

CX. *Of the Irregularities in the Motion of a Satellite arising from the spheroidical Figure of its Primary Planet : In a Letter to the Rev. James Bradley D. D. Astronomer Royal, F. R. S. and Member of the Royal Academy of Sciences at Paris ; by Mr. Charles Walmesley, F. R. S. and Member of the Royal Academy of Sciences at Berlin, and of the Institute of Bologna.*

Reverend Sir,

Read Dec. 14, 1758. **S**INCE the time that astronomers have been enabled by the perfection of their instruments to determine with great accuracy the motions of the celestial bodies, they have been solicitous to separate and distinguish the several inequalities discovered in these motions, and to know their cause, quantity, and the laws according to which they are generated. This seems to furnish a sufficient motive to mathematicians, wherever there appears a cause capable of producing an alteration in those motions, to examine by theory what the result may amount to, though it comes out never so small: for as one can seldom depend securely upon mere guesses for the quantity of any effect, it must be a blameable neglect entirely to overlook it without being previously certain of its not being worth our notice.

Finding therefore it had not been considered what effect the figure of a planet differing from that of a

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sphere

sphere might produce in the motion of a satellite revolving about it, and as it is the case of the bodies of the Earth and Jupiter which have satellites about them, not to be spherical but spheroidal, I thought it worth while to enter upon the examination of such a problem. When the primary planet is an exact globe, it is well known that the force by which the revolving satellite is retained in its orbit, tends to the center of the planet, and varies in the inverse ratio of the square of the distance from it; but when the primary planet is of a spheroidal figure, the same rule then no longer holds: the gravity of the satellite is no more directed to the center of the planet, nor does it vary in the proportion above-mentioned; and if the plane of the satellite's orbit be not the same with the plane of the planet's equator, the protuberant matter about the equator will by a constant effort of its attraction endeavour to make the two planes coincide. Hence the regularity of the satellite's motion is necessarily disturbed, and though upon examination this effect is found to be but small in the moon, the figure of the earth differing so little from that of a sphere, yet in some cases it may be thought worth notice; if not, it will be at least a satisfaction to see that what is neglected can be of no consequence. But however inconsiderable the change may be with regard to the moon, it becomes very sensible in the motions of the satellites of Jupiter both on account of their nearer distances to that planet when compared with its semidiameter, as also because the figure of Jupiter so far recedes from that of a sphere. This I have shewn and exemplified in the fourth satellite; in which case indeed the computation is more exact

exact than it would be for the other satellites: for as my first design was to examine only how far the moon's motion could be affected by this cause, I supposed the satellite to revolve at a distance somewhat remote from the primary planet, and the difference of the equatoreal diameter and the axis of the planet not to be very considerable. There likewise arises this other advantage from the present theory, that it furnishes means to settle more accurately the proportion of the different forces which disturb the celestial motions, by assigning the particular share of influence which is to be ascribed to the figure of the central bodies round which those motions are performed.

I have added at the end a proposition concerning the diurnal motion of the earth. This motion has been generally esteemed to be exactly uniform; but as there is a cause that must necessarily somewhat alter it, I was glad to examine what that alteration could amount to. If we first suppose the globe of the earth to be exactly spherical, revolving about its axis in a given time, and afterwards conceive that by the force of the sun or moon raising the waters its figure be changed into that of a spheroid, then according as the axis of revolution becomes a different diameter of the spheroid, the velocity of the revolution must increase or diminish: for, since some parts of the terraqueous globe are removed from the axis of revolution and others depressed towards it, and that in a different proportion as the sun or moon approaches to or recedes from the equator, when the whole quantity of motion which always remains the same is distributed through the spheroid, the velocity of the diurnal rotation cannot be constantly the same. This

variation however will scarce be observable, but as it is real, it may not be thought amiss to determine what its precise quantity is.

I am sensible the following theory, as far as it relates to the motion of Jupiter's satellites, is imperfect and might be prosecuted further; but being hindered at present from such pursuit by want of health and other occupations, I thought I might send it you in the condition it has lain by me for some time. You can best judge how far it may be of use, and what advantage might arise from further improvements in it. I am glad to have this opportunity of giving a fresh testimony of that regard which is due to your distinguished merit, and of professing myself with the highest esteem,

Reverend Sir,

Your very humble Servant,

Bath, Oct. 21.
1758.

C. Walmsley.

LEMMA I.

Invenire gravitatem corporis longinqui ad circumferentiam circuli ex particulis materiæ in duplicatâ ratione distantiarum inversè attrahentibus constantem.

ESTO NIK (*Vid. TAB. xxxiii. Fig. 1.*) circumferentia circuli, in cujus puncta omnia gravitet corpus longinquum S locatum extra planum circuli. In hoc planum agatur linea perpendicularis SH, et per circuli centrum X ducatur recta HXK secans circumulum in I et K, et SR parallela ad HX: producatu autem SH ad distantiam datam SD, et agantur rectæ DC,

DC, XC, ipſis HX, SD, parallelæ. Tum ductâ chordâ quavis MN ad diametrum IK normali eamque ſecante in L, ex punctis M, N, demittantur in SR perpendicularæ MR, NR, concurrentes in R; junctiſque SM, SN, erit SM = SN, MR = NR, SR = HL. Dicantur jam SD, k ; HX five DC, b ; XL, x ; CX, z ; XI, r ; eritque HL = $b - x$, et SH = $k - z$. Eſt autem SM ad SH ut attractio

$\frac{1}{\overline{SM}^2}$ corporis S verſus particulam M in directione SM ad ejuſdem corporis attractionem in directione SH, quæ proinde erit $\frac{SH}{\overline{SM}^3}$: ſed eſt SR = HL, et

$\overline{SM}^2 = \overline{SR}^2 + \overline{MR}^2 = \overline{SR}^2 + \overline{SH}^2 + \overline{ML}^2$; unde fit

$\frac{SH}{\overline{SM}^3} = \frac{SH}{\overline{HL}^2 + \overline{SH}^2 + \overline{ML}^2}^{\frac{1}{2}}$, et ductâ mn parallelâ ad MN, viſ qua corpus S attrahitur ad arcus quàm minimos Mm , Nn , exponitur per $\frac{SH \times 2 Mm}{\overline{SM}^3} =$

$\frac{SH \times 2 Mm \times \overline{HL}^2 + \overline{SH}^2 + \overline{ML}^2}^{\frac{1}{2}}^{-\frac{3}{2}}$. Eſt autem $\overline{HL}^2 + \overline{SH}^2 + \overline{ML}^2 = kk - 2kz + zz + bb - 2bx + rr$,

hincque ponendo $kk + bb = ll$, $\overline{HL}^2 + \overline{SH}^2 = \overline{ML}^2$ $^{-\frac{3}{2}} =$

$\frac{1}{l^3} + \frac{3kz}{l^3} + \frac{3bx}{l^3} - \frac{3rr}{2l^3} - \frac{3zz}{2l^3} + \frac{15kkzz}{2l^7} + \frac{15kbzx}{2l^7} +$

$\frac{15bbxx}{2l^7}$, neglectis terminis ulterioribus ob longinquitatem quam ſupponimus corporis S. Quarè, ſi ſcribatur d pro circumferentiâ IMKN, gravitas corporis S ad totam illam circumferentiam ſecundum SH,

five fluens fluxionis $SH \times 2 Mm \times \overline{HL}^2 + \overline{SH}^2 + \overline{ML}^2$ $^{-\frac{3}{2}}$

evadit $k - z \times d$ in $\frac{1}{l^3} + \frac{3kz}{l^3} - \frac{3rr}{2l^3} - \frac{3zz}{2l^3} +$

$\frac{15kkzz}{2l^7}$

$\frac{15 k k z z}{2 l^7} + \frac{15 b b r r}{4 l^7}$. Simili modo obtinebitur gravitas ejusdem corporis S secundum SR. *Q. E. I.*

LEMMA II.

Corporis longinqui gravitatem ad Sphæroidem oblatam determinare.

Retentis iis quæ sunt in lemmate superiori demonstrata; esto C centrum sphæroidis, cujus æquatori parallelus sit circulus IMK. Sphæroidis hujus semiaxis major sit a , semiaxis minor b , eorum differentia c , quam exiguam esse suppono; et dicatur D circumferentia æquatoris. Centro C et radio æquali semiaxi minori describi concipiatur circulus qui secet IK in i , eritque gravitas in directione SD, qua urgetur corpus S versus materiam sitam inter circumferentiam IMKN et circumferentiam centro X et radio Xi descriptam, æqualis gravitati in lemmate præcedenti definitæ ductæ in rectam Ii. Sed est $Ii . c :: IX . a$, atque $d . D :: IX . a$; unde $Ii \times d . D \times c :: IX^2 . aa$, hoc est, ex naturâ ellipseos, ob $CX = z$, et $IX = r$, $Ii \times d . D \times c :: bb - zz . bb$, adeoque $Ii \times d = \frac{D \times c}{bb} \times \overline{bb - zz}$, atque $rr = aa - \frac{aaz z}{bb}$; scribi autem potest in sequenti calculo $bb - zz$ pro rr ob parvitatem differentiæ semiaxium in quam omnes termini ducuntur. Gravitas igitur corporis S in materiam inter circumferentias supradictas consistentem exprimetur per $\frac{D \times c}{bb} \times \overline{bb - zz}$ $\times \overline{k - z}$ in $\frac{1}{l^3} + \frac{3kz}{l^5} - \frac{3bb}{2l^5} - \frac{15zz}{4l^5} + \frac{15bbbb}{4l^7} + \frac{45kkzz}{4l^7}$. Et si addatur gravitas in similem materiam

ex

ex alterâ parte centri C ad æqualem à centro distan-
tiam, quia tunc CX five z evadit negativa, gravitas
corporis S in hanc duplicem materiam erit $\frac{D \times c}{bb} \times$

$$\overline{bb} - z z \text{ in } \frac{2k}{l^3} - \frac{6kzz}{l^5} - \frac{3kbb}{l^5} + \frac{15k^3zz}{l^7} + \frac{15hkkbb}{2l^7} -$$

$$\frac{15hbkzz}{2l^7}. \text{ Ducatur jam gravitas hæc in } z, \text{ et sumptâ}$$

gravitatum omnium summâ, factâ $z = b$, gravitatio
tota corporis S in totam materiam globo interiori su-
periore secundum directionem SD æquatori per-

$$\text{pendicularem prodit } D \times c \times \frac{4kb}{3l^3} - \frac{4kb^3}{5l^5} + \frac{2kbb^3}{l^7}.$$

Simili ratiocinio gravitatio corporis S in eandem
materiam secundum directionem SR æquatori pa-

$$\text{rallelam invenitur æqualis } D \times c \times \frac{4bb}{3l^3} + \frac{2bb^3}{5l^5} -$$

$$\frac{2hkkb^3}{l^7}. \text{ Tum si addatur gravitatio corporis S in}$$

globum interiorem, ex unâ parte scilicet $\frac{2b^3kD}{3al^3}$, et

ex alterâ $\frac{2b^3bD}{3al^3}$, habebitur gravitas corporis S in to-
tum sphæroidem. *Q. E. I.*

COROLL.

Igitur gravitas corporis S secundum SD est ad ejus-
dem gravitatem secundum SR five DC in materiam

$$\text{sphæroidis globo interiori incumbentem ut } \frac{2k}{3} - \frac{2kb^2}{5l^2}$$

$$+ \frac{kbb^2}{l^4} \text{ ad } \frac{2b}{3} + \frac{bb^2}{5l^2} - \frac{hkkb^2}{l^4}, \text{ adeoque si gravitas prior}$$

$$\text{exponatur per } k, \text{ posterior exprimetur per } b - \frac{3bb^2}{5l^2}$$

quamproximè. Unde cum fit DC = b , patet gravi-
tatem corporis S in sphæroidem oblatam non tendere

ad

ad centrum C, sed ad punctum *c* rectæ DC in plano æquatoris jacentis vicinius puncto D.

PROPOSITIO I.

PROBLEMA.

Vires determinare quibus perturbatur motus Satellitis circa Primarium suum revolventis.

Exhibeat jam sphærois prædicta planetam quemvis figurâ hac donatum, et corpus S satellitem circa planetam tanquàm primarium gyranter. Quantitas materiæ globo sphæroidis interiori incumbentis æqualis est $\frac{4bbcD}{3^a}$ five $\frac{4bcD}{3}$ proximè, et si materia illa locaretur in centro sphæroidis C, attraheret satellitem S secundum SC vi $\frac{4bcD}{3l^2}$, quæ reducta ad directionem SD fit $\frac{4bckD}{3l^3}$, et ad directionem DC fit $\frac{4bcbD}{3l^3}$. Cum igitur vis $\frac{4bcD}{3l^2}$ non turbat motum satellitis, utpote quæ tendat ad centrum motûs et quadrato distantiae ab eodem centro fit reciproçè proportionalis, vires illæ $\frac{4bckD}{3l^3}$, $\frac{4bcbD}{3l^3}$, in quas resolvitur, etiam motum non turbabunt. Itaque ex vi $D \times c \times \frac{4kb}{3l^3} - \frac{4kb^3}{5l^5} + \frac{2kbhb^3}{l^7}$ auferatur vis $\frac{4bckD}{3l^3}$, et ex vi $D \times c \times \frac{4bb}{3l^3} + \frac{2hb^3}{5l^5} - \frac{2bkkb^3}{l^7}$ auferatur $\frac{4bcbD}{3l^3}$, et remanebunt vires $D \times c \times -\frac{4kb^3}{5l^5} + \frac{2kbhb^3}{l^7}$, $D \times c \times \frac{2hb^3}{5l^5} - \frac{2bkkb^3}{l^7}$, motuum satellitis S perturbatrices. Designetur vis $D \times c \times$

$\frac{2bb^3}{5l^5} - \frac{2bbkb^3}{l^7}$ per rectam Sr (*Fig. 2.*) ac resolvatur in vim Sq tendentem ad centrum planetæ primarii C et ob triangula similia Srq , SDC , æqualem $D \times c \times \frac{2b^3}{5l^4} - \frac{2kkb^3}{l^6}$, existentibus ut priùs, $SD = k$, $DC = b$, $SC = l$; et in vim rq rectæ SD parallelam et æqualem $D \times c \times \frac{2kb^3}{5l^5} - \frac{2k^3b^3}{l^7}$; atque hæc vis posterior subducta ex vi $D \times c \times -\frac{4kb^3}{5l^5} + \frac{2kbb^3}{l^7}$ relinquet $D \times c \times \frac{4kb^3}{5l^5}$ pro vi perturbatrice in directione SD . Unde cum massa tota planetæ fit $\frac{2abD}{3}$, gravitas satellitis tota in planetam erit $\frac{2abD}{3l^2}$ proximè, vel etiam $\frac{2bbD}{3l^2}$, et hæc gravitas est ad vim $D \times c \times \frac{4kb^3}{5l^5}$ ut 1 ad $\frac{6kbc}{5l^3}$.

Deinde vis illius $D \times c \times \frac{4kb^3}{5l^5}$ secundum SD pars ea quæ agit in directione SC est $D \times c \times \frac{4kkb^3}{5l^6}$, quæ addita vi Sq dat $D \times c \times \frac{2b^3}{5l^4} - \frac{6kkb^3}{5l^6}$ vim perturbatricem tendentem ad centrum planetæ primarii, atque hæc vis est ad satellitis gravitatem $\frac{2bbD}{3l^2}$ in primarium ut $\frac{3bc}{5l^2} - \frac{9kkbc}{5l^4}$ ad 1. *Q. E. I.*

COROLL.

Designet CK (*Fig. 3.*) lineam intersectionis planorum æquatoris planetæ et orbitæ satellitis, et resolvatur vis $SD = \frac{6kbc}{5l^3}$, quæ agit perpendiculariter ad

planum æquatoris, in vim DR perpendicularem ad planum orbitæ satellitis, et in vim SR jacentem in eodem plano. Producat SR donec occurrat CK in K, eritque SK normalis ad CK, et planum SDK normale ad planum orbis satellitis; ac propterea ob similia triangula SDK, SRD, si m denotet finum ad radium 1 et n cosinum anguli SKD, inclinationis scilicet orbitæ satellitis ad æquatorem planetæ, erit $DR = SD \times n = \frac{6kbcn}{5l^3}$, et $SR = SD \times m = \frac{6kbcm}{5l^3}$, existente 1 gravitate totâ satellitis in primarium suum. Jam quoniam vis SR jacet in plano orbitæ satellitis, hujus plani situm non mutat; accelerat quidem vel retardat motum satellitis revolventis, sed hæc acceleratio vel retardatio ob brevitatem temporis ad quantitatem sensibilem non exurgit: vis DR eidem plano perpendicularis continuò mutat ejus situm, et motum nodi generat, quem sequenti propositione definiemus.

PROPOSITIO II.

PROBLEMA.

Invenire motum nodi ex prædictâ causâ oriundum.

Per motum nodi in hac propositione intelligo motum intersectionis planorum æquatoris planetæ et orbitæ satellitis; orbitam autem satellitis quamproximè circularem suppono. Esto S locus satellitis in orbe suo SN cujus centrum C, (Fig. 4.) SF arcus centro C descriptus perpendicularis in circulum æquatoris planetæ FN; SB arcus eodem centro descriptus perpendicularis ad orbem SN, atque in SB sumatur lineola Sr æqualis duplo spatio, quod fatelles percurrere posset impellente vi DR in Coroll. præced.

determinatâ, quo tempore in orbe suo describeret arcum quàm minimum pS : per puncta r , p , describatur centro C circulus $rp n$ secans equatorem in n , qui exhibebit situm orbitæ satellitis post illam particulam temporis, nodo N translato in n . Agantur SC , CN , et SH perpendicularis in lineam nodorum CN , et Nm perpendicularis in $rp n$. Jam cum sint lineolæ Sr , Nm , ut sinus arcuum Sp , SN , erit Sp . $Sr :: SH . Nm$; deinde in triangulo rectangulo Nmn habetur $m . 1 :: Nm . Nn$; unde per compo-

sitionem rationum $Sp \times m . Sr :: SH . Nn = \frac{Sr \times SH}{Sp \times m}$:

dato igitur arcu Sp , est Nn five motus nodi ut $Sr \times SH$. In triangulo sphærico rectangulo SFN est sinus anguli N , hoc est, anguli inclinationis orbitæ satellitis ad æquatorem planetæ, ad sinum arcûs SF , ut radius ad sinum arcûs SN , id est, $m . \frac{k}{l} :: 1 . SH$, adeoque

$\frac{k}{l} = m \times SH$; est igitur $\frac{k}{l}$ ut SH . Vis autem Sr per

Coroll. Prop. præced. est ut $\frac{k}{l}$, adeoque ut SH ; quamobrem est $Sr \times SH$, proindeque et Nn , ut \overline{SH}^2 , hoc est, motus horarius nodi vi præfatâ genitus est in duplicatâ ratione distantie satellitis à nodo. Et quoniam summa omnium \overline{SH}^2 , quo tempore fatelles periodum suam absolvit, est dimidium summæ totidem \overline{SC}^2 , ideò motus periodicus est subduplus ejus qui, si fatelles in declinatione suâ maximâ ab æquatore planetæ continuò perstaret, eodem tempore generari posset. Sit igitur fatelles in maximâ suâ declinatione five in quadraturâ cum nodo, eritque SN quadrans circuli, et Nm mensura anguli Npm five Spr , eritque in hoc casu Nn five motus horarius nodi ad Nm , hoc est, ad angulum Spr , ut 1 ad m ;

est autem angulus Spr ad duplum angulum, quem subtendit sinus versus arcus Sp satellitis gravitate in primarium eodem tempore descripti, id est, ad angulum SCp qui est motus horarius satellitis circa primarium, ut vis Sr ad gravitatem satellitis in primarium, hoc est (per Coroll. Prop. I.), ut $\frac{6kbcn}{5l^3}$ ad 1, five, quia est in hoc casu $\frac{k}{l} = m$, ut $\frac{6bcmn}{5l^2}$ ad 1. Unde conjunctis rationibus est motus horarius nodi ad motum horarium satellitis ut $\frac{6bcn}{5l^2}$ ad 1; et si S denotet tempus periodicum solis apparens, et L tempus periodicum satellitis circa primarium suum, cum sit motus horarius satellitis ad motum horarium solis ut S ad L , erit motus horarius nodi ad motum horarium solis ut $\frac{6bcn}{5l^2} \times \frac{S}{L}$ ad 1, et in eadem ratione erit motus nodi annuus ad motum solis annum, hoc est, ad 360° . Quare, si satelles maneret toto anno in maximâ suâ declinatione ab æquatore primarii, vis prædicta ex figurâ sphæroidicâ planetæ primarii proveniens generaret eodem tempore motum nodi æqualem $\frac{6bcn}{5l^2} \times \frac{S}{L} \times 360^\circ$, et ex supradictis motus verus nodi annuus erit hujus subduplus, nempe $\frac{3bcn}{5l^2} \times \frac{S}{L} \times 360^\circ$. *Q. E. I.*

COROLL.

Si computatio instituitur pro lunâ, assumendo mediocrem ejus orbitæ inclinationem ad æquatorem terrestrem, erit n cosinus anguli $23^\circ 28' \frac{1}{4}$; et posito femiaxi terræ $b = 1$, erit distantia lunæ à centro terræ mediocris $l = 60$ circiter, indeque in hypothefi quod
sit

fit differentia semiaxium $c = \frac{1}{229}$, erit $\frac{3bcn}{5l^2} \times \frac{S}{L} \times 360^\circ = 11''\frac{1}{4}$; et si fuerit $c = \frac{1}{177}$, manente terrâ uniformiter densâ, erit ille motus = $15''$. Hic erit motus nodorum annuus lunæ regressivus in plano æquatoris terrestris, qui reductus ad eclipticam, uti postea docebitur, pro vario nodorum situ evadet multò velocior.

Notabilis multò magis erit motus intersectionis orbitarum satellitum Jovis in plano æquatoris Jovialis; et computabitur satis accuratè per formulam supra traditam, modò satelles non sit Jovi nimis vicinus. Sic pro satellite extimo erit $L = 16^d 16^h 32'$, $b = 1$, $l = 25,299$ circiter, semiaxium Jovis differentia $c = \frac{1}{13}$; et positâ orbis hujus satellitis inclinatione ad æquatorem Jovis æquali 3° , erit n cosinus hujus inclinationis, atque inde prodibit $\frac{3bcn}{5l^2} \times \frac{S}{L} \times 360^\circ = 34'$ circiter, motus scilicet nodorum annuus satellitis quarti in plano æquatoris Jovis in antecedentia. Si minùs vel magis inclinatur orbis ad Jovis æquatorem, augeri vel minui debet hic motus in ratione cosinûs hujus inclinationis.

Cæterùm patet motum hunc nodorum in plano æquatoris planetæ primarii, æstimando distantiam satellitis in semidiametris primarii, generatim esse, dato tempore, in ratione compositâ, ex ratione directâ differentiæ semiaxium planetæ et cosinûs inclinationis orbis satellitis ad planetæ æquatorem, conjunctim; et ex ratione inversâ temporis periodici satellitis et quadrati distantiae satellitis à centro planetæ, item conjunctim.

PROPOSITIO III.

PROBLEMA.

Motum nodorum Lunæ supra determinatum ad Eclipticam reducere.

Sunto NAD (Fig. 5.) æquator, AGE ecliptica secans æquatorem in A, E æquinoctium vernal, A autumnale, LGN orbis lunæ secans eclipticam in G et æquatorem in N, LD circulus maximus perpendicularis in æquatorem; et sunt DN, LN, quadrantes circuli. Tempore dato vi prædictâ transferatur intersectio N in n , et describatur circulus Lgn exhibens situm orbis lunaris post illud tempus, secetque eclipticam in g . Ut autem intersectiones N et G sine verborum ambagibus distinguantur, priorem in posterum vocabo *Nodum Æquatorium*, posteriorem *Nodum Eclipticum*. Ductis itaque Nm, Gd, perpendicularibus in orbem lunæ, est $Nn : Nm :: 1 : \sin. GNA$, et $Nm : Gd :: 1 : \sin. LG$, itemque $Gd : Gg :: \sin. Ggd : 1$; unde conjunctis rationibus provenit $Nn : Gg :: \sin. Ggd : \sin. GNA \times \sin. LG$, adeoque $Gg = Nn \times \frac{\sin. GNA \times \sin. LG}{\sin. Ggd}$. Scribantur

s pro sinu et t pro cosinu anguli Ggd , inclinationis scilicet orbitæ lunaris ad eclipticam, ad radium 1, v pro sinu et u pro cosinu arcûs EG, p pro sinu et q pro cosinu obliquitatis eclipticæ; atque per resolutionem trianguli sphærici GAN, habebitur $\cos. GNA = n = \frac{qt + psu}{\sqrt{1 - qqt - 2pqstu - p^2s^2u^2}}$; sed scribi potest 1 pro t , et rejici terminus $p^2s^2u^2$ ob exiguitatem sinûs s anguli

5° 8'½, proindeque erit fin. GNA = $\sqrt{pp - 2pqsu}$; præterea est fin. GNA : fin. GA five $v ::$ fin. GAN five p : fin. GN, ideoque fin. GN five cos. LG = $\frac{pv}{\text{fin. GNA}}$, et fin. LG = $u - \frac{qsuv}{p}$, ac fin. GNA × fin. LG = $pu - qs$ quamproximé. Quare fit Gg = $Nn \times \frac{pu - qs}{s}$, atque hic est motus nodorum lunarium tempore dato in plano eclipticæ: quod si tempus illud datum fit annus solaris, habetur $Nn = \frac{3bcn}{5l^2} \times \frac{S}{L} \times 360^\circ$, unde motus ille eclipticus nodorum annuus, nullâ habitâ ratione mutationis fitûs nodorum ex aliâ causâ per id temporis factæ, fiet $\frac{3bc}{5l^2} \times \frac{qt + psu}{s} \times \frac{pu - qs}{s} \times \frac{S}{L} \times 360^\circ$, vel etiam $\frac{3bcq}{5l^2} \times \frac{pu - qs}{s} \times \frac{S}{L} \times 360^\circ$ proximé. *Q. E. I.*

Quo motum nodi lunaris in hac propositione ad eclipticam reduximus, eodem prorsus ratiocinio motus nodi satellitis cujufvis ad orbitam planetæ primarii reducetur.

COROLL. I.

Exinde liquet nullum esse hunc motum nodi, ubi fin. LG = 0, vel etiam ubi $pu = qs$, quod contingit ubi orbitæ lunaris arcus GN eclipticam et æquatorem æqualis est 90°, five ubi nodi lunares versantur in punctis declinationis lunaris maximæ, five ubi arcus AG, cujus cosinus est u , evadit æqualis 78° 5', id est, ubi nodus ascendens lunæ versatur in 11° 55' Cancrî, vel 18° 5' Sagittarii. Eritque progressivus hic motus, id est, fiet secundum seriem signorum, dum nodus ascendens lunæ transît retrocedendo ab

18° 5' Sagittarii ad 11° 55' Cancrī, regressivus autem in reliquā parte revolutionis; et maximus evadit motus regressivus, ubi $u = -1$, id est, ubi nodus ascendens versatur in principio Arietis; et maximus progressivus, ubi $u = 1$, id est, ubi idem nodus occupat initium Libræ. Itaque cū motus ille nodorum annuus, de quo hīc agitur, universaliter fit æqualis $\frac{3bcq}{5^{12}} \times \frac{pu - qs}{s} \times \frac{S}{L} \times 360^\circ$, hoc est, per Coroll. Prop. 2. æqualis $11'' \frac{1}{2} \times \frac{pu - qs}{s}$ vel $15'' \times \frac{pu - qs}{s}$ prout differentia semiaxium terræ fuerit $\frac{1}{2} \frac{1}{29}$ vel $\frac{1}{177}$, existentibus scilicet p finu et q cosinu anguli $23^\circ 28' \frac{1}{2}$, atque s finu anguli $5^\circ 8' \frac{1}{2}$; eo anno, in cujus medio circiter nodus lunæ ascendens tenuerit principium Arietis, motus nodorum regressivus, qui et maximus, erit $1' 2''$ vel $1' 20''$; ubi verò idem nodus subierit signum Libræ, motus maximus progressivus erit $41''$ vel $53''$. In aliis nodorum positionibus eodem modo computabitur.

COROLL. II.

Si desideretur excessus regressûs nodi supra progressum in integrâ nodi revolutione, sequenti ratione investigabitur. Jungantur equinoctia diametro EA, in quam demittatur perpendicularum GK, et sumpto arcu Gb quem describit nodus eclipticus G quo tempore nodus equatorius N describit arcum Nn, ducatur bc perpendicularis in GK. Per hanc propositionem est $Gg . Nn :: \frac{pu - qs}{s} . 1$, five, quia est $1 . u :: Gb . Gc$, fit $Gg . Nn :: \frac{p \times Gc}{s} - q \times Gb . Gb$; adeoque summa omnium Gg erit ad summam omnium

nium Nn , hoc est, motus nodi ecliptici in integrâ sui revolutione erit ad motum nodi æquatorii eodem tempore factum, ut summa omnium in circulo quantitatum $\frac{p \times Gc}{s} - q \times Gb$ ad summam totidem arcuum Gb , hoc est, ut $-q$ ad 1. Signum autem $-$ denotat motum fieri in antecedentia sive regressum nodi excedere ejusdem progressum. Unde cum motus nodi æquatorii N sit $11''\frac{1}{2}$ vel $15''$ quo tempore nodus eclipticus describit $19^\circ 20'\frac{1}{2}$, motus ille nodi æquatorii tempore nodi ecliptici periodico evadit $11''\frac{1}{2} \times \frac{360^\circ}{19^\circ 20'\frac{1}{2}} = 3' 34''$ vel $15'' \times \frac{360^\circ}{19^\circ 20'\frac{1}{2}} = 4' 39''$; quo pacto prodit motus nodi ecliptici præfatus æqualis $q \times 3' 34''$ vel $q \times 4' 39''$, proindeque est radius ad cosinum obliquitatis eclipticæ ut $3' 34''$ vel $4' 39''$ ad motum quæsitum, nempe $3' 16''$, existente $\frac{1}{2}\frac{1}{9}$ differentiâ axium terræ, vel $4' 16''$ eâ existente $\frac{1}{1}\frac{1}{77}$: atque hic est excessus regressus nodi supra progressum in integrâ nodi revolutione vi prædictâ genitus. Excessu igitur hoc minuatur motus nodi lunaris periodicus 360° , et remanebit motus ille quem generat vis solis.

PROPOSITIO IV.

PROBLEMA.

Variationem inclinationis orbis lunaris ad planum eclipticæ ex figurâ terræ spheroidicâ ortam determinare.

Esto ANH (Fig. 6.) æquator, AG ecliptica, et A punctum æquinoctii autumnalis: sit $NGRM$ orbis lunæ secans eclipticam in G et æquatorem in N , in
VOL. 50. 5 N quo

quo sumantur arcus NL, GR, æquales quadrantibus circuli. Jam si nodus æquatorius N per temporis particulam vi prædictâ transferri intelligatur in n , et per punctum L describatur circulus nLr , exhibebit hic situm orbis lunæ post tempus elapsum, et si in eundem demittantur perpendiculara Nm et Rr , posterius Rr designabit variationem inclinationis orbitæ lunaris ad eclipticam eodem tempore genitam. Est autem $Nn : Nm :: 1 : m$, itemque $Nm : Rr :: 1 : \sin. LR$; sed ob $NL = GR$, est $NG = LR$; unde conjunctis rationibus est $Nn : Rr :: 1 : m \times \sin. NG$; ex quo patet variationem inclinationis momentaneam esse proportionalem sinui distantie nodi lunaris ecliptici à nodo æquatorio. Ad diametrum NM demittatur perpendicularum GK, et existente Gb decremento arcus NG factò quo tempore nodus æquatorius N describit arcum Nn , agatur bk parallela ipsi GK, eritque $1 : GK$ five $\sin. NG :: Gb . Kk$; proindeque jam erit $Nn : Rr :: Gb : m \times Kk$, adeoque summa omnium variationum Rr , quo tempore nodus eclipticus G descripsit arcum MG, genitarum erit ad summam totidem motuum Nn , hoc est, ad motum nodi æquatorii N eodem tempore factum, ut summa omnium Kk ducta in m , ad summam totidem arcuum Gb , id est, ut $m \times MK$ ad MG. Sit NH motus nodi N tempore revolutionis nodi G ab uno equinoctio ad alterum, eritque variatio inclinationis eodem tempore genita, hoc est, variatio tota æqualis $\frac{2m \times NH}{MGN}$.

Unde cum $\frac{NH}{MGN}$ exprimat rationem motus nodi æquatorii ad motum nodi ecliptici, prodit theorema sequens: *Est motus nodi lunaris ecliptici ad motum nodi æquatorii, ut sinus duplicatus inclinationis medio-*

cris orbitæ lunaris ad æquatorem, ad sinum variationis totius inclinationis ejusdem orbitæ ad eclipticam.

In hoc computo inclinationem mediocrem orbis lunaris ad æquatorem, nempe $23^{\circ} 28' \frac{1}{2}$, usurpo, cum in revolutione nodi tantum ex unâ parte augetur, quantum ex alterâ minuitur, et omnes minutias hîc expendere supervacaneum foret. Motus autem nodi lunaris ecliptici est ad motum nodi lunaris æquatorii ut $19^{\circ} 20' \frac{1}{2}$ ad $11'' \frac{1}{2}$ vel $15''$, sive ut 6055 vel 4642 ad 1, unde per theorema supra traditum prodit variatio inclinationis tota æqualis $27''$ vel $35''$, prout differentia axium terræ statuitur $\frac{1}{229}$ vel $\frac{1}{177}$. Hac igitur quantitate augetur inclinatio orbis lunaris ad eclipticam in transitu nodi ascendentis lunæ ab æquinoctio vernali ad autumnale, et tantumdem minuitur in alterâ medietate revolutionis nodi. In loco quolibet G inter æquinoctia variatio inclinationis est ad variationem totam ut sinus versus arcûs MG ad diametrum, ut patet; sive differentia inter semissim variationis totius et variationem quæsitam est ad ipsam semissim variationis totius ut cosinus arcûs MG ad radium, hoc est, ut $u - \frac{q^2vv}{p}$ ad 1. Q. E. I.

PROPOSITIO V.

PROBLEMA.

Motum apsidum in orbe satellitis quamproximè circulari, quatenus ex figurâ planetæ primarii sphaericâ oritur, investigare.

Per propositionem primam vis perturbatrix, quâ trahitur satelles ad centrum planetæ primarii, est ad

satellitidis gravitatem in ipsum primum, ut $\frac{3^{bc}}{5^{l^2}} - \frac{9^{kkbc}}{5^{l^4}}$ ad 1, five, quia per Prop. 2. est $\frac{k}{l} = m \times SH$ (*Fig. 4.*) ponendo scilicet m pro sinu inclinationis orbitæ satellitidis ad æquatorem primarii, et scribendo y pro SH , ut $\frac{3^{bc}}{5^{l^2}} \times 1 - 3m^2y^2$ ad 1; et summa harum virium in totâ circumferentiâ cujus radius est 1, est ad gravitatem satellitidis toties sumptam ut $\frac{3^{bc}}{5^{l^2}} \times 1 - \frac{3m^2}{2}$ ad 1. Vis igitur mediocris, quæ uniformiter agere in satellitem supponi potest, dum revolutionem suam in orbitâ propemodum circulari absolvit, est ad ejus gravitatem in primum ut $\frac{3^{bc}}{5^{l^2}} \times 1 - \frac{3m^2}{2}$ ad 1; atque hac vi movebuntur apfides, si nulla habeatur ratio vis alterius quæ orbis radio est perpendicularis et per medietatem revolutionis satellitidis in unum sensum tendit, per alteram medietatem in contrarium. Jam quia ex demonstratis in hac et primâ propositione sequitur gravitatem satellitidis circa planetam, cujus figura est sphærois oblata, revolventis in distantîâ l generaliter esse ad ejusdem gravitatem in majori distantîâ L , ut $\frac{1}{l^2} + \frac{B}{l^4} \times 1 - \frac{3m^2}{2}$ ad $\frac{1}{L^2} + \frac{B}{L^4} \times 1 - \frac{3m^2}{2}$, existente B quantitate datâ exigui valoris, five ut $\frac{1}{l^2}$ ad $\frac{1}{L^2} - \frac{B}{l^2L^2} \times 1 - \frac{3m^2}{2} + \frac{B}{L^4} \times 1 - \frac{3m^2}{2}$ quamproximé, ideò gravitas satellitidis diminuitur in majori quam duplicatâ ratione distantîæ auctæ quoties m minor est quantitate $\sqrt{\frac{2}{3}}$, id est, ubi inclinatio orbitæ satellitidis ad planetæ æquatorem non attingit 54°

44'; diminuitur autem in minori ratione, quoties est m major quàm $\sqrt{\frac{2}{3}}$, id est, ubi illa inclinatio superat $54^{\circ} 44'$; adeoque in priore casu progrediuntur apfides orbis satellitis, in posteriori regrediuntur. Quantitas autem hujus progressûs vel regressûs sic innotescet.

Per exemplum tertium prop. 45. lib. I. *Princ. Math. Newt.* si vi centripetæ, quæ est ut $\frac{1}{p}$, addatur vis altera ut $\frac{e}{p}$, hoc est, quæ sit ad vim centripetam $\frac{1}{p}$ ut $\frac{e}{p}$ ad 1, angulus revolutionis ab apside unâ ad eamdem erit $360^{\circ} \sqrt{\frac{1+e}{1-e}}$ vel $\frac{360^{\circ}}{1-e}$ quamproximè, existente e quantitate valdè minutâ. Porro cum sit motus satellitis in orbitâ suâ revolventis ad motum apfidis ut $\frac{360^{\circ}}{1-e}$ ad $\frac{360^{\circ}}{1-e} - 360^{\circ}$, hoc est, ut 1 ad e , erit motus apfidis tempore revolutionis satellitis ad sidera æqualis $360^{\circ} \times e$, et hic motus apfidis erit ad ejusdem motum tempore alio quovis dato ut tempus periodicum satellitis ad tempus datum. Est autem in hac nostrâ propositione $e = \frac{3b^2}{5l^2} \times 1 - \frac{3m^2}{2}$; unde datur motus apfidum quæsitus. *Q. E. I.*

COROLL.

Si ad lunam referatur hæc determinatio, habebuntur $b = 1$, $l = 60$, $m =$ sinui anguli $23^{\circ} 28' \frac{1}{2}$, et si fuerit $c = \frac{1}{2} \frac{1}{2} 9$, erit $e = \frac{1}{180} \frac{1}{3} \frac{1}{203}$, atque motus apogæi lunæ spatio centum annorum æqualis $16'$ proximè in consequentia; si fuerit $c = \frac{1}{1} \frac{1}{77}$, erit $e = \frac{1}{139} \frac{1}{3} \frac{1}{74} \frac{1}{2}$, et motus apogæi æqualis $20'$, 7. Hac igitur quantitate minuendus est motus medius apogæi lunæ
prout.

prout observationibus determinatur, ut habeatur motus ille quem generat vis solis.

Pro quarto autem Jovis satellite, erunt $b = 1$, $l = 25,299$, $c = \frac{1}{13}$, $m = \text{finui anguli } 3^\circ$, $e = \frac{1}{139\frac{1}{2}4,7}$; hincque motus apsidis spatio unius anni solaris prodit $33', 95$ vel ferè $34'$ in consequentia, qui tempore annorum decem fit $5^\circ 40'$. Insuper autem notandum est vi solis perturbari motum satellitis simili modo quo perturbatur motus lunæ; ideoque, quoniam vis solis, quâ perturbatur motus lunæ est ad lunæ gravitatem in terram in duplicatâ ratione temporis periodici lunæ circa terram ad tempus periodicum terræ circa solem, hoc est, ut 1 ad $178,725$; pariter vis solis, qua perturbatur motus satellitis Jovialis, est ad ipsius satellitis gravitatem in Jovem in duplicatâ ratione temporum periodicorum satellitis circa Jovem et Jovis circa solem, hoc est, ut 1 ad $67394,6$: vires igitur, quibus perturbantur motus lunæ et satellitis, sunt ad se invicem, relativè ad eorum gravitates in planetas suos primarios ut $\frac{1}{178,725}$ ad $\frac{1}{67394,6}$ sive ut $37,708$ ad 1. Unde cum viribus similibus proportionales sunt motus his viribus dato tempore geniti, si vis prior vel ejusdem vis pars quâlibet motum apsidis generat æqualem $40^\circ 40'\frac{1}{2}$ in orbe lunari annuatim, vis posterior vel ejusdem pars similis et proportionalis motum apsidis eodem tempore generabit æqualem $6'\frac{1}{2}$ in orbe satellitis, atque decem annorum spatio $1^\circ 5'$ in consequentia. Addatur $1^\circ 5'$ ad $5^\circ 40'$; et motus apsidum totus in orbe satellitis extimi Jovialis ex duabus prædictis causis oriundus spatio decem annorum erit $6^\circ 45'$ in consequentia. Observationibus Astronomicis collegit Ill. *Bradleius* hunc motum tempore prædicto esse quasi 6° ; differentia illa qualiscumque

liscumque 45' inter motum observatum et computatum actionibus satellitum interiorum debet ascribi.

SCHOLIUM.

Ex præcedentibus colligere licet motuum lunarium inæqualitates originem suam omnem non ducere ex vi solis, sed earum partem aliquam deberi actioni Telluris quatenus induitur figurâ sphæroidicâ. Sufficiat hîc illarum computasse valorem, et legem, quâ generantur, demonstraſſe: utrum autem hujusmodi correctiones tales sint ut tabulis Astronomicis inscribi mereantur, dijudicent Astronomi.

Item manifestum est præter inæqualitates eas, quæ in motibus satellitum Jovialium ex vi solis et actionibus satellitum in se invicem nascuntur, oriri alias ex figurâ Jovis sphæroidicâ ita notabiles ut Observationes Astronomicas continuò afficere debeant.

De Variatione motûs Terræ diurni.

Si terra globus esset omninò sphæricus quicumque foret revolutionis axis, manente eâdem in globo motûs quantitate, eadem maneret rotationis velocitas: secus autem est, ubi ob vires solis et lunæ terra induit formam sphæroidis oblongæ per aquarum ascensum. Hîc enim non confidero figuram telluris oblatam ob materiæ in æquatore redundantiam, sed sphæricam suppono nisi quatenus per aquarum elevationem et depressionem in sphæroidicam mutatur. Jam verò in sphæroide hujusmodi, quamvis eadem maneat motûs quantitas, mutatâ inclinatione axis transversî ad axem revolutionis, mutabitur revolutionis velocitas, uti satis manifestum est: cum autem axis trans-

transversus tranſit ſemper per ſolem vel lunam, ſingulis momentis mutabit ſitum ſuum reſpectu axis revolutionis ob motum quo hi duo planetae recedunt ab æquatore terreſtri et ad eum viciffim accedunt.

PROBLEMA.

Variationem motûs terræ diurni ex prædictâ cauſâ oriundam inveſtigare.

Exhibeat ſphæroidis oblonga ADCd (Fig. 7.) terram fluidam, cujus centrum T, AC axis transversus jungens centra terræ et ſolis vel lunæ, Dd axis minor, EO diameter æquatoris, et XZ axis motûs diurni. Centro T et radio TD deſcribatur circulus BDd ſecans axem transversum AC in B, et agatur BK perpendicularis in TE: tum ex quovis circuli puncto P ductâ PM ad axem XZ normali quæ ſecet TA in H, fit Ppr circumferentia circuli quam punctum P rotatione ſuâ diurnâ deſcribit, ad cujus quodvis punctum p ducatur Tp et producatnr donec occurrat ſuperficie ſphæroidis in q; deinde demiffâ pG perpendiculari in PM, et GF perpendiculari in TA, ſi per puncta AqC tranſire intelligatur ellipſis ellipſi ADC ſimilis et æqualis, erit ex naturâ curvæ, quia ſphæroidis noſtra parùm admodùm differt à ſphærâ, $pq = AB \times \frac{TF^2}{TP^2}$ quamproximé. Jam deſignet U velocitatem particulæ in terræ æquatore revolventis motu diurno circum axem XZ ad diſtantiam ſemidiametri TP, eritque $\frac{U \times PM}{TP}$ velocitas particulæ P circum Ppr deſcribentis, et cum ſit $TF = \frac{GM - HM \times TK}{TP} + TH$, erit
motus

motus totius lineolæ pq æqualis $pq \times \frac{U \times PM}{TP} =$
 $\frac{U \times AB \times PM}{TP^3} \times \frac{GM - HM \times TK^2}{TP} + TH$, adeoque

summa horum motuum in circuitu circuli Ppr , hoc est, motus superficiei inter circulum Ppr et sphæroidem in directione TP contentæ, æquabitur circumferentiæ hujus

circuli ductæ in $\frac{U \times AB \times PM}{TP^3} \times \frac{TK^2 \times PM^2}{2 TP^2} + \frac{TK^2 \times HM^2}{TP^2}$
 $- \frac{2 TK \times HM \times TH}{TP} + TH^2$ five quia est $HM \cdot TM$

:: $TK \cdot BK$, et $TH \cdot HM$:: $TP \cdot TK$, scribendo D pro circumferentiâ circuli BDd , æquabitur ille motus quantitati $\frac{U \times AB \times D}{2 TP^6} \times \frac{TK^2 \times PM^4 + 2 BK^2 \times TM^2 \times PM^2}{2}$.

Deinde horum motuum summa in toto circuitu globi collecta, hoc est, motus totius materiæ globo BDd incumbentis prodibit æqualis $\frac{U \times AB \times DD}{32} \times$

$\frac{3 TP^2 - BK^2}{TP^2}$. Ubi planeta in plano æquatoris consistit, fit $BK = 0$, et motus prædictus æqualis $\frac{U \times 3 AB \times DD}{32}$. Motus autem globi QPR circa eundem axem est (uti facile demonstratur) $\frac{U \times TP \times DD}{16}$,

adeoque motus terræ totius fit $\frac{U \times TP \times DD}{16} +$
 $\frac{U \times AB \times DD}{32} \times \frac{3 TP^2 - BK^2}{TP^2}$, qui cum idem semper

manere debeat, denotet V velocitatem in superficie æquatoris terrestris ubi planeta versatur in plano æquatoris, eritque $\frac{U \times TP \times DD}{16} + \frac{U \times 3 AB \times DD}{32} =$

$\frac{U \times TP \times DD}{16} + \frac{U \times AB \times DD}{32} \times \frac{3TP^2 - \overline{BK}^2}{TP^2}$; unde
 scribendo 1 pro TP quatenus est radius ad finem
 BK anguli BTK, habetur $V \cdot U :: TP + \frac{3AB}{2} -$
 $\frac{AB \times \overline{BK}^2}{2} \cdot TP + \frac{3AB}{2}$, indeque, quia minima est
 altitudo AB respectu semidiametri TP, $U - V \cdot V ::$
 $AB \times \overline{BK}^2 \cdot 2TP$, et $U - V = V \times \frac{AB \times \overline{BK}^2}{2TP}$: pro
 V autem patet scribi posse velocitatem angularem
 terræ mediocrem quia ab eâ differt quàm minimè et
 ducitur in quantitatem perexiguam $\frac{AB \times \overline{BK}^2}{2TP}$, et

quia tempora revolutionum terræ circa centrum suum
 sint reciprocé ut motus angulares U, V, fiet differen-
 tia revolutionum terræ ubi planeta æquatorem tenet
 et ubi ab æquatore distat angulo BTK, æqualis 23^h
 56' $\times \frac{AB \times \overline{BK}^2}{2TP}$. Quoniam igitur est acceleratio ho-

raria ad motum terræ horarium mediocrem circa cen-
 trum suum ut $AB \times \overline{BK}^2$ ad 2 TP sive (quia est sinus
 p inclinationis eclipticæ ad æquatorem ad radium 1
 ut sinus BK ad finem distantie planetæ ab æquinoc-
 tio, quem finem dico K) ut $AB \times p^2 \times K^2$ ad 2 TP;
 adeoque acceleratio horaria rotationis terræ crescit in
 ratione duplicatâ finis distantie planetæ à puncto
 æquinoctii, et summa omnium illarum acceleratio-
 num, quo tempore transit planeta ab æquinoctio ad
 solstitium, est ad summam totidem motuum hora-
 riorum mediocrium, hoc est, acceleratio tota eo tem-
 pore genita est ad tempus illud ut summa quantitatum
 omnium $AB \times p^2 \times K^2$ in circuli quadrante ad sum-
 mam

mam totidem $2 TP$, id est, quia summa omnium K^2 in circuli quadrante dimidium est summæ totidem quadratorum radii, ut $AB \times p^2$ ad $4 TP$. Quamobrem, si denotet P quartam partem temporis planetæ periodici circa terram, erit acceleratio tota motûs terræ circum axem suum in transitu planetæ ab æquinoctio ad solstitium genita æqualis $\frac{AB \times P \times p^2}{4 TP}$, atque eadem erit retardatio in transitu planetæ à solstitio ad æquinoctium. Unde sponte nascitur hoc Theorema: *Est quadratum diametri ad quadratum sinûs obliquitatis eclipticæ ut quarta pars temporis periodici solis vel lunæ ad tempus aliud; deinde, est semidiameter terræ ad differentiam semiaxium ut tempus mox inventum ad accelerationem quæsitam.*

Ascensus aquæ AB vi solis debitus est duorum pedum circiter, existente semidiametro terræ mediocri $TP = 19615800$, unde prodit per theorema acceleratio terræ circa centrum suum gyantis facta quo tempore incedit sol ab æquinoctio ad solstitium, æqualis $1'' 55^{iv}$ in partibus temporis; et si vi lunæ ascendunt aquæ ad altitudinem octo pedum, acceleratio revolutionis terræ inde orta, quo tempore luna transit ab æquatore ad declinationem suam maximam, erit 34^{iv} : et summa harum accelerationum, quæ obtinet ubi hi duo planetæ in punctis solstitialibus versantur, cùm non superet duo minuta tertia temporis cum semisse sive 37 minuta tertia gradûs, vix observabilis erit. *Q. E. I.*

Cùm igitur tantilla sit hujusmodi variatio in hypothefi sphæricitatis terræ; qualis evaderet, terrâ existente sphæroide oblatâ, frustra quis inquireret.

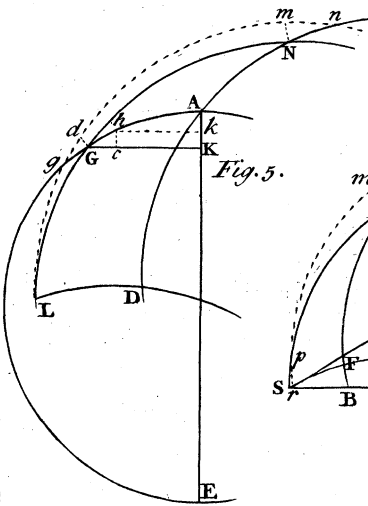
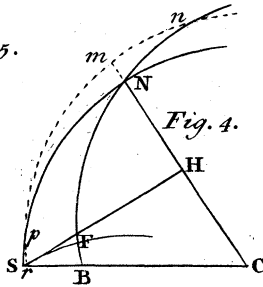
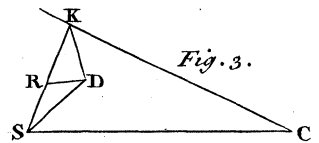
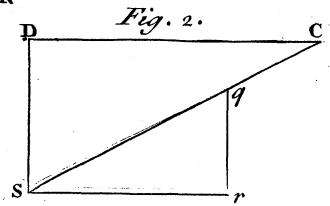
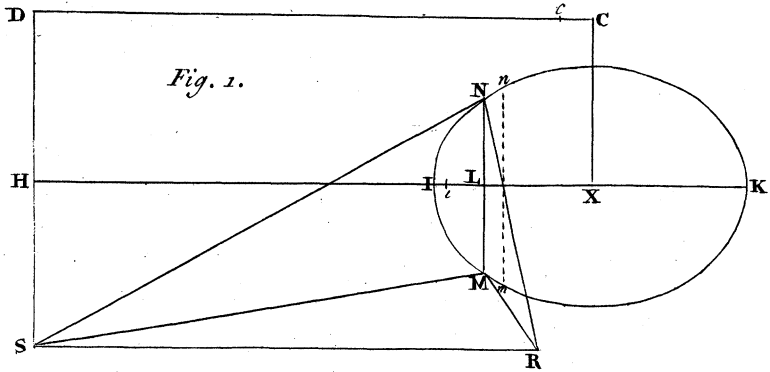


Fig. 7.

