

XXV. *The Properties of the mechanic Powers demonstrated, with some Observations on the Methods that have been commonly used for that Purpose: in a Letter from Hugh Hamilton, D. D. F.R.S. and Fellow of Trinity College, Dublin, to Matthew Raper, Esq; F. R. S.*

S I R, Trinity College, Dublin, 13 July, 1762.

Read April 21, and 28.
1763. **I** Have here ventured to send you some remarks on the methods that have been commonly used in treating of those engines that are called the mechanic powers: and also an account of the principles on which, I think, we may best explain their nature and manner of acting.

The many useful instruments that have been so ingeniously invented, and so successfully executed, and the great perfection to which the mechanic arts are now arrived, would naturally incline one to think that the true principles on which the efficacy and operations of the several machines depend, must long since have been accurately explained. But this is by no means a necessary inference: for, however men may differ in their opinions about the true method of accounting for the effects of the several machines, yet the practical principles of mechanics are so perfectly known by experience and observation, that the artist is thereby enabled to contrive and adjust the movements
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of his engines with as much certainty and success as he could do, was he thoroughly acquainted with the laws of motion, from which these principles may be ultimately derived. However, tho' an enquiry into the true method of deducing the practical principles of mechanics from the laws of motion, should not perhaps contribute much to promote the progress of the mechanic arts, yet it is an enquiry in itself useful, and in some measure necessary: for, since late authors have used very different methods of treating this subject, it may be supposed that no one method has been looked upon as satisfactory and unexceptionable. I should therefore wish to contribute towards having this subject treated with more accuracy than has been hitherto done.

The most general and remarkable theorem in mechanics certainly is this, "That when two weights, by means of a machine counterpoise each other, and are then made to move together, their quantities of motion will be equal". Now an æquilibrium always accompanying this equality of motions, bears such a resemblance to the case wherein two moving bodies stop each other, when they meet together with equal quantities of motion; that Doctor Wallis, and after him most of the late writers, have thought the cause of an æquilibrium in the several machines, might be immediately assigned: by saying, That, since one body cannot produce in another a quantity of motion equal to its own, without losing its own at the same time; two heavy bodies, counteracting each other by means of a machine must continue at rest, when they are so circumstanced that one cannot descend, without causing the other to ascend, at the same time; and with the same

same quantity of motion; and therefore two heavy bodies in such cases must always counterbalance each other. Now, this argument would be a just one, if it could properly be said, that the motion of the ascending body was produced by that of the descending one; but, since the bodies are so connected that one cannot possibly begin to move before the other, I apprehend, that, if the bodies are supposed to move, it cannot be said that the motion of one is produced by that of the other: since whatever force is supposed to move one must be the immediate cause of motion in the other also; that is, both their motions must be simultaneous effects of the same cause, just as if the two bodies were really but one. And therefore if I was to suppose, in this case, that the superior weight of the heavier body (which may be in itself much more than able to sustain the lighter) should overcome the weight of the lighter and produce equal motions in both bodies; I do not think that from thence I could be reduced to the absurdity of supposing, that one body, by its motion, might produce in another, a motion equal to its own, and yet not lose its own at the same time. But those who argue from the equality of motions on this occasion say further, that, since the two bodies must have equal motions when they do move, they must have equal endeavours to move even whilst they are at rest, and therefore these endeavours to move, being equal and contrary, must destroy each other, and the bodies must continue at rest, and consequently balance each other. In answer to this I must observe, that the absolute force with which a heavy body endeavours to descend from a state of rest can only be proportionable to its weight; and therefore I think

it is necessary that some cause should be assigned why (for instance) the endeavour of one pound to descend shall be equal to that of four pounds; and especially as the fulcrum on which both weights act requires no greater force to support it than that of five pounds.

From these considerations I infer, that the reason why very unequal weights may ballance each other, should be assigned not from their having equal *momenta* when made to move together, but by proving *a priori* without considering their motions, that either the reaction of the fixed parts of the machine, or some other cause, so far takes off from the weight of the heavier body as to leave it only just able to support the lighter. However, as this equality of *momenta* which always accompanies an *æquilibrium*, affords a very elegant theorem, it ought to be taken notice of in every treatise of mechanics, and may serve as an index of an *æquilibrium*. But I would not have it applied to a purpose for which it is unfit; as it has been in another instance by Doctor Keil, who from thence gives the reason why water stands at the same height in a narrow tube and a wide vessel with which it communicates. And an argument of the same kind is applied still more improperly by Dr. Rutherford and others, to shew why a drop of water included in a small conical tube will move towards the narrower end; and yet the true ways of accounting for both these phenomena are extremely obvious and easy.

The simple mechanic powers are usually reckoned six, the lever, axle and wheel, pully, wedge, inclined plane, and screw. The only method I have met with of explaining the nature of these machines on one principle, is that which I just now examined; and, as that appears to me unsatisfactory, I shall consider the

the nature of each machine separately in the order I have set them down.

The lever is said to be a right line, inflexible and void of weight. Its fundamental property is this; when any two forces act against each other on the arms of a lever, they will continue in æquilibrium, if their quantities are inversely as the distances between the points to which they are applied and the point round which the lever turns, which point is called the fulcrum or prop.

Several methods have been used, by different authors, to prove, that this property must necessarily belong to the lever. We find, in the works of Archimedes, a proof brought for this purpose, which has since been made use of by several writers of mechanics; who, I find, have somewhat altered the form of his argument, the substance of which is generally expressed as follows. — “ When a cylinder of any uniform
“ matter is supported at its middle point, it will con-
“ tinue at rest; for all the parts on one side must ba-
“ lance those on the other, being exactly equal to
“ them both in weight and situation, so that the
“ whole weight of this cylinder may be looked up-
“ on as acting on the middle point on which it is sup-
“ ported.” From hence it is inferred, that the weight of such a cylinder will act upon whatever supports it, in the same manner as it would do if it was all contracted into the middle point of its axis. If therefore we suppose the cylinder to be distinguished into two unequal cylinders or segments, the distances between the middle points of those segments and the middle of the whole cylinder will be inversely as the lengths of the segments; that is, inversely as their weights: but, as it was said above, the weight of each cylinder acts in the same manner as it would do if contracted

contracted into the middle point of its axis; and therefore, if the weights of these cylinders be contracted into these points, they will continue to support each as before. And thence it is concluded, that any two weights, acting against each other on a line sustained at a fixed point, will counterpoise one another, when they are inversely as the distances of the points on which they act, from the point on which the line rests. To this argument there seems to be a manifest objection; for, when the whole cylinder is distinguished into two segments, part of the weight of the greater segment acts on the same side of the fulcrum with the lesser segment; and therefore when the whole weight of the greater segment is contracted into its middle point on one side of the fulcrum, and acts all of it against the lesser segment, it requires at least some proof to shew, that this contracted weight will be ballanced by the weight of the lesser segment. Mr. Hugen, in his *Miscellaneous Observations on Mechanics* takes notice of this objection to Archimedes's method, which, he says, several mathematicians had endeavoured to remove, but without success. He therefore, instead of this method, proposed one of his own, which depends on a postulatam that he uses in common with Archimedes, that I think ought not to be granted on this occasion; it is this: "When equal bodies are placed on the arms of a lever, the one which is furthest from the fulcrum will prevail and raise the other up". Now this is taking it for granted, in other words, that a small weight placed further from the fulcrum will support or raise a greater one. The cause and reason of which fact must be derived from the demonstration that follows, and therefore this demonstration

stration ought not to be founded on the supposed self-evidence of what is partly the thing to be proved. But perhaps it may be said, that the postulatum may be granted merely on this account; the center of gravity of the two bodies (which in this case is the middle point between them) is not sustained; and therefore the body which is on the same side of the fulcrum with the center of gravity will descend.

In answer to this I must observe, that this property, which the center of gravity has of descending, when not placed directly above or below the point of suspension, cannot be proved to belong to it in any case, nor can we even shew that there is only one center of gravity between two bodies joined by a right line, until it is proved in general that the center of gravity of any two bodies is a point so placed between them that their distances from it are inversely as their weights: but this in effect includes the principal property of the lever, which therefore cannot be proved from any previous supposition, that the center of gravity will descend, even when the bodies are equal, and we know it is the middle point between them.

I must now proceed to consider what Sir Isaac Newton hath delivered on this subject in his Principia, after the 2d cor. to the 3d law of motion which Dr. Clarke (in his notes on Rohault) and all the subsequent writers, have quoted as an elegant proof of the property of the lever; and therefore what appears to me at present an objection to this proof I shall mention with great diffidence, and in hopes of being set right if I am wrong. Sir Isaac supposes two weights, as A and P. TAB. IV. Fig. 1. to hang by threads, from the points M and N, in a wheel or circular plane perpendicular to the horizon,

rizon and movable about its center O ; and then proposes to determine the forces which these weights have to turn the wheel round its center. In order to do this, he supposes that it is indifferent from what points in the perpendicular lines MA and NP the weights are hung, for that they will still have the same power to turn the wheel about its center. His words are: "*Quoniam nil refert utrum filorum puncta K, L, D , affixa sint vel non affixa ad planum rotæ; pondera idem valebunt ac si suspenderentur a punctis K et L , vel D et L* ". Now whether the points of the threads K, L, D , are fixed or not to the plane of the wheel is certainly of importance, as it must make a difference in the points of suspension of the weights, and consequently in the degrees of obliquity with which the weights act; for the lowest point of the thread that is fixed to the plane must be considered as the point from which the weight hangs; as the parts of the thread above that point are quite useless not being at all acted upon. And from thence I shall endeavour to shew that to suppose the weight A will have the same power to turn the wheel from whatever point in the line MA it hangs, is in effect presupposing what is intended to be proved. For it appears, from what he says immediately after, that, when the weight A hangs from the point D , if its whole force be expressed by the line AD , and be resolved into two forces, DC and AC , the former only will have any effect in turning the wheel, as it acts perpendicularly on the radius OD , while the latter is lost, its direction being parallel to OD . But it is evident, that, when the same weight hangs from the point K , as it acts perpendicularly on the radius OK , its whole force

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is exerted to turn the wheel, and none of it lost by oblique action. Therefore the force which the weight A, exerts to oppose the weight P, and turn the wheel when it hangs from D, is, to the force it exerts when it hangs from K, as the line DC to AD, or as OK, to OD, (sim. triang. ADC, DOK) that is the force exerted by the weight A, hanging from the points D, and K, are inversely as the radii OD, and OK. And therefore to suppose, that these two forces will have the same effect in turning the wheel and opposing the weight P, is the same as supposing that two forces will have equal effects in moving the arms of a lever (on which they act perpendicularly) when they are inversely as the lengths of those arms. — But this is the very conclusion Sir Isaac draws from his premises, for he says: “Pondera igitur A & P, quæ sunt reciproce ut radii in directum positi OK, OL, idem pollebunt et sic consistent in æquilibrio, quæ est proprietas notissima libræ vectis et axis in peritrochio”. This property of the lever, which is here expressed in general terms, includes two cases. For the arms of the lever may be either perpendicular or oblique to the directions of the weights. The first of these cases is the simplest, and should be first demonstrated: And I do not see how there can be any room for applying the resolution of forces in demonstrating this case, in which no part of either weight is lost by oblique action. But when this case is proved, we have from thence, by the resolution of forces, an easy way of shewing, in the second case, when the arms of the lever are oblique to the directions of the weights, that the weights will counterballance each other, when they are reciprocally as the perpendicular distances of their
lines

lines of direction from the center of motion. — From either of these cases, we may deduce an obvious reason why the weight *A*, should have the same power to turn the wheel, from whatever point it hangs in the line *MA*; the truth of which, I am persuaded, cannot be proved independent of those cases, and therefore think it ought not to be used as a postulatam in demonstrating the general property of the lever.

Mr. Maclaurin, in his View of Newton's Philosophy, after giving us the methods which Archimedes and Newton have used for proving the fundamental property of the lever, proposes one of his own, which, he says, appears to be the most natural one for this purpose. However as to his method I shall only observe, that from equal bodies sustaining each other at equal distances from the fulcrum, he shews us how to infer that a body of one pound (for instance) will sustain another of two pounds at half its distance from the fulcrum, and from thence that it will sustain one of three pounds at a third of its distance from the fulcrum: and thus he goes on deducing, by a kind of induction, what the proportion is in general between two bodies that sustain each other on the arms of a lever. But this argument (which I do not think by any means satisfactory) he observes cannot be applied when the arms of the lever are incommensurable.

These are the methods of demonstrating the fundamental property of the lever, which are most worth taking notice of; and, since they seem liable to objections, and the other methods I have met with are still more exceptionable, I shall propose a new proof of this property of the lever, which appears to me
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a very simple one, and depends on a postulatum that, I believe, will be readily granted.

If a force be uniformly diffused over a right line, that is, if an equal part of the force acts upon every point of the line, and if the whole force acts according to one and the same plane; this force will be sustained, and the line kept in æquilibrium, by a single force applied to the middle point of the line equal to the diffused force, and acting in a contrary direction.

In order to shorten the following proof, I must premise by way of Lemma, that, if a right line be divided into two segments, the distances between the middle of the whole line, and the middle points of the segments, will be inversely as the segments. This is self evident when the segments are equal; and, when they are unequal, then, since half of the whole line is equal to half of the greater and half of the lesser segment, it is plain that the distance between the middle of the whole line and the middle of one segment must be equal to half of the other segment, so that these distances must be to each other inversely as the segments, all which appears evident from the inspection of TAB. VI. Fig. 2.

Let now the line GH , whose middle point is D , be divided into the unequal segments GL , and LH , whose middle points are C and F , and let two forces or weights, A and B , which are to each other as the segments GL and LH , be applied to their middle points C and F , and let them act perpendicularly on the line GH . Then (by the Lemma) the weights A and B will be to each other inversely as CD , and FD , (the distances of the points C and F , to which

they are applied from the middle of the whole line) if then a third force or weight E, equal to the sum of the forces A and B, be applied to the point D, and acts on the line in an opposite direction; I say these three forces will sustain each other, and keep the line in æquilibrium. For let us suppose the force E to be removed, and instead of it another force, equal also to the sum of A and B, to be uniformly diffused over the whole line G H, and to act directly against the forces A and B, then the part of this force which acts on the segment G L, will be equal to the force A, and therefore will be sustained by it (postulatum); and the other part, which is diffused over the segment L H, will be equal to and sustained by the force B, so that the forces A and B will sustain this diffused force and keep the line in æquilibrium. — Let now two other forces act on this line in opposite directions, one of them the force E acting on the point D, as it was first supposed to do, and the other an uniformly diffused force equal to E (and consequently equal to the other diffused force), then these two additional forces will also ballance each other, and therefore the æquilibrium will still remain. So that the two forces A and B, and a diffused force acting on one side of the line sustain the force E, and a diffused force acting on the other side: but it is manifest, that, in this æquilibrium, the two diffused forces acting on opposite sides are perfectly equivalent, and therefore, if they are taken away from both sides, the æquilibrium must still remain. Hence it appears that the three weights or forces A, B and E, any two of which are (by the construction) to each other inversely as their distances from the third, will sustain each other, and keep the line on which
they

they act in æquilibrium : which is the first and most simple case of the property of the lever ; for here the directions of the weights are supposed to be perpendicular to the line on which they act, and it is evident that, if one of the points C, D or F, be fixed or considered as a fulcrum, that the weights acting on the other two points will continue to support each other. I shall not trouble you with proving the second case of the property of the lever ; it is most easily deduced from the first : for, when two weights act on the arms of a lever in oblique directions, and are to each other inversely as the perpendicular distances of their lines of direction from the center of motion, then, by the resolution of forces, it is easily proved, that the parts of those forces which act perpendicularly on the arms of the lever, and which only are exerted to turn the lever, are to each other inversely as the lengths of those arms ; and therefore by the first case they must ballance each other.

I shall now mention some well known truths in mechanics, which, I think, cannot be proved otherwise than by deducing them from what hath been here demonstrated.

C O R. I.

It appears from hence, that the powers with which any two forces move or endeavour to move the arms of a lever, are as the rectangles, under lines proportional to the forces, and the perpendicular distances of their lines of direction from the fulcrum.

C O R. II.

When therefore two bodies acting on the arms of a lever sustain each other, if one of them be removed farther from the fulcrum, it will preponderate; but if it be brought nearer to the fulcrum, the other weight will prevail: because the product to which its force is proportional will be encreased in the first case, and diminished in the second.

C O R. III.

We learn from hence, to find out the center of gravity of any two bodies joined by an inflexible right line; and to prove that its definition will agree to one point only in the line. For if a point be taken in the line so that the distances of the bodies from it may be inversely as their weights, that point will be their center of gravity, because, when it is sustained, the bodies will be in æquilibrium. But if the line be sustained at any other point, then is the fulcrum removed farther from one body and brought nearer to the other than it was when the bodies ballanced each other; and therefore, by the preceding Cor. that body from which it is removed, or which is on the same side with the center of gravity, will descend. Consequently there is but one point in the line, which being sustained, the bodies will continue in æquilibrium, and therefore but one point only can be their center of gravity. Hence also it appears, that the center of gravity will always descend, when it is not directly above or below the point by which the body is sustained.

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I shall now endeavour to be as concise as possible in what I have to say of the other mechanic powers ; having, I fear, been too tedious in my account of the lever, which however deserves to be particularly considered, since to it may be reduced the ballance, the axle and wheel, and (according to some writers) the pulley.

The ballance I do not consider as a distinct machine, because it is evidently no other than a lever fitted to the particular purpose of comparing weights together, and does not serve for raising weights, or overcoming resistances, as the other machines do.

When a weight is to be raised by means of an axle and wheel it is fastened to a chord that goes round the axle, and the power which, is to raise it is hung to a chord that goes round the wheel. If then the power be to the weight as the radius of the axle to the radius of the wheel, it will just support that weight ; as will easily appear from what was proved of the lever. For the axle and wheel may be considered as a lever, whose fulcrum is a line passing through the center of the wheel and middle of the axle, and whose long and short arms are the radii of the wheel and axle which are parallel to the horizon, and from whose extremities the chords hang perpendicularly. And thus an axle and wheel may be looked upon as a kind of perpetual lever, on whose arms the power and weight always act perpendicularly, tho' the lever turns round its fulcrum. And in like manner when wheels and axles move each other by means of teeth on their peripheries, such a machine is, really, a perpetual compound lever : and, by considering it as such, we may compute the proportion of any power to the weight it.

it is able to sustain by the help of such an engine. And since the radii of two contiguous wheels, whose teeth are applied to each other, are as the number of teeth in each, or inversely as the number of revolutions, which they make in the same time; we may, in the computation, instead of the ratio of these radii, put the ratio of the number of teeth on each wheel; or the inverse ratio of the number of revolutions they make in the same time.

Some writers have thought the nature and effects of the pulley might be best explained by considering a fixed pulley as a lever of the first, and a moveable pulley as one of the second kind. But tho' the pulley may bear being considered in that light; yet, I think, the best and most natural method of explaining its effects (that is, of computing the proportion of any power to the weight it can sustain by means of any system of pulleys) is, by considering that every moveable pulley hangs by two ropes equally stretched, which must bear equal parts of the weight: and therefore when one and the same rope goes round several fixed and moveable pulleys, since all its parts on each side of the pulleys are equally stretched, the whole weight must be divided equally amongst all the ropes by which the moveable pulleys hang. And consequently if the power which acts on one rope be equal to the weight divided by the number of ropes, or double the number of moveable pulleys, that power must sustain the weight.

Upon this principle, the proportion of the power to the weight it sustains by means of any system of the pulleys, may be computed in a manner so easy and natural as must be obvious to every common capacity.

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The proportion, which any power bears to the resisting force it is able to sustain by means of a wedge, has been laid down differently by different authors; some of whom therefore must have been mistaken: and none of them seem to have treated the matter so generally as they might have done. Without examining their several opinions, I shall proceed to consider what proportion a power acting on a wedge must have to the resistance it sustains in three different cases, to which I think all those relating to the wedge may be reduced.

First, When the resisting bodies act perpendicularly on the sides of the wedge, and recede also in lines perpendicular to the sides.

Secondly, When the resisting bodies act on the wedge in oblique directions equally inclined to the sides, and recede in lines perpendicular to the sides.

Thirdly, When the resisting bodies are confined, by planes put under them, or otherwise, to recede in particular directions oblique to the sides.

C A S E I.

Let the æquicrural triangle A, B, C, [TAB. VI. Fig. 3.] represent a wedge on whose sides the two equal resisting forces E and F act perpendicularly with lines of direction meeting at the point D, at which the power P acts perpendicularly on the base A C. Then, since these three forces are supposed to sustain each other and keep the wedge in æquilibrium, they must be to each other as the sides of a triangle to which their directions are parallel, and therefore they will be to each other as the sides of a triangle to which their directions

directions are perpendicular; that is, the sum of the forces E and F will be to the power P , which sustains them, as the sum of the sides of the wedge to the base, or as one side to half of the base; that is, as the radius to the sine of half the vertical angle of the wedge. Hence, when in cleaving timber the wedge fills the cleft, in which case the resistance of the wood acts perpendicularly on the sides of the wedge, the power which drives the wedge must be to the cohesive force of the timber in a proportion somewhat greater than that above mentioned, in order that it may divide the timber, whose parts will then recede in lines perpendicular to the sides of the wedge.

C A S E II.

Let now the resisting forces of E and F be supposed to act obliquely on the sides of the wedge in the directions $E K$ and $F L$, and let these forces be expressed by the lines $E K$ and $F L$, and let each of them be resolved into two forces, expressed respectively by the lines $E G$, $G H$, and $F H$, $H L$, whereof the forces $G H$ and $H L$, by acting parallel to the sides of the wedge, are lost: while the other forces, $E G$ and $F H$, by acting perpendicularly on the sides of the wedge, keep the power P in æquilibrium; therefore by the first case these parts of the whole resisting force are to the power P , as radius to the sine of half the vertical angle of the wedge. But it is evident that the whole resisting force is to its parts expressed by $E G$, $F H$, as radius to the sine of the angle $E K G$, or $F L H$; and therefore (compounding these ratios) the whole resisting force will be to the power which
sustains

sustains it, as the square of radius, to a rectangle under the sine of the angle which the directions of the resisting force make with the sides of the wedge, and the sine of half the vertical angle of the wedge. Now, since the force of the wedge is exerted in lines perpendicular to the surface of its sides, these resisting bodies will naturally recede in that direction; as we suppose them, in this case, free to move in any direction whatever.

C A S E III.

Let us suppose lastly, that the resisting bodies are confined, by planes put under them, to recede in the directions KE , LF , then the power which drives the wedge and the resisting force will be in æquilibrium, when the former is to the latter, as the sine of half the vertical angle of the wedge, to the sine of the angle EKG , FLH , that each side of the wedge makes with the direction in which the resisting force is confined to recede. For in the first case it was proved that the power P , which drives the wedge, is to the force with which it protrudes bodies in directions perpendicular to the sides, as the sine of half the vertical angle of the wedge to radius. Let then the line GE , which is perpendicular to the side AB , express the force with which the power P protrudes the resisting bodies in the directions GE , and HF , and let this force be resolved into two forces, expressed by the lines GO , and OE , one perpendicular and the other parallel to KE , the direction in which the resisting bodies are confined to move; then the force GO is lost, and only OE has effect in protruding the

resisting bodies in the directions KE , and LF . This force therefore being to the force expressed by GE , as the sine of the angle (EGO , or) EKG , to the radius, and the Force GE being (as was said before) to the power P , as the radius to the sine of half the vertical angle of the wedge; it follows, that the force with which the resisting bodies are protruded in the directions KE , and LF , is to the power P , as the sine of the angle EKG , or FLH , which these directions make with the sides of the wedge, to the sine of half the vertical angle of the wedge: and consequently, if the resisting forces, which act on the wedge according to these directions, are to the power P in this proportion, there will be an æquilibrium between them.

Hence we may observe, that, if from D (the middle point in the back of the wedge) a line be drawn, as DA , meeting one of the sides; the resisting forces, which must recede in directions parallel to DA , will be to the power which sustains them, as DB , the height of the wedge, to the line DA ; and this power, if at all increased, will remove these resisting bodies. When therefore the resisting bodies must recede in lines parallel to the back of the wedge, their resistance will be to the power which sustains it, as the height of the wedge, to half the breadth of it's back. This proportion of the power to the resistance in this last mentioned case, is confirmed by an experiment used by Gravesande and others, to shew the nature of the wedge. For, in this experiment, a wedge is drawn down between two cylinders, which roll on rulers parallel to the back of the wedge, and are kept together by weights. And probably it was from their attending to this experiment, without considering other cases, that

that they concluded the same proportion between the power and resistance would obtain in general.

I have already mentioned the proportion which the power that drives the wedge must have to the resistance in cleaving timber, when the wedge exactly fills the cleft; which case however seldom happens; for the wood generally splits to some distance before the wedge. And then, in order that there may be an æquilibrium between the power driving the wedge and the resistance of wood, the former must be to the latter, as the sine of half the vertical angle of the wedge, to the cosine of the angle which the side of the cleft makes with the side of the wedge. The truth of which is easily understood from what was proved in the third case of the wedge; for the cosine of the angle, contained between the side of the cleft and the side of the wedge, is the sine of the angle which the side of the wedge contains with the direction in which the wood recedes; because, as the cleft opens, the wood must recede in lines perpendicular to the sides of the cleft; and in the direction of those lines doth the resistance of the wood act on the sides of the wedge.

The inclined plane is reckoned by some writers among the mechanic powers; and I think with reason, as it may be used with advantage in raising weights.

Let the Line AB [TAB. VI. Fig. 4.] represent the length of an inclined plane, AD its height, and the line BD we may call its base. Let the circular body GEF, be supposed to rest on the inclined plane, and to be kept from falling down it by a string CS tyed to its center C. Then the force with which this body

stretches the string will be to its whole weight, as the sine of ABD , the angle of elevation, to the sine of the angle which the string contains with a line perpendicular to AB the length of the plane. For let the radius CE be drawn perpendicular to the horizon, and CF perpendicular to AB : and from E draw EO parallel to the string and meeting CF in O . Then, as the body continues at rest and is urged by three forces, to wit, by its weight in the direction CE , by the reaction of the plane in the direction FC , and by the reaction of the string in the direction EO ; the reaction of the string, or the force by which it is stretched, is to the weight of the body, as EO to CE : that is, as the sine of (the angle CEF , which is equal to) ABD , the angle of elevation, to the sine of the angle EOC , equal to SCO , the angle which the string contains with the line CF perpendicular to AB , the length of the plane.

When therefore the string is parallel to the length of the plane, the force with which it is stretched, or with which the body tends down the inclined plane, is to its whole weight, as the sine of the angle of elevation, to the radius, or as the height of the plane to the length. And in the same manner it may be shewn that, when the string is parallel to BD , the base of the plane, the force with which it is stretched is to the weight of the body, as AD to BD , that is, as the height of plane to its base. If we suppose the string, which supports the body GEF , to be fastened at S , and that a force, by acting on the line AD , the height of the plane, in a direction parallel to the base BD , drives the inclined plane under the body, and by that means makes it rise in a direction parallel to AD .

Then

Then, from what was proved in the third case of the wedge, it will appear, that this force must be to the weight of the body, as $A D$ to $B D$, or rather in a proportion somewhat greater: if it makes the plane move on and the body rise.

From this last observation we may clearly shew the nature and force of the screw; a machine of great efficacy in raising weights or in pressing bodies closely together. For if the triangle $A B D$ be turned round a cylinder whose periphery is equal to $B D$, then the length of the inclined plane $B A$ will rise round the cylinder in a spiral manner, and form what is called the thread of the screw: and we may suppose it continued in the same manner round the cylinder from one end to the other; and $A D$ the height of the inclined plane will be every where the distance between two contiguous threads of this screw, which is called a convex screw. And a concave screw may be formed to fit this exactly, if an inclined plane every way like the former be turned round the inside of a hollow cylinder, whose periphery is somewhat larger than that of the other. Let us now suppose the concave screw to be fixed, and the convex one to be fitted into it, and a weight to be laid on the top of the convex screw: Then, if a power be applied to the periphery of this convex screw to turn it round, at every revolution the weight will be raised up thro' a space equal to the distance between the two contiguous threads, that is to the line $A D$ the height of the inclined plane $B A$; therefore since this power, applied to the periphery, acts in a direction parallel to $B D$, it must be to the weight it raises as $A D$ to $B D$, or as the distance between two contiguous threads, to the periphery of the convex screw.

N. B. The distance between two contiguous threads is to be measured by a line parallel to the axle; if we now suppose that a hand-spike or handle is inserted into the bottom of the convex screw, and that the power which turns the screw is applied to the extremity of this handle, which is generally the case; then as the power is removed farther from the axis of motion, its force will be so much increased (vide what was said of the lever, cor 1.) and therefore so much may the power itself be diminished. So that the power, which, acting on the end of a handle, sustains a weight by means of a screw, will be to that weight, as the distance between two contiguous threads of the screw, to the periphery described by the end of the handle. In this case we may consider the machine as composed of a screw and a lever, or, as Sir Isaac Newton expresseth it, *Cuneus a vecte impulsus*.

I have now given you my sentiments, as to the principles on which, I think, the efficacy of the mechanic powers may be most properly explained; and hope that, where I have presumed to differ from others, you will think I have some appearance of reason on my side. I find my paper has been drawn out much beyond what I at first expected, and I fear much beyond your patience; and therefore shall detain you no longer than to assure you that I am, Sir,

with the sincerest regard, your

most obedient humble servant,

Hugh Hamilton.

XXVI. *An*

