to be an aſtringent mineral earth; whereas charcoal eaſily takes fire, burns freely without ſmoke, and continues burning, till it conſumes to an aſh; which conſiſts of an alkaline ſalt, and a pure earth, fit for making cuppels; and, by theſe marks, is ſufficiently diſtinguiſhed from all mineral ſubſtances.

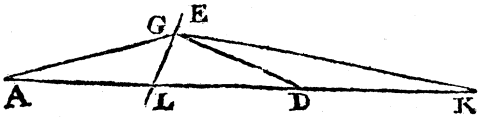
Groſvenor-Street, Jan. 8, 1761. Jer. Milles.

LXXXVI. *De Aberratione Luminis, in Superficiebus et Lentibus Sphæricis refractorum* *.

§ 1.

Read April 2, 1761.

SI radius luminis AG, incidens in ſuperficiem quamcunque refringentem LE, inflectatur ſecundum rectam GK, et quævis recta linea AK occurrat radio incidenti AG in A, refracto GK in K; et rectæ GD, normali ad ſuperficiem refringentem LE, in D; erit rectangulum GK × DA ad rectangulum GA × DK, ut ſinus anguli incidentiæ DGA ad ſinum anguli refractionis DGK. Eſt enim DA ad GA, ut ſin. DGA ad ſin. ADG, et GK : DK :: ſin. GDK : ſin. DGK; quare, ob



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* This Paper, though ſent to England in the ſummer of the year 1760, was, by accident, prevented from being read to the Royal Society, till the 2d of April following.

ſin.

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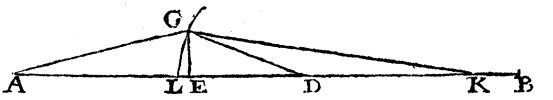
ſin. ADG = ſin. KDG, erit componendo DA × GK : GA × DK : : ſin. DGA : ſin. DGK. Quæ ratio, ſi ponatur ut i ad r, erit i . GA × DK = r . GK × DA.

§ 2.

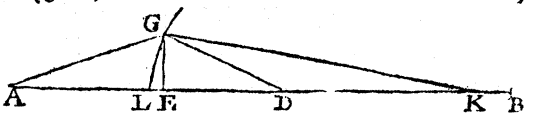
Incidat radius luminis AG in ſuperficiem ſphæricam LG, cujus centrum eſt D,



et refringatur ſecundum rectum GK, quæritur concurſus K radii refracti GK cum axe ſphæræ ALD, poſito arcu LG ſatis parvo.

A puncto incidentiæ G cadat GE normalis ad axem AD, ducaturque radius ſphæræ GD. His factis eſt per elementa, $AGq = ADq + DGq - 2AD \times DE = ADq + DLq - 2AD \times \overline{DL - LE} = \overline{AD - DL}q + 2AD \times LE = ALq + 2AD \times LE$, adeoque $AG = \sqrt{ALq + 2AD \times LE} =$ (ob LE ſatis parvam) $AL + \frac{AD \times LE}{AL}$ quamproxime. Similiter $KGq = KDq + DGq + 2KD \times DE = KDq + DLq + 2KD \times \overline{DL - LE} = \overline{KD + DL}q - 2DK \times LE = KLq - 2KD \times LE$, adeoque $KG = \sqrt{KLq - 2KD \times LE} = KL - \frac{KD \times LE}{KL}$ quamproxime.

Jam vero eſt (§ 1.) i. GA × DK = r . GK × DA, quare ſubſtitutis valoribus modo in-



ventis ipſarum GA et GK, habetur $i . KD \times (AL + \frac{AD \times LE}{AL}) = r . AD \times (KL - \frac{KD \times LE}{KL})$, et

tranſ-

tranſponendo $r.AD \times KL - i.KD \times AL = AD \times KD \times LE \times \left(\frac{i}{AL} + \frac{r}{KL}\right)$.

Ponatur jam $LA = A$, $LK = K$, radius ſphæræ $DL = a$, adeoque $AD = A + a$, et $DK = K - a$. His in noviſſima æquatione ſcriptis, erit $rK \times \overline{A + a} - iA \times \overline{K - a} = \overline{A + a} \times \overline{K - a} \times LE \times \left(\frac{i}{A} + \frac{r}{K}\right)$, unde tranſponendo et dividendo, $K = \frac{iAa}{\overline{i - r}.A - ra} - \frac{\overline{A + a} \times \overline{K - a} \times LE}{\overline{i - r}.A - ra} \times \left(\frac{i}{A} + \frac{r}{K}\right)$.

Jam ſi radius incidens AG fuerit axi AD viciniſſimus, evaneſcet LE, adeoque et terminus $\frac{\overline{A + a} \times \overline{K - a} \times LE}{\overline{i - r}.A - ra} \times \left(\frac{i}{A} + \frac{r}{K}\right)$. Quare in hoc caſu $K = \frac{iAa}{\overline{i - r}.A - ra}$, id eſt, diſtantia foci geometrici ipſi A conjugati à vertice L, erit $\frac{iAa}{\overline{i - r}.A - ra}$, unde aberratio radii refracti GK ab hoc foco erit $\frac{\overline{A + a} \times \overline{K - a} \times LE}{\overline{i - r}.A - ra} \times \left(\frac{i}{A} + \frac{r}{K}\right)$, ſumendo à foco contra directionem curſus radiorum. Quoniam vero aberratio illa ſemper eſt valde parva, erit $\frac{iAa}{\overline{i - r}.A - ra}$ valor prope verus diſtantiæ LK ſive K, adeoque in expreſſione aberrationis ſine ſenſibili errore pro K uſurpari poteſt. Sit itaque diſtantia LB foci geometrici B à vertice ſuperficiei L, ſive $\frac{iAa}{\overline{i - r}.A - ra} = B$,

eritque

eritque K ſive $LB = B - \frac{\overline{A+a}.\overline{B-a}.LE}{\overline{i-r}.A-ra} \times \left(\frac{i}{A} + \frac{r}{K}\right)$ quamproxime.

Dimidia latitudo ſuperficiei refringentis, ſive diſtantia puncti incidentiæ G ab axe, id eſt LG, dicatur L, erit quamproxime $LE = \frac{L^2}{2a}$: quare in formula modo inventa pro LE ſcribendo $\frac{L^2}{2a}$, et ulterius concinnando expreſſionem aberrationis ope æquationis $B = \frac{iAa}{\overline{i-r}.A-ra}$, habetur formula, qua in ſequentibus præcipue utemur: $LK = B - \frac{r.\overline{i-r}.B^2L^2}{2i^3} \times \left(\frac{1}{a} + \frac{1}{A}\right)^2 \times \left(\frac{r}{a} + \frac{i+r}{A}\right)$.

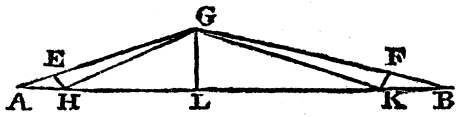
§ 3.

In formulis jam inventis ſigna ſymbolorum illi tantum caſui ſunt accommodata, quem figura exprimit, ubi radius luminis ab axe divergens ponitur incidere in ſuperficiem convexam, et poſt refractionem ad axem convergere. Accommodantur vero ad reliquos problematis caſus, mutando ſignum radii ſphæræ *a*, ſi radius luminis incidat in ſuperficiem concavam, et ſignum ipſius A, ſi radius incidens convergat ad axem. Hoc facto, ſi valor diſtantiæ LB ſive B prodit poſitivus, ſumenda eſt diſtantia illa à vertice L ſecundum directionem curſus radiorum; ſi negativus, contra eandem directionem. Aberratio vero radii à foco B, ſive BK per formulas computata, ſi fuerit poſitiva, ſumenda eſt à foco B contra directionem curſus radiorum; ſi negativa, ſecundum hanc directionem. Si radii incidentes fuerint axi paralleli,

ralleli, facienda eſt A infinita, et ſi B prodit infinita, radii refracti erunt axi paralleli, abſtrahendo ab illorum aberratione.

§ 4.

Si duo radii luminis AG, HG, in angulo quam minimo AGH inter ſe inclinati incidant fere perpendiculariter in idem punctum G ſuperficiei cujuſcunque refringentis LG, et refringantur, prior in GK, poſterior vero in GB, occurrentes lineæ rectæ cuicunque in punctis A, H, K, B; dico eſſe BK ad AH ut eſt $r.LB^q$ ad $i.LA^q$, poſita i ad r ut ſinus incidentiæ ad ſinum refractionis.

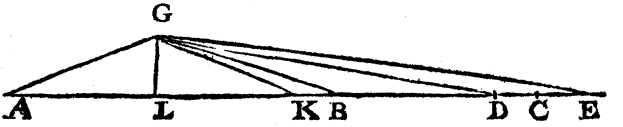


Quoniam enim anguli valde parvi ſunt ſinibus ſuis quamproxime proportionales, erunt in parvis refractionibus anguli incidentiæ et refractionis eorumque adeo differentiæ ſinibus incidentiæ et refractionis proportionales quamproxime. Eſt itaque angulus AGH, utpote differentia angulorum incidentiæ radiorum AG, HG, ad angulum BGK, ſive differentiam angulorum refractionis radiorum GK, GB, ut i ad r. Quare centro G inter hos angulos deſcriptis arcubus HE, KF, erit HD ad KF, ut $i.GH$ ad $r.GK$; ſed eſt $AH = \frac{GH.HE}{GL}$, et $BK = \frac{GK.KF}{GL}$, quare pro HE et KF, ſcribendo earum proportionales $i \times GH$ et $r \times GK$, habetur AH ad BK ut $i \times GH^q$ ad $r \times GK^q$, quamproxime, id eſt, ut $i \times LA^q$ ad $r \times LB^q$.

§ 5.

§ 5.

Data forma lentis refringentis, et data in ejus axe distantia foci radiorum incidentium, invenire punctum concursus radii cujusvis refracti cum axe.



Sit locus lentis in L, ejus axis A L B C, dimidia latitudo L G, sive distantia incidentiæ ab axe, = L; radius convexitatis superficiei primæ, in quam radii incidunt, = *a*; radius convexitatis superficiei secundæ, è qua emergunt, = *b*, radius luminis quivis incidens A G, refractus G K. In axe lentis A B C sit distantia foci radiorum incidentium, sive L A = A, distantia foci geometrici ipsi A conjugati, sive L B = B; distantia foci geometrici C superficiei primæ ipsi A conjugati, sive L C = C, et evidens est, esse etiam idem punctum C focum geometricum superficiei secundæ conjugatum ipsi B. Sit tandem distantia puncti concursus K radii refracti G K cum axe, sive L K = K.

Radius A G in prima superficie refractus tendat versus punctum D, et concipiatur à puncto B, quod est focus lentis ipsi A conjugatis, radius B G incidere

in secundam superficiem, et in ea ita refringi, ut divergat à puncto axeos E. Et dabuntur per § 2. aberrationes C E et C D, adeoque earum summa D E. Sed per § 4. est D E ad B K ut r. L C q ad i. L B q,

 quare

quare $BK = \frac{i.LBq}{r.LCq} \times DE = \frac{iB^2}{rC^2} \times DE$. Jam per § 2. eſt $CD = \frac{r.\overline{i-r}.C^2L^2}{2i^3} \times \left(\frac{1}{a} + \frac{1}{A}\right) \times \left(\frac{r}{a} + \frac{i+r}{A}\right)$ et $CE = \frac{r.\overline{i-r}.C^2L^2}{2i^3} \times \left(\frac{1}{b} + \frac{1}{B}\right) \times \left(\frac{r}{b} + \frac{i+r}{B}\right)$, quarum ſumma DE, ducta in $\frac{i.B^2}{r.C^2}$, dat aberrationem quæſitam $BK = \frac{iB^2}{rC^2} \times DE = \frac{\overline{i-r}.B^2L^2}{2i^2} \left[\left(\frac{1}{a} + \frac{1}{A}\right)^2 \times \left(\frac{r}{a} + \frac{\overline{i+r}}{A}\right) + \left(\frac{1}{b} + \frac{1}{B}\right)^2 \times \left(\frac{r}{b} + \frac{i+r}{B}\right)\right]$, quæ poſt evolutionem terminorum ſubtracta de LB, ſive B, relinquit diſtantiam puncti concurſus $LK = B - \frac{\overline{i-r}.B^2L^2}{2i^2} \times \left\{\begin{array}{l} \frac{r}{a^3} + \frac{i+3r}{Aa^2} + \frac{2i+3r}{A^2a} + \frac{i+r}{A^3} \\ + \frac{r}{b^3} + \frac{i+3r}{Bb^2} + \frac{2i+3r}{B^2b} + \frac{i+r}{B^3} \end{array}\right\}$.

Porro quoniam punctum C eſt focus ipſi A conjugatus reſpectu faciei primæ, erit $LC = \frac{iAa}{\overline{i-r}.A - ra}$; et quoniam idem punctum C eſt focus ipſi B conjugatus reſpectu ſuperficiei ſecundæ, erit $LC = -\frac{iBb}{\overline{i-r}.B - rb}$, per § 2. Hinc $\frac{iAa}{\overline{i-r}.A - ra} = -\frac{iBb}{\overline{i-r}.B - rb}$, adeoque $B = \frac{rAab}{\overline{i-r}.A.\overline{a+b} - rab}$, ſive $\frac{1}{A} + \frac{1}{B} = \frac{i-r}{r} \times \overline{\frac{1}{a} + \frac{1}{b}}$. Dato ſic valore ipſius B, cognoſcitur diſtantia concurſus radii refracti GK cum axe ſive LK, ejuſque aberratio à foco B ſive BK, quæ ob ſignum — ſumenda eſt à foco B contra directionem

directionem curſus radiorum, ſi valorem habet positivum; ſecundum vero, ſi negativum.

Accommodantur vero hæ formulæ ad reliquos problematis caſus eadem ratione ac dictum eſt § 3. de aberratione ſuperficiei ſimplicis.

§ 6.

Expreſſio aberrationis jam inventa, ad formam reducitur uſui magis accommodatam, conjungendo terminos, in quibus a et b eaſdem habent dimenſiones, et reducendo ſingulas partes ope æquationis $\frac{i-r}{r} \times \frac{1}{a} + \frac{1}{b} = \frac{1}{A} + \frac{1}{B} = \frac{1}{P}$, ubi P eſt diſtantia foci principalis. Hac ratione calculum ſubducendo habetur: $\frac{1}{a^3} + \frac{1}{b^3} = \frac{r}{\overline{i-r}.P} \times \left(\frac{r^2}{\overline{i-r}^2.P^2} - \frac{3r}{\overline{i-r}.P.\overline{a+b}}\right)$; $\frac{1}{Aa^2} + \frac{1}{Bb^2} = \frac{r}{\overline{i-r}.P} \times \left(\frac{1}{Aa} + \frac{1}{Bb} - \frac{1}{P.\overline{a+b}}\right)$; $\frac{1}{A^2a} + \frac{1}{B^2b} = \frac{r}{\overline{i-r}.P} \times \left(\frac{i-r}{r} \times \overline{\frac{1}{Aa} + \frac{1}{Bb}} - \frac{1}{P.\overline{A+B}}\right)$; $\frac{1}{A^3} + \frac{1}{B^3} = \frac{r}{\overline{i-r}.P} \times \left(\frac{i-r}{rP^2} - \frac{3.\overline{i-r}}{rP.\overline{A+B}}\right)$.

Singulæ harum partium ductæ in ſuos reſpective coëfficientes, r, $i+3r$, $2i+r$, $i+r$, conjungantur cum ſignis propriis, et habebitur: $LK = LB - BK = B - \frac{B^2L^2}{2iP^2}\left(\frac{i^3 - 2i^2r + 2r^3}{\overline{i-r}^2.P} - \frac{3i+2r}{A+B} + \frac{2.\overline{i+r}}{A+B} \times \overline{\frac{A}{b} + \frac{B}{a}} - \frac{r.\overline{i+2r}}{\overline{i-r}.\overline{a+b}}\right)$, vel etiam $= B - \frac{B^2L^2}{2iP^2} \times \left(\frac{i^3}{\overline{i-r}^2.P} - \frac{3i+2r}{A+B} - \frac{2.\overline{i+r}}{A+B} \times \overline{\frac{A}{a} + \frac{B}{b}} - \frac{r.\overline{i+2r}}{\overline{i-r}.\overline{a+b}}\right)$.

§ 7.

§ 7.

Invenire lentem, quæ radios luminis à data diſtantia advenientis in alia data diſtantia colligat cum minima aberratione, ipſamque aberrationem minimam.

Retentis omnibus ſymbolis ut ſupra, ſumantur differentialia aberrationis modo inventæ, $\frac{B^2L^2}{2i.P^2} \times \left(\frac{i^3}{\overline{i-r}|^2.P} - \frac{3i+2r}{A+B} - \frac{2.\overline{i+r}}{A+B} \times \overline{\frac{A}{a}+\frac{B}{b}} - \frac{r.\overline{i+2r}}{\overline{i-r}.\overline{a+b}}\right)$, poſitis a et b variabilibus, et habebitur $\frac{2.\overline{i+r}}{A+B} \times \overline{\frac{A\,da}{a^2} \times \frac{B\,db}{b^2}} + \frac{r.\overline{i+2r}}{i-r} \times \frac{da+db}{\overline{a+b}|^2} = 0$; ſed ob $\frac{1}{a}+\frac{1}{b} = \frac{r}{\overline{i-r}.P}$, eſt $\frac{da}{a^2}+\frac{db}{b^2} = 0$; quare tollendo per comparationem harum æquationum differentialia, invenitur: $\frac{b-a}{b+a} = \frac{2.\overline{i+r}.\overline{i-r}}{r.\overline{i+2r}} \times \frac{A-B}{A+B}$, adeoque $\frac{a}{b} = \frac{\overline{4r^2+ri-2i^2}.A+i.\overline{2i+r}.B}{\overline{4r^2+ri-2i^2}.B+i.\overline{2i+r}.A}$; unde per æquationem $\frac{1}{a}+\frac{1}{b} = \frac{r}{i-r} \times \overline{\frac{1}{A}+\frac{1}{B}}$, colliguntur radii facierum lentis quæſitæ, ſcil. $a = \frac{2.\overline{i-r}.\overline{i+2r}.AB}{i.\overline{2i+r}.A+\overline{4r^2+ri-2i^2}.B}$, et $b = \frac{2.\overline{i-r}.\overline{i+2r}.AB}{i.\overline{2i+r}.B+\overline{4r^2+ri-2i^2}.A}$; ſive $\frac{1}{a} = \frac{i.\overline{2i+r}}{2.\overline{i-r}.\overline{i+2r}} \times \frac{1}{B} + \frac{4r^2+ri-2i^2}{2.\overline{i-r}.\overline{i-2r}} \times \frac{1}{A}$; et $\frac{1}{b} = \frac{i.\overline{2i+r}}{2.\overline{i-r}.\overline{i+2r}} \times \frac{1}{A} + \frac{4r^2+ri-2i^2}{2.\overline{i-r}.\overline{i+2r}} \times \frac{1}{B}$.

Hic

Hic valores radiorum a et b ſubſtituti in formula aberrationis generali § 6. dant ipſam aberrationem minimam $= \frac{i \,.\, B^2 L^2}{2 \,.\, \overline{i + 2r} \,.\, P^3} \times \left(\frac{P}{A + B} + \frac{r \,.\, \overline{4i - r}}{4 \,.\, \overline{i - r}^2} \right)$.

§ 8.

Poſtquam in § præcedente formam lentis invenimus, quæ in datis circumſtantiis aberrationem minimam producat, et ſimul ipſam ejus aberrationem; rationi conſentaneum eſt, inveſtigationem aberrationis lentium ab hac forma diſcendentium ita aggredi, ut ſimul appareat relatio formarum harum lentium ad illius formam, et aberrationis harum ad illius aberrationem. Solent enim concluſiones prodire concinniores, ſi ad caſum inter reliquos maxime ſingularem et unicum referantur, qualis hic eſt caſus aberrationis minimæ.

Scribatur brevitatis cauſa b pro $\frac{i \,.\, \overline{2i + r}}{2 \,.\, \overline{i - r} \,.\, \overline{i + 2r}}$, et K pro $\frac{4r^2 + ri - 2i^2}{2 \,.\, \overline{i - r} \,.\, \overline{i + 2r}}$, et erit pro determinandis radiis a et b lentis aberrationis minimæ, $\frac{1}{a} = \frac{b}{B} + \frac{K}{A}$, et $\frac{1}{b} = \frac{b}{A} + \frac{K}{B}$, per § 7. Ponatur jam generaliter pro radiis a et b facierum cujuſvis alius lentis, $\frac{1}{a} = \frac{b}{B} + \frac{K}{A} + \frac{x}{P}$, et $\frac{1}{b} = \frac{b}{A} + \frac{K}{B} - \frac{x}{P}$; quos valores ipſarum $\frac{1}{a}$ et $\frac{1}{b}$ tales aſſumo, ut earum ſumma $\frac{1}{a} + \frac{1}{b}$ ſit $= \overline{b + K} \times \overline{\frac{1}{A} + \frac{1}{B}}$, ſive $\frac{r}{i - r} \times \overline{\frac{1}{A} + \frac{1}{B}}$, quemadmodum

dum oportet. Subſtituantur hi valores ipſarum $\frac{1}{a}$ et $\frac{1}{b}$ in formula aberrationis generali, $\frac{B^2 L^2}{2 i P^2} \times \left(\frac{i^3}{P} - \frac{3 i + 2 r}{A + B} - \frac{2 . \overline{i + r}}{A + B} \times \overline{\frac{A}{a} + \frac{B}{b}} - \frac{r . \overline{i + 2 r}}{i - r . \overline{a + b}}\right)$ § 6. et peracto calculo invenietur $\frac{i B^2 L^2}{2 . \overline{i + 2 r} . P^3} \left(\frac{P}{A + B} + \frac{\overline{4 i - r} . r}{4 . \overline{i - r}^2} \frac{\overline{i + 2 r}^2}{i^2} x^2\right)$ pro aberratione lentis, cujus facies lumini obverſa radium habeat a, et facies à lumine averſa radium habeat b, exiſtentibus $\frac{1}{a} = \frac{b}{B} + \frac{K}{A} + \frac{x}{P}$, et $\frac{1}{b} = \frac{b}{A} + \frac{K}{B} - \frac{x}{P}$; P diſtantia foci principalis, A diſtantia foci radiorum incidentium, et B refractorum.

Commodum itaque hic accidit, ut numerus x, qui relationem formæ lentis ad lentem aberrationis minimæ deſignat, idem quoque relationem aberrationis ejus ad aberrationem minimam ratione non minus ſimplici exhibeat.

§ 9.

Si in formulis § præced. exhibitis, pro radiis facierum lentis ejuſque aberratione, ſtatuatur $x = 0$, radit lens ad eam formam, quæ aberrationem dat minimam, et aberratio ejus ad aberrationem minimam. Si vero non fuerit $x = 0$, apparet ex iiſdem formulis, cuivis aberrationis quantitati, quæ major ſit aberratione minima, duas diverſas reſpondere lentis formas, quarum utraque definitur æquationibus $\frac{1}{a} = \frac{b}{B} + \frac{K}{A} + \frac{x}{P}$, et $\frac{1}{a} = \frac{b}{A} + \frac{K}{B} - \frac{x}{P}$, ubi x pro altera

altera positive, altera negative accipitur. Nam in formula aberrationis $\frac{iB^2L^2}{2.\overline{i+2r}.P^3} \times \left(\frac{P}{A+B} + \frac{\overline{4i-r}.r}{4.\overline{i-r}^2} + \frac{\overline{i+2r}^2}{i^2}x^2\right)$ non comparet nisi quadratum ipsius x, quod idem manet, sive radix x sumatur positive, sive negative.

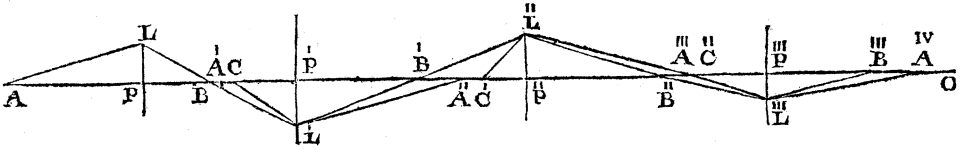
Quoniam aberratio quævis $\frac{iB^2L^2}{2.\overline{i+2r}.P^3} \times \left(\frac{P}{A+B} + \frac{\overline{4i-r}.r}{4.\overline{i-r}^2} + \frac{\overline{i+2r}^2}{i^2}x^2\right)$ est ad aberrationem minimam $\frac{iB^2L^2}{2.\overline{i+2r}.P^3} \times \left(\frac{P}{A+B} + \frac{\overline{4i-r}.r}{4.\overline{i-r}^2}\right)$ in ratione majoris inæquabilitatis, statuatur illa ratio $1+m^2$ ad 1; assumendo m pro numero quovis, et invenitur $x = \pm\frac{mi}{i+2r}\sqrt{\left(\frac{P}{A+B} + \frac{\overline{4i-r}.r}{4.\overline{i-r}^2}\right)}$. Et sic ex data ratione aberrationis alicujus lentis ad aberrationem minimam, datur illi correspondens numerus x, adeoque et radii facierum ejus, per superiora.

Numerus hic x, quoniam simul et formam et aberrationem suæ lentis indicat, dicatur *index* lentis, et scribatur brevitatis causa f pro $\frac{\overline{4i-r}.r}{4.\overline{i-r}^2}$, et g pro $\frac{1+2r}{i}$, ut aberratio sit $\frac{B^2L^2}{2g.P^3} \times \left(\frac{P}{A+B} + f + g^2x^2\right)$.

§ 10.

In linea recta A O, tanquam axe communi, dispositæ intelligantur lentes quotcunque P, $\overset{\text{I}}{P}$, $\overset{\text{II}}{P}$, $\overset{\text{III}}{P}$, &c. quarum foci conjugati sint respective, lentis

primæ A et $\overset{\mathrm{I}}{A}$; ſecundæ $\overset{\mathrm{I}}{A}$ et $\overset{\mathrm{II}}{A}$; tertiæ $\overset{\mathrm{II}}{A}$ et $\overset{\mathrm{III}}{A}$; quartæ $\overset{\mathrm{III}}{A}$ et $\overset{\mathrm{IV}}{A}$, &c.



Radius luminis, veniens à puncto A, refringatur in omnibus his lentibus in punctis L, $\overset{\mathrm{I}}{L}$, $\overset{\mathrm{II}}{L}$, $\overset{\mathrm{III}}{L}$, &c. pergens ſecundum curſum A L $\overset{\mathrm{I}}{L}$ $\overset{\mathrm{II}}{L}$ $\overset{\mathrm{III}}{L}$, et poſt ſingulas refractiones occurrat axi A O in punctis B, $\overset{\mathrm{I}}{B}$, $\overset{\mathrm{II}}{B}$, $\overset{\mathrm{III}}{B}$, &c. aberrans à focis $\overset{\mathrm{I}}{A}$, $\overset{\mathrm{II}}{A}$, $\overset{\mathrm{III}}{A}$, $\overset{\mathrm{IV}}{A}$, &c. longitudinibus $\overset{\mathrm{I}}{A}$ B, $\overset{\mathrm{II}}{A}$ $\overset{\mathrm{I}}{B}$, $\overset{\mathrm{III}}{A}$ $\overset{\mathrm{II}}{B}$, $\overset{\mathrm{IV}}{A}$ $\overset{\mathrm{III}}{B}$, &c. Quæruntur jam hæ aberrationes.

Lentium P, $\overset{\mathrm{I}}{P}$, $\overset{\mathrm{II}}{P}$, $\overset{\mathrm{III}}{P}$, &c. diſtantiæ focorum principalium dicantur reſpective P, $\overset{\mathrm{I}}{P}$, $\overset{\mathrm{II}}{P}$, $\overset{\mathrm{III}}{P}$, &c. et indices x, $\overset{\mathrm{I}}{x}$, $\overset{\mathrm{II}}{x}$, $\overset{\mathrm{III}}{x}$, &c. Diſtantiæ focorum conjugatorum à ſuis reſpective lentibus ſint: pro lente prima P, P A = A, P $\overset{\mathrm{I}}{A}$ = B; pro lente ſecunda $\overset{\mathrm{I}}{P}$, $\overset{\mathrm{I}}{P}$ $\overset{\mathrm{I}}{A}$ = $\overset{\mathrm{I}}{A}$, $\overset{\mathrm{I}}{P}$ $\overset{\mathrm{II}}{A}$ = $\overset{\mathrm{I}}{B}$; pro lente tertia $\overset{\mathrm{II}}{P}$, $\overset{\mathrm{II}}{P}$ $\overset{\mathrm{II}}{A}$ = $\overset{\mathrm{II}}{A}$, $\overset{\mathrm{II}}{P}$ $\overset{\mathrm{III}}{A}$ = $\overset{\mathrm{II}}{B}$; pro lente quarta $\overset{\mathrm{III}}{P}$, $\overset{\mathrm{III}}{P}$ $\overset{\mathrm{III}}{A}$ = $\overset{\mathrm{III}}{A}$, et $\overset{\mathrm{III}}{P}$ $\overset{\mathrm{IV}}{A}$ = $\overset{\mathrm{III}}{B}$, et ita porro: ponatur præterea P L, ſive diſtantia puncti incidentiæ in primam lentem ab axe = L, et erit $\overset{\mathrm{I}}{P}$ $\overset{\mathrm{I}}{L}$ = $\frac{\overset{\mathrm{I}}{A}}{B} \times L$, $\overset{\mathrm{II}}{P}$ $\overset{\mathrm{II}}{L}$ = $\frac{\overset{\mathrm{I}}{A} . \overset{\mathrm{II}}{A}}{B . \overset{\mathrm{I}}{B}} \times L$, $\overset{\mathrm{III}}{P}$ $\overset{\mathrm{III}}{L}$ = $\frac{\overset{\mathrm{I}}{A} . \overset{\mathrm{II}}{A} . \overset{\mathrm{III}}{A}}{B . \overset{\mathrm{I}}{B} . \overset{\mathrm{II}}{B}} \times L$, et ita porro.

I. Vidimus in § 9. aberrationem $\overset{\text{I}}{\text{A}}\text{B}$ lentis primæ P esse $\frac{\text{B}^2\text{L}^2}{2g.\text{P}^3} \times \left(\frac{\text{P}}{\text{A}+\text{B}} + f + g^2x^2\right)$, ubi $f = \frac{\overline{4i-r}.r}{4.\overline{i-r}^2}$, et $g = \frac{i+2r}{i}$.

II. Concipiatur à foco secundæ lentis $\overset{\text{II}}{\text{A}}$ ipsi $\overset{\text{I}}{\text{A}}$ conjugato in illam incidere radius $\overset{\text{II}}{\text{A}}\overset{\text{I}}{\text{L}}$, qui refractus secundum $\overset{\text{I}}{\text{L}}\text{C}$, conveniat cum axe in C, aberrans à foco $\overset{\text{I}}{\text{A}}$ longitudine $\overset{\text{I}}{\text{A}}\text{C}$: et erit per § 9. aberratio $\overset{\text{I}}{\text{A}}\text{C} = \frac{\overset{\text{I}}{\text{A}}{}^4\text{L}^2}{2g.\text{B}^2.\text{P}^3} \times \left(\frac{\overset{\text{I}}{\text{P}}}{\overset{\text{I}}{\text{A}}+\overset{\text{I}}{\text{B}}} + f + g^2\overset{\text{I}}{x}{}^2\right)$, scribendo scilicet $\frac{\overset{\text{I}}{\text{A}}}{\text{B}} \times \text{L}$ pro L, $\overset{\text{I}}{\text{B}}$ pro A, $\overset{\text{I}}{\text{A}}$ pro B, et P pro P. Hæc aberratio $\overset{\text{I}}{\text{A}}\text{C}$, addita ad aberrationem primæ lentis modo inventam, $\overset{\text{I}}{\text{A}}\text{B}$, dat summam BC. Est vero BC ad $\overset{\text{I}}{\text{B}}\overset{\text{II}}{\text{A}}$ ut $\overset{\text{I}}{\text{P}}\overset{\text{I}}{\text{A}}q$ ad $\overset{\text{I}}{\text{P}}\overset{\text{II}}{\text{A}}q$, sive $\overset{\text{I}}{\text{A}}{}^2$ ad $\overset{\text{I}}{\text{B}}{}^2$, id est quantitates aberrationum à focis conjugatis sunt ut quadrata distantiarum focorum à lente, quod facile probari potest. Ergo aberratio $\overset{\text{II}}{\text{A}}\overset{\text{I}}{\text{B}}$, à refractione per duas lentes P et $\overset{\text{I}}{\text{P}}$ producta, habetur $= \frac{\overset{\text{I}}{\text{B}}{}^2}{\overset{\text{I}}{\text{A}}{}^2} \times \text{BC}$

$$= \frac{\overset{\text{I}}{\text{B}}{}^2\text{L}^2}{2g} \times \left[\frac{\text{B}^2}{\text{A}^2.\text{P}^3}\left(\frac{\text{P}}{\text{A}+\text{B}} + f + g^2x^2\right) + \frac{\overset{\text{I}}{\text{A}}{}^2}{\text{B}^2.\overset{\text{I}}{\text{P}}{}^3}\left(\frac{\overset{\text{I}}{\text{P}}}{\overset{\text{I}}{\text{A}}+\overset{\text{I}}{\text{B}}} + f + g^2\overset{\text{I}}{x}{}^2\right)\right].$$

III. Concipiatur ſimiliter à foco interiori $\overset{III}{A}$ lentis tertiæ $\overset{II}{P}$ incidere in illam radium $\overset{III}{A}\overset{II}{L}$, cumque refractum convenire cum axe in puncto $\overset{I}{C}$, et aberrare à foco exteriore $\overset{II}{A}$ longitudine $\overset{II}{A}\overset{I}{C}$, quæ per § 9. eſt $\frac{\overset{I}{A}^2 . \overset{II}{A}^2 . L^2}{2g . B^2 . \overset{I}{B}^2 . \overset{II}{P}^3} \times \left(\frac{\overset{II}{P}}{\overset{II}{A} + \overset{II}{B}} + f + g^2 \overset{II}{x}^2\right)$. Hæc addita ad aberrationem duarum priorum lentium modo inventam, $\overset{II}{A}\overset{I}{B}$, dat ſummam $\overset{II}{A}\overset{I}{B} + \overset{II}{A}\overset{I}{C} = \overset{I}{B}\overset{I}{C}$, deinde fiat ut $\overset{II}{A}^2$ ad $\overset{II}{B}^2$; ita hæc ſumma $\overset{I}{B}\overset{I}{C}$ ad aberrationem trium lentium $\overset{III}{A}\overset{II}{B} = \frac{\overset{II}{B}^2 . L^2}{2g} \times \left[\frac{B^2 . \overset{I}{B}^2}{\overset{II}{A}^2 . \overset{I}{A}^2 . P^3} \left(\frac{P}{A + B} + f + g^2 x^2\right) + \frac{\overset{I}{B}^2 \overset{I}{A}^2}{B^2 . \overset{II}{A}^2 . \overset{I}{P}^3} \left(\frac{\overset{I}{P}}{\overset{I}{A} + \overset{I}{B}} + f + g^2 \overset{I}{x}^2\right) + \frac{\overset{I}{A}^2 . \overset{II}{A}^2}{\overset{I}{B}^2 . B^2 . \overset{II}{P}^3} \left(\frac{\overset{II}{P}}{\overset{II}{A} + \overset{II}{B}} + f + g^2 \overset{II}{x}^2\right)\right]$.

IV. Eadem ratione invenitur aberratio $\overset{IV}{A}\overset{III}{B}$ poſt refractionem per quatuor lentes P, $\overset{I}{P}$, $\overset{II}{P}$, $\overset{III}{P}$, $= \frac{\overset{III}{B}^2 . L^2}{2g} \times \left[\frac{B^2 . \overset{I}{B}^2 . \overset{II}{B}^2}{\overset{III}{A}^2 . \overset{II}{A}^2 . \overset{I}{A}^2 . P^3} \times \left(\frac{P}{A + B} + f + g^2 x^2\right) + \frac{\overset{I}{B}^2 . \overset{II}{B}^2 . \overset{I}{A}^2}{B^2 . \overset{III}{A}^2 . \overset{II}{A}^2 . \overset{I}{P}^3} \times \left(\frac{\overset{I}{P}}{\overset{I}{A} + \overset{I}{B}} + f + g^2 \overset{I}{x}^2\right) + \frac{\overset{II}{B}^2 . \overset{I}{A}^2 . \overset{II}{A}^2}{\overset{I}{B}^2 . B^2 . \overset{III}{A}^2 . \overset{II}{P}^3} \times \left(\frac{\overset{II}{P}}{\overset{II}{A} + \overset{II}{B}} + f + g^2 \overset{II}{x}^2\right) + \frac{\overset{I}{A}^2 . \overset{II}{A}^2 . \overset{III}{A}^2}{\overset{II}{B}^2 . \overset{I}{B}^2 . B^2 . \overset{III}{P}^3} \left(\frac{\overset{III}{P}}{\overset{III}{A} + \overset{III}{B}} + f + g^2 \overset{III}{x}^2\right)\right]$. Et eadem ratione progredi licet ad definiendam

definiendam aberrationem radiorum poſt tranſitum per plures lentes.

In his computationibus omnes lentes concipiuntur convexæ, et foci radiorum incidentium ponuntur jacere ante lentes ſuas, refractorum vero poſt lentes reſpectu curſus radiorum, quemadmodum figura exhibet. Formulæ autem allatæ ad quoſvis alios caſus facile accommodantur, mutando ſigna ſymbolorum, prout requirit mutatio hypotheſeos, quod monuiſſe ſufficiat.

§ 11.

Datis poſitionibus et focorum principalium diſtantiis lentium quotcunque, oporteat invenire formas earum, quæ efficiant, ut radii luminis, à dato in communi earum axe puncto advenientes, poſt refractionem colligantur in ultimo foco ſine abberratione, ſi fieri poteſt.

Solutio problematis generaliter abſolvitur, ponendo expreſſionem aberrationis, datæ multitudini lentium convenientem, (§ 10.) nihilo æqualem, quo ipſo habetur æquatio definiens rationem requiſitam indicum ſingularum lentium x, $\overset{\text{I}}{x}$, $\overset{\text{II}}{x}$, $\overset{\text{III}}{x}$, &c. Et quoniam relatio omnium indicum unica hac æquatione continetur, patet multum hic locum eſſe arbitrariæ nonnullorum aſſumtioni, prout requirunt vel ſuadent aperturæ lentium aliæque circumſtantiæ ex ſcopo inſtrumenti dijudicandæ, de quibus hic non eſt ſermo. In genere tantum cavendum eſt, ne aſſumtio ita fiat, ut aliquis indicum prodeat imaginarius. Determinatis ſic indicibus x, $\overset{\text{I}}{x}$, $\overset{\text{II}}{x}$, $\overset{\text{III}}{x}$, &c. inveniuntur radii facierum per § 8. Dicatur ſcilicet radius

faciei

faciei anterioris ſingularum lentium a, poſterioris b; diſtantia foci principalis à lente, P; diſtantia foci radiorum incidentium ante lentem, A; diſtantia foci radiorum refractorum poſt lentem, B; et index ad lentem pertinens, x; et erit $\frac{1}{a} = \frac{b}{B} + \frac{K}{A} + \frac{x}{P}$, et $\frac{1}{b} = \frac{b}{A} + \frac{K}{B} - \frac{x}{P}$, ubi $b = \frac{i \,.\, \overline{2i + r}}{2 \,.\, \overline{i - r} \,.\, \overline{i + 2r}}$, et $K = \frac{4r^2 + ri - 2i^2}{2 \,.\, \overline{i - r} \,.\, \overline{i + 2r}}$, unde dabuntur ſingularum lentium radii a et b.

Si nullus detur valor realis indicum x, $\overset{I}{x}$, $\overset{II}{x}$, $\overset{III}{x}$, &c. id quod accidit, ſi omnes æquationis termini ad unam partem rejecti fuerint ejuſdem ſigni, caſus propoſitus eſt impoſſibilis.

§ 12.

Exemplum I. Ex duabus lentibus teleſcopium componere, quod ſit liberum ab aberratione, et objecta viſa amplificet in data ratione n ad 1. Abſtrahitur autem hic a reliquis boni teleſcopii requiſitis.

Lentes duæ primæ P et $\overset{I}{P}$ repræſentent binas lentes teleſcopii quæſiti, P quidem objectivam et $\overset{I}{P}$ ocularem (vid. fig. § 10.). Aberratio harum duarum lentium (§ 10. caſ. 11.) nihilo æqualis poſita dat æquationem generalem: $g^2 \times \left(\frac{B^2 \,.\, x^2}{\overset{I}{A}{}^2 \,.\, P^3} + \frac{\overset{I}{A}{}^2 \,.\, \overset{I}{x}{}^2}{B^2 \,.\, \overset{I}{P}{}^3}\right) + f \times \left(\frac{B^2}{\overset{I}{A}{}^2 \,.\, P^3} + \frac{\overset{I}{A}{}^2}{B^2 \,.\, \overset{I}{P}{}^3}\right) + \frac{B^2}{\overset{I}{A}{}^2 \,.\, P^2 \,.\, \overline{A + B}} + \frac{\overset{I}{A}{}^2}{B^2 \,.\, \overset{I}{P}{}^2 \,.\, \overline{\overset{I}{A} + \overset{I}{B}}} = 0$. Quoniam per conſtitutionem teleſcopii, radii luminis

minis in lentem objectivam incidentes, et è lente oculari emergentes, ſunt axi teleſcopii paralleli, erit A infinita, adeoque $B = P$; item $\overset{1}{B}$ infinita, adeoque $\overset{1}{A} = \overset{1}{P}$. Quare deletis duobus ultimis æquationis terminis, utpote evaneſcentibus, et in reliquis in ſcribendo P et $\overset{1}{P}$ pro B et $\overset{1}{A}$, habetur pro caſu hujus exempli æquatis $g^2 \times (P x^2 + \overset{1}{P} \overset{1}{x}{}^2) + f \times (P + \overset{1}{P}) = 0$, relationem indicum x et $\overset{1}{x}$ definiens, requiſitam ad id, ut aberratio radiorum è lente oculari emergentium nulla ſit. Jam ſi in hac æquatione P et $\overset{1}{P}$ ponantur ejuſdem ſigni, ſive utraque lens convexa, vel utraque concava, nullus valor realis haberi poteſt pro x et $\overset{1}{x}$, adeoque problema in hoc caſu erit impoſſibile. Statuatur itaque $\overset{1}{P}$ negativa, ſive lens ocularis concava, ſumendo pro exponente amplificationis objecti teleſcopio viſi, n, numerum quemvis poſitivum, et faciendo $\overset{1}{P} = -\frac{1}{n} P$, ut patet ex opticis. Scripto hoc valore pro $\overset{1}{P}$ in æquatione indicum, evadit illa $g^2 \times (\overset{1}{x}{}^2 - n x^2) = f \times \overline{n - 1}$. Horum itaque indicum utrumlibet pro lubitu aſſumere licet, modo ne $\overset{1}{x}$ fiat minor, quam $\frac{\sqrt{\overline{n - 1} \cdot f}}{g}$, quod redderet indicem x imaginarium. Altero indicum ſic aſſumto, alter determinatur per allatam æquationem $g^2 \times (\overset{1}{x}{}^2 - n x^2) = f \cdot \overline{n - 1}$. Quo facto radii facierum utriuſque lentis prodeunt per § 11. ut ſequitur:

Radius faciei anterioris lentis objectivæ $= \frac{P}{b + x}$, posterioris $= \frac{P}{K - x}$;

Radius faciei anterioris lentis ocularis $= \frac{\overset{I}{P}}{K + \overset{I}{x}}$, posterioris $= \frac{\overset{I}{P}}{b - \overset{I}{x}}$, ubi $\overset{I}{P}$ sumenda est negativa, sive $\overset{I}{P} = -\frac{I}{n} P$, ut dictum est.

Pro quovis itaque pari valorum correspondentium indicum x et $\overset{I}{x}$, quatuor prodeunt diversæ problematis solutiones, ob signa duplicia ipsarum x et $\overset{I}{x}$, unde duæ prodeunt formæ utriusque lentis, quarum quælibet combinari potest cum binis formis alterius lentis.

Si detur vel assumatur forma alterutrius lentis, forma alterius per supradicta commode determinatur. Sit v. gr. lens objectiva plano-convexa, habens faciem convexam antrorsum versam, planam retrorsum. Radius faciei ejus posterioris $\frac{P}{K - x}$ statuatur infinitus, et erit $x = K$, adeoque radius faciei, anterioris $\frac{P}{b + x} = \frac{P}{b + K} = \frac{i - r}{r} \times P$. Pro x scribatur K in æquatione indicum $\overset{I}{x}^2 - n x^2 = \frac{\overline{n - I} \cdot f}{g^2}$, et erit $\overset{I}{x} = \sqrt{\frac{\overline{n - I} \cdot f}{g^2} + n K^2}$, unde dantur radii facierum lentis ocularis, idque dupliciter, ob signum duplex valoris ipsius $\overset{I}{x}$.

Vidimus

Vidimus in ſolutione hujus problematis, aberrationem à duabus lentibus productam non poſſe evaneſcere, ſi utraque fuerit convexa, vel utraque concava, cujuſcumque ſint formæ et quomodocunque componantur. Idem quoque obtinet in quacunque lentium multitudine, quæ omnes ſint convexæ, vel omnes concavæ. Omnes enim lentes, quarum facies ſuperficierum ſphæricarum ſunt ſegmenta, aberrationes a focis ſuis eo producunt, quod radios luminis refractione nimium inflectunt; unde facile perſpicitur, lentem convexam, convexis additam, vel concavam concavis, errores a prioribus productos augere. Quare ut aberratio a foco ultimo evaneſcat, debebunt lentium aliæ eſſe convexæ, aliæ concavæ, quo nimiæ radiorum incurvationes in unam partem corrigantur per nimias incurvationes factas in partem contrariam.

§ 13.

Exemplum II. Propoſitum ſit inveſtigare formas binarum lentium, quæ juxta ſe poſitæ radios incidentes axi parallelos in data à lentibus diſtantia R colligant, citra aberrationem ex figura facierum ſphærica oriundam.

Quoniam in caſu hujus exempli radii incidentes ponuntur paralleli erit in æquatione (§ 10. caſ. 2.) pro relatione binorum indicum x et $\overset{1}{x}$, A infinita, adeoque $B = P$. Et quoniam lentes ponuntur juxta ſe poſitæ, erit earum diſtantia $B + \overset{1}{A} = 0$, ſive $\overset{1}{A} = -B = -P$. Præterea eſt etiam $\frac{1}{\overset{1}{B}} = \frac{1}{P} + \frac{1}{\overset{1}{P}}$, unde $\overset{1}{B} - P = -\frac{P^2}{P + \overset{1}{P}}$. Subſtitutis itaque his

 valoribus,

valoribus, erit æquatio pro relatione indicum in casu præsenti: $g^2 . (\overset{1}{P}{}^3 . x^2 + P^3 . \overset{1}{x}{}^2) + f . \overline{P^3 + \overset{1}{P}{}^3} = P . \overset{1}{P} . \overline{P + \overset{1}{P}}$, sive $P^3 \overset{1}{x}{}^2 + \overset{1}{P}{}^3 x^2 = \frac{P + \overset{1}{P}}{g^2} \times (\overline{1 + f} . P \overset{1}{P} - f . P^2 + \overset{1}{P}{}^2)$, quæ assumto numero quovis n, et statuendo $\overset{1}{P} = n P$, evadit $\overset{1}{x}{}^2 + n^3 x^2 = \frac{n + 1}{g^2} \times (n - \overline{n^2 - n + 1} . f)$. Nè autem casus propositus fiat impossibilis, ob utramque lentem convexam, vel utramque concavam, per n semper intelligi debet numerus negativus. Assumatur jam pro Cubitu alteruter indicum x et $\overset{1}{x}$, et dabitur alter per æquationem allatam.

Determinato vero utroque indice, radii facierum erunt ut sequitur:

Radius faciei anterioris lentis primæ $= \frac{P}{h + x}$, posterioris $= \frac{P}{K - x}$;

Radius faciei anterioris lentis secundæ $= \frac{\overset{1}{P}}{\overline{n + 1} . h - n K + \overset{1}{x}}$, posterioris $= \frac{\overset{1}{P}}{\overline{n + 1} . K - n h - \overset{1}{x}}$, ubi est $n P = \overset{1}{P}$, posito n numero negativo.

Distantia foci principalis systematis binarum lentium P et $\overset{1}{P}$ dicatur R, et quoniam est $\frac{1}{R} = \frac{1}{P} + \frac{1}{\overset{1}{P}}$, erit $P = \frac{n + 1}{n} R$, et $\overset{1}{P} = \overline{n + 1} . R$, quare in expressionibus radiorum scribendo hos valores pro P et $\overset{1}{P}$, habebuntur:

Radius faciei anterioris lentis primæ $= \frac{\frac{n+1}{n} R}{h + x}$, posterioris $= \frac{\frac{n+1}{n} R}{K - x}$;

Radius faciei anterioris lentis secundæ $= \frac{\overline{n+1} . R}{\overline{n+1} . h - nK + \overset{1}{x}}$, posterioris $= \frac{\overline{n+1} . R}{\overline{n+1} . K - nh - \overset{1}{x}}$.

Et hæ lentes juxta se positæ radios incidentes axi parallelos in distantia à lente, R, colligent sine aberratione, si distantia foci lentis concavæ major fuerit distantia foci lentis convexæ; si autem contra dispergentur radii sine aberratione à puncto ante lentem, cujus distantia positive sumta est R.

Pro casu singulari hujus exempli, ponamus requiri formas binarum lentium ita comparatarum, ut distantia foci principalis lentis concavæ sit ad distantiam foci principalis lentis convexæ, ut 3 ad 2, et ut radii colligantur in distantia R à lentibus. Si lens anterior debet esse convexa, erit $n = -\frac{3}{2}$, et æquatio definiens relationem indicum lentis convexæ x et lentis concavæ $\overset{1}{x}$, $8\overset{1}{x}^2 - 27x^2 = \frac{19f + 6}{g^2}$. Determinatis vero decenter indicibus x et $\overset{1}{x}$, radii facierum erunt, ut sequitur:

Radius faciei anterioris lentis primæ $= \frac{R}{3h + 3x}$, posterioris $= \frac{R}{3K - 3x}$;

Radius faciei anterioris lentis secundæ $= -\frac{R}{3K - h + 2\overset{1}{x}}$, posterioris $= -\frac{R}{3h - K - 2\overset{1}{x}}$.

 Si

Si vero lens anterior debet esse concava, erit $n = -\frac{2}{3}$, et relatio indicum $8x^2 - 27\overset{1}{x}^2 = \frac{19f+6}{g^2}$, radii vero facierum ut sequitur:

Radius faciei anterioris lentis primæ $= -\frac{R}{2b+2x}$, posterioris $= -\frac{R}{2K-2x}$;

Radius faciei anterioris lentis secundæ $= \frac{R}{b+2K+3\overset{1}{x}}$, posterioris $= \frac{R}{K+2b-3\overset{1}{x}}$.

§ 14.

Exemplum III. Propositum sit, telescopium ex tribus lentibus componere, quarum prima P et secunda $\overset{1}{P}$ juxta se positæ vitrum objectivum duplex constituant, cujus distantia foci principalis sit R; tertia vero sive ocularis $\overset{II}{P}$, habeat distantiam foci $\overset{II}{P} = \frac{R}{m}$, ut potentia amplificandi designetur per numerum m. Ponatur præterea $\overset{1}{P} = nP$, intelligendo per n numerum datum negativum, eumque unitate majorem, si lens convexa anterior est collocanda, minorem si posterior, ut radii post refractionem in vitro objectivo convergant, quemadmodum expositum est in exemplo præcedente. Quæruntur autem formæ harum trium lentium, ut telescopium liberum sit ab omni erratione ex figura illarum sphærica oriunda.

Aberratio à tribus lentibus P, $\overset{1}{P}$, $\overset{II}{P}$, oriunda (§ 10. cas. III.) nihilo æqualis posita, dat $\frac{B^2 . \overset{1}{B}^2}{\overset{II}{A}^2 . \overset{1}{A}^2 . P^3} \left(\frac{P}{A+B} \right.$

+

$$+f+g^2x^2)+\frac{\overset{I}{B}{}^2.\overset{I}{A}{}^2}{\overset{II}{B}{}^2.\overset{II}{A}{}^2.\overset{I}{P}{}^3}\left(\frac{\overset{I}{P}}{\overset{I}{A}+\overset{I}{B}}+f+g^2\overset{I}{x}{}^2\right)$$

$$+\frac{\overset{I}{A}{}^2.\overset{II}{A}{}^2}{\overset{I}{B}{}^2.B^2.\overset{II}{P}}\left(\frac{\overset{II}{P}}{\overset{II}{A}+\overset{II}{B}}+f+g^2\overset{II}{x}{}^2\right)=0.$$

Symbola hanc æquationem ingredientia per hypothesin hujus exempli ita determinantur: ob A infinitam est $\frac{P}{A+B}=0$, et $B=P$. Ob $\overset{II}{B}$ infinitam est $\frac{\overset{II}{P}}{\overset{II}{A}+\overset{II}{B}}=0$, et $\overset{II}{A}=\overset{II}{P}$. Ob lentes P et $\overset{I}{P}$ juxta se positas; est earum distantia $\overset{I}{A}+B=0$, sive $\overset{I}{A}=-B=-P$. Ob $\frac{1}{R}=\frac{1}{P}+\frac{1}{\overset{I}{P}}$, et $\overset{I}{P}=nP$, est $P=\frac{n+1}{n}R$, et $\overset{I}{P}=\overline{n+1}.R$. Præterea est $\overset{I}{B}=R$, et $\overset{II}{P}=\frac{R}{m}$. His itaque valoribus substitutis, et æquatione ordinata, habetur relatio indicum x, $\overset{I}{x}$, $\overset{II}{x}$, expressa ut sequitur: $g^2\times(n^3x^2+\overset{I}{x}{}^2+\frac{\overline{n+1}^3}{m}\overset{II}{x}{}^2)$ $+f\times(n^3+1+\frac{\overline{n+1}^3}{m})=n.\overline{n+1}$. Determinatis indicibus x, $\overset{I}{x}$, $\overset{II}{x}$, habentur per § 11. radii facierum: videlicet,

Radius faciei anterioris lentis primæ $=\frac{\overline{n+1}.R}{n.\overline{h+x}}$, posterioris $=\frac{\overline{n+1}.R}{n.\overline{K-x}}$;

Radius faciei anterioris lentis secundæ $=\frac{\overline{n+1}.R}{\overline{n+1}.h-nK+\overset{I}{x}}$, posterioris $=\frac{\overline{n+1}.R}{\overline{n+1}.K-nh-\overset{I}{x}}$;

Radius

Radius faciei anterioris lentis tertiæ $= \frac{R}{m \cdot K + \overset{II}{x}}$, posterioris $= \frac{R}{m \cdot h - \overset{II}{x}}$.

Ita si vitrum objectivum hujus telescopii componendum sit ex duabus lentibus, convexa et concava, quarum distantiæ focales sint ut 3 et 2, debeatque primo lens convexa anterior collocari, oportebit sumere $n = -\frac{3}{2}$, et erit æquatio pro indicibus:

$$g^2 \times \left(27x^2 - 8\overset{I}{x^2} + \frac{\overset{II}{x^2}}{m}\right) + f \times \left(19 + \frac{1}{m}\right) + 6 = 0,$$

et radii facierum:

Radius faciei anterioris lentis primæ $= \frac{R}{3h + 3x}$, posterioris $= \frac{R}{3K + 3x}$;

Radius faciei anterioris lentis secundæ $= -\frac{R}{3K - h + 2\overset{I}{x}}$, posterioris $= -\frac{R}{3h - K - 2\overset{I}{x}}$;

Radius faciei anterioris lentis tertiæ $= \frac{R}{mK + m\overset{II}{x}}$, posterioris $= \frac{R}{mh - m\overset{II}{x}}$.

Si vero lens concava anterior est collocanda, debet sumi $n = -\frac{2}{3}$, unde æquatio pro indicibus prodit

$$g^2 \times \left(-8x^2 + 27\overset{I}{x^2} + \frac{\overset{II}{x^2}}{m}\right) + f \times \left(19 + \frac{1}{m}\right) + 6 = 0,$$

et consequenter:

Radius faciei anterioris lentis primæ $= -\frac{R}{2h + 2x}$, posterioris $= -\frac{R}{2K - 2x}$;

Radius

Radius faciei anterioris lentis ſecundæ $= \frac{R}{h + 2K + 3\overset{I}{x}}$, poſterioris $= \frac{R}{K + 2h - 3\overset{I}{x}}$;

Radius faciei anterioris lentis tertiæ $= \frac{R}{mK + m\overset{II}{x}}$, poſterioris $= \frac{R}{mh - m\overset{II}{x}}$.

§ 15.

Vidimus in ſolutione generati noviſſimi problematis (§ ii.) et ſubjunctis exemplis (§ 12, 13, 14.) relationem indicum x, $\overset{I}{x}$, $\overset{II}{x}$, &c. unica tantum æquatione definiri, adeoque judicio artificis plurimum eſſe relictum in commoda eorum aſſumtione facienda. Duæ autem ſunt conſiderationes, quibus hæc aſſumtio utcunque dirigi poſſit. Prima eſt, ut nullus indicum fiat nimis magnus, ſive, quod eodem recidit, ut formæ lentium quamproxime accedant ad eas, quæ in datis circumſtantiis aberrationem dant omnium minimam. Hac enim ratione errores in forma lentium forte commiſſi minus nocebunt, quemadmodum ex natura minimi notum eſt. Altera eſt, ut ſingulæ lentes, quam fieri poteſt, proxime evadant æqualiter utrinque convexæ, vel æqualiter concavæ, quo fiet, ut majores aperturæ ſint patientes. Quantitates enim aberrationum ſupra definitæ non ſunt niſi quamproxime veræ, et tanto magis fallunt, quanto facies lentium fuerint majora ſuarum ſphærarum ſegmenta. Binæ hæ regulæ, etiamſi ſibi invicem ſæpius adverſuntur, nec facile definiri queat, quantum in quovis dato caſu uni vel alteri ipſarum ſit tribuendum; non dubito tamen, quin judicioſo artifici ſint profuturæ

profuturæ ad mediam viam tenendam inter utramque.

Eædem quoque conſiderationes impediunt, quo minus certi aliquid definire auſim, de maximo poſſibili effectu inſtrumentorum opticorum ſecundum hæc principia compoſitorum, vel de modo perfectiſſima componendi. Regulæ huc pertinentes tutiſſime exſpectantur ab experientia, modo dirigatur à theoria.

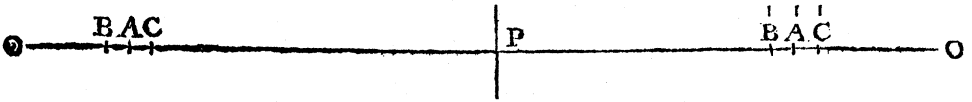
§ 16.

Expoſitis jam breviter quæ dicenda habuimus de aberrationibus radiorum homogeneorum refractorum, ortis a lentium figura ſphærica, ut et de modo illas corrigendi in inſtrumentis opticis: liceat etiam pauca addere de altera illa aberrationis ſpecie, quæ a diverſa radiorum refrangibilitate oritur, quæ objecta trans lentes viſa coloribus inficit, et maximam confuſionis partem producit. Creditum hucuſque communiter fuit hoc vitium nulla arte emendabile, donec ingenioſiſſimus artifex Londinenſis, Dollandus, experimentis inſtitutis feliciter deprehenderet, varia dari vitrorum genera, quæ, licet fere æqualiter refringant certum quoddam radiorum genus, plurimum tamen diſcrepent viribus refringendi reliqua radiorum genera, adeoque et radios heterogeneos a ſe invicem ſeparandi: quo eximio invento et theoriam et praxim optices inſigniter auxit. Diſcimus enim hinc, rationes refractionem radiorum diverſi generis nullo modo a ſe invicem dependere, ut hucuſque fuit creditum; adeoque fruſtra quæri regulam, qua ex datis refractionibus radiorum diverſi generis in uno aliquo medio inveniantur refractiones eorumdem in alio; vel

vel qua ex data refractione unius alicujus radii in quovis medio, inveniuntur refractiones reliquorum in eodem medio. Praxis quoque telescopiorum dioptricorum insigne incrementum hujus inventi beneficio nacta est. Scilicet sagacissimus inventor modum inde derivavit corrigendi aberrationes ex separatione radiorum heterogeneorum oriundas, componendo vitrum objectivum ex duabus lentibus, una convexa, altera concava, quæ ita sint comparatæ, ut una alterius effectum in radiis separandis destruat, quemadmodem in Vol. L. Part. II. p. 733. et seq. Transactionum Philosophicarum exposuit. Quod artificium ita breviter explicabimus.

§ 17.

Sit P lens quæcunque, habens radios facierum suarum a et b; distantiam foci principalis mediocriter refrangibilium P; et ax in rectam O O.



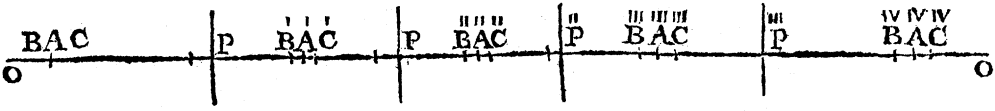
Ratio refractionis radiorum mediocriter refrangibilium ex aëre in lentem sit N ad 1; radiorum maxime refrangibilium $N + n$ ad 1, et radiorum minime refrangibilium $N - n$ ad 1, existente n numero valde parvo respectu ipsius N. In axe lentis O O sint puncta A, B, C foci radiorum in lentem P incidentium, A mediocriter refrangibilium, B maxime et C minime refrangibilium, quorum focorum distantiæ A B, A C sint valde parvæ, et ponantur puncta $\overset{1}{A}$, $\overset{1}{B}$, $\overset{1}{C}$, foci conjugati ipsorum A, B, C respective, pro sua quique radiorum specie, ita ut sint B C, $\overset{1}{B}\overset{1}{C}$,

 dissipa-

diſſipationes radiorum heterogeneorum circa focos conjugatos A et $\overset{1}{A}$. Quæratur earum relatio.

Quoniam B et $\overset{1}{B}$ ſunt foci conjugati radiorum maxime refrangibilium, quorum ratio refractionis eſt $N + n$ ad 1, erit $\frac{1}{PB} + \frac{1}{P\overset{1}{B}} = \overline{N + n - 1} \cdot \overline{\frac{1}{a} + \frac{1}{b}}$. Similiter quoniam puncta C et $\overset{1}{C}$ ſunt foci conjugati radiorum minime refrangibilium, quorum ratio refractionis eſt $N - n$ ad 1, erit $\frac{1}{PC} + \frac{1}{P\overset{1}{C}} = \overline{N - n + 1} \cdot \overline{\frac{1}{a} + \frac{1}{b}}$. Subtrahatur hæc poſterior æquatio à priore, et habebitur $\frac{1}{PB} - \frac{1}{PC} + \frac{1}{P\overset{1}{B}} - \frac{1}{P\overset{1}{C}} = 2n \cdot \overline{\frac{1}{a} + \frac{1}{b}}$, ſive conjungendo binos terminos: $\frac{PC - PB}{PB \times PC} + \frac{P\overset{1}{C} - P\overset{1}{B}}{P\overset{1}{B} \times P\overset{1}{C}} = 2n \cdot \overline{\frac{1}{a} + \frac{1}{b}}$, quæ pro $PB \times PC$ et $P\overset{1}{B} \times P\overset{1}{C}$, ſcribendo PAq et $P\overset{1}{A}q$, et pro $\frac{1}{a} + \frac{1}{b}$ ſcribendo $\frac{1}{\overline{N - 1} \cdot P}$, evadit $\frac{\overset{1}{B}\overset{1}{C}}{P\overset{1}{A}q} = \frac{2n}{\overline{N - 1} \cdot P} + \frac{BC}{PAq}$, exhibens relationem inter diſſipationes conjugatas BC et $\overset{1}{B}\overset{1}{C}$.

His præmiſſis, intelligantur lentes quotcunque P, $\overset{1}{P}$, $\overset{II}{P}$, $\overset{III}{P}$, &c. quarum diſtantiæ focorum principalium reſpectu radiorum mediocriter refrangibilium ſint P, $\overset{1}{P}$, $\overset{II}{P}$, $\overset{III}{P}$, &c. reſpective, ordine diſpoſitæ in axe earum



earum communi O O. Rationes refractionum radiorum mediocriter refrangibilium in his lentibus ordine ſumtis ſint reſpective, N ad 1, $\overset{\text{I}}{\text{N}}$ ad 1, $\overset{\text{II}}{\text{N}}$ ad 1, $\overset{\text{III}}{\text{N}}$ ad 1, &c. radiorum maxime refrangibilium $\text{N} + n$ ad 1, $\overset{\text{I}}{\text{N}} + \overset{\text{I}}{n}$ ad 1, $\overset{\text{II}}{\text{N}} + \overset{\text{II}}{n}$ ad 1, $\overset{\text{III}}{\text{N}} + \overset{\text{III}}{n}$ ad 1, &c. et radiorum minime refrangibilium $\text{N} - n$ ad 1, $\overset{\text{I}}{\text{N}} - \overset{\text{I}}{n}$ ad 1, $\overset{\text{II}}{\text{N}} - \overset{\text{II}}{n}$ ad 1, $\overset{\text{III}}{\text{N}} - \overset{\text{III}}{n}$ ad 1, &c. Puncta A, $\overset{\text{I}}{\text{A}}$, $\overset{\text{II}}{\text{A}}$, $\overset{\text{III}}{\text{A}}$, $\overset{\text{IV}}{\text{A}}$, &c. ſint foci conjugati lentium reſpectu radiorum mediocriter refrangibilium, puncta B, $\overset{\text{I}}{\text{B}}$, $\overset{\text{II}}{\text{B}}$, $\overset{\text{III}}{\text{B}}$, $\overset{\text{IV}}{\text{B}}$, &c. reſpectu radiorum maxime refrangibilium, C, $\overset{\text{I}}{\text{C}}$, $\overset{\text{II}}{\text{C}}$, $\overset{\text{III}}{\text{C}}$, $\overset{\text{IV}}{\text{C}}$, &c. reſpectu minime refrangibilium, ita ut BC, $\overset{\text{I}}{\text{B}}\overset{\text{I}}{\text{C}}$, $\overset{\text{II}}{\text{B}}\overset{\text{II}}{\text{C}}$, $\overset{\text{III}}{\text{B}}\overset{\text{III}}{\text{C}}$, $\overset{\text{IV}}{\text{B}}\overset{\text{IV}}{\text{C}}$, &c. ſint diſſipationes ſucceſſivæ radiorum heterogeneorum in axe lentium. Dicantur tandem diſtantiæ focorum conjugatorum à ſuis reſpective lentibus: PA = A, $\text{P}\overset{\text{I}}{\text{A}} = \text{B}$; $\overset{\text{I}}{\text{P}}\overset{\text{I}}{\text{A}} = \text{A}$, $\text{P}\overset{\text{I}}{\text{A}} = \text{B}$; $\overset{\text{II}}{\text{P}}\overset{\text{II}}{\text{A}} = \text{A}$, $\overset{\text{II}}{\text{P}}\overset{\text{II}}{\text{A}} = \overset{\text{II}}{\text{B}}$; $\overset{\text{III}}{\text{P}}\overset{\text{III}}{\text{A}} = \overset{\text{III}}{\text{A}}$, $\overset{\text{III}}{\text{P}}\overset{\text{IV}}{\text{A}} = \overset{\text{III}}{\text{B}}$, &c.

Jam ſi in æquatione pro relatione diſſipationum conjugatarum modo inventa, $\frac{\overset{\text{I}}{\text{B}}\overset{\text{I}}{\text{C}}}{\text{P}\overset{\text{I}}{\text{A}}^{q}} = \frac{2n}{\overline{\text{N}-1}\,.\,\text{P}} + \frac{\text{BC}}{\text{PA}^{q}}$, per BC et $\overset{\text{I}}{\text{B}}\overset{\text{I}}{\text{C}}$ intelligantur ſucceſſive ſingula paria diſſipationum conjugatorum, et per PA et $\text{P}\overset{\text{I}}{\text{A}}$ diſtantiæ focorum conjugatorum ipſis reſpondentes;

 item

item per P, N et n eorum valores ſingulis lentis proprii, habebuntur tot æquationes, quot ſunt lentes: videlicet; $\overset{\mathrm{I}}{\mathrm{B}}\overset{\mathrm{I}}{\mathrm{C}} = \mathrm{B}^2 \times \left(\frac{2n}{\mathrm{N}-1 \,.\, \mathrm{P}} + \frac{\mathrm{BC}}{\mathrm{A}^2}\right)$; $\overset{\mathrm{II}}{\mathrm{B}}\overset{\mathrm{II}}{\mathrm{C}} = \overset{\mathrm{I}}{\mathrm{B}}{}^2 \times \left(\frac{2\overset{\mathrm{I}}{n}}{\overset{\mathrm{I}}{\mathrm{N}}-1 \,.\, \overset{\mathrm{I}}{\mathrm{P}}} + \frac{\overset{\mathrm{I}}{\mathrm{B}}\overset{\mathrm{I}}{\mathrm{C}}}{\overset{\mathrm{I}}{\mathrm{A}}{}^2}\right)$; $\overset{\mathrm{III}}{\mathrm{B}}\overset{\mathrm{III}}{\mathrm{C}} = \overset{\mathrm{II}}{\mathrm{B}}{}^2 \times \left(\frac{2\overset{\mathrm{II}}{n}}{\overset{\mathrm{II}}{\mathrm{N}}-1 \,.\, \overset{\mathrm{II}}{\mathrm{P}}} + \frac{\overset{\mathrm{II}}{\mathrm{B}}\overset{\mathrm{II}}{\mathrm{C}}}{\overset{\mathrm{II}}{\mathrm{A}}{}^2}\right)$; $\overset{\mathrm{IV}}{\mathrm{B}}\overset{\mathrm{IV}}{\mathrm{C}} = \overset{\mathrm{III}}{\mathrm{B}}{}^2 \times \left(\frac{2\overset{\mathrm{III}}{n}}{\overset{\mathrm{III}}{\mathrm{N}}-1 \,.\, \overset{\mathrm{III}}{\mathrm{P}}} + \frac{\overset{\mathrm{III}}{\mathrm{B}}\overset{\mathrm{III}}{\mathrm{C}}}{\overset{\mathrm{III}}{\mathrm{A}}{}^2}\right)$; et ita porro, ſi plures fuerint lentes. Ponantur puncta B, A, C, coincidere, ſive ex primo foco A prodire radium compoſitum, ut diſſipatio prima BC ſit nulla, et reducendo has æquationes, ponendo $\mathrm{BC} = 0$, habentur valores diſſipationum ſucceſſivarum, pro quovis lentium numero:

1. $\overset{\mathrm{I}}{\mathrm{B}}\overset{\mathrm{I}}{\mathrm{C}} = \mathrm{B}^2 \times \frac{2n}{\mathrm{N}-1 \,.\, \mathrm{P}}$.

2. $\overset{\mathrm{II}}{\mathrm{B}}\overset{\mathrm{II}}{\mathrm{C}} = \overset{\mathrm{I}}{\mathrm{B}}{}^2 \times \frac{2\overset{\mathrm{I}}{n}}{\overset{\mathrm{I}}{\mathrm{N}}-1 \,.\, \overset{\mathrm{I}}{\mathrm{P}}} + \frac{\overset{\mathrm{I}}{\mathrm{B}}{}^2 \,.\, \mathrm{B}^2}{\overset{\mathrm{I}}{\mathrm{A}}{}^2} \times \frac{2n}{\mathrm{N}-1 \,.\, \mathrm{P}}$.

3. $\overset{\mathrm{III}}{\mathrm{B}}\overset{\mathrm{III}}{\mathrm{C}} = \overset{\mathrm{II}}{\mathrm{B}}{}^2 \times \frac{2\overset{\mathrm{II}}{n}}{\overset{\mathrm{II}}{\mathrm{N}}-1 \,.\, \overset{\mathrm{II}}{\mathrm{P}}} + \frac{\overset{\mathrm{II}}{\mathrm{B}}{}^2 \,.\, \overset{\mathrm{I}}{\mathrm{B}}{}^2}{\overset{\mathrm{II}}{\mathrm{A}}{}^2} \times \frac{2\overset{\mathrm{I}}{n}}{\overset{\mathrm{I}}{\mathrm{N}}-1 \,.\, \overset{\mathrm{I}}{\mathrm{P}}} + \frac{\overset{\mathrm{II}}{\mathrm{B}}{}^2 \,.\, \overset{\mathrm{I}}{\mathrm{B}}{}^2 \,.\, \mathrm{B}^2}{\overset{\mathrm{II}}{\mathrm{A}}{}^2 \,.\, \overset{\mathrm{I}}{\mathrm{A}}{}^2} \times \frac{2n}{\mathrm{N}-1 \,.\, \mathrm{P}}$.

4. $\overset{\mathrm{IV}}{\mathrm{B}}\overset{\mathrm{IV}}{\mathrm{C}} = \overset{\mathrm{III}}{\mathrm{B}}{}^2 \times \frac{2\overset{\mathrm{III}}{n}}{\overset{\mathrm{III}}{\mathrm{N}}-1 \,.\, \overset{\mathrm{III}}{\mathrm{P}}} + \frac{\overset{\mathrm{III}}{\mathrm{B}}{}^2 \,.\, \overset{\mathrm{II}}{\mathrm{B}}{}^2}{\overset{\mathrm{III}}{\mathrm{A}}{}^2} \times \frac{2\overset{\mathrm{II}}{n}}{\overset{\mathrm{II}}{\mathrm{N}}-1 \,.\, \overset{\mathrm{II}}{\mathrm{P}}} + \frac{\overset{\mathrm{III}}{\mathrm{B}}{}^2 \,.\, \overset{\mathrm{II}}{\mathrm{B}}{}^2 \,.\, \overset{\mathrm{I}}{\mathrm{B}}{}^2}{\overset{\mathrm{III}}{\mathrm{A}}{}^2 \,.\, \overset{\mathrm{II}}{\mathrm{A}}{}^2} \times \frac{2\overset{\mathrm{I}}{n}}{\overset{\mathrm{I}}{\mathrm{N}}-1 \,.\, \overset{\mathrm{I}}{\mathrm{P}}} + \frac{\overset{\mathrm{III}}{\mathrm{B}}{}^2 \,.\, \overset{\mathrm{II}}{\mathrm{B}}{}^2 \,.\, \overset{\mathrm{I}}{\mathrm{B}}{}^2 \,.\, \mathrm{B}^2}{\overset{\mathrm{III}}{\mathrm{A}}{}^2 \,.\, \overset{\mathrm{II}}{\mathrm{A}}{}^2 \,.\, \overset{\mathrm{I}}{\mathrm{A}}{}^2 \,.\, \mathrm{A}^2} \times \frac{2n}{\mathrm{N}-1 \,.\, \mathrm{P}}$.

Et ita deinceps, ſi plures fuerint lentes.

§ 18.

§ 18.

Datis positionibus lentium quotcunque in communi axe, una cum singularum legibus refractionum pro radiis omnis generis, oporteat invenire relationem inter distantias focorum principalium earundem lentium, quæ efficiat, ut radii heterogenei à quovis puncto advenientes vel paralleli, post refractionem in omnibus lentibus emergant, sine dissipatione à diversa refrangibilitate radiorum oriunda, si fieri potest.

Solutio problematis generaliter absolvitur ponendo expressionem dissipationis, datæ multitudini lentium convenientem, § 17, nihilo æqualem, quo ipso habetur æquatio definiens relationum distantiarum focalium, P, $\overset{I}{P}$, $\overset{II}{P}$, $\overset{III}{P}$, &c. requisitam ad id, ut dissipatio in foco ultimo evanescat. Ita in casu binarum lentium habebitur: $\frac{\overset{I}{n}}{\overset{I}{N}-1 \,.\, \overset{I}{P}} + \frac{B^2}{\overset{I}{A}^2} \times \frac{n}{N-1 \,.\, P} = 0$; in casu trium lentium erit: $\frac{\overset{II}{n}}{\overset{II}{N}-1 \,.\, \overset{II}{P}} + \frac{\overset{I}{B}^2}{\overset{II}{A}^2} \times \frac{\overset{I}{n}}{\overset{I}{N}-1 \,.\, \overset{I}{P}} + \frac{\overset{I}{B}^2 \,.\, B^2}{\overset{II}{A}^2 \,.\, \overset{I}{A}^2} \times \frac{n}{N-1 \,.\, P} = 0$; in casu quatuor lentium: $\frac{\overset{III}{n}}{\overset{III}{N}-1 \times \overset{III}{P}} + \frac{\overset{II}{B}^2}{\overset{III}{A}^2} \times \frac{\overset{II}{n}}{\overset{II}{N}-1 \times \overset{II}{P}} + \frac{\overset{II}{B}^2 \,.\, \overset{I}{B}^2}{\overset{III}{A}^2 \,.\, \overset{II}{A}^2} \times \frac{\overset{I}{n}}{\overset{I}{N}-1 \times \overset{I}{P}} + \frac{\overset{II}{B}^2 \,.\, \overset{I}{B}^2 \,.\, B^2}{\overset{III}{A}^2 \,.\, \overset{II}{A}^2 \,.\, \overset{I}{A}^2} \times \frac{n}{N-1 \times P} = 0$; et ita porro, si plures fuerint lentes.

Ita

Ita ſi in æquatione pro binis lentibus, $\frac{\overset{1}{n}}{\overset{1}{N} - 1 . \overset{1}{P}}$ $+ \frac{B^2}{\overset{1}{A}^2} \times \frac{n}{N - 1 . P} = 0$, ponatur diſtantia lentium $B + \overset{1}{A} = 0$, ut evadant contiguæ, quemadmodum in vitro objectivo Dollondiano; habetur $\frac{\overset{1}{n}}{\overset{1}{N} - 1 . \overset{1}{P}}$ $+ \frac{n}{N - 1 . P} = 0$. Unde patet alterutram lentium P et $\overset{1}{P}$ faciendam eſſe concavam, alteram convexam, et diſtantias earum focales affirmative ſumtas P et $\overset{1}{P}$ faciendas eſſe in ratione $\frac{n}{N - 1}$ ad $\frac{\overset{1}{n}}{\overset{1}{N} - 1}$, quæ ratio evadit n ad $\overset{1}{n}$, ſi vires refractivæ mediæ utriuſque lentis fuerint æquales.

Similiter ſit conſtruendum teleſcopium ex tribus lentibus, quarum duæ priores P et $\overset{1}{P}$ juxta ſe poſitæ conſtituant vitrum objectivum, tertia $\overset{II}{P}$ ſit ocularis, et potentia amplificandi exprimatur numero m. Quoniam lentes priores P et $\overset{1}{P}$ ſunt juxta ſe poſitæ, erit earum diſtantia $B + \overset{1}{A} = 0$. Quoniam per conſtitutionem teleſcopii radii in primam lentem incidentes ſunt paralleli, erit $B = P$, et $\frac{1}{\overset{1}{B}} = \frac{1}{P} + \frac{1}{\overset{1}{P}}$. Quoniam radii ex ultima lente emergunt paralleli, erit $\overset{II}{A} = \overset{II}{P}$. Et tandem, quia potentia amplificandi

 indicatur

indicatur numero m, erit $\overset{\text{I}}{B} = m\overset{\text{II}}{P}$. Subſtitutis itaque his valoribus in æquatione pro tribus lentibus ſupra allata, tranſit illa in hanc: $\frac{n}{\overline{N-1}.P} + \frac{\overset{\text{I}}{n}}{\overset{\text{I}}{\overline{N-1}}.\overset{\text{I}}{P}} + \frac{\overset{\text{II}}{n}}{\overset{\text{II}}{\overline{N-1}}.m^2\overset{\text{II}}{P}} = 0$. Et hæc æquatio una cum æquatione $\frac{\text{I}}{P} + \frac{\text{I}}{\overset{\text{I}}{P}} - \frac{\text{I}}{m\overset{\text{II}}{P}} = 0$, determinat rectationem diſtantiarum focalium P, $\overset{\text{I}}{P}$, $\overset{\text{II}}{P}$, qualis ad id requiritur, ut radiorum emergentium diſſipatio à diverſa refrangibilitate nulla ſit. Comparando ſcilicet has æquationes, habetur $\left(\frac{nm}{\overline{N-1}} + \frac{\overset{\text{II}}{n}}{\overset{\text{II}}{\overline{N-1}}}\right)\overset{\text{I}}{P} + \left(\frac{\overset{\text{I}}{nm}}{\overset{\text{I}}{\overline{N-1}}} - \frac{\overset{\text{II}}{n}}{\overset{\text{II}}{\overline{N-1}}}\right) P = 0$, et $\left(\frac{n}{\overline{N-1}} - \frac{\overset{\text{I}}{n}}{\overset{\text{I}}{\overline{N-1}}}\right) \times m^2\overset{\text{II}}{P} + \left(\frac{\overset{\text{I}}{nm}}{\overset{\text{I}}{\overline{N-1}}} + \frac{\overset{\text{II}}{n}}{\overset{\text{II}}{\overline{N-1}}}\right) \times P = 0$. Eadem ratione procedendum eſt in aliis caſibus.

S. Klingenſtierna.

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