

by a *Mad Dog*, or hot Fomentations, might be of greater Service than cold Applications: For a cold Bath shuts the Pores, as a warm one opens them; therefore the Blood being allowed to be greatly inflamed in this Case, and *Dampier's Powder* being a very hot Medicine, it is reasonable to think, that when a Patient takes it, the setting him up to the Chin in hot Water for some Hours, would help the Operation of the *Powders*, by diluting the Blood, and relaxing the Pores.

VII. *A brief Account by Mr. John Eames, F. R. S. of a Work entitled, The Method of Fluxions and Infinite Series, with its Application to the Geometry of Curve Lines, by the Inventor Sir Isaac Newton, Kt. &c. Translated from the Author's Latin Original not yet made publick. To which is subjoin'd a perpetual Comment upon the whole, &c. by John Colson, M. A. and F. R. S.*

THIS Posthumous Work of our late excellent President, a Translation of which we have now received from the Hand of the learned and ingenious Mr. *Colson*, has been long and impatiently expected by the curious in these Matters; and now it appears, I believe it will fully answer, if not exceed, those Expectations, as well as confirm the Reputation the Author

thor has so justly acquired by his other Writings. For it is wrote with the same Genius and *Acumen*, it explains the Principles of his Method of Fluxions with great Clearness and Accuracy, and applies those Principles to very general and scientific Speculations in the higher Geometry. And farther to explain this Work, and to supply such Things, for the Use of common Readers, which the Author, according to his usual Brevity, has often omitted; the Translator has thought fit to give us a Comment on a good Part of the Work, and has promised the rest at a proper Season. His Fitness for such an Undertaking is well known to the learned World, into which he was many Years ago introduced by a very good Judge, as a Person who was *reconditioris Analyseos peritissimus*.

This Text may very well be divided into three Parts: An Introduction, containing the Method of Infinite Series; The Method of Fluxions and Fluents; and lastly, The Application of both to the most considerable Problems of the higher Geometry. The Comment consists of very valuable and curious Annotations, Illustrations, and Supplements, in order to make the whole a *complete Institution* for the Use of Learners. I shall take a kind of comparative View of the Text and Comment together.

The great Author, in what is called the Introduction, teaches the Rudiments of his Method of Infinite Converging Series, which is preparatory to that of Fluxions. In this he shews how all Compound Algebraical Quantities may be resolved into Series of simple Terms, which will converge to those compound Quantities, or rather to their Roots; just as in com-

mon Decimal Arithmetick, any complicate Number whatever, rational or furd, may be profecuted and exhibited to what Degree of Accuracy we please, by decimal Parts continued *in infinitum*. And this general Arithmetick is here applied to the finding of the Roots of all kinds of Algebraical Equations, whether pure or affected.

And this Doctrine is carried on still farther by Mr. *Colson* in his Comment. He pursues the Author's Hint, that vulgar Arithmetick and Algebra, decimal Fractions and infinite Series, have the same common Foundation, and compose together but one uniform Science of Computation. For, as in our vulgar Arithmetick, when rightly explain'd, we express and compute all Numbers by the Root *Ten*, and its several Powers and their Reciprocals, together with a Set of certain known and small Coefficients; so in this more universal Arithmetick of Infinite Series, we do the same thing in effect, by means of any Root assumed at Pleasure, its Powers and their Reciprocals, disposed in a regular descending Order, together with any Coefficients, as it may happen. And when these Series duly converge, they will as truly exhibit by their Aggregate the Quantity required, as a Decimal Fraction infinitely continued will approximate to its proper *Quæsitum*. This gives him Occasion to expatiate largely upon the Nature and Construction of Arithmetical Scales, particular and general; and to inquire into the Nature and Formation of Infinite Series, and their Circumstances of Convergency and Divergency. To explain which he shews, that in every Series there is always a Supplement to be understood, when it is not exhibited. This Supplement sums up the Series, and
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makes it stop at a finite Number of Terms, in Series that either converge or diverge. Whence in diverging Series it must necessarily be found and admitted, or otherwise the Conclusion will not be true; but in converging Series, where it can seldom be known, it may safely be omitted, because it continually diminishes with the Terms of the Series, and finally becomes less than any assignable Quantity.

The Nature of infinite Series being thus display'd, he applies them to the Resolution of all kinds of Algebraical Equations. He explains in a very general Manner, the Author's famous Artifice, for finding the *Forms of the Series* for the Roots, and their initial Approximations, by means of a Parallelogram and Ruler, and shews its Application in all Cases. Then he invents many ways of Analysis, by which the Roots are further prosecuted, and may be produced to any Degree of Accuracy required. Also many other Speculations are added, to compleat the Doctrine of Series; particularly a very general and useful Theorem, for the Solution of all affected Equations in Numbers.

From the Resolution of Equations and the Doctrine of Infinite Series, which finishes the first Part of this Work, Sir *Isaac Newton* proceeds to lay down the Principles of his Method of Fluxions, which is the chief Design of the present Treatise. This Method he founds upon the abstract or rational Mechanics, by supposing all Mathematical Quantities to be generated, as it were, by local Motion, and therefore to have relative Velocities of Increase or Decrease, which Velocities he calls *Fluxions*. And the Quantities so generated by a continual Flux he calls *Flu-*

ents or flowing Quantities; the Relation of which Fluents is always express'd by some Algebraical Equation, either given or required. If this Equation be given, and the Relation of the Fluxions is required, it constitutes the *direct Method of Fluxions*; but when the contrary, 'tis the *inverse Method of Fluxions*.

Sir *Isaac*, in his first Problem, which takes in the direct Method of Fluxions, shews how to find the Relation of the Fluxions in a very general Manner, and by a great Variety of Solutions. This way of resolving the Problem is peculiar to this Work. He likewise extends it to Equations involving several Fluents, which accommodates it to those Cases, wherein any complex or irrational Quantities may be found, or Quantities that are geometrically irreducible. Then he demonstrates the Principles of his Method, or the Precepts of Solution, from the Nature of Moments or vanishing Quantities, and from the obvious Properties of Equations, which involve indeterminate Quantities.

The Commentator much enlarges upon this whole Doctrine; he enters into the Reason and Use of this Multiplicity of Solutions, and shews it is a necessary Result from the different Forms the same given Equation may acquire. But especially he takes the Author's Demonstration into strict Examination, endeavours farther to illustrate and enforce its Evidence, and to clear it from all the Objections that either have or may be urged against it. He even contends, that tho' the Moments and vanishing Quantities of the Author could be proved to be impossible, as has been suggested by some Mathematicians, yet even then they
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would be sufficient for all the purposes of Fluxions, and he produces Instances of a like Nature from other parts of Mathematicks. And tho' the Author, Sir *Isaac Newton*, in his present Treatise, does not directly mention second Fluxions, or those of higher Orders; yet the ingenious Commentator thinks proper to extend his Inquiries to these Orders of Fluxions, demonstrates their Theory, gives Rules and Examples for deriving their Equations, proves their relative Nature, and even exhibits them to View by Geometrical Figures. This last he does chiefly in what he calls the Geometrical and Mechanical Elements of Fluxions; and he contrives a very general Method, by means of Curve-lines and their Tangents, to make Fluxions and Fluents the Objects of Sense and ocular Inspection; and thereby he illustrates and verifies the received Methods of deriving their Equations in all Cases.

In the Author's second Problem, or the Relation of the Fluxions being given to determine the Relation of the Fluents, which includes the inverse Method of Fluxions, he begins with a particular Solution of it. He calls this Solution particular, because it extends only to such Cases, wherein the given Fluxional Equation either has been, or might have been, derived from some previous finite Algebraical Equation. Then he shews how we may return directly to this Equation. But this is seldom the Case of such Fluxional Equations, whose Fluents or Roots are proposed to be found. For they have commonly Terms either redundant or deficient, by which they cannot be brought under this particular Solution. Therefore to answer this Case also, he gives us a general Solution, in which he extracts the Roots of any proposed

proposed Fluxional Equation, by several ingenious Methods of Analysis. And here it is chiefly, that he calls his Method of Infinite Series to his Assistance; for the Fluent, or Root, will here always be exhibited by a Series. And to find the Fluent in finite Terms, when it can be done, requires particular Expedients, as we shall see afterwards.

Mr. *Colson* in his Comment upon this Part of the Work is very full and explicit. He explains and applies the Author's particular Solution; but is much more copious in explaining the Examples, and clearing up the Difficulties and Anomalies of the general Solution. This is chiefly perform'd by introducing several new and simple Methods of Analysis, or Processes of Resolution; and by applying the Author's Artifice of the Ruler and Parallelogram mention'd before, to these Fluxional Equations: By which means not only the Forms of the Series are determin'd, and their initial Approximations, as has been observ'd above; but likewise all the Series may be found, that can be derived from the same Fluxional Equation. The Commentator concludes by giving us a very general Method for resolving all Equations, whether Algebraical or Fluxional; which Method requires no foreign Assistance, or no subsidiary Operations, which all other Methods do. It is founded upon the Use and Admission of the higher Orders of Fluxions, and is exemplify'd by the Solution of several useful Problems. Here the Comment leaves us, but we will go on with our Author.

Having thus taught us the Method of Fluxions, both direct and inverse, he proceeds to apply this Method to some very curious and general Problems,
chiefly

chiefly in the Geometry of Curve-lines. As first, he determines the *maxima* and *minima* of Quantities in all Cases, and proposes some elegant Problems to illustrate this Doctrine. Then he teaches us to draw Tangents to Curves, whether Geometrical or Mechanical, and that after a great Variety of Ways, or however the Nature of the Curve may be defined. Here likewise he proposes some Questions, to exercise and improve the Learner: Then is very particular upon finding the Quantity of Curvature, at any Point of a given Curve, whether Geometrical or Mechanical, or in determining the Centre and the Radius of Curvature: To which several other curious Speculations are subjoin'd of a like Nature. Here he communicates a very elegant and intirely new Problem, for determining the Quality of the Curvature, at any Point of a given Curve; or how the Curvature proceeds in respect of its greater or less Inequability.

Afterwards he goes on to the Quadrature of Curves, which chiefly gives occasion to apply the inverse Method of Fluxions. And first he shews how, by the direct Method, to find as many Curves as you please, (or to determine their Equations) the Areas of which shall be capable of an exact Quadrature. Then he shews how to find as many Curves as you please, which, tho' not capable of a just Quadrature, yet their Areas may be compared to those of the Conic Sections, or of such other Curves as shall be assign'd. Lastly, He shews how to determine in general the Area of any Curve that shall be proposed, chiefly by the Method of Infinite Series; where many curious and useful Speculations are occasionally introduced and inserted: As how to ascertain the Limits of an
Area,

Area, when thus found analytically; how commodiously to square the Circle, the Ellipsis, or Hyperbola, and how to apply the Quadrature of this last to the computing a Canon of Logarithms; the Construction of Tables for the ready finding of Quadratures, or the Comparifon of Areas, and how to apply them to the solving of other like Problems; the forming of Constructions, and demonstrating Theorems by Fluxions; the approximating to Areas mechanically, and fuch-like.

From finding of Areas he proceeds to the *Rectification of Curves*; and first he shews how to find as many Curves as you please, whose Curve-lines are capable of an exact Rectification. Then he teaches us to find as many Curves as we please, whose Curve-lines, tho' not capable of a just Rectification, yet may be compared with the Lengths of any Curve-lines assign'd, or with the Areas of any Curve, when reduced to the Order of Lines. Lastly, He determines the Lengths of any Curve in general, and gives several proper Examples of it. All which elegant Speculations are managed with admirable Skill, great Subtilty, and fine Contrivance.