabolic form is very exact for this stronger component, and as an illustration of the degree of exactness obtained, we append one calculation made from a single plate—which was somewhat smaller than the majority. The contour could be measured with some certainty to $\frac{1}{10}$ of a millimetre, by placing a scale along any required distance and observing through a magnifying lens. The true, or "normal" breadth is, by the preceding section, the apparent breadth on the distorted curve, and is denoted by $2x$. The height above a fixed base line is y. The following were the values of x and y for arbitrarily selected points:—

mm.	mm.	mm.	mm.
$2x = 0{\cdot}0$,	$y = 13{\cdot}2$,	$2x = 4{\cdot}0$,	$y = 10{\cdot}0$,
$2x = 2{\cdot}8$,	$y = 11{\cdot}8$,	$2x = 4{\cdot}4$,	$y = 9{\cdot}4$,
$2x = 3{\cdot}4$,	$y = 11{\cdot}0$,	$2x = 4{\cdot}7$,	$y = 8{\cdot}6$.

The total change of height between two orders distant 88·5 mm. was 11·4 mm., giving a rate 11·4/88·5 per mm. The contour should, therefore, possess the equation,

$$-kx^2 = y-h-\frac{11{\cdot}4x}{88{\cdot}5},$$

and calculating from the second and fifth values of corresponding co-ordinates in the above list, we find,

$$k = 0{\cdot}847, \qquad h = 13{\cdot}3 \text{ mm.}$$

The calculated height is therefore 13·3 against the measured value 13·2, so that the result is accurate to 1 part in 130. With $y = 8{\cdot}6$, we find by calculation, $x = 2{\cdot}38$ against the observed value 2·35. The other values are reproduced with similar accuracy, and we conclude that the undistorted curve is strictly parabolic, and that the *probability distribution of intensity is correct for the case of the Hydrogen lines in the ordinary discharge.* The intensity at a distance $\lambda-\lambda_0$ from the maximum at λ_0 is proportional to $\exp -k^2(\lambda-\lambda_0)^2$ where k^2 is some constant. The plate to which the above typical calculation relates is, in fact, much smaller than the others, so that the accuracy there obtained is a minimum. It is much greater in the calculations from the other plates, and we may, therefore, conclude that calculated distances are only subject to a possible error of about 1 part in 100 at most, and an error even less if the mean of several calculations is taken. The material is, therefore, at hand for a very accurate determination of the separation in H_α and H_β.

The actual plates were taken slightly out of focus, in order to remove the trouble caused by the grain of the plate. The exactness of the parabolic forms which the

normal contours take removes the necessity for giving a detailed investigation of the effect of this procedure. It is evident that, as could be proved otherwise, it introduces no appreciable change in the distances between axes of parabolas.

(XVI.) *Isolation of Components of a Line.*

The photograph for H_α is found to consist of two parabolas—slightly distorted—as in the figure (fig. 6), where the normal parabolas are shown, intersecting at P. The axes are a_1 and a_2 and σ is their separation on the photograph whose contour is APBC. Near P, of course, since intensities are additive, the parabolic forms are lost, and points close to P should not be selected for measurement. Calculation shows that this region round P is, in fact, nearly negligible, but we do not exhibit the calculation, for it is somewhat tedious. The equation for the parabolic form above P can be found with accuracy as in the last section for the distorted or normal curve. It is convenient to use the latter. We can then calculate the half breadth x of the normal curve at Q, and thence the apparent one x' by the formula

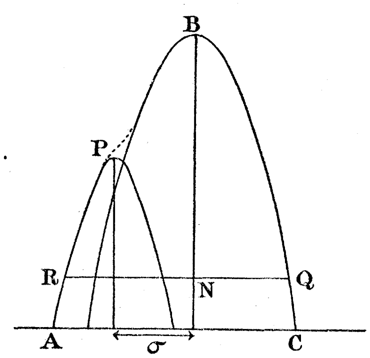


Fig. 6.

$$x' = x - \frac{\alpha x^2}{2d_0^2}.$$

The whole breadth QR of the contour may be measured, and thence the apparent RN in the figure, equal to x''. This can be reduced to the normal value x (say), which is $\sigma + X$, where σ is the separation and X the co-ordinate of R relative to the axis of its own parabola. By finding X and the heights for various points, σ may be deduced and thence $\delta\lambda$. There is, however, a simpler method of procedure. The axis of the upper parabola, when made normal in the longitudinal direction, is the locus of the central points of the various breadths, and can be drawn on the curve. Since RAYLEIGH's theory has been shown by a direct method to be applicable to these experiments, we may at once assume that k^2 in the law of intensity is the same for both components of H_α. The equations of the separate parabolas, with reference to the axis of y just constructed, should therefore be

$$k\xi^2 = h_1 - y + \beta\xi,$$

$$k(\xi - \sigma)^2 = h_2 - y + \beta\xi,$$

where h_1 and h_2 are their heights, and β is the rate of change of height of the maxima per millimetre. For the same height y, if ξ_1 and ξ'_1 are two abscissæ,

$$k\{\xi_1^2 - (\xi'_1 - \sigma)^2\} = h_1 - h_2 + \beta(\xi_1 - \xi'_1)$$

and for two other abscissæ ξ_2, ξ'_2, measured on the normal scale,

$$k\{\xi_2^2-(\xi'_2-\sigma)^2\} = h_1-h_2+\beta(\xi_2-\xi'_2),$$

and, by subtraction, h_1-h_2 is eliminated. Investigation shows at once, as in the following numerical examples, that the term in β in the result of this subtraction cannot affect even the third decimal place in the calculated separation and we may, therefore, omit it at once.

Thus finally,

$$\sigma = (\xi'^2_1-\xi_1^2-\xi'^2_2+\xi_2^2)/2(\xi'_1-\xi'_2).$$

These magnitudes ξ are true normal breadths, deduced from the observed values by correction for dispersion.

(XVII.) *Separation of the Components of* H_α.

In a series of fringes for H_α, the intervals between one fringe and its neighbours on either side were 14·0, 15·3 mm., and the next interval in the direction in which they decrease was 12·7. The law of maxima stated already is fulfilled, for these numbers are in arithmetical progression. The appropriate normal separation for the fringe in question, now the subject of examination, is the mean of 14·0 and 15·3, or 14·66. In other words, a separation 14·66 mm. corresponds to the interval $\delta\lambda = \epsilon = 0{\cdot}4301$ Å.U. for H_α. Moreover, $\alpha = 1{\cdot}3$, $d_0 = 2(14{\cdot}0)-15{\cdot}3 = 12{\cdot}7$.

A real normal breadth ξ on the right of the axis of the main component indicates, therefore, an apparent breadth x, where

$$x = \xi-\frac{\alpha\xi^2}{2d_0^{\,2}} = \xi-0{\cdot}0048\xi^2, \qquad \xi = x+0{\cdot}0048x^2,$$

and the same breadth ξ on on the left, an apparent breadth x' where

$$x' = \xi+0{\cdot}0048\xi^2, \qquad \xi = x'+0{\cdot}0048x'^2.$$

We can also calculate the real breadths on the right from the equation to the parabola for the main component. Actual measurements of the total breadths at the points $y = 19{\cdot}4$ cm., $y = 16{\cdot}5$ cm. give $2\xi = 4{\cdot}3$, $2\xi = 6{\cdot}4$ respectively. These are points which depend only on the main component. The corresponding equation of the main parabola becomes

$$\xi^2 = 1{\cdot}969(22{\cdot}0+0{\cdot}1\xi-y),$$

when the coefficient 0·1 is calculated. It agrees completely with the value derived by comparison with neighbouring fringes, which is 0·105, and the whole interpretation of the curves is justified. At the heights $y = 12{\cdot}8$ cm., $y = 11{\cdot}0$ cm., we thus find directly from the equation,

$$\xi_1 = 4{\cdot}355 \text{ mm.}, \qquad \xi_2 = 4{\cdot}753 \text{ mm.},$$

and when these are corrected to apparent breadths, the results agree with observation to one-tenth of a millimetre—the observations giving the apparent breadths on the right of the main axis.

The breadths on the left, when measured and corrected to normal dispersion, become

$$\xi'_1 = 5{\cdot}49 \text{ mm., corresponding to } y = 12{\cdot}8, \quad \xi_1 = 4{\cdot}35,$$

$$\xi'_2 = 6{\cdot}17 \text{ mm.,} \qquad ,, \qquad y = 11{\cdot}0, \quad \xi_2 = 4{\cdot}75.$$

Thus

$$\sigma = \{(5{\cdot}49)^2-(4{\cdot}35)^2-(6{\cdot}17)^2+(4{\cdot}75)^2\}/2\,(5{\cdot}49-6{\cdot}17),$$

or, on reduction, $\sigma = 4{\cdot}53$ mm. But 14·66 mm. corresponds to $\delta\lambda = \epsilon = 0{\cdot}4301$ Å.U., and the separation in wave-length is therefore

$$\delta\lambda = (4{\cdot}53 \times 0{\cdot}430)/14{\cdot}66 = 0{\cdot}133 \text{ Å.U.},$$

which is very close to BUISSON and FABRY's value 0·132 Å.U. Three determinations have been made in this way, by the selection of different heights, and the others give values $\delta\lambda = 0{\cdot}130$, $\delta\lambda = 0{\cdot}134$ Å.U., the general mean being 0·1323 Å.U. BUISSON and FABRY's value is thus confirmed in an absolute manner, even to the third significant figure.

We found it necessary to avoid points too near the base of the photograph, where the successive fringes tended to merge into one another by the increase in breadth of the parabolas. It was impossible on this account to test the possible existence of a third component as given by SOMMERFELD's theory. Its separation would only be 0·01, and its intensity as calculated by SOMMERFELD* would necessitate the appearance of its maximum in this particular region.

(XVIII.) *The Structure of* H_β.

H_β is also broadened symmetrically in the normal spectrum, and follows the probability law of intensity with extreme accuracy. The following are details of a typical calculation of its upper portion—representing the main component—made from a larger plate than that quoted for H_α.

For this plate, $\alpha = 0{\cdot}5$ mm., $d_0 = 9{\cdot}75$ mm., and the rate of change of height of the maxima per millimetre = 0·111 mm. The height of the curve being h_1, its equation when corrected longitudinally should become

$$-kx^2 = y-h_1-0{\cdot}111x.$$

Co-ordinates measured were, in millimetres,

$$\begin{array}{rcccc} 2x = & 3{\cdot}4, & 5{\cdot}3, & 6{\cdot}7, & 7{\cdot}6, \\ y = & 14{\cdot}0, & 11{\cdot}9, & 9{\cdot}8, & 8{\cdot}0, \end{array}$$

* *Loc. cit.*

and calculating the constants k and h_1 from the first and last, the equation becomes

$$0\cdot5394x^2 = 15\cdot368+0\cdot111x-y.$$

When $x = 2\cdot65$, $3\cdot35$, we find $y = 11\cdot9$, $9\cdot7$, by calculation from this formula, against the observed values 11·9, 9·8. The agreement is almost exact, and the stronger component of H_β follows the same law as that of H_α. It is, moreover, evident that the error of measurement in x cannot exceed about one-tenth of a millimetre at any point, and we may credit, therefore, the final separation of the components on the plate with this rough degree of accuracy. The scale of the plate being, as stated above, 9·5 mm. for a separation ϵ, equal to 0·22, the error in the deduced separation could be as great as $(0\cdot1\times0\cdot22)/9\cdot75 = 0\cdot002$ Å.U., but this should be practically the limit of error in any individual determination.

A simple inspection of the photographs indicates that there can be no second component of H_β with a separation comparable to that in H_α. If the series H_α, H_β, ... is Diffuse or Sharp, the separation in H_β should be 0·07 Å.U., and if it is Principal, 0·03 Å.U., according to a preceding calculation. The second component which can actually be seen on the photographs does not at first sight appear likely to lead to the former value, and accurate measurements show that the latter is the true value. A typical calculation is as follows :—

At one definite height, the breadths of the pattern on the right and left of the centre were $x_1 = 3\cdot9$, $x_2 = 4\cdot3$, and at another height $x'_1 = 4\cdot4$, $x'_2 = 5\cdot0$, the points being selected away from the junction of the curves. The true breadths on the normal scale are, if $\beta = \alpha/d_0^{\,2}$,

$$\xi_1 = 3\cdot9\,(1+1\cdot95\beta), \qquad \xi'_1 = 4\cdot3\,(1-2\cdot15\beta),$$
$$\xi_2 = 4\cdot4\,(1+2\cdot2\beta), \qquad \xi'_2 = 5\cdot0\,(1-2\cdot5\beta),$$

and neglecting, as is legitimate, the effect of the correction for height, the separation σ is given by

$$\sigma = (\xi'^2_1-\xi^2_1-\xi'^2_2+\xi^2_2)/2\,(\xi'_1-\xi'_2),$$

which becomes, to order β

$$\sigma = (2\cdot36-71\beta)/(1\cdot4-6\cdot5\beta).$$

But for this plate, $\beta = 0\cdot5/(9\cdot75)^2 = 0\cdot0053$, and finally $\sigma = 1\cdot98/1\cdot37 = 1\cdot44$ mm. But on the plate, 9·75 mm. corresponds to $\delta\lambda = \epsilon = 0\cdot23$ Å.U., and σ thus corresponds to $\delta\lambda = 0\cdot033$ Å.U., which is the separation in H_β.

There is thus no doubt that the Balmer series is a Principal series. The limit of error to be expected in the results, as above, is roughly 0·002 Å.U., and the true separation for a Principal series is 0·030 Å.U., so that the result is within the limit of error. Not only, therefore, can the method be applied to the elucidation of the nature of this series, but it can separate lines whose interval is only 0·010 Å.U., or less with

precision, although such lines cannot be seen as resolved on account of their width. Even when the correction involving β is not applied, so that account is not taken of the variation of dispersion within the small breadth of the line, the result is $\delta\lambda = 0{\cdot}039$ Å.U. This would be sufficient to indicate that there was no Diffuse series component, and it gives an estimate of the comparatively small degree of dependence of the result on irregularity of dispersion.

(XIX.) *Discussion.*

We may at once state that we have at present no theory to offer for the results obtained, but it would appear that the changes in relative intensity which we have found are facts which must be seriously considered in theories of the origin of spectra in relation to the structure of the atom. It cannot be said that spectroscopy has already passed from the descriptive to the rational state. We do not propose to discuss in detail the bearing of our results on the application of the quantum theory to the origin of spectra, but attention may be drawn to certain considerations. It would appear from the well-known relation $Ve = h\nu$ that the distribution of intensity in a spectrum should be a simple function of the distribution of velocity among the electrons in the discharge tube, but this view appears to be negatived by the fact that different series behave in different ways, and we may probably conclude from this that the range of velocities of the electrons is not the sole factor which determines the emission in line spectra; this view appears to be strengthened by such phenomena as the apparently anomalous effect of Neon on the distribution of intensity in the Balmer series.*

As regards the Balmer series, the separations of the components which we have found indicate that it is of the Principal type, and lend no direct support to the theoretical considerations advanced by SOMMERFELD. In any case it can no longer be regarded as a Diffuse or Sharp series.

We do not venture to speculate further on the results we have obtained, which can only be regarded as preliminary in their scope, but we hope that the methods of investigation which we have outlined may lead to the detailed knowledge which must be essential to the further elucidation of spectrum series.

(XX.) *Summary.*

1. A method has been found for the accurate determination of the photographic intensities of spectrum lines, and the reduction of such intensities to absolute values by comparison with the continuous black body radiation of the Carbon arc.

* The remarks already made in a previous communication (*loc. cit.*) as to the sources of error attaching to photographic estimates of intensity appear to be applicable to some results obtained by RAU ('Sitz. Phys. Med. Ges. Wurtzburg,' February, 1914), whose conclusions appear to imply that a critical voltage, related to the frequency, is necessary to produce spectrum lines.

2. A study has been made of the relative intensity distribution in the spectra of Helium and Hydrogen under different conditions of excitation.

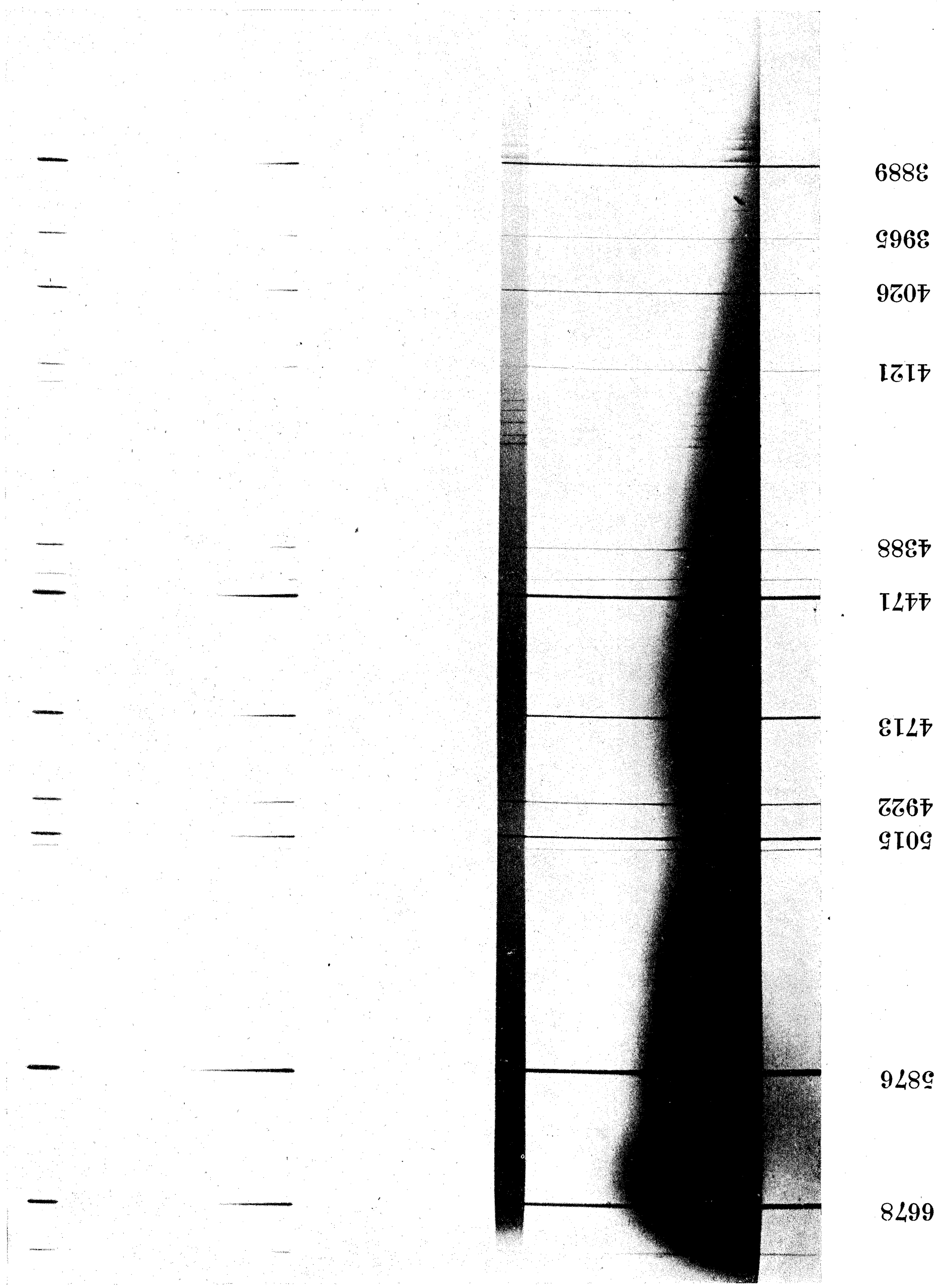
3. It has been found that under certain specified conditions there is a transfer of energy from the longer to the shorter wave-lengths in any given series, and that under such conditions the associated series, and in particular the Diffuse series, are relatively enhanced at the expense of the Principal series.

4. It has also been found that the distribution of intensity found in certain celestial spectra can be approximately reproduced in the laboratory.

5. A study has been made of the separations of the components of lines of the Balmer series of Hydrogen, and the mean values of the separations of the doublets constituting the lines H_α and H_β have been found to be respectively 0·132 Å.U., and 0·033 Å.U. These values are consistent with the separations appropriate to Principal series, and the first is in precise agreement with the value deduced by BUISSON and FABRY.

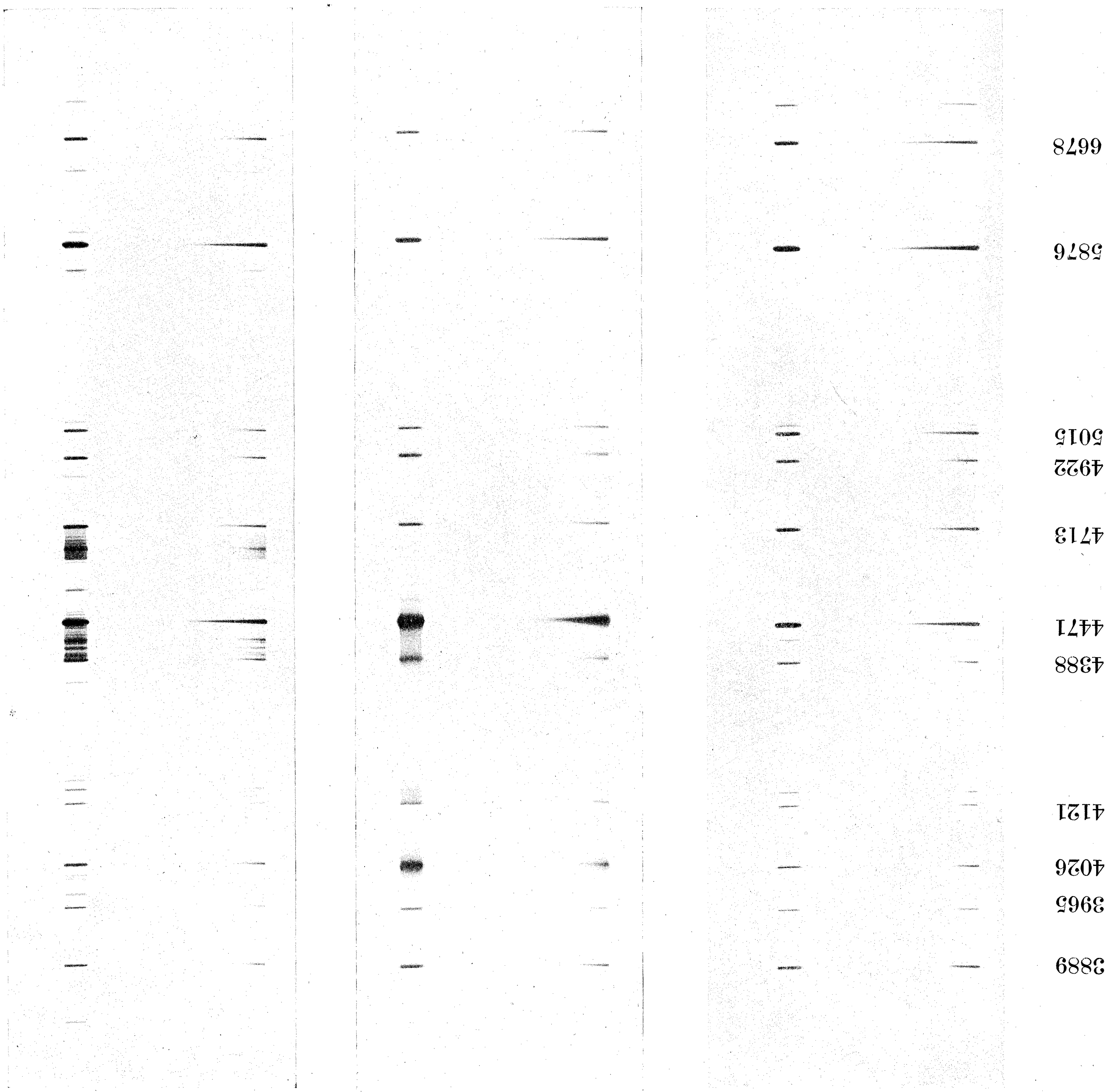
Note on the Band Spectrum of Helium.—We have made a number of observations of the pinkish glow which surrounds the cathode in a Helium tube excited by the ordinary discharge. The cathode glow in Helium presents a very striking appearance when viewed through coloured screens which transmit only a narrow region of the spectrum. Through a violet screen the dark space around the electrode is clearly defined and surrounded by a uniform glow. Viewed through a screen which transmits red rays only, the appearance is entirely changed, the "dark space" being in fact the region in which the red rays are predominant. The band spectrum of Helium is relatively strong in the cathode glow, though not intrinsically so strong as with the "bulb discharge."* In this connection we may mention that we have definite evidence from the relative intensities of the bands in the "bulb discharge" and the cathode glow that this spectrum is capable of sub-division. Apart from theoretical considerations, this circumstance may be of importance in modifying the conclusion that the spectrum in question is absent from celestial spectra.

* CURTIS, 'Roy. Soc. Proc.,' A, vol. 89, p. 146, 1913.



B.
Helium
Ordinary
Discharge.

A.
+ Crater
of Carbon
Arc.

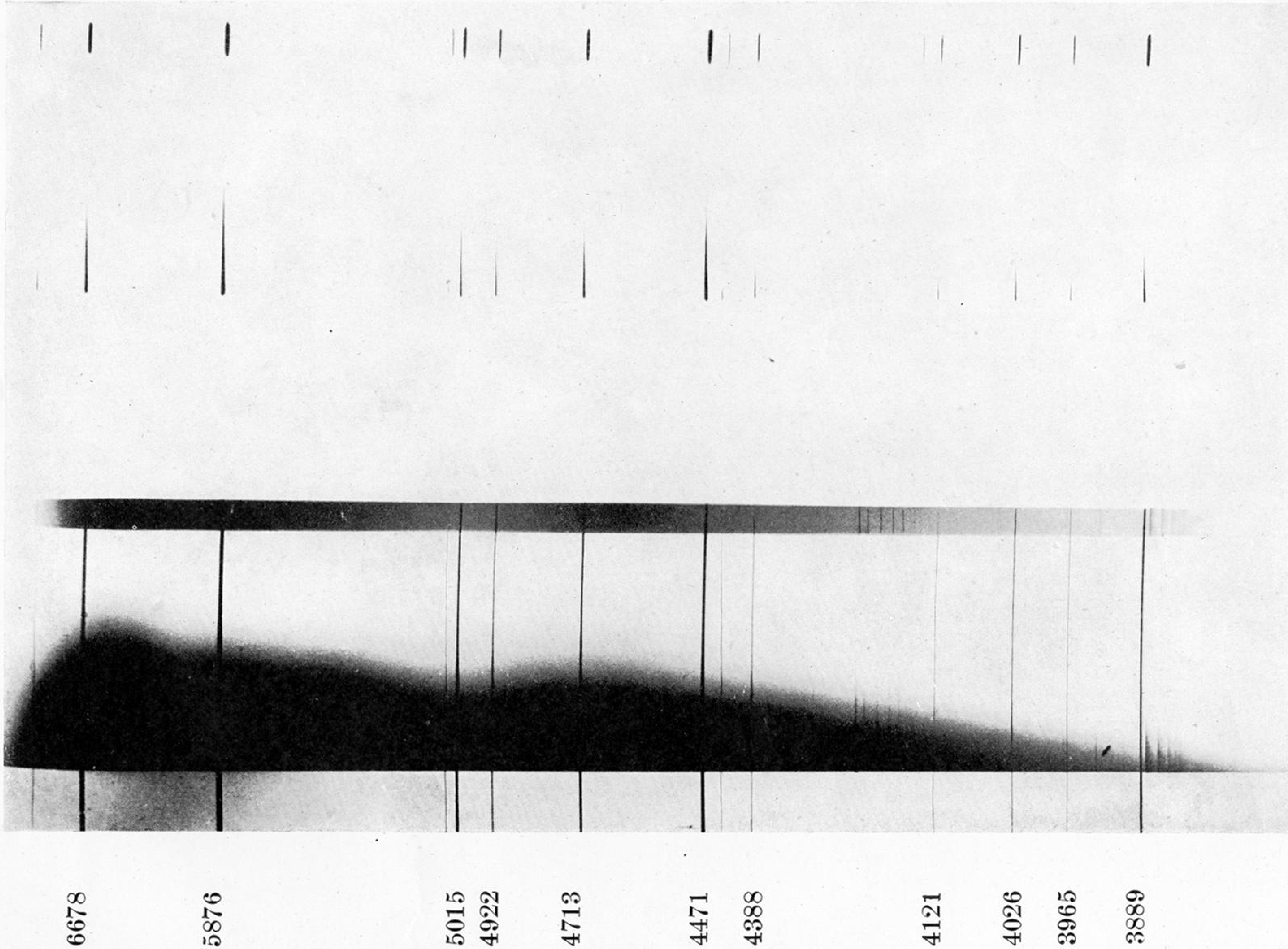


Helium (bulb).

Helium (capacity).

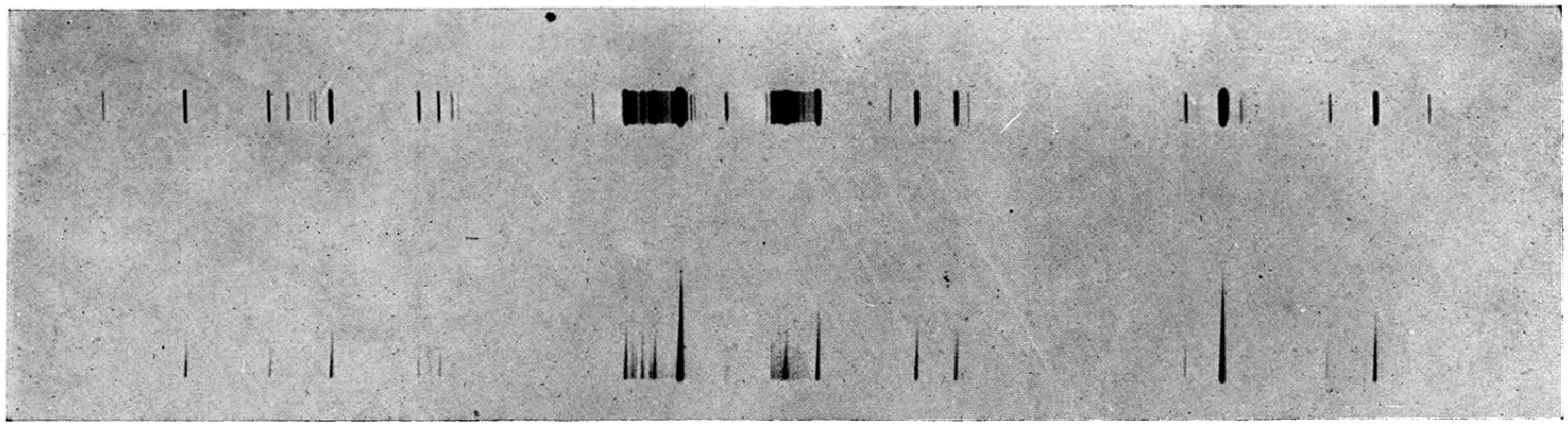
Helium (ordinary discharge).

B.
Helium
Ordinary
Discharge

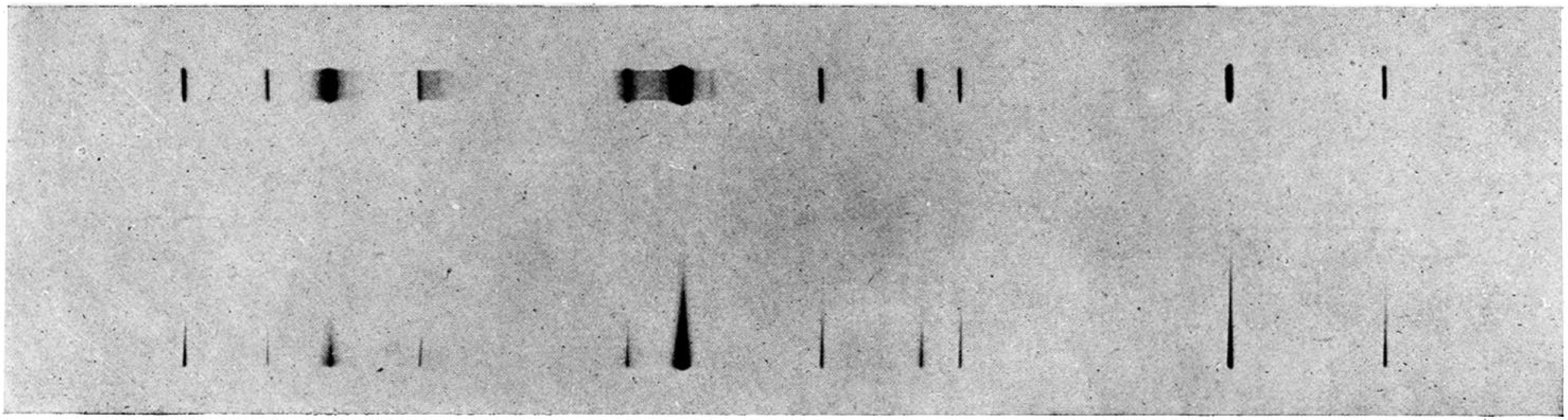


A.
+ Crater
of Carbon
Arc.

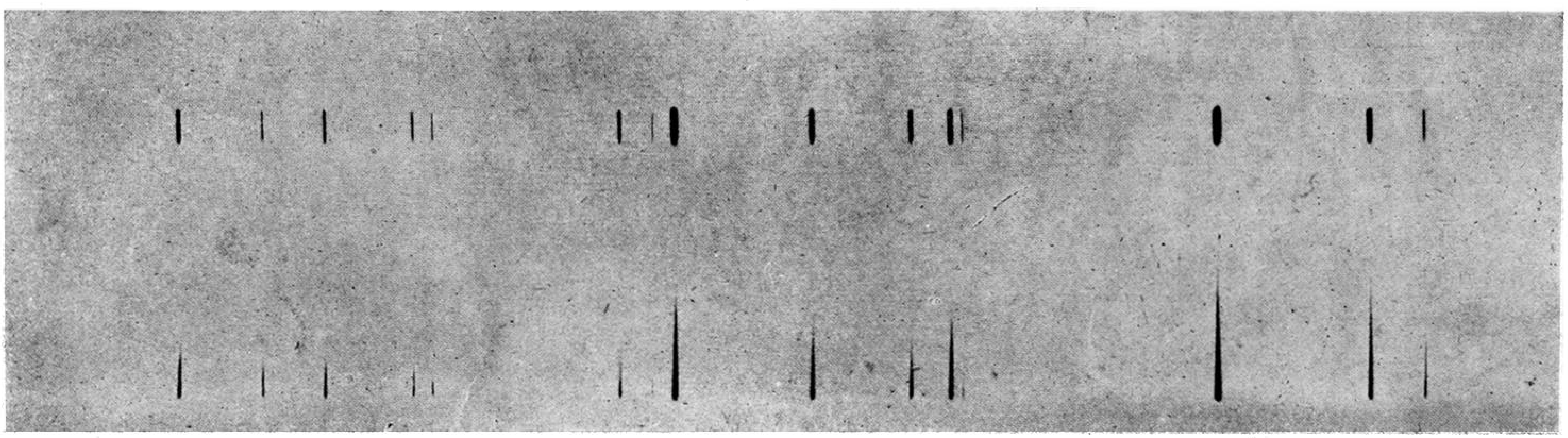
Helium
(bulb).



Helium
(capacity).



Helium
(ordinary
discharge).



3889 3965 4026 4121 4388 4471 4713 4922 5015 5876 6678